

BS ISO 22514-2:2017



BSI Standards Publication

# Statistical methods in process management — Capability and performance

Part 2: Process capability and performance of time-dependent process models

**National foreword**

This British Standard is the UK implementation of ISO 22514-2:2017. It supersedes BS ISO 22514-2:2013 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee SS/4, Statistical Process Management.

A list of organizations represented on this committee can be obtained on request to its secretary.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

© The British Standards Institution 2017.  
Published by BSI Standards Limited 2017

ISBN 978 0 580 94907 4

ICS 03.120.30

**Compliance with a British Standard cannot confer immunity from legal obligations.**

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 28 February 2017.

**Amendments/corrigenda issued since publication**

Date	Text affected
------	---------------

---

---

---

**Statistical methods in process  
management — Capability and  
performance —**

Part 2:  
**Process capability and performance of  
time-dependent process models**

*Méthodes statistiques dans la gestion de processus — Aptitude et  
performance —*

*Partie 2: Aptitude de processus et performance des modèles de  
processus dépendants du temps*





## **COPYRIGHT PROTECTED DOCUMENT**

© ISO 2017, Published in Switzerland

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized otherwise in any form or by any means, electronic or mechanical, including photocopying, or posting on the internet or an intranet, without prior written permission. Permission can be requested from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office  
Ch. de Blandonnet 8 • CP 401  
CH-1214 Vernier, Geneva, Switzerland  
Tel. +41 22 749 01 11  
Fax +41 22 749 09 47  
copyright@iso.org  
www.iso.org

# Contents

Page

<b>Foreword</b> .....	<b>iv</b>
<b>Introduction</b> .....	<b>v</b>
<b>1 Scope</b> .....	<b>1</b>
<b>2 Normative references</b> .....	<b>1</b>
<b>3 Terms definitions, symbols and abbreviated terms</b> .....	<b>1</b>
3.1 Symbols.....	1
3.2 Abbreviated terms.....	2
<b>4 Process analysis</b> .....	<b>3</b>
<b>5 Time-dependent distribution models</b> .....	<b>3</b>
<b>6 Process capability and performance indices</b> .....	<b>14</b>
6.1 Methods for determination of performance and capability indices — Overview.....	14
6.1.1 General.....	14
6.1.2 Calculation of location.....	15
6.1.3 Calculation of dispersion.....	16
6.1.4 Calculation of $X_{0,135}$ % and $X_{99,865}$ %.....	17
6.2 One-sided specification limits.....	18
6.3 Use of different calculation methods.....	19
<b>7 Reporting process performance/capability indices</b> .....	<b>20</b>
<b>Bibliography</b> .....	<b>21</b>

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html)

The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This second edition of ISO 22514-2 cancels and replaces the corrected version of the first edition (ISO 22514-2:2013), of which it constitutes a minor revision.

The changes compared to the previous edition are as follows:

- the symbols and indices in  $C_{pk_L}$ ,  $C_{pk_U}$ ,  $P_{pk_L}$  and  $P_{pk_U}$  have been improved;
- in [Table 2](#), row "Location", column "C", the letter "s" has been replaced by "s/r";
- in [Table 2](#), row "Location", column "D", the capital letter "S" has been replaced by "s/r";
- in [Table 3](#), row "Location method label", rows "3" and "4" – in Formulae (13) and (14) the usage of indices has been improved and it is more precise now;
- editorial adjustments have been made to comply with the latest edition of the ISO/IEC Directives, Part 2, 2016.

A list of all parts in the ISO 22514- series, published under the general title *Statistical methods in process management — Capability and performance*, can be found on the ISO website.

## Introduction

Many standards have been created concerning the quality capability/performance of processes by international, regional and national standardization bodies and also by industry. All of them assume that the process is in a state of statistical control, with stationary, normally distributed processes. However, a comprehensive analysis of production processes shows that, over time, it is very rare for processes to remain in such a state.

In recognition of this fact, this document provides a framework for estimating the quality capability/performance of industrial processes for an array of standard circumstances. These circumstances are categorized based on the stability of the mean and variance, as to whether they are constant, changing systematically, or changing randomly. As such, the quality capability/performance can be assessed for very differently shaped distributions with respect to time.

In other parts of ISO 22514 more detailed information about calculations of indices can be found. It should be noted that where the capability indices given in this document are computed they only form point estimates of their true values. It is therefore recommended that wherever possible the indices' confidence intervals are computed and reported.





# Statistical methods in process management — Capability and performance —

## Part 2:

# Process capability and performance of time-dependent process models

## 1 Scope

This document describes a procedure for the determination of statistics for estimating the quality capability or performance of product and process characteristics. The process results of these quality characteristics are categorized into eight possible distribution types. Calculation formulae for the statistical measures are placed with every distribution.

The statistical methods described in this document only relate to continuous quality characteristics. They are applicable to processes in any industrial or economical sector.

**NOTE** This method is usually applied in case of a great number of serial process results, but it can also be used for small series (a small number of process results).

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 5479, *Statistical interpretation of data — Tests for departure from the normal distribution*

## 3 Terms definitions, symbols and abbreviated terms

For the purposes of this document, the terms and definitions given in ISO 3534-2 and ISO 22514-1, and the symbols and abbreviated terms given below, apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

### 3.1 Symbols

$C_p$	process capability index
$C_{pk}$	minimum process capability index
$C_{pk_L}$	lower process capability index
$C_{pk_U}$	upper process capability index
$c_4$	constant based on subgroup size $n$

$\Delta$	dispersion of the process
$\Delta_L$	difference between $X_{\text{mid}}$ and $X_{0,135\%}$ of the distribution of the product characteristic
$\Delta_U$	difference between $X_{99,865\%}$ and $X_{\text{mid}}$ of the distribution of the product characteristic
$d_2$	constant based on subgroup size $n$
$k$	number of subgroups of the same size $n$
$\mu$	average location of the process
$L$	lower specification limit
$M_{l,d}$	calculation methods with location method label $l$ and dispersion method label $d$
$N$	sample size
$p_L$	lower fraction nonconforming
$p_t$	total fraction nonconforming
$p_U$	upper fraction nonconforming
$P_p$	process performance index
$P_{pk}$	minimum process performance index
$P_{pk_L}$	lower process performance index
$P_{pk_U}$	upper process performance index
$R_i$	range of the $i$ th subgroup
$s$	standard deviation, realized value
$\sigma$	standard deviation, population
$S$	standard deviation, sample statistic
$S_i$	observed sample standard deviation of the $i$ th subgroup
$S_t$	standard deviation, with the subscript "t" indicating total standard deviation
$U$	upper specification limit
$X_{0,135\%}$	0,135 % distribution quantile
$X_{99,865\%}$	99,865 % distribution quantile
$X_{50\%}$	50 % distribution quantile
$X_{\text{mid}}$	distribution midpoint

### 3.2 Abbreviated terms

ANOVA	analysis of variance
SPC	statistical process control

## 4 Process analysis

The purpose of process analysis is to obtain knowledge of a process. This knowledge is necessary for controlling the process efficiently and effectively so that the products realized by the process fulfil the quality requirement. It is a general assumption of this document that a process analysis has been carried out and subsequent process improvements have been implemented.

The behaviour of a characteristic under consideration can be described by the distribution, the location, the dispersion and the shape, parameters of which are time-dependent functions, in general. Different models of such resulting distributions the parameters of which are time-dependent functions are discussed in [Clauses 6](#) and [7](#). To indicate whether a time-dependent distribution model fits, statistical methods [e.g. estimating parameters, analysis of variance (ANOVA)] including graphical tools (e.g. probability plots, control charts) are used.

The values of the characteristics under consideration are typically determined on the basis of samples taken from the process flow. The sample size and frequency should be chosen depending on the type of process and the type of product so that all important changes are detected in time. The samples should be representative for the characteristic under consideration. To assess the stability of the process a control chart should be used. Information on the use of control charts can be found in ISO 7870-2.

## 5 Time-dependent distribution models

The instantaneous distribution characterizes the behaviour of the characteristic under investigation during a short interval. Usually, it is the time interval during which the sample (e.g. the subgroup) can be taken from the process. Observing the process continuously in time for a longer time interval the output from the process is called the resulting process distribution and it is described by a corresponding time-dependent distribution model that reflects

- the instantaneous distribution of the characteristic under consideration, and
- the changes of its location, dispersion and shape parameters during the time interval of process observation.

In practice, the resulting distribution can be represented by the whole data set, e.g. when SPC is applied, by all subgroups gained during the interval of the process observation.

Time-dependent distribution models can be classified into four groups according to whether the location and dispersion moments are constant or changing (see [Table 1](#)).

- a) A process whose location and dispersion are constant is in time-dependent distribution model A. In this case only, all the means and variances of the instantaneous distributions are equal to each other and they are equal to the resulting distribution.
- b) If the dispersion of a process is changing with time, but the location stays constant, the process is said to be in time-dependent distribution model B.
- c) If the dispersion is constant, but the location is changing, we have time-dependent distribution model C.
- d) Otherwise, we have time-dependent distribution model D.

