INTERNATIONAL STANDARD

ISO 21771

First edition
2007-09-01

Gears — Cylindrical involute gears and gear pairs — Concepts and geometry

Engrenages — Roues et engrenages cylindriques à développante — Concepts et géométrie

Reference number
ISO 21771:2007(E)

Contents

Page

Foreword iv

1 Scope 1

2 Normative references 1

3 Symbols, subscripts and units 2
3.1 Symbols 2
3.2 Subscripts 8
3.3 Units 9

4 Individual cylindrical gears 10
4.1 Concepts for an individual gear 10
4.2 Reference surfaces, datum lines and reference quantities 12
4.3 Involute helicoids 17
4.4 Angular pitch and pitches 21
4.5 Diameters of gear teeth 24
4.6 Gear tooth height 24
4.7 Tooth thickness, space width 25

5 Cylindrical gear pairs 27
5.1 Concepts for a gear pair 27
5.2 Mating quantities 28
5.3 Calculation of the sum of the profile shift coefficients 31
5.4 Tooth engagement 32
5.5 Backlash 40
5.6 Sliding conditions at the tooth flanks 41

6 Tooth flank modifications 44
6.1 Tooth flank modifications which restrict the usable flank 44
6.2 Transverse profile modifications 45
6.3 Flank line (helix) modifications 48
6.4 Flank face modifications 49
6.5 Descriptions of modifications by functions 51

7 Geometrical limits 52
7.1 Counterpart rack tooth profile 53
7.2 Machining allowance 54
7.3 Deviations in tooth thickness 55
7.4 Generating profile shift, generating profile shift coefficient 56
7.5 Generated root diameter 57
7.6 Usable area of the tooth flank, tip and root form diameter 57
7.7 Undercut 59
7.8 Overcut 59
7.9 Minimum tooth thickness at the tip circle of a gear 59

Annex A (informative) Calculations related to tooth thickness 60

Bibliography 83

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 21771 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771 cancels and replaces ISO/TR 4467:1982, of which it constitutes a technical revision.

Gears — Cylindrical involute gears and gear pairs — Concepts and geometry

1 Scope

This International Standard specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included.

It also covers the concepts and parameters for cylindrical gear pairs with parallel axes and a constant gear ratio, which consist of cylindrical gears according to it. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

The equations given are not restricted to the pressure angle, $\alpha_P = 20°$.

The standard is structured as follows.

— Listing of symbols and nomenclature for a unique description of gears and gear pairs (see Clause 3).

— Equations and explanations of the relevant values for defining a cylindrical gear and its tooth system. The equations for determination of the nominal values for zero-deviation gear description parameters are stated for radial tooth dimensions (gear tooth heights), the distance between flanks of the same hand, the distance between flanks of opposite hand, as well as the tooth flank characterizing parameters (see Clause 4).

— Equations and explanations of the relevant values for defining cylindrical gear pairs. The equations for the essential parameters characterizing the engagement conditions of the unloaded gear pair are listed (see Clause 5).

— Equations and suggestions for desired flank modifications (see Clause 6).

— Concepts and recommendations needed for a unique geometrical definition of the intended results from manufacture (Clause 7).

— Equations for determination of the nominal values or the limiting values for the most used inspection methods for tooth thickness (see Annex A).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*

ISO 1328-2:1997, *Cylindrical gears — ISO system of accuracy — Part 2: Definitions and allowable values of deviations relevant to radial composite deviations and runout information*

3 Symbols, subscripts and units

3.1 Symbols

Symbol	Description	Used in
a_{w}	centre distance of a cylindrical gear pair	5.2.3
a_0	centre distance in the generating gear unit	7.5
a_{L}	centre distance for tooth flank engagement	A.8
b	facewidth	4.2.8
b_{F}	usable facewidth	4.2.8
b_{M}	contact line overlap (for measuring base tangent length)	A.2.1
b_{w}	active facewidth (the facewidth used)	5.4.7.2
c	tip clearance	5.2.7
c_{F}	form over dimension	5.4.4
d	reference diameter	4.2.4
d_{a}	tip diameter	4.5.3
d_{a0}	tip diameter of tool	7.5
d_{aM}	tip diameter of overcut cylindrical gears	A.9
d_{b}	base diameter	4.3.10
d_{b0}	base diameter of the pinion-type cutter	7.6
d_{f}	root diameter (nominal dimension)	4.5.4
d_{fE}	root diameter produced	7.5
d_{f0}	root diameter of the pinion-type cutter	A.9
d_{v}	V-circle diameter	4.5.1
d_{w}	working pitch diameter	5.2.5
d_{y}	Y-circle diameter	4.3.3
d_{Fa}	tip form diameter	7.6
d_{Fa0}	tip form diameter of the pinion-type cutter	7.6
d_{Ff}	root form diameter	7.6
d_{K}	diameter of circle through centre of ball	A.5
d_{M}	diameter of a measuring circle	A.2.1
d_{Na}	active tip diameter	5.4.1
d_{Nf}	start of active profile diameter (SAP diameter, active root diameter)	5.4.1
e_{t}	space width on the reference cylinder	4.7.3
e_{yt}	space width on the Y-cylinder	4.7.3

Symbol	Description	Used in
e_{P}	space width of the standard basic rack tooth profile	4.2.3
g_{a}	length of addendum path of contact	5.4.5.2
g_{f}	length of dedendum path of contact	5.4.5.2
g_{α}	length of path of contact	5.4.5.2
$g_{\alpha y}$	distance of a point Y from pitch point C	5.6.1
g_{β}	arc of contact	5.4.7.4
h	tooth depth (between tip line and root line)	4.6.1
h_{a}	addendum	4.6.2
h_{aP}	addendum of the standard basic rack tooth profile	Figure 4
h_{aP0}	addendum of the tool standard basic rack tooth profile	7.5
h_{f}	dedendum	4.6.2
h_{fP}	dedendum of the standard basic rack tooth profile	Figure 4
h_{fP0}	dedendum of the tool standard basic rack tooth profile	A.9
h_{w}	working depth of teeth in a gear pair	5.2.6
h_{FfP}	depth of dedendum form of the standard basic rack tooth profile	Figure 4
h_{K}	radius of the tip corner chamfering or tip corner rounding	6.1.2
h_{P}	tooth depth of standard basic rack tooth profile	Figure 4
inv	involute function	4.3.9
j_{bn}	contact backlash	5.5
j_{r}	radial backlash	5.5
j_{t}	circumferential backlash at the reference circle	5.5.2
j_{wt}	circumferential backlash at the pitch circle	5.5
k	number of teeth, spaces or pitches in a span (e.g. number of teeth spanned)	A.2.1
k	addendum modification coefficient	4.5.2
l_{max}	path of engagement	5.4.8
$\sum l$	sum of path of contact	5.4.8
m_{n}	normal module	4.2.7
m_{t}	transverse module	4.2.7
m_{x}	axial module	4.2.7
n_{a}	number of revolutions of driving gear (rpm)	5.2.2
n_{b}	number of revolutions of driven gear (rpm)	5.2.2
p	pitch, pitch on the reference cylinder	Figure 4

Symbol	Description	Used in
p_{bn}	normal pitch on the base cylinder	4.4.5
p_{bt}	transverse pitch on the base cylinder	4.4.5.1
p_{en}	normal base pitch on the path of contact	4.4.5.2
p_{et}	transverse base pitch on the path of contact	4.4.5.1
p_n	normal pitch	4.4.2.2
p_t	transverse pitch	4.4.2.1
p_x	axial pitch	4.4.4
p_{yn}	normal pitch on the Y-cylinder	4.4.3
p_{yt}	transverse pitch on the Y-cylinder	4.4.3
p_z	lead	4.3.2
q	machining allowance on tooth flank	7.2
q_{Fs}	undercut	Figure 24
s_{aK}	residual tooth thickness at tip with tip corner chamfering or tip corner rounding	6.1.2
s_{bn}	normal tooth thickness on the base circle	A.2.2
s_n	normal tooth thickness on the reference circle	4.7.5
s_{ni}	minimum normal tooth thickness on the reference circle	7.3
s_{ns}	maximum normal tooth thickness on the reference circle	7.3
s_t	transverse tooth thickness on the reference circle	4.7.1
s_{yn}	normal tooth thickness on the Y-cylinder	4.7.5
s_{yt}	transverse tooth thickness on the Y-cylinder	4.7.1
s_P	tooth thickness of the standard basic rack tooth profile	4.2.3
u	gear ratio	5.2.1
v_g	sliding speed	5.6.1
v_{ga}	sliding speed at the addendum	5.6.1
v_{gf}	sliding speed at the dedendum	5.6.1
v_n	normal speed	5.6.1
x	profile shift coefficient	4.2.9
x_E	generating profile shift coefficient	7.4
x_{Emin}	generating profile shift coefficient at undercut limit	7.7
x_L	profile shift coefficient of master gear	A.8
z	number of teeth	4.1.5
z_a	number of teeth of driving gear	5.2.2

Symbol	Description	Used in
z_b	number of teeth of driven gear	5.2.2
z_L	number of teeth of master gear	A.8
z_0	number of teeth of pinion-type cutter	7.6
A	starting point of meshing	5.4.3
B	starting point of single tooth contact on driving gear	5.4.5.1
C	pitch point, depth of relief for modifications	5.4.3
C_{ay}	modification of the profile	6.5
$C_{\beta y}$	modification of the flank line	6.5
$C_{\Sigma y}$	modification of the flank surface	6.5
$C_{\alpha a}$	amount of tip relief	6.2.1
$C_{\alpha f}$	amount of root relief	6.2.1
C_{Ea}	tip amount of triangular end relief modification	6.4.2
C_{Ef}	root amount of triangular end relief modification	6.4.2
$C_{i,j}$	amount of modification at point (i,j)	6.4.1
$C_{H\alpha}$	amount of transverse profile slope modification	6.2.2
C_{α}	amount of profile crowning (barrelling)	6.2.3
$C_{\beta I}$, $C_{\beta II}$	amount of end relief	6.3.1
C_{β}	amount of flank line crowning	6.3.3
$C_{H\beta}$	amount of flank line slope modification	6.3.2
D_M	measuring ball or measuring cylinder diameter	A.5
D	end point of single tooth contact point on driving gear	5.4.5.1
E	end point of meshing	5.4.3
E_{sn}	normal tooth thickness deviation limit (or allowance)	A.9
E_{sni}	lower deviation limit for tooth thickness	7.3
E_{sns}	upper deviation limit for tooth thickness	7.3
K_g	sliding factor	5.6.2
K_{ga}	sliding factor at tooth tip	5.6.2
K_{gf}	sliding factor at tooth root	5.6.2
L_{AE}	roll length	6.2
L_{Ca}	tip relief roll length	6.2.1
L_{Cf}	root relief roll length	6.2.1
L_{CI}, L_{CII}	length of end relief	6.3.1
L_{Ea}	tip roll length of triangular end relief modification	6.4.2

Symbol	Description	Used in
L_{Ef}	root roll length of triangular end relief modification	6.4.2
M_{dK}	dimension over balls	A.7
M_{dZ}	dimension over cylinders	A.7.1
M_{rK}	radial single-ball dimension	A.5
M_{rZ}	radial single-cylinder dimension	A.6
N	number of tooth or pitch	4.1.6
O	centre of a circle	Figure 10
S_{α}	twist of the transverse profile	6.4.3
S_{β}	twist of the flank line	6.4.3
T_{sn}	tooth thickness tolerance	Figure 37
T	contact point of tangent (lines of engagement) at base circle	Figure 10
U	involute point of origin	4.3.7
W_{k}	base tangent length over k measured teeth or measured spaces	A.2.1
Y	any point on a tooth flank or involute	4.3.5
α_{n}	normal pressure angle	4.3.6
α_{t}	transverse pressure angle	4.3.5
α_{wt}	working transverse pressure angle of gear pair	5.2.4
α_{wt0}	working transverse pressure angle in the generating gear unit	7.6
α_{yn}	normal pressure angle at the Y-cylinder	4.3.6
α_{yt}	transverse pressure angle at the Y-cylinder	4.3.5
α_{Ff}	pressure angle at root form circle	7.6
α_{K}	pressure angle at circle through centre of ball	A.5
α_{Kt}	transverse pressure angle at a point at circle through centre of ball	A.5
α_{Mt}	transverse pressure angle at a point at measuring circle	A.5
α_{P}	pressure angle of the standard basic rack tooth profile	4.3.6
α_{P0}	pressure angle of the tool basic rack tooth profile	7
α_{L}	working transverse pressure angle for double-flank engagement	A.8
α_{vt}	transverse pressure angle at the V-cylinder	A.5
β	helix angle	4.3.3
β_{b}	base helix angle	4.3.3
β_{y}	helix angle at Y-cylinder	4.3.3
δ_{w}	angle of rocking for span measurement	A.2.1

Symbol	Description	Used in
γ	lead angle at reference cylinder	4.3.3
γ_{y}	lead angle at Y-cylinder	4.3.3
ε_{α}	transverse contact ratio	5.4.7.1
ε_{β}	overlap ratio	5.4.7.3
ε_{γ}	total contact ratio	5.4.7.5
ζ	specific sliding	5.6.3
ζ_{f}	specific sliding at end points of path of contact	5.6.3
η	space width half angle at reference circle	4.7.4
η_{b}	base space width half angle	4.7.4
η_{y}	space width half angle at Y-circle	4.7.4
ξ_{y}	rolling angle of the involute at point Y	4.3.7
ξ_{Fa0}	rolling angle at tip form circle of pinion-type cutter	7.6
ξ_{Ff}	rolling angle at root form circle	7.6
ξ_{Na}	rolling angle at active tip circle	5.4.1
ξ_{Nf}	rolling angle at active root circle	5.4.1
ρ_{fP}	root radius on the standard basic rack tooth profile	Figure 4
ρ_{y}	radius of curvature of the involute at point Y	4.3.8
τ	angular pitch	4.4.2
φ_{j}	backlash angle	5.5.2
φ_{α}	transverse angle of transmission	5.4.7.1
φ_{β}	overlap angle	5.4.7.3
φ_{γ}	total angle of transmission	5.4.7.5
ψ	tooth thickness half angle at reference circle	4.7.2
ψ_{b}	base tooth thickness half angle	4.7.2
ψ_{y}	tooth thickness half angle at Y-circle	4.7.2
ω_{a}	angular velocity of driving gear	5.2.2
ω_{b}	angular velocity of driven gear	5.2.2
$\sum x$	sum of profile shift coefficients	5.3
$\sum x_{\mathrm{E}}$	sum of profile shift coefficient, non-zero backlash	5.3

3.2 Subscripts

Subscript	Description	Used in [b]
—	[a]	
a	for quantities associated with the tip of a tooth or for the driving gear	5.2.2
b	for quantities associated with the base cylinder	4.3.10
b	for quantities associated with the driven gear	5.2.2
e	for quantities associated with the plane of action	
f	for quantities associated with the root	
g	for “sliding”	
i	for the lower limit in the case of deviations	
k	for a number of teeth, spaces, pitches or spans	
l	for “left-hand”	
m	for a mean value	
max	for a maximum value	
min	for a minimum value	
n	for quantities in a normal section	4.2.6.2
r	for “right-hand”	
s	relating to “tooth thickness”, for the upper limit in the case of deviations	
t	for quantities in a transverse section	4.2.6.1
v	for quantities associated with the V-cylinder	4.5.1
w	for quantities associated with the pitch cylinder and working values of a gear pair	
x	for quantities in an axial section	4.2.6.3
y	for values at a point Y (on the Y-cylinder)	
E	relating to “generating” (e.g. quantities generated on the cylindrical gear) or “generator”	
F	for quantities determining form circles and maximum usable flank area	
K	for quantities resulting from corner chamfering or for ball dimensions	
L	for designating a master gear	
L	for designating left flanks	4.1.8.2
M	for designating a measured value	
N	for active circles	
P	for quantities of the standard basic rack tooth profile	
P0	for quantities of the tool standard basic rack tooth profile	
R	for designating right flanks	4.1.8.2
V	for working side, for rough gear cutting	
W	for measuring base tangent length	

Subscript	Description	Used in [b]
Z	for quantities associated with cylinder dimensions	
α	for quantities associated with contact	
β	for quantities associated with a tooth trace	
γ	for total contact ratio	
$\sum$	for "sum"	
0	for quantities associated with the generating tool or the generating gear unit	7
1	for quantities associated with the pinion (smaller gear) of a gear pair	5.1.3
2	for quantities associated with the wheel (larger gear) or internal gear, used for designating a coefficient relating to the module	5.1.3
I	for locating face	4.2.1
II	for the face opposite the locating face	4.2.1

[a] No subscript designates quantities associated with the reference cylinder.

[b] Used with the symbols listed in 3.1 or as additions.

3.3 Units

The quantities dealt with in this International Standard are to be stated in the following units:

— modules, lengths and linear dimensions in millimetres (mm);

— angles which are to be used in equations in radians (rad);

— angles which can be used for entries or to display results in degrees (°);

— angular velocity in radians per second (rad/s).

