BRITISH STANDARD

BS 978-1:1968

Incorporating Amendment No. 1

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Specification for

Fine pitch gears —

Part 1: Involute Spur and Helical Gears

UDC 621.833.1

Co-operating organizations

The Mechanical Engineering Industry Standards Committee, under whose supervision this British Standard was prepared, consists of representatives from the following Government departments and scientific and industrial organizations:

Associated Offices' Technical Committee
Association of Consulting Engineers*
Association of Mining Electrical and Mechanical Engineers*
Board of Trade
British Chemical Plant Manufacturers' Association
British Compressed Air Society
British Electrical and Allied Manufacturers' Association*
British Gear Manufacturers' Association*
British Internal Combustion Engine Manufacturers' Association
British Iron and Steel Federation
British Mechanical Engineering Federation
British Pump Manufacturers' Association
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Electricity Council, the Central Electricity Generating Board and the Area Boards in England and Wales
Engineering Equipment Users' Association
Gas Council
Institute of Marine Engineers*
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Ministry of Transport
National Coal Board
National Physical Laboratory (Ministry of Technology)
Radio Industry Council
Royal Institute of British Architects

The Government department and scientific and industrial organizations marked with an asterisk in the above list, together with the following, were directly represented on the committee entrusted with the preparation of this standard:

British Horological Institute
Electronic Engineering Association
Institute of British Foundrymen
Institution of Engineering Inspection
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Milling Cutter and Reamer Association
Ministry of Defence, Army Department
Ministry of Defence, Navy Department
Ministry of Technology
Navy and Vickers Gearing Research Association
North-East Coast Institution of Engineers and Shipbuilders
Society of Motor Manufacturers and Traders Ltd.
University of Sheffield
Manufacturers of fine pitch gears

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Contents

Foreword

This standard makes reference to the following British Standards:

BS 978, *Fine pitch gears.*

BS 978-2, *Cycloidal type gears.*

BS 978-PD 3376, *Addendum No. 1 to BS 978-2, Double circular arc type gears.*

BS 978-3, *Bevel gears.*

BS 978-5, *Hobs and cutters.*

First published in 1952 and revised in 1962, this British Standard has been further revised under the authority of the Mechanical Engineering Industry Standards Committee.

In the 1962 edition, dual flank composite testing was introduced in the form of an appendix to the specification. As this method of test became more widespread, users of the standard were becoming confused between the application of the composite tolerances and application of the individual elemental tolerances. The prime reason for revision, therefore, has been to give dual flank testing the importance it now deserves by placing it in the mandatory part of the standard as the main accuracy testing procedure. Also care has been taken to ensure that the user is not misled by attempting to reconcile composite errors with elemental errors. It is necessary to retain the elemental tolerances because these will continue to be used in some fields as acceptance criteria and because they are essential as the basis for manufacturing gears and cutting tools[1]. Recognizing also that single flank testing is used reference to this is made in an appendix.

Opportunity has been taken to effect improvements in presentation and in the examples of calculations, and the terminology and notation is in accordance with that agreed within the International Organization for Standardization (ISO).

This standard is a companion work to BS 978, "*Fine pitch gears*", Part 2, "*Cycloidal type gears*" (including addendum PD 3376, "*Double circular arc type gears*"), Part 3, "*Bevel gears*", and Part 5, "*Hobs and cutters*".

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 1 to 29 and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

[1] BS 978, "*Fine pitch gears*", Part 5, "*Hobs and cutters*".

1 General

1.1 Scope

This British Standard relates to involute spur, helical and crossed helical gears, having diametral pitches finer than 20, in four accuracy grades. Examples of application are:

Class A. Astronomical apparatus and scientific instruments; colour printing machinery; control systems (when multiple trains of gears are involved this grade would generally be used for low speed components where the effect of positional accuracy is important).

Class B. Navigational instruments; high speed components of control systems where quietness and smooth running are essential; machine tool speed control; photographic instruments.

Class C. General purpose instruments; accounting and calculating machinery; counters; clockwork mechanisms.

Class D. Textile machinery; switch operating gear; clockwork mechanisms where accuracy is not the primary consideration.

It is recommended that all spur, helical and crossed helical gears for use in horology should be produced to this British Standard with the possible exception of gears for weight or spring-driven mechanisms in which the pinion is the driven member and has 12 or fewer teeth. In such cases it may be preferable to use gears conforming to BS 978-2[2)].

1.2 Terminology and notation

As the first step towards achieving international understanding of gears terminology, the terms and definitions of Draft ISO Recommendation No. 888[3)], "International vocabulary of gears", have been applied, together with corresponding notation derived from Draft ISO Recommendation No. 889, "International gear notation, symbols for geometrical data"[3)].

The notation used is as follows:

a Centre distance
a_v Virtual centre distance
d Reference circle diameter
d_1 Reference circle diameter: pinion
d_2 Reference circle diameter: wheel
d_a Tip diameter
d_{a1} Tip diameter: pinion
d_{a2} Tip diameter: wheel
d_R Measuring pin diameter
d_b Base circle diameter
j_n Normal backlash
j^a Coefficient for calculation of backlash
l Length of arc
P_n Normal diametral pitch
s Arc tooth thickness
s_1 Arc tooth thickness: pinion
s_2 Arc tooth thickness: wheel
x Addendum modification coefficient
x_1 Addendum modification coefficient: pinion
x_2 Addendum modification coefficient: wheel
Δx_1 Secondary addendum modification coefficient: pinion
Δx_2 Secondary addendum modification coefficient: wheel
y Centre distance modification coefficient

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2) BS 978, "*Fine pitch gears*", Part 2, "*Cycloidal type spur gears*".

3) To be incorporated in BS 2519, "*Glossary of terms for toothed gearing*".

z Number of teeth
z_1 Number of teeth: pinion
z_2 Number of teeth: wheel
z_v Virtual number of teeth
α_n Normal pressure angle
α_{nw} Normal pressure angle, operating
α_t Transverse pressure angle
α_{tw} Transverse pressure angle, operating
β Helix angle at reference cylinder
β_b Helix angle at base cylinder

[a] BS 978, "*Fine pitch gears*", Part 5, "*Hobs and cutters*".

1.3 Preferred range of normal diametral pitches

The pitch shall be selected from the following preferred range.

Table 1 — Normal diametral pitch P_n (inch units)

	32	48	100	160
24	36	64	120	180
28	40	80	140	200

1.4 Basic rack tooth profile

The shape and proportions of the basic rack tooth for spur helical and crossed helical gears shall be as shown either in Figure 1 or Figure 2. In the case of helical and crossed helical gears, the shape and proportions of the tooth are on a section at right angles to the pitch cylinder helix.

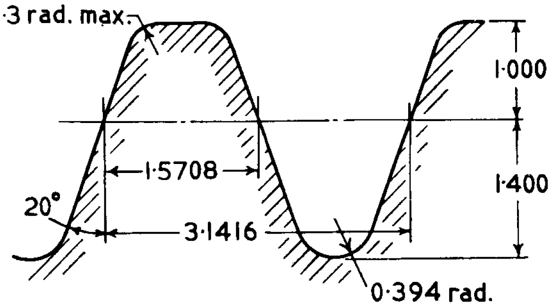

Figure 1 — Basic rack tooth profile for unit normal diametral pitch

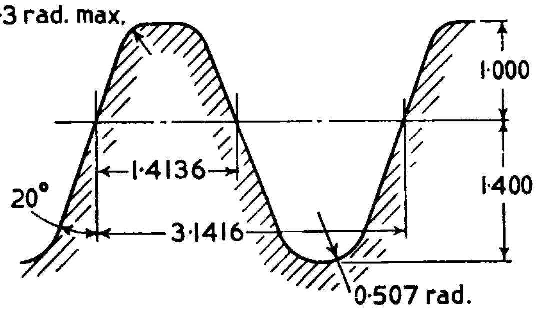

Figure 2 — Basic rack tooth profile for unit normal diametral pitch with backlash allowance

Figure 1 shows the basic rack usually employed where it is desired to have minimum backlash, as in servomechanisms.

Figure 2 is used where it is desired to have backlash to avoid any risk of jamming and where the load is usually uni-directional, for example, in clockwork mechanisms. When non-topping hobs are used it is permissible to use a hob conforming to Figure 1 basic rack and to cut at greater depth than normal to obtain the required tooth thickness.

For internal gears (Figure 1 and Figure 2 basic racks) the addendum shall normally be 0.850 instead of 1.000. In cases when the tips of the teeth have a radius of 0.25 or greater (up to 0.3 max. as shown) the 1.000 addendum may be applied.

2 Design details

2.1 Outside diameter and centre distance

For the normal case when both z_1 sec^{3}β and z_2 sec^{3}β are greater than 17 the dimensions are as follows:

$$d_1 = \frac{z_1 \sec\beta}{P_n} \quad (1) \qquad d_2 = \frac{z_2 \sec\beta}{P_n} \quad (2)$$

$$d_{a1} = \frac{z_1 \sec\beta + 2}{P_n} \quad (3) \qquad d_{a2} = \frac{z_2 \sec\beta + 2}{P_n} \quad (4)$$

$$a = \frac{(z_1 + z_2)\sec\beta}{2P_n} \quad (5)$$

(For spur gears, secβ = 1)

2.2 Modification of addendum and centre distance

2.2.1 For an explanation of the principles followed in connection with addendum modification see Appendix A.

2.2.2 Where no limitations are imposed, the value of x for any gear whose virtual number of teeth z_v is less than 17 is given by the formulae:

$$x = 1 - \frac{z_v \sin^2 \alpha_n}{2} \text{ (See Table 2 and Figure 3 and Figure 4)} \quad (6)$$

where $z_v = z \sec\beta \sec^2\beta_b \approx z \sec^3\beta$ (7)

This gives values in accordance with the lower limiting line XX in Figure 3 and a small and normally negligible amount of undercutting is thereby permitted.

The addendum modification coefficient for helical pinions from 6 to 16 teeth and helix angles up to 45° is shown in Figure 4.

