

BS ISO 22514-8:2014



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Statistical methods in process management — Capability and performance

Part 8: Machine performance of a multi-state production process

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National foreword

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**Statistical methods in process
management — Capability and
performance —**

Part 8:
**Machine performance of a multi-state
production process**

*Méthodes statistiques dans la gestion de processus — Aptitude et
performance —*

Partie 8: Aptitude machine d'un procédé de production multimodal



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Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 22514 consists of the following parts, under the general title *Statistical methods in process management — Capability and performance*:

- *Part 1: General principles and concepts*
- *Part 2: Process capability and performance of time-dependent process models*
- *Part 3: Machine performance studies for measured data on discrete parts*
- *Part 4: Process capability estimates and performance measures [Technical Report]*
- *Part 6: Process capability statistics for characteristics following a multivariate normal distribution*
- *Part 7: Capability of measurement processes*
- *Part 8: Machine performance of a multi-state production process*

Introduction

The methodology introduced through this part of ISO 22514 provides the platform for producing the items required for building a long-term process capability and its leading, for a given product characteristic. This can, for example, make it possible to

- define the in-process or mid-process sampling procedure,
- predict, for batch furnaces, a process capability variation range covering all the parts in the batch load, once a recorded partial load variation has been characterized beforehand, and
- follow, for multi-cavity casting, the changes of extreme variation field based on different positions in the mould, each variation of the mould cavities have been characterized beforehand.

Statistical methods in process management — Capability and performance —

Part 8: Machine performance of a multi-state production process

1 Scope

This part of ISO 22514 aims to define the evaluation method to quantify the short-term capability of a production process (capacity of the production tool, widely termed capability), i.e. the machine performance index, to ensure compliance to a toleranced measurable product characteristic, when said process does not feature any kind of sorting system.

If the production process integrates a sorting system, then this one (clearing away nonconforming parts) should be analysed independently.

This part of ISO 22514 does not aim to define evaluation methods of the capability of a production process that is gauged through long-term observation (capability process or performance process indices).

This part of ISO 22514 defines

- the principles guiding the development of indicators for quantifying capability, and
- the statistical methods to be employed.

The characteristics used to evaluate production process capability have statistical distributions, and it is presumed, a priori, that at least one of these distributions is multi-modal. A distribution is presumed to be multimodal if it results from the marked effect of at least one cause inducing a significant difference between the produced items.

This part of ISO 22514 applies, for example, to characteristics generated by processes such as the following:

- multi-cavity casting: simultaneously producing several identical parts from a mould featuring several cavities.

Since each cavity has its own geometry and its own position in the mould architecture, it can create a systemic difference on the output result;

- multi-fixture machining: a part produced at the same time, but the produced parts are positioned in relation to the production tool by different fixture systems.

Since each fixture has its own geometry, mount clamps, etc., it can create a systematic difference on the output result;

- batch load treatments: heat treatment applied at the same time on a set of identical parts (the batch load), distributed within a pre-defined space of furnace. The position of an item of the batch relative to the furnace can influence the output result.

Each cavity, fixture, or position in the batch load corresponds to a different state. The multi-state process can be understood as the result of the combination of different states within the same process (e.g. cavity, fixture, position in the batch load).

NOTE It needs to be ensured that such systematic differences, if any, constitute only a very small proportion of permissible error so that their impact is harmless and do not affect the capabilities of the process.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534 (all parts), *Statistics — Vocabulary and symbols*

ISO 5725 (all parts), *Accuracy (trueness and precision) of measurement methods and results*

ISO 22514-3, *Statistical methods in process management — Capability and performance — Part 3: Machine performance studies for measured data on discrete parts*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

**3.1
production tool**
machine or production machinery performing all the operations necessary for production, from delivered supplies through product deliverable

**3.2
process**
set of interrelated or interacting activities which transforms inputs into outputs

Note 1 to entry: Inputs to a process are generally outputs of other processes.

Note 2 to entry: Processes in an organization are generally planned and carried out under controlled conditions to add value.

Note 3 to entry: A process where the conformity of the resulting product cannot be readily or economically verified is frequently referred to as a “special process”.

Note 4 to entry: This set includes all the factor resources necessary: production tools, labour, operating procedures, maintenance, etc.

[SOURCE: ISO 9000:2005, 3.4.1]

**3.3
equipment (or tools)**
interchangeable component of a production tool for producing different products, and which cannot be considered a wear-out component

EXAMPLE Mould of an injection-moulding machine — Counter-example: in machining, a cutting tool cannot be considered as a piece of equipment.

**3.4
process operation**
step in the production process leading to a final or intermediate product status

**3.5
toleranced characteristic of the product**
quantitative characteristic of the product, and for which the *upper specification limits* (ISO 3534-2) and/or *lower specification limits* (ISO 3534-2) are prescribed

3.6 **dispersion interval (or dispersion of a characteristic)** interval within which all items are produced

Note 1 to entry: Where the dispersion interval is estimated based on statistical methods, it is estimated based on its reference interval (ISO 3534-2).

Note 2 to entry: Any one process carries as many dispersion intervals as it does characteristics produced. For example, a product presenting four different characteristics, i.e. length, width, height, and weight, is produced including systematic (controllable) and random (uncontrollable) sources of variation by a single production tool in a single operation. This operation is thus associated with four different dispersion intervals.

3.7 **intrinsic dispersion interval (or instantaneous dispersion)** observable dispersion interval for a characteristic observed on produced items over a period during which the process implementation parameters have not varied: same operator, same method, same equipment, same batch of homogeneous raw materials, same temperature, etc.

Note 1 to entry: The underlying statistical distribution is called intrinsic (or instantaneous) distribution.

Note 2 to entry: In the event of drift in a process setting (such as drift caused by tool wear-out), it is common practice to include this drift in the production dispersion instead of integrating it into the estimate of intrinsic dispersion.

Note 3 to entry: This intrinsic dispersion interval is also called instantaneous dispersion because it affects production at a given point in time.

Note 4 to entry: Any one process operation carries as many intrinsic dispersion intervals as it does characteristics produced. A product presenting four different characteristics, i.e. length, width, height, and weight, is produced in a single operation. This operation is thus associated with four different intrinsic dispersion intervals.

Note 5 to entry: The intrinsic dispersion interval is same as natural or inherent spread.

Note 6 to entry: In some industries, the intrinsic dispersion interval is called “production tool dispersion”; the production tool featuring machine and its equipment.

3.8 **intrinsic factor** internal condition to the production process and involved in the intrinsic dispersion interval, and which has different aspects, each produced item comes across only one of these aspects

EXAMPLE 1 The cavities of a mould; each cavity defines one aspect of the “cavity” factor; machining fixtures assumed to be identical; each fixture defines one aspect of the “fixture” factor.

EXAMPLE 2 If two different speeds are applied during the same production process to manufacture the product (a first run at low speed followed by a second run at high speed), then the speed is not an intrinsic factor.

3.9 **process state** specific configuration of the full set of intrinsic factors, where each intrinsic factor takes one of its states

EXAMPLE See [Figure 1](#).

This setup involves six states:

- State C1 F1;
- State C2 F1;
- State C3 F1;
- State C1 F2;
- State C2 F2;

— State C3 F2.

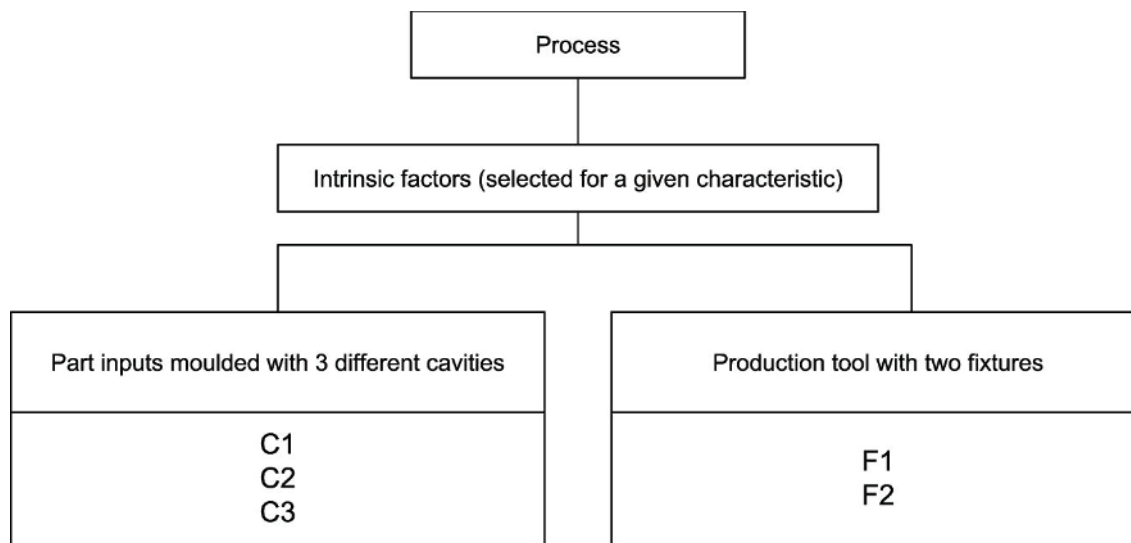


Figure 1 — Process with two intrinsic factors (cavity identifier, fixture identifier)

Note 1 to entry: One process state generates one statistical distribution. The statistical distributions tied to the different states of the process can be similar or different.

Note 2 to entry: For a simple process that only produces one part at a time using single equipment and a single quality of inputs, there will, a priori, be only one state. From the moment that several parts are produced simultaneously under different conditions (different equipment, different positions in the batch load, etc.), there will, a priori, be different states.

Note 3 to entry: If the production process simultaneously handles a set of p parts, then there can be p different states.