NOTE The notation $|z|$, denotes the absolute value, which is always positive, e.g. $|-50| = +50$. The expression $\frac{z}{|z|}$ is used to extract the sign of the tooth number and is convenient for programming. In particular, it is used often to determine the appropriate sign for an element of an expression; the result is 1 for external gears and -1 for internal gears.

4 Individual cylindrical gears

In this clause, the geometry of gear teeth is described using a generation process based on zero backlash engagement with a basic rack. The relationships are valid for any basic rack, but the standard basic rack (see ISO 53) is used for illustration. The standard basic rack tooth profile of the tooth system has straight flanks. Its datum line is the straight line on which the nominal dimensions of tooth thickness and space width are defined as equal to half the pitch. The standard basic rack tooth profile has the same pressure angles for the left and right flanks and the addendum plus bottom clearance equal to the dedendum. The helix angles for all the tooth flanks of a gear have the same nominal value.

4.1 Concepts for an individual gear

4.1.1 Gear, cylindrical gear, external gear, internal gear

A gear is a rotationally symmetrical object (gear blank) with a tooth system worked into the rim. A cylindrical gear is a gear with a cylindrical reference surface. A distinction is made between external and internal gears according to the radial arrangement of the teeth in each case. The tips of the teeth point outwards in an external gear and inwards in an internal gear.

4.1.2 Tooth system, external teeth, and internal teeth

The tooth system refers to all the teeth and space widths around the rim of a gear. As in 4.1.1, a distinction is made between internal and external gear teeth.

4.1.3 Tooth and space

A tooth is a geometrical element on the gearwheel body that enables the transmission of force and motion. The form and dimensions of the teeth and the distance between consecutive teeth are defined by the tooth system parameters. The space is the gap between two consecutive teeth.

4.1.4 Tooth system parameters

The nominal dimensions of involute cylindrical gear teeth are uniquely determined by the diameter of the reference cylinder, the associated basic rack and its position in relation to the reference circle. The nominal dimensions are defined by the following parameters, which are independent of each other:

— number of teeth, z;

— standard basic rack tooth profile;

— normal module, m_n;

— helix angle, β, and flank direction;

— profile shift coefficient, x;

— tip diameter, d_a;

— facewidth, b.

4.1.5 Number of teeth and sign of number of teeth

The number of teeth around the rim of the gearwheel is denoted by z.

The number of teeth, z, of an external cylindrical gear must be taken as a positive value in the following equations while the number of teeth, z, in an internal cylindrical gear is to be taken as a negative value.

In the case of segments, the number of teeth, z, used in calculations is the number that there would be on the whole circumference.

4.1.6 Tooth number

When numbering teeth, the designations tooth 1, tooth 2, etc. are to be defined on a transverse surface (datum face) viewed in an agreed direction so that the teeth are numbered in ascending order (moving in a clockwise direction). If the letter N is used to denote a reference tooth, the next tooth in the direction of counting is denoted by $N + 1$ and the previous tooth going in the opposite direction by $N - 1$. Tooth No. z is followed by tooth 1 in the direction of counting, see Figure 1.

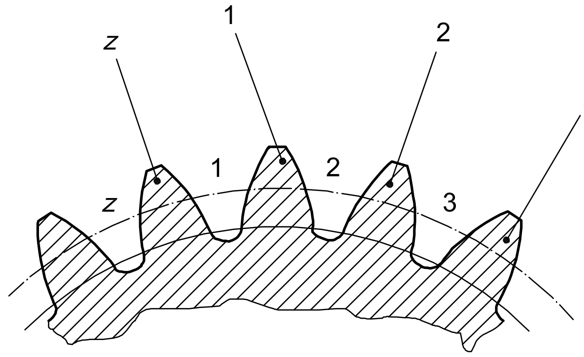


Figure 1 — Numbering of teeth and spaces on datum face

4.1.7 Top land and bottom land

4.1.7.1 Top land

The top land of a tooth is the outermost (innermost in the case of internal gears) periphery of the tooth concentric to the reference cylinder, see Figure 2.

4.1.7.2 Bottom land

The bottom land is the innermost (outermost in the case of internal gears) periphery of the space width concentric to the reference cylinder, see Figure 2.

4.1.8 Tooth flanks and flank sections

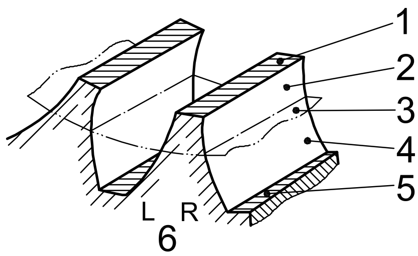
4.1.8.1 Tooth flank

Tooth flanks are those parts of the surface of a tooth that are located between the top land and the bottom land, see Figure 2.

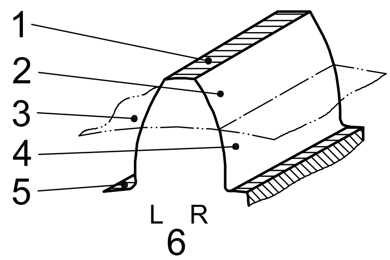
4.1.8.2 Right flank, left flank

The right flank (or left flank) is the tooth flank that an observer sees on the right-hand (or left-hand) side when viewing the datum face of a tooth when it is pointing upwards. This definition applies to both external and internal gears, see Figure 2.

Right flank parameters are indicated by the subscript R and left flank parameters by the subscript L.



a) Internal tooth



b) External tooth

Key

1 top land
2 addendum flank
3 reference cylinder
4 dedendum flank
5 bottom land
6 datum face

Figure 2 — Top land, bottom land and tooth flank with division (internal and external teeth)

4.1.8.3 Addendum flank, dedendum flank

The addendum flank (or dedendum flank) is that part of a tooth flank that is located between the reference cylinder and the top land (or the bottom land), see Figure 2.

4.1.8.4 Usable flank

The usable flank is that part of a tooth flank that can be used to engage with a mating flank. On a cylindrical gear, it is part of the involute helicoid including any flank modifications.

4.2 Reference surfaces, datum lines and reference quantities

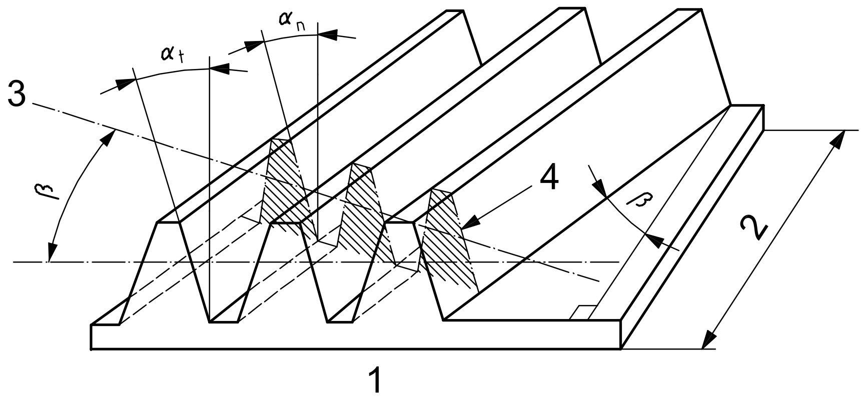
4.2.1 Reference surface, datum surface, datum face

The reference surface of the teeth is an imaginary surface to which the geometrical parameters relate. In the case of cylindrical gears, the reference surface is termed the *reference cylinder*.

The agreed front of the gear (usually used for text or suitably marked) is used as the datum face. Parameters which relate to the datum face are denoted by the subscript I while parameters which relate to the opposite face are denoted by the subscript II.

4.2.2 Reference rack

The reference rack is the rack that can be produced using the same gear-cutting tool, gear-cutting method and pitch point (pitch axis) as the actual cylindrical gear. It is characterized by its profile, the direction of its teeth in relation to the pitch axis of the generating gear unit, tip plane, root plane and facewidth, see Figure 3.



Key

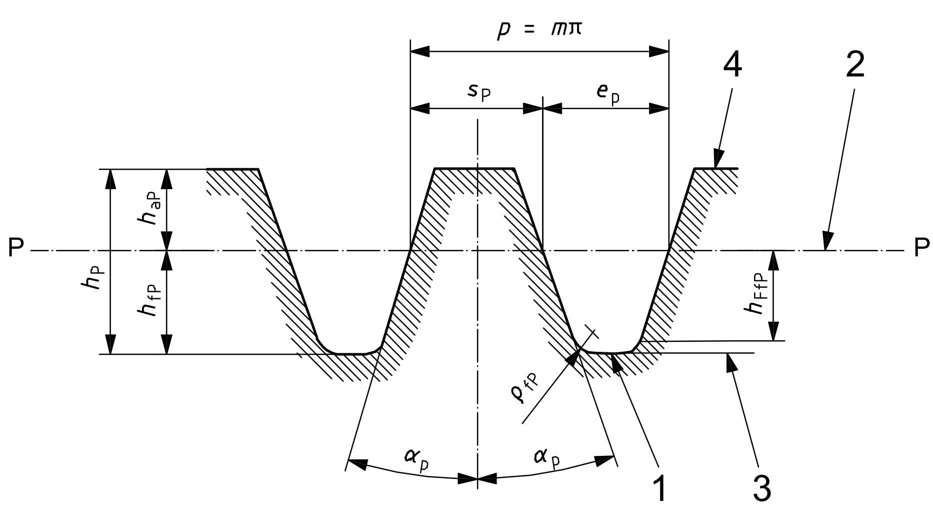
1 transverse section
2 facewidth
3 normal section
4 standard basic rack tooth profile

Figure 3 — Concepts and parameters relating to reference rack

4.2.3 Basic rack tooth profile for involute gear teeth

The basic rack tooth profile is defined in a normal section. The flanks of the basic rack tooth profile of involute teeth are straight lines. Tooth thickness, s_P, and space width, e_P, on the datum line of the basic rack (P-P) in the reference plane are equal, see Figure 4.

The standard basic rack tooth profile for involute teeth is standardized in ISO 53.



Key

1 basic rack profile
2 datum line
3 root line
4 tip line

Figure 4 — Terms and parameters relating to basic rack tooth profile in normal section

4.2.4 Reference cylinder, reference circle, reference diameter

The reference cylinder is the reference surface for the cylindrical gear teeth. Its axis coincides with the axis of the gear (gear axis). The reference circle is the intersection of the reference cylinder with a transverse plane section. The reference diameter, d, is determined by

$$d = |z| m_{\mathrm{t}} = \frac{|z| m_{\mathrm{n}}}{\cos\beta} \tag{1}$$

4.2.5 Gear axis

The axis of a gear (gear axis) is the axis that is defined by the geometrical axis of the support surfaces.

4.2.6 Sections through a cylindrical gear

4.2.6.1 Transverse section, transverse profile

The sectioning of cylindrical gear teeth by a plane perpendicular to the gear axis yields a transverse section. For helical gears, quantities in the transverse section are denoted by the subscript t. The intersection of a tooth with a transverse plane is termed the transverse profile.

4.2.6.2 Normal section, normal profile

The sectioning of involute helical gear teeth by a surface perpendicular to the flank lines of the involute helicoid yields a normal surface. The normal surface is curved three-dimensionally.

Quantities on the normal surface are denoted by the subscript n. The intersection of a tooth with a normal surface is termed the normal profile.

4.2.6.3 Axial section, axial profile

The sectioning of cylindrical gear teeth by a plane containing the gear axis yields an axial section.

Quantities in the axial section are denoted by the subscript x. The intersection of a tooth with an axial plane is termed the axial profile.

4.2.6.4 Cylindrical section, flank lines

The flank lines are lines of intersection of the right and left flanks with a cylinder that has an axis which coincides with the gear axis. Hence, right and left flank lines are to be distinguished.

The reference flank line (tooth trace) is the line of intersection of the flank with the reference cylinder. The base flank line is the line of intersection of the involute flank — possibly imagined as extended — with the base cylinder. The base flank line is a helix on the base cylinder. The origin of the involute helicoid is a base flank line. The tip flank line is the line of intersection of the involute flank — possibly imagined as extended — with the tip cylinder.

The flank lines are helices in the case of helical gear teeth and straight lines in the case of spur gear teeth.

4.2.7 Module

The module of a basic rack is found as the pitch of the rack divided by the number π (see Figure 4). The normal module, m_{n}, of the cylindrical gear is found as the module of the standard basic rack tooth profile (module series ISO 54).

For a helical gear, the transverse module, m_{t}, is found as

$$m_{\mathrm{t}} = \frac{m_{\mathrm{n}}}{\cos\beta} \tag{2}$$

and the axial module, m_{x}, as

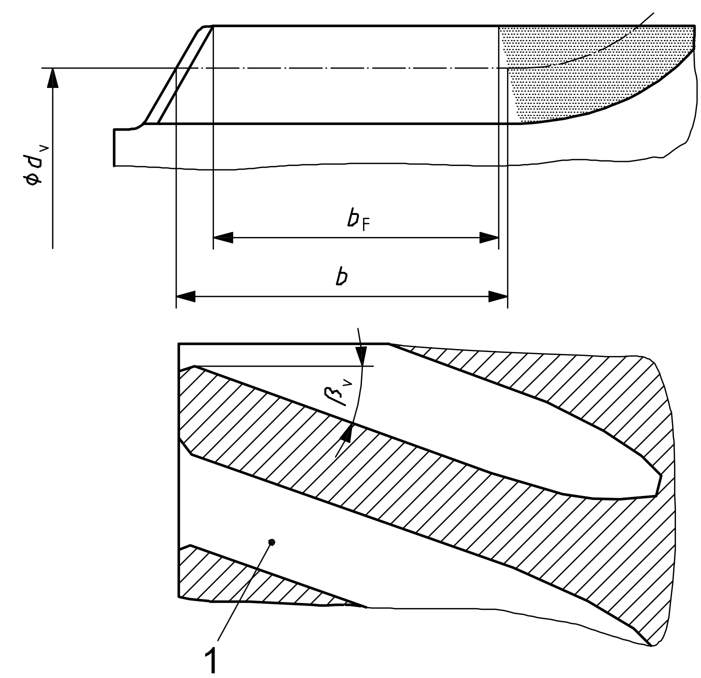
$$m_{\mathrm{x}} = \frac{m_{\mathrm{n}}}{\sin\beta} = \frac{m_{\mathrm{n}}}{\cos\gamma} = \frac{m_{\mathrm{t}}}{\tan\beta} \tag{3}$$

For a spur gear, the module is $m = m_{\mathrm{t}} = m_{\mathrm{n}}$.

4.2.8 Facewidth

The facewidth, b, is the length of the toothed part of the cylindrical gear measured in the axial direction on the V-cylinder. (See 4.5.1.)

The usable facewidth, b_{F}, is the distance between two transverse sections that contain the fully developed height of the tooth flank. (See Figure 5.)



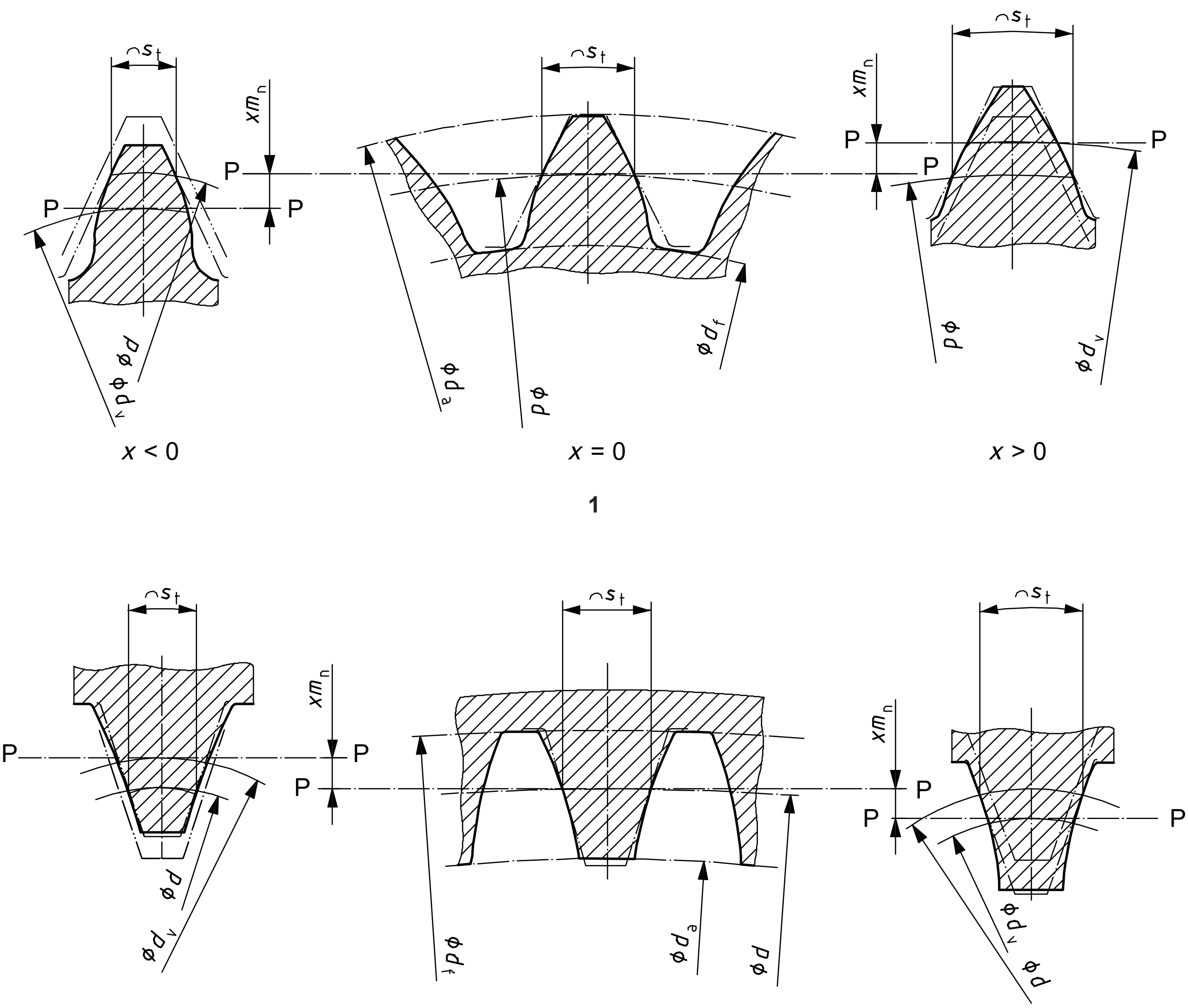
Key

1 developed view of V-cylinder

Figure 5 — Facewidth b, usable facewidth b_{F}

4.2.9 Profile shift, profile shift coefficient and sign of profile shift

The profile shift, xm_n, for involute gear teeth is the displacement of the basic rack datum line from the reference cylinder. The magnitude of the profile shift can be made non-dimensional by dividing by the normal module, and it is then expressed by the profile shift coefficient, x. Positive profile shift increases the tooth thickness on the reference cylinder. (See Figure 6.)