Table 1 — Classification of time-dependent distribution models

Process-standard deviation $s(t)$	Constant				Process average $\mu(t)$			
	A		B		C			
Constant	A1	A2	B		C1	C2	C3	C4
	Short time distribution	Normal distributed	Not normal distributed - unimodal	Resulting distribution		Random	Random	Systematic (e.g. trend)
Normal distributed		Not normal distributed - unimodal	Normal distributed			Normal distributed	Normal distributed	Normal distributed
Resulting distribution	Any shape - unimodal		Resulting distribution		Normal distributed	Not normal distributed - unimodal	Any shape	Any shape (e.g. multimodal)
	Any shape - unimodal				D			

For changing moments, the models can be classified according to whether the changes are random, systematic or both.

NOTE Model A2 is known as *stationary* in time-series analysis literature and model A1 is known as *second order stationary*.

Table 2 summarizes the basic features of individual time-dependent distribution models; their graphical representations are given in Figures 1 to 8. There are subclasses of time-dependent distribution models A and C which are introduced due to their practical importance. They differ in the shape of the resulting distribution and in the cause of the process being in an out-of-control state.

**Table 2 — Basic features of time-dependent distribution models**

Characteristic	Time-dependent distribution models <sup>a</sup>							
	A1	A2	B	C1	C2	C3	C4	D
Location	c	c	c	r	r	s	s/r	s/r
Dispersion	c	c	s/r	c	c	c	c	s/r
Instantaneous distribution	nd	1m	nd	nd	nd	as	as	as
Resulting distribution	nd	1m	1m	nd	1m	as	as	as
Figure	1	2	3	4	5	6	7	8
<b>Location/dispersion:</b>								
c parameter remains constant								
r parameter changes randomly only								
s parameter changes systematically only								
<b>Instantaneous/resulting distribution:</b>								
nd normally distributed								
1m not normally distributed, one mode only								
as any shape								
<sup>a</sup> The choice of the model is a result of process analysis.								

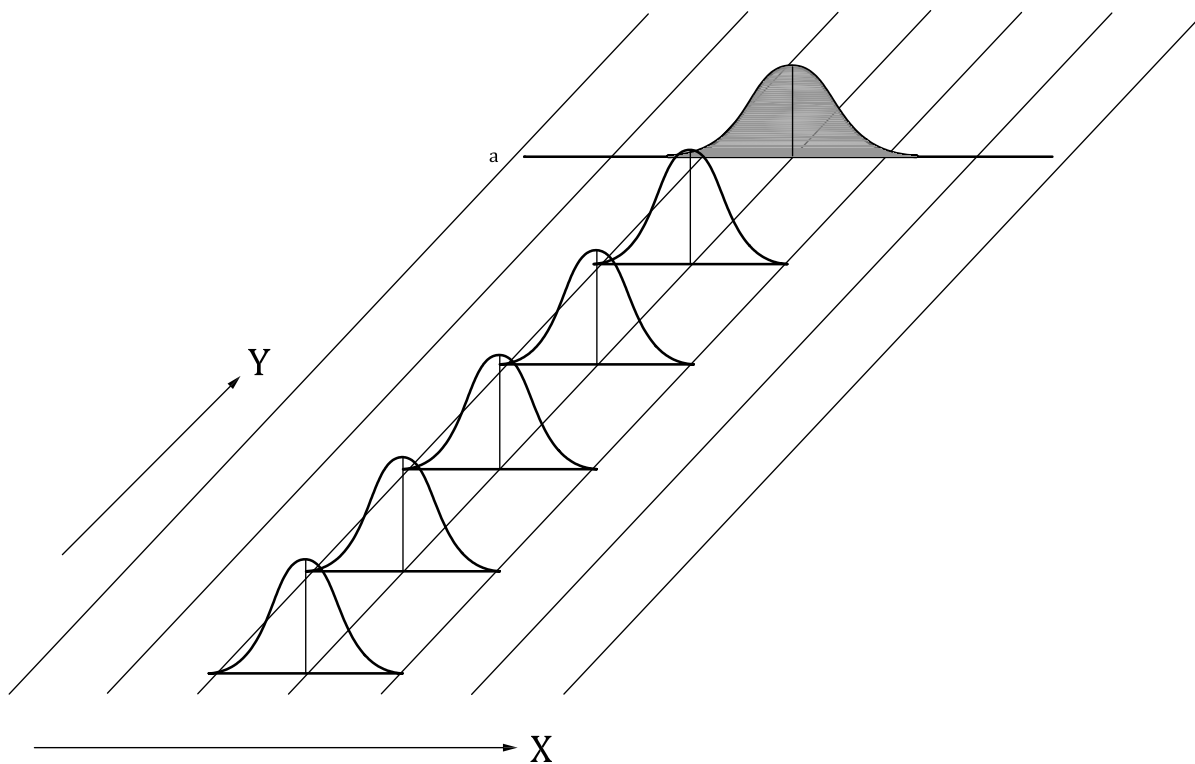
For each time-dependent distribution model, several instantaneous distributions are shown as a function of time; the related resulting distribution is shown as well. These distributions are not drawn to scale.

The choice of models and their verification requires extensive data analysis. This will usually require the use of statistical software.

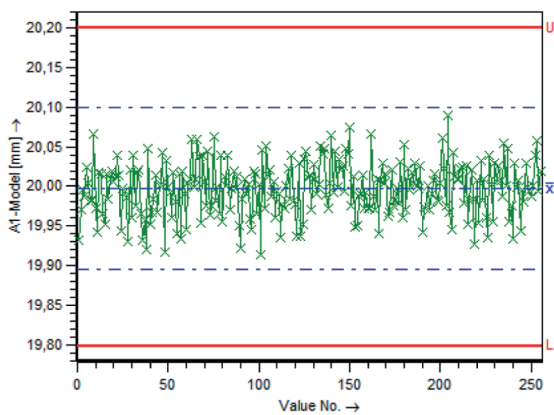
Time-dependent distribution model A1 (see Figure 1) has the following characteristics (e.g. the measured length of an item from a process in a state of statistical control):

- location: constant;
- dispersion: constant;
- instantaneous distribution: normally distributed;
- resulting distribution: normally distributed.

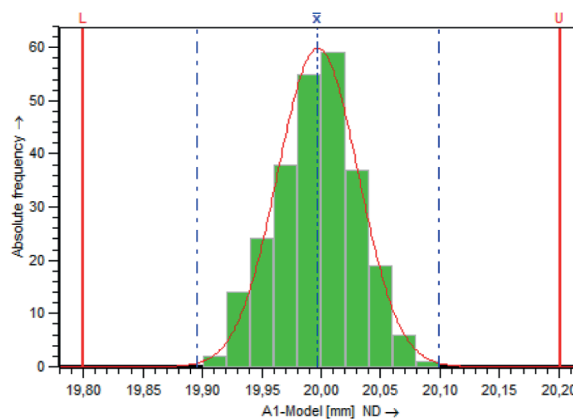
This process is under statistical control.



**a) Time-dependent distribution model A1**



**b) Example of run chart model A1**



**c) Example of histogram model A1**

**Key**

X characteristic value

Y time

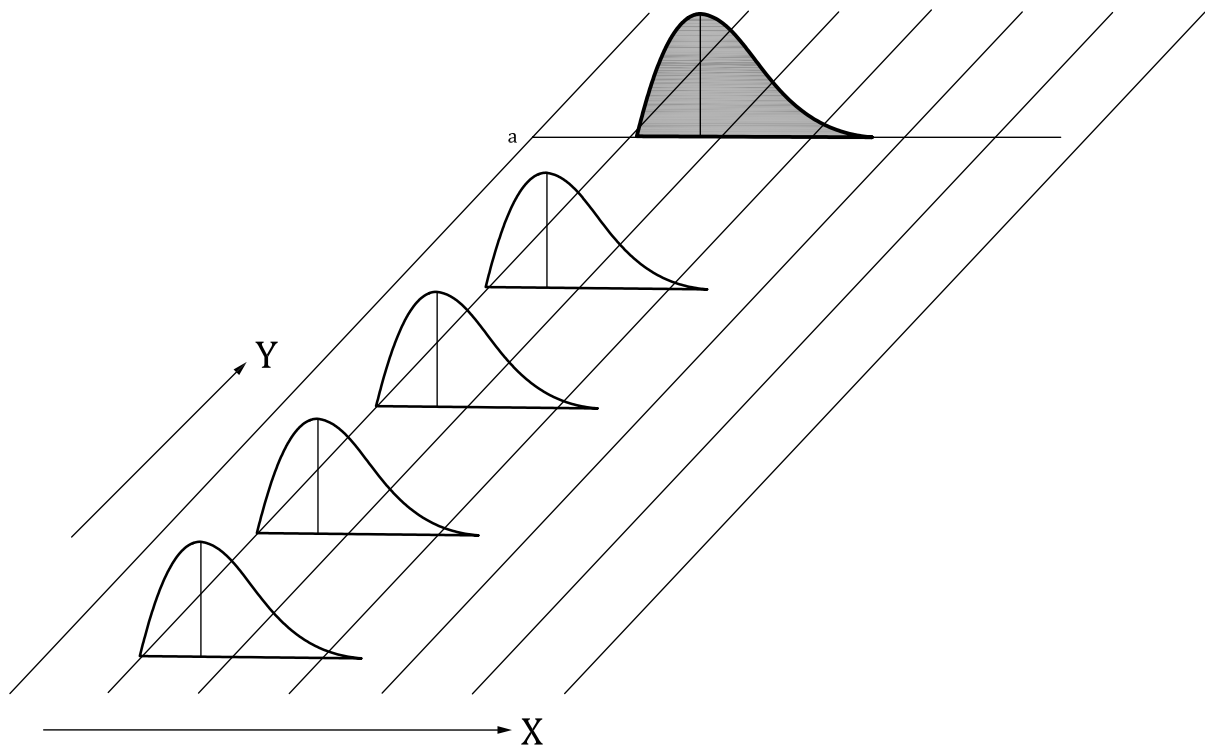
a Resulting distribution.