2.2.3 When the pinion is modified as above and there are no other limitations, the wheel is unmodified and its dimensions are obtained from the formulae in **2.1**; but the dimensions of the pinion and centre distance are as follows:

$$d_{a1} = \frac{z_1 \sec\beta + 2(1 + x_1)}{P_n} \quad (8)$$

$$s_1 = \frac{1{\cdot}5708 + 0{\cdot}7279x_1}{P_n} \text{ (Figure 1 type gears)} \quad (9)$$

$$s_1 = \frac{1{\cdot}4136 + 0{\cdot}7279x_1}{P_n} \text{ (Figure 2 type gears)} \quad (10)$$

$$a = \frac{(z_1 + z_2)\sec\beta + 2x_1}{2P_n} \quad (11)$$

Parallel axis external gear pairs designed as above have backlash at this centre distance. (See Appendix A.)

Table 2 — Data for spur pinions of unit diametral pitch having fewer than 17 teeth

No. of teeth z_1	Addendum modification coefficient x_1	Tip diameter d_{a1}	Increase in circular tooth thickness at standard pitch dia.	Recommended minimum number of teeth in mating wheel z_2
8[a]	0.5321	11.0642	0.3874	26
9[a]	0.4736	11.9472	0.3448	25
10	0.4151	12.8302	0.3022	24
11	0.3566	13.7132	0.2596	23
12	0.2982	14.5963	0.2170	22
13	0.2397	15.4793	0.1744	21
14	0.1812	16.3623	0.1319	20
15	0.1227	17.2453	0.0893	19
16	0.0642	18.1284	0.0467	18

[a] Pinions having 8 or 9 teeth generated from basic rack Figure 1 have narrow crest width, as also have pinions of 10 and 11 teeth generated from basic rack Figure 2. 10 is the smallest number of teeth that can be generated at full depth from basic rack Figure 2.

2.2.4 Enlarged spur pinions with minimum backlash. Spur pinions enlarged according to Table 2 may be engaged with wheels in which x_2 is zero, at centre distance derived from Table 3. Gears derived from Figure 1 basic rack will have no backlash at this centre distance. A small and negligible reduction of tip-to-root clearance is obtained at this centre distance.

Table 2 is not applicable to helical gears, for which the formulae in Appendix A are required.

Alternatively, spur pinions enlarged according to Table 2 or helical pinions enlarged according to Figure 4 may be engaged with wheels in which $x_2 = -x_1$, in which case the centre distance is as obtained in **2.1**, and

$$s_2 = \frac{1{\cdot}5708 - 0{\cdot}7279x_1}{P_n} \quad \text{(Figure 1 type gears)} \qquad (12)$$

$$s_2 = \frac{1{\cdot}4136 - 0{\cdot}7279x_1}{P_n} \quad \text{(Figure 2 type gears)} \qquad (13)$$

$$d_{a2} = \frac{z_2 \sec\beta + 2(1 - x_1)}{P_n} \qquad (14)$$

Such gears derived from Figure 1 basic rack will have zero backlash at nominal dimensions.

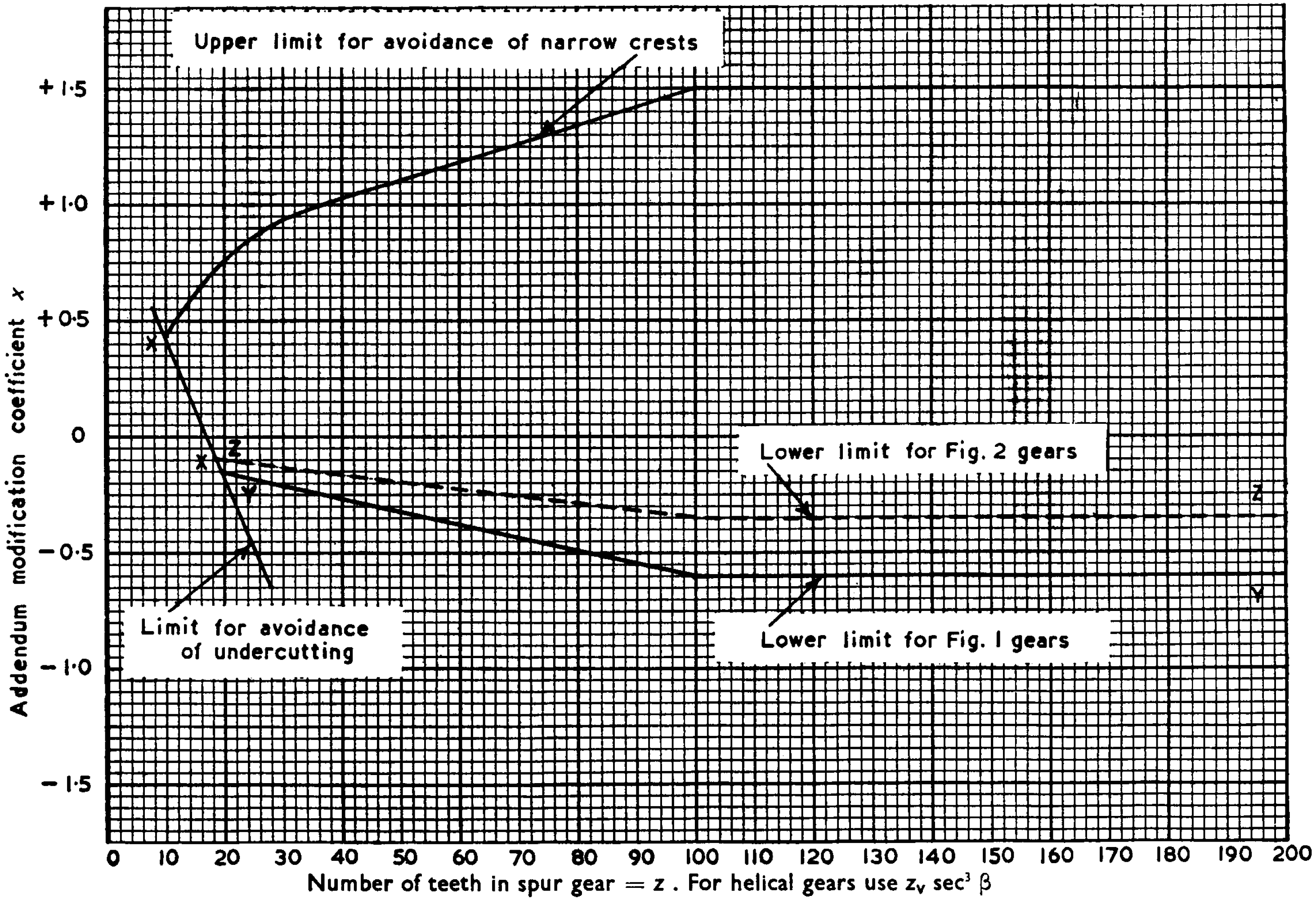

Figure 3 — Limiting values of addendum modification coefficient

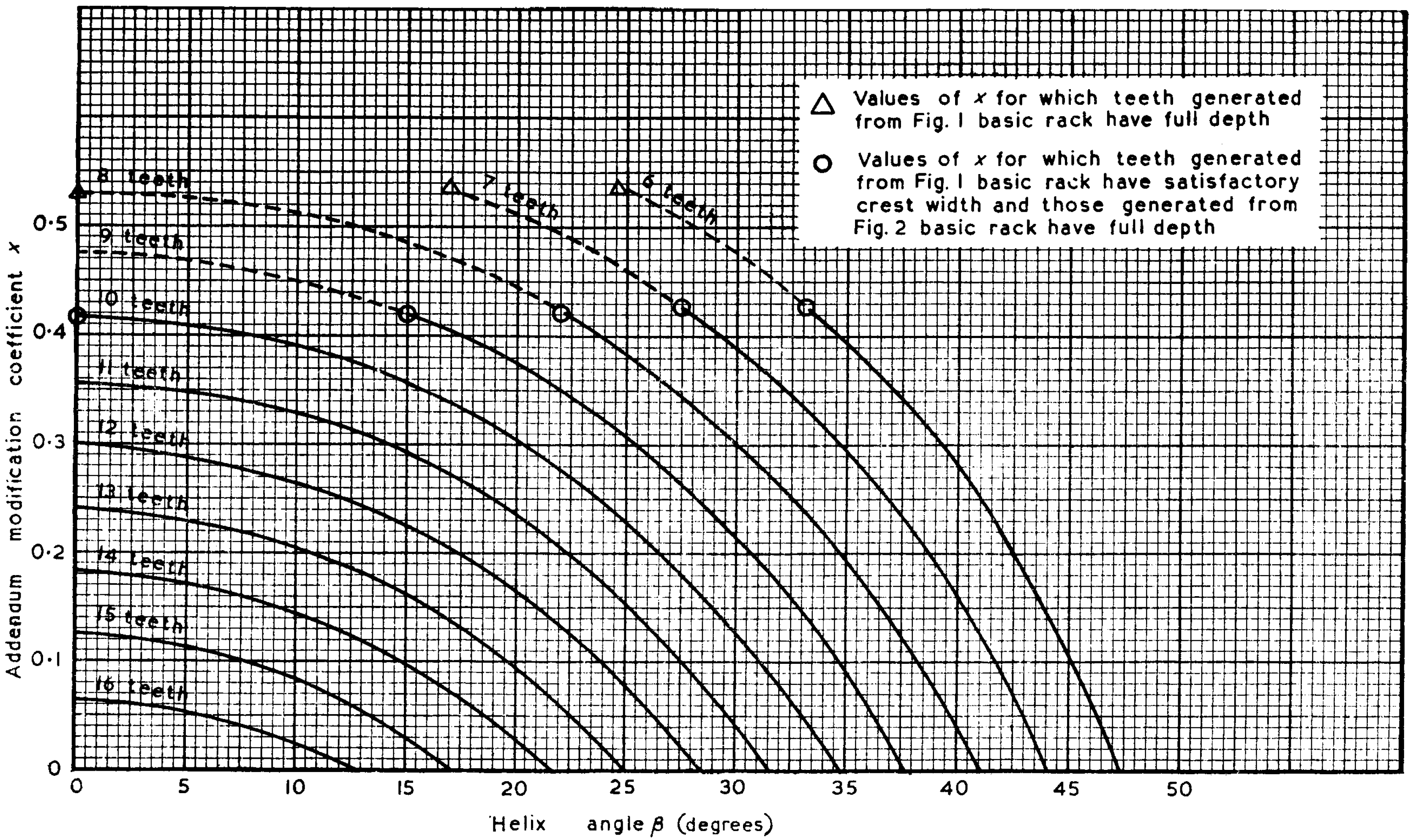

Figure 4 — Minimum permissible addendum modification coefficient for helical pinions (α_n = 20°)