Key

⌒ measurement along an arc

P–P datum line of basic rack

1 external gear

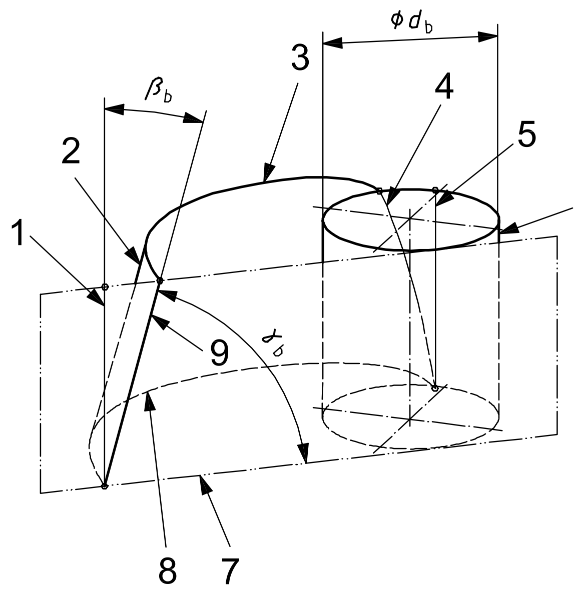
2 internal gear

Figure 6 — Profile shift for external and internal gear teeth

4.3 Involute helicoids

4.3.1 Generator of involute helicoids

In developing the base cylinder surface as a plane, a flank line on the base cylinder describes an involute helicoid. The straight line inclined to the axial line in the developed surface (base cylinder tangential plane) is the generator of the involute helicoid, see Figure 7.



Key

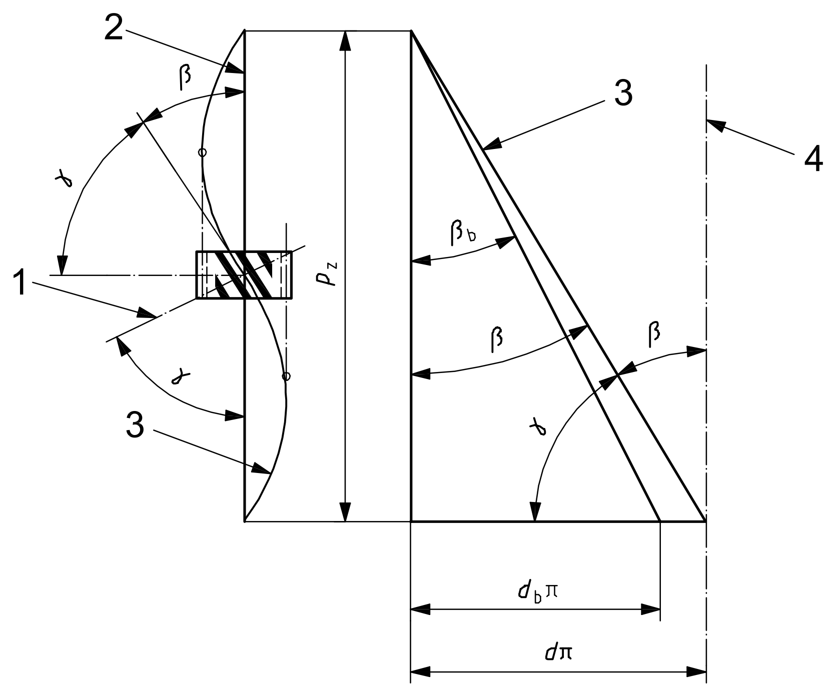
1 developed axial line
2 involute helicoid
3 involute
4 base helix
5 base cylinder axial line
6 base cylinder
7 developed base cylinder envelope
8 involute
9 straight line generator

Figure 7 — Base cylinder with generator and involute helicoids

4.3.2 Lead

The lead, p_z, is the distance between successive intersections of an axial line with the involute helicoid, see Figure 8. The lead is independent of the diameter of the cylinder.

$$p_z = \frac{|z| m_n \pi}{\sin\beta} = \frac{|z| m_t \pi}{\tan\beta} = |z| p_x \tag{4}$$



$\beta + \gamma = 90°$

Key

1 normal plane
2 reference cylinder envelope line, gear axis
3 reference trace
4 projection of the gear axis

Figure 8 — Lead triangle, lead, helix angle, lead angle

4.3.3 Helix angle, lead angle

The helix angle, β, is the angle between a tangent to a reference helix and the reference cylinder envelope line through the tangent contact point. In special cases, the helix angle, β_R, of right flanks may differ from the helix angle, β_L, of left flanks; however, all equations are based on equal helix angles.

The relationship between β and the base helix angle, β_b (helix angle on the base cylinder), is found from Equations (5) to (7):

$$\tan\beta_b = \tan\beta \cos\alpha_t \tag{5}$$

$$\sin\beta_b = \sin\beta \cos\alpha_n \tag{6}$$

$$\cos\beta_b = \cos\beta \frac{\cos\alpha_n}{\cos\alpha_t} = \frac{\sin\alpha_n}{\sin\alpha_t} = \frac{\sin\alpha_{yn}}{\sin\alpha_{yt}} = \cos\alpha_n \sqrt{\tan^2\alpha_n + \cos^2\beta} \tag{7}$$

On a cylinder with arbitrary diameter, d_y, the helix angle, β_y, is found from Equations (8) to (10):

$$\tan\beta_y = \tan\beta\frac{d_y}{d} = \tan\beta\frac{\cos\alpha_t}{\cos\alpha_{yt}} = \tan\beta_b\frac{d_y}{d_b} = \frac{\tan\beta_b}{\cos\alpha_{yt}} \tag{8}$$

$$\sin\beta_y = \sin\beta\frac{\cos\alpha_n}{\cos\alpha_{yn}} = \frac{\sin\beta_b}{\cos\alpha_{yn}} \tag{9}$$

$$\cos\beta_y = \frac{\tan\alpha_{yn}}{\tan\alpha_{yt}} = \frac{\cos\alpha_{yt}\cos\beta_b}{\cos\alpha_{yn}} \tag{10}$$

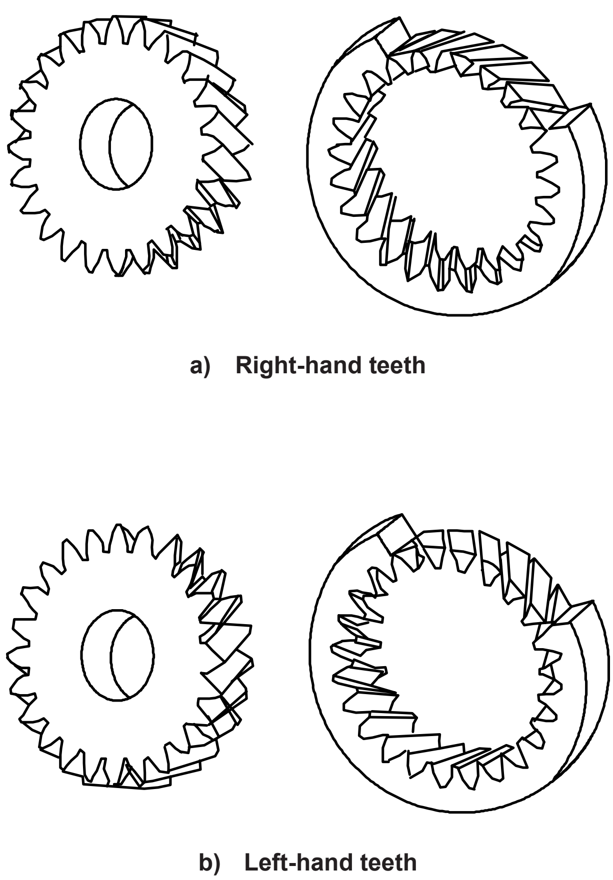
The lead angle, γ, is the angle at which the normal plane crosses the gear axis, see Figure 8. It is also the angle between a tangent to a reference helix (reference flank line) and the transverse section through the tangent contact point:

$$\gamma_y = 90° - \beta_y \tag{11}$$

For spur gears, β = 0° and γ = 90°.

4.3.4 Flank direction

The flank direction is right-handed if the flank line describes a right-hand helix and left-handed if the flank line describes a left-hand helix. (See Figure 9.)



a) Right-hand teeth

b) Left-hand teeth

Figure 9 — Direction of helix

4.3.5 Transverse pressure angle at a point, transverse pressure angle

In a transverse section, the tangent to the involute at the arbitrary point Y is inclined to the radius to that point by the transverse pressure angle, α_{yt}:

$$\cos\alpha_{yt} = \frac{d_b}{d_y} = \frac{d}{d_y}\cos\alpha_t \tag{12}$$

(See Figure 10.)

The transverse pressure angle, α_t, is the acute angle between the tangent to the involutes at their point of intersection with the reference circle and the radius through this point of intersection. It is expressed by

$$\cos\alpha_t = \frac{d_b}{d} \tag{13}$$

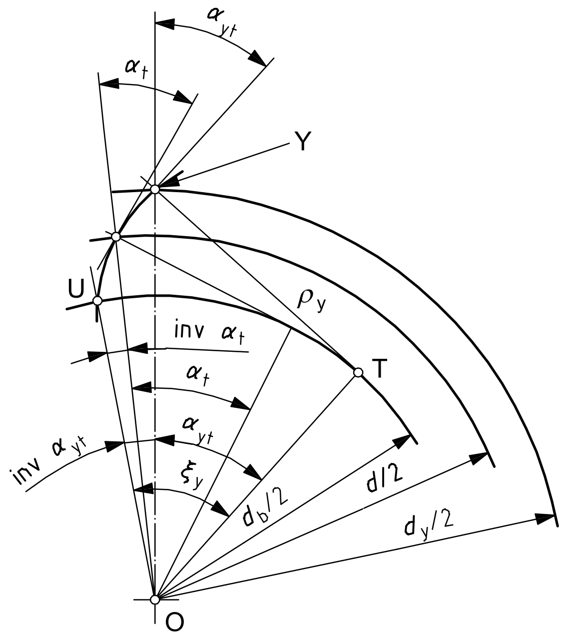


Figure 10 — Parameters relating to involute

4.3.6 Normal pressure angle at a point, normal pressure angle

In the normal section of the involute helicoid, the tangent to this section at an arbitrary point Y is inclined to the radius through Y by the normal pressure angle at that point, α_{yn}. The corresponding angle of inclination at the reference cylinder is the normal pressure angle, α_n; this is equal to the pressure angle, α_P, of the standard basic rack tooth profile.

$$\tan\alpha_n = \tan\alpha_t \cos\beta \tag{14}$$

$$\tan\alpha_{yn} = \tan\alpha_{yt} \cos\beta_y \tag{15}$$

For a spur gear, $\alpha_n = \alpha_t$ and $\alpha_{yn} = \alpha_{yt}$.

4.3.7 Roll angle of the involute

The angle at the centre over the base circle arc from the origin, U, of the involute to the contact point, T, of the tangent from point Y to the base circle is the roll angle, ξ_y, of the involute, see Figure 10. The base circle arc, UT, is equal to the tangent portion, YT, hence

$$\xi_y = \tan \alpha_{yt} \tag{16}$$

4.3.8 Radius of curvature of the involute, length of roll

The tangent portion, YT, is the radius of curvature, ρ_y, of the involute at point Y and at the same time the length of roll, L_y, belonging to point Y, i.e. the developed base circle arc from the involute origin, U. In the triangle OTY it is the side opposite the transverse pressure angle, α_{yt}, at the centre of the circle O:

$$L_y = \rho_y = \frac{z}{|z|} \frac{d_b}{2} \xi_y = \frac{z}{|z|} \frac{d_b}{2} \tan \alpha_{yt} = \frac{z}{|z|} \frac{\sqrt{d_y^2 - d_b^2}}{2} \tag{17}$$

(See Figure 10.)

4.3.9 Involute function

The angular difference, $\xi_y - \alpha_{yt}$, is termed the involute function of angle α_{yt} and is denoted by inv α_{yt} (to be read as "involute α_{yt}"):

$$\operatorname{inv} \alpha_{yt} = \xi_y - \alpha_{yt} = \tan \alpha_{yt} - \alpha_{yt} \tag{18}$$

(See Figure 10.)

4.3.10 Base cylinder, base circle, base diameter

The base cylinder is that cylinder coaxial with the gear axis that is determinative for the generation of the involute helicoids, see Figure 10. Quantities associated with the base cylinder are denoted by the subscript b.

The base circle is the intersection of the base cylinder with a plane of transverse section. The involutes from the base circle form the transverse profiles of the gearing. The base diameter, d_b, is given by

$$d_b = d \cos \alpha_t = |z| m_t \cos \alpha_t = \frac{|z| m_n \cos \alpha_t}{\cos \beta} = \frac{|z| m_n}{\sqrt{\tan^2 \alpha_n + \cos^2 \beta}} \tag{19}$$

$$d_b = |z| m_n \frac{\cos \alpha_n}{\cos \beta_b} \tag{20}$$

4.4 Angular pitch and pitches

4.4.1 Angular pitch

The angular pitch, τ, is that angle laying in transverse sections that result from the dividing of the complete periphery of a circle into z equal parts.

$$\tau = \frac{2\pi}{|z|} = \frac{2 p_{yt}}{d_y} \text{ in radians} \tag{21}$$

$$\tau = \frac{360}{|z|} \text{ in degrees} \tag{22}$$

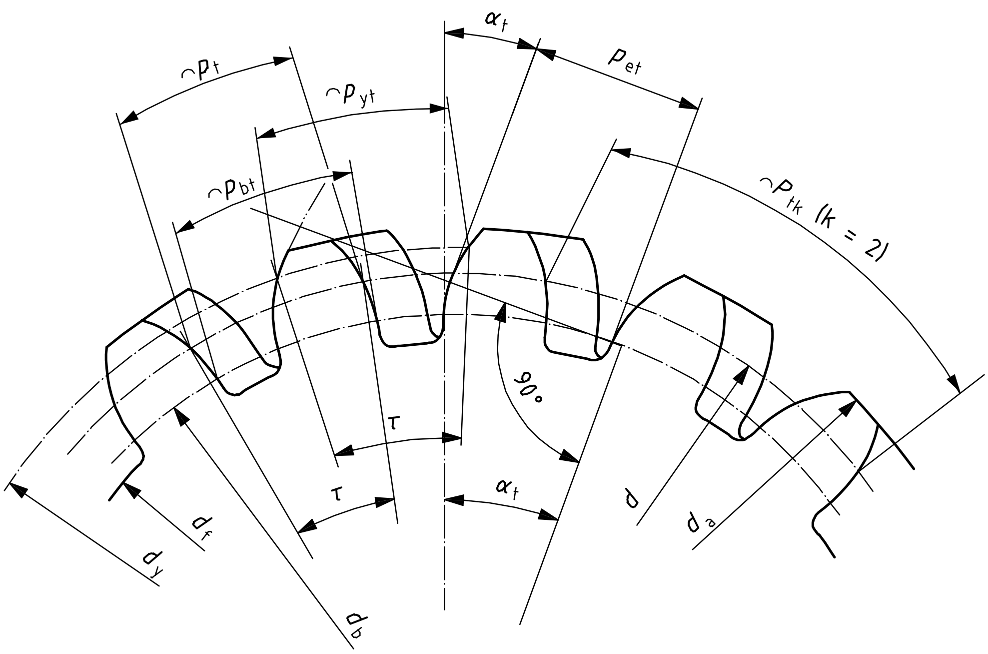
4.4.2 Pitches on the reference cylinder

4.4.2.1 Transverse pitch

The (reference cylinder) transverse pitch, p_t, is the length of the reference circle arc between two successive equal-handed tooth flanks (right or left flanks):

$$p_t = \frac{\pi m_n}{\cos\beta} = \frac{d}{2}\tau = \frac{\pi d}{|z|} = \pi m_t \tag{23}$$

(See Figure 11.)