**Figure 1 — Graphical representation of time-dependent distribution model A1**

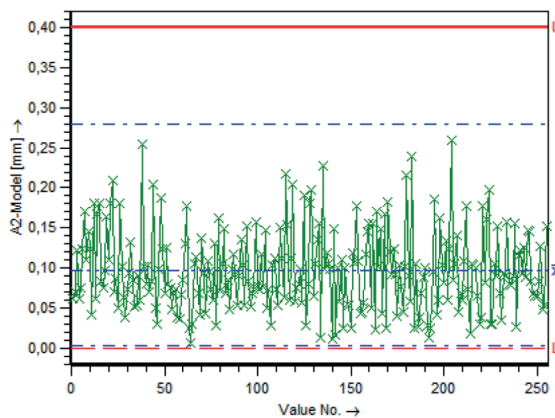
Time-dependent distribution model A2 (see [Figure 2](#)) has the following characteristics (e.g. the surface roughness of an item as an example for a physically limited characteristic):

- location: constant;
- dispersion: constant;
- instantaneous distribution: not normally distributed, unimodal;
- resulting distribution: not normally distributed, unimodal.

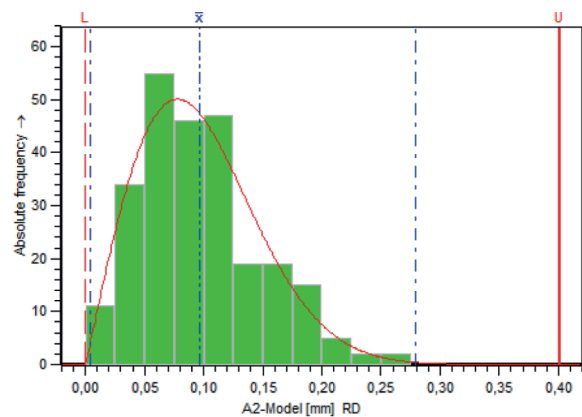
This process is under statistical control.



a) Time-dependent distribution model A2



b) Example of run chart model A2



c) Example of histogram model A2

**Key**

X characteristic value

Y time

a Resulting distribution.

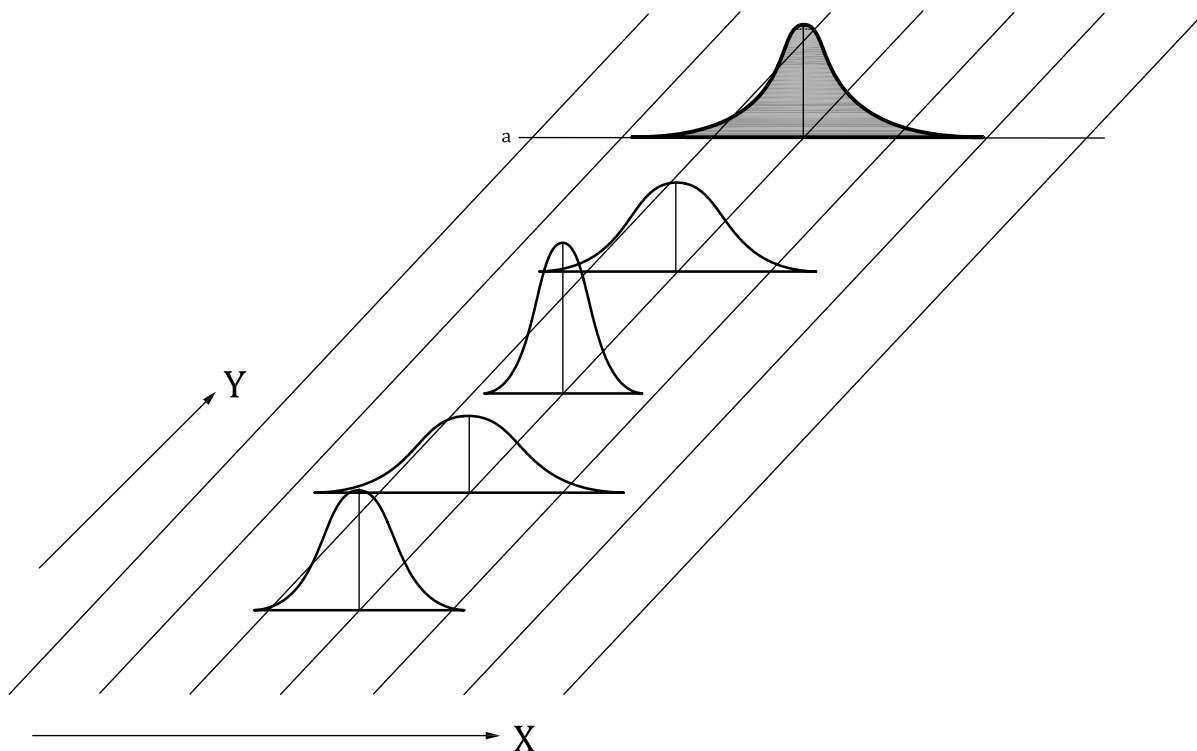
**Figure 2 — Graphical representation of time-dependent distribution model A2**

Time-dependent distribution model B (see [Figure 3](#)) has the following characteristics (e.g. different wear of the spindles on a multiple-spindle automatic machine with equal centring):

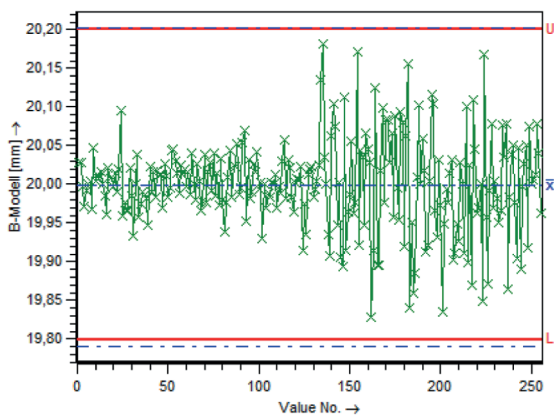
- location: constant;
- dispersion: systematic or random variation;
- instantaneous distribution: normally distributed;

— resulting distribution: not normally distributed, unimodal.

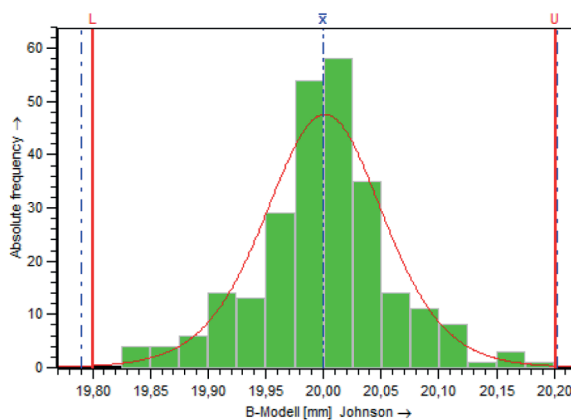
This process is not under statistical control.



a) Time-dependent distribution model B



b) Example of run chart model B



c) Example of histogram model B

**Key**

X characteristic value

Y time

a Resulting distribution.