3.10 multi-state (production) process

process through which different states generate statistical distributions that have different dispersion widths and/or dispersion locations (see [Figure 2](#))

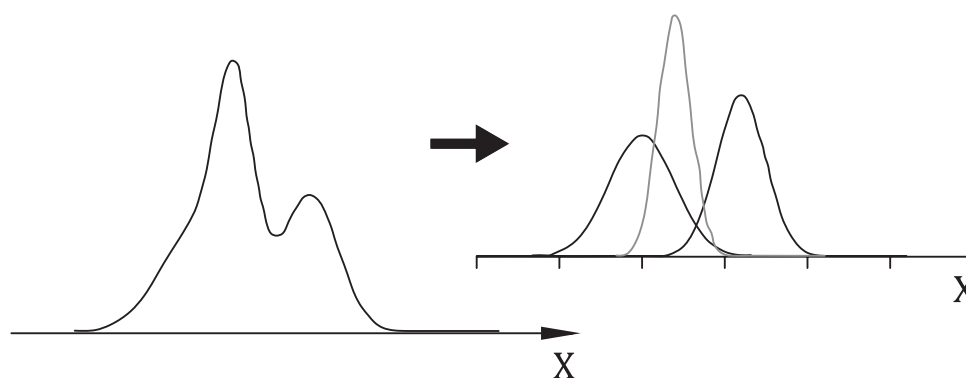


Figure 2 — Example illustrating the distribution of a multi-state process and its breakdown into states that each follow Gaussian distribution

3.11 local intrinsic dispersion (interval)

observable intrinsic dispersion interval associated with one of the process states
 EXAMPLES

— local intrinsic dispersion interval connected to mould No. 2 on cavity No. 5;

- local intrinsic dispersion interval at coordinates $X = 500, Y = 500, Z = 500$ inside the batch furnace;
- local intrinsic dispersion interval on the left-hand side of the conveyer over a given time slot in a continuous furnace.

3.12

global intrinsic dispersion (interval)

observable intrinsic dispersion interval when pooling the results of the combination of different process states

3.13

production dispersion (interval)

observable dispersion interval for a characteristic, observed on produced items over a typical representative period during which the different process implementation parameters may have varied

Note 1 to entry: The parameter that has varied in the production dispersion interval, can be, for example,

- change of operators,
- settings, or
- change in raw material batch, etc.

Note 2 to entry: The underlying statistical distribution is called production distribution.

Note 3 to entry: The production timeframe observed is not standardized. Depending on how the capability indices are used, this time frame is set either internally by the product manufacturer or contractually by agreement between supplier and customer.

3.14

measurement uncertainty

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand (quantity-of-interest)

Note 1 to entry: The parameter defining an interval bounding the result of a measurement and that can be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand, is called enlarged uncertainty.

[SOURCE: adapted to the VIM]

Note 2 to entry: This part of ISO 22514 works on the basis that enlarged uncertainty is evaluated at a 95,44 % confidence level, as is common practice.

4 Symbols and abbreviations

For the purposes of this document, the following symbols and abbreviations apply.

<i>L</i>	lower specification limit
<i>U</i>	upper specification limit
<i>T</i>	specified tolerance ($= U - L$)
<i>D_i</i>	generic term used to denote the width of intrinsic dispersion
<i>D_{ig}</i>	width of the global intrinsic dispersion
<i>k</i>	number of investigated states
<i>n</i>	sampling size (number of measured parts per state), which is considered the same for all states
<i>N</i>	total sample size (total number of parts sampled: $N = n \cdot k$)

Di_j	width of the local intrinsic dispersion in state j
Di_l	width of the local intrinsic dispersion when widths are not state-dependent
$Di_{l,j}$	half-width of the bottom-slope side of the local intrinsic dispersion interval in state j (see Figure 3)
$Di_{u,j}$	half-width of the top-slope side of the local intrinsic dispersion interval in state j (see Figure 3)
$Di_{l,el}$	half-width of the bottom-slope side of the local intrinsic dispersion interval of the state with the smallest dispersion bound
$Di_{u,er}$	half-width of the top-slope side of the local intrinsic dispersion of the state with the highest dispersion bound
Di_i	half-width of the bottom-slope side of the local intrinsic dispersion when widths are not state-dependent
Di_u	half-width of the top-slope side of the local intrinsic dispersion when widths are not state-dependent
$X_{\alpha \%}$	quantile at α % of the distribution without taking into account the different states
$X_{\alpha \%,j}$	quantile at α % of the distribution of state j
\bar{x}_j	mean of state j
\tilde{x}_j	median of state j
$\overline{\bar{X}}$	mean of the values of samples, once outliers have been eliminated
Δm	range of extreme locations of local intrinsic dispersion intervals when this difference is significantly different from zero
Δm^*	maximal value of the observable Δm when the relative locations of the different local intrinsic dispersion intervals vary independently of each other over time
Δa	amplitude of the outlier (expressed algebraically), in the event an outlier is recorded

5 Preliminary technical analysis of the process

5.1 General

The preliminary analysis is designed to determine the intrinsic factors and their aspects, and thus, to define the process states liable to be found.

For a process operation, an assumption should be made, per characteristic, on whether or not the process produces a multimodal distribution. If it is assumed to be multimodal, then it will be necessary to define the states liable to be found.

5.2 Identification of intrinsic factors

The analyst tasked with this upstream preliminary analysis shall first define the intrinsic factors liable to act as sources of differentiation and thus generate different statistical distributions. The analyst shall also define the various different aspects possible for each intrinsic factor identified.

EXAMPLES

- part positioning in relation to the machine (multi-station fixtures, heat treatment load);
- product-machine interface (different fixtures, different clampings, etc.);

- different mounting patterns (guide-slots, die cavities, positions on the conveyor, etc.).

5.3 Determination of process-specific states

The process analyst shall integrate different constraint factors (economic, organizational, etc.) so as to shortlist only the specific states deemed representative of extreme process states.

EXAMPLES

- for a heat treatment process involving a batch furnace, where each load contains 300 parts, the process analyst could, for example, select four positions from the 300 states possible: one at the furnace gate, one near the back of the furnace, and two near the centre (one at load centre and one at the top of the load), which the analyst's knowledge of the furnace tells them when are the extreme state effects;
- for a cylinder head machining process for which 400 fixtures (adapters) are used, the process analyst could only select six fixtures. Based on an analysis of the geometric surveys taken on these fixtures (defects tied to the support bases, defects tied to clamping quality, etc.), the analyst shall select a series of occurrences tied to these surveys (extreme cases, possibly combined with some intermediate cases).

One process operation can generate different observable characteristics. These characteristics are often impacted differently in different process states. This can guide the analyst to select states according to categories of characteristics.

At this point, the analyst shall arbitrate between the options to select the states. Their selection shall also take into account the process knowledge needed in order to lead root cause analysis as a step toward building a process tracking system.

The number of states envisaged, a priori, at this early stage can be revised when defining which sampling plan to apply. The aim is to determine the best trade-off between number of states investigated and the quality of the knowledge on each of these states.

Examples are presented in [Annex A](#).

6 Preliminary verifications before calculating the machine performance indices

6.1 Measurement system

Regardless of the process to qualify and the characteristic, it is first necessary to start by verifying that the measurement uncertainty is compatible with the pre-set capability objectives defining the maximum permissible global intrinsic dispersion.

This is done by running an estimate of measurement uncertainty taking into account the various sources of error: repeatability, reproducibility, bias error, etc.

This step can be determined using the GUM (Guide to the expression of uncertainty in measurement) and the ISO 5725 series of International Standards. The analysts can turn to an approach of the measurement uncertainty estimation set out, for example, in ISO 22514-7 or in the guide MSA (measurement systems analysis).

In order to move on to qualify the process, the measurement uncertainty shall not be too high. Excessively high measurement uncertainty impacts significantly on any estimates of real product dispersions, making it difficult to differentiate between process states.

A necessary prerequisite is that the enlarged uncertainty shall be less, for example, than a sixth of the maximum acceptable global intrinsic dispersion in order to make sure the judgement on the production tool does not become overly biased.

6.2 Definition of the sampling plan for estimating global intrinsic dispersion

The sampling plan is dictated by the number N of parts, on which measurements will be realized. It is often the case that N is set at around 30 or 50. In order to eliminate any time-related bias, the N parts shall be manufactured under the most identical conditions possible (although obviously excluding state switchovers), which therefore means in a short time span.

The number of states to be investigated is denoted as k . Number k can be different from the number initially planned for in the preliminary analysis, as N is often imposed. For each state, the sample size shall be identical and is equal to n . The n value shall never, under any circumstances, be less than 3 (5 is the value most widely used). N is calculated as $n \times k$.

If the production process simultaneously handles a set of products, then a sample is compiled of produced parts through different production cycles, where only the outputs from one of the selected states is studied. Since sample size is set at n , there will be n measurement campaigns with k measurements per campaign.

EXAMPLE In the case of a batch-load furnace heat treatment, for the validation of intrinsic dispersion, $n = 6$ parts per load will be sampled from $k = 5$ different loads. A sample of five parts is put together using parts sampled from the same in-furnace location but from five different loads. Two of the samples will be taken from the edges of the load but from opposite locations, plus two from the load centre and two from an intermediate position. Where the process involves particularly voluminous loads, the analyst can opt to take a higher number of samples but from a lower number of loads (parts per sample).

NOTE Definition of the sampling plan for estimating production dispersion (see ISO 3534-2).

Once global intrinsic dispersion has been estimated, the next step is to identify the process states leading to distributions that present extreme dispersion widths or locations. These states will serve as the basis for estimating production dispersion.

EXAMPLE In the case of a continuous furnace treating several parts in parallel or a batch-load furnace heat treatment, the very first step is to validate the intrinsic dispersion. It is only when the intrinsic dispersion has been validated that it becomes possible, if necessary, to validate production dispersion. The definition of the sample for validating production dispersion is actually dependent on the results recorded during the intrinsic dispersion validation step.