Key

⌒ measurement along an arc

Figure 11 — Diameter, angular pitch, transverse pitches on helical cylindrical gear

4.4.2.2 Normal pitch

The (reference cylinder) normal pitch, p_n, is the length of the helix arc between two successive equal-handed tooth flanks (right or left flanks) on the reference cylinder in the normal section of the gear:

$$p_n = \pi m_n = p_t \cos\beta \tag{24}$$

(See Figure 12.)

4.4.3 Pitches on any cylinder

It is necessary to distinguish between the transverse pitch, p_{yt}, and the normal pitch, p_{yn}, on a cylinder of any diameter, d_y, (Y-cylinder):

$$p_{yt} = \frac{d_y}{2}\tau = \frac{\pi d_y}{|z|} = \frac{d_y}{d} p_t \tag{25}$$

$$p_{yn} = p_{yt} \cos\beta_y \tag{26}$$

4.4.4 Axial pitch

The axial pitch, p_x, of a helical gear is the portion of a generation line of a cylinder concentric with the gear axis between two successive equal-handed tooth flanks (right or left flanks), see Figure 12. The axial pitch is independent of the diameter of the cylinder. Axial pitch does not apply to spur gears. It is expressed by

$$p_x = \frac{\pi m_n}{\sin\beta} = \pi m_x = \frac{p_z}{|z|} = \frac{\pi m_t}{\tan\beta} = \frac{p_{yt}}{\tan\beta_y} = \frac{p_{yn}}{\sin\beta_y} \tag{27}$$

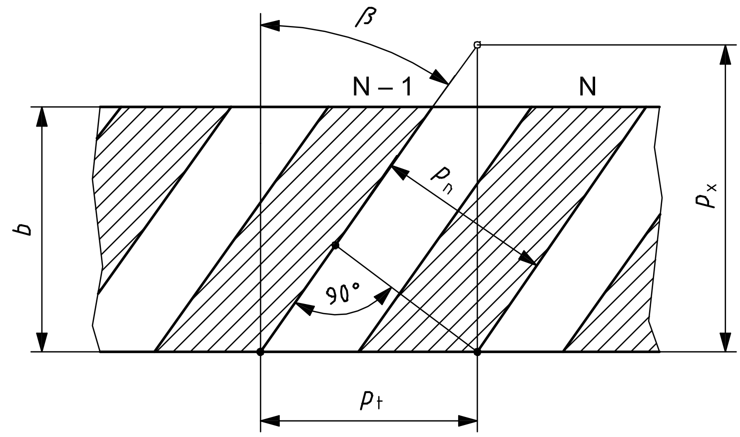


Figure 12 — Geometrical relations between transverse, normal and axial pitch in a developed view of a reference cylinder

4.4.5 Base pitch

The distance between successive equal-handed tooth flanks (right or left flanks) on the developed base cylinder tangential plane is the base pitch.

— Transverse base pitch:

$$p_{bt} = \frac{d_b}{2}\tau = p_t\cos\alpha_t = p_{yt}\cos\alpha_{yt} = \frac{\pi d_b}{|z|} = \frac{d_b}{d}p_t \tag{28}$$

— Normal base pitch:

$$p_{bn} = p_n\cos\alpha_n = p_{bt}\cos\beta_b \tag{29}$$

4.4.5.1 Transverse base pitch on the path of contact

The transverse base pitch on path of contact, p_{et}, is the distance between two parallel tangents in a transverse section which contact two successive equal-handed tooth flanks:

$$p_{et} = p_{bt} \tag{30}$$

(See Figure 11.)

4.4.5.2 Normal base pitch on the plane of contact

The normal base pitch, p_{en}, is the distance between two parallel tangential planes which contact two successive equal-handed tooth flanks:

$$p_{en} = p_{bn} \tag{31}$$

4.5 Diameters of gear teeth

The position of the standard basic rack tooth profile relative to the reference cylinder gives rise to the following cylindrical surfaces and diameters with respect to the gear teeth.

4.5.1 V-cylinder, V-circle diameter

The V-cylinder is the cylinder which is tangent to the reference plane of the basic rack in its generating position (see Figure 6). Its nominal diameter, d_{V} (V-circle diameter), is

$$d_{\mathrm{V}} = d + 2\frac{z}{|z|}xm_{\mathrm{n}} \tag{32}$$

4.5.2 Tip alteration coefficient

A change to the addendum relating to the addendum determined in the standard basic rack tooth profile is expressed by the tip alteration. The tip alteration is made non-dimensional by dividing by the normal module, and it is then expressed as the tip alteration coefficient, k.

The value to be used for k is signed.

A negative value yields a shorter addendum for either external or internal gears.

4.5.3 Tip cylinder, tip circle, tip diameter

The tip cylinder is the cylinder that defines the tips of the gear tooth system. A transverse section yields the tip circle. The nominal dimension of the tip diameter, d_{a}, is

$$d_{\mathrm{a}} = d + 2\frac{z}{|z|}(xm_{\mathrm{n}} + h_{\mathrm{aP}} + km_{\mathrm{n}}) \tag{33}$$

4.5.4 Root cylinder, root circle, root diameter

The root cylinder is the cylindrical envelope surface that forms the bottom of the tooth space. A transverse section yields the root circle. The nominal dimension of the root diameter, d_{f}, is

$$d_{\mathrm{f}} = d - 2\frac{z}{|z|}(h_{\mathrm{fP}} - xm_{\mathrm{n}}) \tag{34}$$

4.6 Gear tooth height

4.6.1 Tooth depth

The tooth depth, h, of cylindrical gear (or rack) teeth is the difference between tip and root radius:

$$h = \frac{|d_{\mathrm{a}} - d_{\mathrm{f}}|}{2} = h_{\mathrm{aP}} + km_{\mathrm{n}} + h_{\mathrm{fP}} \tag{35}$$

4.6.2 Addendum, dedendum

The addendum, h_a, and the dedendum, h_f, of a cylindrical gear are stated on the basis of the reference circle. Their values are calculated from Equations (36) and (37):

$$h_a = \frac{|d_a - d|}{2} = h_{aP} + xm_n + km_n \tag{36}$$

$$h_f = \frac{|d - d_f|}{2} = h_{fP} - xm_n \tag{37}$$

4.7 Tooth thickness, space width

The equations in this section yield the tooth thickness and space width and their half angles for any value of x. If x is the nominal profile shift factor then nominal sizes for the tooth thickness and space width and their half angles result. If the generating profile shift factor, x_E, is used, then generated sizes for the tooth thickness and space width and their half angles result.

See Annex A for tooth thickness measuring methods.

4.7.1 Transverse tooth thickness

The transverse tooth thickness, s_{yt}, in the transverse section is the length of the circular arc of diameter, d_y, between the two involute helicoids of a tooth:

$$s_{yt} = d_y \psi_y = d_y \left[\psi + \frac{z}{|z|}(\mathrm{inv}\,\alpha_t - \mathrm{inv}\,\alpha_{yt}) \right] = d_y \left[\frac{\pi + 4x\tan\alpha_n}{2|z|} + \frac{z}{|z|}(\mathrm{inv}\,\alpha_t - \mathrm{inv}\,\alpha_{yt}) \right] \tag{38}$$

(See Figure 13.)

The transverse tooth thickness, s_t, on the reference circle is produced from

$$s_t = d\psi = d\left(\frac{\pi + 4x\tan\alpha_n}{2|z|} \right) = \frac{m_n}{\cos\beta}\left(\frac{\pi}{2} + 2x\tan\alpha_n \right) \tag{39}$$

4.7.2 Tooth thickness half angle

Tooth thickness angles are angles at the centre in a transverse section which are enclosed by the radii bounding a transverse tooth thickness, see Figure 13. The corresponding tooth thickness half angles, ψ_y, for any transverse tooth thickness, s_{yt}, are expressed by

$$\psi_y = \frac{s_{yt}}{d_y} = \psi + \frac{z}{|z|}(\mathrm{inv}\,\alpha_t - \mathrm{inv}\,\alpha_{yt}) \tag{40}$$

The following tooth thickness half angle applies to the reference circle:

$$\psi = \frac{\pi + 4x\tan\alpha_n}{2\,|z|} \tag{41}$$

In the case of the base circle, the base tooth thickness half angle is produced by

$$\psi_b = \psi + \frac{z}{|z|}\mathrm{inv}\,\alpha_t \tag{42}$$

4.7.3 Space width

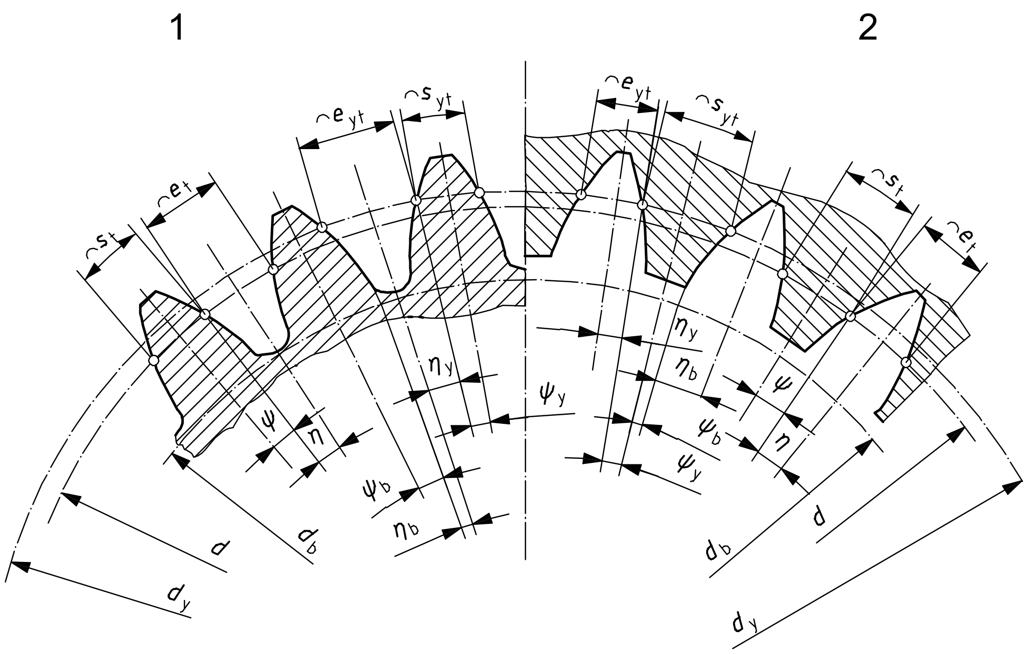
The space width, e_{yt}, in the transverse section is the length of a circular arc of diameter, d_y, between the two involute helicoids of a space width:

$$e_{yt} = d_y \eta_y = d_y \left[\eta - \frac{z}{|z|} (\operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt}) \right] = d_y \left[\frac{\pi - 4x \tan \alpha_n}{2|z|} - \frac{z}{|z|} (\operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt}) \right] \quad (43)$$

(See Figure 13.)

The space width, e_t, on the reference circle is produced by

$$e_t = d\eta = d \left(\frac{\pi - 4x \tan \alpha_n}{2|z|} \right) = \frac{m_n}{\cos \beta} \left(\frac{\pi}{2} - 2x \tan \alpha_n \right) \quad (44)$$



Key

⌒ measurement along an arc
1 external gear
2 internal gear

Figure 13 — Tooth thickness and space width (external and internal gear teeth)

4.7.4 Space width half angle

Space width angles are angles at the centre in a transverse section which are enclosed by the radii bounding a space width, see Figure 13. The corresponding space width half angles, η_y, for any space width, e_{yt}, are produced by

$$\eta_y = \frac{e_{yt}}{d_y} = \eta - \frac{z}{|z|} (\operatorname{inv} \alpha_t - \operatorname{inv} \alpha_{yt}) \quad (45)$$

The following space width half angle applies to the reference circle:

$$\eta = \frac{\pi - 4x \tan \alpha_n}{2|z|} \tag{46}$$

At the base circle, the base space width half angle is produced by

$$\eta_b = \eta - \frac{z}{|z|} \operatorname{inv} \alpha_t \tag{47}$$

4.7.5 Normal tooth thickness

The normal tooth thickness is the tooth thickness in a normal section of the gear teeth. It is the length of the helical arc on the respective cylinder between the two involute helicoids of a tooth. The normal tooth thickness, s_{yn}, for any cylinder is calculated using

$$s_{yn} = s_{yt} \cos \beta_y \tag{48}$$

The following normal tooth thickness applies to the reference cylinder:

$$s_n = s_t \cos \beta = m_n \left(\frac{\pi}{2} + 2x \tan \alpha_n\right) \tag{49}$$

4.7.6 Normal space width

The normal space width is the space width in a normal section of the gear teeth. It is the length of the helical arc on the respective cylinder between the two involute helicoids of a space width. The normal space width, e_{yn}, for any cylinder is calculated using

$$e_{yn} = e_{yt} \cos \beta_y \tag{50}$$

The following normal space width applies to the reference cylinder:

$$e_n = e_t \cos \beta = m_n \left(\frac{\pi}{2} - 2x \tan \alpha_n\right) \tag{51}$$

5 Cylindrical gear pairs

The basic prerequisites for meshing of a cylindrical gear pair (or rack and pinion) according to this International Standard are

— identical standard basic rack tooth profiles for gear and mating gear (rack), and

— the same base helix angle with appropriate hands of the helices.

5.1 Concepts for a gear pair

5.1.1 Mating gear, mating flank

The mating gear in a gear pair is the gear which meshes with the other gear in question. The mating flanks for the tooth system of the other gear in question are the contacting tooth flanks of the mating gears.

5.1.2 Working flank, non-working flank

The tooth flank which transmits the torque during meshing is called the working flank. The other flank of this tooth is the non-working flank.

5.1.3 External gear pair

The mating of two external cylindrical gears (external gears) gives an external gear pair. The mating of an external gear with a rack gives a rack and pinion.

In the case of an external gear pair, the subscript 1 is used in equations for the smaller gear (pinion) and the subscript 2 for the larger gear (wheel). When the gears are of the same size, the subscripts can be allocated as desired. In the case of an external gear pair with helical gear teeth, one gear has a left-handed and the other gear (mating gear) a right-handed flank direction.

5.1.4 Internal gear pair

The mating of an external cylindrical gear (external gear) with an internal cylindrical gear (internal gear) gives an internal gear pair.

In the case of an internal gear pair, the subscript 1 is used in equations for the external gear and the subscript 2 for the internal gear. In the case of an internal gear pair with helical gear teeth, both gears have the same flank direction. Both are either right-handed or left-handed.

5.2 Mating quantities

5.2.1 Gear ratio

The gear ratio, u, of a gear pair is the ratio of the number of teeth of the wheel (or internal gear), z_2, to the number of teeth of the pinion, z_1:

$$u = \frac{z_2}{z_1}, \; |u| \geqslant 1 \tag{52}$$

5.2.2 Driving gear, driven gear, transmission ratio

The driving gear introduces rotation to the gear pair and effects the rotation of the driven gear.

The transmission, i, of a gear pair is the ratio of the angular speed (rotational speed) of the driving gear (subscript a) to that of the driven gear (subscript b):

$$i = \frac{\omega_a}{\omega_b} = \frac{n_a}{n_b} = -\frac{z_b}{z_a} \tag{53}$$

In the case of an external gear pair, the two cylindrical gears rotate in opposite directions, i.e. their angular speeds or rotational speeds have opposite signs; the transmission ratio is negative. In the case of an internal gear pair, the two cylindrical gears have the same direction of rotation, i.e. their angular speeds or rotational speeds have the same sign; the transmission ratio is positive. If it is necessary to make a distinction, ratios such that $|i| > 1$ are said to be “speed reducing ratios” while ratios such that $|i| < 1$ are said to be “speed increasing ratios”.

5.2.3 Line of centres, centre distance

In a transverse section of two mating gears, the line which connects the two axes is called the *line of centres*. The centre distance, a_w, is the working distance between the gear axes of the two gears on the line of centres, see Figure 14.

