

# Physical testing of rubber —

## Part 5: Guide to the application of rubber testing to finite element analysis

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## Foreword

This British Standard has been prepared by Technical Committee PRI/22.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

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### Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, pages 1 to 43 and a back cover.

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## 1 Scope

This part of BS 903 gives recommendations for test procedures and guidance on appropriate methods for determining model parameters from test data for use in finite element analysis (FEA) of rubber. It covers stress–strain characterization, mechanical failure, friction, thermal properties and heat build-up.

It is applicable to solid vulcanized rubbers of hardness 20 to 80 IRHD for which the deformation is predominantly elastic, and to cellular materials formed using such rubbers. It might prove useful for other materials which can be deformed elastically to large strain.

## 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BS 903-A1, *Physical testing of rubber — Part A1: Determination of density.*

BS 903-A2, *Physical testing of rubber — Part A2: Method for determination of tensile stress-strain properties.*

BS 903-A4, *Physical testing of rubber — Part A4: Determination of compression stress–strain properties.*

BS 903-A10 (BS ISO 132), *Rubber, vulcanized or thermoplastic — Part A10: Determination of flex cracking and crack growth (De Mattia).*

BS 903-A14, *Physical testing of rubber — Part A14: Method for determination of modulus in shear or adhesion to rigid plates — Quadruple shear method.*

BS 903-A21, *Physical testing of rubber — Part A21: Determination of rubber to metal bond strength.*

BS 903-A42 (BS ISO 3384), *Physical testing of rubber — Part A42: Determination of stress relaxation in compression at ambient and at elevated temperatures.*

BS 903-A51, *Physical testing of rubber — Part A51: Determination of resistance to tension fatigue.*

BS 903-A61 (BS ISO 15113), *Physical testing of rubber — Part A61: Determination of the frictional properties.*

BS 7608, *Code of practice for fatigue design and assessment of steel structures.*

BS EN 12667, *Thermal performance of building materials and products — Determination of thermal resistance by means of guarded hot plate and heat flow meter methods — Products of high and medium thermal resistance.*

BS ISO 34-1, *Physical testing of rubber — Determination of tear strength — Part 1: Trouser, angle and crescent test pieces.*

BS ISO 4664, *Rubber — Guide to the determination of dynamic properties.*

BS ISO 23529, *Rubber — Physical test methods — Preparation and conditioning of test pieces and preferred test conditions.*

## 3 Terms and definitions

For the purposes of this British Standard, the following terms and definitions apply.

### 3.1

#### **finite element analysis**

#### **FEA**

numerical method of analysing a product in which the numerical calculations are carried out for discrete, linked elements of the product

NOTE FEA is usually carried out using a commercial software package which allows visual simulation of the product to aid creation of the finite element model and viewing of the results of the analysis.

### 3.2

#### **deformation mode**

deformation arising through a particular relationship of the principal extension ratios

NOTE Examples are uniaxial tension, uniaxial compression, pure shear, simple shear or equibiaxial extension.

### 3.3

#### **deviatoric deformation**

deformation involving change of shape with no associated change of volume

### 3.4

#### **volumetric deformation**

deformation involving change of volume with no associated change of shape

### 3.5

#### **extension ratio**

ratio of deformed to undeformed length of a line element of material in a specified direction

NOTE 1 This is also known as a stretch ratio.

NOTE 2 The term extension is used to cover all types of deformation; thus a compression is defined with an extension ratio less than one.

### 3.6

#### **hyperelastic**

<material> deforming to high strains under the application of stresses, and returning to the original shape when the stresses are reduced, such that there is no loss of energy

### 3.7

#### **incompressible**

<material> deforming without a change in volume

NOTE For infinitesimal strains, incompressibility is equivalent to a Poisson's ratio of 0.5. For large deformations, the condition is that  $\lambda_1\lambda_2\lambda_3 = 1$ .

### 3.8

#### **model**

<finite element> computer simulation of a product or structure, containing all the information necessary to carry out a finite element analysis

### 3.9

#### **model**

<material> description, expressed mathematically, of some aspect of the behaviour of the material, which is implemented in the FEA package

### 3.10

#### **principal strain axis**

one of three orthogonal directions within the material along which the deformation results only in changes in length of a line element of the material without any rotation of the element, other than that associated with any rigid body rotation

### 3.11

#### **principal stress**

stress acting on a surface element of material normal to a principal stress axis

### 3.12

#### **principal extension ratio**

$\lambda_i$

ratio of deformed to undeformed length of an element of material in the direction of one of the principal strain axes

### 3.13

#### **principal stress axis**

one of three orthogonal directions in the material with the property that the stress acting on a surface normal to it has the same direction

**3.14****stable**

<model> showing an increase in stress for an increase in extension

NOTE This guide applies only to elastic materials for which unstable models are not physically possible.

**3.15****stiffness**

ratio of force to deflection of a test piece or product

NOTE Dynamic stiffness is the ratio of the amplitude of the periodic force to that of an applied sinusoidal deformation.

**3.16****strain energy function**

mathematical expression for the strain energy per unit unstrained volume (strain energy density) of the hyperelastic material in terms of its deformation

**3.17****strain invariant**

$I_1, I_2, I_3$

function of the principal extension ratios which is independent of the choice of co-ordinate axes

NOTE The most commonly used are defined in Clause 4.

**4 Symbols**

The following symbols are used with the following definitions throughout this part of BS 903. Other symbols are defined in the clauses in which they appear.

$I_1$	is the first strain invariant, defined such that $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ ;
$I_2$	is the second strain invariant, defined such that $I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2$ ;
$I_3$	is the third strain invariant, defined such that $I_3 = \lambda_1^2\lambda_2^2\lambda_3^2$ ;
$W$	is the strain energy density, as defined in <b>3.16</b> ;
$\lambda_1, \lambda_2, \lambda_3$	are the three principal extension ratios.

**5 Introduction to FEA**

FEA has become a popular tool for design engineers and others. It provides a means of obtaining a computer simulation of the behaviour of a product from which useful quantitative predictions may be made. Some of the uses of FEA for products containing rubber are:

- to simulate the response of the product under a system of applied forces or deflections;
- to optimize the design (shape) of a component;
- to identify regions where failure can occur;
- to estimate the sealing pressure of a seal;
- to optimize the cure time and temperature of a bulky product.

The widespread use of FEA among engineers has been aided by the development of a number of commercial FEA software packages, which enable those without a specialist knowledge of mathematics or numerical analysis methods to carry out FEA. As well as the program which performs the analysis (the solver), many such software packages also contain programs which allow a visual simulation of the product on the computer screen (the pre- and post-processor). These are very useful for generating the correct input for the analysis (the FEA model) and for viewing the results, such as the deformed shape of the product, or the temperature distribution following a thermal analysis.

Relatively simple, so-called “linear”, FEA packages are available and are suitable for carrying out analyses where the strains are small, such as stress analysis of metals. Two difficulties make such packages unsuitable for analysing rubber products: they assume that the strains are very small, and that the material is compressible (Poisson’s ratio significantly less than 0.5). Instead, for mechanical analyses of rubber products, so called “non-linear” FEA packages are required. Non-linear packages are also required for most transient thermal analyses, such as simulating heat flow during vulcanization.

As with any computer simulation, the accuracy of the analysis is heavily dependent on the quality of the information supplied to the computer. The skill of the FEA practitioner lies in selecting the best analysis method, and also in using sensible models for describing the behaviour of the materials, with accurately measured values of the model parameters. Adequate models for some features of rubber behaviour are not yet provided with any existing commercial FEA package, notably for:

- a) dynamic properties of rubber (see **6.3**);
- b) the Mullins effect (see **6.3**);
- c) heat build-up (see Clause **10**).

The purpose of this guide is to give some background information on the material models available in FEA packages for modelling rubber, and to recommend suitable test methods for measuring the model parameters.

## 6 Stress–strain behaviour

### 6.1 Solid (incompressible) rubbers

#### 6.1.1 Introduction

Traditionally, rubber is modelled as a perfect hyperelastic material. This means that features such as set, hysteresis and strain softening are ignored. Commercial FEA packages are gradually introducing models which do take account of these features, but such models do not always provide a realistic description of the material behaviour, so should be used with caution. Further details are given in **6.3**.

#### 6.1.2 Hyperelastic models

Commercial FEA packages normally provide a choice of hyperelastic models expressed as strain energy functions, and the user is required to input one or more parameters of the function. The strain energy function is a mathematical expression for the amount of energy stored in the material, arising from work done in deforming it. In its most general form, it may be written as:

$$W = W(I_1, I_2, I_3) \quad (1a)$$

or

$$W = W(\lambda_1, \lambda_2, \lambda_3) \quad (1b)$$

where the symbols have the meanings given in Clause 4. Stress–strain equations may be obtained directly from equation (1) by differentiation and, conversely, experimental stress–strain curves may be used to fit the parameters of the strain energy function. The necessary equations are given in Annex A.

Many FEA packages also provide a curve-fitting procedure in which the user may enter tables of experimental stress–strain data and the program provides the best-fit values of the parameters for the chosen function (see **6.7**). Alternatively, the function need not have an explicit form, but values required to fit a particular experimental data set may be obtained numerically (see **6.1.4.7**).

#### 6.1.3 Choosing a hyperelastic model

**6.1.3.1** If a model has a large number of parameters which are fitted from a limited amount of test data it is possible that the model will be unstable and unrepresentative of any real material for strains or deformation modes outside those covered by the test data. Therefore the model with the fewest number of fitted parameters which is able to give a satisfactory description of the rubber behaviour should be used.

**6.1.3.2** Hyperelastic models do not account for the effects of set, hysteresis, strain softening and frequency. However, these features usually have a significant effect on the stress–strain behaviour of filled rubbers and thus experimental stress–strain curves used to fit the parameters of the hyperelastic model are dependent on these features. It is unnecessary and unwarranted to attempt to obtain a fit to the experimental data which is much better than the quality of the experimental data, bearing in mind its dependence on features which cannot be modelled. Furthermore, other imperfections and approximations in the FEA can also outweigh imperfections in the material model.



**6.1.3.3** For solid rubbers, the bulk modulus is a few thousand times larger than the shear modulus so it is normally reasonable to model the material as fully incompressible. Mathematically, incompressibility is defined by setting  $I_3 = 1$ . Hence, the strain energy function for an incompressible hyperelastic material is a function of only two variables and is normally expressed as:

$$W = W(I_1, I_2) \quad (2a)$$

or

$$W = W(\lambda_1, \lambda_2) \quad (2b)$$

In **6.1.4** all the models have been expressed in a form which neglects compressibility. However, most commercial FEA programs allow the user to enter a value for the bulk modulus, and take account of compressibility with an additional volumetric term in the strain energy function. Thus they assume that the volumetric deformation does not influence the deviatoric deformation. There is little experimental evidence to indicate the form which the volumetric term should take; in any case, for most analyses, the volumetric deformation will be very small compared to the deviatoric deformation so its exact form is unlikely to be critical. Exceptions are given in **6.5.1**. Most FEA programs consider cellular rubbers or rubbers with a cellular filler, such as cork, separately, and modelling these is discussed in **6.2**.

**6.1.3.4** It is often useful to assume that the strain energy density is a function of  $I_1$  only. Hence:

$$W = W(I_1) \quad (2c)$$

This assumption has been validated for filled natural rubber [1]. If this assumption is made at the outset, a considerable saving in the number of experimental tests required to characterize the rubber is possible since, in principle, a single test geometry is all that is required to fit the function  $W(I_1)$ , it not even being necessary to have a preconceived functional form with a fixed number of parameters. For further elaboration on this point, see **6.4.1**. However, further tests serve to validate the assumption for a particular rubber and provide a check of the accuracy of the test.

**6.1.3.5** Some models are designed to model particular features of the stress–strain curve such as the high modulus at low strains, observed in filled rubbers, or the high modulus at high strains due to finite extensibility of the long chain molecules. Not all models will give a good representation of all the features of the stress–strain behaviour of a particular rubber. The model and fitting procedure should be chosen to give a good representation of the behaviour in the strain range of interest.

## **6.1.4 Appraisal of incompressible hyperelastic models**

### **6.1.4.1 Introduction**

All the models listed in **6.1.4.2** to **6.1.4.6** are available in at least one commercial FEA package. The advantages and disadvantages of each model are discussed, so that the FEA practitioner can make a sensible choice for any specific application. Examples of fits of some of these models to an experimental data set, to demonstrate some of the issues discussed in **6.1.3** and **6.1.4** are given in Annex C.

### **6.1.4.2 Mooney-Rivlin models**

**6.1.4.2.1** These models are truncations of the infinite Taylor series expansion for a strain energy function given by [2]:

$$W = \sum_{i=j=0}^{\infty} C_{ij}(I_1 - 3)^i (I_2 - 3)^j \quad (3)$$

where

$W$ ,  $I_1$  and  $I_2$  are defined in Clause 4;

$C_{ij}$  are constants.

**6.1.4.2.2** The neo-Hookean model [3] is obtained from equation (3) by setting all coefficients other than  $C_{10}$  to zero. Hence, the strain energy function is given by:

$$W = C_{10}(I_1 - 3) \quad (4)$$

This model gives linear behaviour in simple shear with the one constant being equal to half the shear modulus. This model is recommended for general use in FEA, when precise details of the non-linear behaviour are unknown or can be neglected. The model may be derived from the statistical theory of rubber elasticity, giving  $C_{10} \approx NkT$  where  $N$  is the number of active chains in the network,  $k$  is Boltzmann's constant and  $T$  is absolute temperature, and thus has a molecular, rather than purely phenomenological, basis.

**6.1.4.2.3** The Mooney model [4] is given by:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (5)$$

This model was developed to provide a more accurate representation of the behaviour of unfilled rubber in uniaxial tension than the neo-Hookean model, which tends to predict too high a stress at moderate strains if fitted at small strain. However, the Mooney model gives a poorer representation of the behaviour in compression. Also, fits to a limited amount of experimental data sometimes yield unstable functions (see 6.1.3.1). Hence, the Mooney model is not normally recommended.

**6.1.4.2.4** The Yeoh model [5] is given by:

$$W = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3 \quad (6)$$

This model is capable of modelling some of the non-linear features of rubber. Normally  $C_{10}$  and  $C_{30}$  are positive and  $C_{20}$  is negative. Where this criterion is satisfied, the model will normally be stable but this is not assured. It is recommended when a more accurate representation of non-linear stress-strain behaviour than can be provided by the neo-Hookean model is required.

**6.1.4.2.5** Commercial FEA programs often allow use of functions with more terms of the infinite series. Usually, these functions contain terms in  $I_1$  and  $I_2$ . Due to the large number of fitting constants, it is generally possible to obtain a good fit to a particular set of experimental data. However, these functions are liable to give unstable representations of the behaviour for deformations outside the scope of the fit, unless an extensive range of data is used for the fit, and careful checks of stability are made. Furthermore, inelastic effects, such as hysteresis and stress softening, are often significant, so precise agreement with particular sets of experimental data is usually unwarranted. Hence, these models are not normally recommended.

#### 6.1.4.3 The Ogden model

The Ogden strain energy function [6] takes the form:

$$W = \sum_{n=1}^{\infty} \frac{\mu_n}{\alpha_n} \left\{ \lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + (\lambda_1 \lambda_2)^{-\alpha_n} - 3 \right\} \quad (7)$$

where

$\mu_n$  and  $\alpha_n$  are constants.

The coefficients,  $\mu_n$  and  $\alpha_n$ , need not be integers or positive. Stability can be assured if  $\mu_n \alpha_n > 0$  for all  $n$ . The Ogden function reduces to the neo-Hookean form if  $n = 1$  and  $\alpha_1 = 2$ . A better representation of the behaviour might be achieved with more terms, but care is needed to ensure that the behaviour is sensible for strains and deformation modes other than those used for the fit. Three terms are generally sufficient.

