

BS 5700:2015



BSI Standards Publication

# Guide to the selection of charting methods and capability assessment for use in statistical process control

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### Summary of pages

This document comprises a front cover, an inside front cover, pages *i* to *ii*, pages 1 to 24, an inside back cover and a back cover.

## Foreword

### Publishing information

This British Standard is published by BSI Standards Limited, under licence from The British Standards Institution, and came into effect on 30 September 2015. It was prepared by Technical Committee SS/4, *Statistical process management*. A list of organizations represented on this committee can be obtained on request to its secretary.

### Supersession

This British Standard supersedes BS 5700:1984, which is withdrawn.

### Use of this document

As a guide, this British Standard takes the form of guidance and recommendations. It should not be quoted as if it were a specification or a code of practice and claims of compliance cannot be made to it.

### Presentational conventions

The guidance in this standard is presented in roman (i.e. upright) type. Any recommendations are expressed in sentences in which the principal auxiliary verb is "should".

*Commentary, explanation and general informative material is presented in smaller italic type, and does not constitute a normative element.*

### Contractual and legal considerations

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

**Compliance with a British Standard cannot confer immunity from legal obligations.**

## Introduction

BS 5700 is a guide for any organization wanting to use control charts and capability analysis to control a process, report performance, or measure results of an improvement programme. The application of control charts and capability analysis is a vital step in the management and improvement of processes. Business processes can be many and varied, including applications in manufacturing and in services, ranging from health and finance to customer care.

When followed correctly, statistical process control (SPC) can deliver a definable, measureable, and recognized way to monitor processes and produce accurate performance reporting. These are important tools in the delivery of improvement programmes.

The application of SPC offers many benefits to an organization beyond that of controlling and improving processes – SPC has an important role in developing staff and helping them to engage with processes. For the methods to be effective, it is important that senior management and other stakeholders act in a consistent and proactive manner on the messages arising from the application of the methods.

To improve a process it needs to be understood and to do so, take data from a key characteristic that describes the process. For example, if dimensions such as height, length and thickness are to be monitored, the data needs to be charted. For this type of data, variable charts are used. If the characteristic of interest is a count, such as the number of scratches or a proportion such as the number of parcels with incorrect addresses, an attribute chart is appropriate. It is also important to measure process performance outcomes and assess capability of the process so improvement can be assessed. This British Standard acts as a guide to selecting the correct type of chart, performance measurement and capability analysis for the process and the relevant standard to refer to.

*NOTE If the wrong characteristic for measurement has been selected, success is not guaranteed.*

## 1 Scope

This British Standard provides guidance on the selection and application of control chart methods that are expanded upon in all parts of BS 5701, BS 5702 and BS ISO 7870.

This British Standard also provides information on capability and performance assessment as given in BS ISO 22514 (all parts) for the purpose of monitoring, understanding, reporting and improving processes.

## 2 Terms, definitions and symbols

### 2.1 Terms and definitions

For the purposes of this British Standard, the terms and definitions given in BS ISO 3534 (all parts) apply.

*NOTE However, the terminology used in quality control is subject to a continuing process of development in response to market demand.*

### 2.2 Symbols

For the purposes of this British Standard the following symbols apply.

$C_p$	capability index
$C_{pk}$	minima of $C_{pkU}$ and $C_{pkL}$

$C_{pkL}$	lower capability index
$C_{pkU}$	upper capability index
$c$	count number of non-conformities
$\bar{c}$	average count number of non-conformities
$D$	demerit score
$D_3$	coefficient to calculate the lower control limit of a range chart
$D_4$	coefficient to calculate the upper control limit of a range chart
$d$	distance on a V-mask
$d_2$	coefficient to estimate standard deviation from average sample range
$i$	index counter
$k$	number of subgroups
$L$	lower specification limit
$L_{CL}$	lower control limit
$n, N$	subgroup, sample size
$np$	number non-conforming
$P_p$	performance index
$P_{pk}$	minima of $P_{pkU}$ and $P_{pkL}$
$P_{pkL}$	lower performance index
$P_{pkU}$	upper performance index
$p$	proportion non-conforming
$\bar{p}$	average proportion non-conforming
$R$	range
$s$	standard deviation
$U$	upper specification limit
$U_{CL}$	upper control limit
$V$	variance
$X_i$	the $i^{\text{th}}$ reading in a subgroup or sample
$\bar{x}, \bar{X}$	subgroup, sample, mean value
$\bar{\bar{x}}, \bar{\bar{X}}$	average of the subgroup means
$\Sigma$	summation
$\mu$	population mean
$\sigma$	population standard deviations
$\sigma_t$	total standard deviation
$\sigma_{\bar{x}}$	Standard error of the mean

## 3 Control charts, performance and capability analysis

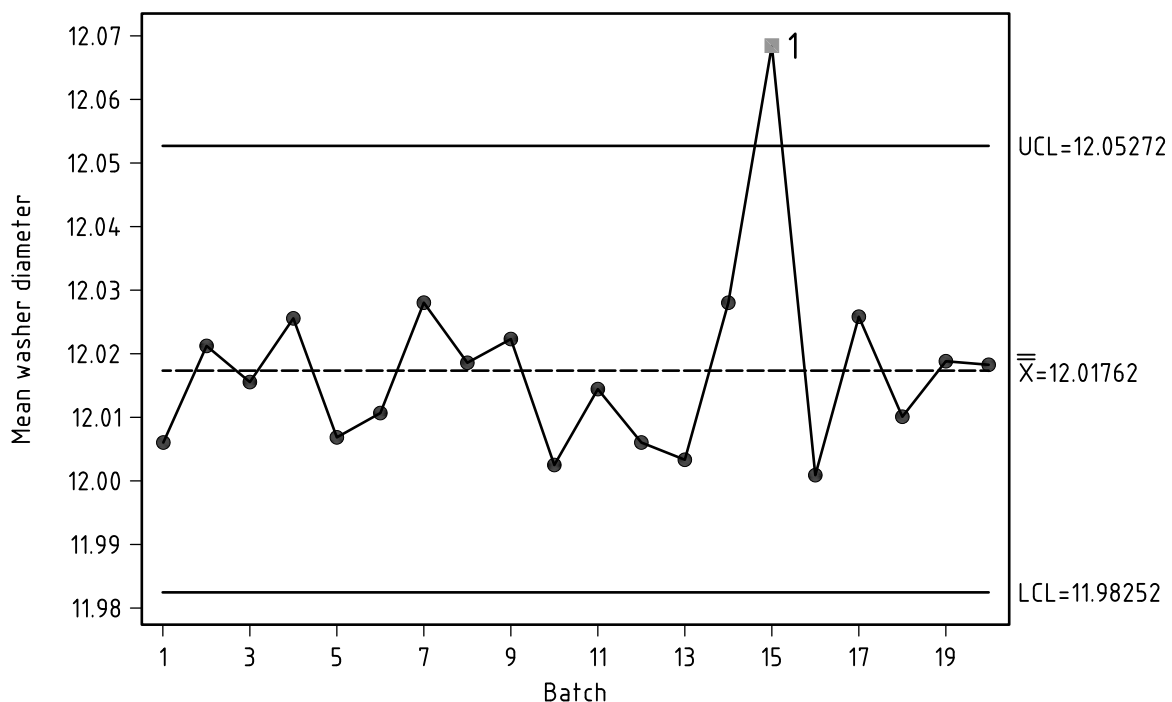
### 3.1 Control charts

#### 3.1.1 General

Observations of the process should be made at regular intervals and plotted. If the control limits which are set at the process mean  $\pm 3$  standard errors of the process variation are exceeded then the process is deemed to be out of control and to have changed as a consequence of an assignable cause.

Figure 1 illustrates a process variable (in this case the interior diameter of a washer specified to be  $[12.00 \pm 0.06]$  mm).

Figure 1 Example of an  $\bar{X}$  control chart of washer diameters



**NOTE** This chart was constructed from drawing samples of five washers every 30 minutes and plotting the average on the chart (further detail on chart construction is given in 5.3). The process mean, derived from sample measures was found to be 12.01 mm with a standard error of 0.0133 mm. This sets the control lines at 11.97 mm and 12.05 mm that are within specification limits indicating that this might be a capable process, but it has gone out of control.

Assignable causes can also occur if there is a systematic pattern within the control lines, meaning that if the process is behaving as desired, points should appear at random within the control lines. For a continuous variable that is distributed according to the normal distribution this would mean that if there are no assignable causes, the chance of exceeding the control lines is very small (less than a probability of 0.003). In addition, the user should expect around two thirds of the observations to be within  $\pm 1$  standard error of the process mean and 95% of the sample means to lie within  $\pm 2$  standard errors of the process average (almost equivalent to a 95% confidence interval). The width between the control lines is reflective of the inherent variation (or common cause variation) in the process.

*NOTE For further discussion of assignable and common cause variation see Deming [1] and Wheeler [2]. When these charts were developed, sample measures were taken by the process operator, at regular intervals, who then calculated the mean and the range (the difference between the highest and lowest sample measurement, taken as a simplified measure of variability). The operator then plotted these calculated values onto the charts. This had an additional benefit of involving the operator and allowing him or her to take responsibility (See Shewhart [3] and Wheeler [2]).*

BS 5701, BS 5702 and BS ISO 7870-4 detail the theory and use of control charts and explain how the performance of these charts is assessed by using the measure “average run length” (ARL) that a chart will take to detect a change.