7 Estimation of global intrinsic dispersion and calculation of machine performance indices

7.1 General

Global intrinsic dispersion results from a mixing of local intrinsic dispersions, integrating their locations and widths plus the possibility that a value detected can be an outlier yet having a real physical effect. Each local intrinsic dispersion linked to a given state is defined by its central tendency (location) and its width. This makes it possible to define the upper and lower bounds of the local intrinsic dispersion of each state. For each state indexed j , the following estimators should be calculated (see [Figure 3](#)):

- the location of the lower bound of the local intrinsic dispersion in state j : $X_{0,135 \% ,j}$;
- the location of the upper bound of the local intrinsic dispersion in state j : $X_{99,865 \% ,j}$;
- which gives the width of the local intrinsic dispersion, $Di_j = X_{99,865 \% ,j} - X_{0,135 \% ,j}$;
- and the half-widths of dispersion:
 - lower-bound local intrinsic dispersion, $Di_{l,j} = X_{50 \% ,j} - X_{0,135 \% ,j}$;
 - upper-bound local intrinsic dispersion, $Di_{u,j} = X_{99,865 \% ,j} - X_{50 \% ,j}$;

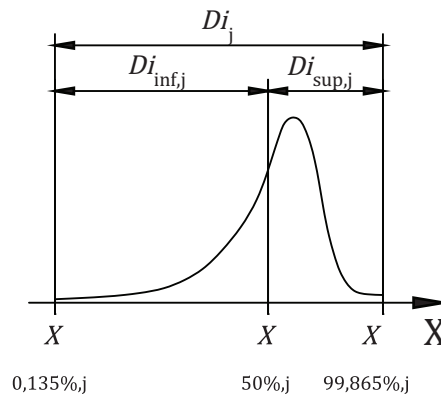


Figure 3 — Parameters estimated for each state in relation to each local intrinsic dispersion

The multi-state process has to be split up into a set of states before capability indices related to global instantaneous dispersion can be determined (there is no indicator available for local instantaneous dispersion).

In order to achieve this splitting up, it is first necessary to check that there are no outliers liable to corrupt the later estimates produced. Next, the widths of local intrinsic dispersions shall be estimated before their locations can be compared. Statistical hypothesis testing is performed for this cross-comparison step.

The statistical tests applied in this part of ISO 22514 employ a default 5 % significance level. A 5 % significance level corresponds to the risk to reject wrongly the hypothesis.

If the calculated value of the statistic is greater than the tabulated value at 5 % level of significance, then the hypothesis is rejected, otherwise accepted.

7.2 Verification on the absence of outliers in the set of made measurement results

For each drawn sample on a process state, a Grubbs' test for outliers is applied (see [Annex B](#)) and then repeated a second time but on the full data set (all process states collected).

If an outlier is detected, it shall be eliminated from the sample, after which the Grubbs' test is repeated again and again if necessary, up until no more outliers are detected. Care needs to be taken that no more than one third of the data gets eliminated in the process.

Whatever the results of this Grubbs' test for outliers, it is important not to question the whole set of values from all samples; instead, continue with the analysis.

Each time an outlier is detected, determine whether the outlier is in order by

- a data transcription error,
- a measurement error, and
- a physical reality.

Depending on the case (dealt with in order)

- a) if the outlier is a data transcription error: if the original intended value is known, it shall be inserted to replace the outlier value. Otherwise, the following rule (b) is applied.
- b) if the outlier is a measurement error: if the part concerned can be re-measured, then the outlier value shall be replaced by the new result of measurement; otherwise, it shall be excluded. Furthermore, the measurement process shall need to be made reliable.

- c) if the outlier is a physical reality: first, estimate the effect of the outlier by calculating the difference between the outlier value and the mean of the other values for the state concerned. This difference is given the symbol Δa . Second, this outlier value is excluded from the statistical treatments tied to local intrinsic dispersion (width and location), but to end with, it is, nevertheless, integrated into the estimation of global intrinsic dispersion.

If the test reveals several outliers related to a physical reality, then a more advanced analysis shall need to be led on the causes of the outliers in order to be in a position to even consider estimating global intrinsic dispersion.

7.3 Determination of the widths of local intrinsic dispersions

The width of intrinsic dispersion is estimated (via its reference interval) for each process state, on the data for any one given sample. This width is split into two half-widths (upper and lower), which can be different if the distributions are asymmetrical (see [Figure 4](#)).

Depending on the type of characteristic under analysis, it should be assigned an a priori distribution. The same characteristic for a given production process is considered to share the same type of distribution, regardless of process state.

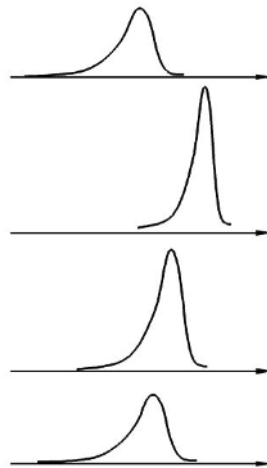


Figure 4 — Illustrations of state distributions

These distribution selections are not tested for goodness-of-fit test, the size n of each sample is often too low. The choices are thus made based on the type of characteristic under analysis and on the analyst's experience.

EXAMPLES

- for a dimensional characteristic, a normal distribution;
- for a geometric characteristic, a Rayleigh distribution, a non-uniform distribution, a Weibull distribution, or a Galton distribution (log-normal off-centred);
- for a characteristic defined as the impurity burden in a solution, a Galton distribution.

Thus, the width of local intrinsic dispersion D_{ij} is then estimated for each process state j .

The various different estimations, each tied to a process state, are then cross-compared. The aim is to answer the question, "Can we accept (within a specified risk) that the widths of the local intrinsic dispersions are identical, regardless of the process state considered?"

To answer this question, it is recommended to cross-compare variances using a Bartlett's test, if there are more than two process states, or a Fisher's test, if there are only two process states (see [Annex B](#)).

If this hypothesis can be accepted, then all the widths of local intrinsic dispersions are considered equal to a value that is calculated according to the system given in [Annex B](#).

If this hypothesis cannot be accepted, there are two possible scenarios:

- to identify the cause of the difference and eliminate this cause from the production process, in which case the previous protocol should be repeated, either partially or in full depending on the cause suggested (cylinder head, for example);
- to accept this cause as being inherent to the production process, in which case it is possible to consider whether or not to pool together several different local intrinsic dispersions (that all share the same causal effect). This makes it possible to define the final conclusive retained dispersion widths, by providing a narrower confidence interval (see the example set out in [Annex A.2](#)).

NOTE 1 [Annex B](#) also gives the conventional limits for the Bartlett's test.

NOTE 2 The Bartlett's test can be dropped and replaced by a Levene's test.

7.4 Determination of the locations of local intrinsic dispersions

The analysis of the locations of local intrinsic dispersions shall not be performed until the analyst has first determined the widths of these dispersions and, if these widths are deemed non equal, led the indispensable process investigations.

The location of local intrinsic dispersion $X_{50\%,j}$ is estimated (via its mean \bar{x}_j or its median \tilde{x}_j) for each process state, on the data for any one given sample. By default, this location is estimated via the median, but where the distribution is symmetrical or close to symmetrical, the mean shall be preferred to the median as location estimator.

The different location estimations, each in relation to a process state, are cross-compared if and only if the widths of local dispersions have been deemed identical. If this is not the case, then the location estimations are not cross-compared if there are more than two process states.

If the locations of dispersion intervals are compared by running a goodness of fit test, then the aim of the fit-test is to answer the question, "Can we accept (within a specified significance level) that the locations of the local intrinsic dispersions are identical, regardless of the process state considered?"

To answer this question, it is recommended to use the following:

- an F-test if more than two states are being tackled, and if the variances of the states have been estimated to be the same;
- if only two states are being tackled, and if the variances of the states have been estimated to be:
 - the same, then a *t*-test, and
 - different, then an aspin-welch's test (*t*-test with unequal variances, also called paired *t*-test).

[Annex B](#) describes these tests and the limits to their use.

If this hypothesis can be accepted, then all the locations of local intrinsic dispersions are considered equal to the mean, in this case, the difference between the locations of local dispersion interval is considered equal to zero ($\Delta m = 0$).

If this hypothesis can no longer be accepted, then the analyst should look to integrate the observed differences between the extreme means or between the extreme medians into the estimation of the range of extreme locations of local intrinsic dispersion intervals (Δm).

If the hypothesis of equal locations has not been tested (more than two states and unequal variances), the range of extreme locations of local intrinsic dispersion intervals (Δm) is considered equal to the range in the extreme locations of these states.

7.5 Global intrinsic dispersion: type and estimation

Global intrinsic dispersion corresponds to the range covered by all the local intrinsic dispersions combined together (widths and locations), plus the effect of any outliers having a physical reality (see [Figure 5](#)).

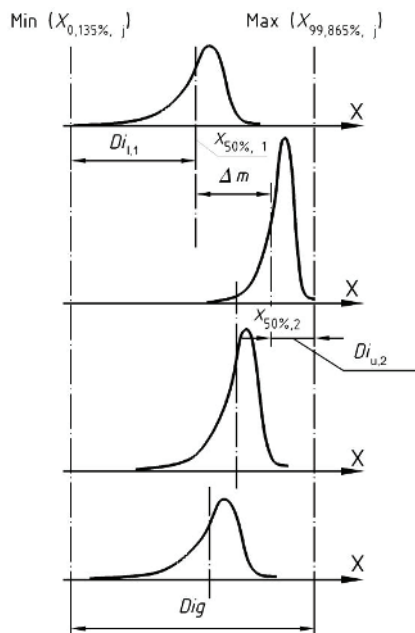


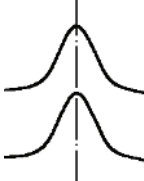
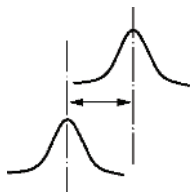
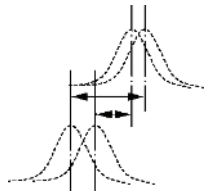
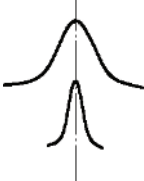
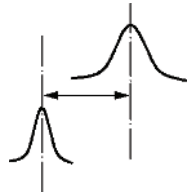
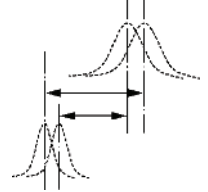
Figure 5 — Illustration of global intrinsic distribution based on local intrinsic distributions

The results on the homogeneity of the widths and the locations of local intrinsic dispersions, as obtained via the previous sections, can be used to determine the type of global intrinsic dispersion that matches the production process.

In cases where the locations of the local dispersion intervals are estimated as different ($\Delta m \neq 0$), then analysis should be conducted to decide whether this difference Δm is liable to vary quickly or, conversely, whether is expected to remain stable over time. This analysis has an impact on how capability indices will be calculated.