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

z_1 = 10; x_1 = 0.4151

z_2	10	20	30	40	50	60	70	80	90	100
0	10.6845	15.3814	20.3887	25.3934	30.3963	35.3987	40.4004	45.4019	50.4032	55.4043
1	11.1460	15.8822	20.8893	25.8938	30.8966	35.8980	40.9005	45.9020	50.9033	55.9044
2	11.6060	16.3851	21.3898	26.3941	31.3968	36.3991	41.4007	46.4022	51.4034	56.4045
3	12.0644	16.8839	21.8904	26.8944	31.8971	36.8993	41.9008	46.9022	51.9035	56.9046
4	12.5213	17.3846	22.3909	27.3947	32.3974	37.3994	42.4010	47.4024	52.4036	57.4046
5	12.9767	17.8854	22.8913	27.8950	32.8976	37.8996	42.9012	47.9026	52.9038	57.9047
6	13.4307	18.3861	23.3918	28.3953	33.3978	38.3998	43.4013	48.4027	53.4039	58.4048
7	13.8833	18.8868	23.8922	28.8956	33.8981	38.9000	43.9014	48.9028	53.9040	58.9049
8	14.3795	19.3875	24.3927	29.3958	34.3982	39.4000	44.4016	49.4029	54.4041	59.4050
9	14.8804	19.8881	24.8931	29.8961	34.8985	39.9002	44.9017	49.9031	54.9042	59.9051

z_2	110	120	130	140	150	160	170	180	190	200
0	60.4052	65.4059	70.4066	75.4071	80.4076	85.4080	90.4083	95.4086	100.4089	105.4092
1	60.9052	65.9060	70.9066	75.9071	80.9076	85.9080	90.9084	95.9087	100.9090	105.9092
2	61.4053	66.4060	71.4067	76.4072	81.4076	86.4080	91.4084	96.4087	101.4090	106.4092
3	61.9054	66.9062	71.9067	76.9072	81.9077	86.9081	91.9084	96.9087	101.9090	106.9093
4	62.4055	67.4062	72.4068	77.4073	82.4077	87.4081	92.4085	97.4088	102.4091	107.4093
5	62.9056	67.9062	72.9068	77.9073	82.9078	87.9082	92.9085	97.9088	102.9091	107.9093
6	63.4056	68.4064	73.4069	78.4074	83.4078	88.4082	93.4085	98.4088	103.4091	108.4093
7	63.9057	68.9063	73.9069	78.9074	83.9078	88.9082	93.9086	98.9088	103.9091	108.9093
8	64.4058	69.4064	74.4070	79.4075	84.4079	89.4083	94.4086	99.4089	104.4092	109.4094
9	64.9058	69.9065	74.9070	79.9075	84.9079	89.9083	94.9086	99.9089	104.9092	109.9094

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

$z_1 = 11$; $x_1 = 0.3566$

z_2	10	20	30	40	50	60	70	80	90	100
0	11.1461	15.8315	20.8366	25.8402	30.8427	35.8445	40.8460	45.8471	50.8480	55.8488
1	11.6060	16.3321	21.3370	26.3405	31.3429	36.3447	41.3461	46.3472	51.3481	56.3488
2	12.0644	16.8327	21.8374	26.8408	31.8431	36.8448	41.8462	46.8473	51.8482	56.8489
3	12.5213	17.3333	22.3378	27.3410	32.3433	37.3450	42.3463	47.3474	52.3483	57.3490
4	12.9767	17.8338	22.8382	27.8413	32.8435	37.8451	42.8464	47.8474	52.8483	57.8490
5	13.4307	18.3343	23.3385	28.3416	33.3437	38.3453	43.3465	48.3475	53.3484	58.3491
6	13.8833	18.8348	23.8389	28.8418	33.8438	38.8454	43.8466	48.8476	53.8485	58.8492
7	14.3344	19.3353	24.3392	29.3421	34.3440	39.3456	44.3468	49.3477	54.3486	59.3492
8	14.8302	19.8358	24.8395	29.8423	34.8442	39.8457	44.8469	49.8478	54.8486	59.8493
9	15.3309	20.3362	25.3399	30.3425	35.3444	40.3458	45.3470	50.3479	55.3487	60.3493

z_2	110	120	130	140	150	160	170	180	190	200
0	60.8494	65.8500	70.8504	75.8508	80.8512	85.8515	90.8518	95.8520	100.8522	105.8523
1	61.3495	66.3500	71.3505	76.3509	81.3512	86.3515	91.3518	96.3520	101.3522	106.3523
2	61.8495	66.8500	71.8505	76.8509	81.8513	86.8516	91.8518	96.8520	101.8522	106.8523
3	62.3496	67.3501	72.3506	77.3510	82.3513	87.3516	92.3518	97.3520	102.3522	107.3523
4	62.8496	67.8502	72.8506	77.8510	82.8513	87.8516	92.8518	97.8520	102.8522	107.8524
5	63.3497	68.3502	73.3506	78.3510	83.3514	88.3516	93.3518	98.3520	103.3522	108.3524
6	63.8498	68.8502	73.8507	78.8511	83.8514	88.8517	93.8519	98.8521	103.8523	108.8524
7	64.3498	69.3503	74.3507	79.3511	84.3514	89.3517	94.3519	99.3521	104.3523	109.3524
8	64.8498	69.8504	74.8508	79.8512	84.8514	89.8517	94.8519	99.8521	104.8523	109.8524
9	65.3499	70.3504	75.3508	80.3512	85.3515	90.3518	95.3519	100.3521	105.3523	110.3524

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

z_1 = 12; x_1 = 0.2982

z_2	10	20	30	40	50	60	70	80	90	100
0	11.6060	16.2806	21.2844	26.2868	31.2884	36.2896	41.2905	46.2913	51.2919	56.2925
1	12.0644	16.7811	21.7847	26.7870	31.7885	36.7897	41.7906	46.7914	51.7920	56.7925
2	12.5213	17.2815	22.2850	27.2872	32.2886	37.2898	42.2907	47.2914	52.2921	57.2926
3	12.9767	17.7826	22.7852	27.7874	32.7888	37.7899	42.7908	47.7915	52.7921	57.7926
4	13.4307	18.2827	23.2855	28.2876	33.2889	38.2900	43.2909	47.2916	53.2922	58.2927
5	13.8833	18.7828	23.7858	28.7877	33.7890	38.7901	43.7909	48.7916	53.7922	58.7927
6	14.3344	19.2831	24.2860	29.2879	34.2891	39.2902	44.2910	49.2917	54.2923	59.2928
7	14.7841	19.7835	24.7862	29.7880	34.7892	39.7903	44.7911	49.7918	54.7923	59.7928
8	15.2796	20.2838	25.2864	30.2882	35.2893	40.2904	45.2912	50.2918	55.2924	60.2928
9	15.7801	20.7841	25.7866	30.7883	35.7894	40.7905	45.7912	50.7919	55.7924	60.7929

z_2	110	120	130	140	150	160	170	180	190	200
0	61.2929	66.2933	71.2936	76.2939	81.2942	86.2944	91.2946	96.2948	101.2949	106.2950
1	61.7930	66.7934	71.7937	76.7940	81.7942	86.7944	91.7946	96.7948	101.7949	106.7950
2	62.2930	67.2934	72.2937	77.2940	82.2942	87.2944	92.2946	97.2948	102.2949	107.2950
3	62.7930	67.7934	72.7937	77.7940	82.7942	87.7944	92.7946	97.7948	102.7949	107.7950
4	63.2931	68.2934	73.2938	78.2940	83.2942	88.2944	93.2946	98.2948	103.2950	108.2950
5	63.7931	68.7935	73.7938	78.7941	85.7943	88.7945	93.7947	98.7949	103.7950	108.7951
6	64.2932	69.2935	74.2938	79.2941	84.2943	89.2945	94.2947	99.2949	104.2950	109.2951
7	64.7932	69.7935	74.7938	79.7941	84.7943	89.7945	94.7947	99.7949	104.7950	109.7951
8	65.2932	70.2936	75.2939	80.2941	85.2943	90.2945	95.2947	100.2949	105.2950	110.2951
9	65.7933	70.7936	75.7939	80.7942	85.7944	90.7946	95.7948	100.7949	105.7950	110.7951