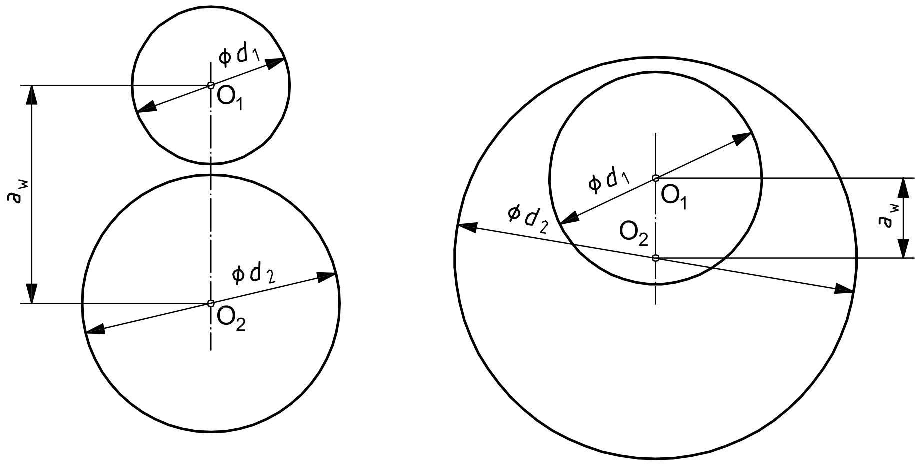


Figure 14 — Line of centres, centre distance

5.2.4 Working transverse pressure angle

The working transverse pressure angle, α_{wt}, is that pressure angle whose vertex lies on the pitch circle (working pitch circle). When a_w is known, α_{wt} is calculated from

$$\alpha_{wt} = \arccos\left[\left|z_1 + z_2\right|\left(\frac{m_n \cos\alpha_t}{2a_w \cos\beta}\right)\right] \tag{54}$$

Alternatively, for the particular case of zero backlash, α_{wt} results from

$$\operatorname{inv}\alpha_{wt} = \operatorname{inv}\alpha_t + \frac{2\tan\alpha_n}{z_1 + z_2}\left(x_1 + x_2\right) \tag{55}$$

5.2.5 Pitch point, pitch cylinders, pitch circles, pitch diameter, pitch axis

The pitch point divides the centre distance in the ratio of the tooth numbers. The pitch cylinders (pitch circles) are those cylinders (circles) which pass through the pitch point. The pitch diameter is the diameter of the pitch circle. The pitch axis is the axis through the pitch point, parallel to the axis of a pitch cylinder.

NOTE During operation, the peripheral velocities on the pitch cylinders are the same.

The pitch circles established during the operation of a cylindrical gear pair (gear pair in a gear unit) are termed *working pitch circles* (d_w). (See Figures 15 and 17.) The pitch circles established by a generating cutter during the generating of a tooth system in a generating gear unit are termed *generating pitch circles*.

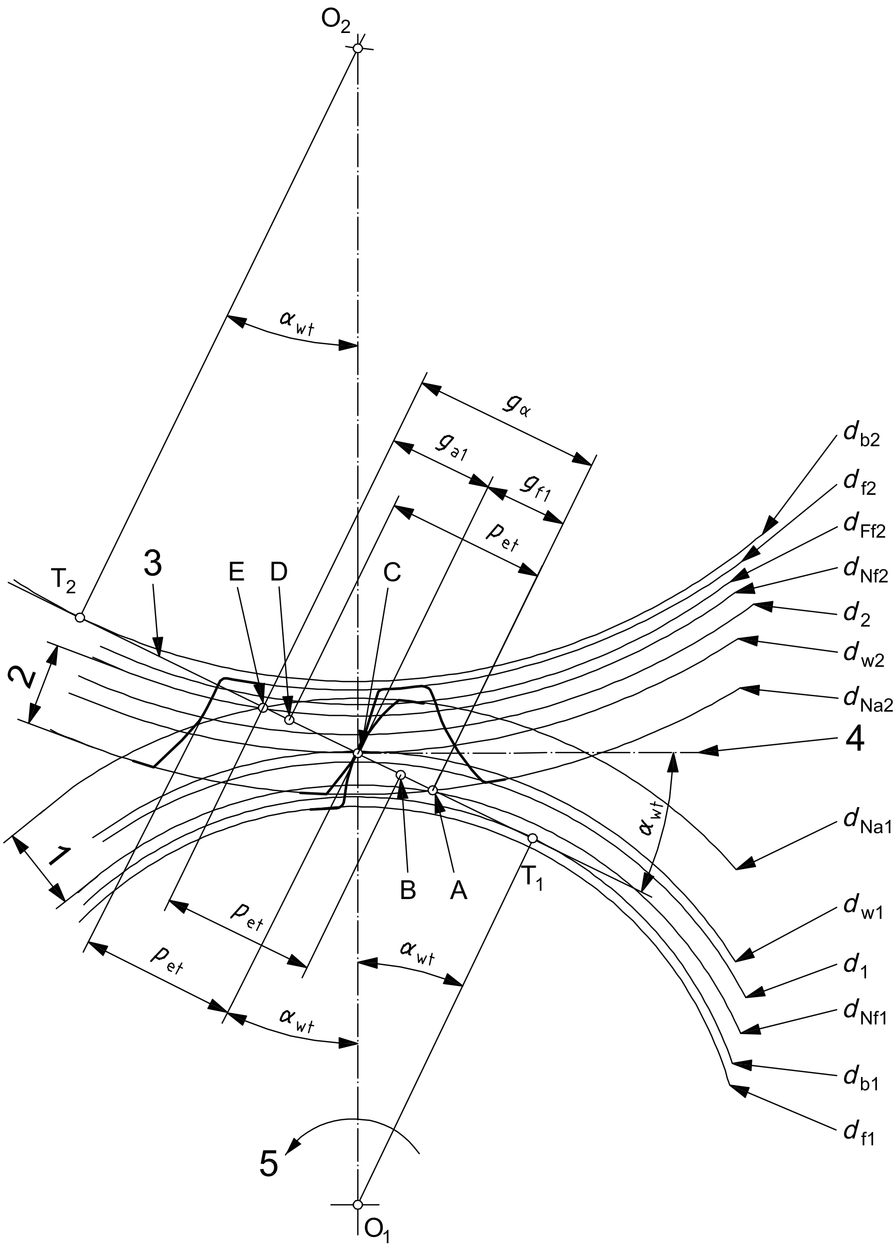
The diameters of the working pitch circles are calculated from Equations (56) and (57):

$$d_{w1} = \frac{2a_w}{\frac{z_2}{z_1} + 1} = d_1 \frac{\cos\alpha_t}{\cos\alpha_{wt}} = \frac{d_{b1}}{\cos\alpha_{wt}} \tag{56}$$

$$d_{w2} = \frac{2a_w}{\frac{z_1}{z_2} + 1} = d_2 \frac{\cos\alpha_t}{\cos\alpha_{wt}} = \frac{d_{b2}}{\cos\alpha_{wt}} \tag{57}$$

This gives

$$a_\mathrm{w} = \frac{1}{2}\left(d_\mathrm{w2} + \frac{z_2}{|z_2|} d_\mathrm{w1}\right) \tag{58}$$



Key

1 radial part of active flank gear 1

2 radial part of active flank gear 2

3 line of action

4 tangent to pitch circles

5 direction of rotation of driving pinion

NOTE See 5.4.5.1 for description of lettered points.

Figure 15 — Meshing conditions and active ranges on working flanks in a transverse section of an external gear pair

5.2.6 Working depth

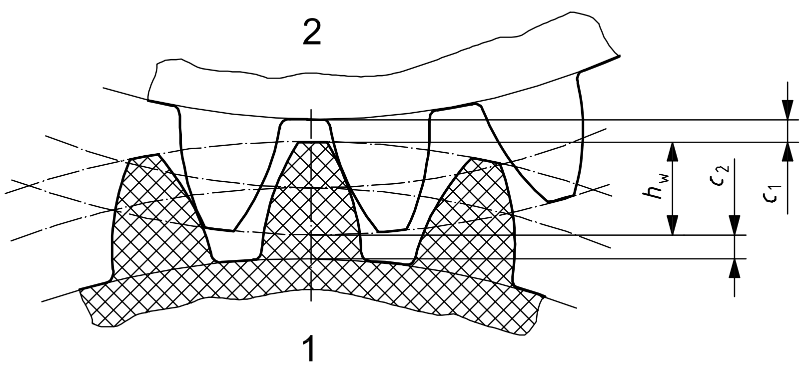
The working depth, h_w, of a gear pair is the overlap of the tip circles of the two cylindrical gears on the line of centres:

$$h_\mathrm{w} = \frac{d_\mathrm{a1} + \frac{z_2}{|z_2|} d_\mathrm{a2}}{2} - \frac{z_2}{|z_2|} a_\mathrm{w} \tag{59}$$

(See Figure 16.)

5.2.7 Tip clearance

The tip clearance, c, is the distance by which the tip circle of a gear is separated from the root circle of the mating gear, see Figure 16.



Key

1 pinion

2 gear wheel

Figure 16 — Working depth, h_w, and tip clearances c_1 and c_2 of a gear pair

The actual clearance follows from the centre distance, a_w, the manufactured tip diameter, d_a, and the generated root diameter, d_fE. For a pinion it is

$$c_1 = \frac{z_2}{|z_2|}\left(a_\mathrm{w} - \frac{d_\mathrm{fE2}}{2}\right) - \frac{d_\mathrm{a1}}{2} \tag{60}$$

and for a wheel

$$c_2 = \frac{z_2}{|z_2|}\left(a_\mathrm{w} - \frac{d_\mathrm{a2}}{2}\right) - \frac{d_\mathrm{fE1}}{2} \tag{61}$$

5.3 Calculation of the sum of the profile shift coefficients

The sum of the profile shift coefficients which corresponds to the zero backlash condition is related to the basic tooth parameters and centre distance by Equation (62), with a_wt from Equation (54):

$$\sum x = x_1 + x_2 = \frac{(z_1 + z_2)(\mathrm{inv}\,\alpha_\mathrm{wt} - \mathrm{inv}\,\alpha_\mathrm{t})}{2\tan\alpha_\mathrm{n}} \tag{62}$$

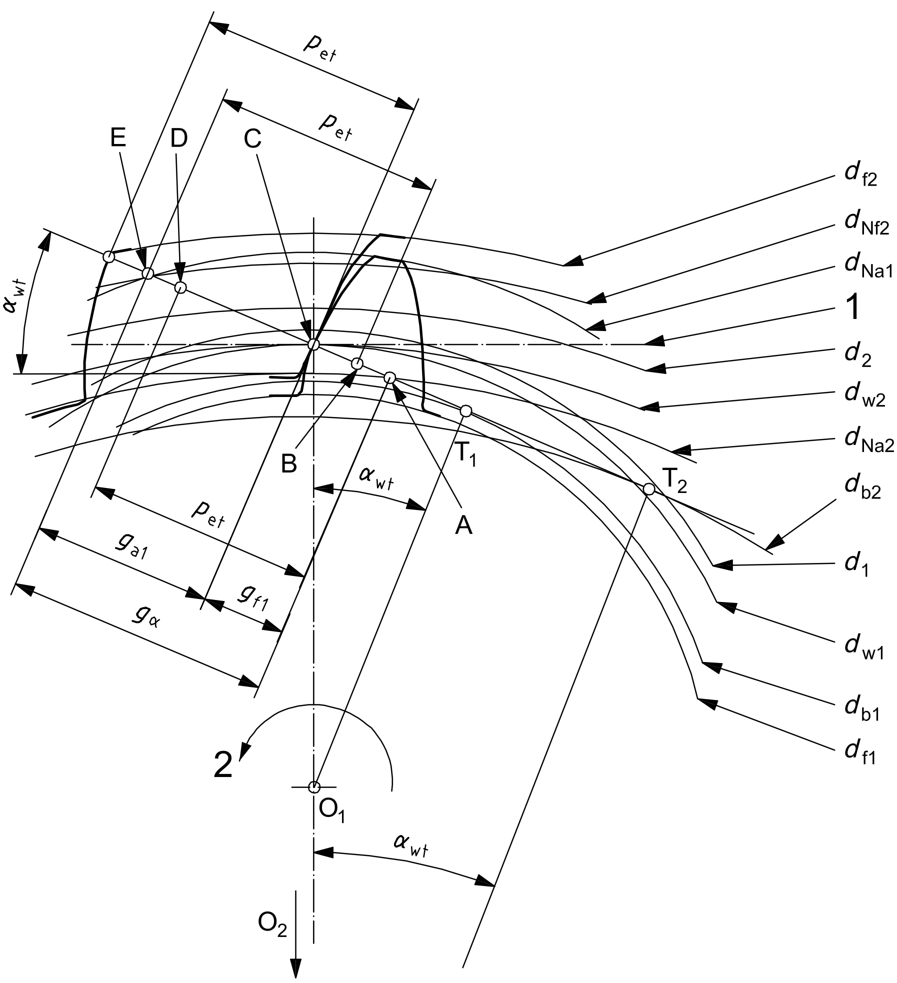
In the case of non-zero backlash, the normal backlash, j_{bn} (see 5.5.1), is included in the calculation:

$$\sum x_E = x_{E1} + x_{E2} = \frac{(z_1 + z_2)(\mathrm{inv}\,\alpha_{wt} - \mathrm{inv}\,\alpha_t)}{2\tan\alpha_n} - \frac{j_{bn}}{2m_n \sin(\alpha_n)} \tag{63}$$

The way in which $\sum x = x_1 + x_2$ is distributed between the two gears may be decided on the basis of aspects such as permissible stress, sliding velocities or other specified dimensions of the gear teeth such as root diameter.

5.4 Tooth engagement

Tooth engagement refers to the meshing of a gear (or a rack) with its mating gear. The tooth engagement is influenced by the geometry of the gear pair (or rack and pinion), the mutual contact of the tooth flanks and the sliding conditions, see Figures 15 and 17.



Key

1 tangent to pitch circles

2 direction of rotation of driving pinion

NOTE See 5.4.5.1 for a description of points A, B, C, D and E.

Figure 17 — Meshing conditions and active ranges on working flanks in a transverse section of an internal gear pair

5.4.1 Start of involute, active area of the tooth flanks, start of active profile and active tip diameters

The root form diameter, d_{Ff}, is the start of the involute portion of the profile. For an external gear it is the greatest of the base diameter, d_{b}, or the diameter of the intersection of the flank with the root fillet or trochoid (taking into account undercut if necessary).

In a defined gear pair the active tip diameter, d_{Na}, of a gear may be governed either by its tip form diameter, d_{Fa}, or by the start of involute of its mate (i.e. $d_{\text{Na}} \leqslant d_{\text{Fa}}$). The start of active profile (active root) diameter, d_{Nf}, may be governed either by the diameter of start of involute or by the tip form diameter of its mate. The active area of the flank extends from the active tip diameter to the active root diameter and so is dependent on the characteristics of both gears and the centre distance.

The following applies to a gear pair.

Usually, root contact on both gears is limited by the tip form diameter of the mating gear ($d_{\text{Na}} = d_{\text{Fa}}$). In this case:

$$d_{\text{Nf1}} = \sqrt{\left(2 a_{\text{w}} \sin\alpha_{\text{wt}} - \frac{z_2}{|z_2|}\sqrt{d_{\text{Fa2}}^2 - d_{\text{b2}}^2}\right)^2 + d_{\text{b1}}^2} \tag{64}$$

$$d_{\text{Nf2}} = \sqrt{\left(2 a_{\text{w}} \sin\alpha_{\text{wt}} - \sqrt{d_{\text{Fa1}}^2 - d_{\text{b1}}^2}\right)^2 + d_{\text{b2}}^2} \tag{65}$$

However, if d_{Ff} is greater than the quantity calculated by the corresponding equation above, then:

$$d_{\text{Nf1}} = d_{\text{Ff1}} \tag{66}$$

$$d_{\text{Nf2}} = d_{\text{Ff2}} \tag{67}$$

(See 7.6 for d_{Ff}.)

If $d_{\text{Nf1}} = d_{\text{Ff1}}$, then

$$d_{\text{Na2}} = \sqrt{\left(2 a_{\text{w}} \sin\alpha_{\text{wt}} - \sqrt{d_{\text{Ff1}}^2 - d_{\text{b1}}^2}\right)^2 + d_{\text{b2}}^2} \tag{68}$$

otherwise, $d_{\text{Na2}} = d_{\text{Fa2}}$.

If $d_{\text{Nf2}} = d_{\text{Ff2}}$, then

$$d_{\text{Na1}} = \sqrt{\left(2 a_{\text{w}} \sin\alpha_{\text{wt}} - \frac{z_2}{|z_2|}\sqrt{d_{\text{Ff2}}^2 - d_{\text{b2}}^2}\right)^2 + d_{\text{b1}}^2} \tag{69}$$

otherwise, $d_{\text{Na1}} = d_{\text{Fa1}}$.