#### 6.1.4.4 Models that give a high stiffness at large strains

**6.1.4.4.1** The models in **6.1.4.4.2** and **6.1.4.4.3** are specifically formulated to give a very high stiffness at large strains, corresponding to the finite extensibility of the rubber molecules.

**6.1.4.4.2** The Gent strain energy function [7] takes the form:

$$W = -\frac{1}{2}G(I_m - 3)\ln\{1 - (I_1 - 3)/(I_m - 3)\} \quad (8)$$

where

$G$  is a constant (the shear modulus at small strain);

$I_m$  is a constant (the maximum possible value of  $I_1$ ). Typically  $I_m$  is about 100.

This model is devised to reduce to the neo-Hookean model at small strains. It provides a straightforward way, with only one extra constant, of modelling rubber at high strains, but it is important to ensure that  $I_m$  is large enough never to be exceeded in the analysis; otherwise the analysis will fail.

**6.1.4.4.3** The Arruda-Boyce model [8] is given by:

$$W = G \sum_{i=1}^n \frac{C_i}{\lambda_m^{2i-2}} (I_1^i - 3^i) \quad (9)$$

where

$G$  is a constant (the shear modulus at small strain);

$\lambda_m$  is a constant (the maximum possible extension ratio of a single molecule);

$C_i$  are the constants arising from a series expansion of an inverse Langevin function.

Thus,  $C_1 = 1/2$ ;  $C_2 = 1/20$ ;  $C_3 = 11/1050$ ;  $C_4 = 19/7000$ ;  $C_5 = 519/673750$ .

This model also reduces to the neo-Hookean model at small strain.

#### 6.1.4.5 Model derived from molecular considerations

The Kilian or Van der Waals model [9] is given by:

$$W = G \left\{ -(\lambda_m^2 - 3)[\ln(1 - \eta) + \eta] - \frac{2}{3}a \left( \frac{\tilde{I} - 3}{2} \right)^{\frac{3}{2}} \right\} \quad (10)$$

where

$$\tilde{I} = (1 - \beta)I_1 + \beta I_2 \quad (11)$$

$$\eta = \sqrt{(\tilde{I} - 3)/(\lambda_m^2 - 3)} \quad (12)$$

$G$  and  $\lambda_m$  have the definitions given in **6.1.4.4.3**

$a$  is the global interaction parameter which may be estimated from:

$$\alpha = \frac{2C_{01}}{3G} + \frac{\lambda_m^2}{\lambda_m^3 - 1} \quad (13)$$

where

$C_{01}$  is the second Mooney-Rivlin parameter;

$\beta$  is a linear mixture parameter, combining  $I_1$  and  $I_2$  into one parameter. Hence  $0 < \beta < 1$ .  
If  $\beta = 0$ , the model is a function of  $I_1$  only.

This model is an attempt to derive a strain energy function from molecular considerations. However, its rather complicated form and the relatively large number of parameters make it less convenient than some other models, so it is not recommended for general use.

#### 6.1.4.6 Marlow model [10]

This model does not assume an explicit form for the strain energy function, except that it is a function of  $I_1$  only. Instead, the strain energy density is calculated numerically from the data from one experimental test, interpolating between data points to give a complete description of the material. The strain energy density is assumed to be a function of  $I_1$  only in order to enable predictions in a general strain state to be made.

#### 6.1.4.7 User-defined models

Commercial FEA packages often allow the user to define a strain energy function by writing a user-subroutine. The user is required to provide expressions, in terms of  $I_1$ ,  $I_2$  and user-defined parameters, for  $W$  and its derivatives with respect to  $I_1$  and  $I_2$ .

#### 6.1.4.8 Summary

The best choice of model depends on the purpose of the analysis, the range of strains present in the component and the availability of experimental characterization data. For many purposes, the neo-Hookean model is sufficient. Only one parameter, the shear modulus, need be measured or estimated. An improved fit may generally be achieved with the Yeoh model. If the application involves large strains (greater than about 200 %) the Gent or Arruda-Boyce model are recommended. For these latter three models, a characterization test in a single mode of deformation is sufficient. The three-term Ogden model, and higher order Mooney-Rivlin type models require more characterization tests. They can give a precision comparable to, or perhaps better than, the Yeoh or Gent models, but only if the fitting is carried out with care to avoid instabilities; for this reason they are not recommended.

## 6.2 Cellular rubbers

Rubbers with a small volume fraction of voids can be modelled with the hyperelastic models given in 6.1.4, with a suitable value for the bulk modulus. In addition, special foam models are provided in some FEA programs for modelling isotropic, hyperelastic highly compressible materials. An inherent difficulty in the use of an isotropic model for significantly compressible cellular rubbers is that their behaviour in tension differs markedly from their behaviour in compression. In tension the stiffness rises with increasing strain. In compression, the initial stiffness, at less than about 5 % strain, is the same as in tension, thereafter the cells buckle, giving a plateau where there is little increase in stress with increasing strains. At high compressive strains, corresponding to collapsed cells, the stiffness rises steeply. This means that it is essential that experimental characterization tests are carried out in similar deformations to those applicable in the analysis; for example, if compression dominates the material should be characterized in compression. Both deviatoric tests (see 6.4) and dilatational tests (see 6.5) are required to characterize the material.

## 6.3 Limitations of hyperelastic models and alternative modelling techniques

### 6.3.1 General

The traditional assumption that rubber behaves as a perfectly elastic material which may be described by a hyperelastic material model ignores many features of the real behaviour of typical rubbers. The most significant departures from hyperelastic behaviour are:

- a) hysteresis. There is a loss of energy on loading and unloading so that the loading and unloading curves do not follow the same path; this also gives rise to damping or loss factor. For some unfilled rubbers damping is related to creep, a gradual increase in deformation under a load, stress relaxation, a gradual decrease in force under a constant deformation, and recoverable set through viscoelastic theory. For filled rubbers the interrelationships are not simple and the additional departures from elastic behaviour described in b), c) and d) are more evident.
- b) the Mullins effect [11, 12]. The rubber is softer on second and subsequent loadings at strains less than the previous maximum. If strained above the previous maximum there is a marked stiffening near the previous maximum strain as the stress approaches its level for the unstrained material. Most softening occurs on the first cycle but there is some further softening on subsequent cycles. There is usually some slow recovery from this effect but, for the time scales of many applications, it may be considered to be largely irreversible.
- c) permanent set. After deforming and relaxing the rubber does not return to its original dimensions but retains some permanent deformation. For some rubbers, cyclic loading increases the creep above the level expected from tests under constant load; this phenomenon is also akin to set.
- d) the Payne or Fletcher-Gent effect [13, 14]. The dynamic stiffness of rubber tends to fall as the strain amplitude is increased, especially if the rubber incorporates reinforcing filler or is a block copolymer. The high stiffness at small amplitude recovers quite rapidly after subjection of the material to a higher strain amplitude. An associated effect is the relatively high stiffness of a small "dynamic" strain superimposed on a larger "static" strain.

Models which aim to take account of one or more of these features are under development. Some have been implemented in some commercial packages but not all are adequate for the purpose for which they are designed. Furthermore, some FEA packages allow the user to define and implement a model using a user-subroutine, though such an exercise is not trivial. At the moment, it is often better to use a hyperelastic material model, with the rubber characterized under the most appropriate conditions (see 6.4.1, 6.4.2, and 6.4.3). Exceptions are applications involving slow relaxation of forces under a static deformation, such as relaxation in seals, and applications involving damping using viscoelastic materials. For these applications the viscoelastic model described in 6.3.2 is recommended.

### 6.3.2 Large strain viscoelasticity

A model which is based on linear viscoelasticity theory, modified to large strains, is implemented in some FEA programs. It is able to model stress relaxation and the frequency dependence of the dynamic modulus. It successfully models only those departures from elasticity given in 6.3.1 a). Experimental data, giving the dependence of modulus with time, from a stress relaxation experiment are usually required. Appropriate test methods are given in 6.6.2. Alternatively, some FEA programs can formulate the model from dynamic properties, such as from a table giving values of storage modulus and loss factor as a function of frequency. Suitable test methods are described in 6.6.3.

## 6.4 Recommended deviatoric tests for obtaining parameters for hyperelastic models

### 6.4.1 Introduction

Useful tests are uniaxial tension, uniaxial compression, equibiaxial tension, pure (planar) shear and simple shear. These tests are described in 6.4.4.

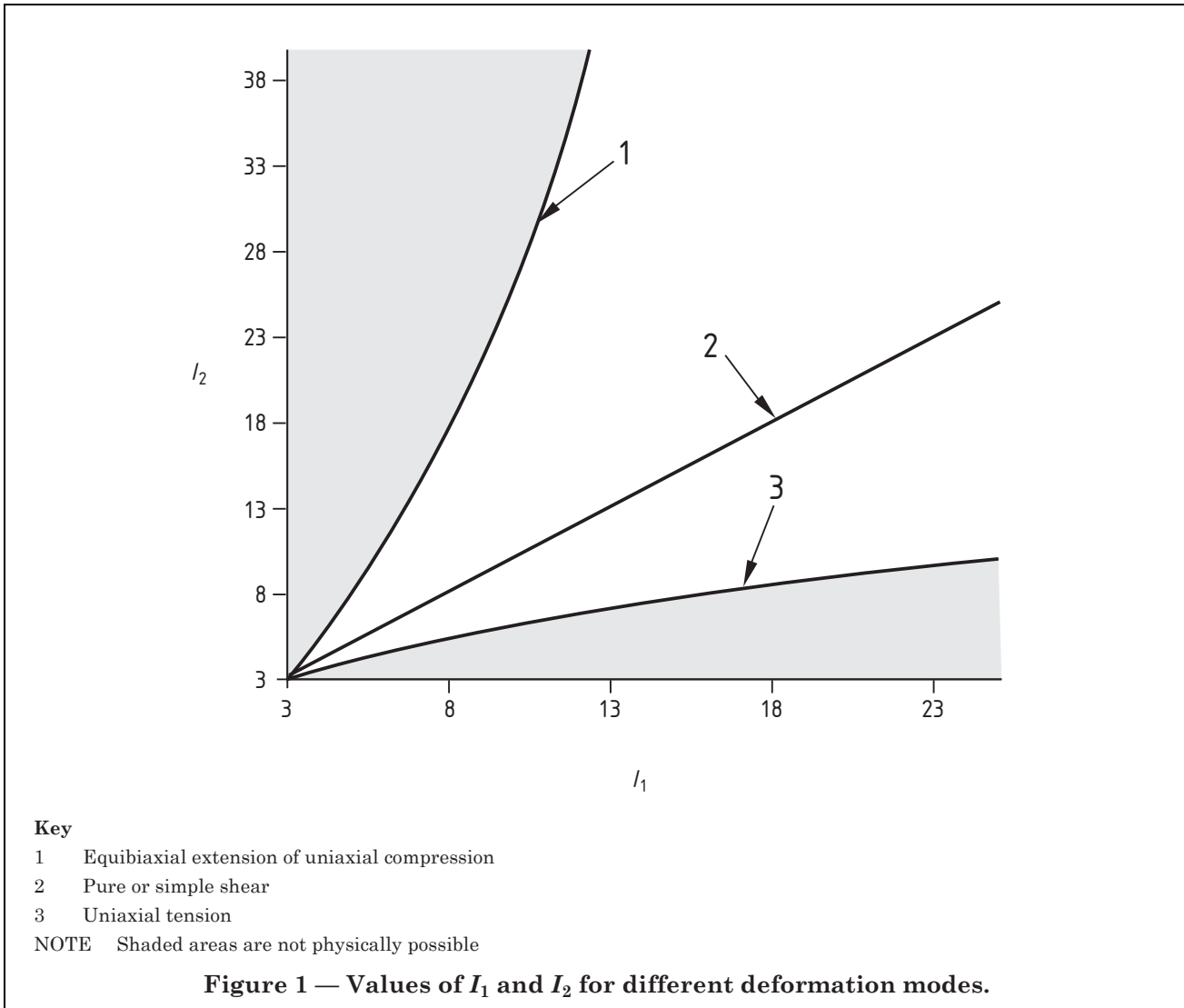


Figure 1 shows the values of  $I_1$  and  $I_2$  which are covered by the tests. A comprehensive test schedule would require a fully biaxial test rig, allowing different stretches in two perpendicular directions on a sheet of rubber, in order to map the whole of the  $I_1$   $I_2$  space, (the area shown unshaded on Figure 1). Ensuring compatibility of strain history and rate effects would be very difficult. Alternatively, a simplifying assumption could be made, such as that  $W$  is a function of  $I_1$  only, or that  $W$  is a preconceived function of  $I_1$  and  $I_2$ , requiring only a specified number of parameters to be determined. If either assumption is made, a fit may be achieved from a test in a single mode of deformation. However, choosing  $W$  as a preconceived function of  $I_1$  and  $I_2$ , and fitting from a test in a single mode of deformation is very dangerous because the fit may be apparently improved by independently adjusting the parameters of both the  $I_1$  and  $I_2$  terms. This effectively increases the number of adjustable parameters and can result in very unrealistic predictions for other modes of deformation, although the use of a stability check should avoid such extreme behaviour.

This problem can only be avoided by carrying out additional tests in other modes of deformation to ensure that the fit gives a realistic dependence for both  $I_1$  and  $I_2$ . Fortunately, it appears that the first option,  $W$  as a function of  $I_1$  only, is often satisfactory, and then testing in a single mode of deformation is adequate. In any case, assuming  $W$  is a function of  $I_1$  only is robust and will at worst impair the fit rather than lead to very unrealistic predictions in other modes of deformation. Nevertheless, further tests would serve to

validate the assumption that  $W$  is a function of  $I_1$  only, provide a check of the experimental accuracy of the tests and enable a greater range of strains to be covered than is possible from a single test. If tests in more than one deformation mode are to be carried out, it is sensible to choose tests which cover as much of the  $I_1, I_2$  space of Figure 1 as possible. To this end, the following points should be taken into account. For an incompressible material:

- a) equibiaxial tension and uniaxial compression are equivalent deformations. No additional information, except for checks of the accuracy of the test methods, and the applicability of an incompressible hyperelastic model, may be obtained from carrying out both tests;
- b) pure (planar) and simple shear are equivalent.

Any two or three tests, which correspond to different lines on Figure 1, provide a reasonably comprehensive evaluation of the  $I_1$  and  $I_2$  dependence of  $W$ .

Another consideration when choosing test methods is their experimental simplicity and the range of strain which may be covered. Uniaxial tension, pure shear and simple shear are all straightforward to carry out, and require no sophisticated equipment except a test machine, nor exceptionally precise measurement, although simple shear test pieces require moulding and bonding to end-pieces. These test methods are recommended for providing accurate data with the minimum effort or expense. Uniaxial compression is slightly more difficult to carry out accurately since it requires smooth, parallel test platens, effective lubrication of the test piece and careful measurement of deflection. Moreover, it is not possible to reach high strains. The alternative, an equibiaxial tension test, is also difficult. One technique, equibiaxial straining of a flat sheet (see 6.4.4.4.2), requires a relatively elaborate test jig. The other, inflation of a sheet (see 6.4.4.4.3), also requires some special equipment and demands precise measurements on curved surfaces to be made, thus is liable to be the most inaccurate of the tests at modest strains. However, it is possible to reach very high extensions by this method and thus it is a useful technique for modelling applications involving very large strains.

Test conditions should be chosen to reflect as far as possible the conditions of the product to be modelled. Thus the test speed, test temperature and strain range should, if possible, be comparable to those experienced by the product. The effect of strain rate on stress is normally quite small; a tenfold increase in strain rate typically gives less than 10 % increase in modulus. In respect of strain range it is important to bear in mind that some parts of the product might experience local strains that are much larger than the overall strain, thus the maximum strains in the characterization tests should be large enough to reflect this.