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

$z_1 = 13$; $x_1 = 0.2397$

z_2	10	20	30	40	50	60	70	80	90	100
0	12.0644	16.7282	21.7306	26.7322	31.7333	36.7341	41.7347	46.7352	51.7356	56.7359
1	12.5213	17.2285	22.2308	27.2323	32.2334	37.2342	42.2348	47.2353	52.2357	57.2360
2	12.9767	17.7288	22.7310	27.7325	32.7335	37.7343	42.7348	47.7353	52.7357	57.7360
3	13.4307	18.2291	23.2312	28.2326	33.2336	38.2344	43.2349	48.2353	53.2357	58.2360
4	13.8833	18.7293	23.7314	28.7327	33.7337	38.7344	43.7349	48.7354	53.7358	58.7361
5	14.3344	19.2296	24.2316	29.2328	34.2338	39.2344	44.2350	49.2354	54.2358	59.2361
6	14.7841	19.7298	24.7317	29.7329	34.7338	39.7345	44.7350	49.7355	54.7358	59.7361
7	15.2324	20.2300	25.2318	30.2330	35.2339	40.2346	45.2351	50.2355	55.2358	60.2362
8	15.7276	20.7302	25.7320	30.7331	35.7340	40.7346	45.7351	50.7355	55.7359	60.7362
9	16.2280	21.2304	26.2321	31.2332	36.2340	41.2347	46.2352	51.2356	56.2359	61.2362

z_2	110	120	130	140	150	160	170	180	190	200
0	61.7362	66.7365	71.7367	76.7369	81.7371	86.7372	91.7373	96.7374	101.7375	106.7376
1	62.2363	67.2366	72.2368	77.2370	82.2371	87.2372	92.2373	97.2374	102.2375	107.2376
2	62.7363	67.7366	72.7368	77.7370	82.7371	87.7372	92.7373	97.7374	102.7375	107.7376
3	63.2363	68.2366	73.2368	78.2370	83.2371	88.2372	93.2373	98.2374	103.2375	108.2376
4	63.7364	68.7366	73.7368	78.7370	83.7372	88.7372	93.7374	98.7374	103.7376	108.7376
5	64.2364	69.2366	74.2368	79.2370	84.2372	89.2373	94.2374	99.2375	104.2376	109.2376
6	64.7364	69.7366	74.7368	79.7370	84.7372	89.7373	94.7374	99.7375	104.7376	109.7376
7	65.2364	70.2367	75.2369	80.2371	85.2372	90.2373	95.2374	100.2375	105.2376	110.2376
8	65.7365	70.7367	75.7369	80.7371	85.7372	90.7373	95.7374	100.7375	105.7376	110.7377
9	66.2365	71.2367	76.2369	81.2371	86.2372	91.2373	96.2374	101.2375	106.2376	111.2377

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

$z_1 = 14$; $x_1 = 0.1812$

z_2	10	20	30	40	50	60	70	80	90	100
0	12.5213	17.1746	22.1760	27.1769	32.1775	37.1780	42.1784	47.1787	52.1789	57.1791
1	12.9767	17.6748	22.6761	27.6770	32.6776	37.6781	42.6784	47.6787	52.6789	57.6791
2	13.4307	18.1749	23.1762	28.1770	33.1776	38.1781	43.1785	48.1787	53.1789	58.1791
3	13.8833	18.6751	23.6763	28.6771	33.6777	38.6781	45.6785	48.6788	53.6790	58.6792
4	14.3344	19.1752	24.1764	29.1772	34.1777	39.1782	44.1785	49.1788	54.1790	59.1792
5	14.7841	19.6754	24.6765	29.6773	34.6778	39.6782	44.6786	49.6788	54.6790	59.6792
6	15.2324	20.1755	25.1766	30.1773	35.1778	40.1782	45.1786	50.1788	55.1790	60.1792
7	15.6793	20.6756	25.6766	30.6774	35.6779	40.6783	45.6786	50.6788	55.6790	60.6792
8	16.1740	21.1758	26.1767	31.1774	36.1779	41.1783	46.1787	51.1789	56.1791	61.2792
9	16.6744	21.6759	26.6768	31.6775	36.6780	41.6784	46.6787	51.6789	56.6791	61.6792

z_2	110	120	130	140	150	160	170	180	190	200
0	62.1792	67.1794	72.1794	77.1796	82.1797	87.1798	92.1799	97.1799	102.1800	107.1800
1	62.6793	67.6794	72.6795	77.6796	82.6797	87.6798	92.6799	97.6799	102.6800	107.6800
2	63.1793	68.1794	73.1795	78.1796	83.1797	88.1798	93.1799	98.1799	103.1800	108.1800
3	63.6793	68.6794	73.6795	78.6796	83.6797	88.6798	93.6799	98.6799	103.6800	108.6800
4	64.1793	69.1794	74.1795	79.1796	84.1797	89.1798	94.1799	99.1799	104.1800	109.1800
5	64.6793	69.6794	74.6795	79.6796	84.6797	89.6798	94.6799	99.6799	104.6800	109.6800
6	65.1793	70.1794	75.1795	80.1796	85.1797	90.1798	95.1799	100.1800	105.1800	110.1801
7	65.6793	70.6794	75.6795	80.6796	85.6797	90.6798	95.6799	100.6800	105.6800	110.6801
8	66.1793	71.1794	76.1795	81.1796	86.1798	91.1798	96.1799	101.1800	106.1800	111.1801
9	66.6793	71.6794	76.6795	81.6796	86.6798	91.6799	96.6799	101.6800	106.6800	111.6801

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

$z_1 = 15$; $x_1 = 0.1227$

z_2	10	20	30	40	50	60	70	80	90	100
0	12.9767	17.6195	22.6202	27.6207	32.6210	37.6212	42.6214	47.6215	52.6216	57.6217
1	13.4307	18.1196	23.1203	28.1207	33.1210	38.1212	43.1214	48.1215	53.1216	58.1217
2	13.8833	18.6197	23.6203	28.6207	33.6210	38.6212	43.6214	48.6215	53.6216	58.6217
3	14.3344	19.1198	24.1204	29.1208	34.1210	39.1212	44.1214	49.1215	54.1216	59.1217
4	14.7841	19.6199	24.6204	29.6208	34.6211	39.6213	44.6214	49.6215	54.6216	59.6217
5	15.2324	20.1200	25.1205	30.1208	35.1211	40.1213	45.1214	50.1216	55.1216	60.1218
6	15.6793	20.6200	25.6205	30.6208	35.6211	40.6213	45.6215	50.6216	55.6217	60.6218
7	16.1248	21.1201	26.1205	31.1209	36.1211	41.1213	46.1215	51.1216	56.1217	61.1218
8	16.6192	21.6201	26.6206	31.6209	36.6212	41.6214	46.6215	51.6216	56.6217	61.6218
9	17.1193	22.1202	27.1206	32.1209	37.1212	42.1214	47.1215	52.1216	57.1217	62.1218

z_2	110	120	130	140	150	160	170	180	190	200
0	62.6218	67.6218	72.6219	77.6220	82.6220	87.6220	92.6221	97.6221	102.6221	107.6222
1	63.1218	68.1219	73.1219	78.1220	83.1220	88.1220	93.1221	98.1221	103.1221	108.1222
2	63.6218	68.6219	73.6219	78.6220	83.6220	88.6220	93.6221	98.6221	103.6221	108.6222
3	64.1218	69.1219	74.1219	79.1220	84.1220	89.1220	94.1221	99.1221	104.1221	109.1222
4	64.6218	69.6219	74.6219	79.6220	84.6220	89.6220	94.6221	99.6221	104.6221	109.6222
5	65.1218	70.1219	75.1219	80.1220	85.1220	90.1220	95.1221	100.1221	105.1222	110.1222
6	65.6218	70.6219	75.6219	80.6220	85.6220	90.6221	95.6221	100.6221	105.6222	110.6222
7	66.1218	71.1219	76.1220	81.1220	86.1220	91.1221	96.1221	101.1221	106.1222	111.1222
8	66.6218	71.6219	76.6220	81.6220	86.6220	91.6221	96.6221	101.6221	106.6222	111.6222
9	67.1218	72.1219	77.1220	82.1220	87.1220	92.1221	97.1221	102.1221	107.1222	112.1222

Table 3 — Centre distances for spur gear pairs of unit diametral pitch for the conditions given in clause 2.2.4

$z_1 = 16$; $x_1 = 0.0642$

z_2	10	20	30	40	50	60	70	80	90	100
0	13.4307	18.0633	23.0635	28.0636	33.0637	38.0638	43.0638	48.0639	53.0639	58.0639
1	13.8833	18.5634	23.5635	28.5636	33.5637	38.5638	43.5638	48.5639	53.5639	58.5639
2	14.3344	19.0634	24.0635	29.0636	34.0637	39.0638	44.0638	49.0639	54.0639	59.0639
3	14.7841	19.5634	24.5636	29.5636	34.5637	39.5638	44.5638	49.5639	54.5639	59.5639
4	15.2524	20.0634	25.0636	30.0657	35.0637	40.0638	45.0638	50.0639	55.0639	60.0639
5	15.6793	20.5634	25.5636	30.5637	35.5637	40.5638	45.5638	50.5639	55.5639	60.5639
6	16.1248	21.0635	26.0636	31.0637	36.0638	41.0638	46.0638	51.0639	56.0639	61.0639
7	16.5688	21.5635	26.5636	31.5637	36.5638	41.5638	46.5638	51.5639	56.5639	61.5639
8	17.0633	22.0635	27.0636	32.0637	37.0638	42.0638	47.0638	52.0639	57.0639	62.0639
9	17.5633	22.5635	27.5636	32.5637	37.5638	42.5638	47.5639	52.5639	57.5639	62.5639

z_2	110	120	130	140	150	160	170	180	190	200
0	63.0639	68.0640	73.0640	78.0640	83.0640	88.0640	93.0640	98.0640	103.0640	108.0641
1	63.5639	68.5640	73.5640	78.5633	83.5640	88.5640	93.5640	98.5640	103.5640	108.5641
2	64.3639	69.0640	74.0640	79.0640	84.0640	89.0640	94.0640	99.0640	104.0640	109.0641
3	64.5640	69.5640	74.5640	79.5640	84.5640	89.5640	94.5640	99.5640	104.5640	109.5641
4	65.0640	70.0640	75.0640	80.0640	85.0640	90.0640	95.0640	100.0640	105.0640	110.0641
5	65.5640	70.5640	75.5640	80.5640	85.5640	90.5640	95.5640	100.5640	105.5640	110.5641
6	66.0640	71.0640	76.0640	81.0640	86.0640	91.0640	96.0640	101.0640	106.0641	111.0641
7	66.5640	71.5640	76.5640	81.5640	86.5640	91.5640	96.5640	101.5640	106.5641	111.5641
8	67.0640	72.0640	77.0640	82.0640	87.0640	92.0640	97.0640	102.0640	107.0641	112.0641
9	67.5640	72.5640	77.5640	82.5640	87.5640	92.5640	97.5640	102.5640	107.5641	112.5641

3 Accuracy of gear blanks and gears

3.1 General

The measurement of individual elemental errors in fine pitch gears is sometimes difficult and often involves the use of equipment which may not readily be available. Furthermore, the individual limits of tolerance may be so very minute as to render them unrealistic as inspection criteria.