**Figure 3 — Graphical representation of time-dependent distribution model B**

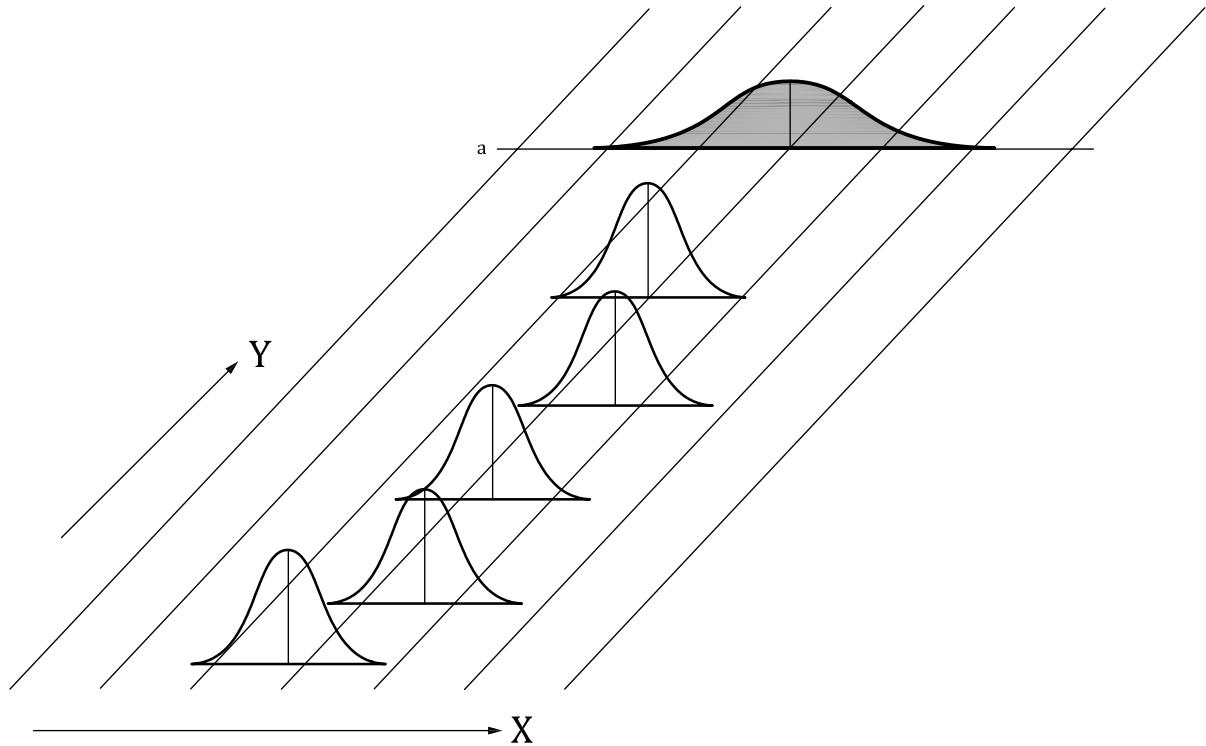
Time-dependent distribution model C1 (see [Figure 4](#)) has the following characteristics (e.g. different centring of workholding fixtures):

- location: random (normally distributed);
- dispersion: constant;

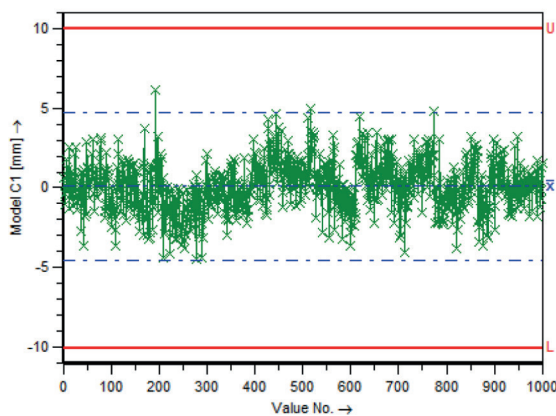


- instantaneous distribution: normally distributed;
- resulting distribution: normally distributed.

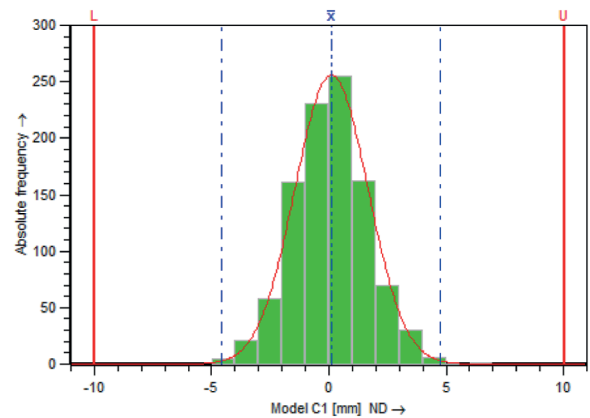
This process is not under statistical control.



a) Time-dependent distribution model C1



b) Example of run chart model C1



c) Example of histogram model C1

**Key**

X characteristic value

Y time

a Resulting distribution.

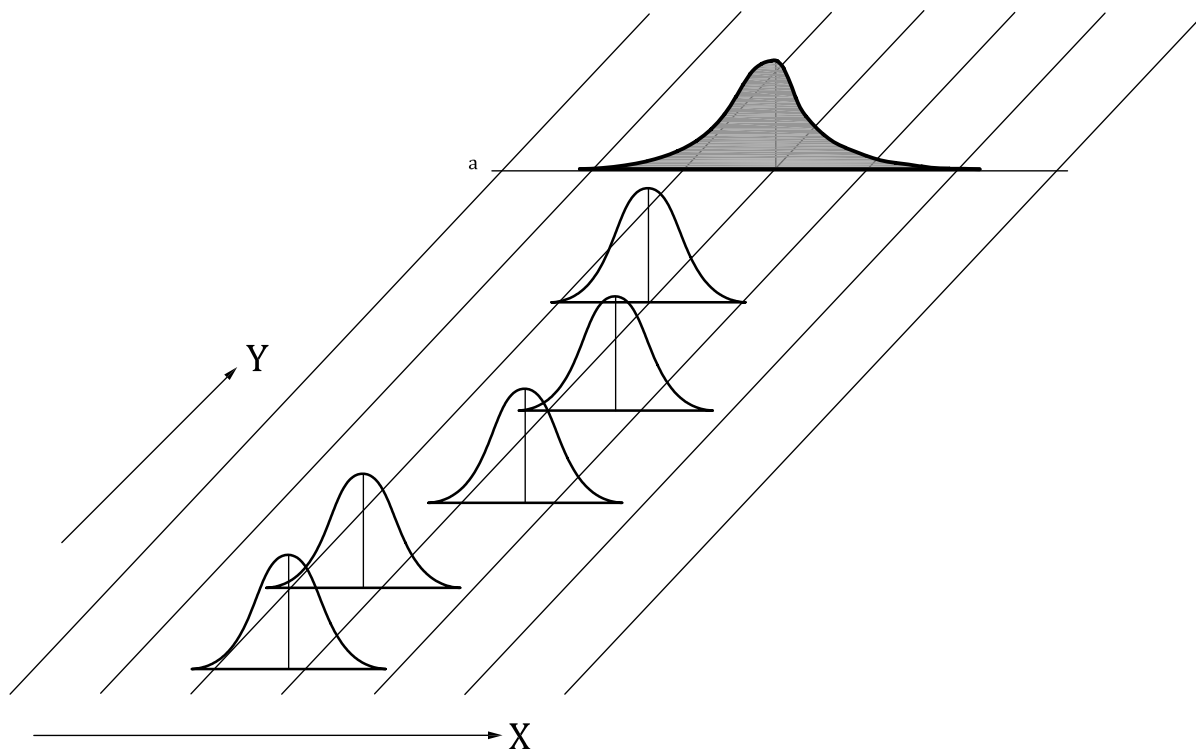
**Figure 4 — Graphical representation of time-dependent distribution model C1**

Time-dependent distribution model C2 (see [Figure 5](#)) has the following characteristics (e.g. fixed tools):

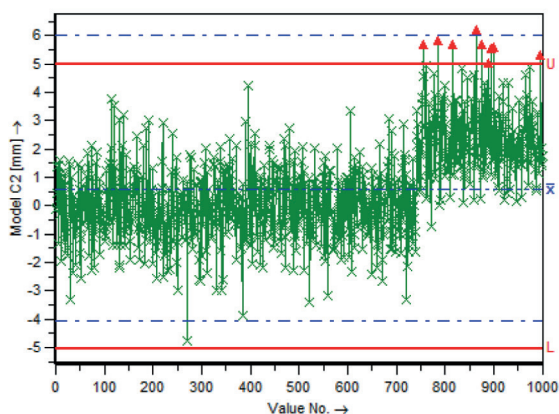
- location: random (not normally distributed, unimodal);
- dispersion: constant;

- instantaneous distribution: normally distributed;
- resulting distribution: not normally distributed, unimodal.

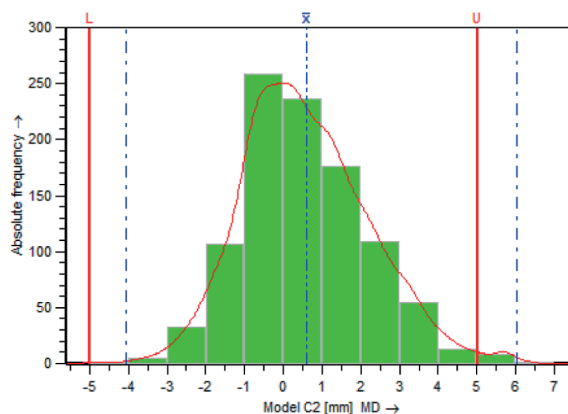
This process is not under statistical control.



a) Time-dependent distribution model C2



b) Example of run chart model C2



c) Example of histogram model C2

**Key**

- X characteristic value                      Y time                      a Resulting distribution.