The roll angle (see 4.3.7), $\xi_{\text{Nf}} = \tan\alpha_{\text{Nf}}$, can be used to obtain the active root diameter of the external gear (z_1) used by the mating gear (z_2) as,

$$d_{\text{Nf1}} = \frac{d_{\text{b1}}}{\cos\alpha_{\text{Nf1}}} \tag{70}$$

with α_{Nf1} from

$$\xi_{\mathrm{Nf1}} = \frac{z_2}{z_1}\left(\xi_{\mathrm{wt}} - \xi_{\mathrm{Na2}}\right) + \xi_{\mathrm{wt}} \tag{71}$$

$$\xi_{\mathrm{Na2}} = \tan\arccos\frac{d_{\mathrm{b2}}}{d_{\mathrm{Na2}}} \tag{72}$$

and the usable root diameter of the internal gear (z_2) used by the pinion (z_1) as

$$d_{\mathrm{Nf2}} = \frac{d_{\mathrm{b2}}}{\cos\alpha_{\mathrm{Nf2}}} \tag{73}$$

with α_{Nf2} from

$$\xi_{\mathrm{Nf2}} = \frac{z_1}{z_2}\left(\xi_{\mathrm{wt}} - \xi_{\mathrm{Na1}}\right) + \xi_{\mathrm{wt}} \tag{74}$$

$$\xi_{\mathrm{Na1}} = \tan\arccos\frac{d_{\mathrm{b1}}}{d_{\mathrm{Na1}}} \tag{75}$$

5.4.2 Plane of action, zone of action, contact line

The plane of action of a cylindrical gear pair is tangent to the base cylinders of the gear and mating gear. In the case of an external gear pair, the plane of action passes between the base cylinders. The intersection of the two planes of action (one for each tooth flank) is parallel to the gear axes, and is the pitch axis (see 5.2.5). The zones of action are the parts of the planes of action which are limited by the usable tip cylinders of the gear and mating gear and by the facewidth and, in the case of external gears, can be further limited by the start of the involute. A zone of action is linked to the flank that is normal to it. Hence, one of the planes of action is linked to the right flanks and the other to the left flanks.

At any instant in time, the intersection of the zone of action with the corresponding tooth flanks of a gear pair is known as the *contact line*. With the rotation of the gears around their axes, the contact lines move through the zone of action. On tooth flanks, the contact lines are identical to the generators of flank and mating flank, see 4.3.1.

5.4.3 Line of action, path of contact, point of contact

Lines of action are where the planes of action intersect transverse sections. According to 5.4.2, it is necessary to distinguish between the right flank line of action and the left flank line of action. A line of action is inclined to the common tangent to the pitch circles at the pitch point (pitch circle tangent) by the working transverse pressure angle, α_{wt} (see Figures 15 and 17), and it contacts the two base circles at the points T_1 and T_2.

A path of contact is that part of the line of action which is within the zone of action. The starting point, A, of the path of contact is at or near the tip circle of the driven gear. The finishing point, E, of the path of contact is at or near the tip circle of the driving gear.

NOTE In Figures 15 and 17, only the line of action of the working flanks is shown in each case.

The lines of action intersect the centre line at pitch point C (see 5.2.5). Pitch point C is also the point at which the two lines of action intersect.

A point of contact is a point where a path of contact intersects the corresponding tooth flanks in a specific working position of the two gears. It is a point on the contact line.

5.4.4 Form over dimension

The form over dimension, c_F, is the radial distance between active root diameter and form root diameter.

$$c_F = \frac{1}{2} \frac{z_2}{|z_2|} \left(d_{Nf} - d_{Ff} \right) \tag{76}$$

5.4.5 Designations and values relating to the line of action

5.4.5.1 Special points on the line of action

Special points on the line of action (see Figures 15 and 17) are as follows:

T_1 is the point of contact between the line of action and the base circle of pinion (d_{b1});

T_2 is the point of contact between the line of action and the base circle of wheel (d_{b2});

C is the pitch point, the intersection of the line of action with the line of centres.

In Figures 15 and 17 the pinion is the driving gear and ε_α is less than 2. Special points on the path of contact are the following:

A is the starting point of engagement, the point at which the line of action intersects the active tip diameters, d_{Na}, of the driven gear;

B is the inner point of single pair contact on the driving gear, outer point of single pair contact on the driven gear; where $\varepsilon_\alpha < 2$, it is the point within the path of contact which is one transverse base pitch away from point E;

D is the outer point of single pair contact on the driving gear, inner point of single pair contact on the driven gear; where $\varepsilon_\alpha < 2$, it is the point within the path of contact which is one transverse base pitch away from point A;

E is the end point of engagement, the point at which the line of action intersects the active tip diameter, d_{Na}, of the driving gear.

5.4.5.2 Length of the path of contact

The length, g_α, of the path of contact (length between points A and E on the contact lines, which is also defined as L_{AE} in ISO 1328-1) of two mating cylindrical gears is

$$g_\alpha = \frac{1}{2} \left[\sqrt{d_{Na1}^2 - d_{b1}^2} + \frac{z_2}{|z_2|} \left(\sqrt{d_{Na2}^2 - d_{b2}^2} - 2 a_w \sin \alpha_{wt} \right) \right] \tag{77}$$

The length of the path of contact when a cylindrical gear (subscript 1) is mated with a rack is

$$g_\alpha = \frac{1}{2} \left(\sqrt{d_{Na1}^2 - d_{b1}^2} - d_{b1} \tan \alpha_t \right) + \frac{h_{aP} - x_1 m_n}{\sin \alpha_t} \tag{78}$$

The path of contact is divided by pitch point C into the approach path of contact (portion of the path of contact at the root flank of the driving gear between the root circle, d_{Nf1}, and the pitch point) and the recess path of contact (portion of the path of contact at the tip flank of the driving gear between the pitch point and the tip circle, d_{Na1}), see Figures 15 and 17. These portions of the path of contact are sometimes referred to as the tip (addendum) path of contact, g_a, and root (dedendum) path of contact, g_f, of the gears.

For the case where pinion is the driving gear and wheel the driven gear, the approach path of contact is equal to the dedendum path of contact, g_{f1}, of pinion, which is equal to the addendum path of contact, g_{a2}, of wheel:

$$g_{f1} = \overline{AC} = \frac{1}{2}\frac{z_2}{|z_2|}\left(\sqrt{d_{Na2}^2 - d_{b2}^2} - d_{b2}\tan\alpha_{wt}\right) = g_{a2} \tag{79}$$

The recess path of contact is equal to the addendum path of contact, g_{a1}, of pinion, which is equal to the dedendum path of contact, g_{f2}, of wheel:

$$g_{a1} = \overline{CE} = \frac{1}{2}\left(\sqrt{d_{Na1}^2 - d_{b1}^2} - d_{b1}\tan\alpha_{wt}\right) = g_{f2} \tag{80}$$

For the opposite case (wheel driving, pinion driven), in Equations (79) and (80), g_a and g_f are to be interchanged.

5.4.5.3 Radii of curvature of the tooth flanks

The following segments of the lines of action give rise to the radii of curvature of the tooth flanks in the transverse plane (see Figures 15 and 17):

$$\overline{T_1C} = \rho_{C1} = \frac{1}{2}\sqrt{d_{w1}^2 - d_{b1}^2} = \frac{1}{2}d_{b1}\tan\alpha_{wt} \tag{81}$$

$$\overline{T_2C} = \rho_{C2} = \frac{1}{2}\frac{z_2}{|z_2|}\sqrt{d_{w2}^2 - d_{b2}^2} = \frac{1}{2}\frac{z_2}{|z_2|}d_{b2}\tan\alpha_{wt} \tag{82}$$

$$\overline{T_2A} = \rho_{A2} = \frac{1}{2}\frac{z_2}{|z_2|}\sqrt{d_{Na2}^2 - d_{b2}^2} \tag{83}$$

$$\overline{T_1E} = \rho_{E1} = \frac{1}{2}\sqrt{d_{Na1}^2 - d_{b1}^2} \tag{84}$$

$$\overline{T_1B} = \rho_{B1} = \rho_{E1} - p_{et} \tag{85}$$

$$\overline{T_2D} = \rho_{D2} = \rho_{A2} - p_{et} \tag{86}$$

$$\overline{T_1T_2} = \rho_{C1} + \rho_{C2} = \frac{z_2}{|z_2|}a_w\sin\alpha_{wt} = \rho_{A1} + \rho_{A2} = \rho_{E1} + \rho_{E2} \tag{87}$$

Equations (83) to (86) apply if pinion is the driving gear and wheel the driven gear. In the opposite case, A and E, as well as B and D, are to be interchanged in Figures 15 and 17 and in Equations (83) to (86).

NOTE The values obtained for the curvature radii of an internal gear and for the segment $\overline{T_1T_2}$ of an internal gear pair are negative values.

5.4.6 Tooth interference

Interference conditions occur if parts of the tooth flanks or top lands of a gear come into contact with non-involute flank sections or the root on the mating gear. Additional meshing difficulties can be caused by contacts by a tooth tip with a non-working flank, which is a particular problem with internal gears. Also see 7.8.

NOTE Such complexities are not covered in this International Standard.

5.4.7 Overlaps

5.4.7.1 Transverse angle of transmission, transverse contact ratio

The transverse angle of transmission, φ_α, is the centre angle through which a gear of a gear pair rotates from start to finish of engagement of one active tooth flank transverse profile with its mating profile. The transverse angle of transmission of pinion and gear is given as follows:

$$\varphi_{\alpha 1} = \frac{2 g_\alpha}{d_{b1}} = |u| \, \varphi_{\alpha 2} \tag{88}$$

$$\varphi_{\alpha 2} = \frac{2 g_\alpha}{d_{b2}} = \frac{\varphi_{\alpha 1}}{|u|} \tag{89}$$

The transverse contact ratio, ε_α, is the ratio of the transverse angle of transmission, φ_α, to the angular pitch, τ, or the ratio of the path of contact to the transverse normal base pitch:

$$\varepsilon_\alpha = \frac{\varphi_{\alpha 1}}{\tau_1} = \frac{\varphi_{\alpha 2}}{\tau_2} = \frac{g_\alpha}{p_{et}} = \frac{g_f + g_a}{p_{et}} \tag{90}$$

where $p_{et} = p_{bt}$.

5.4.7.2 Active facewidth

The active facewidth, b_w, is the overlapping section of the usable facewidths of the gear pair, see Figure 18 and Figure 5.

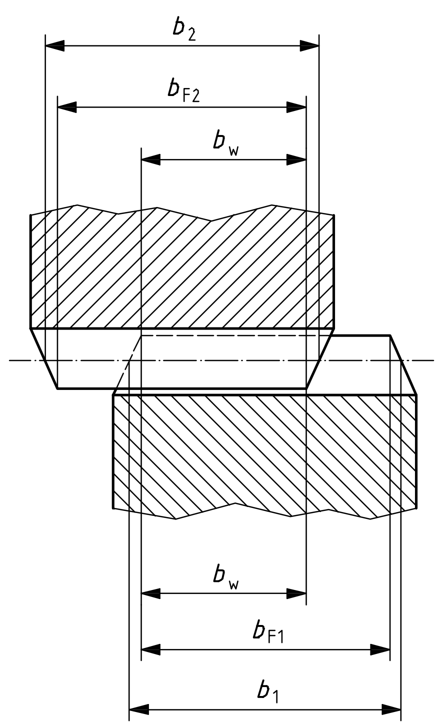


Figure 18 — Active facewidth, b_w

5.4.7.3 Overlap angle, overlap ratio

The overlap angle, φ_β, is the angle between the two axial planes enclosing the end points of a tooth trace of the gear pair:

$$\varphi_{\beta 1} = \frac{2\, b_\mathrm{w}\, \tan\beta}{d_1} = \frac{2\, b_\mathrm{w}\, \sin\beta}{m_\mathrm{n}\, z_1} = |u|\, \varphi_{\beta 2} \tag{91}$$

$$\varphi_{\beta 2} = \frac{2\, b_\mathrm{w}\, \sin\beta}{m_\mathrm{n}\, z_2} = \frac{\varphi_{\beta 1}}{|u|} \tag{92}$$

(See Figure 19.)

The overlap ratio, ε_β, is the ratio of the overlap angle, φ_β, to the angular pitch, τ, or the ratio of the facewidth, b, to the axial pitch, p_x.

$$\varepsilon_\beta = \frac{\varphi_{\beta 1}}{\tau_1} = \frac{\varphi_{\beta 2}}{\tau_2} = \frac{b}{p_\mathrm{x}} = \frac{b\, \sin\beta}{m_\mathrm{n}\, \pi} = \frac{b\, \tan\beta}{p_\mathrm{t}} = \frac{b\, \tan\beta_\mathrm{b}}{p_\mathrm{et}} \tag{93}$$

5.4.7.4 Overlap length

The overlap length, g_β, of a helical gear is the reference circle arc belonging to the overlap angle, φ_β:

$$g_\beta = r\, \varphi_\beta = b_\mathrm{w}\, \tan\beta \tag{94}$$

5.4.7.5 Total angle of transmission, total contact ratio

The total angle of transmission, φ_γ, is the angle at the centre of a gear in a gear pair through which the gear rotates from start to finish of contact of one of its flanks with its mating flank. It is equal to the sum of the transverse angle of transmission and the overlap angle:

$$\varphi_{\gamma 1} = \varphi_{\alpha 1} + \varphi_{\beta 1} = |u|\, \varphi_{\gamma 2} \tag{95}$$

$$\varphi_{\gamma 2} = \varphi_{\alpha 2} + \varphi_{\beta 2} = \frac{\varphi_{\gamma 1}}{|u|} \tag{96}$$

The total contact ratio, ε_γ, is the ratio of the total angle of transmission to the angular pitch. It is equal to the sum of the transverse contact ratio and the overlap ratio:

$$\varepsilon_\gamma = \frac{\varphi_{\gamma 1}}{\tau_1} = \frac{\varphi_{\gamma 2}}{\tau_2} = \varepsilon_\alpha + \varepsilon_\beta \tag{97}$$

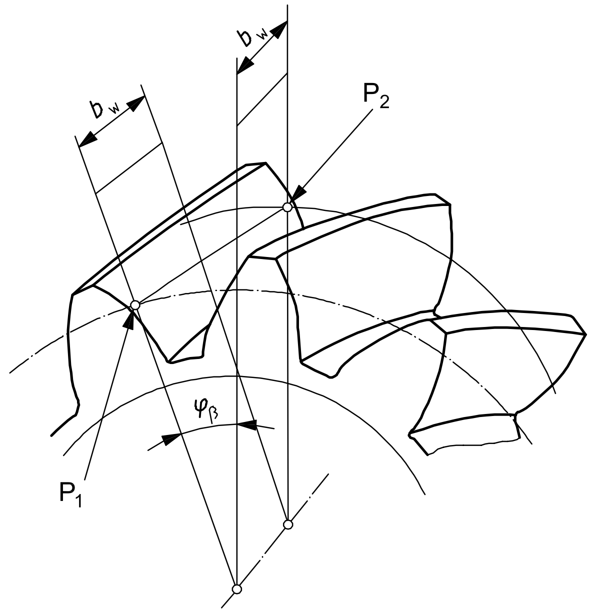


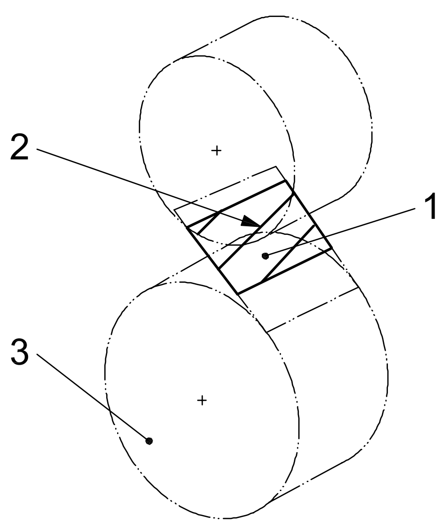
Figure 19 — Overlap angle of gear pair, φ_β

5.4.8 Contact line, sum of the contact line lengths

The contact line is the theoretical line at an instant in time where the flanks of a tooth pair of the gear and mating gear touch; l_{max} is the maximum length of such a contact line of a flank pair.

When spur-toothed gears are in contact, the individual tooth pair contact line length remains constant. When tooth traces are not modified (e.g. by crowning), the value of l_{max} is equal to the facewidth, b_w; see 5.4.7.2.

In the case of helical gears, the contact lines are within the zone of action and are at angle β_b to the pitch axis, see Figure 20. The length of the contact line changes with the rolling of the flank pair. It starts as a point contact at the beginning of engagement, reaches its maximum value, l_{max}, in a working position or in a certain range of rotational angles and is subsequently reduced to the point contact at the end of the engagement of the flank pair.



Key

1 zone of action
2 helical contact lines
3 base cylinder

Figure 20 — Zone of action

The maximum length of a contact line, l_{max}, is given by

$$l_{max} = \frac{g_\alpha}{\sin \beta_b} \tag{98}$$

or

$$l_{max} = \frac{b_w}{\cos \beta_b} \tag{99}$$

whichever is less.

NOTE If $\frac{b_w}{\cos\beta_b} > \frac{g_a}{\sin\beta_b}$, then the contact line extends across the whole active range of the transverse profile but not across the whole facewidth; while if it is less, then the contact line extends across the whole facewidth but across only part of the active range of the transverse profile.

The sum of the individual contact lines, $\sum l$, is the total length of all the contact lines which occur at the same time when the gear pair is in an instantaneous working position in the zone of action.

5.5 Backlash

The backlash is the clearance between the non-working flanks of the teeth of a gear pair when the working flanks are in contact.

There is a distinction between the normal backlash, j_{bn}, circumferential backlash, j_{wt}, and radial backlash, j_r; see Figure 21.

5.5.1 Normal backlash

Normal backlash, j_{bn}, is the shortest distance between the non-working flanks of the teeth of a gear pair when the working flanks are in contact with zero force. It is defined in the plane of action of the non-working flanks.