### 3.1.2 Count and proportion charts covered in BS 5701

Generally, those who manage processes use qualitative or attribute data such as the number of blemishes on the paint work of a car, the number of errors in the initial processing of a legal document, or the proportion of a production batch that is in some way defective. Charts to allow the monitoring of such attributes are count ( $c$ ) and proportion ( $p$ ) charts and are introduced in Clause 4 and are covered in BS 5701.

### 3.1.3 Mean and range (or standard deviation) charts covered in BS 5702

For continuous measures, the user also needs to monitor variation in the process as well as changes in the process average. For this, range ( $R$ ) or standard deviation ( $s$ ) charts should be used. These, along with  $\bar{X}$  charts, are covered in BS 5702.

### 3.1.4 Cusum charts covered in BS ISO 7870-4

The charts described in 3.1.1 and 3.1.2 tend to perform well if changes in the process are reasonably substantial. When changes are small and gradual, a chart that plots the cumulative deviation from the process average performs better and should be used. These charts are called cusum charts and are detailed in BS ISO 7870-4, and are covered in more detail in Clause 4 and Clause 5.

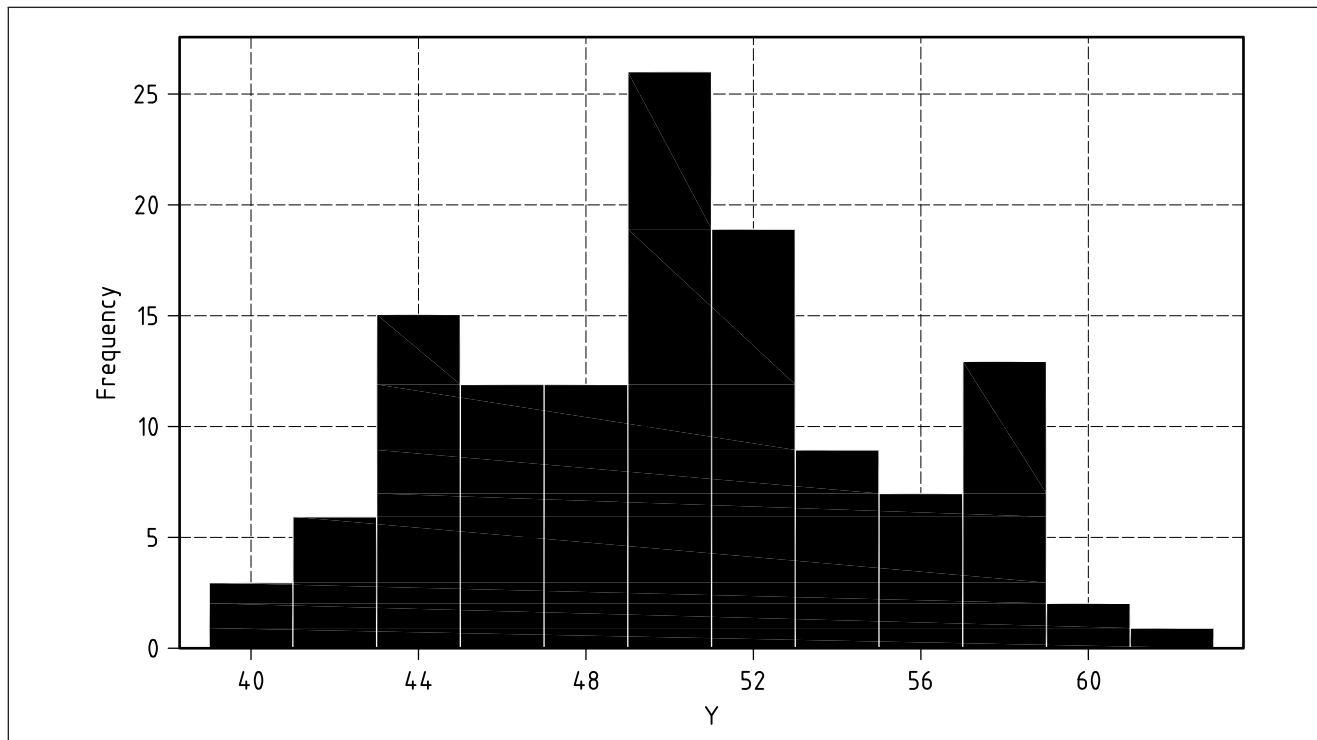
## 3.2 Performance

### 3.2.1 Continuous variables

PD ISO/TR 22514-4 describes the performance assessment of continuous variables. An example of a histogram of continuous data is shown in Figure 2.

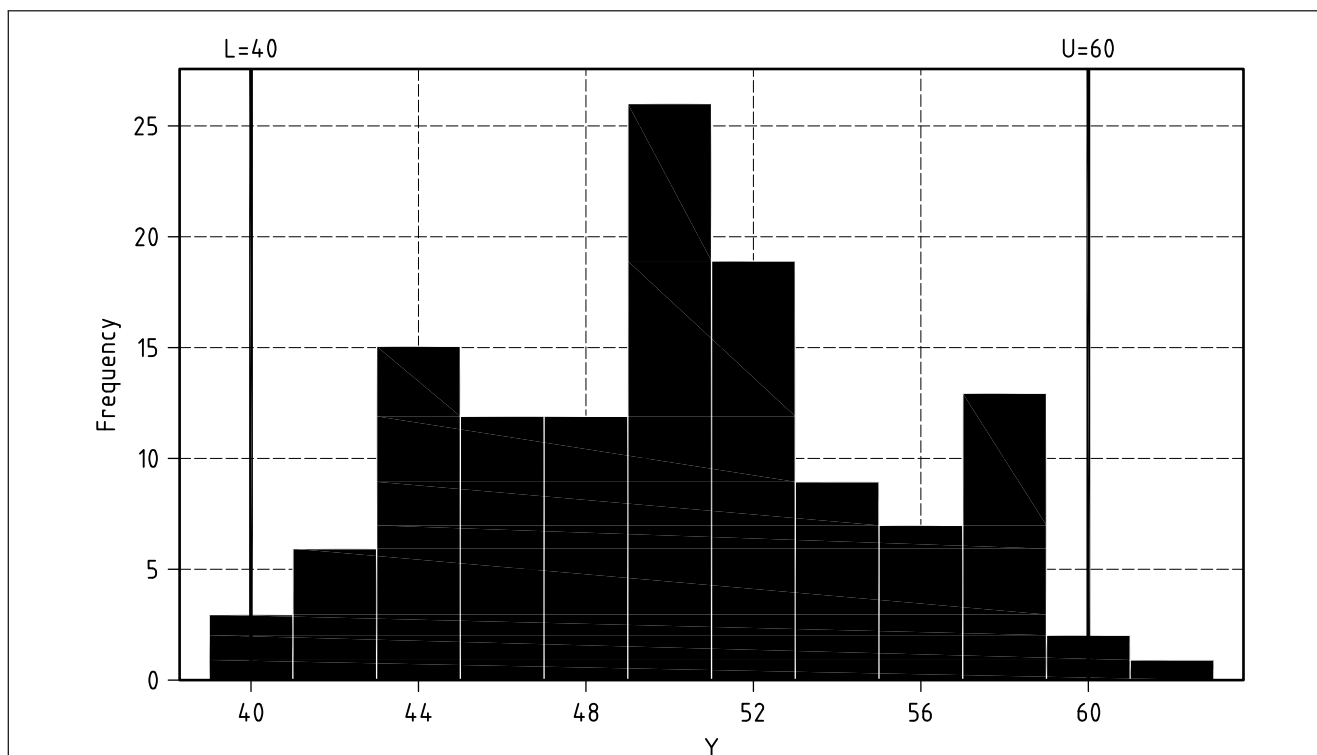


Figure 2 Example of a histogram of continuous data



In Figure 2, there is no order for the data and no control chart was used. Therefore, if the specification limits for the process are 40 to 60, when superimposed over the histogram, as in Figure 3, the output of the process clearly exceeds the specification limits.