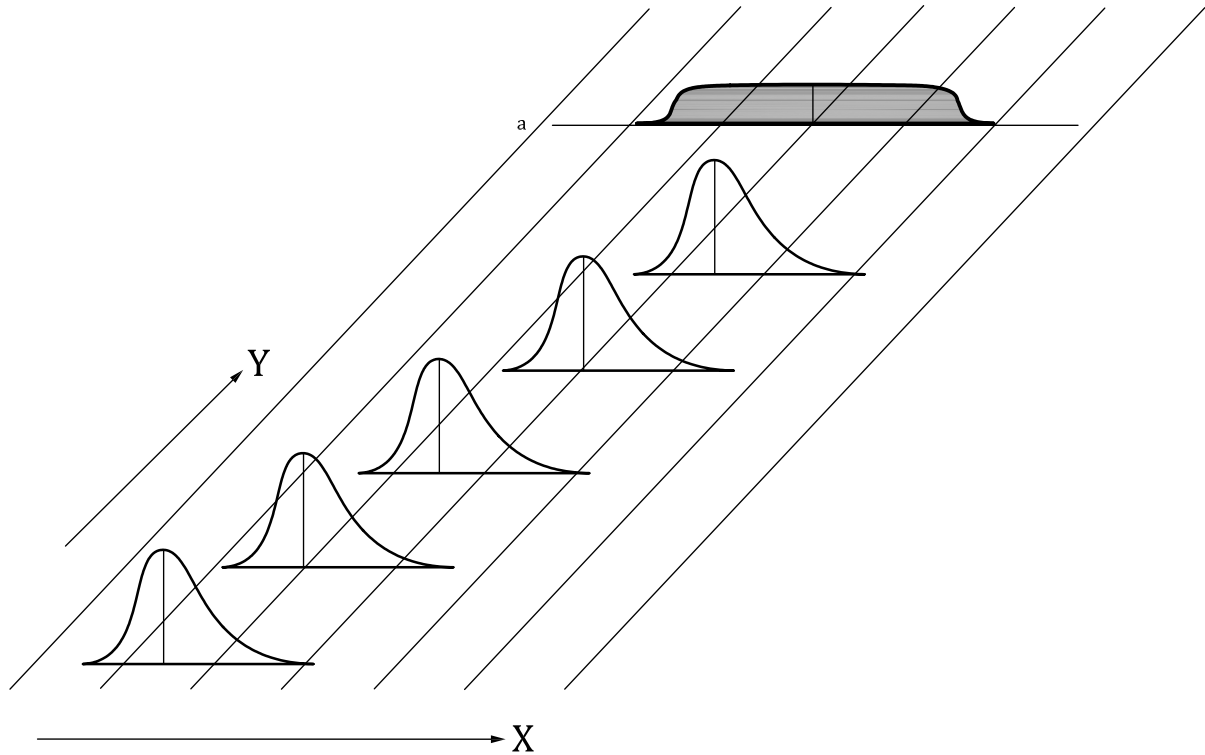
**Figure 5 — Graphical representation of time-dependent distribution model C2**

Time-dependent distribution model C3 (see [Figure 6](#)) has the following characteristics:

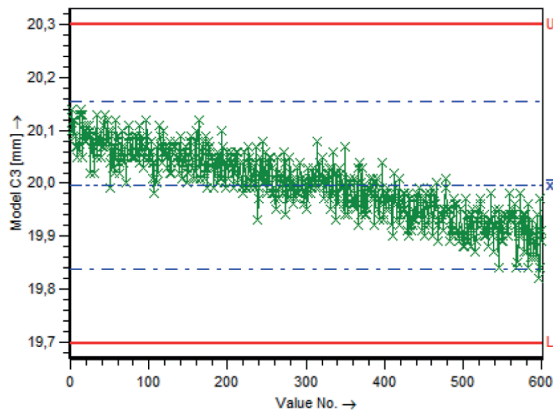
- location: function oriented (e.g. trend, caused by wear, and cycle);
- dispersion: constant;

- instantaneous distribution: any shape whatever;
- resulting distribution: any shape whatever.

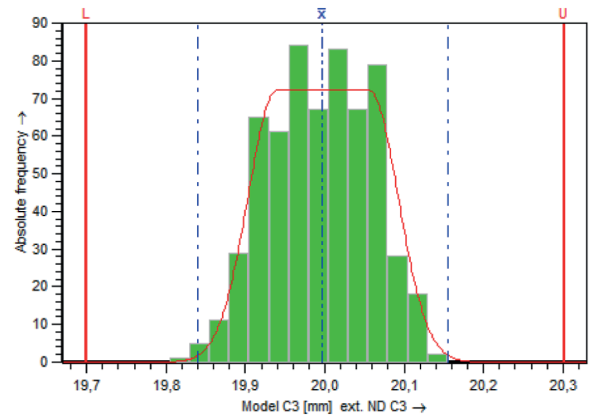
This process is not under statistical control.



a) Time-dependent distribution model C3



b) Example of run chart model C3



c) Example of histogram model C3

**Key**

X characteristic value

Y time

a Resulting distribution.

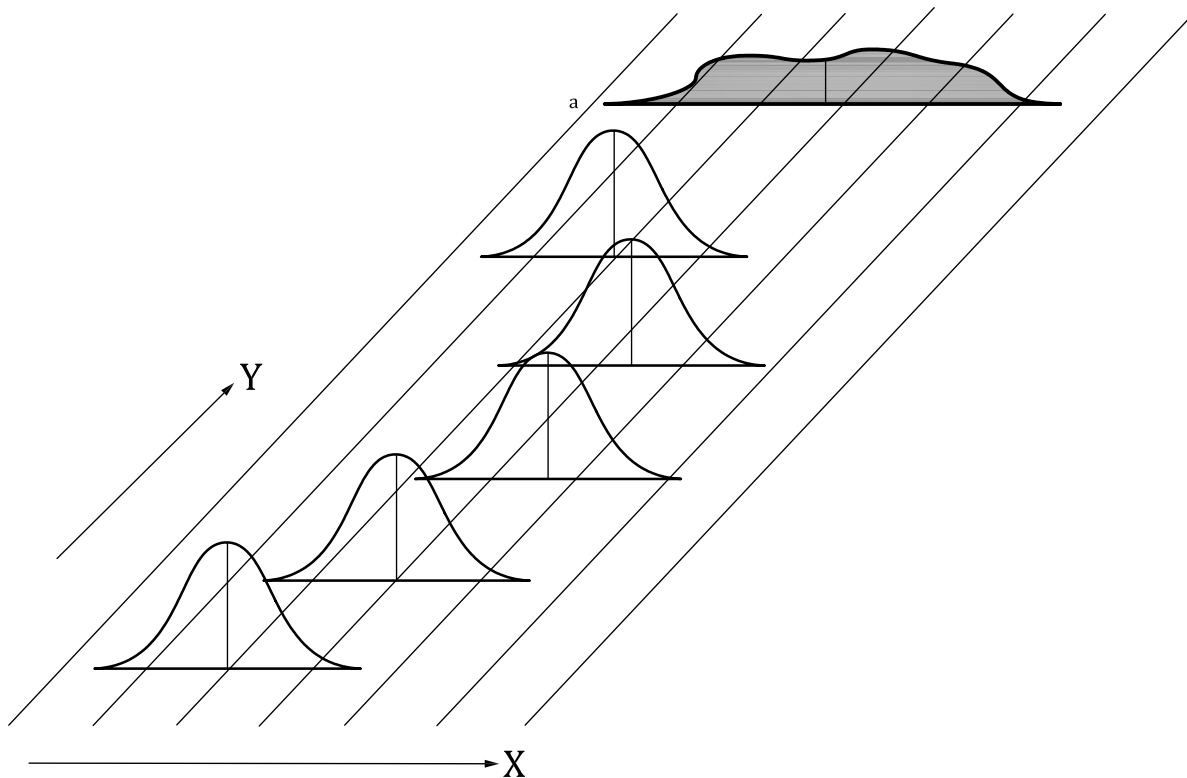
**Figure 6 — Graphical representation of time-dependent distribution model C3**

Time-dependent distribution model C4 (see [Figure 7](#)) has the following characteristics:

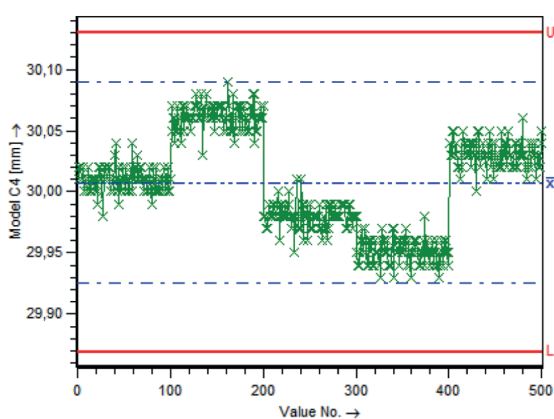
- location: systematic and random change (e.g. tool changes or change of batches);
- dispersion: constant;

- instantaneous distribution: any shape whatever;
- resulting distribution: any shape whatever.