5.5.2 Backlash angle, circumferential backlash

The backlash angle, φ_j, is the angle of rotation through which the gear can be rotated, while the mating gear is stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact; see Figure 21. The backlash angle for each gear is found from the normal backlash, j_{bn}:

$$\varphi_{j1} = \frac{2}{m_n \, z_1 \cos \alpha_n} \, j_{bn} \tag{100}$$

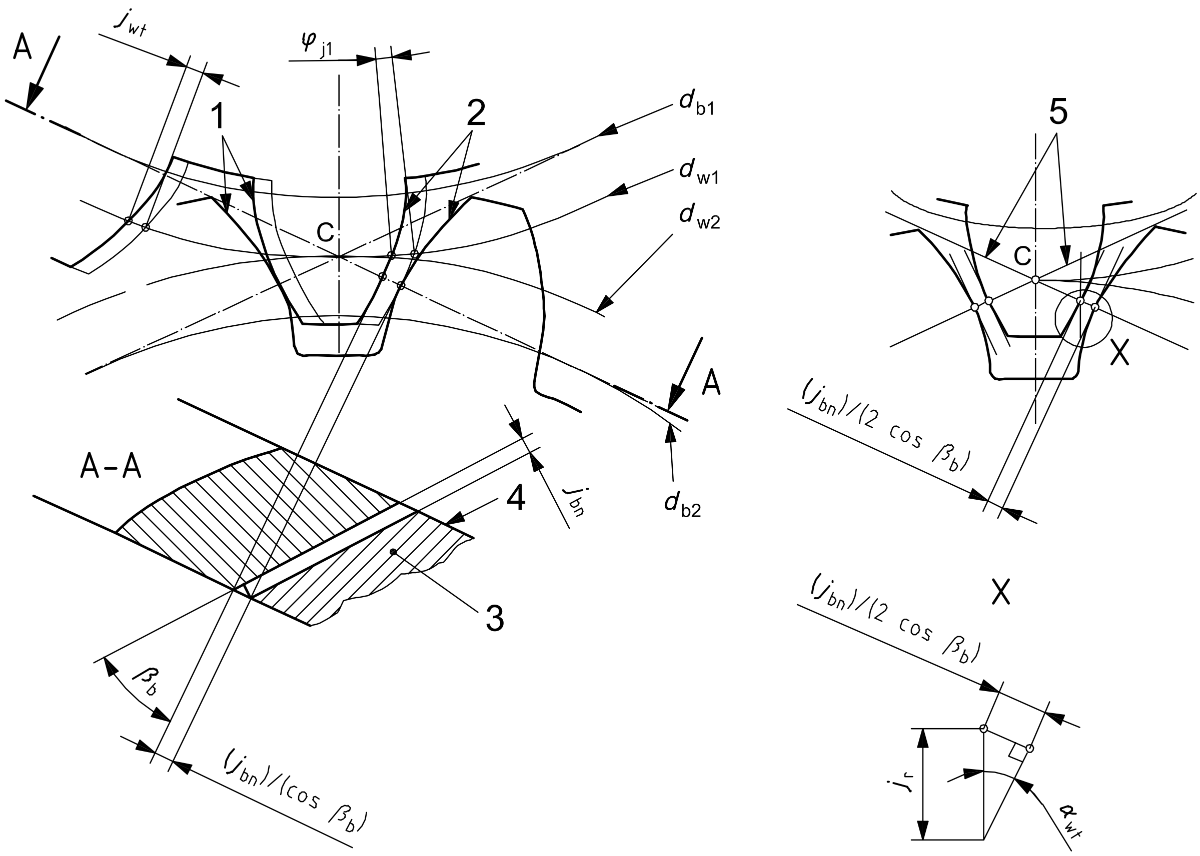
$$\varphi_{j2} = \frac{2}{m_n \, |z_2| \cos \alpha_n} \, j_{bn} \tag{101}$$

The circumferential backlash, j_{wt}, is the length of the pitch circle arc through which each of the two gears can be rotated, whilst the other is held stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact. Its magnitude is given by

$$j_{wt} = \frac{1}{\cos \alpha_{wt} \cos \beta_b} \, j_{bn} \tag{102}$$

or in relation to the length of the reference circle arc by

$$j_t = \frac{1}{\cos \beta \cos \alpha_n} \, j_{bn} \tag{103}$$



Key

1 working flanks
2 non-working flanks
3 plane of action non-working flanks
4 end face
5 plane of action

Figure 21 — Diagram showing backlash when working flanks are in contact (left-hand side) and with symmetrical positioning of tooth in space of mating gear (right-hand side)

5.5.3 Radial backlash

The radial backlash, j_r, is the difference in the centre distance in the working condition of the gear pair and the centre distance produced if one of the gears is moved towards the centre line until zero-backlash engagement of the flank pairs occurs; see Figure 21 (right-hand side).

The relation between circumferential backlash, j_{wt}, and radial backlash, j_r, is as follows:

$$j_r = \frac{1}{2 \tan \alpha_{wt}} j_{wt} \tag{104}$$

5.6 Sliding conditions at the tooth flanks

5.6.1 Sliding speed

At a point of contact of two tooth flanks in engagement, the sliding speed, v_g, is the difference of the speeds of the two transverse profiles in the direction of the common tangent.

At the point of contact Y, see Figures 22 and 23, the two transverse profiles have the normal speed:

$$v_n = \frac{1}{2}\omega_1 d_{b1} \tag{105}$$

This yields the sliding speed:

$$v_g = \pm\,\omega_1 \left(\frac{\rho_{y2}}{u} - \rho_{y1}\right) \tag{106}$$

where the curvature radii, ρ_{y1} and ρ_{y2}, are to be determined using equation (17).

The distance, $g_{\alpha y}$, between Y and C is

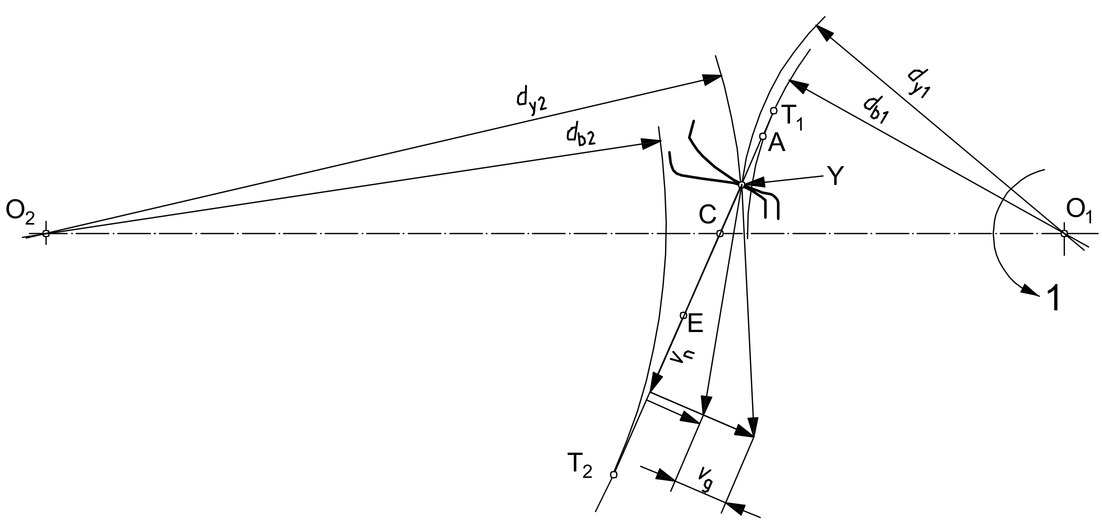
$$g_{\alpha y} = \left|\rho_{c1} - \rho_{y1}\right| = \left|\rho_{c2} - \rho_{y2}\right| \tag{107}$$

where |~| denotes absolute value.

Hence

$$v_g = \left|\omega_1\, g_{\alpha y} \left(1 + \frac{1}{u}\right)\right| \tag{108}$$

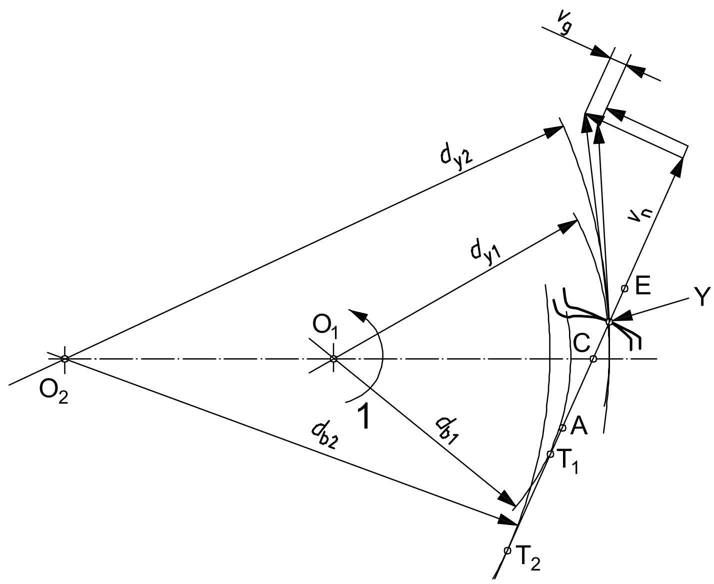
NOTE $g_{\alpha y}$ is always positive. Since u is positive for an external gear pair and negative for an internal gear pair, it usually follows that the sliding speed is greater for external gear teeth than for internal gear teeth.



Key

1 direction of rotation of driving pinion

Figure 22 — Sliding speed, v_g, at point of contact Y on external gear pair



Key

1 direction of rotation of driving pinion

Figure 23 — Sliding speed, v_g, at point of contact Y on internal gear pair

The sliding speed is proportional to distance $g_{\alpha y}$ and equal to zero at the pitch point. It reaches its maximum values at the end points A and E of the path of contact:

$$v_{gf} = \left| \omega_1 \ g_f \left(1 + \frac{1}{u}\right) \right| \tag{109}$$

$$v_{ga} = \left| \omega_1 \ g_a \left(1 + \frac{1}{u}\right) \right| \tag{110}$$

with g_f and g_a according to Equations (79) and (80).

5.6.2 Sliding factor

The sliding factor, K_g, is the ratio of sliding speed, v_g, to the velocity, v_t, of the pitch circles:

$$K_g = \frac{v_g}{v_t} = \frac{2\ g_{\alpha y}}{d_{w1}} \left(1 + \frac{1}{u}\right) \tag{111}$$

The maximum values for K_g are attained at end points A and E of the path of contact:

— at A:

$$K_{gf} = \frac{2\ g_f}{d_{w1}} \left(1 + \frac{1}{u}\right) \tag{112}$$

— at E:

$$K_{ga} = \frac{2\ g_a}{d_{w1}} \left(1 + \frac{1}{u}\right) \tag{113}$$

5.6.3 Specific sliding

The specific sliding, ζ, is the ratio of the sliding speed to the speed of a transverse profile in the direction of the tangent to the profile. The tangential velocity is equal to $\rho_y \omega$. Equation (106) yields:

$$\zeta_1 = 1 - \frac{\rho_{y2}}{u\ \rho_{y1}} \tag{114}$$

$$\zeta_2 = 1 - \frac{u\ \rho_{y1}}{\rho_{y2}} \tag{115}$$

The maximum values of ζ are reached at end points A and E of the path of contact:

— at A:

$$\zeta_{f1} = 1 - \frac{\rho_{A2}}{u\ \rho_{A1}} \tag{116}$$

— at E:

$$\zeta_{f2} = 1 - \frac{u\ \rho_{E1}}{\rho_{E2}} \tag{117}$$

using the curvature radii, ρ_A and ρ_E, according to 4.3.8 and 5.4.5.3.

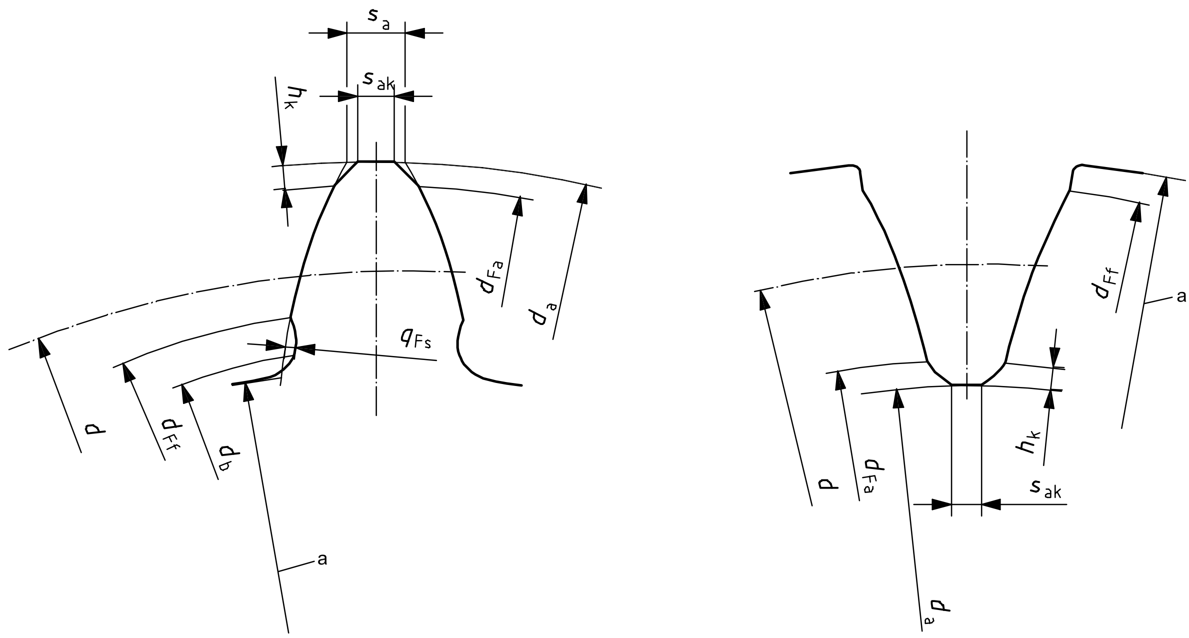
6 Tooth flank modifications

Tooth flank modifications are desired alterations to the tooth flank face compared with the main geometry described in 4.3. Superimposing the nominal modifications on the main geometry produces the nominal tooth flank. The modifications can be defined in characteristic profiles of the tooth flank or in relation to the flank face. Modification depths are always given in the transverse section and normal to the involute of the main geometry.

6.1 Tooth flank modifications which restrict the usable flank

6.1.1 Pre-finish flank undercut

Pre-finish (root) relief is a planned, generated, undercut (e.g. using a protuberance tool) of the transverse profile of a tooth flank in the area of the root. The magnitude of the relief, q_{FS}, is the greatest distance between the root rounding and the involute imagined as extended to the base circle, see Figure 24. Below this, the datum is a line from the involute origin to the gear centre.



[a] d_f or d_{fE}.

Figure 24 — Spur cylindrical gear with undercut and tip corner chamfering

6.1.2 Tip corner chamfering, tip corner rounding

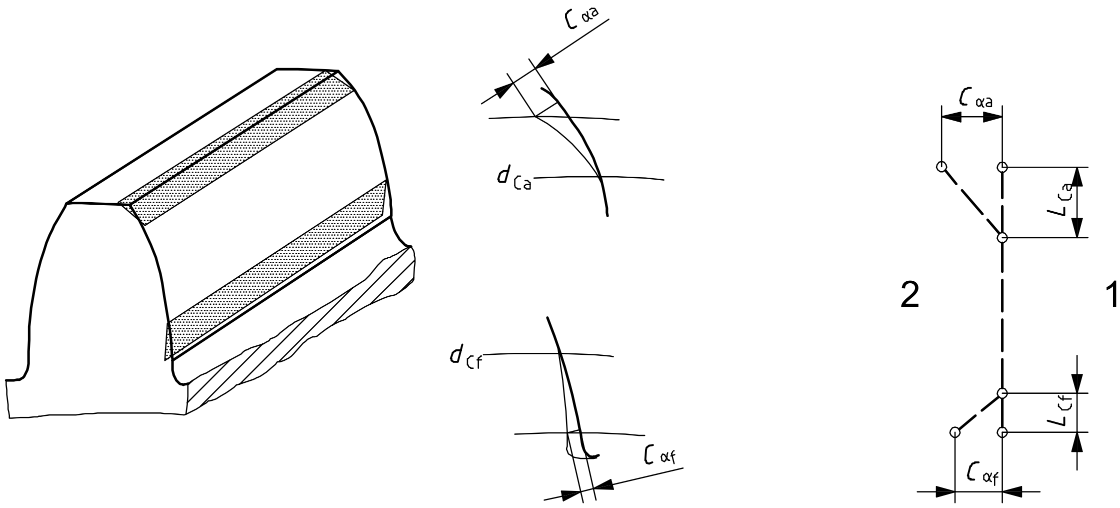
Tip corner chamfering and tip corner rounding are reliefs of the transverse profile which restrict the usable area of the tooth flank. Tip corner chamfering is the chamfer arising through removal of the tip corner. In the case of tip corner rounding, this corner is radiused in the normal plane. The radial height, h_K, and the residual tooth thickness at the tip, s_{aK}, are given as the dimensions of this modification, see Figure 24, and are different for chamfering and rounding.