#### 6.4.2 Methods for standardizing strain history effects

Because rubber, especially filled rubber, is not a perfect hyperelastic material, the stress–strain data will be dependent on strain history. Further information on strain history effects such as the Mullins effect are given in 6.3.1. Methods of carrying out characterization tests are recommended as follows.

- a) Use only data from the first loading of a previously unstrained test piece. This technique does not address the phenomenon of softening due to previous straining, and thus will lead to models which overestimate the stiffness of products that have been pre-strained. However, it avoids confusion or error due to an inappropriate prestraining procedure, and is very simple to define and carry out.
- b) Stretch the test piece one or more times to a predetermined strain, then use the data from the next loading. This technique allows for stress-softening. However, the magnitude of the prestrain affects the stress–strain behaviour in subsequent loadings. The time between prestraining and testing has some, though usually a small, effect on the stress–strain behaviour. A single prestrain is representative only of a product in a largely uniform state of strain. Normally, some parts of the product are subjected to higher strains than other parts and thus it is preferable for the prestrain to depend on the strain.
- c) Stretch the test piece a fixed number of times to successively increasing strains, and use only the values at the peak of the final stretch at each strain. A typical test might be to stretch the test piece three times to 5 %, 10 %, 20 %, 50 %, 100 %, 150 %, 200 %, 250 % and 300 % strain. This technique allows for strain softening in a way which takes account of non-uniform strains in a product. Its disadvantages are that a relatively elaborate test procedure is needed and only a few data points are generated, so that the fit to a material model might be rather crude. Also, the stress–strain curve is a composition of many curves, rather than one obtainable directly from the rubber.

Method a) is recommended for applications in which the product experiences occasional deformations of varying magnitude. Method c) is recommended for applications in which the product is subjected to regular cyclic deformations.

If methods b) or c) are chosen, there is a further complication. After each stretch, the test piece will not return to its original length but will retain some permanent set. The strains and stresses in subsequent stretches may be determined by:

- i) using the original test piece dimensions;
- ii) by redefining the length and cross-sectional area using the new unstrained length.

NOTE If option ii) is chosen it is applied as a retrospective correction, to the test data, rather than for specifying the test deflections. It is only possible to specify the test deflections after subtracting the permanent set if a time-lapse between cycles is incorporated in the test method.

Option ii) makes the data handling more cumbersome, but is more realistic so is recommended for materials with a large permanent set used in circumstances where some prestraining is likely to have taken place.

### 6.4.3 Ensuring consistency between tests in different modes of deformation

If data from tests in more than one mode of deformation are used to fit a hyperelastic model, the following issues should be considered.

- a) The magnitude of any prestrains or maximum strains specified in the test procedure should be consistent between different tests. The recommended procedure is to strain to the same value of  $I_1$ . Characterization using prestrains cannot be carried out in a logically consistent way in more than one deformation mode if  $W$  is taken as being dependent on  $I_2$  as well as  $I_1$ .
- b) The strain rates in the different tests should be comparable. The recommended procedure is to strain to a set value of  $I_1$  (say equivalent to 100 % uniaxial tensile strain) in equal times. This avoids the unnecessary elaboration of variable rate tests.
- c) Samples prepared from the same rubber formulation can differ if the processing and vulcanization are not identical. If possible, the test pieces for the different tests should be cut from the same rubber sheet. Otherwise, care should be taken to ensure that all test pieces have been vulcanized to the same level, and that flow during processing is similar.

### 6.4.4 Deviatoric test methods

#### 6.4.4.1 General guidelines

For all the tests detailed in this sub-clause preparation and conditioning of the rubber from which the test piece is cut should be in accordance with BS ISO 23529. The test speed and maximum strain(s) should be selected in accordance with the advice and recommendations of 6.4.1, 6.4.2 and 6.4.3. The test machine should be capable of operating at these speeds and accommodating these deflections, and thus its specification may differ from those given in BS 903-A2 (for tension), A4 (for compression) and A14 (for shear).

In all cases the force-deflection curve(s) required for the fit to a hyperelastic model should be recorded in graphical or digital form. Since the data are normally required in digital form for the curve-fitting procedure, a digital recording system is preferred. If method a) or b) of 6.4.2 is used the resolution of the digital recording device and test machine should be sufficient for the force and deflection at one-tenth of the maximum strain to be recorded to a precision of  $\pm 2$  % and the capture rate sufficient to capture at least 10 evenly spaced points. If method c) of 6.4.2 is used, the resolution should be sufficient for the force and deflection at the first maximum to be recorded to a precision of  $\pm 2$  %. The capture rate should be sufficient to ensure that the point captured at the first maximum is within 1 % of the maximum, and is on the loading and not the unloading curve.

#### 6.4.4.2 Uniaxial tension

The principal extension ratios are  $\lambda$ ,  $1/\sqrt{\lambda}$  and  $1/\sqrt{\lambda}$  where  $\lambda$  is greater than one. BS 903-A2 provides a suitable test method.

NOTE BS 903-A2 provides procedures for calculating tensile strength, elongation at break and the stress at a given elongation. These values are not required for FEA characterization and thus may be ignored. Only the force-deflection curve need be recorded.

For characterization up to stresses lower than the tensile strength, a parallel sided strip may be used instead of the dumbbell or ring test pieces specified in BS 903-A2. The length of the strip should be at least ten times its width. Its extension may be obtained directly from the cross-head movement of the test machine. Suggested dimensions of the strip are 2 mm  $\times$  5 mm  $\times$  150 mm. These proportions meet the requirement for a large length to width ratio, which ensures that the rubber is effectively in uniaxial extension, and a length large enough to make machine compliance and imprecision of clamping points



insignificant. Because of its long length, this test piece is better than the dumbbell test piece for small or moderate (less than 100 %) strain measurements which are often the ones of interest.

#### 6.4.4.3 Uniaxial compression

The principal extension ratios are  $\lambda$ ,  $1/\sqrt{\lambda}$  and  $1/\sqrt{\lambda}$  where  $\lambda$  is less than one. Redefining the extension ratios in terms of the extension ratio in the direction of the “bulge”, i.e.  $\lambda^* = 1/\sqrt{\lambda}$  gives principal extension ratios of  $1/\lambda^{*2}$ ,  $\lambda^*$  and  $\lambda^*$  where  $\lambda^* > 1$ . Thus uniaxial compression gives a deformation which is equivalent to equibiaxial tension (see 6.4.4.4). BS 903-A4 provides a suitable test method.

NOTE Only method A of BS 903-A4 (compression between lubricated plates) is suitable; compression between bonded plates gives a non-uniform strain state in the test piece which is not suitable for fitting a hyperelastic model.

If the test piece is properly lubricated, it might be ejected when strained. To prevent this, it should be constrained by a small pin, about 1 mm long, projecting from the centre of one of the plates. Good lubrication and accurate measurement of the compressive strain are important.

#### 6.4.4.4 Equibiaxial tension

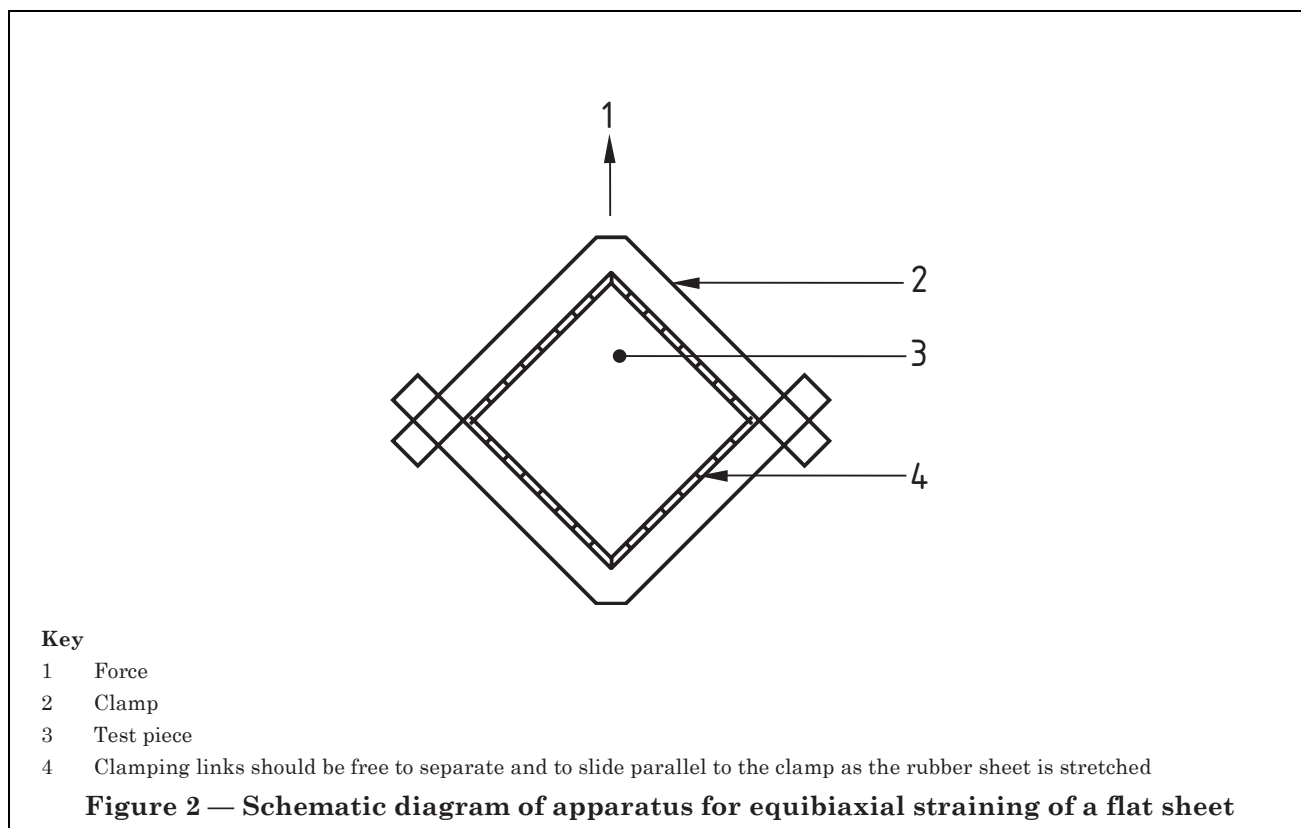
##### 6.4.4.4.1 General

The principal extension ratios are  $\lambda$ ,  $\lambda$  and  $1/\lambda^2$  where  $\lambda$  is greater than one. Suitable test methods are:

- a) equibiaxial straining of a flat sheet (6.4.4.4.2);
- b) inflation of a sheet (6.4.4.4.3).

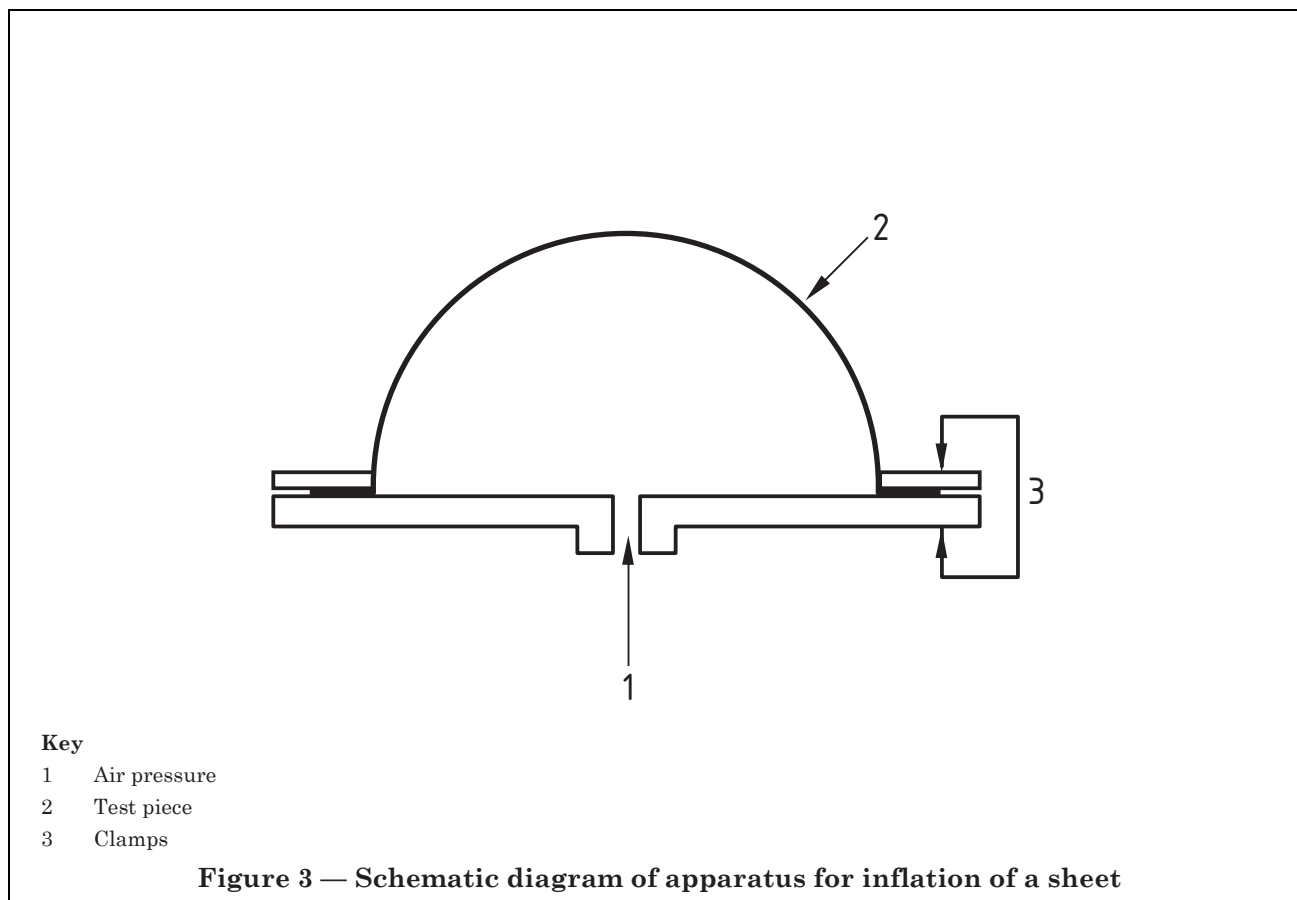
##### 6.4.4.4.2 Equibiaxial straining of a flat sheet

A square rubber sheet is stretched equally in two perpendicular directions using a special rig. A typical design of rig is shown in Figure 2.



The rig should be constructed such that the force measurements are not distorted by friction between sliding members of the rig. Since clamping causes distortion of the edges of the rubber sheet, extensions should be measured away from the sheet edge, for example with an optical following device measuring a grid drawn on the unstrained test piece.

## 6.4.4.4.3 Inflation of a sheet



A flat rubber sheet is inflated through a circular clamp as shown in Figure 3. Near the apex of the sheet the rubber is in a state of equibiaxial strain. Only points within about  $10^\circ$  of the apex conform closely to the equibiaxial strain state, so measurements should be taken within this region.