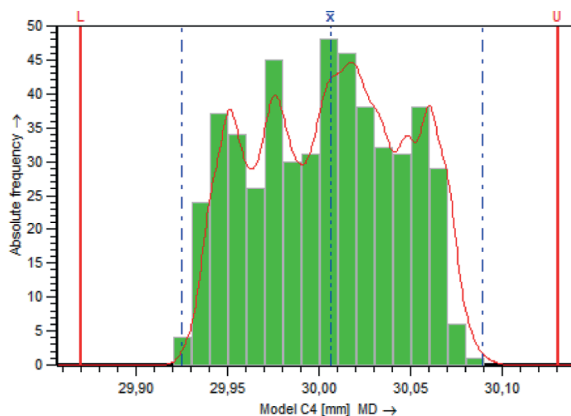
This process is not under statistical control.



a) Time-dependent distribution model C4



b) Example of run chart model C4



c) Example of histogram model C4

**Key**

X characteristic value

Y time

a Resulting distribution.

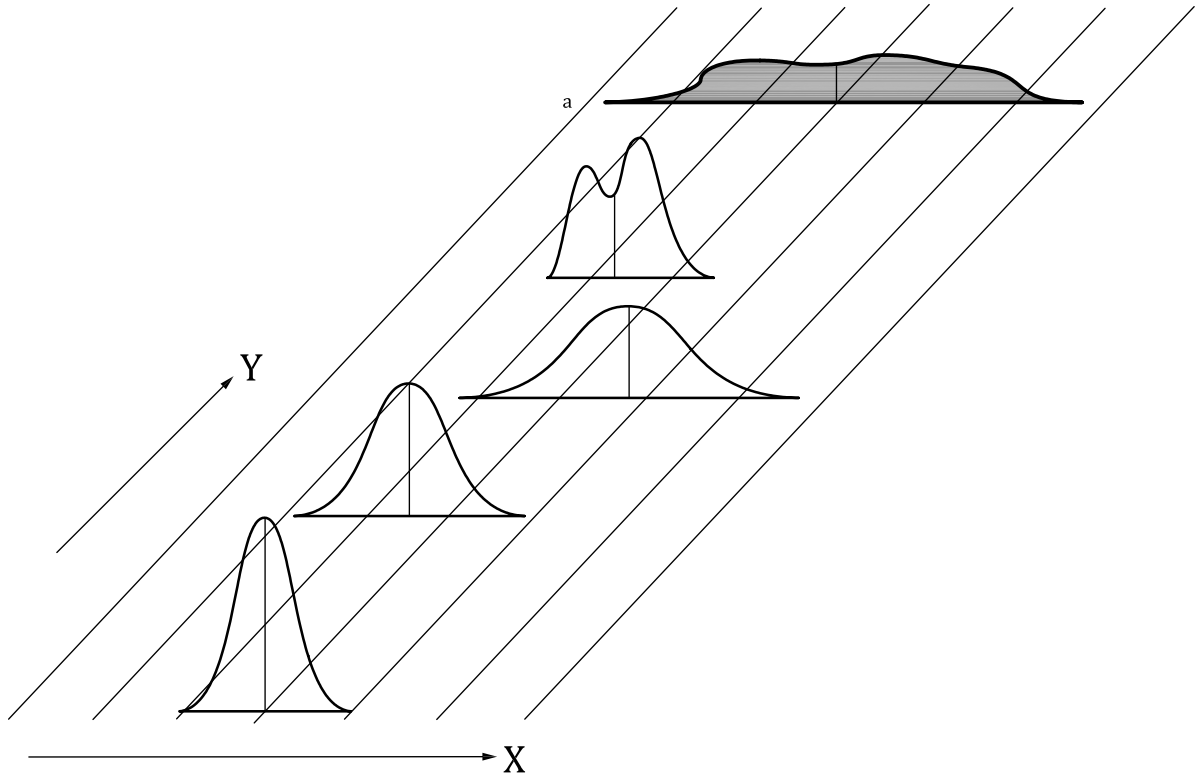
**Figure 7 — Graphical representation of time-dependent distribution model C4**

Time-dependent distribution model D (see [Figure 8](#)) has the following characteristics (e.g. multi-stream processes):

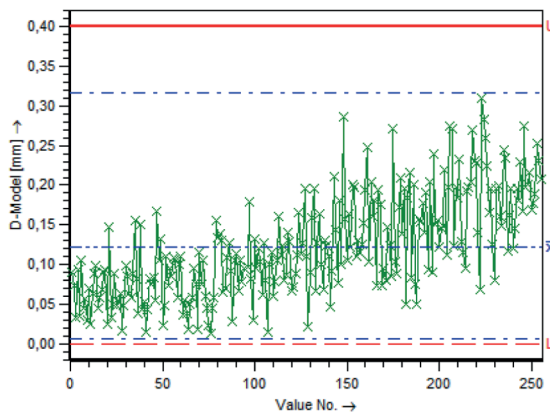
- location: systematic and random change;

- dispersion: systematic and random change;
- instantaneous distribution: any shape whatever;
- resulting distribution: any shape whatever.

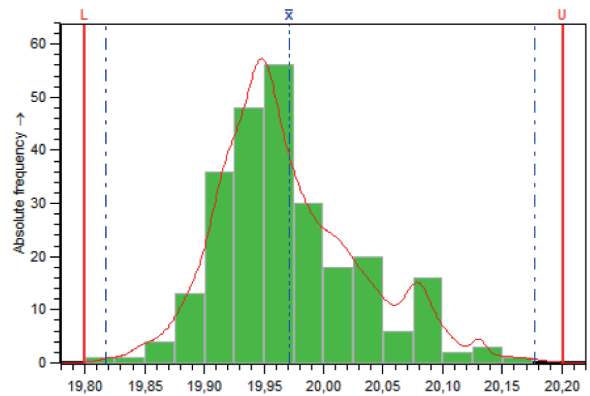
This process is not under statistical control.



a) Time-dependent distribution model D



b) Example of run chart model D



c) Example of histogram model D

**Key**

X characteristic value

Y time

a Resulting distribution.

**Figure 8 — Graphical representation of time-dependent distribution model D**

## 6 Process capability and performance indices

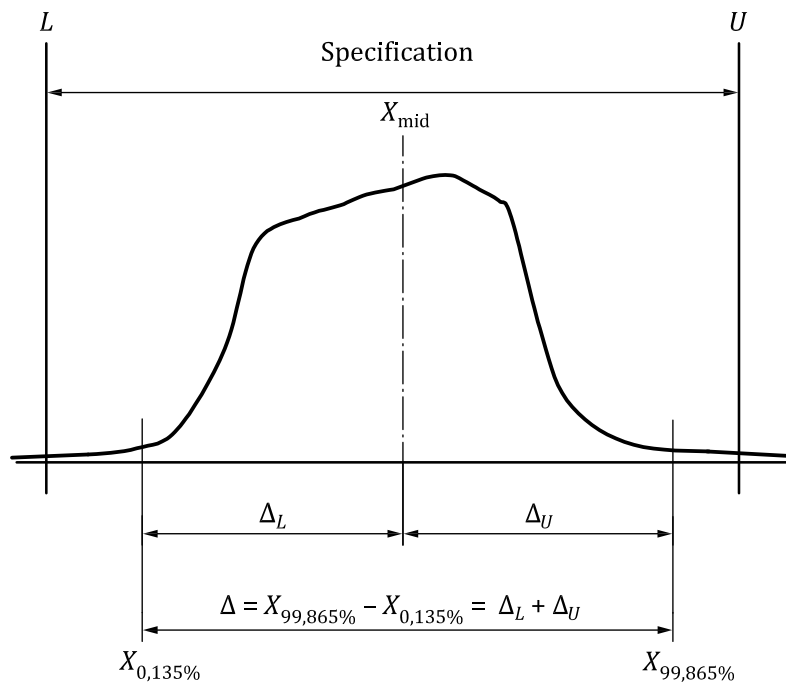
### 6.1 Methods for determination of performance and capability indices — Overview

#### 6.1.1 General

As detailed in the preceding clauses, the basis for determination of process capability and performance statistics is the distribution of characteristic values of a product characteristic.

The calculation of the performance indices, as well as the capability indices is based on the location and dispersion of characteristic values with respect to the tolerance.

A general graphical representation is shown in [Figure 9](#).