The acceptance criteria of this British Standard are therefore based on dual flank composite error testing, by which product gears are rotated in close mesh with master gears of known error, and the results given in terms of the measured variations in centre distance appropriate to the combination of master gear and product gear.

It is necessary, however, to appreciate that the variation in centre distance so measured, may include the combined effects of profile, tooth thickness, adjacent pitch and tooth alignment errors but will not necessarily be the sum of such errors. Under some circumstances, therefore, it may be necessary to control and assess elemental errors and, for this reason, coupled with the need to provide an adequate basis for accurate manufacture of the related hobs and cutters, the limits of tolerance for each possible contributing source of error are also given.

Therefore, the test methods embraced within this British Standard are:

a. limits of tolerance on gear blanks (when applicable)

b. dual flank composite testing

c. assessment of elemental errors

d. combinations of *b* and *c*.

However, recognizing the development of the single flank composite testing procedure, tolerances are given in Appendix D.

Additionally, pin dimensions for taking measurements over pins in respect of spur gears, are given in Appendix E. It should be appreciated, however, that owing to radial run-out errors, the tooth thickness measured over pins will not necessarily equate with tooth thickness at the reference circle.

The actual testing procedures to be adopted shall be the subject of agreement between purchaser and manufacturer and requirements shall be stated clearly on drawings (see Section 5 and Figure 6) and on purchase orders and contracts.

IMPORTANT NOTE. Generally, a pair of mating gears are of identical accuracy grade but, by agreement between purchaser and manufacturer, finer or coarser grades may be adopted for the pinion or wheel, or for certain elements of a pinion or wheel.

3.2 Dual flank composite testing — test pressures and limits of tolerance

The product gear shall be rotated in close mesh with a calibrated master gear under just sufficient force to ensure adequate contact whilst permitting free rotation. Forces appropriate to the pitch suitable for brass and steel product gears are given in Table 4. The errors thus revealed shall not exceed the appropriate limits of tolerance as prescribed in Table 5.

Table 4 — Forces appropriate to the pitch suitable for brass and steel product gears

	24 to 80 P_n	over 80 to 160 P_n	over 160 P_n
Master worm	8 ozf	4 ozf	2 ozf
Master wheel	16 ozf	8 ozf	4 ozf
NOTE These values may need to be reduced when checking product gears in soft materials such as plastics or aluminium.			

Table 5 — Limits of tolerance

		Class A	Class B	Class C	Class D
		in	in	in	in
Max. tooth to tooth composite error tolerance	10 to 19 teeth	0.0004	0.0007	0.0010	0.0025
	20 to 29 teeth	0.0003	0.0006	0.0009	0.0023
	over 30 teeth	0.0002	0.0005	0.0009	0.0023
Max. total composite error tolerance		0.0006	0.0010	0.0016	0.0040
Max. departure from the centre distance appropriate to this combination of master gear and product gear when meshed with a theoretically correct master gear		– 0.0003 – 0.0011	– 0.0005 – 0.0019	– 0.0005 – 0.0027	– 0.0005 – 0.0060
Applicable to a pitch range of[a]		24 to 200 P_n	24 to 160 P_n	24 to 80 P_n	24 to 40 P_n
NOTE Where for any reason it is desired to use a tooth thickness which is not that given for the Figure 1 basic rack, the centre distance tolerance zones given above apply. [a] Finer pitches are permissible provided that the contact ratio of the gear pair is not less than 1.1.					

3.3 Limits of tolerance on blanks and gear elements

3.3.1 Gear blanks. Limits and tolerances on gear blanks shall be in accordance with Table 6. Where a topping hob or cutter is used the limits on blank diameter may not apply.

Table 6 — Limits of tolerance on gear blanks

Item	Class A	Class B	Class C	Class D
Blank diameter when used for location	h7 of BS 1916[a]	h7 of BS 1916[a]	h8 of BS 1916[a]	h9 of BS 1916[a]
	in	in	in	in
Radial runout of periphery (or reference surface) relative to datum when used for setting or measurement of teeth (FIM[b]) Axial runout at reference diameter (FIM[b]) Flatness of end faces	0.000 2	0.000 2	0.000 3	0.000 1 d_a + 0.001

[a] BS 1916, "*Limits and fits for engineering*".
[b] Full indicated movement.

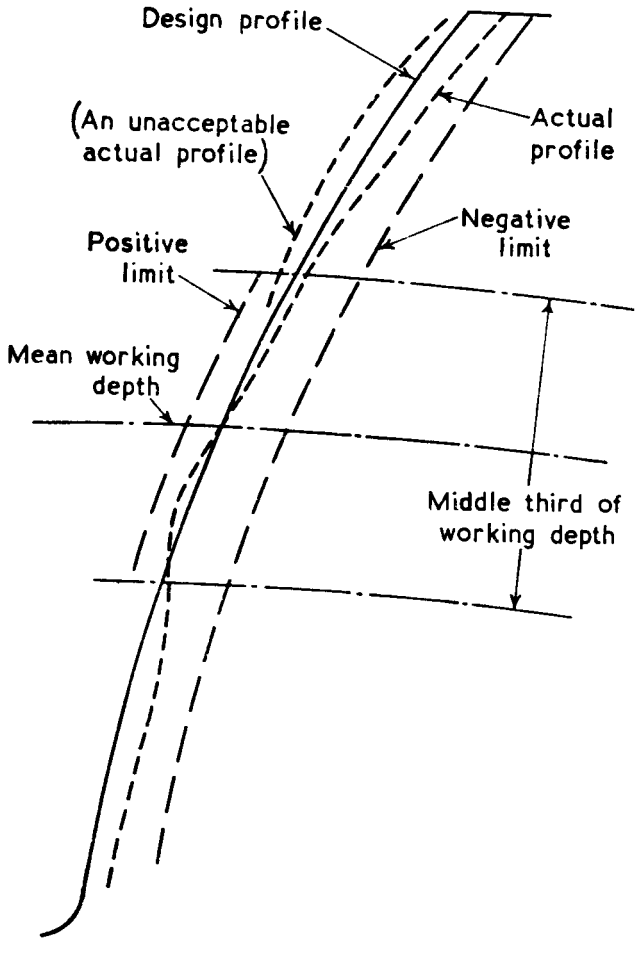

Figure 5 — Tolerance zone for profile error

3.3.2 Profile. Limits of tolerance on tooth profile are given in Table 7. The limits given are in relation to the design profile that coincides with the actual profile at mid working depth (see Figure 5).

Table 7 — Limits of tolerance on tooth profile

Class	Limits of tolerance		Generally applicable to a pitch range of
	Middle third of working depth	Outside middle third of working depth	
	in	in	
A	+ 0.000 05 – 0.000 10	0 – 0.000 1	24 to 200 inc.
B	+ 0.000 05 – 0.000 20	0 – 0.000 2	24 to 160 inc.
C	+ 0.000 10 – 0.000 30	0 – 0.000 3	24 to 80 inc.
D	+ 0.000 10 – 0.000 70	0 – 0.000 7	24 to 40 inc.

3.3.3 Pitch. Limits of tolerance on pitch (adjacent and cumulative) are given in Table 8.

Table 8 — Limits of tolerance on pitch

Class	Limits of tolerance	Generally applicable to a pitch range of
	in	
A	$0.000\ 2\ \sqrt{l} + 0.000\ 1$	24 to 200 inc.
B	$0.000\ 32\ \sqrt{l} + 0.000\ 16$	24 to 160 inc.
C	$0.000\ 5\ \sqrt{l} + 0.000\ 25$	24 to 80 inc.
D	$0.000\ 8\ \sqrt{l} + 0.000\ 4$	24 to 40 inc.

where l = any selected length of arc, including an arc length of one pitch, but not greater than $\frac{\pi d}{2}$.

NOTE Limits of pitch tolerance are given with reference to the axis of rotation.

3.3.4 Tooth alignment. The tolerances are listed in Table 9.

Table 9 — Limits of tolerance on tooth alignment

Face width		Class A	Class B	Class C	Class D
Over	up to and inc.				
in	in	in	in	in	in
	0.5	0.000 16	0.000 25	0.000 4	0.000 6
0.5	1	0.000 2	0.000 32	0.000 5	0.000 8
1	2	0.000 25	0.000 4	0.000 6	0.001 0
2	4	0.000 32	0.000 5	0.000 8	0.001 3

3.3.5 Radial run-out of teeth. Tolerances are given in Table 10.

Table 10 — Limits of tolerance on radial run-out of teeth

	Class A		Class B		Class C	Class D
Pitch range generally applicable	**24 to 100 P_n inc.**	**over 100 to 200 P_n inc.**	**24 to 100 P_n inc.**	**over 100 to 160 P_n inc.**	**24 to 80 P_n inc.**	**24 to 40 P_n inc.**
	in	in	in	in	in	in
Tolerance	0.000 3	0.000 2	0.000 4	0.000 3	0.000 6	0.001 6
NOTE Where backlash is allowable the limit for radial run-out does not apply provided the pitch tolerances are within the limits specified in **3.3.3**.						

3.3.6 Tooth thickness. Limits of tolerance for tooth thickness are given in Table 11.

Table 11 — Limits of tolerance on tooth thickness

Class	Limits of tolerance		Generally applicable to a pitch range of
	Figure 1 gears	Figure 2 gears	
A	– 0.000 3 – 0.000 6	± 0.000 25	24 to 200 inc.
B	– 0.000 6 – 0.001 0	± 0.000 4	24 to 160 inc.
C	– 0.000 9 – 0.001 6	± 0.000 8	24 to 80 inc.
D	– 0.001 6 – 0.003 0	± 0.001 6	24 to 40 inc.
NOTE 1 The above tolerances should be related to those of Table 5 when the dual flank method of composite testing is employed. NOTE 2 Measuring pin diameters for external and internal spur gears are given in Appendix E.			