The different types of global intrinsic dispersion are defined in [Table 1](#).

Table 1 — Definitions of different types of global intrinsic dispersion

Type of process	Widths of local intrinsic dispersions (D_{ij})	Estimation of the differences in locations of local intrinsic dispersions (Δm)	Type of global intrinsic production dispersion	Illustration
Uni-modal	equal	$\Delta m = 0$	See ISO 22514-3	
	equal	$\Delta m \neq 0$ and is deemed constant	Type 1	
		$\Delta m \neq 0$ and is deemed variable	Type 2	
Multi-modal	different	$\Delta m = 0$	Type 3	
		$\Delta m \neq 0$ and is deemed constant	Type 4	
		$\Delta m \neq 0$ and is deemed variable	Type 5	
NOTE The illustrations in this table show the distributions of two extreme states and, where appropriate, their variations (variable differences in location).				

The bounds of local intrinsic dispersion intervals shall be cross-compared on both bottom-slope side and top-slope side in order to define the lowest-performance state on each side (denoted g on the bottom slope and d on top slope).

If there are no outliers having a real physical effect, the global intrinsic dispersion is equal to the interval between the extreme-left lower bound and the extreme-right upper bound (the extreme-left and extreme-right locations of the points $X_{0,135 \% ,j}$ and $X_{99,865 \% ,j}$).

If there is an outlier having a real physical effect, the Δa value shall be integrated in the estimation of global intrinsic dispersion.

There are two scenarios:

- either the outlier Δa can only occur in one direction, in which case:
 - if Δa is negative, it shall be added as an absolute value, but only to all the $Di_{l,j}$ values,
 - if Δa is positive, it shall only be added to all the $Di_{u,j}$ values;
- the outlier Δa can occur in both directions, in which case:
 - the absolute value for Δa shall be added to all the $Di_{l,j}$ values and to all the $Di_{u,j}$ values.

This operation modifies the widths of the local intrinsic dispersions of each state: $Di_j = Di_{l,j} + Di_{u,j}$. It is these new estimations that shall be integrated into the calculation of the capability indicators.

7.6 Calculation of capability indices P_m and P_{mk}

Estimations of multi-state process capability indices cannot be globalized without picturing all the states in their entirety. These indices are therefore equally dependent on extreme states.

The standards in place today hinge around two types of indices (see ISO 22514-1):

- indications of potential, which compare the width of the dispersion with the width of the tolerances limits (i.e. the tolerance interval), i.e. P_m ;
- indicators that also integrate the location of the dispersion: i.e. P_{mk} ;

Table 2 — Calculation of capability indicators

Type of global intrinsic production dispersion	Estimation of P_m or P_p depending on the type of production process observation	Estimation of P_{mk} or P_{pk} depending on the type of production process observation $P_{mk} = \min(P_{mk,l}; P_{mk,u})$
Type 1 $\Delta m \neq 0$ is constant Widths of local Di are equal	$\frac{T - \Delta m}{Di_l + Di_u}$	$P_{mk,u} = \frac{U - \max(X_{50\%,j})_{1 \leq j \leq k}}{Di_u}$
Type 2 $\Delta m \neq 0$ is variable Widths of local Di are equal	$\frac{T}{Di_l + Di_u + \Delta m^*}$	$P_{mk,l} = \frac{\min(X_{50\%,j})_{1 \leq j \leq k} - L}{Di_l}$
Type 3 $\Delta m = 0$ Widths of local Di are unequal	$\frac{T}{\max(Di_j)_{1 \leq j \leq k}}$	$P_{mku} = \frac{U - X_{50\%}}{\max(Di_{u,j})_{1 \leq j \leq k}}$ $P_{mkl} = \frac{X_{50\%} - L}{\max(Di_{l,j})_{1 \leq j \leq k}}$

Δm^* When the relative locations of the various different local intrinsic dispersions are considered to vary independently of each other over time, it is possible to overestimate the difference Δm observed in the calculation of P_m . This overestimation is defined by the process analysts who use their experience to evaluate the maximum Δm liable to be encountered. This deviation is denoted via the symbol Δm^* .

Table 2 (continued)

Type of global intrinsic production dispersion	Estimation of P_m or P_p depending on the type of production process observation	Estimation of P_{mk} or P_{pk} depending on the type of production process observation $P_{mk} = \min(P_{mk,l}; P_{mk,u})$
Type 4 $\Delta m \neq 0$ is constant Widths of local Di are unequal	$\frac{T - \Delta m}{Di_{l,el} + Di_{u,er}}$	$P_{mku} = \frac{U - \max(X_{50\%,j})_{1 \leq j \leq k}}{\max(Di_{u,j})_{1 \leq j \leq k}}$
Type 5 $\Delta m \neq 0$ is variable Widths of local Di are unequal	$\frac{T}{\max(Di_{l,j})_{1 \leq j \leq k} + \max(Di_{u,j})_{1 \leq j \leq k} + \Delta m^*}$	$P_{mku} = \min(P_{mku,j})_{1 \leq j \leq k}$ <p>where</p> $P_{mku,j} = \frac{U - X_{50\%,j}}{Di_{u,j}}$ $P_{mkl} = \min(P_{mkl,j})_{1 \leq j \leq k}$ <p>where</p> $P_{mkl} = \frac{X_{50\%,j} - L}{Di_{l,j}}$
Δm^* When the relative locations of the various different local intrinsic dispersions are considered to vary independently of each other over time, it is possible to overestimate the difference Δm observed in the calculation of P_m . This overestimation is defined by the process analysts who use their experience to evaluate the maximum Δm liable to be encountered. This deviation is denoted via the symbol Δm^* .		

7.7 Acceptance thresholds for the machine performance indices

Machine performance, process capability, and process performance indices have objectives that are set according to various different criteria:

- reasons for which the dispersions estimated during the analysis are liable to evolve, and quantification of the likely evolution (the aim is to ensure that all products will stay conform to specification over the long term);
- difficulties involved in attempting to bring global intrinsic dispersion back to its original state;
- degree of guaranteed conformity according to how important the characteristic is to customer expectations;
- policy adopted to determine the tolerance interval.

The acceptance thresholds to be applied can take into account the quality of the statistical estimators incorporating the size of the samples using for determining states, or possibly even the full sample size (see ISO 22514-2).

It is important to understand the difference between target machine performance indices and acceptance threshold.

However, at this point, it is useful to underline that when it is calculated, a machine performance index from a sample having, for example, a sampling size of 30, the machine performance index P_m is estimated by \hat{P}_m . If a 95 % confidence interval of the machine performance index (P_m) is to be calculated then with a sample size of 30 items, the performance machine index is contained in an interval of plus or minus 26 % of the value of the estimation of the machine performance index P_m (calculated value) around the same value.

This is what prompts the recommendation to use at least 30 product copies in order to minimize the risks while maintaining a workably low number of measurements. As the number of states increases, the number of produced items shall also be increased.

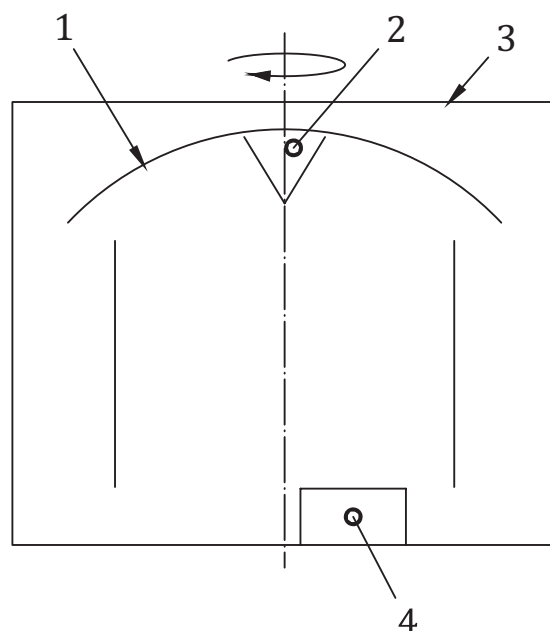
Annex A (informative)

States qualifying a processing process

A.1 Qualification of a processing process using a vacuum chamber

A.1.1 Overview of the process

The principle guiding the processing of parts in a vacuum chamber is illustrated in [Figure A.1](#). Injection guns spray a material onto products attached to a rotary bell that has clear-marked positions.



Key

- 1 rotary bell
- 2 vacuum chamber
- 3 diffuser
- 4 coatings evaporator

Figure A.1 — Sketch of the principle of the coating system device

Various different responses are measured: coating thickness, visual appearance metrics, etc.

The products targeted for processing are positioned on a bell jar. The bell jar spins at a constant rotation speed throughout the process. The process is controlled by an automated system.

One rotary bell contains an average of 150 products.

A.1.2 Characteristic targeted for analysis and threshold

This example focuses on a single characteristic, the thickness of the material coating, which is measured in microns and for which the post-treatment tolerance interval specified is 25,0 μm to 45,0 μm .

In the example quantified here, the inspection system has an expanded uncertainty of 1.

The target set for production tool capability is over 1,33.

Expanded uncertainty is less than $(T/1,33)/6 = 2,5$.

A.1.3 Intrinsic factors

A single intrinsic factor has been identified for this operation: product position on the bell jar.

Given that there are 150 possible positions, there are 150 states.

This leads into the following question, “Is part position in the rotary bell an influential factor (on average or on dispersion)?”

A.1.4 Process states

The process analyst suggests selecting only three of these 150 states.

The process states selected are

- product positioned at the centre of the bell, in position 1 (state C),
- product positioned intermediately on the bell, in position 50 (state I), and
- product position at the outer perimeter of the bell, in position 145 (state P).

Each of these states is assumed to be normally distributed.

A.1.5 Sampling plan

The following sampling plan is applied.

Over 10 successive production cycles on 150 products, parts are sampled at each cycle from the identified positions, i.e. three parts per cycle to form 10 samples.

NOTE It is important not to confuse sample with state. This is why the analyst shall not sample 10 parts from the outer bell periphery during the same cycle, as there would be no means of assessing the cycle-to-cycle fluctuations that are intended to be estimated.