**Figure 9 — Graphical representation of the general geometric method**

In [Figure 9](#),  $X_{mid}$  indicates the location of the process and  $\Delta$  indicates the dispersion of the process. Their exact definitions, depending on the method, will be given later. The dispersion is bounded by the lower reference limit  $X_{0,135\%}$ , and the upper reference limit  $X_{99,865\%}$ . Then we have

$$\Delta_L = X_{mid} - X_{0,135\%} \quad (1)$$

and

$$\Delta_U = X_{99,865\%} - X_{mid} \quad (2)$$

The process performance indices are defined by ratios of length of a geometric parameter of the distribution to the specified tolerance.

Process performance index:

$$P_p = \frac{U-L}{\Delta} \quad (3)$$

Lower process performance index:

$$P_{pk_L} = \frac{X_{\text{mid}} - L}{\Delta_L} \quad (4)$$

Upper process performance index:

$$P_{pk_U} = \frac{U - X_{\text{mid}}}{\Delta_U} \quad (5)$$

Minimum process performance index:

$$P_{pk} = \min(P_{pk_L}, P_{pk_U}) \quad (6)$$

If a process is shown to be in the state of statistical control, a capability index can be assigned. The formulae are the same as for the corresponding performance index.

Capability index:

$$C_p = \frac{U-L}{\Delta} \quad (7)$$

Lower capability index:

$$C_{pk_L} = \frac{X_{\text{mid}} - L}{\Delta_L} \quad (8)$$

Upper capability index:

$$C_{pk_U} = \frac{U - X_{\text{mid}}}{\Delta_U} \quad (9)$$

Minimum capability index:

$$C_{pk} = \min(C_{pk_L}, C_{pk_U}) \quad (10)$$

There are different estimators for the location,  $\mu$ , and the dispersion,  $\Delta$ , of a given data set.

**IMPORTANT — It should be emphasized that a quantitative comparison of the performance or capability indices calculated according to the different methods is not meaningful and should not be done.**

### 6.1.2 Calculation of location

The location of the process,  $X_{\text{mid}}$ , can be calculated using one of the formulae given in [Table 3](#).

**Table 3 — Different methods for calculation of location**

Location method label, <i>l</i>	Calculation method of location/Formula $M_{l,d}$	No.
1	$\hat{X}_{\text{mid}} = \bar{x} = \frac{1}{k \cdot n} \sum x_i$	(11)
2	$\hat{X}_{\text{mid}} = \tilde{x} = X_{50\%} = \begin{cases} x_{\left(\frac{n+1}{2}\right)} & ; n \text{ odd} \\ \frac{1}{2} \left[ x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)} \right] & ; n \text{ even} \end{cases}$ order statistic $x_i$	(12)
3	$\hat{X}_{\text{mid}} = \bar{\bar{x}} = \frac{1}{k} \sum_{j=1}^k \bar{x}_j$	(13)
4	$\hat{X}_{\text{mid}} = \tilde{\tilde{x}} = \frac{1}{k} \sum_{j=1}^k \tilde{x}_j$	(14)
$x_i$ individual values $n$ number of values $\bar{x}_j$ average of the $j$ th subgroup $k$ number of subgroups of size $n$ $\tilde{x}_j$ median of the $j$ th subgroup		

### 6.1.3 Calculation of dispersion

The dispersion of the process can be calculated using one of the formulae given in [Table 4](#).



**Table 4 — Different methods for calculation of dispersion**

Dispersion method label, $d$	Calculation method of dispersion/Formula $M_{l,d}$	No.
1	$\hat{\Delta} = X_{99,865\%} - X_{0,135\%};$ $\hat{\Delta}_U = X_{99,865\%} - X_{\text{mid}}; \hat{\Delta}_L = X_{\text{mid}} - X_{0,135\%}$	(15)
2	$\hat{\Delta} = 6\hat{\sigma}; \hat{\Delta}_U = 3\hat{\sigma}; \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = \sqrt{\frac{\sum s_i^2}{k}}$	(16)
3	$\hat{\Delta} = 6\hat{\sigma}; \hat{\Delta}_U = 3\hat{\sigma}; \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = \frac{\sum s_i}{k \cdot c_4}$	(17)
4	$\hat{\Delta} = 6\hat{\sigma}; \hat{\Delta}_U = 3\hat{\sigma}; \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = \frac{\sum R_i}{k \cdot d_2}$	(18)
5	$\hat{\Delta} = 6\hat{\sigma}; \hat{\Delta}_U = 3\hat{\sigma}; \hat{\Delta}_L = 3\hat{\sigma}$ where $\hat{\sigma} = s_t = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$	(19)

$s_i^2$  variance of the  $i$ th subgroup  
 $s_i$  standard deviation of the  $i$ th subgroup  
 $k$  number of subgroups of size  $n$   
 $R_i$  range of the  $i$ th subgroup  
 $s_t$  standard deviation of the whole data set

See ISO 7870-2 for tables of  $c_4$  and  $d_2$  coefficients.

$M_{l,d}$  is used as a symbol for the calculation method. The subscript  $l$  refers to an equation for calculation of the estimator for the location  $\mu$  [Formulae (11) to (14)]. The subscript  $d$  refers to an equation for calculation of the estimator for the dispersion  $\Delta$  [Formulae (15) to (19)].

#### 6.1.4 Calculation of $X_{0,135\%}$ and $X_{99,865\%}$

The three procedures that can be used to estimate the  $X_{0,135\%}$  and  $X_{99,865\%}$  are the following.

- Fit a distribution to the combined data set, and estimate them from the fitted resulting distribution.
- Estimate them directly from the combined data set. In order to obtain reliable estimate of  $X_{0,135\%}$  and  $X_{99,865\%}$  in this procedure, the size of the given data set must be large. For instance, for a combined sample sizes of 1 000,  $X_{0,135\%}$  and  $X_{99,865\%}$  are taken to be the minimum and maximum value of the data set.
- Estimate them from a probability plot in accordance to ISO 5479). If the data do not form a normal distribution it may become necessary to employ a different worksheet.

The symbol for the calculation of an index should be  $M_{l,d}$ , where  $l$  defines the calculation method for location and  $d$  defines the calculation method for the dispersion.

EXAMPLE The calculation method  $M_{1,2}$  is based on calculation of average and variance.

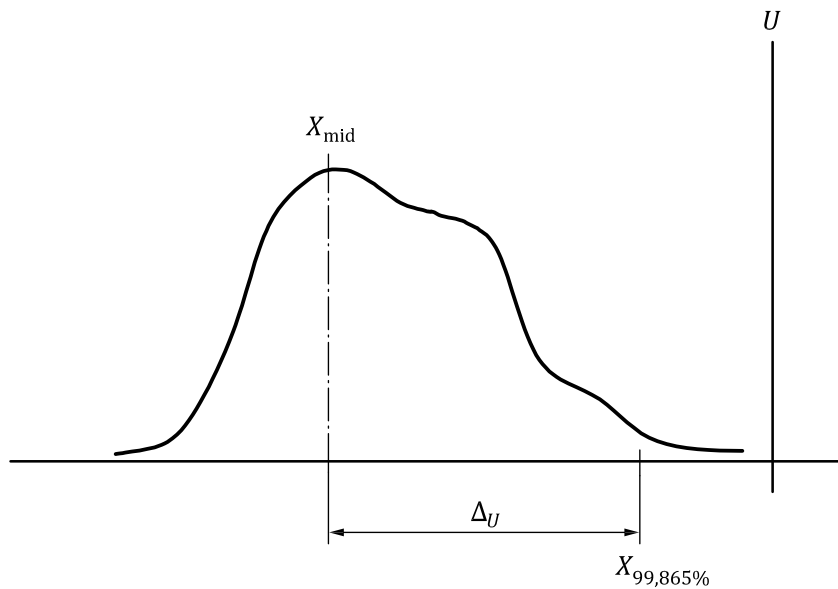
- The estimator  $\hat{\Delta}$  for  $d = 1$  is the most general one, it may be used under all conditions.
- The estimators  $\hat{\Delta}$  for  $d = 2, 3$  and  $4$  estimate the subgroup spread only. They should be used for process model A1 only because they neglect the differences between subgroups.

— The estimators  $\hat{\Delta}$  for  $d = 2, 3, 4$  and  $5$  assume that the data are normally distributed. Otherwise, their result is biased depending on the type of distribution.

NOTE  $\hat{\Delta}$  is also called the *reference interval*.

## 6.2 One-sided specification limits

One-sided specification limits can be treated in the same manner as two-sided specification limits. See [Figure 10](#).



**Figure 10 — Graphical representation of the calculation method  $\Delta_U$**

In the case of an upper specification limit, we have the following.

Upper process performance index:

$$P_{pk_U} = \frac{U - X_{mid}}{\Delta_U} \quad (16)$$

Minimum process performance index:

$$P_{pk} = P_{pk_U} \quad (17)$$

If a process is proven to be in the state of statistical control, a capability index can be assigned. The formulae are the same as for the corresponding performance index.