4 Limits and tolerances of assembly

4.1 Centre distance

Limits of tolerance for centre distance of mounting shall be as given in Table 12.

Table 12 — Limits of tolerance on centre distance of mounting

Class	Limits of tolerance	Generally applicable to a pitch range of
	in	in
A	$\pm (0.000\ 08\sqrt{a} + 0.000\ 08)$	24 to 200 inc.
B	$\pm (0.000\ 16\sqrt{a} + 0.000\ 16)$	24 to 160 inc.
C	$\pm (0.000\ 32\sqrt{a} + 0.000\ 32)$	24 to 80 inc.
D	$\pm (0.000\ 6\sqrt{a} + 0.000\ 6)$	24 to 40 inc.
where a is the centre distance in inches.		

4.2 Depth of engagement

It may be found necessary, in the finer pitches of gears, to resort to graded inspection and selective assembly in order to maintain a satisfactory minimum depth of engagement and a suitable contact ratio.

Table 13 and Table 14 give recommended minimum values for working depth, based on contact ratios not less than 1.0 where the teeth have rounded tips or approximately 1.2 where the rounding is absent.

The larger of the two values obtained from the tables should be taken.

These data are applicable to spur gears. They will be applicable to helical or crossed helical gears if the number of teeth in each gear is multiplied by $\sec^3 \beta$.

It should be noted that backlash is increased when the working depth is less than nominal, and in circumstances in which backlash has to be limited, tolerances affecting depth of engagement are governed by consideration of the backlash rather than by contact ratio.

Table 13 — Minimum depth of engagement for spur gear pairs with 28 to 33 teeth ($z_2 + z_1$)

No. of teeth in pinion (z_1)	Minimum depth of engagement	Sum of number of teeth ($z_2 + z_1$)	Minimum depth of engagement
10–12	$1.85/P_n$	28–29	$1.80/P_n$
13–14	$1.80/P_n$	30–32	$1.75/P_n$
15–16	$1.70/P_n$	33	$1.7/P_n$

Table 14 — Minimum depth of engagement for spur gear pairs with more than 33 teeth ($z_2 + z_1$)

No. of teeth in pinion (z_1)	Minimum depth of engagement	Sum of number of teeth ($z_2 + z_1$)	Minimum depth of engagement
8	$1.8/P_n$	34–59	$1.70/P_n$
9	$1.75/P_n$	40–49	$1.65/P_n$
10–11	$1.70/P_n$	50 and above	$1.60/P_n$
12–16	$1.65/P_n$		
17 and above	$1.60/P_n$		

5 Information to be given on drawings

In accordance with the requirements of BS 308[4)] the following is recommended as a standard form showing essential information required on drawings for workshops and office reference (see Figure 6 which is for illustration purposes only).

All manufacturing dimensions should be shown in the body of the drawing except those relating to gear cutting which should be shown in a table, either at the right hand side or bottom of the drawing.

The following information should be given in the table:

Type of gear
Number of teeth
Normal diametral pitch
Basic rack tooth form
Class of gear
Reference circle diameter
Whole depth of tooth
Tooth thickness on reference circle
Hand
Lead

[4)] BS 308, "*Engineering drawing practice*".

Helix angle at reference cylinder

Centre distance with mating gear (optional)

Drawing No. of mating gear (optional).

A table concerning inspection data and auxiliary information may also be included. When inserting information in this table care must be exercised to avoid conflict between requirements for accuracy of individual elements and dual flank testing requirements.

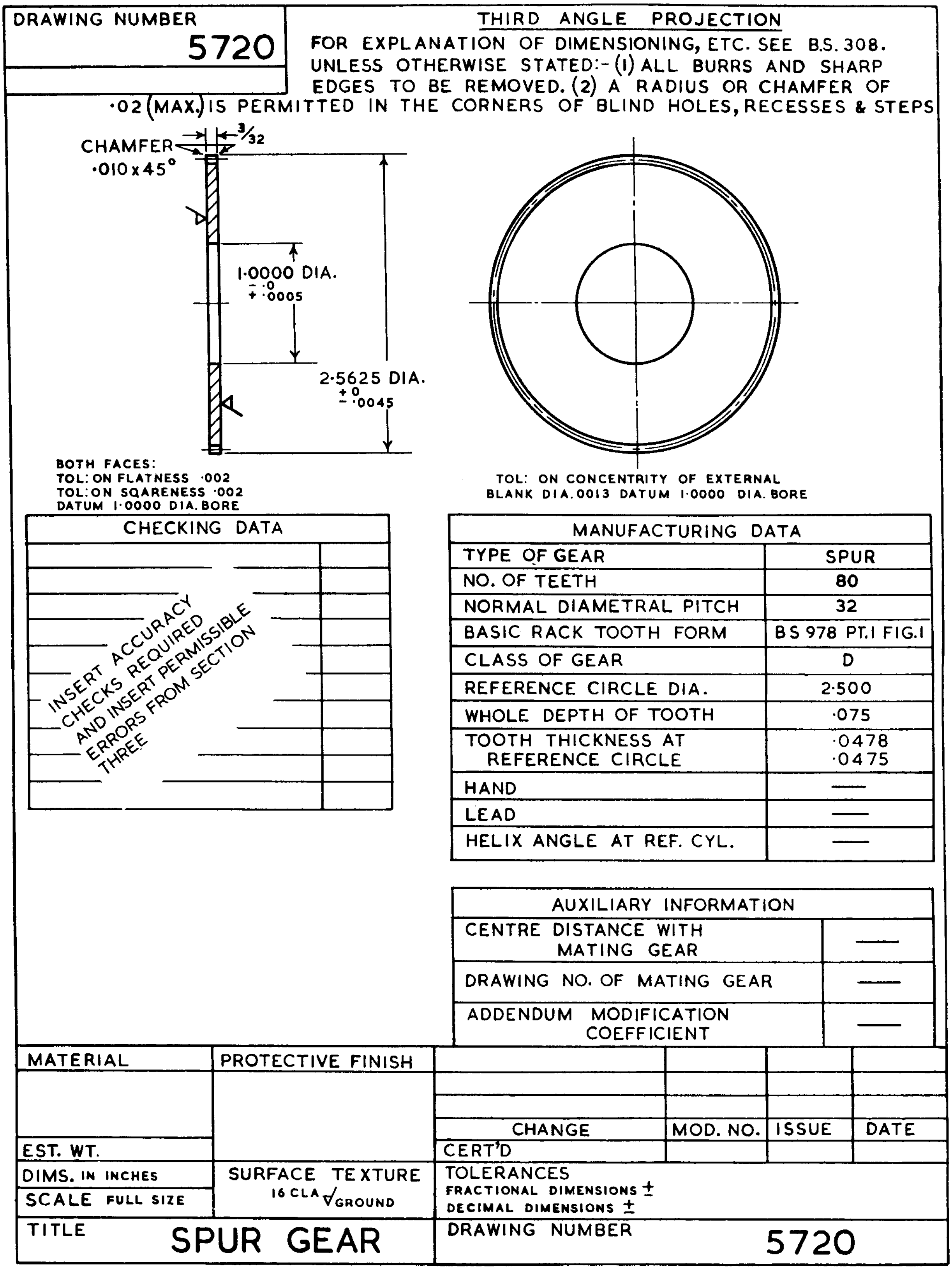

Figure 6 — Example of information to be given on drawings

Appendix A Centre distance and addendum modification

A.1 Introduction

The centre distance of a pair of gears may have any value within certain limits since two gears generated by similar tools with equal base pitch will mesh correctly. The working pressure angle, the pitch diameters and the backlash will vary with change in centre distance.

By taking advantage of these features in the design and manufacture of involute gears, the pitches listed in Table 1 will be adequate for most purposes. Gear diameters and centre distances may be varied by the choice of a suitable value for the addendum coefficients x_1 and x_2 within the upper and lower limits laid down by Figure 3.

The case where there is no addendum modification is dealt with in **2.1**. In other cases the rules given apply to gears generated from the Figure 1 basic rack tooth shape. They apply substantially to the Figure 2 basic rack tooth shape also except where reference is made to close mesh and minimum backlash.

The chart given in Figure 3 has been prepared with the following points in mind:

1) *Upper limits of addendum modification coefficient.*

Too large a value will lead to an undesirably narrow crest width which should be not less than $0.3/P_n$.

If small wheels and pinions with excessively enlarged addenda are meshed together, the contact ratio becomes too low.

2) *Lower limits of addendum modification coefficient.*

Where x has a smaller value than the following the gears become undercut by generation:

For Figure 1 type gears $x = 1.14 - \dfrac{z \sin^2\alpha_t}{2 \cos\beta}$

Figure 2 type gears $x = 1.066 - \dfrac{z \sin^2\alpha_t}{2 \cos\beta}$

The root fillets of gears extend a minimum distance from the root circle when x is slightly greater than + 1. As x varies from this value the root fillets extend progressively further up the teeth. A limiting negative value of x is reached when the root fillets are just clear of the path of contact during engagement.

A.2 Addendum modification coefficient x_1 and x_2

A.2.1 *Gears at predetermined centre distance*

When the centre distance is predetermined the addendum modification coefficient x_1 and x_2 may have any value permitted by Figure 3 provided that x_1 is within the limits imposed by equations (30) and (31).