Dimensional measurements are needed to calculate the extension ratio and the radius of curvature at the apex from which the nominal stress is determined from the equation:

$$s = Pr/(2t)$$

where

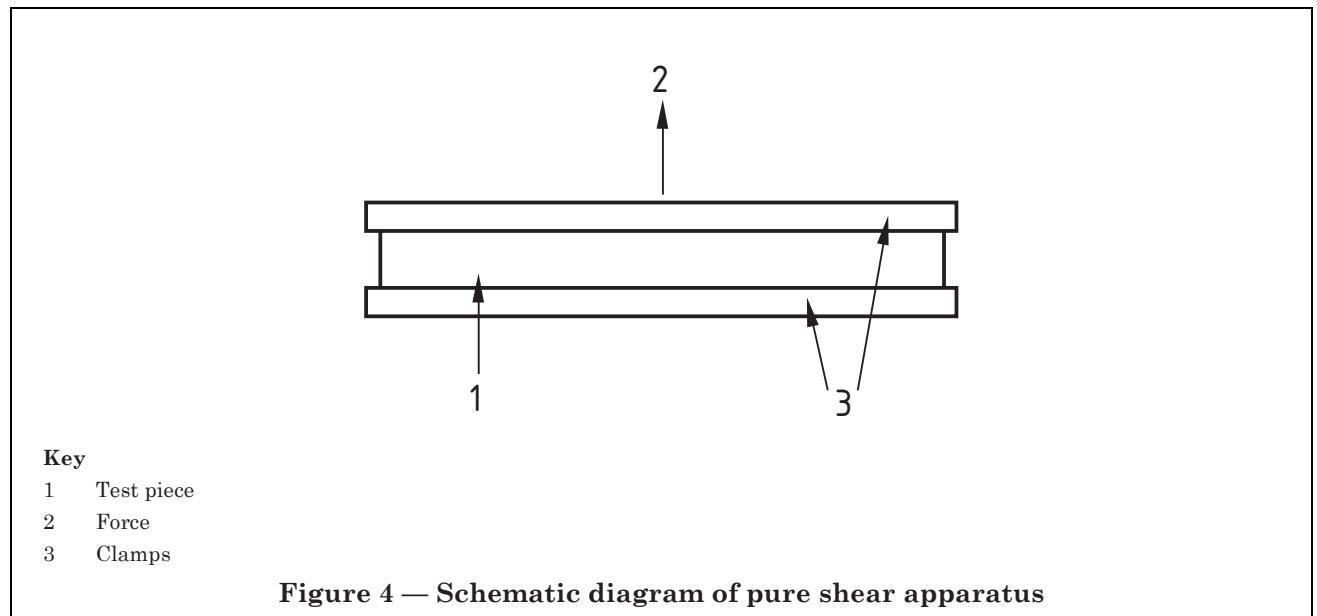
- $s$  is the nominal stress;
- $P$  is the inflation pressure;
- $r$  is the radius of curvature of the sheet at the apex;
- $t$  is the initial thickness of the sheet.

An optical method is generally used to obtain the measurements. For example,  $r$  could be determined from photographs of the inflated sheet viewed from the side, and  $\lambda$  from photographs of a plane view of the apex of the sheet, on which a suitable grid has been drawn, using trigonometric corrections if necessary.

The pressure of air or other inflating fluid should be measured with an accurate pressure gauge. If a dense fluid such as water is used there will be a distortion of the inflated shape due to the weight of the fluid. The pressure–extension curve for inflation of a sheet is highly non-linear, and, at moderate extensions, a very

large increase in extension occurs with very little change in pressure. This means that in order to obtain measurements at equally spaced extensions, control of the fluid flow, as well as the pressure, is required.

#### 6.4.4.5 Pure (planar) shear



The pure shear test piece is a strip of rubber, clamped along its length and strained perpendicular to its length (Figure 4). The principal extension ratios are  $\lambda$ ,  $1/\lambda$  and 1. The height of the strip in the straining direction should be no more than one-fifth of its longest dimension. Recommended dimensions (excluding the rubber inside the clamps) are 20 mm × 2 mm × 200 mm. The thickness of the test piece should be measured with a suitable gauge to an accuracy of  $\pm 0.01$  mm. Five measurements should be taken over the width of the test piece and the mean used in the calculation of cross-sectional area. The longest dimension and unstrained height should be measured to an accuracy of  $\pm 0.5$  %.

#### 6.4.4.6 Simple shear

A suitable test method is given in BS 903-A14. Other test pieces such as the double bonded shear test piece (BS ISO 4664) are also suitable. A circular double bonded shear test piece may be strained in any direction, including symmetrically about the unstrained state. In order to avoid significant bending of the rubber elements in double or quadruple shear test pieces, the thickness of each rubber element should not exceed one-quarter of its width or diameter.

The curve fitting procedure in at least one commercial FEA package does not accommodate simple shear test data. However, since simple shear and pure shear are equivalent deformations (see Figure 1), it is possible to convert simple shear data into an equivalent pure shear form, which is accepted by the curve fitting procedure. A method for doing this is given in Annex B.

#### 6.4.5 Summary

From a theoretical standpoint all the tests are equally acceptable. The number of tests required depends primarily on the choice of strain energy function (see 6.1.3 and 6.4.1). The test conditions, such as strain range, depend primarily on the application. For fitting to robust models, such as functions of  $I_1$  only, a single test is sufficient and the simpler ones, such as uniaxial tension, are recommended unless the application involves very large strains, in which case an additional test in equibiaxial extension is desirable.

### 6.5 Recommended test methods for obtaining a value for the bulk modulus

#### 6.5.1 Introduction

Since the bulk modulus of a solid rubber is a few thousand times its shear modulus, normally the deformation is overwhelmingly deviatoric (change in shape) rather than dilatational (change in volume). Thus, in most cases, it is reasonable to assume that the rubber is incompressible, or to assume a large value

of 2 000 MPa to 3 000 MPa for the bulk modulus. In a few circumstances, volumetric compression is important and the bulk modulus should be measured experimentally. These circumstances are:

- a) when the rubber is significantly prevented from bulging, such as when enclosed in a recess, or when thin layers are bonded to rigid plates;
- b) when the rubber is cellular or contains a compressible filler such as cork.

Because of the relatively high coefficient of thermal expansion of rubber, there is a significant difference (around 15 %) between the bulk modulus of solid rubber under isothermal and adiabatic conditions; their relationship can be derived from thermodynamics:

$$\frac{1}{B_a} = \frac{1}{B_i} - \frac{\theta V_o \alpha^2}{C} \quad (14)$$

where

$B_i$  is the bulk modulus under isothermal conditions;

$B_a$  is the bulk modulus under adiabatic conditions;

$\theta$  is the absolute temperature;

$V_o$  is the volume of unit mass under standard conditions of pressure and temperature;

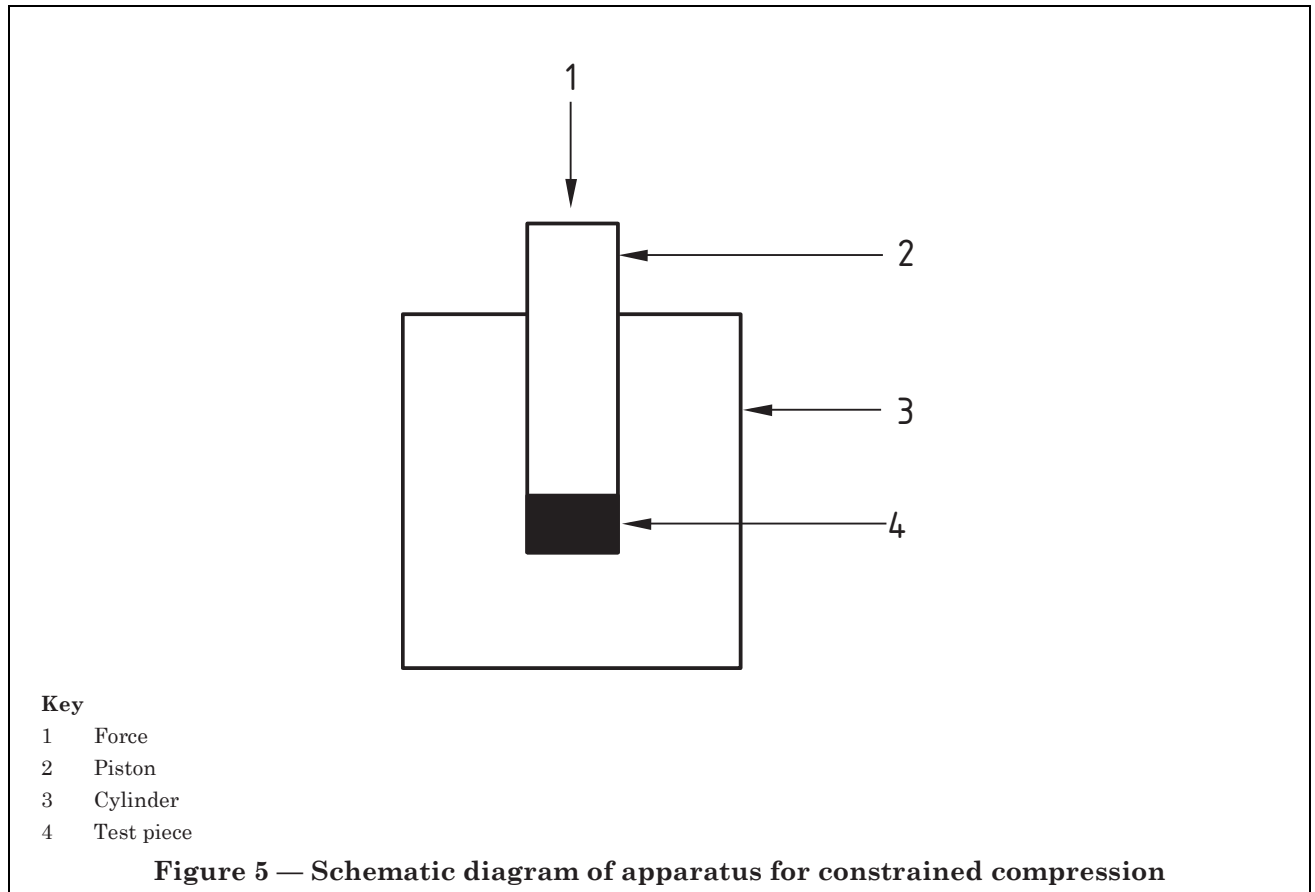
$\alpha$  is the volumetric coefficient of thermal expansion;

$C$  is the specific heat capacity.

The distinction between  $B_a$  and  $B_i$  must be borne in mind when choosing the test piece size and the rate of volumetric compression. If compression is achieved in a time significantly shorter than  $\kappa l^2$  where  $\kappa$  is the thermal diffusivity, defined by equation (37), and  $l$  is the effective radius of the test piece, then conditions will effectively be adiabatic.

## 6.5.2 Dilatational test methods

### 6.5.2.1 Constrained compression



A cylinder of rubber, which is prevented from bulging, is compressed. A suitable apparatus is shown in Figure 5. This test method is most suitable for almost incompressible materials, such as solid rubbers. The test piece should be lubricated with a suitable lubricant, which does not swell the rubber, such as detergent or low viscosity silicone oil. The deflection of the piston should be measured directly with a linear variable differential transformer (LVDT) or similar device; it is not advisable to rely on the cross-head displacement, as the machine compliance might be significant. It is sufficient for solid or nearly solid rubbers to assume a constant value for the bulk modulus, which should be obtained from the slope of the force–deflection curve from the equation:

$$B = \frac{kl}{A} \quad (15)$$

where

$B$  is the bulk modulus;

$k$  is the tangent stiffness obtained from the slope of force–deflection curve. The initial slope of the graph might be smaller, due to “lead-in” associated with imperfections of the fit of the test piece in the apparatus. Thereafter, the graph will normally be approximately linear, and the tangent stiffness may be measured at any suitable strain, such as 5 %;

$l$  is the height of the deformed test piece at the deflection at which  $k$  is measured;

$A$  is the cross-sectional area of the test piece, which should be taken as the cross-sectional area of the cylindrical cavity in which the rubber is constrained.

If the material is significantly compressible, equation (15) is not sufficiently accurate because it ignores the deviatoric component of the deformation in the confined compression test, so the method given in 6.5.2.2 should be used instead.

#### 6.5.2.2 *Compression in a fluid*

Materials which are significantly compressible, such as foam rubbers, may be compressed by immersing a test piece in a pressurizable container of a fluid which does not swell or interact with the rubber. The fluid pressure is increased and the volumetric compression is equated with the volume of fluid pumped into the container. Thus a means of measuring both the pressure and flow of the fluid is required. If the fluid is liable to penetrate the rubber, such as in the case of an open cell foam, it might be necessary to enclose the test piece in a bag which should allow air to escape without requiring appreciable force. This technique is recommended for materials which are much more compressible than the fluid; if this condition is not satisfied the compression should be corrected to allow for the compression of the fluid.

The bulk modulus is given by the slope of a plot of pressure against change in volume:

$$B = P/\Delta V \quad (16)$$

where

$B$  is the bulk modulus;

$P$  is the hydrostatic pressure;

$\Delta V$  is the change in volume.

This technique is recommended for measuring the dilatational behaviour of significantly compressible cellular rubbers for use with foam models. In this case, it might be preferable to fit the parameters using the curve-fitting program provided by the FEA package for which a curve of  $P$  against  $\Delta V$  is required, rather than a single value of bulk modulus.

## 6.6 Recommended tests for obtaining parameters for large strain viscoelastic models

### 6.6.1 *General guidelines*

The choice of test method depends mainly on the application. Stress relaxation experiments cover longer time-scales than dynamic tests. Hence, stress relaxation data are appropriate for modelling events which take place over long times, such as stress relaxation of seals, whereas dynamic tests are more appropriate for modelling products subjected to vibrations.

### 6.6.2 *Stress relaxation test method*

A test piece is held at constant extension and the change in force with time is recorded. Usually the force will decay rapidly at first, and then more slowly, so it should be recorded more frequently near the beginning of the test. The timescale and temperature of the test should reflect that of the application. Any of the test pieces and test geometries described in 6.4.4.2 to 6.4.4.6 are suitable in principle, but the simpler geometries, such as uniaxial tension and simple shear, are recommended. Stress relaxation in compression is covered by BS 903-A42 Method A. The stiffness of rubber depends on temperature; for this reason it is important that the test temperature is controlled to within  $\pm 1$  °C.

### 6.6.3 *Dynamic test method*

Suitable test methods, and definitions of dynamic properties are given in BS ISO 4664. Forced vibration test methods are recommended. The test temperatures, frequencies and amplitudes should be chosen to reflect the service conditions of the product.

## 6.7 Fitting procedures

Some FEA programs provide a curve fitting procedure which calculates the constants for a hyperelastic model directly from stress–strain data. The types of data which they accept are all or some of uniaxial tension, uniaxial compression, biaxial extension, pure (planar) shear, simple shear and volumetric compression. In the absence of such a curve-fitting procedure, a least-squares fit to the experimental data may be carried out, as explained in Annex A.

Most curve-fitting programs give equal weight to each data point. Thus roughly equal numbers of data points from each test should be used to ensure that the fitting procedure does not favour the results of one test over those of another.

Strain energy density functions with a large number of parameters, and functions of both  $I_1$  and  $I_2$ , rather than  $I_1$  only, might predict unstable behaviour for strains or deformations outside those covered by the test data. Some FEA programs have a stability check which should be used. If such a check is not provided, a visual check of stability should be made by plotting the stress–strain predictions of the model for different deformations, including equibiaxial extension, up to high strains. If the slope of the plots becomes negative at high strains, then the function is unstable.

## 7 Mechanical failure

### 7.1 Introduction

FEA aimed at prediction of mechanical failure of rubber requires no input from testing other than the stress–strain model (Clause 6), and for preliminary design work a neo-Hookean model is recommended [equation (4)]. However the results from the FEA must be set in a wider context of the failure characteristics of the material, and this is the subject of this clause.

FEA for materials other than rubber, such as steel, is often synonymous with stress analysis, because the only objective is to quantify the maximum stress values, and, through modification of the design, to reduce them to levels low enough to ensure adequate fatigue life according to an S–N curve for the material (e.g. BS 7608). Stress analysis is not an appropriate term for FEA of rubber products because there are usually additional objectives to prediction of mechanical failure (see Clause 5), and stress values are not an adequate guide to quantitative prediction of mechanical failure.