A.1.6 Measurements obtained

Table A.1 — Measurements obtained

Cycles	Samples		
	State P	State I	State C
1	26,3	31,5	35,6
2	25,8	32,3	35,1
3	27,3	30,0	36,3
4	28,1	32,4	37,4
5	26,9	31,3	36,0
6	26,4	31,1	35,5
7	27,4	29,4	36,6
8	26,5	29,6	37,3
9	27,7	31,5	35,9
10	24,7	32,5	37,9

A.1.7 Analysis of the results of the measurement campaign

A.1.7.1 Screening for outliers

The Grubbs' test for outliers is performed on three subsamples of 10 values and on all 30 values.

For the 10-value subsamples, the critical value of Grubbs' test statistic at 5 % significance level = 2,289 947 and the calculated values of Grubbs' test statistic calculated for each 10-measurement subsample are 2,016, 1,539, and 1,671.

Across the full data set of 30 measurements, the critical value of Grubbs' test statistic at 5 % significance level = 2,908 (rounded value of 2,908 47) and the calculated value of Grubbs' test statistic = 1,624.

Conclusion: no outliers detected, which means no values to be eliminated for the later analyses.

A.1.7.2 Influence of state

Table A.2 — Statistics per state

	State P	State I	State C
Mean	26,710	31,160	36,360
Standard deviation	0,997	1,143	0,922

Test for homogeneity of the widths of local intrinsic dispersions using Bartlett's test. The critical value of Bartlett's test statistic at 5 % significance level is 5,991 and the calculated value of Bartlett's test statistic is 0,414 (calculated *p*-value for Bartlett's test statistic is 0,813 which is greater than 0,05).

Conclusion: it can be accepted that the widths of local intrinsic dispersions are identical, and their standard deviation can be estimated as 1,01.

Goodness of fit test for homogeneity of the locations of local intrinsic dispersions, Fisher's test:

- The critical value of Fisher's test statistic at 5 % significance level is 3,35 and the calculated value of Fisher's test statistic is 222 (calculated *p*-value for Fisher's test statistic is lower than 0,001 and by consequence not greater than 0,05);
- Conclusion: it cannot be accepted that the locations of local intrinsic dispersions are identical. Difference in locations is estimated at $\Delta m = 9,65 \mu\text{m}$ (range of the local location estimation: 36,36 – 26,71).

An illustration of the widths and the location of the local intrinsic dispersions for respectively the states P, I, and C is given, relatively to the data given in [Tables 1](#) and [2](#), to show that the test determine the locations of local intrinsic dispersions are clearly different and their widths are relatively comparable. (see [Figure A.2](#) below).

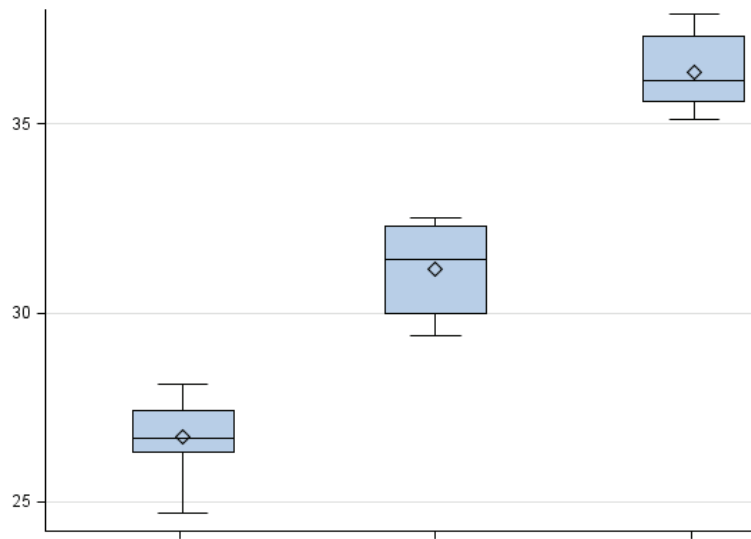


Figure A.2 — Widths and the location of the local intrinsic dispersions

A.1.7.3 Capability indices

Another sample was taken from outer periphery position, P (Position 120); there was no difference between outer periphery positions. It is therefore concluded that the mean difference Δm is tied to position and to device configuration. This therefore places us in a type-1 configuration, constant mean difference, with widths of local intrinsic dispersions considered equal.

The difference is

$$P_m = \frac{T - \Delta m}{Di_1} = \frac{20 - 9,65}{6 \times 1,01} = 1,69$$

The toughest local intrinsic dispersion with the worst performance is the outer-periphery dispersion:

$$P_{mk} = P_{mk,l} = \frac{\min(X_{50\%,j})_{1 \leq j \leq k} - L}{Di_1} = \frac{26,71 - 25}{3 \times 1,01} = 0,56 \quad (\text{with estimation of } X_{50\%} \text{ via the mean value})$$

This is not acceptable (being $P_{mk} < 1,33$).

At this juncture, since P_m is satisfactory (being $P_m > 1,33$), there are two solutions possible:

- coat more material by increasing processing time. However, although this sounds an easy solution to implement, it carries added processing time and material costs;
- change the process configuration to bring the means at centre closer to the means at the outer periphery. This solution needs more effort to implement, but is more cost efficient.

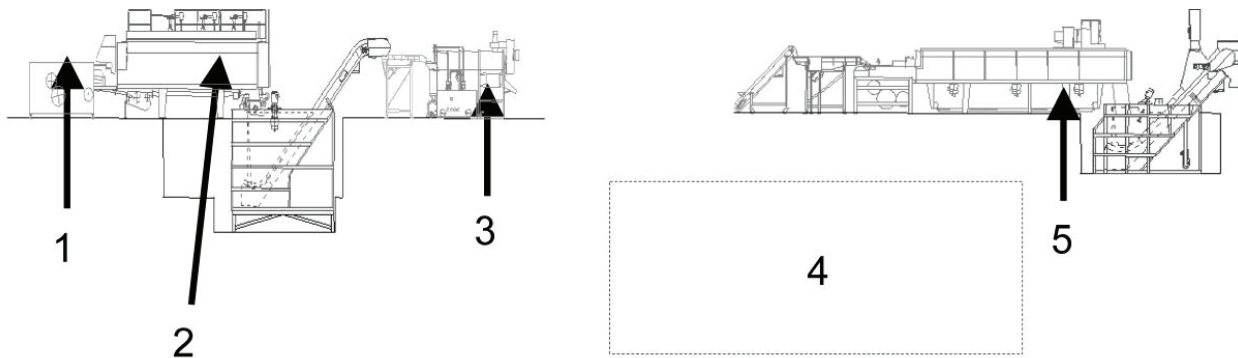
A.2 Qualification of a heat treatment process using a continuous conveyor furnace assumed to be a multi-state process

The continuous conveyor furnace is a production tool capable of being adapted for use in heat treatment processing.

This example works with a per-campaign treatment. It uses P_p and P_{pk} indicators rather than P_m and P_{mk} since the full campaign is considered a production line.

A.2.1 Overview of the process

The principle guiding the processing of parts in a conveyor furnace is illustrated in the figure below:



Key

- 1 load
- 2 hardening furnace
- 3 wash-down machine or wash-down/rinse-down tank
- 4 certain processes (e.g. bainitic hardening) do not employ the tempering furnace
- 5 tempering furnace

Figure A.3 — Sketches of principle for continuous-furnace heat treatments

The steel parts are loaded in the hardening furnace on a moving conveyor. The conveyor travels at a certain speed through a tunnel furnace which is configured with pre-set operating settings (temperatures, gas flow rates, etc.); the parts are processed in the furnace (austenitization step) before being oil-quenched in a step-dedicated oil.

They are then degreased and, where necessary, tempered following the same procedural principle as for the austenitization step. The aim of this processing step is to provide the required mechanical characteristics.

The operation selected for process qualification is the operation during which any slight variability can have a major impact on the metallurgical and mechanical properties of the parts, i.e. the austenitization step.

A.2.2 Characteristic targeted for analysis

This example tracks a single characteristic: Rockwell hardness, for which the specified tolerances (post-hardening) are [55 HRC, 60 HRC].

In the example quantified here, the hardness measurement process has an expanded measurement uncertainty of 0,5 HRC. This value remains under $1T/6$ ($5/6 = 0,84$), therefore, the measurement system is, a priori, acceptable.

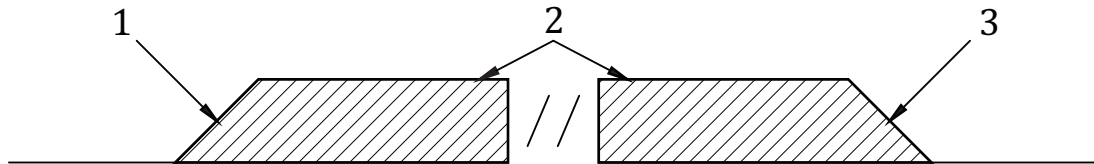
A.2.3 Intrinsic factors

There are a total of two intrinsic factors identified for the process operation selected

- part position on the conveyor (left - middle - right), and
- timepoint during the campaign (also termed “series” or “convoy”) where the part is processed (start - main body of production - end).

NOTE Each time there is a change of batch or reference part, the conveyor is completely emptied before launching the following series:

- Is part position on the conveyor an influential factor (on average or on dispersion)?
- Is there an influence of the head or tail of the series (or “campaign” or “convoy”, depending on the business terminology used), such as that due to energy input per part?
- Are there any in-production fluctuations?



Key

- 1 start of the series
- 2 main body of production
- 3 end of series

Figure A.4 — Part stacking during a continuous treatment process

A.2.4 Process states

Given that under normal operational process conditions, the parts are all piled up, the process analyst designs the following two-step process qualification approach:

- phase one: cross-analysis of process behaviour at campaign start and campaign end;
- phase two: building on the results of the phase-one analysis, the analyst defines sampling protocol and process states to analyse the main body of production.

NOTE The left-side, middle, and right-side states can only be comfortably implemented at campaign start or campaign end. States tied to in-pile position are difficult to capture since it would be necessary to arrange the parts while identifying their positions within the pile.