Nominal tip diameters and tooth thicknesses are then obtained from:

$$d_{a1} = d_1 + \frac{2(1 + x_1)}{P_n} \tag{15}$$

$$d_{a2} = d_2 + \frac{2(1 + x_2)}{P_n} \tag{16}$$

Figure 1 type gears, $$s_1 = \frac{1{\cdot}5708 + 0{\cdot}7279x_1}{P_n} \tag{17}$$

$$s_2 = \frac{1{\cdot}5708 + 0{\cdot}7279x_2}{P_n} \tag{18}$$

Figure 2 type gears, $$s_1 = \frac{1{\cdot}4136 + 0{\cdot}7279x_1}{P_n} \tag{19}$$

$$s_2 = \frac{1{\cdot}4136 + 0{\cdot}7279x_2}{P_n} \tag{20}$$

If the centre distance modification coefficient is equal to the sum of the addendum modification coefficients of pinion and wheel then:

$$y = x_1 + x_2 = \left(a - \frac{d_1 + d_2}{2}\right)P_n \tag{21}$$

and y has a positive or negative value other than zero;

Figure 1 type external gears have backlash at nominal dimensions

Figure 1 type internal gears have interference at nominal dimensions

Figure 1 type crossed helical gears have zero backlash at nominal dimensions.

The normal backlash (or interference) obtained is given by

$$j_n = j^* \left(\frac{y}{aP_n}\right)^2 a \tag{22}$$

where j^* is obtained from Figure 7.

For helical gears the normal backlash is obtained by using a_v in place of a in formula (22) and Figure 7, where

$$a_v = a/\cos^2\beta \tag{23}$$

NOTE For internal gears the values of a, z_2 and d_2 are to be regarded as negative, and x_2 as negative when its sense is to enlarge the wheel. All formulae are then applicable.

If it is desired to reduce (or increase) backlash it will be necessary to increase (or decrease) tooth thickness by a secondary addendum modification Δx_1 for the pinion and Δx_2 for the wheel.

$$\frac{\Delta x_1 + \Delta x_2}{P_n} = \frac{\text{desired reduction in normal backlash}}{0.6840} \tag{24}$$

Δx_1 and Δx_2 are then added to the initial values of x_1 and x_2 obtained by equations (21), (30), (31) and used in equations (15) – (20).

In following this procedure it is necessary, if $x_1 + x_2$ have values toward the upper limits of Figure 3, to examine the root clearance and if insufficient, to modify the outside diameter of one or both gears by amounts;

$$\frac{2\ \Delta x_1}{P_n} \text{ and } \frac{2\ \Delta x_2}{P_n}$$

A.2.2 ***Gears having zero backlash requirement***

Calculation of values of $x_1 + x_2$ (or a) for gears (other than crossed helical gears) which require zero backlash at nominal dimensions.

Given a, to find $(x_1 + x_2)$.

$$\cos\alpha_{tw} = \frac{(d_1 + d_2)\cos\alpha_t}{2a} \tag{25}$$

$$(x_1 + x_2) = \frac{(\text{inv}\alpha_{tw} - \text{inv}\alpha_t)(z_1 + z_2)}{2\tan\alpha} \tag{26}$$

$$\text{where } \tan\alpha_t = \frac{\tan\alpha}{\cos\beta} \tag{27}$$

Given $(x_1 + x_2)$, to find a.

$$\mathrm{inv}\alpha_{tw} = \mathrm{inv}\alpha_t + \frac{2\tan\alpha(x_1 + x_2)}{z_1 + z_2} \tag{28}$$

$$a = \frac{(d_1 + d_2)\cos\alpha_t}{2\cos\alpha_{tw}} \tag{29}$$

NOTE $\mathrm{inv}\alpha_t$ and $\mathrm{inv}\alpha_{tw}$ may be obtained from tables of involute functions. An involute function of an angle is the small difference between the tangent of the angle and the value of the angle in radians.

A.3 Limiting values of x_1 and x_2

Where x_1 and x_2 have been fixed by the foregoing formulae, x_1 should lie between the following values:

$$\text{Minimum } x_1 = \frac{z_1}{z_2 + z_1}(x_1 + x_2) \tag{30}$$

$$\text{Maximum } x_1 = \frac{z_1}{z_2 + z_1}(x_1 + x_2) + 0.4 \tag{31}$$

In general, most favourable action is obtained when a high value of x_1 is used with the pinion driving the wheel and a low value of x_1 with the wheel driving the pinion.

Where a negative value of the sum $(x_1 + x_2)$ arises the value of x_2 may be below the relevant line, XX or YY, of Figure 3 provided that the sum of minimum values of $(x_1 + x_2)$ allowed by the graph is more negative than the sum required.

Appendix B Examples of calculations

Example 1. External spur gears at predetermined centre distance which differs from half the sum of the reference circles and a normal amount of backlash is permissible.

Given	*Required*
$z_2 = 63$	x_1 and x_2
$z_1 = 20$	d_{a2} and d_{a1}
$P_n = 24$	
$a = 1.8$ in	
Pinion driving	
Figure 1 gears.	

From formula (21), $x_1 + x_2 = (1.8 \times 24) - \frac{(63 + 20)}{2}$

$= 1.7$

From formula (30), x_1 min. $= \frac{20}{83} \times 1.7$

$= 0.41$

From formula (31), x_1 max. $= \frac{20}{83} \times 1.7 + 0.4$

$= 0.81$

From Figure 3, x_1 must not exceed 0.75. Selecting this value of 0.75 for x_1 then $x_2 = 1.7 - 0.75$

$$= 0.95$$

From formula (16), $d_{a2} = \dfrac{63}{24} + \dfrac{2(1 + 0.95)}{24}$

$$= 2.788 \text{ inches}$$

From formula (15), $d_{a1} = \dfrac{20}{24} + \dfrac{2(1 + 0.75)}{24}$

$$= 0.979 \text{ inch}$$

Ans: $d_{a1} = 0.979$ $d_{a2} = 2.788$ $x_1 = 0.75$ $x_2 = 0.95$

Example 2. External spur gears at predetermined centre distance and minimum backlash.

Given	*Required*
$P_n = 36$	z_1, z_2, x_1, x_2
$a = 2.55$	s_1 and s_2
Ratio = 4 : 1	
Pinion driving	
Figure 1 gears.	

Two different solutions, A and B, are given. A is derived by involute functions and B by the method of Figure 7.

Solution A. By examination, tooth numbers which give a near approach to the required centre distance are 36 and 144. With these tooth numbers and unmodified gears the centre distance would be:

$$\frac{z_1 + z_2}{2P_n} = \frac{36 + 144}{2 \times 36} = 2{\cdot}5 \text{ inches}$$

From (25) $\cos\alpha_w = \dfrac{2.5}{2.55} \times 0.939\ 69 = 0.921\ 265$

Whence $\alpha_w = 22.89°$ and $\text{inv}\alpha_w = 0.022\ 70$ $\text{inv}\alpha = 0.014\ 904$ and $0.022\ 700 - 0.014\ 904 = 0.007\ 796$

From (26) $x_1 + x_2 = \dfrac{0.007\ 796 \times 180}{2 \times 0.363\ 97} = 1.928$

From (30) x_1 (min.) $= \dfrac{36}{180} \times 1.928 = 0.3856$

From (31) (x_1 max.) $= \dfrac{36}{180} \times 1.928 + 0.4 = 0.7856$, say 0.78

As the pinion is driving the maximum value of x_1 may be taken and from Figure 3, x_1 can be 0.78.

Therefore $x_2 = 1.928 - 0.78 = 1.148$

From (17 and 18)

$$s_1 = \frac{1.5708 + 0.7279\ (0.78)}{36} = 0.0594 \text{ inch}$$

$$s_2 = \frac{1.5708 + 0.7279\ (1.148)}{36} = 0.0668 \text{ inch}$$

Note that s_2 cannot be gauged at the reference diameter, but can be used for calculation of diameter over gauge pins.

Ans. $z_1 = 36$, $z_2 = 144$, $x_1 = 0.78$, $x_2 = 1.148$, $s_1 = 0.0594$ in, $s_2 = 0.0668$ inch.

Solution B. Taking alternative tooth numbers, $z_1 = 37$ and $z_2 = 148$, the centre distance of unmodified gears would be: $\dfrac{z_1 + z_2}{2P_n} = \dfrac{185}{72} = 2.569\ 45$ inches

From (21) $x_1 + x_2 = (2.55 - 2.569\ 45)\ 36 = -\ 0.700$

From **A.3**, this value may be used without restriction,

whence from (31), x max. $= \dfrac{37}{185}(-\ 0.7) + 0.4 = 0.26$ and $x_2 = -\ 0.700 - 0.260 = -\ 0.960$.

To check the amount of backlash obtained, using Figure 7

$$\frac{y}{P_n} = \frac{-\ 0.700}{36} = -\ 0.019\ 45$$

$$\frac{y}{aP_n} = \frac{-\ 0.019\ 45}{2.55} = -\ 0.0076 \text{ whence } j^* = 2.63$$

From (22) $j_n = \dfrac{2.63}{2.55}(0.019\ 45^2) = 0.000\ 39$ inch.

This is an acceptably small amount and no increase in x_1 or x_2 is justified.

From (17) and (18)

$$s_1 = \frac{1.5708 + 0.7279 \times 0.26}{36} = 0.048\ 86 \text{ inch}$$

$$s_2 = \frac{1.5708 - 0.7279 \times 0.96}{36} = 0.024\ 22 \text{ inch}$$

Note that the reference circle is near the tip of the wheel and it will not be practicable to gauge s_2, except by gauge pin measurement.

Ans. $z_1 = 37$, $z_2 = 148$, $x_1 = 0.26$, $x_2 = -\ 0.96$, $s_1 = 0.048\ 86$ inch, $s_2 = 0.024\ 22$ inch.

Example 3. External spur gears with no limitations other than avoidance of undercutting and backlash. (**2.2.2**.)