Hysteresis can result in heat build-up in rubber products undergoing arduous mechanical working, and is addressed separately in Clause 10. Mechanical failure of rubber, in the absence of such heat build-up, is believed to result from the growth of pre-existing internal flaws, occurring at a rate that depends on the amount of energy available to generate the fracture surfaces. This is known as the fracture mechanics, or energetics, approach. Except in some especially simple cases, the rate of crack growth cannot be directly related to field variables such as strain energy density, maximum principal stress, or maximum principal strain in the locality of the initiating flaw, so S–N or Wöhler curves generated from simple material fatigue experiments (see 7.4) often cannot be used to make reliable quantitative predictions of fatigue life of products. The field variables are, however, a useful output from FEA for making qualitative improvements in the design from the point of view of resistance to mechanical failure. For example, in a well-designed product the strain energy density should be as uniform as possible, while the maximum (tensile) value of principal stress gives a good guide to the location and orientation (normal to the maximum tensile stress) in which a crack can first appear.

According to the energetics approach, an FEA based analysis of mechanical failure will require the stress–strain model for the rubber and two additional inputs from experiment – the size of initiating flaws or cracks and the crack growth rate as a function of the energy available for fracture. The objective of the FEA would be to calculate the latter, and is briefly discussed in 7.5. Before discussing experimental characterization of the material through crack growth (7.3) and fatigue (7.4) tests, the foundations of the energetics approach will be outlined in 7.2.

This clause does not cover cellular rubbers, since very little literature exists on their mechanical strength. Nevertheless, it appears that the energetics approach, together with parameters describing the microstructure, may be used to relate the mechanical strength of a cellular rubber to that of the solid rubber from which it is formed [15]. In particular, the largest pores serve as flaws, so that materials with a uniform pore distribution, and no abnormally large pores, are anticipated to have the highest tensile strength.

### 7.2 The energetics, or fracture-mechanics approach

#### 7.2.1 Criterion for mechanical failure

The basic assumption is that the energy needed to increase a crack surface area by  $\delta A$  is  $T\delta A$ , where  $T$  is a characteristic of the material and the particular conditions such as temperature, rate and chemical environment, but is independent of the geometric detail. The symbol  $T$  was chosen to signify “tearing energy” since the first application of the approach was to establish a criterion for tearing of rubber [16], and  $\delta A$  is taken to be the area of just one of the two new surfaces generated by the incremental fracture.

For a perfectly elastic material, deformed with fixed displacements at the boundaries during the crack growth  $\delta A$  so that the external forces do no work in the process, it is clear that  $T$  is supplied by the “strain energy release rate”  $(\partial U/\partial A)_l$  where  $U$  is the total strain energy, and the partial differentiation (with fixed length  $l$ ) signifies that the derivative is evaluated with fixed boundary displacements. Thus the basic equation expressing the balance of energy is:

$$T = \left( \frac{\partial U}{\partial A} \right)_l \quad (17)$$

For convenience,  $T$  is usually defined to be the right-hand side of equation (17), which may be referred to as the energy available for tearing. For stable crack growth this is equal to the energy required for tearing, but the material response is in general more usefully thought of as the rate of crack growth corresponding to the energy,  $T$ , available for tearing. Evaluation of the right-hand side of equation (17) for a particular geometry involving finite strains is not usually tractable without resort to numerical methods such as FEA. The energetics approach has also been adopted for materials such as metals for which classical, linear elasticity is applicable. For such materials the “stress intensity factor” for a crack in a particular stress field or geometry is often used rather than the strain energy release rate as the driving force for fracture. The two quantities are interrelated for through-cracks in plates (or sheets) by:

$$\left( \frac{\partial U}{\partial A} \right)_l = \frac{K^2}{E'} \quad (18)$$

where

$K$  is the stress intensity factor;

$E'$  is the extensional modulus.  $E' = E$  for plane stress, and  $E' = E/(1 - \nu^2)$  for plain strain where  $E$  is Young's modulus and  $\nu$  is Poisson's ratio.

Thus, at small strains, the values of strain energy release rates for some rubber test pieces can be calculated from tabulated values for stress intensity factors for other materials. However, the high extensibility of rubber causes departures from such formulae at the usual working strains. High extensibility has the consequence that no distinction is made between different cracking modes in rubber, and that experimental results are much less sensitive to the perfection of boundary conditions at the clamped ends of the test pieces than is the case for more rigid materials.

### 7.2.2 Material characteristics

The energy,  $T$ , required to propagate a crack is related to the hysteresis of the material, as it is strained cyclically up to very high strains around the crack-tip and relaxed as the crack moves on.

It is found that, in the absence of special effects such as ozone, cracks grow only if the energy available for tearing,  $T$ , exceeds a threshold value  $T_0$ . The value of  $T_0$  is about  $0.05 \text{ kJ} \cdot \text{m}^{-2}$  for typical unfilled rubbers.

For  $T > T_0$ , the crack growth rate depends on  $T$ :

$$\frac{dc}{dt} = f(T) \quad (19)$$

where

$c$  is the crack length;

$t$  is time.

For a critical value  $T_c$  of  $T$  the crack growth rate becomes very large, and catastrophic tearing occurs as soon as  $T$  reaches this value. For strain-crystallizing rubbers such as natural rubber and polychloroprene,  $dc/dt$  is effectively zero for  $T$  significantly below  $T_c$ , but non-strain crystallizing rubbers suffer time dependent crack growth at all  $T > T_0$ .



Under cyclic stress conditions, the crack grows faster than anticipated from the “time dependent” crack behaviour, each cycle involving an increment in crack length during loading in addition to any contribution from time-dependent crack growth over the entire cycle. It is found that the cyclic crack growth rate,  $dc/dn$ , where  $n$  is the number of cycles, depends strongly on the maximum value,  $T_{\max}$ , of  $T$  during the cycle, and, for strain-crystallizing rubbers and rubbers with reinforcing filler, also quite strongly on the minimum value,  $T_{\min}$ , of  $T$  during the cycle. However, the dependence on the detailed time-history of the cycle is weak, unless, for a non-strain crystallizing rubber, the frequency is sufficiently low that time-dependent crack growth makes a significant contribution. We may thus often assume:

$$\frac{dc}{dn} = g(T_{\max}, T_{\min}) \quad (20)$$

### 7.2.3 Fatigue

To use the energetics approach and FEA to predict the mechanical fatigue life of a product, the following steps are made:

- a) use FEA to identify the probable position where a flaw is likely to develop into a crack, and the orientation of the crack. Calculate the dependence of  $T$  on  $c$ :

$$T = F(c) \quad (21)$$

More detail is given in 7.5.

- b) find the crack growth rate for the material, as discussed in 7.3.

$$dc/dn = g(T) = g(F(c)) \quad (22)$$

- c) Find the effective initial flaw size,  $c_0$ , in the material as discussed in 7.4, and choose a condition such as  $c_f = 3$  mm, at which the product is deemed to have failed.

- d) the number of cycles to failure,  $N$ , is given by:

$$N = \int_{c_0}^{c_f} \frac{dn}{dc} dc = \int_{c_0}^{c_f} \frac{1}{g(F(c))} \quad (23)$$

As discussed in 7.4, simple fatigue experiments, together with the crack growth characteristic,  $g(T)$  (see 7.3), may be used to find the effective initial flaw size,  $c_0$ , by regarding it as a fitting parameter in equation (23).

## 7.3 Test pieces for characterizing the crack growth behaviour of rubber

### 7.3.1 Introduction

There are no suitable standard methods for characterizing the crack growth behaviour using the energetics approach. The tension fatigue test (BS 903-A51) contains no methodology for inserting a crack, or measuring its length, although it is sometimes used in this way. The De Mattia test (BS 903-A10), with or without an initial cut, is not suitable because the geometry does not lend itself to a convenient expression for the strain energy release rate [equation (17)]. In principle this shortcoming could be overcome by using FEA to calculate the strain energy release rate, but it is simpler to use one of the test methods outlined in 7.3.2 for which simple analytical expressions for the strain energy release rate have been derived.

According to the energetics approach, all of the test methods presented in 7.3.2 should yield the same results, characteristic of the material, if plotted as  $dc/dt$  or  $dc/dn$  versus  $T$ . This has usually been found to be the case, but in some cases agreement is imperfect, and it is good practice to work with more than one type of test piece to gauge the confidence level of the approach in a particular application.

### 7.3.2 Fracture test pieces for rubber

Several test pieces for characterizing fracture of rubber are common to those discussed in 6.4 for stress-strain characterization. However, the standard of precision required for fracture tests is not so great (see 7.3.6), so stipulations regarding test piece proportions (e.g. tensile) may be less stringent. A comprehensive set of fracture test pieces is given in Table 1. Test pieces A, B, C, D and F may be cut from 2 mm thick sheet, but for test pieces E and G use of thinner sheet (0.5 mm) is recommended.

Table 1 — Fracture test pieces for rubber

NOTE All dimensions refer to the unstrained state. Unless otherwise specified,  $h$  is the thickness of test pieces from sheet, orthogonal to the plane of the diagrams.  $W$  is the strain energy density in the rubber far enough away from the crack and the edge of the test piece for the strains to be uniform.

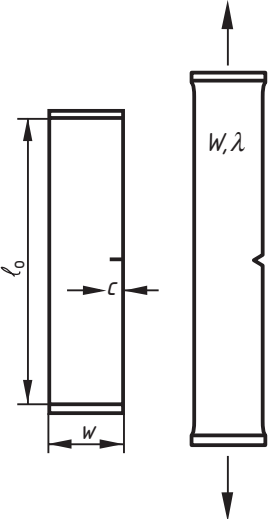
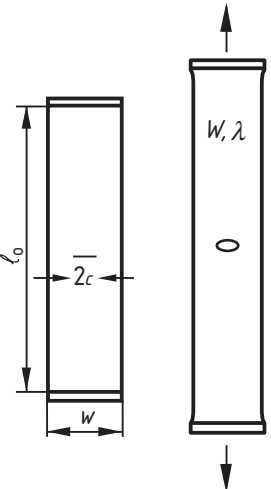
Test piece	Expression for $T$ and conditions of validity	Characteristics
<p>A. Edge crack of length <math>c</math> in tensile strip [16 to 18]</p> 	$T = \frac{2\pi}{\sqrt{\lambda}} Wc$ $c < 0.2w$ $l_0 > 4w$	<ul style="list-style-type: none"> <li>• Crack growth is directionally fairly stable and can easily be measured optically</li> <li>• <math>T</math> is proportional to <math>c</math>, so a single test piece covers a range of <math>T</math> values</li> <li>• Low values of <math>T</math> are attainable</li> <li>• Fairly high frequencies can be used</li> </ul>
<p>B. Central crack of length <math>2c</math> in tensile strip [17]</p> 	$T \approx \frac{2\pi}{\sqrt{\lambda}} Wc$ $2c < 0.2w$ $l_0 > 4w$	<p>Similar to A, but more difficult to introduce an initiating crack</p>

Table 1 — Fracture test pieces for rubber (continued)

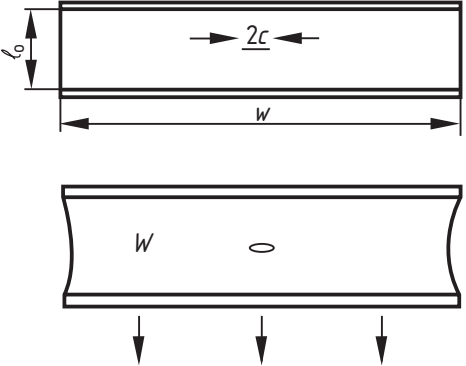
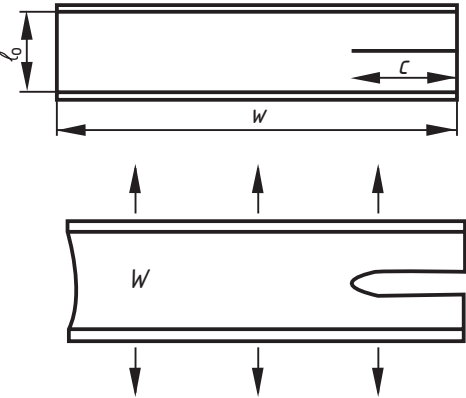
Test piece	Expression for $T$ and conditions of validity	Characteristics
<p>C. Short crack in pure shear [19]</p> 	$T \approx 2\pi Wc$ $c < 0.15l_0$ $w > 5l_0$	<ul style="list-style-type: none"> <li>• Similar to A, or B, but more difficult to set up an accurate strain, and so harder to study low values of <math>T</math></li> </ul>
<p>D. Long crack in pure shear [16, 19]</p> 	$T = Wl_0$ $0.75l_0 < c < w - 3l_0$ $w > 5l_0$	<ul style="list-style-type: none"> <li>• Crack growth is directionally fairly stable and can easily be monitored optically</li> <li>• <math>T</math> is independent of <math>c</math></li> <li>• Fairly high frequencies can be used</li> <li>• It is difficult to set up experiments at low <math>T</math>, and very good temperature control is required to avoid influence of thermal expansion on <math>T</math> for low <math>T</math></li> <li>• The crack should be long enough for the rubber at the edge containing the crack to be fully relaxed</li> </ul>

Table 1 — Fracture test pieces for rubber (continued)

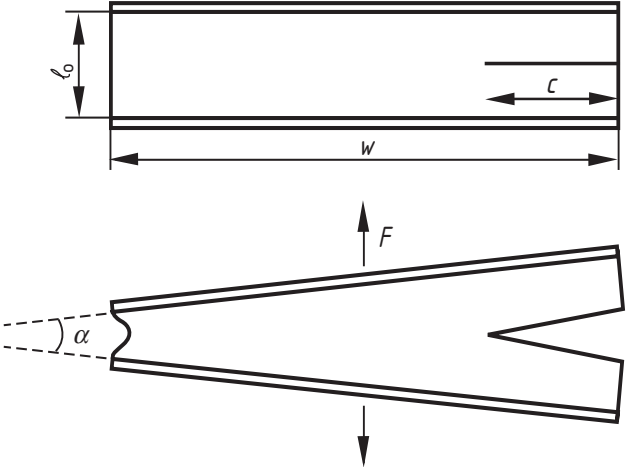
Test piece	Expression for $T$ and conditions of validity	Characteristics
<p data-bbox="237 177 562 204">E. Angled test piece [20]</p>  <p data-bbox="237 730 936 906">The test piece is a parallel sided strip, clamped along its longest edges (top figure). When inserted into the test machine, the clamps are tilted (lower figure) such that the rubber at the edge opposite to the crack is fully relaxed. The crack should be long enough for the rubber at the edge containing the crack to also be fully relaxed</p>	$T \approx \frac{2F}{h} \sin \frac{\alpha}{2}$	<ul style="list-style-type: none"> <li>• Crack growth is directionally fairly stable and can easily be monitored optically or deduced from clamp separation necessary to keep <math>F</math> constant</li> <li>• <math>T</math> is independent of <math>c</math> at constant <math>F</math></li> <li>• <math>T</math> depends on <math>F</math>, not on stress–strain properties, hence imperfect elasticity is not of direct importance</li> <li>• Fairly high frequencies can be used</li> <li>• Test pieces need to be significantly less than 2 mm thick to minimize buckling force</li> <li>• Very good temperature control is required to avoid influence of thermal expansion on <math>T</math> for low <math>T</math></li> <li>• A special machine is needed</li> </ul>

Table 1 — Fracture test pieces for rubber (continued)