The selected process states in phase one of the analyses are the following:

- for the start of a series, the position of parts across the width of the conveyor, for which three aspects were selected (states B_L , B_M , and B_R);
- for the end of a series, the position of parts across the width of the conveyor, for which three aspects were selected (states E_L , E_M , and E_R);

The selected process states in phase two of the analysis are the following:

- if phase one identified that the “position across the width of the conveyor” factor has no effect on the location or the width of local intrinsic dispersions, then parts are sampled randomly to form a sample set at a timepoint t ;
- if this is not the case, then a more detailed experimental protocol needs to be designed, involving the identification of all part positions. Since it is extremely difficult to recover all the parts pre-identified and pre-positioned on the conveyor, this heavier experimental protocol shall only be envisaged if the position-related differences are assessed to be important in terms of the tolerance interval ($\Delta m > IT/6$).

A.2.5 Sampling plan

The following sampling plan is applied:

- within any one given series, three parts are sampled for each of the six states, B_L, B_M, B_R, E_L, E_M, and E_R. This first sample set shall gauge the influence of part position on the conveyor and of the timepoint at which they enter the cycle;
- within at least five series, three parts are randomly sampled during the steady output rate. The analysis of this second sample set shall further refine the influence of the timepoint at which parts enter the cycle and make it possible to determine capability indicators for the selected process operation.

A.2.6 Phase-one analysis (start — end of series + on-conveyor position)

A.2.6.1 Measurements obtained (start — end of series)

Table A.3 — Measurements obtained (start — end of series)

Beginning of the series			End of the series		
Left	Middle	Right	Left	Middle	Right
59	58,4	58,3	58,3	58,3	58,3
58,2	58,5	58,6	58,6	58,6	58,6
58,7	58,6	58,5	58,5	58,5	58,5
58,5	58,5	59	59	59	59
58,4	58,4	58,6	58,6	58,6	58,6
58,9	58,4	58,6	58,6	58,6	58,6

Before continuing with the analysis, the process analyst shall check that there are no outliers liable to hamper the later analyses. This is done by running one Grubbs' test for outliers on six subsamples of six values and another on all 36 values.

For the 6-value subsamples, the critical value of Grubbs' test statistic at 5 % significance level is 1,887 (rounded value of 1,887 147) and the calculated values of Grubbs' test statistic calculated for each 6-measurement subsample are 1,361, 1,633, 1,754, 1,754, 1,754, and 1,754.

Across the full data set of 36 measurements, the critical value of Grubbs' test statistic at 5 % significance level is 2,991 (rounded value of 2,990 584) and the calculated value of Grubbs' test statistic = 1,940.

In conclusion, no outliers were detected, which means no values to be eliminated for the later analyses. The analyst can validate the measurements and continue with the qualification process.

A.2.7 Results analysis (beginning — end of series)

Table A.4 — Statistics per state

State	B _L	B _M	B _R	E _L	E _M	E _R
Mean	58,617	58,467	58,600	58,600	58,600	58,600
Standard deviation	0,306	0,082	0,228	0,228	0,228	0,228
Range	0,8	0,2	0,7	0,7	0,7	0,7

The first step is to test whether the hypothesis of homogeneity of the widths of local intrinsic dispersions can be accepted. A Bartlett's test is run for this purpose:

- The critical value of Bartlett's test statistic at 5 % significance level is 11,070 and the calculated value of Bartlett's test statistic is 6,470 (calculated p -value for Bartlett's test statistic is 0,263 which is greater than 0,05).
- Conclusion: it can be accepted that the widths of local intrinsic dispersions are identical and their standard deviation can be estimated as 0,227 HRC.

The second step is to test whether the hypothesis of homogeneity of the locations of local intrinsic dispersions can be accepted. A Fisher's test is run for this purpose:

- The critical value of Fisher's test statistic at 5 % significance level is 2,53 and the calculated value of Fisher's test statistic is 0,369 (calculated p -value for Fisher's test statistic is 0,66 which is greater than 0,05).

We can therefore accept that the pairs (position of parts across the width of the conveyor, campaign start, or campaign end) have no effect

- on the width of local intrinsic dispersions and
- on the location of local intrinsic dispersions.

The analyst decides to continue with the investigation by confirming that only the time state is observed in the phase-two analysis.

NOTE 1 The analyst could have opted to segment this analysis into the following three sub-analyses:

- a) at campaign start, cross-compare the locations and widths of local intrinsic dispersions at all three positions;
- b) at campaign end, cross-compare the locations and widths of local intrinsic dispersions at all three positions;
- c) compare campaign start against campaign end.

This would have led the analyst to the same conclusion. However, had the six-state analysis revealed a significant difference, then the analyst would have had to lead a technical analysis on this difference, as described above, to correlate the deviation recorded with an effect factor, campaign start or end, or position on the conveyor.

NOTE 2 For each factor analysed, if the test for homogeneity of variances fails, it still remains possible to test for equality of means, provided that the test for equality of variances is positive at 99 % (rather than 95 %).

A.2.8 Phase-two analysis (main body of production)

A.2.8.1 Measurements obtained — phase two

Table A.5 — Measurements obtained — Phase two

	Any position across the width of the conveyor						
	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7
Main body of production	58,1	58,3	58,0	58,0	57,8	57,7	58,2
	57,8	57,6	57,9	57,8	58,0	57,3	57,8
	58,2	57,5	58,3	58,5	57,4	57,0	58,2

Before continuing with the analysis, the process analyst shall check that there are no outliers liable to hamper the later analyses in this second phase. This is done by running one Grubbs' test for outliers on seven subsamples of three values and another on all 21 values.

For the 3-value subsamples, the critical value of Grubbs' test statistic at 5 % significance level is 1,154 and the calculated values of Grubbs' test statistic calculated for each 3-measurement subsample are 1,121, 1,147, 1,121, 1,109, 1,091, 1,044, and 1,155.

Across the full data set of 21 measurements, the critical value of Grubbs' test statistic at 5 % significance level is 2,734 and the calculated value of Grubbs' test statistic is 2,327.

Sample 7 presents a significantly higher outlier value at 5 % significance level but not at a 1 % significance level, and the test remains negative for the full 21-measurement data set.

In conclusion, the analyst considers that no outliers were detected, which means no values to be eliminated for the later analyses. The analyst can validate the measurements and continue with the qualification process.

NOTE 1 An ANOVA analysis can be used to evaluate the variances from various components.

NOTE 2 At a 5 % significance level, if there is no outlier, the risk of raising the alarm is 5 %. As the test is run seven times, the probability that it will yield at least one positive response is equal to $1 - 0,957 = 30\%$, which is why the analyst does not take the value as an outlier.

A.2.8.2 Results analysis — Phase-two analysis (main body of production)

Table A.6 — Statistics per sample

Sample Nbr.	1	2	3	4	5	6	7
Mean	58,033	57,800	58,067	58,100	57,733	57,333	58,067
Standard deviation	0,208	0,436	0,208	0,361	0,306	0,351	0,231

The first step is to test whether the hypothesis of homogeneity of the widths of local intrinsic dispersions can be accepted. A Bartlett's test is run for this purpose:

- The critical value of Bartlett's test statistic at 5 % significance level is 12,59 and the calculated value of Bartlett's test statistic is 1,71 (calculated *p*-value for Bartlett's test statistic is 0,94 which is greater than 0,05).
- Conclusion: it can be accepted that the widths of local intrinsic dispersions are identical and their standard deviation can be estimated as 0,306 HRC.

The second step is to test whether the hypothesis of homogeneity of the locations of local intrinsic dispersions can be accepted. A Fisher's test is run for this purpose:

- The critical value of Fisher's test statistic at 5 % significance level is 2,85 and the calculated value of Fisher's test statistic is 2,42 (calculated *p*-value for Fisher's test statistic is 0,094 which is greater than 0,05).

Thus, in this experiment, it can be accepted that positions in time and across the conveyor

- have no effect on the widths of local intrinsic dispersions and
- have no effect on the locations of local intrinsic dispersions.

NOTE For this calculation, the process is accepted as stable, but the phenomenon should be monitored closely.

The analyst then decides to compare the results obtained between campaign-start and campaign-end transient output rates and the "steady" output rate (main body of production), with each output rate corresponding to a state.

Table A.7 — Comparison of results between campaign-start and campaign-end transient output rates and the “steady” output rate

Steady-rate state	Start state	End state
58,1	59,0	58,3
57,8	58,2	58,6
58,2	58,7	58,5
58,3	58,5	59,0
57,6	58,4	58,6
57,5	58,9	58,6
58,0	58,4	58,3
57,9	58,5	58,6
58,3	58,6	58,5
58,0	58,5	59,0
57,8	58,4	58,6
58,5	58,4	58,6
57,8	58,3	58,3
58,0	58,6	58,6
57,4	58,5	58,5
57,7	59,0	59,0
57,3	58,6	58,6
57,0	58,6	58,6
58,2		
57,8		
58,2		

Statistical tests shall be run on the homogeneity between widths and locations of local intrinsic dispersions.

The first test run is a Bartlett’s test:

- The critical value of Bartlett’s test statistic at 5 % significance level is 5,991 and the calculated value of Bartlett’s test statistic is 7,270 (calculated p -value for Bartlett’s test statistic is 0,026 which is not greater than 0,05);
- Conclusion: it can be accepted that the widths of local intrinsic dispersions are not identical.

NOTE The p -value being upper than 1 %, this can lead us into running a Fisher’s test, even though, a priori, the variances cannot be considered equal at a 5 % significance level.

The second step is to test whether the hypothesis of homogeneity of the locations of local intrinsic dispersions can be accepted. A Fisher’s test is run for this purpose:

- The critical value of Fisher’s test statistic at 5 % significance level is 3,15 and the calculated value of Fisher’s test statistic is 42,91 (calculated p -value for Fisher’s test statistic is lower than 0,001, and which is not greater than 0,05).

It can therefore be accepted that the output rate (transient — steady) causes a mean difference that the analyst considers significant and variable and a width difference in intrinsic width dispersions. Global intrinsic dispersion is therefore a type-5 configuration ($\Delta m \neq 0$ is variable and the local D_i widths are unequal).

A.2.9 Calculation of treatment process capability indicators

Since global intrinsic dispersion was estimated as type 5, the analyst applies the formulae given in [Clause 7](#).