Given	*Required*
$z_1 = 10$	x_1 and x_2
$z_2 = 60$	s_1 and s_2
$P_n = 36$	d_{a1} and d_{a2}
	a

Two solutions are available, A) and B):

A) From Table 2, $x_1 = 0.4151$, $x_2 = 0$,

$$s_1 = \frac{1{\cdot}5708 + 0{\cdot}3022}{36} = 0{\cdot}052\ 03 \text{ inch}$$

$$s_2 = \frac{1{\cdot}5708}{36} = 0{\cdot}043\ 63 \text{ inch}$$

From (8) $d_{a1} = \dfrac{10 + 2(1{\cdot}4151)}{36} = 0{\cdot}3564$ inch

$$d_{a2} = \frac{60 + 2}{36} = 1{\cdot}7222 \text{ inch}$$

From Table 3, $a = \dfrac{35{\cdot}3987}{36} = 0{\cdot}983\ 30$ inch

B) $x_1 = 0.4150$, $x_2 = -\ 0.4150$

whence d_{a1} and s_1 are as obtained for solution A).

From (12) $s_2 = \dfrac{1{\cdot}5708 - 0{\cdot}3022}{36} = 0{\cdot}035\ 24$ inch

$$d_{a2} = \frac{60 + 2(1 - 0{\cdot}4151)}{36} = 1{\cdot}699\ 17 \text{ inch}$$

From (5) $a = \dfrac{10 + 60}{2 \times 36} = 0{\cdot}972\ 22$ inch

Ans. (A and B): $x_1 = 0.4150$, $s_1 = 0.0520$ inch, $d_{a1} = 0.3564$ inch.

A; $x_2 = 0$, $s_2 = 0.043\ 63$ inch, $d_{a1} = 1.7222$ inch, $a = 0.9833$ inch.

B; $x_2 = -0.4150$, $s_2 = 0.035\ 24$ inch, $d_{a2} = 1.699\ 17$ inch, $a = 0.9722$ inch.

Example 4. Helical gears at predetermined centre distance and minimum backlash.

Given	*Required*
$z_1 = 22$	d_{a1}
$z_2 = 67$	d_{a2}
$P_n = 40$	x_1
$\beta = 18°$	x_2
$a = 1.1562$	

Pinion driving
Figure 1 type gears.
(Solution using involute functions)

Obtain,
$$\sec\beta = 1.051\ 46$$
$$\sec^3\beta = 1.162$$
$$\tan\alpha = 0.363\ 97$$
$$\cos\alpha = 0.939\ 69$$

From (1) and (2)
$$d_1 = \frac{22 \times 1.051\ 46}{40} = 0.5783$$
$$d_2 = \frac{67 \times 1.051\ 46}{40} = 1.7612$$

From (27) $\mathrm{Tan}\alpha_t = 0.363\ 97 \times 1.051\ 46 = 0.3827$

whence $\alpha_t = 20.942°$

$\cos\alpha_t = 0.933\ 94$

$\mathrm{inv}\alpha_t = 0.017\ 196$

From (25)

$$\mathrm{Cos}\alpha_{tw} = \frac{(1.7612 + 0.5783)\ 0.933\ 94}{2 \times 1.1562} = 0.944\ 84$$

$\alpha_{tw} = 19.119°$

$\mathrm{inv}\alpha_{tw} = 0.012\ 963$

From (26) $x_1 + x_2 = \dfrac{(0.012\ 963 - 0.017\ 196)\ (67 + 22)}{2 \times 0.363\ 97} = -0.517\ 54$

From (30)

$$x_1\ (\text{min.}) = \frac{22}{22 + 67}\ (-\ 0.517\ 54) = -\ 0.1279$$

x_2 (max.) $= -0.517\ 54 + 0.1279 = -0.3896$

From (31)

x_1 (max.) $= -0.1279 + 0.4 = 0.2721$

x_2 (min.) $= -0.517\ 54 - 0.2721 = -0.789\ 64$

To find the range of values of x_1 and x_2 which are permissible find z_{v1} and z_{v2}, i.e. from (7)

$z_{v1} = 22 \times 1.162 = 25.6$

$z_{v2} = 67 \times 1.162 = 78$

From Figure 3 we chose the value -0.48 for x_2 leaving $x_1 = -0.517\ 54 + 0.48 = -0.0375$.

From (15) and (16)

$$d_{a1} = 0.5783 + \frac{2\ (1 - 0.0375)}{40} = 0.6264 \text{ inch}$$

$$d_{a2} = 1.7612 + \frac{2\ (1 - 0.48)}{40} = 1.7872 \text{ inch.}$$

Ans. $x_1 = -0.0375$, $x_2 = -0.48$, $d_{a1} = 0.6264$ inch, $d_{a2} = 1.7872$ inches.

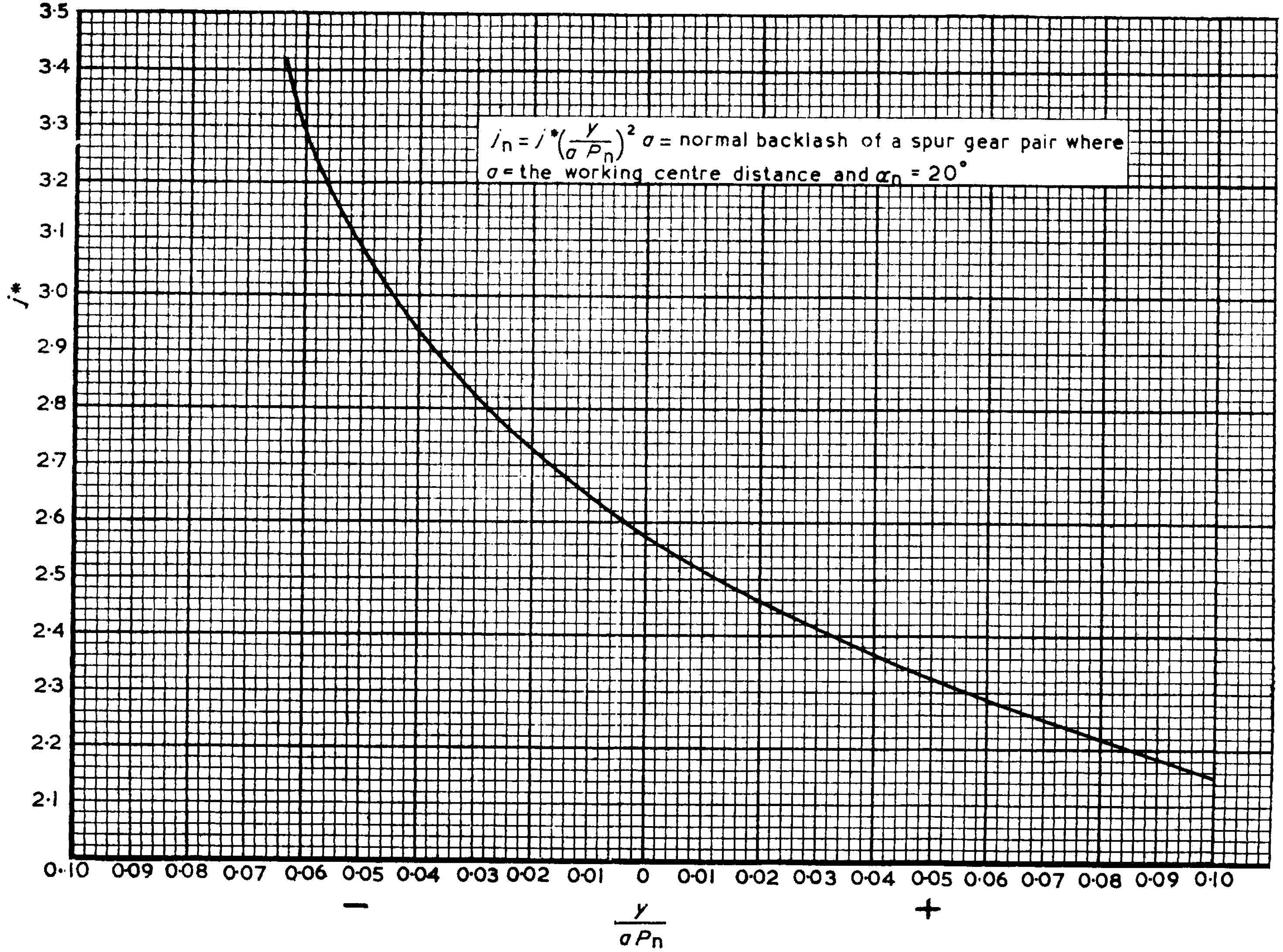

Figure 7 — Chart for normal backlash when $x_1 + x_2 = y$

Appendix C Comparable diametral pitches and metric modules

Diametral pitch	Module
in	mm
24	1.058
28	0.907
32	0.794
36	0.706
40	0.635
48	0.529
64	0.397
80	0.318
100	0.254
120	0.212
140	0.181
160	0.159
180	0.141
200	0.127

Appendix D Single flank composite testing: limits of tolerance

When single flank testing is applied, the limits of tolerance are derived as follows:

1) *Tooth-to-tooth* composite error. For all grades, the tolerance is derived from: single pitch tolerance + tooth profile tolerance.

2) *Total composite error*. For all grades, the tolerance is derived from: maximum cumulative pitch tolerance + tooth profile tolerance; where the maximum cumulative pitch tolerance is derived from the value of $l = \pi d/2$.

The values derived represent errors in angular transmission when a product gear is rotated in mesh with a calibrated master gear under similar conditions to those prescribed in **3.1**.

Appendix E Calculation of measuring pin dimensions for external and internal spur gears

In order to overcome the difficulty of resetting on the gear-cutting machine gears which are found to be outside dual flank tolerances in respect of size, it is necessary either to carry out dual flank testing on the machine or to adopt other methods of test at that stage of manufacture. The former method is used only in exceptional cases, whilst the practice of measurement over pins is a practical method for this purpose although insufficient in itself to dispose of the final dual flank test.

For external and internal spur gears the measuring pin diameter d_R should be calculated from the following formulae. Further information on measurement of pins is given in B.R.6001 (1) "Precision gearing for control systems and armaments"*.

For external gears of 7 to 17 teeth with positive addendum modification:

$$d_R = \frac{1.92}{P_n}$$

For external gears: $d_R = \frac{1.728}{P_n}$

For internal gears: $d_R = \frac{1.44}{P_n}$

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