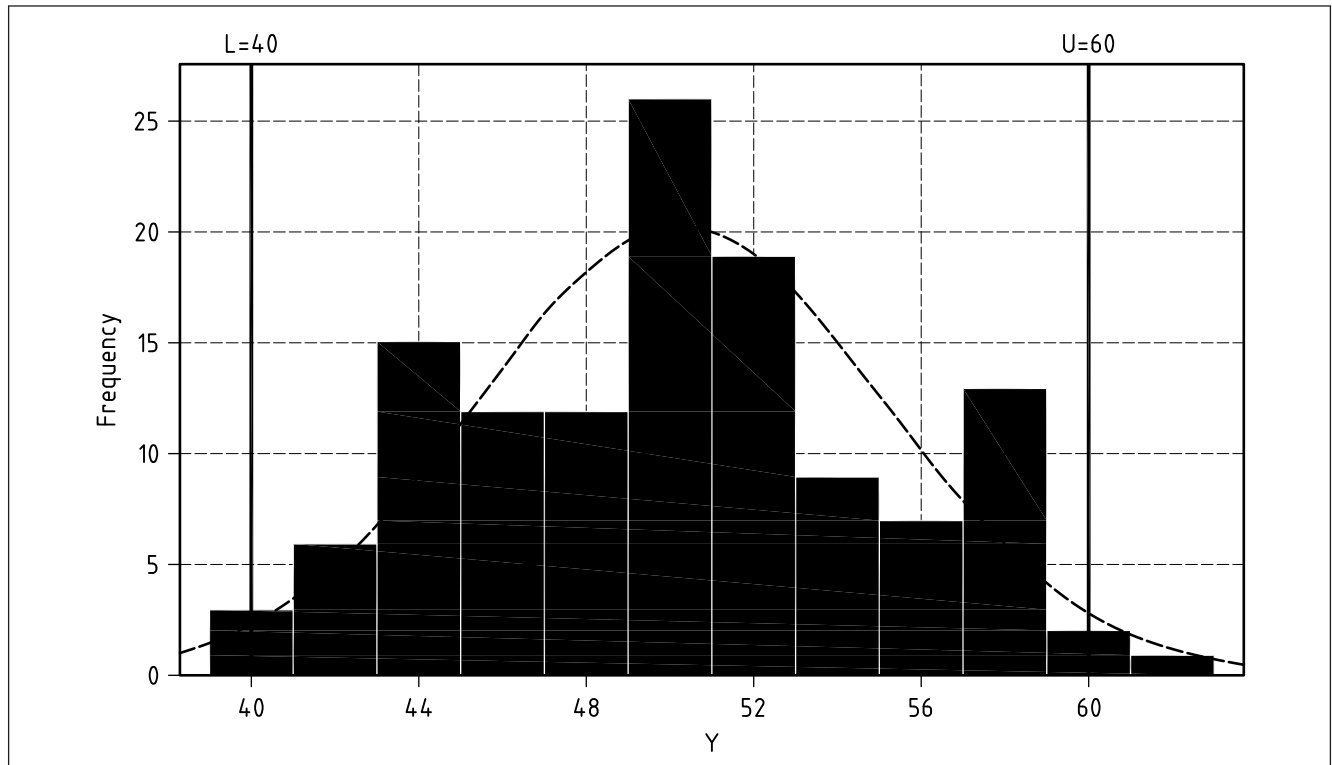
Figure 3 Example of a histogram of continuous data with specification limits superimposed



NOTE It is the end user's decision to specify what the actual permissible proportion of results will be, but most contractual circumstances would allow a maximum of 0.0134% defective per limit, i.e. 0.0268% in total.

As detailed in PD ISO/TR 22514-4, a suitable model should be found that describes the total output. For the data in Figures 2 and 3 the normal distribution provides a good model and it has been superimposed over the data and shown in Figure 4.

Figure 4 Example of a histogram of continuous data with specification limits and a normal distribution model displayed



PD ISO/TR 22514-4 defines indices that are typically used for performance assessment.

In the case of the example data shown in Figure 4 being modelled by the normal distribution, the equations are:

$$P_p = \frac{U - L}{6\sigma_t}$$

$$P_{pkU} = \frac{U - \mu}{3\sigma_t}$$

$$P_{pkL} = \frac{\mu - L}{3\sigma_t}$$

If a process has an index of 1.33 it would be regarded in most circumstances to be barely acceptable. The higher the index value the better.

From the data given in Figure 3 and Figure 4, the mean is 50.13 and the standard deviation is 4.95. Thus the indices are:

$$P_p = \frac{60 - 40}{6 \times 4.95} = 0.67$$

$$P_{pkU} = \frac{60 - 50.13}{3 \times 4.95} = 0.66$$

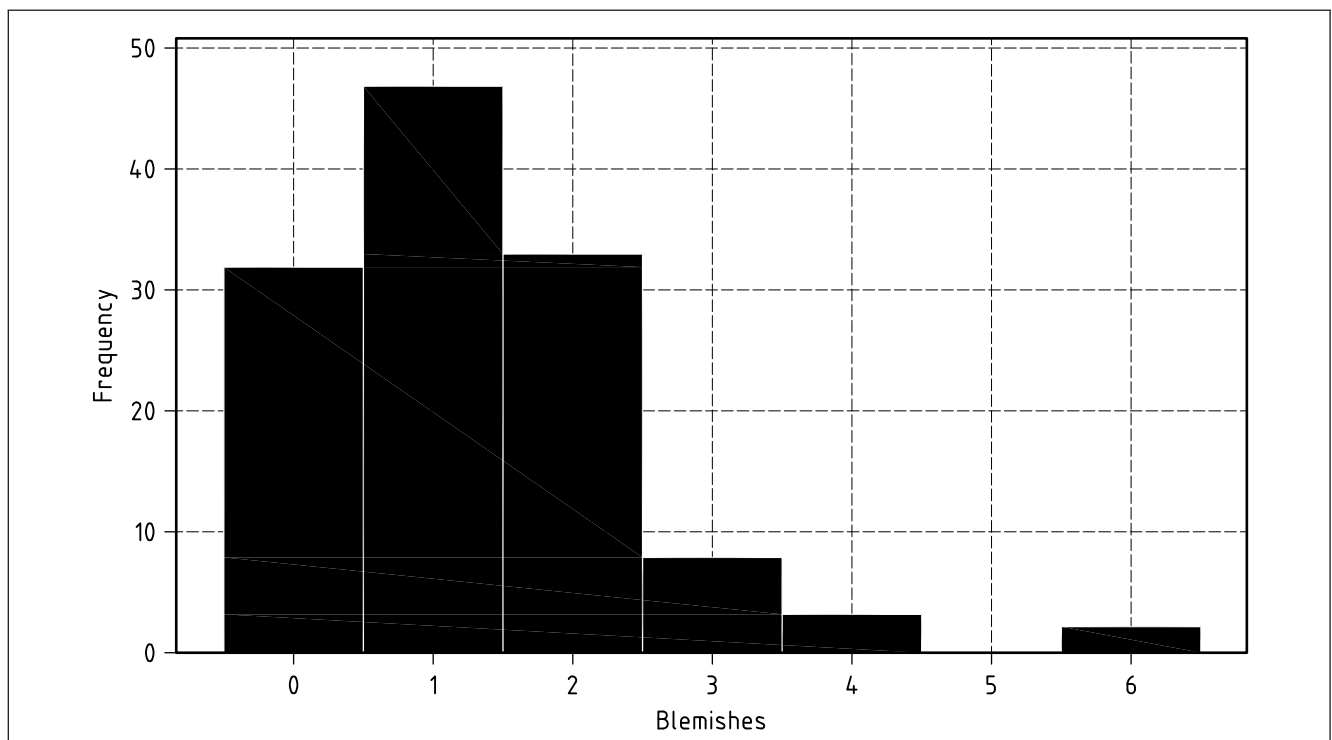
$$P_{pkL} = \frac{50.13 - 40}{3 \times 4.95} = 0.68$$

The low value of these indices indicates a large proportion of non-conforming items have been produced.

### 3.2.2 Discrete variables

If the variable in question is of a discrete type, e.g. the number of blemishes found per 10 m length of a carpet, as shown in Figure 5, the equations referred to in PD ISO/TR 22514-4 should not be used. Instead, it is usual to simply state the achieved average level of the non-conformity. In this example the average is 1.3 blemishes per 10 m length.

Figure 5 Example of a histogram of discrete data



### 3.3 Process capability

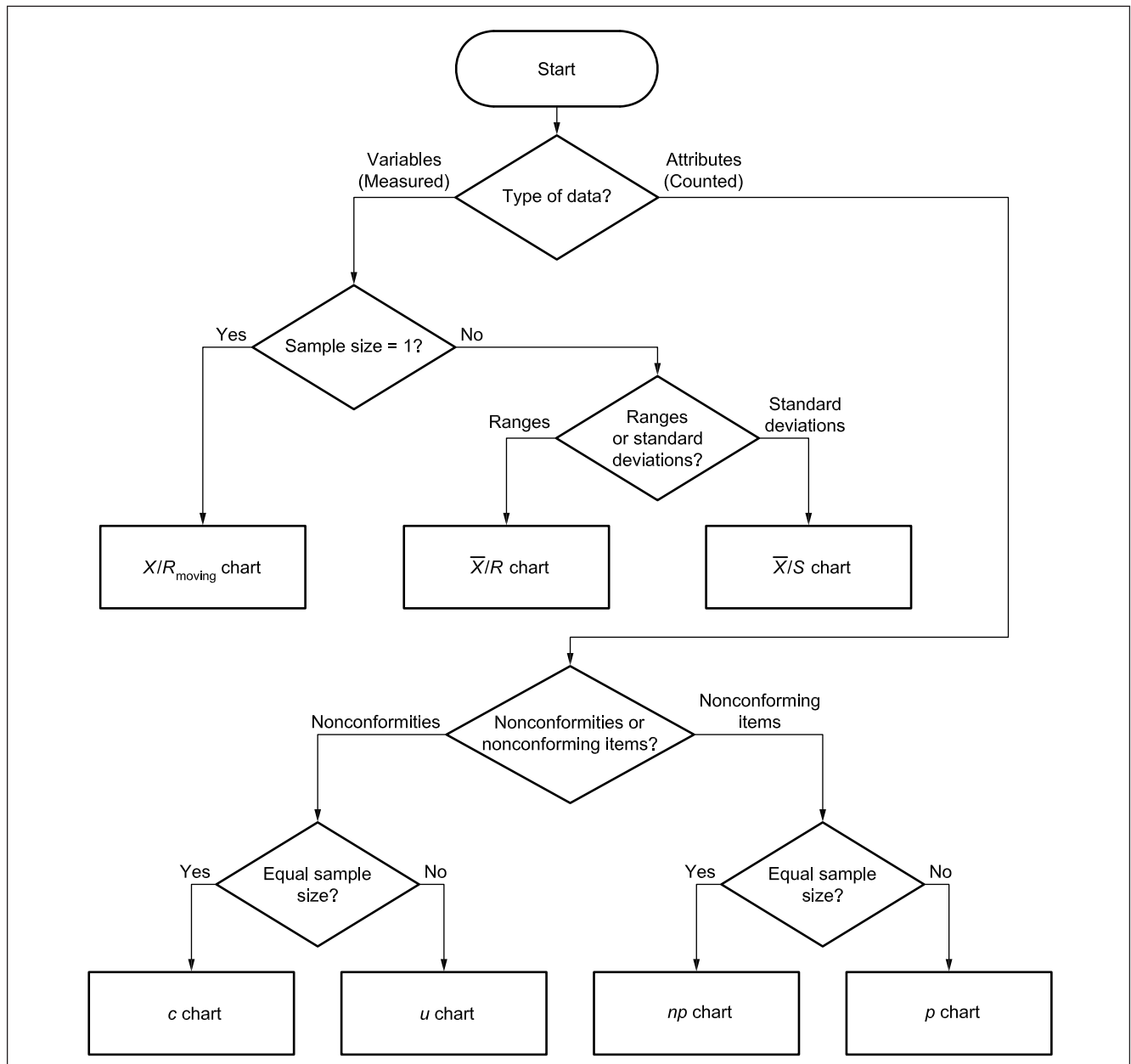
If a process is in control then process capability can be assessed. Process capability is the degree to which customer agreed specifications, or sometimes internally set specifications, are met. Capability can be assessed for continuous data and methods which are under development for discrete data. See BS ISO 25514 for more information.