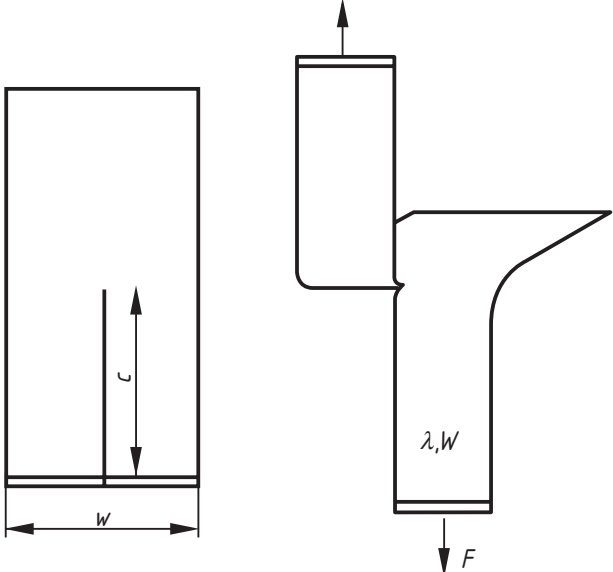
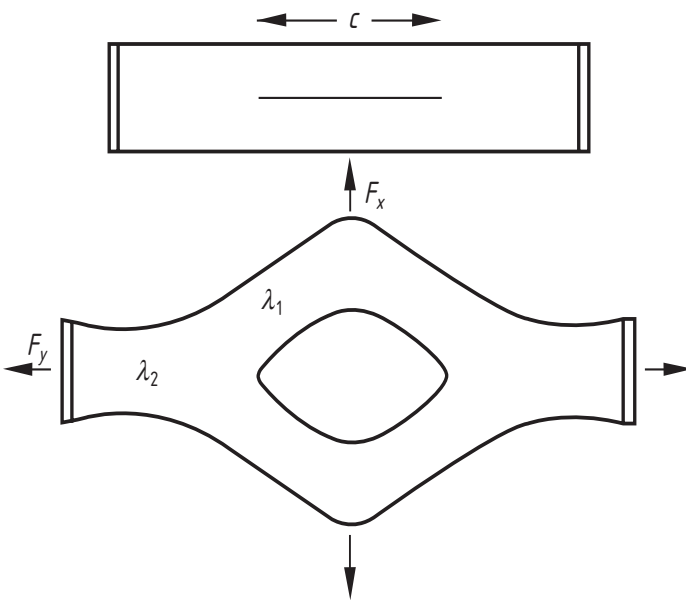
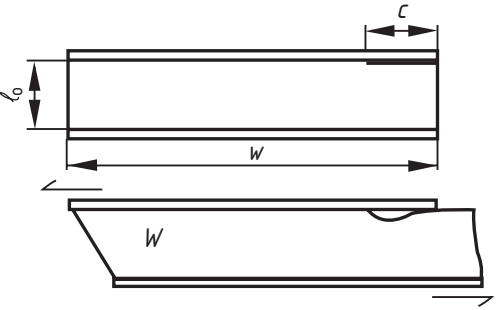
Test piece	Expression for $T$ and conditions of validity	Characteristics
<p>F. Trouser test piece [16]</p> 	$T = \frac{2\lambda F}{h} - wW \approx \frac{2F}{h}$	<ul style="list-style-type: none"> <li>• Crack growth is not directionally very stable</li> <li>• <math>T</math> depends on <math>F</math>, not stress-strain properties, hence imperfect elasticity is not of direct importance</li> <li>• Energy release rate is independent of crack length</li> <li>• Low values of <math>T</math> are not easily attainable</li> <li>• High frequencies are not easily attainable</li> </ul>

Table 1 — Fracture test pieces for rubber (continued)

Test piece	Expression for $T$ and conditions of validity	Characteristics
<p>G. Split [21]</p> 	$T = \frac{\bar{\lambda}(\sqrt{F_x^2 + F_y^2} - F_y)}{h}$ $\bar{\lambda} = \frac{\lambda_1 + \lambda_2}{2}$	<ul style="list-style-type: none"> <li>• Crack growth is directionally fairly stable</li> <li>• Energy release rate depends on <math>F</math>, not stress-strain properties, hence imperfect elasticity is not of direct importance</li> <li>• Energy release rate is independent of crack length, <math>c</math></li> <li>• Low values of <math>T</math> are attainable</li> <li>• Test piece should be significantly less than 2 mm thick to minimize buckling force</li> <li>• A special machine is needed</li> </ul>
<p>H. Simple shear [22]</p> 	$T = Wl_0$ $2.5l_0 < c < w - 3l_0$	<ul style="list-style-type: none"> <li>• Crack growth is directionally fairly stable but is difficult to monitor</li> <li>• The test piece is a thick block, not a sheet</li> <li>• <math>T</math> depends on the direction of shear, so in fully reversed cycles the value of <math>T</math> not only passes through zero but does not reach the same magnitude in each direction</li> </ul>

**Table 1 — Fracture test pieces for rubber (continued)**

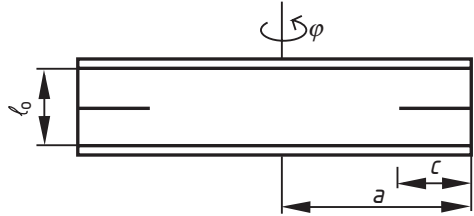
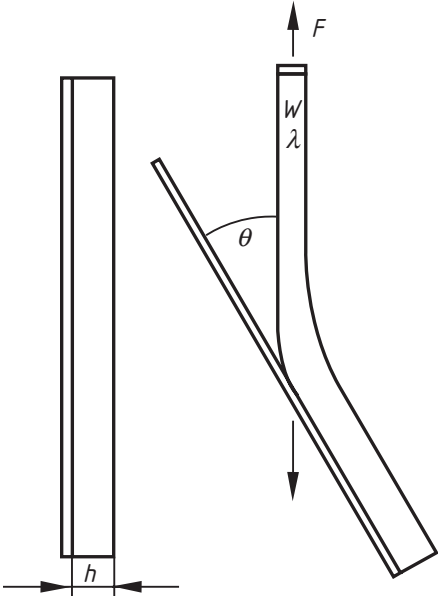
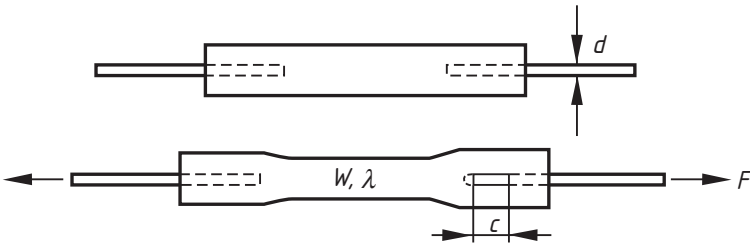
Test piece	Expression for $T$ and conditions of validity	Characteristics
<p>I. Twisted disc [23]</p> 	$T \approx \frac{\left(a - c + \frac{l_0}{3}\right)^2 G \phi^2}{2l_0}$ <p>where <math>G</math> is the shear modulus</p>	<ul style="list-style-type: none"> <li>• Crack growth is directionally stable</li> <li>• <math>T</math> depends on <math>c</math>, so a single test piece covers a broad range of <math>T</math></li> <li>• The test piece is not a thin sheet</li> <li>• The formula for <math>T</math> assumes a neo-Hookean strain energy function</li> </ul>

Table 1 — Fracture test pieces for rubber (continued)

Test piece	Expression for $T$ and conditions of validity	Characteristics
<p>J. Peel [24]</p> 	$T = \frac{F\lambda}{w} - \frac{F\cos\theta}{w} - Wh$ <p>where <math>w</math> is the width of the test piece orthogonal to the plane of the diagram</p>	<ul style="list-style-type: none"> <li>Crack growth is not directionally very stable unless the peel angle <math>\theta</math> is small (<math>30^\circ</math> or less)</li> </ul>
<p>K. Rod pull-out [25]</p> 	$T = \frac{F(\lambda - 1) - AW}{\pi d}$ $dL/2 < A$ <p>where</p> <p><math>L</math> is the embedded length of the rod.</p> <p><math>A</math> is the cross-sectional area of the test piece, orthogonal to the cylindrical rods.</p> <p><math>c</math> is the distance along the rod where the bond has failed.</p>	<ul style="list-style-type: none"> <li>Crack growth is fairly stable, around the periphery of the rod</li> <li>Friction contributes to the pull-out force unless the criterion regarding the maximum rod diameter is adhered to</li> </ul>



For a crack of length  $c$  through the thickness,  $h$ , of a sheet test piece,  $\delta A = h\delta c$  so that:

$$T = -\frac{1}{h} \left( \frac{\partial U}{\partial c} \right)_l \quad (24)$$

Evaluating this equation for test pieces containing cracks and undergoing finite strains has been done in several special cases, designated A, B, C, D in Table 1.

The expressions for strain energy release rate of the test pieces shown in Table 1 A, B, C and D require knowledge of the strain energy density,  $W$ , of the rubber in the region of approximately uniform strain far away from the perturbing influence of the crack. If the strain energy function for the rubber is known,  $W$  may be calculated from the deformation, but otherwise an auxiliary test is needed,  $W$  being calculated from the area under the force–extension graph for the extension used in the test, using a test piece with no crack, and dividing by the volume of rubber.

For another set of test pieces, it is convenient to consider  $U$  as being a function of the positions of the external boundaries – here we choose two orthogonal displacements  $x, y$  in place of  $l$  – as well as of the crack length  $c$ , so that:

$$dU = \left( \frac{\partial U}{\partial c} \right)_{x,y} dc + F_x dx + F_y dy \quad (25)$$

where two of the partial derivatives have been identified with the forces  $F_x$  and  $F_y$  in the  $x$  and  $y$  directions, respectively:

$$F_x = \left( \frac{\partial U}{\partial x} \right)_{c,y} \quad (26)$$

$$F_y = \left( \frac{\partial U}{\partial y} \right)_{c,x}$$

It follows that the strain energy release rate is given by:

$$-\frac{1}{h} \left( \frac{\partial U}{\partial c} \right)_{x,y} = \frac{-1}{h} \left\{ \left( \frac{\partial U}{\partial c} \right)_{F_x, F_y} - F_x \left( \frac{\partial x}{\partial c} \right)_{F_x, F_y} - F_y \left( \frac{\partial y}{\partial c} \right)_{F_x, F_y} \right\} \quad (27)$$

A set of test pieces shown in Table 1 as E, F and G have been devised for which the partial derivatives on the right-hand side of equation (27) may be evaluated, based on the geometry and, for  $(\partial U / \partial c)_F$ , on auxiliary measurements. The strain energy release rate may then be calculated from measurements of the forces  $F_x$  and  $F_y$ .

The use of the trouser test piece, Table 1 F, for determining the tear strength is covered by BS ISO 34-1. However, this latter standard gives no guidance on crack growth measurements or on expressing results in terms of the tearing energy. Moreover, because the geometry recommended in BS ISO 34-1 for the trouser test piece has legs of small cross-sectional area ( $7.5 \times 2 \text{ mm}^2$ ) they will undergo significant stretching. This means that the approximation for the strain energy release rate based on neglecting the leg extension:

$$T \approx \frac{2F}{h} \quad (28)$$

is usually very poor.

The test pieces shown in Table 1 A to G are all based on sheet rubber, and it is desirable to verify that the same characteristics of crack growth rate versus  $T$  are found for three dimensional test pieces; two such test pieces are given as Table 1, H and I. The equation for  $T$  for test piece I is unlike those for the other test pieces, in that the neo-Hookean form for  $W$  is assumed in its derivation,  $G$  representing the shear modulus ( $G = 2C_{10}$  of 6.1.4.2.2).

### 7.3.3 Test pieces for characterizing bond and near-bond failure

The energetics approach is applicable to bond, or near-bond, failure as well as to cohesive failure within the bulk of the material. The proximity of the rigid material, to which the rubber is bonded, can modify the failure characteristics of the material, and this should be borne in mind when using test pieces H and I of Table 1 for determining the crack growth characteristics of the rubber. If experiment shows the failure not only to be parallel to the bond, but at or very near the interface, the results may be regarded as giving the bond fracture characteristics. BS 903-A14 advocates the quadruple shear test piece for assessing adhesion, but for the present purposes it is necessary also to stipulate that a sufficiently long initiating crack be inserted at the appropriate locus, shown in Table 1 H; also, cyclic loading conditions are usually of greater interest to the service conditions than a single shear load to failure as specified in BS 903-A14.

Two more adhesion tests, amenable to interpretation using the energetics approach, are included in Table 1 as J and K. Test piece J, the peel test piece, is covered by BS 903-A21, but again this latter standard would have to be augmented by choosing appropriate (e.g. cyclic) load conditions and expressing the results in terms of crack growth rate versus strain energy release rate.

### 7.3.4 Comparison of the test pieces and effect of crack tip sharpness.

Brief notes on the attributes, positive and negative, of the different test pieces are given in Table 1. The most convenient test pieces are A for bulk properties and J, using a low peel angle such as 30°, for near-bond failure. However, it is important to note that it is good practice to validate the energetics approach by using more than one test piece type for each major characterization programme.

Differences in crack tip sharpness result in different growth rates for the same energy release rate. The crack growth rate should only be regarded as characteristic of the material once sufficient crack growth has occurred for a natural crack tip roughness to develop, so initial rapid crack growth from a crack introduced with a razor blade should be excluded from reported results. Similar, but less marked, transients occur when the energy release rate is changed since the natural crack tip roughness increases as the energy release rate increases. In some circumstances discrepancies between results from different test pieces – for example between one for which  $T$  is constant and one for which  $T$  is proportional to  $c$  – may be attributed to this phenomenon.

Forking of the crack tip is an extreme form of blunting. It is part of the strength characteristics of the material, associated particularly with the use of reinforcing filler and non-relaxing cyclic loading conditions. Procedures for dealing with it should be guided by whether or not it plays a role in the application.

### 7.3.5 Dealing with imperfections in elasticity

As presented in 7.2.1, the energetics approach is based on an assumption of perfectly elastic behaviour of the rubber. This is not entirely consistent with the known behaviour, or with the idea discussed in 7.2.2, that the fracture toughness is related to hysteresis in the region near the crack tip. For materials that exhibit significant hysteresis for all stress–strain cycles, there is a need to define  $W$  in the equations given in Table 1 in an unambiguous manner.

For test pieces A, B, C, D, H and I the fracture energy,  $T$ , is supplied by release of the far-field strain energy, so it is apparent that the retraction energy (the area under the force–retraction plot) is required. If the retraction energy depends on the rate of retraction there is still an ambiguity. The relevant rate would be expected to depend on the rate of advance of the crack tip and the length scale, in the region of the crack tip, over which the strain energy density changes from the far-field value to the relaxed value in the lee of the crack, but this is generally unknown. It is recommended therefore that, for these test pieces, the retraction strain energy density is used, as measured in a stress–strain testing procedure using the same rate of loading as in the crack growth tests, and using the same rate of unloading and, for cyclic studies, the same number of cycles. The test piece used for this purpose should be an auxiliary one, not containing a crack, but in practice for cyclic crack growth tests the convenience of working with the same test piece for determining  $W$  as for determining  $dc/dn$  may outweigh the error in  $W$  caused by the presence of a crack.

For test pieces F and J, the strain energy density appearing in the equations for  $T$  in Table 1 represents an offset to the predominant supply of  $T$  by the externally applied force, the energy stored in the region with energy density  $W$  actually increasing when the crack grows. It is therefore recommended in these cases that the extension strain energy density be used.