As the characteristic is assumed to be normally distributed, the analyst is in a position to estimate the local intrinsic dispersions and the distribution quantiles. The specified tolerances (post-hardening) are [55 HRC, 60 HRC].

Table A.8 — Tabulated calculation of capability indicators

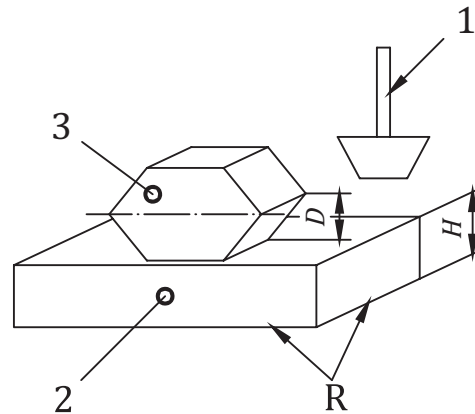
State	Body of production	Transient
No. of state j	1	2
Sample size n_j	21	36
Mean	57,876	58,581
Standard deviation	0,371	0,216
Δm	0,705	
Δm^*	0,705	
$X_{0,135 \% ,j}$	56,763	57,933
$X_{99,865 \% ,j}$	58,989	59,229
el	1	
er	2	
$Di_{u,j}$	1,113	0,648
$Di_{l,j}$	1,113	0,648
$Di_{l,g}$	1,113	
$Di_{u,d}$	0,648	
$P_{mkl,j}$	2,58	5,53
$P_{mku,j}$	1,91	2,19
P_{mku}	1,91	
P_{mkl}	2,58	
P_{mk}	1,91	
P_m	2,25	

Given the objective of the capability indicator and its threshold, the analyst is able to declare the treatment process as fully capable.

A.3 Qualification of a multi-state machining operation

A.3.1 Overview of the process

The parts are mounted on adapters that are moved into position opposite machine tool units.



Key

- 1 tool
- 2 adapter, having its height, H
- 3 machined part, with a characteristic D to control

Figure A.5 — Sketch of the principle of the machining operation under study

Each adapter is mounted with a different copy of the same part.

The adapter is successively moved into position opposite each machine tool unit so that the part is made to undergo each operation in its production process. Each machine tool unit produces different part characteristics. There will obviously be a different capability indicator for each characteristic produced.

On any one machine tool, for a characteristic produced, if the adapter does not affect the result and if the parts delivered are homogeneous (i.e. they present characteristics that, a priori, are normally distributed), then there is every reason to assume a uni-modal distribution. However, if the adapter shapes the characteristic, then the analyst can assume multi-modal distribution. In [Figure A.5](#), the dimension, D , is dependent on the thickness, H , of the adapter if the tool, when performing its task, is referenced in relation to the machine frame (reference R) rather than in relation to the top face of the adapter. Since not all adapters are strictly identical (down to the nearest micron), then before the machining operation, the machine should be re-adjusted to the top face of the adapter.

Where there are 400 adapters in action, there are 400 potential states.

A.3.2 Analysis led

Each adapter is taken geometrically, one by one. The analyst can then confidently consider that the adapter effect will also generate a normal distribution because it can adopt such a large number of aspects.

At this juncture, the analyst has two solutions:

- Sample parts randomly. If process observation reveals a normal distribution, then the calculations can be performed using the usual standards. However, if a result proves non-satisfactory, it will be impossible to determine the relative weight of the adapters and the machine itself;
- Organize the structure of the sampling plan to simultaneously determine the relative weight of the adapters and the machine and decide on whether to accept or refuse to qualify the tool. The cross-comparison of relative weights will decisively shape the adapter monitoring plan for when the production tool is in operational use. If certain adapter parameters turn out to have a non-negligible influence on the result, then it will be essential to ensure that these parameters do not show within-series variation.

The analyst has obviously opted for the second solution. The analyst has identified two adapter parameters liable to create differences in the end product; the parallelism between the inner face of the support and the three-product bearing points and the Y dimension of a clamping position locator.

The analyst therefore selected six (of the 400) adapters based on readings of their geometric characteristics and indicated in the inspection report sent with the adapter batch delivery:

- adapter 1: excellent parallelism — nominal geometric location;
- adapter 2: excellent parallelism — maximum geometric location;
- adapter 3: very poor parallelism (tapered forward) — minimum geometric location;
- adapter 4: very poor parallelism (tapered forward) — maximum geometric location;
- adapter 5: very poor parallelism (tapered backward) — minimum geometric location;
- adapter 6: very poor parallelism (tapered backward) — maximum geometric location.

The analyst estimated that the adapters selected represented the extreme cases liable to be found in practice.

For tool acceptance, the analyst decided to accept that mean differences could be observed (provided that the capability coefficients fitted with the analyst's specifications) but that any differences in dispersion widths would disqualify the tool. These decisions were based on the rationale that if certain adapters generated different widths, then the cause was either parts being poorly clamped on the adapters (this parameter was not considered influential at the analysis) or a potential interference between the adapters and the unworked sections of the part, in which case certain part copies could sometimes present highly divergent measurements.

The characteristic employed in this example is a specification symbolized in the diagram below. The characteristic that is actually read on the measurements is an algebraic deviation from the precise theoretical position of 20 mm in relation to specified reference point A, and this deviation is subsequently added to the theoretical set-point value. This gives a tolerance interval that equates to $20 \text{ mm} \pm 0,2 \text{ mm}$.

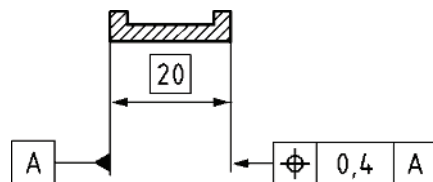


Figure A.6 — Definition of the characteristic targeted for analysis

The deviation read-out shall only be taken at a position along the face, which introduces a component related to a form defect of the toleranced surface into the degree of measurement uncertainty.

The statistical state of a distribution selected is considered a normal distribution, based on the fact that the characteristic measured can be likened to a dimensional characteristic.

In the example quantified here, measurement uncertainty was estimated to be 0,02 mm, i.e. less than a sixth of the tolerance interval ($0,4/6 = 0,067 \text{ mm}$).

A.3.3 Sampling plan

As tool wear-out across 30 parts is assumed to have a negligible effect on the output result, the following sampling plan was selected:

- produce 30 successive parts on the production tool, in the following order:
 - part 1 on adapter A1;

- part 2 on adapter A2;
- part 3 on adapter A3;
- part 4 on adapter A4;
- part 5 on adapter A5;
- part 6 on adapter A6;
- part 7 on adapter A1;
- part 8 on adapter A2;
- part 12 on adapter A6;
- part 13 on adapter A1; etc.

Drift in tool wear-out is therefore included in local intrinsic dispersion.

- the samples will therefore be formed of copies produced from the same adapter:
 - sample 1: parts 1, 7, 13, 19, and 25;
 - sample 2: parts 2, 8, 14, 20, and 26;
 - sample 6: parts 6, 12, 18, 24, and 30.

The tests are thresholded at 5 %, as recommended herein.

Given the company's quality policy and the technology implemented, acceptance thresholds for P_m and P_{mk} are set at 1,3.

A.3.4 Measurements obtained

Table A.9 — Measurements obtained on the different adapters

A1	A2	A3	A4	A5	A6
20,12	20,11	20,14	20,12	20,08	20,01
20,11	20,13	20,11	20,12	20,07	20,03
20,11	20,11	20,12	20,11	20,06	20,01
20,12	20,10	19,95	20,13	20,09	20,02
20,10	20,10	20,11	20,12	20,09	20,05

A Grubbs' test is performed to validate the measurements obtained.

The Grubbs' test applied sample-by-sample indicates the 19,95 value is an outlier.

For the 5-value subsamples, the critical value of Grubbs' test statistic at 5 % significance level = 1,715 (rounded value of 1,715 036) and the calculated values of Grubbs' test statistic for adapter 3 is at 1,766 (rounded value of 1,766 1).

NOTE Applying the Grubbs' test to the full value-set results in the same outcome (the calculated values of Grubbs' test statistic is 3,093 (rounded value of 3,092 8) for the critical value of Grubbs' test statistic at 5 % significance level is 2,908 (rounded value of 2,908 47). The test should ideally be performed sample-by-sample, then on the full value-set. As soon as one of the signals is tagged, an outlier is declared.

The outlier shall be eliminated in order to process the results. An investigation revealed that this outlier was not a measurement error or a data transcription error but was due to the presence of a foreign body between the part and the adapter.

This outlier value is eliminated in order to process the measurements. Until effective counter-measures are identified that can prevent this outlier value from re-occurring, a minimum deviation between 19,95 and 20,12 (the sample mean after the outlier is excluded) shall be added to the dispersion width. The Δa value equals $19,95 - 20,12 = -0,17$.

The analyst tasked with qualification can refuse to qualify the process if outliers are detected and if the risk of outliers leading to nonconformities is deemed unacceptable.

Given the product manufacturing process, the analyst determines that the Δa value has to be negative (it is physically impossible for the Δa value to be positive).

The Grubbs' test applied after eliminating the 19,95 value does not find any other outliers. The analyst can validate the measurements and continue with the qualification process.

It is entirely possible that the company's policy prompts it to adopt the position that it will never qualify a process until action has been taken to minimize the potential amplitude of outliers.

A.3.5 Results analysis and subsequent action

After outliers have been eliminated, the previous table becomes [Table A.10](#).

Table A.10 — Measurements obtained after eliminating outliers

A1	A2	A3	A4	A5	A6
20,12	20,11	20,14	20,12	20,08	20,01
20,11	20,13	20,11	20,12	20,07	20,03
20,11	20,11	20,12	20,11	20,06	20,01
20,12	20,10	20,11	20,13	20,09	20,02
20,10	20,10		20,12	20,09	20,05

The Bartlett's test accepts homogeneity of standard deviations. The critical value of Bartlett's test statistic at 5 % significance level is 11,070 (rounded value of 11,070 498) and the calculated value of Bartlett's test statistic is 3,430 (rounded value of 3,429 742) [calculated p -value for Bartlett's test statistic is 3,430 (rounded value of 3,429 742) which is greater than 0,05].