### 3.4 How to choose an appropriate chart and capability analysis

#### 3.4.1 General

When the data is a set of continuous measures such as temperature, weight, length or diameter then the user should choose from a set of charts known as variable charts as illustrated in Figure 6.

Figure 6 Flow chart for control chart selection



### 3.4.2 $\bar{X}$ and range charts

If the user can take sample batches of a process characteristic, they should consider using  $\bar{X}$  and range charts. If it is only possible to obtain individual measures then the user should choose individual and moving range charts instead.

### 3.4.3 Cusum charts

When the user is interested in small gradual changes then cusum should be used.

*NOTE* There are several more specialized charts, notably exponentially weighted moving average charts (EWMA) which are also useful for detecting small changes (and additionally are advised when the data is auto correlated), and tool wear charts which are used when a process drift is expected.

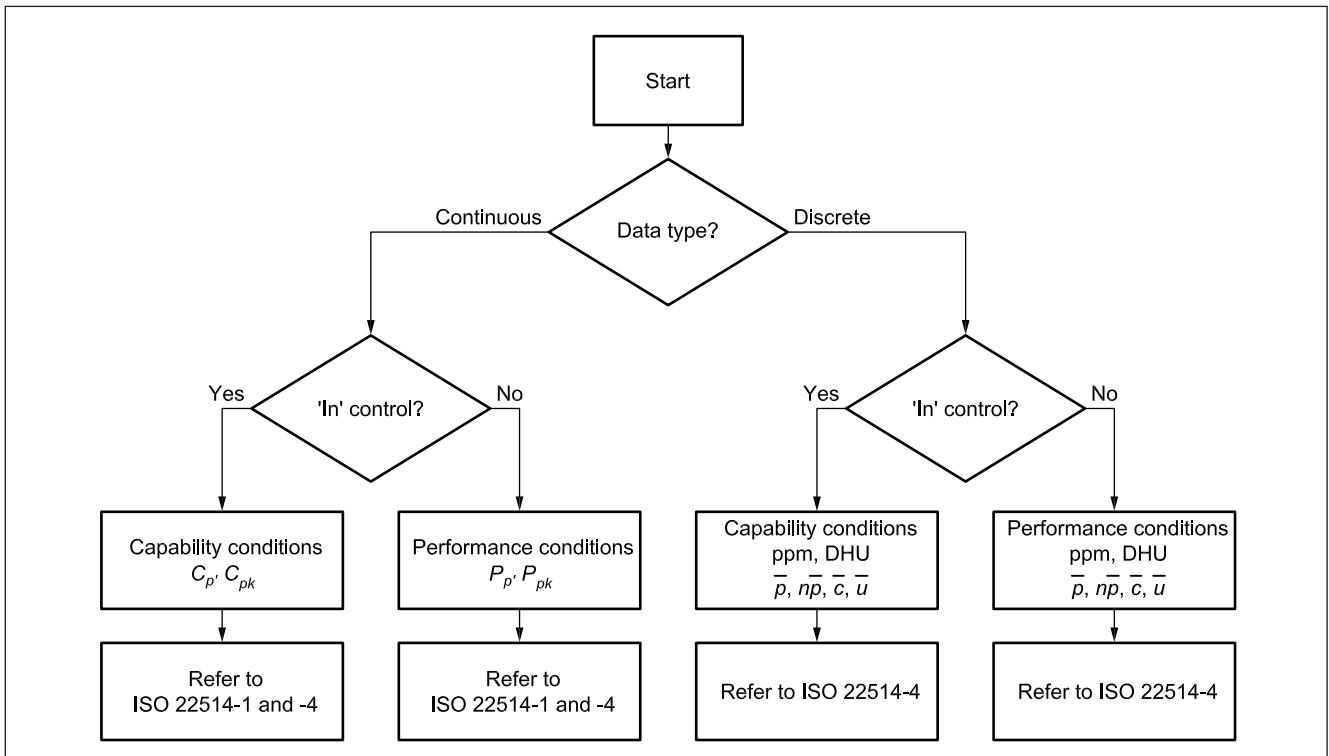
**3.4.4 Attribute charts**

When the data consists of attributes such as the number or proportion of occurrences, or the count or the proportion of non-conforming items, then an attribute chart should be used.

**3.4.5 Classification for capability and performance measures**

There is a similar classification for capability and performance measures and the method to be used might be selected using the flow chart shown in Figure 7.

Figure 7 Flow chart for selecting capability and performance measures



**4 Charting discrete data**

**4.1 General**

Discrete data is the subject of BS 5701-4, providing guidance on which chart to select (see also Figure 6) and how to set up an attribute control chart when subjective judgments are involved, such as determining whether a particular flaw or imperfection is present or not. In order to give examples of these charts, a chart formed from counts of non-conforming outputs should first be considered (see 4.2 and 4.3).

**4.2 Counts of non-conforming (c) chart**

c charts should be used to monitor the number of non-conformities and how simple count data is obtained.

*NOTE* Count data is often modelled by the Poisson distribution (see BS ISO 3534).

For example, Table 1 shows counts of non-conformities in a monthly newsletter. The non-conformities might be spelling mistakes, missing items, printing errors, pages in the wrong order, etc. A year's worth of observations are displayed.

Table 1 Non-conformity counts in a monthly newsletter

Month	Count of non-conformities
January	10
February	8
March	12
April	4
May	8
June	2
July	0
August	5
September	8
October	9
November	6
December	5

The average number of non-conformities is determined by:

$$\bar{c} = \frac{\sum_{i=1}^n c_i}{n} = \frac{77}{12} = 6.42$$

The control lines are found from:

$$U_{CL_c} = \bar{c} + 3\sqrt{\bar{c}}$$

$$U_{CL_c} = 6.42 + 3\sqrt{6.42} = 14.02$$

$$L_{CL_c} = \bar{c} - 3\sqrt{\bar{c}}$$

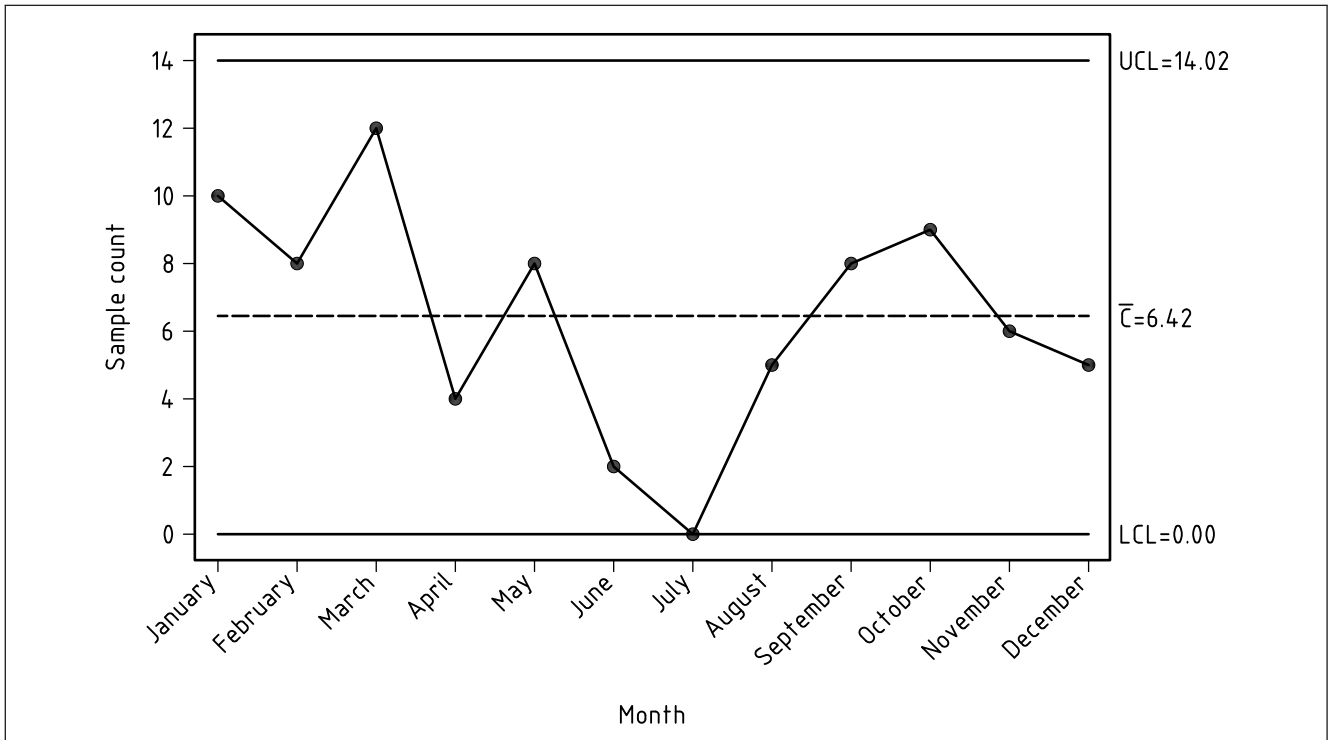
$$L_{CL_c} = 6.42 - 3\sqrt{6.42} = -1.18$$

*NOTE 1* As the lower control limit is in this case negative, the lower control limit is either given as zero or not recorded.

The resulting c chart is shown in Figure 8.

*NOTE 2*  $\sqrt{\bar{c}}$  is the standard deviation of data distributed according to the Poisson distribution (see BS ISO 3534).

Figure 8 Example of a c chart of the counts of non-conformities in a monthly newsletter



### 4.3 Proportion non-conforming (p) chart

This chart is a plot of the proportion defective, or non-conforming, in a batch. The sample sizes need to be much larger than for variable charts since the information recorded is only the presence or absence of a characteristic and a larger sample is required to get a reliable estimate of the mean proportion and its standard deviation. The chart utilizes properties of the Binomial distribution.

The procedure for constructing a p chart is as follows:

- a) take a sample of the outcomes;
- b) record the number non-conforming;
- c) work out the proportion non-conforming (p) for each sample as the number non-conforming divided by the sample size taken;
- d) repeat steps a), b) and c) approximately twenty times;
- e) calculate the average proportion non-conforming:

$$\bar{p} = \frac{\sum p_i}{k}$$

determine the control lines as follows:

$$U_{CL_p} = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$L_{CL_p} = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

if  $L_{CL_p} < 0$  then  $L_{CL_p} = 0$

**NOTE** n is the sample size and the part under the square root sign is the formula for the variance of the Binomial distribution to which proportions belong.

f) plot the chart.

As an example, samples of 100 printed circuit boards were taken and the numbers of non-conforming boards were recorded. The numbers of non-conforming in twenty batches are displayed in Table 2.

Table 2 Non-conforming circuit boards

Batch number	Number non-conforming	$p$
1	5	0.05
2	5	0.05
3	2	0.02
4	3	0.03
5	2	0.02
6	4	0.04
6	3	0.03
7	1	0.01
8	7	0.07
9	8	0.08
10	5	0.05
11	0	0.00
12	9	0.09
13	0	0.00
14	10	0.10
15	5	0.05
16	3	0.03
17	8	0.08
18	7	0.07
19	0	0.00
20	12	0.12

The formula is as follows:

$$\bar{p} = 0.047$$

$$U_{CL_p} = \bar{p} + 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.047 + 3 \sqrt{\frac{0.047(1-0.047)}{100}} = 0.110$$

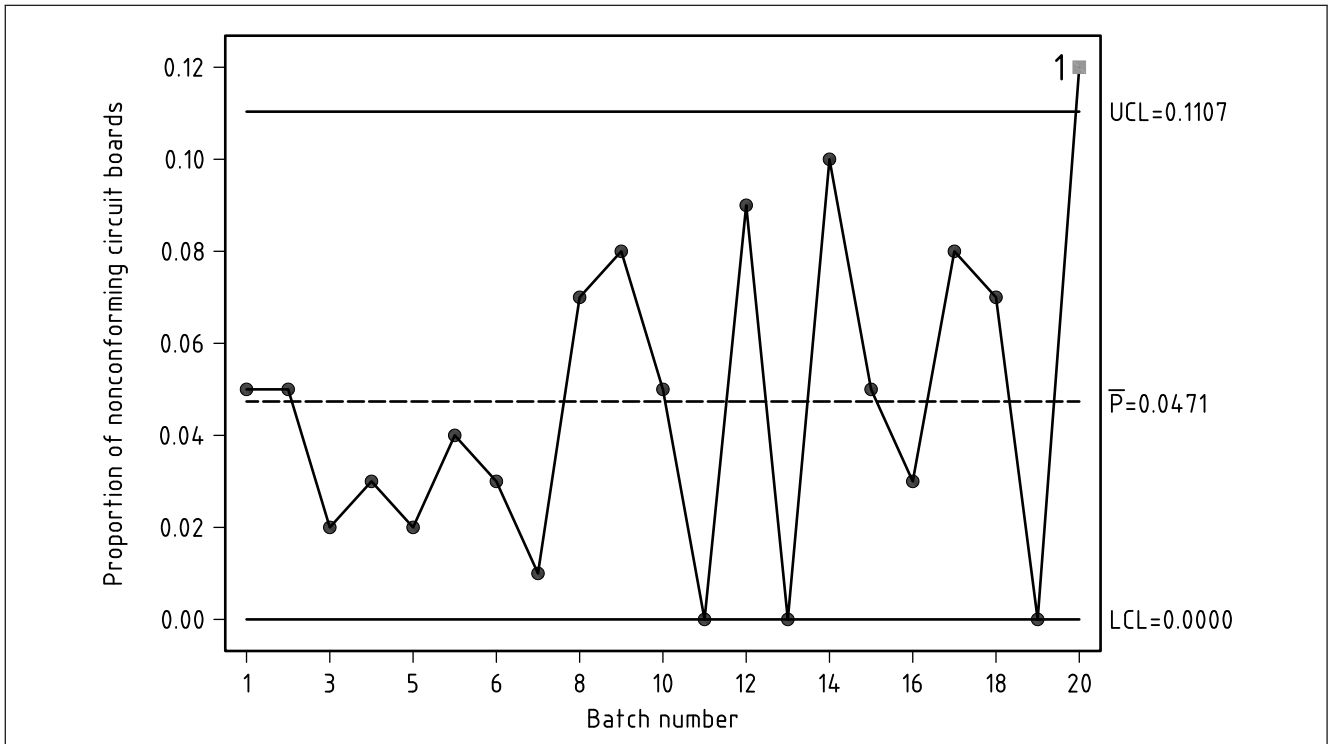
$$L_{CL_p} = \bar{p} - 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.047 - 3 \sqrt{\frac{0.047(1-0.047)}{100}} = -0.016$$

so  $L_{CL_p} = 0.000$

The chart is then as shown in Figure 9.



Figure 9 Example of a *p* chart of the proportion non-conforming



**4.4 Charts for the cusum of non-conformities**

An alternative way to portray how non-conforming outcomes deviate from a target is to use a cusum. This target could be zero or the process average of non-conforming outcomes, i.e. 6.42 in the count example in 4.2 or 0.047 in the proportions example in 4.3. To see if there are any patterns to the deviations from the target these deviations are successively summed and the cumulative sum, or cusum, is plotted against the time period of observation. The slopes of these trends are then observed. If the slope is upwards (positive), then the observations are tending to be greater than the target and if the slope is downwards (negative), then the deviations are tending to be less than the target. If there is no particular trend to the cumulative sums, then the deviations are zero as the process is on target.

To exemplify this, consider a round of golf on an 18-hole golf course. For each hole the target is called the "par" – the number of golf strokes that the hole should be achieved in. The par, the actual score and the computation of the cusum are listed in Table 3.