### 7.3.6 Errors in experimental determination of the crack growth characteristics

Strength properties are statistical rather than precisely definable quantities. It is therefore to be expected that there is a scatter of around 30 % in crack growth rate for a given value of strain energy release rate. Errors associated with estimation of the  $T$ -values, for example the effects of imperfect elasticity discussed in 7.3.5, are probably of a similar order. Very little work on the latter has been published, the exception being a comparison [26] of results for pure shear (D of Table 1) and trouser (F of Table 1) test pieces for time-dependent cracking in unvulcanized natural rubber, a material whose elasticity is far from perfect. It was found that agreement was good only if the retraction strain energy was used for the pure shear test piece. A feel for the uncertainty in  $T$  may be obtained by comparing values calculated using the extension and the retraction values for  $W$ , and where these are very large a special study of the kind given in [26] is called for.

### 7.4 Material test for fatigue of rubber

BS 903-A51 provides a suitable method of determining the resistance to cyclic fatigue in tension. Assuming that the crack growth characteristics of the material are known (e.g. equation (20) with  $T_{\min} = 0$ ), the effective flaw size,  $c_0$ , may be determined from the number of cycles to failure by choosing  $c_0$  to fit equation (23) where  $T$  is expressed in terms of  $c$ ,  $W$  and  $\lambda$  using the equation given in Table 1 A.

For large flaws, such that all crack growth occurs in the power law region:

$$\frac{dc}{dn} = g(T) \approx BT^n \quad (29)$$

where

$B$  and  $n$  are fitting constants.

It follows, with  $c_f \gg c_0$  and  $n > 1$ , that

$$c_0 \approx \left\{ (1-n)BN \left( \frac{6}{\sqrt{\lambda}} W \right)^{-n} \right\}^{\frac{1}{1-n}} \quad (30)$$

When equation (30) is used, the resulting value of  $c_0$  should be used to calculate the initial value of  $T$ , to check that it is large enough to be consistent with the power law region of  $g(T)$ . If not, the integration in equation (23) should be carried out without this simplification, if necessary numerically.

It is good practice to verify that the effective flaw size is independent of the strain used in the fatigue test, verifying the applicability of the fracture mechanics approach to predict fatigue life.

Fatigue results may also be used to construct Wöhler or S–N type plots, by plotting the logarithm of the number of cycles to failure against the logarithm of maximum stress, strain or energy density. Such results give a guide to the lifetime of a product, the tensile fatigue number of cycles to failure being read off them for the same maximum stress, strain or strain energy as occurs in the product. However, for strain-crystallizing rubbers such as natural rubber and polychloroprene, there is a strongly beneficial effect of non-relaxing conditions on the fatigue life, and this is also true to some extent for non-crystallizing rubbers incorporating reinforcing filler. Thus, in addition to the uncertain basis on which tensile fatigue is used as a guide to fatigue in the more complex modes of deformation in products, the tensile fatigue tests of BS 903-A51 would need, as a minimum, to be supplemented with non-relaxing tests.

### 7.5 Calculation of strain energy release rate using FEA

It is generally straightforward to find values of field variables such as strain energy density, maximum principal stress and maximum principal strain from FE models of rubber products. Such results are useful for design optimization, predicting where mechanical failure will start and in what direction the initiating crack will grow (orthogonal to the maximum principal stress, if tensile), but serve only as a crude guide to fatigue life, through the use of S–N or Wöhler plots.

The test pieces in Table 1 are recommended as training and validation benchmarks for strain energy release rate analysis, the prediction of FEA being compared to those of the formulae given in Table 1. The strain energy release rate may be calculated from the difference in total energies for two models differing only in crack length. Satisfaction of equation (17) requires that the model boundaries do not move when

the crack is introduced. Thus boundary conditions based on a fixed displacement, rather than a fixed load, facilitate calculation of the strain energy release rate. The total strain energy may be reported directly by the FEA solver or calculated from the area under the force–deformation plot. Some finite strain FEA packages provide a direct solution for the strain energy release rate based on the J-integral [27] or Lorenzi [28] contour integral methods but in some packages these calculations may be incorrect for large strain problems. Considerable experience might be required to achieve convergence for a model of a product containing a crack, and to study crack propagation requires a remesh for each crack advance. Thus, unless the crack advance procedure is automated in some way, there might be relatively little advantage in effort in using the J-integral method compared to a direct calculation of strain energy release rate.

To remove the tedium of repetitive meshing with different lengths of crack to simulate propagation, software packages are available that handle the meshing of the crack, and call an appropriate FEA package as a subroutine to solve for the strain energy release rate.

To simulate crack propagation in 3D FEA models, the crack front should be advanced locally in the direction that maximizes the release of strain energy. Advancing in the direction normal to the maximum principal stress is easier to implement and should give similar results. Propagation can result in the crack surface becoming non-planar, making the modelling of 3D crack growth very difficult.

## 8 Friction

### 8.1 Introduction

Rubber has unusually high friction, as a consequence of its very low elastic modulus and its viscoelastic behaviour. It is easily deformed under a contact pressure, resulting in a large real area of contact and high friction. Under sufficiently high loads, the real area of contact becomes equal to the apparent contact area, and the frictional stress becomes approximately constant, a condition known as “saturation”.

Most FEA packages incorporate friction models based on Coulomb’s law in which slip occurs if:

$$\tau > \mu P \quad (31)$$

where

$\tau$  is the tangential stress along the interface;

$P$  is the pressure normal to the interface;

$\mu$  is the coefficient of friction.

For materials which obey Coulomb’s law,  $\mu$  is independent of normal load and velocity, but most FEA programs allow a variable  $\mu$ , although in some this is only permitted through use of a user subroutine. This is useful since rubber does not obey Coulomb’s law and the coefficient of friction is dependent on the normal stress, sliding velocity and temperature. For example at “saturation”,  $\tau$  is constant; thus it is readily seen from equation (31) that the coefficient of friction is inversely proportional to the normal pressure. A modified version of the main model may allow for the possibility that a force “barrier” must be overcome for the transition from sticking to sliding. This is useful for cases where so-called “static” friction is higher than sliding or “dynamic” friction, such as where lubrication is more effective once sliding begins. Thus the purpose of friction testing for FEA is to provide a value or set of values for the coefficient of friction over the range of normal pressures, velocities and temperatures applicable to the analysis. Adequate estimates of appropriate normal pressures and sliding velocities may be obtained from simple hand calculations or from preliminary analyses with a crude value for  $\mu$ .

## 8.2 Test methods

BS 903-A61 provides suitable test methods for determining the coefficient of friction for contact involving rubber, and background information on the factors affecting rubber friction.

NOTE Method B of BS 903-A61 concerns changes in the coefficient of friction which occur after continuous long term sliding. It is not normally feasible to allow for changes in  $\mu$  over time in FEA, but use of long term, rather than initial, values of  $\mu$  is appropriate where this is representative of the service conditions.

The normal load(s) required for the friction tests may be obtained from:

$$N = PA \quad (32)$$

for planar contact and from:

$$N = \pi^3 P^3 \left( \frac{9R}{16E} \right)^2 \quad (33)$$

for contact between a rubber ball and planar track where

- $N$  is the normal load;
- $P$  is the normal pressure (estimated as suggested in 8.1);
- $A$  is the apparent contact area of the rubber test piece;
- $R$  is the radius of the rubber ball;
- $E$  is the Young's modulus of the rubber ball.

NOTE For contact between a rubber ball and planar track, the pressure distribution over the area of contact will be non-uniform. Applying a normal force,  $N$ , calculated according to equation (33), will result in a mean pressure of  $P$ .

The test speed(s), temperature(s) and lubrication, if any, should also be chosen to be representative of the service conditions of the product.

## 9 Thermal properties

### 9.1 General

Heat transfer during vulcanization, and other processes involving temperature changes, can be modelled with FEA by performing a transient thermal analysis. Taking the simple example of heat flow in an isotropic material, with no temperature dependence of the thermal properties, and no evolution or absorption of heat by the rubber, the heat flow within the body is defined by the differential equation:

$$\frac{\partial \theta}{\partial t} = \kappa \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad (34)$$

where

- $\theta$  is the temperature;
- $t$  is the time;
- $\kappa$  is the thermal diffusivity;
- $x$ ,  $y$  and  $z$  are the co-ordinates of position.

For a cure analysis, heat evolved as the reaction progresses can be significant. Thus the governing differential equation becomes:

$$\frac{\partial \theta}{\partial t} = \kappa \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) + \frac{1}{C} \frac{dQ}{dt} \quad (35)$$

where

$C$  is the specific heat capacity;  
 $dQ/dt$  is the rate of heat evolution per unit mass.

At the surface of the body, the flux,  $F$ , is given by:

$$F = H(\theta_s - \theta_0) = -K \frac{\partial \theta}{\partial n} \quad (36)$$

where

$H$  is the film coefficient (the inverse of the thermal resistance);  
 $K$  is the thermal conductivity;  
 $(\theta_s - \theta_0)$  is the temperature difference between the body at the surface and the surroundings;  
 $n$  is the direction of the normal to the surface.

The thermal diffusivity is related to the thermal conductivity by the equation:

$$\kappa = K/\rho C \quad (37)$$

where

$\kappa$ ,  $K$  and  $C$  have the definitions already given in this clause;  
 $\rho$  is the density.

## 9.2 Thermal conductivity

Various apparatuses exist for measurement of thermal conductivity. The use of guarded hot plate and heat flow meter apparatus is described in BS EN 12667, and these methods are suitable for use with rubber. They are “steady-state” methods and involve heating two surfaces of a rubber test piece to different temperatures and monitoring the heat flow across the rubber once thermal equilibrium has been reached. The geometry of the test piece and design of the apparatus is such that the heat flow is unidirectional. The influence of the thermal resistance at the boundary between the apparatus platens and the rubber may be eliminated by making the measurement on two test pieces of different thickness.

Alternatively, the thermal conductivity may be calculated, using equation (37), from direct measurements of thermal diffusivity. This method involves measurement of the temperature changes of a test piece before an equilibrium state has been reached and are thus termed “transient”. The temperature changes are often applied to one surface of a test piece suddenly, although values deduced from gradual heating have also been reported. Various test piece shapes are possible; symmetry considerations favour slabs, spheres or cylinders. There is no British Standard governing this method, and users are advised to give careful consideration to the magnitude of experimental and computational errors; this method is inherently less accurate than the steady-state methods.

## 9.3 Specific heat capacity

The specific heat capacity may be measured by calorimetry. The most widespread method is differential scanning calorimetry (DSC). In this method, a constant heating rate, typically 20 K/min is applied to the test piece. The rate of heat flow into the test piece (compared to the heat flow into a second identical sample holder) is measured continuously. Three scans are required; the first of an empty test piece holder to establish an accurate baseline, the second of the test piece, and the third of a standard test piece of accurately known specific heat capacity over the temperature range of interest. The use of a standard eliminates errors associated with temperature dependence of the machine calibration, which might arise if a direct measurement is carried out. At any temperature, the specific heat capacity of the test piece,  $C$ , is given by:

$$C = \frac{C_s M_s \Delta H'}{M \Delta H_s} \quad (38)$$

where

- $C_s$  is the specific heat capacity of the standard at the same temperature;  
 $M_s$  is the mass of the standard;  
 $\Delta H'$  is the measured rate of heat flow into the test piece, relative to the baseline;  
 $M$  is the mass of the test piece;  
 $\Delta H'_s$  is the measured rate of heat flow into the standard relative to the baseline.

At the start and end of heating at constant rate, there will be changing heating rates at different places in the test piece. These transient effects will give spurious readings at the start of the run; their duration depends on the size and thermal diffusivity of the test piece. Thus the heating run should start at a temperature at least 20 K below that for which data are required. Thereafter, the heating rate of all points in the test piece is uniform, but the temperature inside the test piece will lag behind that of the test piece holder and temperature sensor; this will lead to errors if the test piece is large or if the heat capacity has a strong temperature dependence.

The specific heat capacity of a rubber of known formulation may be estimated from the specific heat capacities of the ingredients, if these are known, from the equation:

$$C = \sum V_i C_i \quad (39)$$

where

- the  $V_i$  are the mass fractions of the ingredients;  
the  $C_i$  are the corresponding specific heat capacities.

#### 9.4 Density

The method of hydrostatic weighings given in BS 903-A1 should be used.

#### 9.5 Contact resistances and film coefficients

Since rubber is a deformable material, it normally forms good thermal contact with other solid materials. The thermal contact resistance is low, and may generally be neglected for contact between a clean rubber surface and another clean surface. The presence of surface layers, such as dirt or metal oxide films, might increase the surface resistance considerably. Thus, it is important that experimental determinations of contact resistance are made between materials whose surface conditions accurately match those of the modelled product.

There are no well documented test methods for measuring thermal contact resistances between rubber and other materials. A possible technique would be to measure the thermal conductivity of a few multiple layer "sandwiches" of the two materials by one of the methods given in 9.2, from which the thermal resistance may be deduced from the total resistance if the thermal conductivities of the two materials are already known.

When the rubber is in contact with a fluid, such as air or steam, the heat transfer depends on whether the surface is vertical or horizontal, its dimensions, and the temperature and speed of the fluid. Formulae and methods for obtaining the heat transfer film coefficients are given in standard chemical engineering texts.

#### 9.6 Heat of vulcanization

The heat of vulcanization may be measured by DSC of an unvulcanized sample, followed by a second scan of the same, now vulcanized, sample to establish the baseline. The heat of vulcanization is given by the area under the exotherm on a plot of heat capacity against temperature. It is important that the DSC samples come from mixes in which the curatives are well-dispersed, to ensure the values obtained are representative. Other thermal processes can also occur in the same temperature range, such as melting or softening of solid compounding ingredients, or evaporation of plasticizers; in this case identification of the cure exotherm might be more difficult.

## 10 Heat build-up

If a rubber product is subjected to rapid strain cycling, hysteretic losses in the rubber are converted to heat. If the heat is generated more rapidly than it is dissipated, the product becomes hot and degradation of the rubber can follow.

In order to model this behaviour with FEA, the program must first calculate the hysteretic energy loss and equate this to heat generation at every point in the product, then calculate the flow of heat. The heat flow calculations and material properties required are as described in Clause 9. The conversion of hysteretic losses to heat is more problematic; currently commercial FEA programs have not implemented this for their large strain linear viscoelastic models, so the user would need to estimate it or calculate it with a user-subroutine. Normally a “modular” approach is adopted, in which the mechanical problem is solved for a small number of cycles first, and the calculated heat generation rate is then fed into a thermal analysis in which the rubber is assumed to be motionless. This avoids the need to use a very large number of increments to model the movement of the rubber over thousands of cycles. If necessary, further iterations of the two analyses may be carried out to allow for the temperature dependence of the mechanical properties. In view of the limitations of the commercial FEA packages in providing suitable mechanical models, further discussion of modelling heat build-up is beyond the scope of this guide.



## Annex A (informative)

### Stress-extension relationships in simple deformations and parameter optimization

The relationships between the strain energy function and the nominal stress are:

$$\sigma_T = 2(\lambda_T - \lambda_T^{-2}) \left( \frac{\partial W}{\partial I_1} + \frac{1}{\lambda_T} \frac{\partial W}{\partial I_2} \right) \text{ for uniaxial tension and compression} \quad (\text{A.1})$$

$$\sigma_P = 2(\lambda_P - \lambda_P^{-3}) \left( \frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \right) \text{ for pure shear} \quad (\text{A.2})$$

$$\sigma_B = 2(\lambda_B - \lambda_B^{-5}) \left( \frac{\partial W}{\partial I_1} + \lambda_B^2 \frac{\partial W}{\partial I_2} \right) \text{ for equibiaxial extension} \quad (\text{A.3})$$

$$\sigma_S = 2\gamma \left( \frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \right) \text{ for simple shear} \quad (\text{A.4})$$

where

$\sigma_T$  is the nominal stress in the direction of stretching in uniaxial tension or squashing in uniaxial compression;

$\sigma_P$  is the nominal stress in the direction of stretching in pure shear;

$\sigma_B$  is the nominal stress in the direction of stretching in equibiaxial tension;

$\sigma_S$  is the shear stress in simple shear;

$\lambda_T$  is the extension ratio in the direction of stretching in uniaxial tension or squashing in uniaxial compression;

$\lambda_P$  is the extension ratio in the direction of stretching in pure shear;

$\lambda_B$  is the extension ratio in the direction of stretching in equibiaxial tension;

$\gamma$  is the shear strain in simple shear;

$W$ ,  $I_1$  and  $I_2$  are defined in Clause 4.