NOTE The Bartlett's test applied without eliminating outliers would have indicated heterogeneous standard deviations. This is an outlier-induced effect. The critical value of Bartlett's test statistic at 5 % significance level is 11,070 and the calculated value of Bartlett's test statistic is 34,44.

The F-test indicates non-homogenous means. The critical value of F-test statistic at 5 % significance level is 2,62 and the calculated value of F-test statistic is 46,85 (calculated p -value for F-test statistic is lower than 0,001, which is not greater than 0,05).

Analysis of the sample means indicates that the means of samples A5 and especially A6 are different with the other means.

Table A.11 — Statistics per state

	A1	A2	A3	A4	A5	A6
Sample size	5	5	4	5	5	5
Mean	20,112	20,110	20,086	20,120	20,078	20,024
Standard deviation	0,0084	0,0122	0,0770	0,0071	0,0130	0,0167
Range	0,02	0,03	0,03	0,02	0,03	0,04

NOTE If only the first four samples are taken into account, the F-tests indicates homogeneity of the means. The critical value of F-test statistic at 5 % significance level is 3,239 (rounded value of 3,238 872) and the calculated value of F-test statistic is 1,24 (calculated p -value for Bartlett's test statistic is 0,33 which is greater than 0,05).

This leads to the conclusion that the parallelism of adapter faces has a strong influence over the mean output results obtained if the non-parallel border is tapered backward (no influence when tapered forward). The effect is obviously worsened when at minimum geometric location.

The mean differences are related to adapter geometries. This means they are, by definition, stable over time.

A.3.6 Calculation of capability indices

As conclusion of previous steps, the means are considered different and stable over the time and the local intrinsic standard deviations are considered identical. As the adapters were selected in relation to extreme geometries, the global intrinsic dispersion is therefore type 1 ($\Delta m \neq 0$ is constant and local D_i widths are equal).

- local intrinsic standard deviation = 0,012 3 with 23 degrees of freedom;
- maximum mean = 20,120;
- minimum mean = 20,024;
- hence, $\Delta m = 0,096$;
- $D_{i_u} = 3 \times 0,012\ 3 = 0,036\ 9$;
- $D_{i_l} = 3 \times 0,012\ 3 + 0,17 = 0,206\ 9$ (integrating correction for outliers);
- $D_{ig} = 0,036\ 9 + 0,206\ 9 = 0,243\ 8$;
- $P_m = (0,4 - 0,096)/0,243\ 8 = 1,25$;
- $P_{mku} = (20,2 - 20,120)/0,036\ 9 = 2,17$;
- $P_{mkl} = (20,024 - 19,8)/0,206\ 9 = 1,08$;
- $P_{mk} = \min (2,17, 1,08) = 1,08$.

The analyst states a P_m of 1,25 and a P_{mk} of 1,08 with an outlier effect equal to 0,17. Given the acceptance thresholds set, qualification is rejected.

Annex B (normative)

Statistical tests

The tests detailed in this part of ISO 22514 are the Grubbs' test, Bartlett's test, and Fisher's test for application to k-sample tests on homogeneity of means.

The conventional *t*-test, paired *t*-test (also called aspin-welch's test or *t*-test with unequal variances), and Fisher's exact test are detailed in the ISO 16269 series and are not borrowed in this part of ISO 22514.

B.1 Grubbs' test

In a sample of *n* individuals from a data set assumed to be roughly normally distributed, a value is assumed to be an outlier if the quantity value

$$G = \frac{\max(x_{\max} - \bar{x}; \bar{x} - x_{\min})}{s} \tag{B.1}$$

where

\bar{x} is the mean of the sample;

s the standard deviation of the dataset estimated from the sample:

$$\left(s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \right) \text{ is greater than the threshold } \frac{n-1}{\sqrt{n}} \cdot \sqrt{\frac{t^2_{[(1-\alpha/2)/n; n-2]}}{t^2_{[(1-\alpha/2)/n; n-2]} + n-2}}$$

where

t is the student's, *t*;

α the significance level of the test.

This test is reserved to sample sizes of three or more and with a range of at least three times the measurement resolution. Furthermore, if sample size is three, then none of the measurements shall be identical.

Table B.1 — Example of values measured within a Grubbs' test framework

Values measured	138
	140
	137
	180
Sample size, <i>n</i>	4
Mean	148,75
Standard deviation, <i>s</i>	20,87

Table B.1 (continued)

Values measured	138
	140
	137
	180
Min	137
Max	180
Student's, <i>t</i> value	8,860
Threshold	1,481
Indicator, <i>G</i>	1,497
Outlier	Value = 180

B.2 Bartlett's test

In *k* samples indexed *j*, where *j* varies from 1 to *k*, each sample comprising *n_j* individuals, each sample presents a variance, *s_j²*, where

$$s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{i,j} - \bar{x}_j)^2}{v_j}, \quad \bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{i,j}}{n_j}, \quad \text{and } v_j = n_j - 1$$

Mean variance is estimated by

$$s^2 = \frac{\sum_{j=1}^k v_j \cdot s_j^2}{v} \quad \text{where } v = \sum_{j=1}^k v_j$$

If the samples are drawn from datasets sharing the same variance σ^2 , then indicator *B*

$$B = \frac{v \cdot \ln(s^2) - \sum_{j=1}^k v_j \cdot \ln(s_j^2)}{c}$$

has an approximately χ^2 distribution, with (*k*-1) degrees of freedom.

The bias correction *c* is such that

$$c = 1 + \frac{\left(\sum_{j=1}^k \frac{1}{v_j} \right) - \frac{1}{v}}{3 \cdot (k-1)}$$

NOTE Some users do not employ bias correction and instead take *c* = 1.

Employing this test supposes that the distributions of each data set from which the samples are taken are not too far from normally distributed, or are at least uni-modal, it is understood that the full set of observations can be multi-modal.

This test shall only be employed on samples whose size does not vary $\pm 50\%$ and on sample sizes of three or more.

Furthermore, employing this test also supposes that each sample reflects variability in the end products, i.e. that the measurement resolution does not create too much rounding out. In practice, once sample range is less than or equal to two scale markings on the measurement device, the variances are forced up to a value that is higher than that calculated but nevertheless, credible given the observed range, and only for the purposes of running the Bartlett's test. This is done by highlighting a variance in [Table B.2](#) below:

Forced variance = $d * \text{resolution}^2$ (it is to consider when calculated variance is lower than this value).

Table B.2 — The d values

Range values ^a	Sample size									
	3	4	5	6	7	8	9	10	>10	
0	0,25	0,19	0,16	0,14	0,13	0,12	0,12	0,11	0,10	
1	1	0,74	0,63	0,56	0,52	0,49				
2	2,25	1,67	1,41	No overestimation of the variance calculated						

^a The range values are given as number of scale markings on the measurement device.

[Table B.2](#) has been built on the basis that real values are uniformly distributed between the two upper-lower bounds possible at a given measurement resolution.

Table B.3 — Example, at 5 % significance level

Values measured	A1	A2	A3	Globalized
		143	143	
	140	140	135	
	137	140	137	
	139	141	137	
		145	136	
Sample size	4	5	5	14
Degrees of freedom	3	4	4	11
Mean	139,75	141,80	136,20	
Standard deviation	2,50	2,17	0,84	
Range	6	5	2	
Variance	6,25	4,70	0,70	3,67
ln (variance)	1,83	1,55	-0,36	1,30
Bias correction	1,127	Bartlett indicator		3,58
Critical value	5,991	Homogeneous variances		

Table B.4 — Example with forced variance — Resolution 0,1

Values measured	A1	A2	A3	Globalized
		143,1	140,2	
	143,1	140,2	140,0	
	143,1	140,2	140,2	
	143,1	140,1	140,3	
	143,1		140,6	
Sample size	5	4	5	14

Table B.4 (continued)

Values measured	A1	A2	A3	Globalized
	143,1	140,2	140,2	
	143,1	140,2	140,0	
	143,1	140,2	140,2	
	143,1	140,1	140,3	
	143,1		140,6	
Degrees of freedom	4	3	4	11
Mean	143,10	140,18	140,26	
Standard deviation	0	0,05	0,219	
Range	0	0,1	0,6	
Variance	0,000	0,002	0,048	0,01995
Forced variance	0,0016	0,0074	0,048	0,0205
ln (variance)	-2,80	-1,13	-1,32	0315
Bias correction	1,127	Bartlett indicator		8,70
Critical value	5,991	Non-homogeneous variances		

The variance in sample A1 is forced by the variance factor $0,16 \cdot 0,1^2$.

The variance in sample A2 is forced by the variance factor $0,74 \cdot 0,1^2$.

The Bartlett's test indicates heterogeneous variances. Samples A1 and A3 are far too different.

B.3 F-test — Application to the comparison of sample means

The F-test is used to compare two standard deviations in samples derived from two roughly normally-distributed populations.

If the two populations share the same standard deviation, the value s_1^2/s_2^2 follows an F-distribution at n_1-1 and n_2-1 degrees of freedom, where s_1 and s_2 are the estimated standard deviations of the 1st and 2nd populations, respectively, these estimations being calculated using the formula given in [Clause 4](#) and where n_1 and n_2 are the sample sizes of the 1st and 2nd populations, respectively.

- The F-distribution is known and tabulated. Consequently, if the ratio s_1^2/s_2^2 calculated from two variances falls within the regular values (at a given significance level test) of the probability distribution tabulated, then the hypothesis of equal variances between two populations is accepted. Otherwise, the hypothesis of equal standard deviations is rejected.

B.3.1 Application to homogeneity of means

The aim of the test is to gain a global comparison between samples.

Based on k samples, each sample counting n parts, observations are made on k mean values and k standard deviation estimations.

If the hypothesis of equal standard deviations can be accepted (Bartlett's test – see [B.2](#)), then the mean standard deviation is calculated from the mean variance using the formula given in this annex.

If the samples are drawn from identical populations, the statistic $\frac{n \cdot s_x^2}{s^2}$ * follows an F-distribution at $k-1$, and $nk-k$ degrees of freedom, where s^2 is the mean variance as defined in [B.2](#).

By extension, if the sample sizes are slightly different (i.e. only differing by one unit), then this rule shall be applied by replacing n with \bar{n} , mean sample size.

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