Table 3 Example of a cusum of golf data (1 of 2)

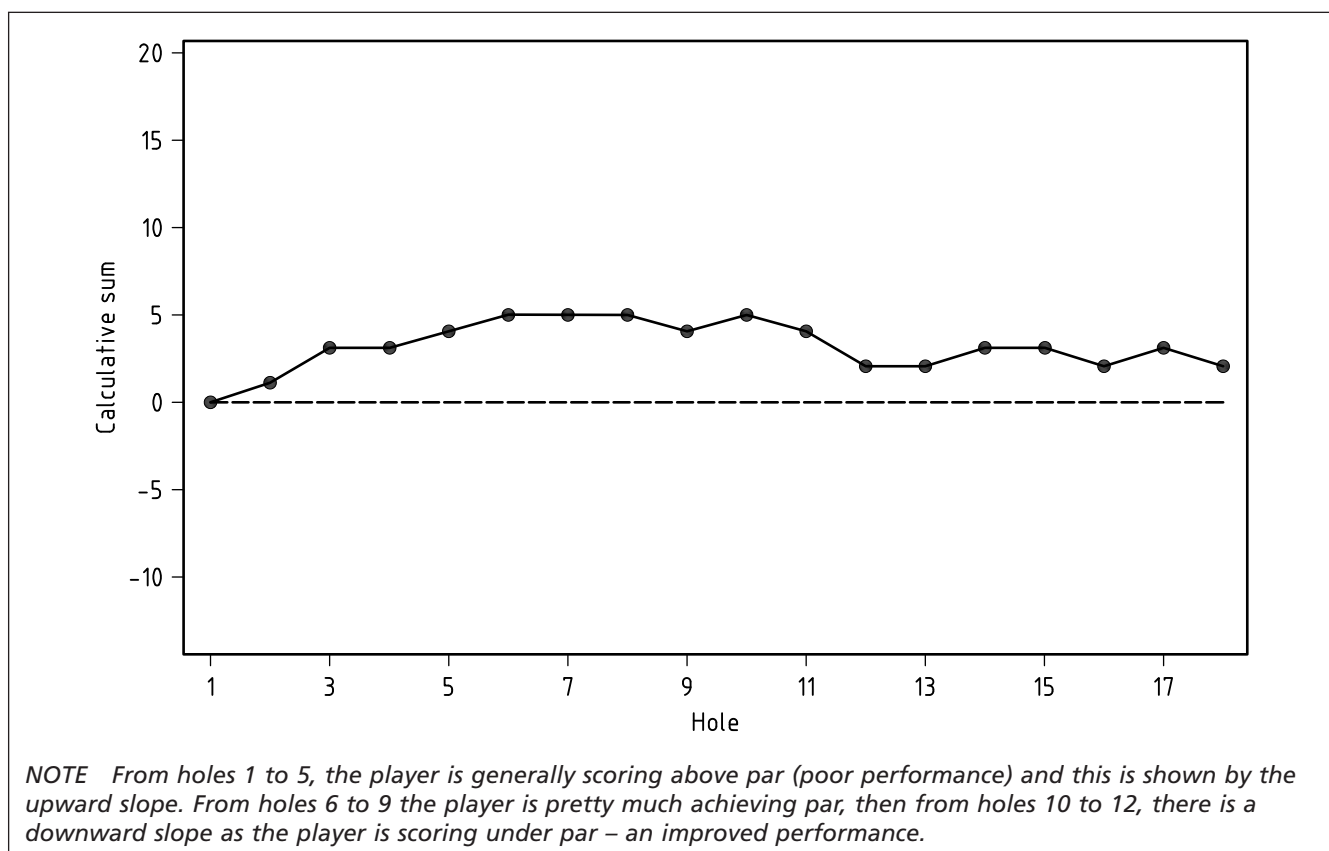
Hole	Par	Score	Score - minus par	Cusum
1	4	4	0	0
2	5	6	1	1
3	5	7	2	3
4	6	6	0	3
5	3	4	1	4
6	4	5	1	5
7	7	7	0	5
8	6	6	0	5
9	5	4	-1	4
10	4	5	1	5

Table 3 Example of a cusum of golf data (2 of 2)

Hole	Par	Score	Score - minus par	Cusum
11	5	4	-1	4
12	7	5	-2	2
13	5	5	0	2
14	5	6	1	3
15	4	4	0	3
16	6	5	-1	2
17	8	9	1	3
18	4	3	-1	2

The cusum is now plotted against the hole number as shown in Figure 10.

Figure 10 Example cusum chart for golf attribute data



There are various tools which can help interpret cusum slopes and these are outlined in 5.4.

#### 4.5 Charts for discrete variables or attributes and the relevant standard(s)

A summary of the commonly used charts for variables and the relevant standard is presented in Table 4.

Table 4 Charts for discrete variables or attributes and which standard to refer to

Chart	Description	Where to find
<i>c</i>	Chart for count of non-conformities in an item	BS 5701-2:2003, 5.1
<i>u</i>	Chart for the average number of non-conformities per inspection unit	BS 5701-2:2003, 5.2
<i>D</i>	Demerit chart – chart of several characteristics at the same time	BS 5701-2:2003, 5.3
<i>np</i>	This type of chart is suitable for data where items in the sample are classified as either “pass” or “reject” and the ample size is constant	BS 5701-2:2003, 5.4
<i>p</i>	Chart of the proportion of non-conformities per batch of items – batch size need not be constant	BS 5701-2:2003, 5.5
Cusum	Chart for cumulative counts of the number of non-conformities to detect small changes	BS ISO 7870-4:2011

## 5 Charts for continuous variables

### 5.1 General

Continuous variables are usually measures of dimensions such as height, width, length, weight, temperature, pressure, etc. These charts are the subject of BS 5702. Although it is possible to produce charts for individual readings, typically, successive batches of several measures is obtained and two charts are formed. One chart is based on the mean to monitor the location of the process. Variability of the process is monitored by plotting the standard deviation or range of successive batches of readings.

### 5.2 The mean and range chart ( $\bar{X}$ and *R* charts)

Control lines are placed around the process mean (the mean of all the samples) and the process average range. The charts are monitored and violations of the control lines indicate that the process might be out of control (as does any regular patterns in the chart). Setting the control lines utilizes properties of the normal distribution and the central limit theorem to enable inferences from small samples to be made. One of the main inferences is that the probability of being above the upper control line or below the lower control line in a process which is in control is extremely small (<0.003). Thus when the lines are exceeded the process is deemed to be “out of control”.

### 5.3 Procedure for constructing $\bar{X}$ and *R* charts

#### 5.3.1 General

This procedure is detailed in BS 5702. In summary, the procedure to establish  $\bar{X}$  and *R* charts is to take around twenty samples at regular intervals from the process when it is thought to be stable.

Often the size of each sample is small, at around four or five. To detect changes in process, frequent small samples should be taken rather than infrequent large samples. For example the data collected on a particular dimension, such as weights of chocolates in grams, might be presented as in Table 5.

Table 5 Sampled data

Sample number	Reading					$\bar{X}$	$R$	Variance
	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$			
1	2.80	2.11	2.99	2.77	2.95	2.72	0.89	0.13
2	2.22	2.65	3.52	2.29	3.62	2.86	1.39	0.45
3	2.69	2.95	2.99	2.56	3.47	2.93	0.91	0.12
4	3.26	3.27	3.71	2.77	3.48	3.30	0.94	0.12
5	3.37	1.93	3.28	3.05	2.95	2.92	1.44	0.33
6	2.01	2.98	3.70	3.95	3.93	3.31	1.94	0.68
7	3.78	2.72	3.36	2.76	2.19	2.96	1.59	0.38
8	3.01	2.68	2.40	2.17	3.51	2.76	1.34	0.28
9	3.44	3.53	3.94	3.28	3.26	3.49	0.68	0.08
10	3.14	3.52	2.53	3.76	3.89	3.37	1.36	0.30
11	2.35	2.39	2.80	3.20	3.33	2.81	0.98	0.20
12	2.35	3.37	1.40	2.49	3.54	2.63	2.15	0.75
13	3.19	2.68	2.11	3.44	3.51	2.99	1.40	0.35
14	2.79	3.72	3.23	2.66	2.64	3.01	1.08	0.22
15	2.98	3.91	2.21	3.51	3.97	3.32	1.76	0.54
16	3.18	2.73	3.52	3.66	3.65	3.35	0.93	0.16
17	2.56	3.55	3.64	2.71	2.80	3.05	1.08	0.25
18	4.04	2.84	2.01	3.07	3.20	3.03	2.03	0.53
19	2.93	2.27	2.77	2.01	3.26	2.65	1.25	0.26
20	2.70	1.94	3.81	2.48	3.19	2.83	1.87	0.51
					Mean	3.01	1.35	0.33

The  $\bar{X}$  column is the mean of each row, i.e.:

$$\bar{X} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{X} \pm 3.09\sigma_{\bar{X}}$$

The column  $R$  is the range of each row, i.e. the maximum value in each row minus the minimum value in the row.

The mean of the  $\bar{X}$  now computed to give the overall mean  $\bar{\bar{X}}$ . This gives the centre line for the  $\bar{X}$  control chart. The mean of all the ranges ( $R$ ) gives  $\bar{R}$  which serves as the centre line of the range chart.