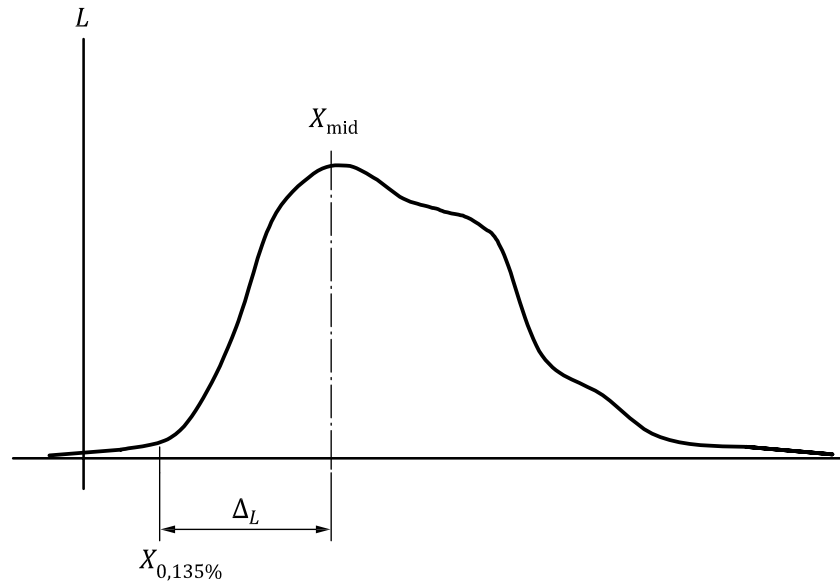
Upper capability index:

$$C_{pk_U} = \frac{U - X_{mid}}{\Delta_U} \quad (18)$$

Minimum capability index:

$$C_{pk} = C_{pk_U} \quad (19)$$

$X_{99,865\%}$  and  $X_{mid}$  are estimated as in method  $M_{2,1}$ . See [Figure 11](#).



**Figure 11 — Graphical representation of calculation method  $\Delta_L$**

In the case of a lower specification limit, we have the following.

Lower process performance index:

$$P_{pk_L} = \frac{X_{mid} - L}{\Delta_L} \quad (20)$$

Minimum process performance index:

$$P_{pk} = P_{pk_L} \quad (21)$$

If a process is proven to be in the state of statistical control, a capability index can be assigned. The formulae are the same as for the corresponding performance index.

Lower capability index:

$$C_{pk_L} = \frac{X_{mid} - L}{\Delta_L} \quad (22)$$

Minimum capability index:

$$C_{pk} = C_{pk_L} \quad (23)$$

$X_{0,135\%}$  and  $X_{mid}$  are estimated as in method  $M_{2,1}$ .

### 6.3 Use of different calculation methods

For a specific time-dependent distribution model not all calculation methods can be used. [Table 5](#) shows the combination of models and calculation methods.

**Table 5 — Process capability indices**

	Time model	A1	A2	B	C1	C2	C3	C4	D
<b>Location calculation</b>	1	a		a					
	2	a	a	a	a	a	a	a	a
	3	a							
	4	a	a	a					
<b>Dispersion calculation</b>	1	a	a	a	a	a	a	a	a
	2	a							
	3	a							
	4	a							
	5	a	a	a	a				a

<sup>a</sup> Indicates those methods which could be used for the calculation of indices.

## 7 Reporting process performance/capability indices

If process performance/capability statistics are used for process qualification, they shall be reported with relation to this document. The calculation methods for location and dispersion and the number of values used as basis for the calculation and also the measurement uncertainty shall be stated.

Other information may be added, including

- frequency of sampling,
- time and duration of data taking; choice of time distribution model justification, and
- technical conditions (batches, operation, tools).

An example is given in [Table 6](#).

**Table 6 — Example of report of calculated process capability indices**

Process performance/capability index	$C_p = 1,68$
Minimum process performance/capability index	$C_{pk} = 1,47$
Calculation method	$M_{1,1}$
Number of values used for the calculation	2 000
Measurement uncertainty	0,002 mm
Time distribution model	A1
Calculation method $M_{1,1}$ means that the capability calculation is done using the average and the reference interval as estimators for location and dispersion.	

## Bibliography

- [1] ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*
- [2] ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*
- [3] ISO 7870-2, *Control charts — Part 2: Shewhart control charts*
- [4] ISO 9000:2015, *Quality management systems — Fundamentals and vocabulary*
- [5] ISO 22514-1, *Statistical methods in process management — Capability and performance — Part 1: General principles and concepts*
- [6] ISO 22514-4:2016, *Statistical methods in process management — Capability and performance — Part 4: Process capability estimates and performance measures*
- [7] Kotz, S. and Lovelace, C.R. (1998). *Process Capability Indices in Theory and Practice*. Arnold, London





# British Standards Institution (BSI)

BSI is the national body responsible for preparing British Standards and other standards-related publications, information and services.

BSI is incorporated by Royal Charter. British Standards and other standardization products are published by BSI Standards Limited.

## About us

We bring together business, industry, government, consumers, innovators and others to shape their combined experience and expertise into standards-based solutions.

The knowledge embodied in our standards has been carefully assembled in a dependable format and refined through our open consultation process. Organizations of all sizes and across all sectors choose standards to help them achieve their goals.

## Information on standards

We can provide you with the knowledge that your organization needs to succeed. Find out more about British Standards by visiting our website at [bsigroup.com/standards](http://bsigroup.com/standards) or contacting our Customer Services team or Knowledge Centre.

## Buying standards

You can buy and download PDF versions of BSI publications, including British and adopted European and international standards, through our website at [bsigroup.com/shop](http://bsigroup.com/shop), where hard copies can also be purchased.

If you need international and foreign standards from other Standards Development Organizations, hard copies can be ordered from our Customer Services team.

## Copyright in BSI publications

All the content in BSI publications, including British Standards, is the property of and copyrighted by BSI or some person or entity that owns copyright in the information used (such as the international standardization bodies) and has formally licensed such information to BSI for commercial publication and use.

Save for the provisions below, you may not transfer, share or disseminate any portion of the standard to any other person. You may not adapt, distribute, commercially exploit, or publicly display the standard or any portion thereof in any manner whatsoever without BSI's prior written consent.

## Storing and using standards

Standards purchased in soft copy format:

- A British Standard purchased in soft copy format is licensed to a sole named user for personal or internal company use only.
- The standard may be stored on more than 1 device provided that it is accessible by the sole named user only and that only 1 copy is accessed at any one time.
- A single paper copy may be printed for personal or internal company use only.

Standards purchased in hard copy format:

- A British Standard purchased in hard copy format is for personal or internal company use only.
- It may not be further reproduced – in any format – to create an additional copy. This includes scanning of the document.

If you need more than 1 copy of the document, or if you wish to share the document on an internal network, you can save money by choosing a subscription product (see 'Subscriptions').

## Reproducing extracts

For permission to reproduce content from BSI publications contact the BSI Copyright & Licensing team.

## Subscriptions

Our range of subscription services are designed to make using standards easier for you. For further information on our subscription products go to [bsigroup.com/subscriptions](http://bsigroup.com/subscriptions).

With **British Standards Online (BSOL)** you'll have instant access to over 55,000 British and adopted European and international standards from your desktop. It's available 24/7 and is refreshed daily so you'll always be up to date.

You can keep in touch with standards developments and receive substantial discounts on the purchase price of standards, both in single copy and subscription format, by becoming a **BSI Subscribing Member**.

**PLUS** is an updating service exclusive to BSI Subscribing Members. You will automatically receive the latest hard copy of your standards when they're revised or replaced.

To find out more about becoming a BSI Subscribing Member and the benefits of membership, please visit [bsigroup.com/shop](http://bsigroup.com/shop).

With a **Multi-User Network Licence (MUNL)** you are able to host standards publications on your intranet. Licences can cover as few or as many users as you wish. With updates supplied as soon as they're available, you can be sure your documentation is current. For further information, email [subscriptions@bsigroup.com](mailto:subscriptions@bsigroup.com).

## Revisions

Our British Standards and other publications are updated by amendment or revision.

We continually improve the quality of our products and services to benefit your business. If you find an inaccuracy or ambiguity within a British Standard or other BSI publication please inform the Knowledge Centre.

## Useful Contacts

### Customer Services

**Tel:** +44 345 086 9001

**Email (orders):** [orders@bsigroup.com](mailto:orders@bsigroup.com)

**Email (enquiries):** [cservices@bsigroup.com](mailto:cservices@bsigroup.com)

### Subscriptions

**Tel:** +44 345 086 9001

**Email:** [subscriptions@bsigroup.com](mailto:subscriptions@bsigroup.com)

### Knowledge Centre

**Tel:** +44 20 8996 7004

**Email:** [knowledgecentre@bsigroup.com](mailto:knowledgecentre@bsigroup.com)

### Copyright & Licensing

**Tel:** +44 20 8996 7070

**Email:** [copyright@bsigroup.com](mailto:copyright@bsigroup.com)

### BSI Group Headquarters

389 Chiswick High Road London W4 4AL UK