Hence, the form of the stress, stretch ratio relationship for a strain energy function  $W(I_1, I_2)$  may be obtained by differentiating the function and substituting into the relationships given in equations (A.1) to (A.4). Values of the coefficients may then be obtained from a least-squares fit to the experimental data. So that relative rather than absolute departures are minimized, it is appropriate to use, as the objective function to be minimized, the sum of the square of relative errors:

$$\sum \left( \frac{\sigma_a - \sigma_f}{\sigma_f} \right)^2 \quad (\text{A.5})$$

where

$\sigma_e$  is the experimental value of stress;

$\sigma_f$  is the analytical value at the same  $\lambda$ ;

If experimental data have been obtained from more than one type of test, the fit may be made to one set of data, and a check made of the accuracy of the fit to the other sets of data. This procedure is adequate for functions of  $I_1$  only. With more complicated strain energy functions, such as functions of  $I_1$  and  $I_2$ , a more sophisticated procedure is needed to provide best-fit values of all the coefficients.

**Annex B (informative)****Relationship between stress in simple shear and pure shear**

It may be shown that the extension ratio in the direction of stretching in pure shear,  $\lambda_P$ , is given by:

$$\lambda_P = \frac{1}{2}(\gamma + \sqrt{\gamma^2 + 4}) \quad (\text{B.1})$$

where

$\gamma$  is the shear strain in simple shear.

From equations (A.2) and (A.4) the nominal stress in the same direction,  $\sigma_P$ , is given by:

$$\sigma_P = \frac{\sigma_S}{\gamma}(\lambda_P - \lambda_P^{-3}) \quad (\text{B.2})$$

where  $\gamma$  is defined previously and  $\sigma_P$ ,  $\sigma_S$ , and  $\lambda_P$  are defined in Annex A.

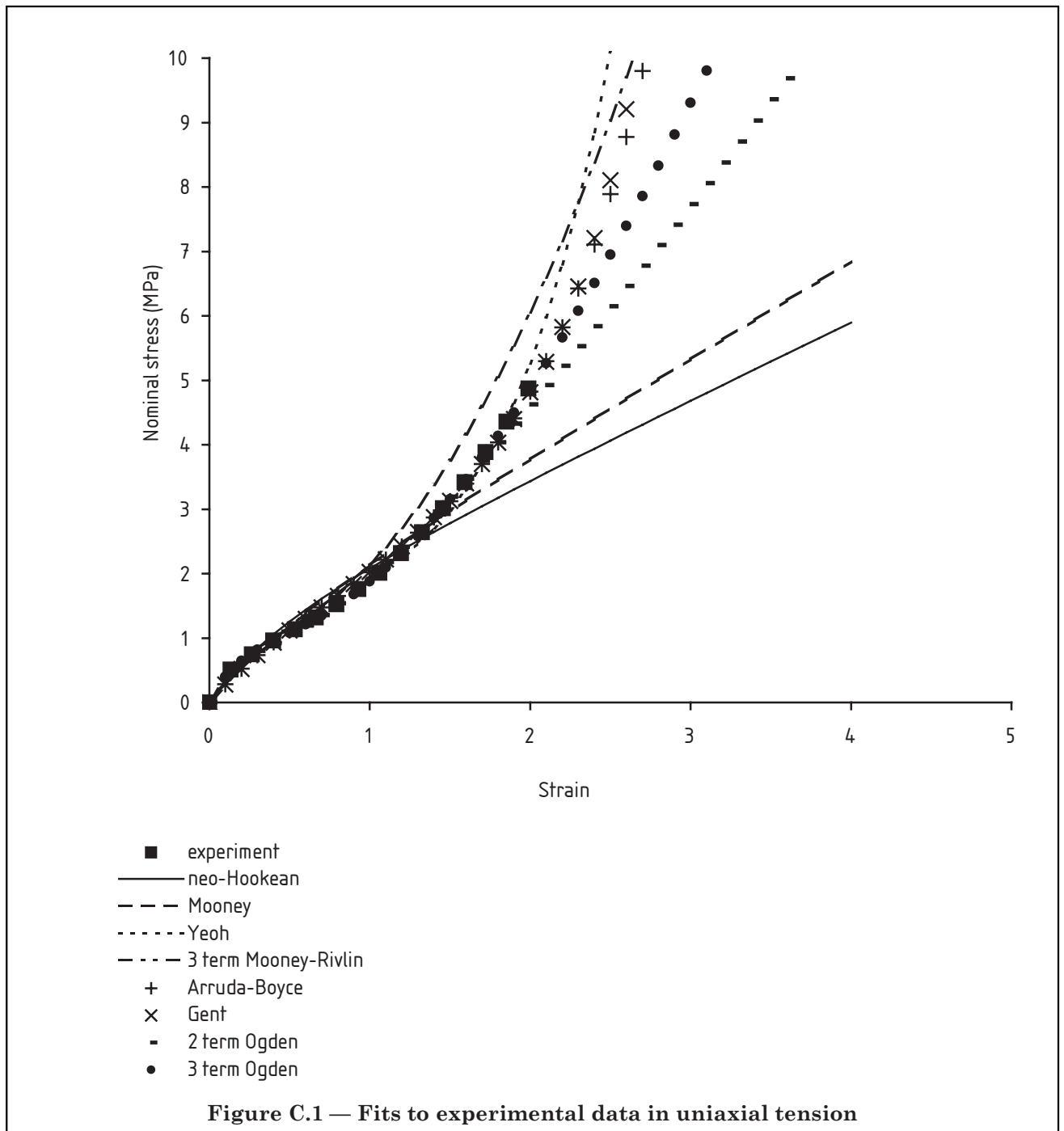
Hence, simple shear data may be converted to a pure shear form by use of equations (B.1) and (B.2) to obtain  $\lambda_P$  and  $\sigma_P$  from  $\gamma$  and  $\sigma_S$ .

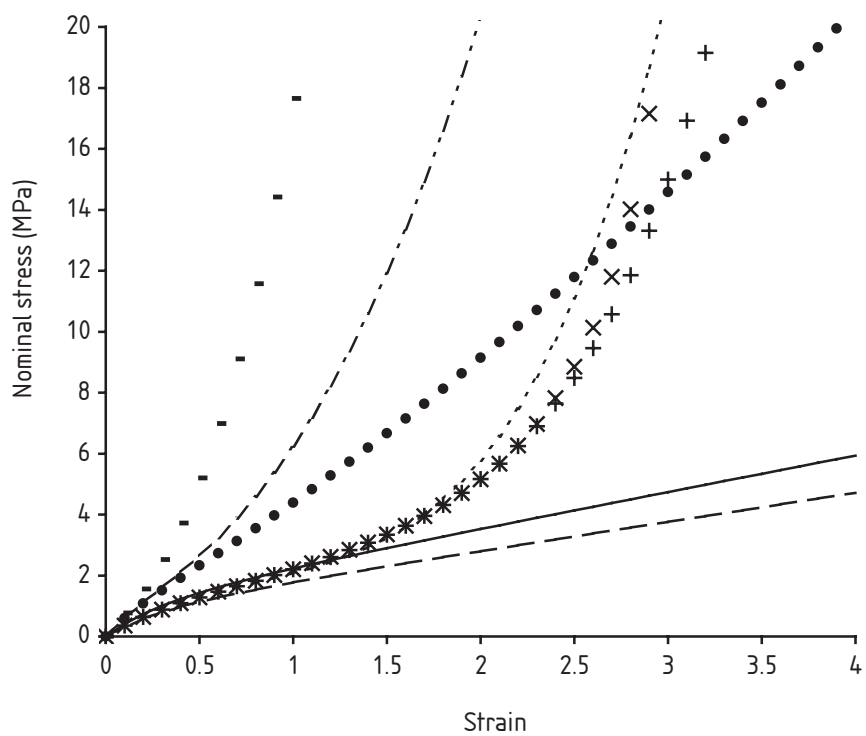
**Annex C (informative)****An example of fitting models to experimental data**

Some of the models discussed in 6.1.4 were fitted to the stress–strain curve generated from a first cycle uniaxial tensile test on a carbon black-filled natural rubber vulcanizate. The test data were recorded from zero to 200 % strain. The curve fitting was carried out using the routine supplied as part of a major commercial non-linear FEA package. No attempt was made to satisfy any available criteria to ensure stability of the models. The fits to the uniaxial tension data, with an extrapolation to 400 % strain, are shown in Figure C.1. The strain in the figures is expressed as extension ratio. The model constants are given in Table C.1. The predictions of the models (with the same fitted constants) of the behaviour in pure shear and equibiaxial extension are shown in Figure C.2 and Figure C.3 respectively.

**Table C.1 — Constants derived from fits to uniaxial tension data**

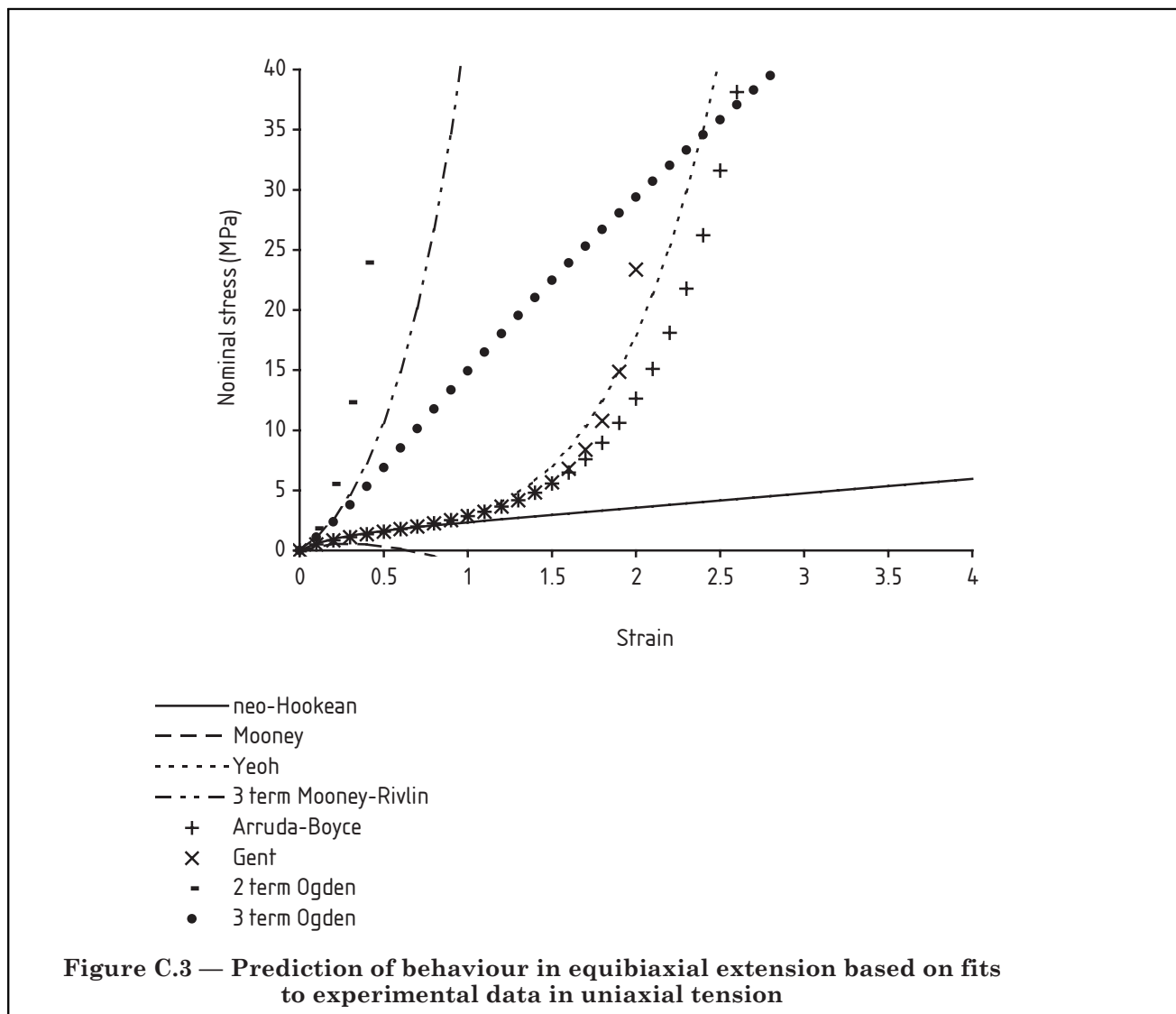
Model	Fitted constants
neo-Hookean	$C_{10} = 0.594$
Mooney	$C_{10} = 0.745; C_{01} = -0.273$
Yeoh	$C_{10} = 0.569; C_{20} = -0.018 \text{ 1}; C_{30} = 0.004 \text{ 4}$
3 term Mooney-Rivlin	$C_{10} = -0.588; C_{01} = 1.435; C_{11} = 0.181$
Arruda-Boyce	$G = 0.898; \lambda_m = 2.25$
Gent	$G = 1.02; I_m = 20.3$
2 term Ogden	$\mu_1 = 2.16; \alpha_1 = -4.28; \mu_2 = -28.1; \alpha_2 = 0.188;$
3 term Ogden	$\mu_1 = 2.16; \alpha_1 = 2.42; \mu_2 = -35.5; \alpha_2 = -0.634; \mu_3 = -44.5; \alpha_3 = 0.547$





- neo-Hookean
- - - Mooney
- .....Yeoh
- . - . 3 term Mooney-Rivlin
- + Arruda-Boyce
- × Gent
- 2 term Ogden
- 3 term Ogden

**Figure C.2 — Prediction of behaviour in pure shear based on fits to experimental data in uniaxial tension**



It may be seen from Figure C.1 that all models provide an adequate fit to the data within the strain range of the experimental data. Thereafter the models diverge: the Yeoh, Gent and Arruda-Boyce models predict rather stiff behaviour, whereas the neo-Hookean and Mooney models show no increase in stiffness. Typically, the real behaviour would lie between these extremes. For the Arruda-Boyce model, the value for the maximum chain extension,  $\lambda_m$ , determined from the fit, is much lower than is physically realistic. The stiffening, for all models capable of capturing it, would be predicted more accurately if test data to higher strains were used for the fit, emphasising the need to ensure that the characterizations tests cover the full strain range of interest.

In the other modes of deformation (Figure C.2 and Figure C.3), the predictions of the models which are functions of  $I_1$  only (neo-Hookean, Yeoh, Gent and Arruda-Boyce) agree quite closely, at least at moderate strains, and their predictions seem sensible. The other models deviate widely from these predictions, with the Ogden and 3 term Mooney-Rivlin model suggesting unrealistically stiff behaviour. Moreover, the Mooney model is unstable as it predicts a negative stiffness in equibiaxial extension. It also predicts, implausibly, that the rubber is softer in pure shear, where it is constrained laterally, than in uniaxial tension, where it is not. Although an improvement to the Mooney fit would be possible by making use of a stability check, these examples show the advantage of restricting the model to a function of  $I_1$  only. At high strains, the lack of high strain fitting data in tension is likely to have impaired the fit of the Yeoh and Arruda-Boyce models. Furthermore, analyses with the Gent model will fail if  $I_1$  ever exceeds  $I_m$ ; its utility might be improved if a higher value for  $I_m$  is adopted.

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