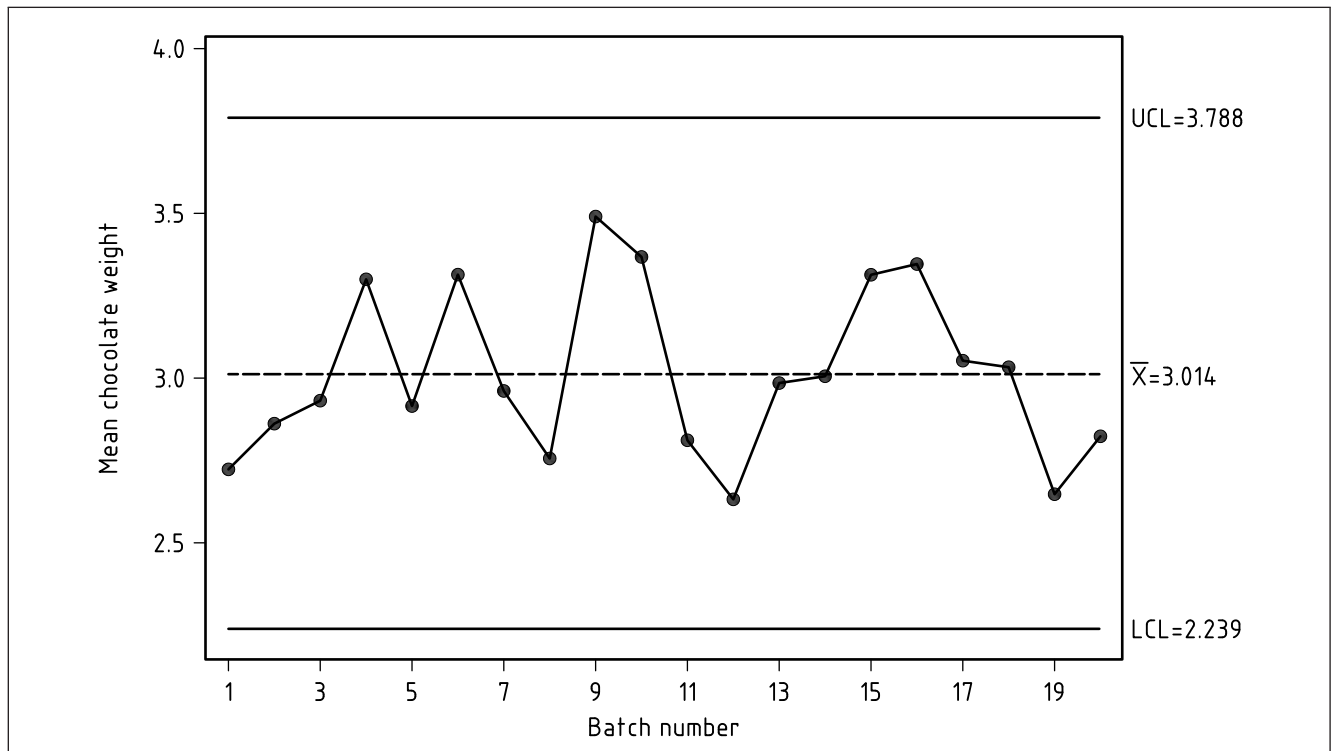
### 5.3.2 Obtaining the control lines for $\bar{X}$ and $R$ charts

The detail of obtaining the control for  $\bar{X}$  charts is detailed in BS 5702. For the  $\bar{X}$  chart the control lines are established from:

$$\bar{\bar{X}} \pm 3.09\sigma_{\bar{X}}$$

The upper control line is 3.81 and the lower control line is 2.22.

The  $\bar{X}$  chart can now be constructed as shown in Figure 11.

Figure 11 Example  $\bar{X}$  chart

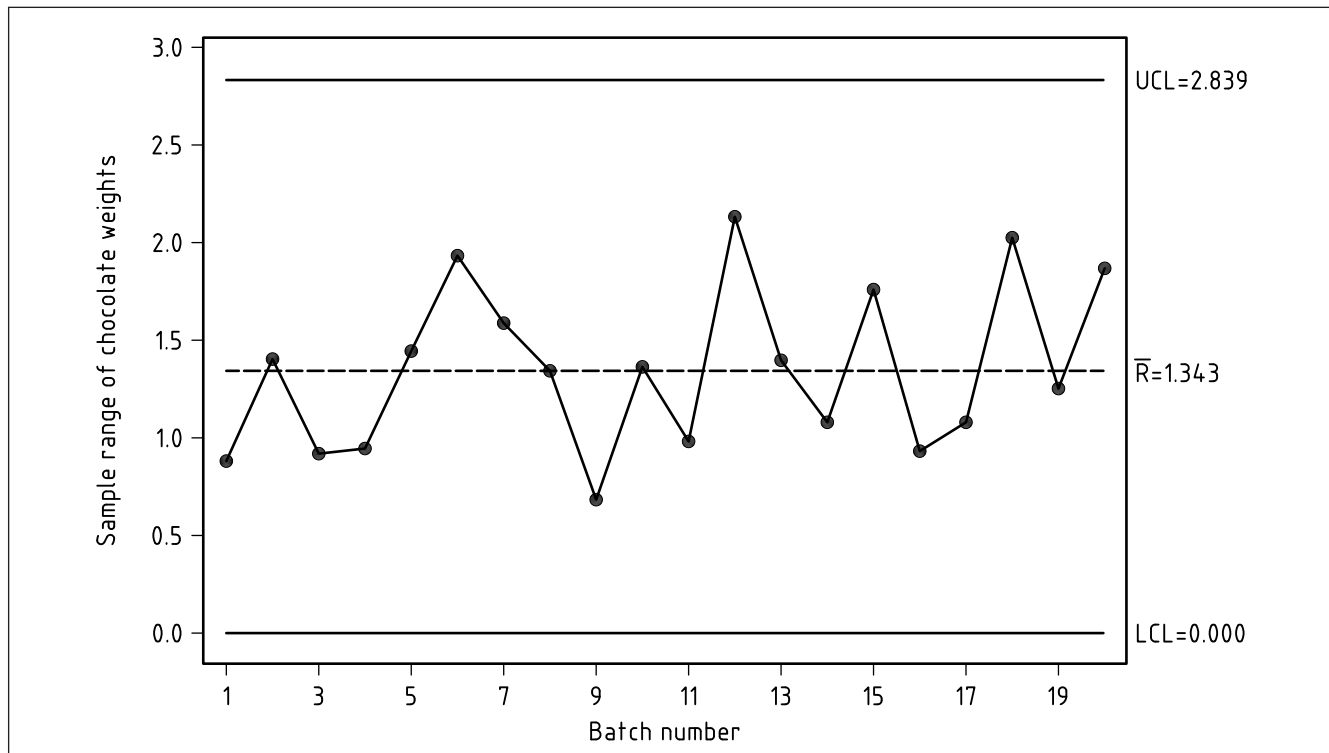
For the  $R$  chart the upper control limit is 2.85. The lower control limit for this case is regarded as 0.

*NOTE* In Figure 12 it is suggested by both the  $\bar{X}$  and  $R$  charts that this process is "in control". Now the control lines are established the chart is ready for use. Means and ranges of new samples are added to the chart and tested against the control lines. At a minimum weekly interval but probably more frequently, the chart should be re-established and new control lines computed; this is further explained in BS 5702.

### 5.3.3 Obtaining the control lines for other types of variables chart

Other charts and how to construct, interpret and use these charts for variables is fully explained in BS 5702 and this standard also gives the background theory and advice on selecting sample sizes. Relations to assessing process capability are also discussed in this standard.

Figure 12 Example R chart



#### 5.4 Cusum charts for continuous variables

Cusum charts are often formed from accumulations of deviations from a process target and the parameter of interest is the slope or gradient of the line rather than its level. This means that the charts are good for detecting small gradual drifts in the process.

*NOTE* These charts have had applications in budgetary control, monitoring energy use and water or fuel leakage.

BS ISO 7870-4 describes general purpose methods of decision-making using cusum techniques for monitoring, control and retrospective analysis. The standard starts by reviewing the fundamentals of cusum-based decision-making followed by a review of the types of cusum decision schemes.

BS ISO 7870-4 provides guidance on quantifying and minimizing the risks involved in decision-making arising from the use of the cusum method. This leads to better and more consistent decision-making under conditions of uncertainty. For example, consider the following example of the data on total protein content of blood plasma for which the target value is 45 mg. The data is shown in Table 6.

Table 6 Protein content of blood plasma (1 of 2)

Sample	Protein content
1	46.7
2	46.0
3	44.2
4	44.3
5	45.2
6	45.7
7	42.7
8	43.7

Table 6 Protein content of blood plasma (2 of 2)

Sample	Protein content
9	43.9
10	46.2
11	46.7
12	46.0
13	45.0
14	45.5
15	45.5
16	45.8
17	46.2
18	47.0
19	46.8
20	46.1
21	45.9
22	45.9
23	45.7
24	45.7
25	46.0
26	45.8

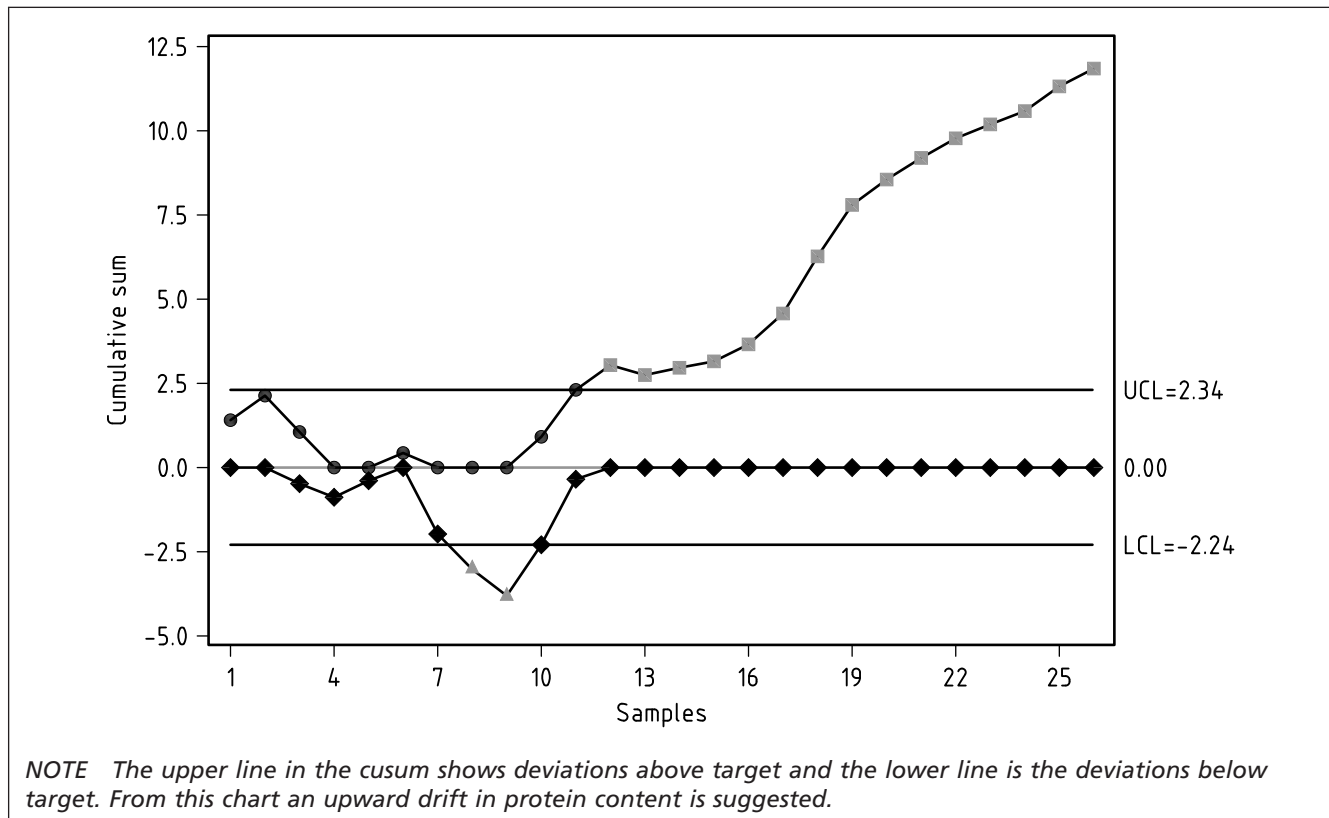
A cusum of the data in Table 6 is displayed in Figure 13.

### 5.5 Determining when "out of control" exists

There are numerous ways of determining when a slope is such as to represent an "out of control" situation. One is the protractor which displays the slope for one, two and three standard deviation changes in the process. The other is a V-mask. These devices are fully described in BS ISO 7870-4.

Cusum charts can also be interpreted in tabular form and details of this are given in BS ISO 7870-4.

Figure 13 Example of a cusum for protein content data



## 5.6 Charts for continuous variables and where to find them

A short description of commonly used charts for continuous variables and their relevant standard are given in Table 7.

Table 7 Charts for continuous variables and which standard to refer to

Chart	Description	Where to find
$\bar{X}$ and $R$ charts (also median and standard deviation)	Charts for mean and range of samples of process characteristics. Charts for other measures of location	BS 5702-1
Individual and moving range charts	Charts for individual and the moving range of a continuous process characteristic	BS 5702-2
Small runs and mixed batches	Charts for short runs and for short runs of mixed batches	BS 5702-3
EWMA	Chart for monitoring means of a process characteristic – good for detecting small changes or when the series of measures are auto-correlated.	BS ISO 7870-6 <sup>A)</sup>
Cusum	Chart for cumulative sums of a continuous process characteristic to detect small changes.	BS ISO 7870-4

<sup>A)</sup> In preparation.

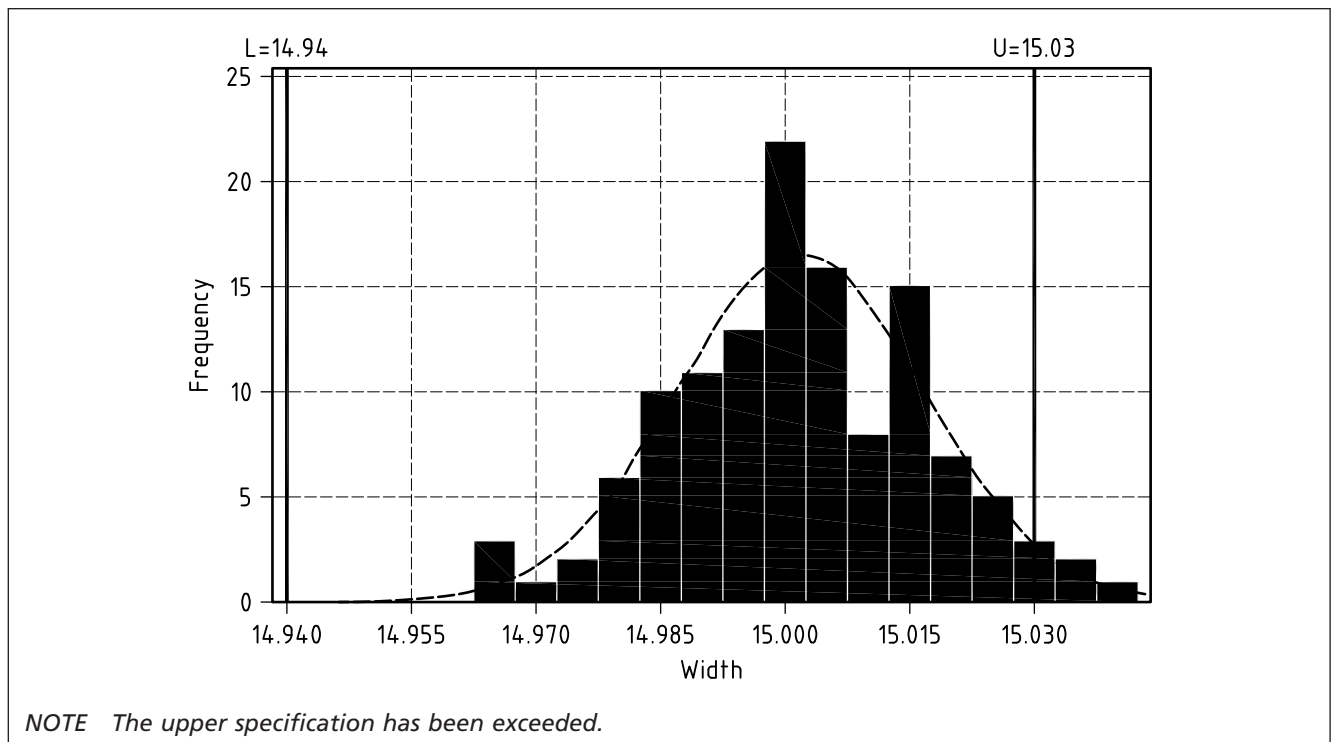


## 6 Process capability

### 6.1 Continuous variables

Consider the following example of a dimension of a component. An  $\bar{X}$  control chart and  $R$  control chart indicate that the data is in a state of statistical control. There are 25 subgroups of data and with a subgroup size of five, this gives 125 individual data points. A histogram of the data, together with the associated specification limits, can be seen in Figure 14.

Figure 14 Example of a histogram of data with specification limits and normal distribution model displayed



In the same way that PD ISO/TR 22514-4 describes the general equations for performance indices, it also gives the general equations for capability indices.

In the case of the example data shown in Figure 14 being modelled by the normal distribution the equations are:

$$\hat{C}_p = \frac{U - L}{6\hat{\sigma}}$$

$$\hat{C}_{pkU} = \frac{U - \bar{X}}{3\hat{\sigma}}$$

$$\hat{C}_{pkL} = \frac{\bar{X} - L}{3\hat{\sigma}}$$

Where  $\hat{\sigma}$  represents the within subgroup standard deviation and can be estimated using:

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

From the data,  $\bar{X}$  is 15.002 and  $\hat{\sigma}$  is 0.0155 and this gives:

$$C_p = \frac{15.03 - 14.94}{6 \times 0.0155} = 0.97$$

$$C_{pkU} = \frac{15.03 - 15.002}{3 \times 0.0155} = 0.60$$

$$C_{pkL} = \frac{15.002 - 14.94}{3 \times 0.0155} = 1.33$$

*NOTE* With a  $C_p$  of 0.97 the "process" is not capable and means there is no target that will achieve all of the output within the tolerance. As it will be possible to rework those "oversize" it is clear that aiming at about 15.000 gives virtually nothing below the lower specification limit ( $C_{pkL}$  of 1.33). All of the out-of-specification is above the upper specification limit ( $C_{pkU}$  of 0.60) leading to approximately 3.4% rework in the long run.

## 6.2 Discrete variables

If the variable of interest is a discrete variable, e.g. the number of leaking pipes per batch, a control chart (e.g. a  $p$ -chart) should indicate an "in-control" state. Thereafter, the capability is stated as the average performance, e.g.  $\bar{p}$ , the average proportion of leaking pipes. From this a calculation can be made to give either the percentage leaking or parts per million leaking. As before, because of capability conditions, this is predictive of future performance.

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