6.2 Transverse profile modifications

In the following, L_{AE} is used to define the roll length for compatibility with ISO 1328-1. L_{AE} is equivalent to length of path of contact, g_α.

6.2.1 Tip and root relief

Tip and root reliefs are the continuously increasing reliefs of the transverse profile of the main geometry from defined points in each case (diameter, length of roll, roll angle) in the direction of the tip or root (mostly involute). See Figure 25.



Key

d_{Ca} tip relief datum diameter

L_{Ca} tip relief roll length

$C_{\alpha a}$ amount of tip relief

d_{Cf} root relief datum diameter

L_{Cf} root relief roll length

$C_{\alpha f}$ amount of root relief

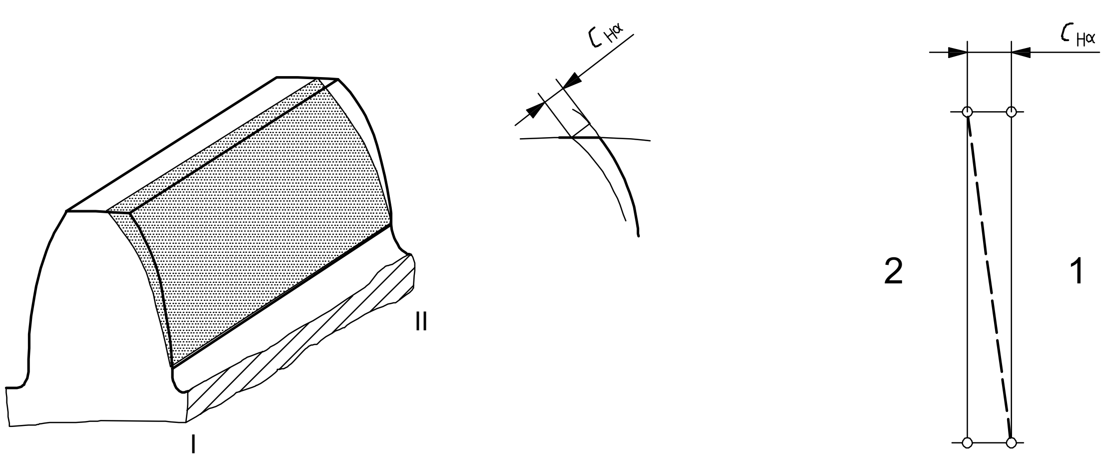
1 space

2 tooth

Figure 25 — Tip and root relief

6.2.2 Transverse profile slope modification, $C_{H\alpha}$

This is similarly defined as for tip or root relief, except that $C_{H\alpha}$ extends over the whole width of the face. See Figure 26.



Key

$C_{H\alpha}$ amount of transverse profile slope modification

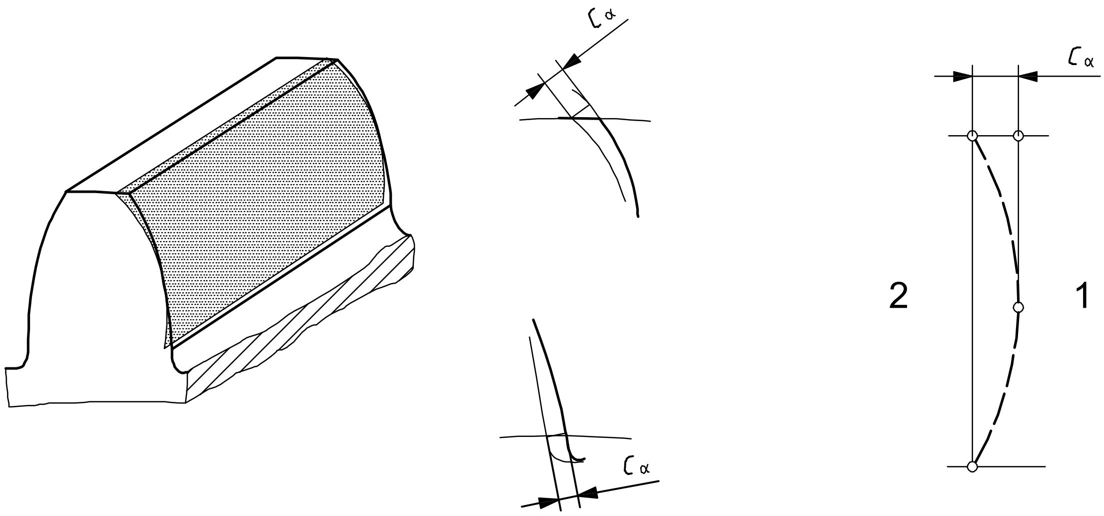
1 space

2 tooth

Figure 26 — Transverse profile slope modification

6.2.3 Profile crowning (barrelling), C_α

Profile crowning is the continuously increasing relief of the transverse profile from a common defined point of the main geometry (diameter, length of roll, roll angle) in the direction of the tip and root of the gear teeth. See Figure 27.



Key

C_α amount of profile crowning

1 space

2 tooth

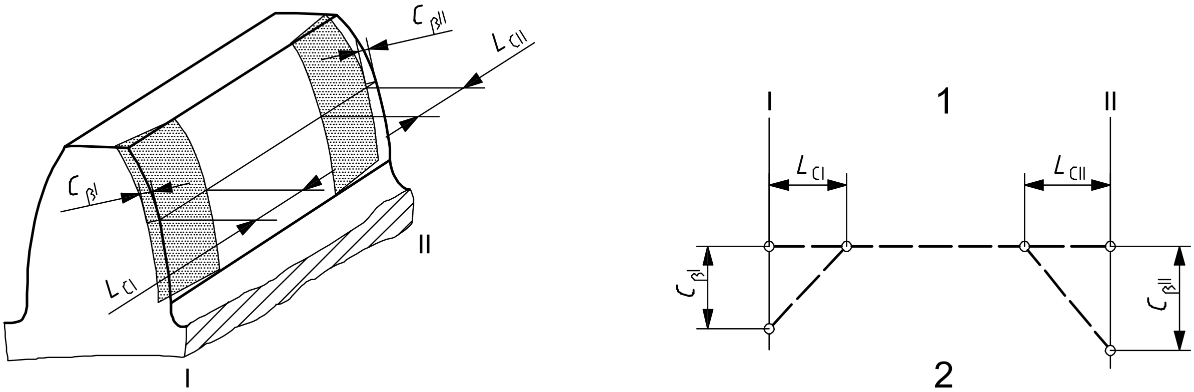
Figure 27 — Profile crowning

Profile crowning is generally defined with respect to the centre of the length of roll of the usable flank and has a parabolic form passing though the points defined by C_α.

6.3 Flank line (helix) modifications

6.3.1 Flank line end relief

Flank line end reliefs are continuously increasing reliefs of the flank line from defined points of the main geometry in each case in the direction of the datum faces (linear or parabolic). See Figure 28.



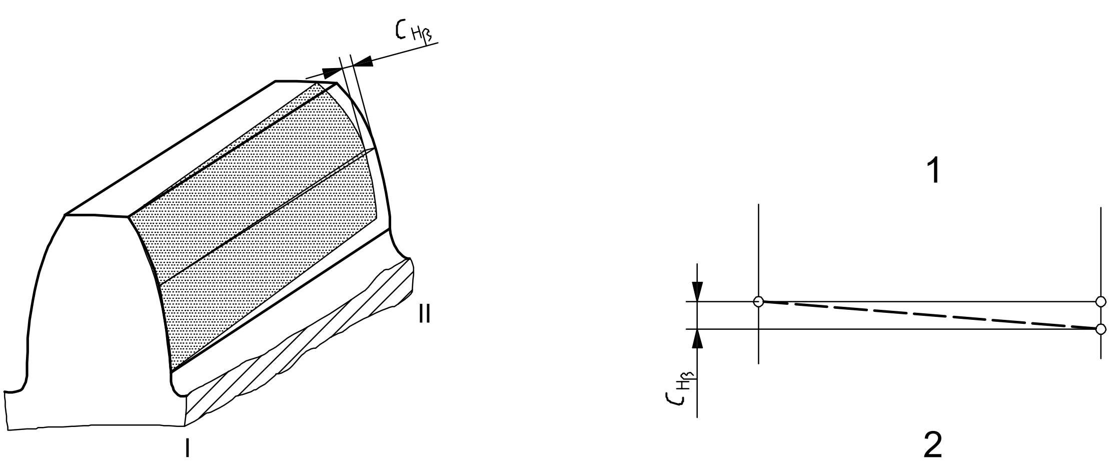
Key

L_{CI}	length (datum face)
$C_{\beta\mathrm{I}}$	amount of end relief (datum face)
L_{CII}	length (non-datum face)
$C_{\beta\mathrm{II}}$	amount of end relief (non-datum face)
1	space
2	tooth

Figure 28 — Flank line end relief

6.3.2 Flank line (helix) slope modification, $C_{\mathrm{H}\beta}$

This is similarly defined as for end relief, but L_{CI} or L_{CII} extends across the whole facewidth. It is not necessarily linear. See Figure 29.



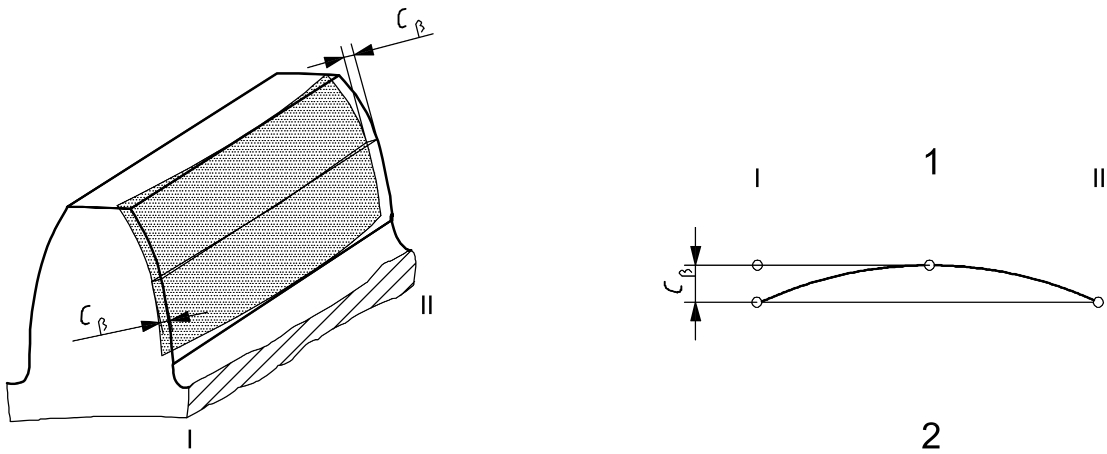
Key

$C_{\mathrm{H}\beta}$	amount of flank line slope modification
1	space
2	tooth

Figure 29 — Flank line slope modification

6.3.3 Flank line (helix) crowning, C_β

Flank line crowning is the continuously increasing relief of the flank line from a common defined point of the main geometry, symmetrically in the direction of both ends of the tooth (arc-shaped or parabolic). See Figure 30.



Key

C_β amount of the flank line crowning

1 space

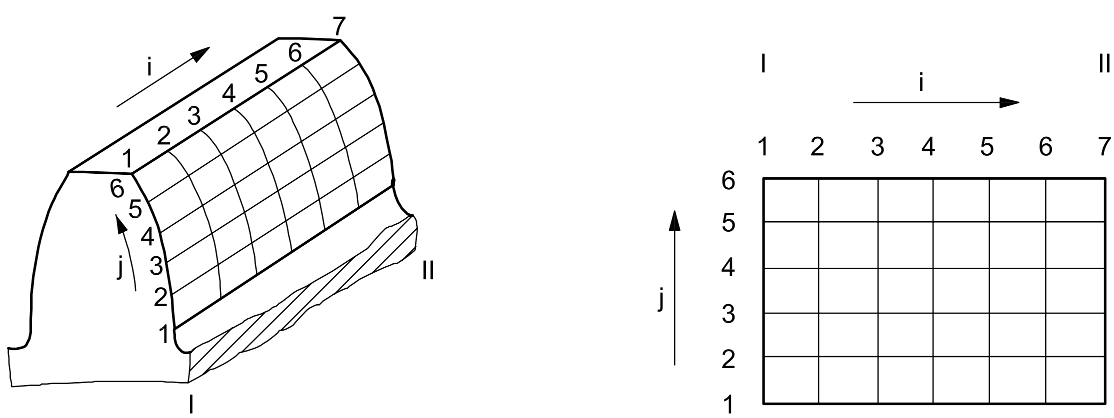
2 tooth

Figure 30 — Flank line crowning

6.4 Flank face modifications

6.4.1 Topographical modifications

The desired deviation from the unmodified involute helicoid is determined in relation to each point of intersection on a grid laid over the tooth flank of the main geometry. See Figure 31.



Key

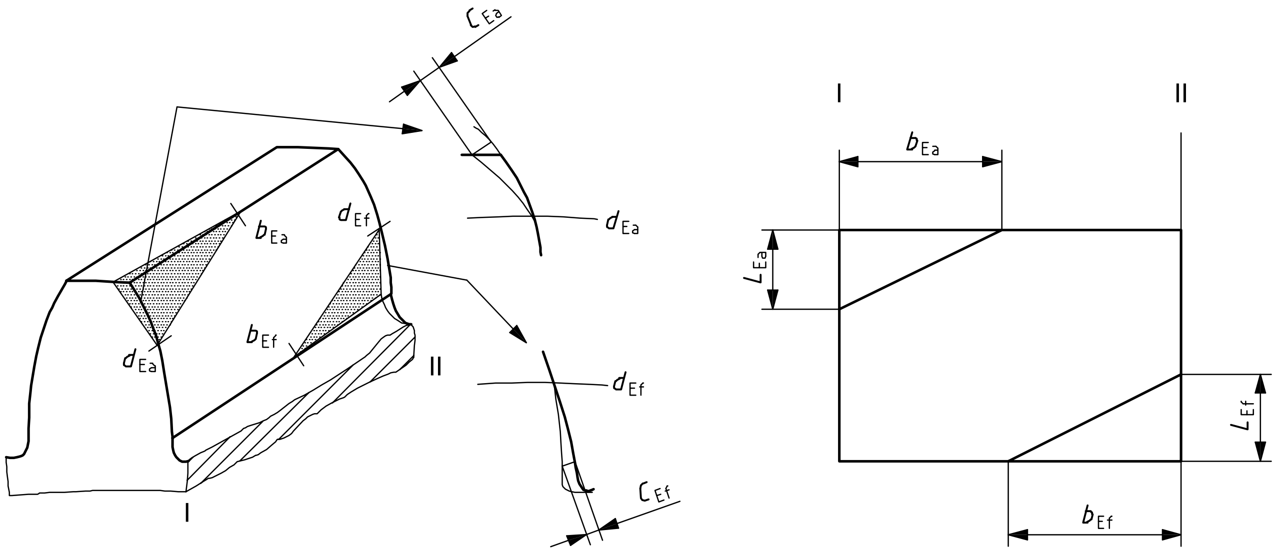
$C_{i,j}$ amount of modification on point (i,j)[a]

[a] Transverse section i; flank line j. If necessary, interpolated points between the defined points (i,j) of a grid can be generated.

Figure 31 — Topographical modification

6.4.2 Triangular end relief

Triangular end reliefs are continuously increasing reliefs of the tooth flanks generally perpendicular to the generators of the main geometry (along the lines of contact) from a defined roll angle in the direction of the start or end of roll on the tooth flank. See Figure 32.



Key

C_{Ea} amount of modification (tip)

d_{Ea} datum diameter (tip)

L_{Ea} roll length (tip)

b_{Ea} length of relief (tip)

C_{Ef} amount of modification (root)

d_{Ef} datum diameter (root)

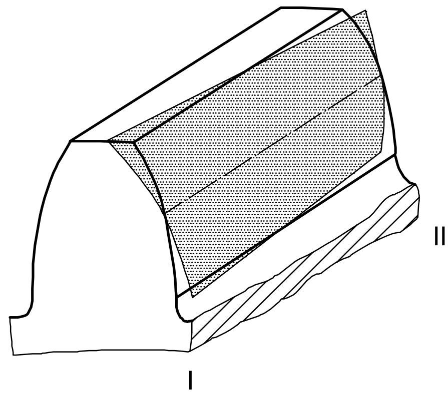
L_{Ef} roll length (root)

b_{Ef} length of relief (root)

Figure 32 — Triangular end relief

6.4.3 Flank twist

Twist is an effect on a flank described as a rotation of the transverse profile along a helix. There is a distinction between twist of the transverse profile, S_α, and of the flank line, S_β. If not otherwise defined, it changes linearly from the beginning to the end of the useable flank. The sign of the flank twist is very important, but is not defined here. See Figure 33.



Twist of transverse profile, S_α:

$$\left|S_\alpha\right| = \left|C_{\mathrm{H}\alpha\mathrm{I}} - C_{\mathrm{H}\alpha\mathrm{II}}\right| \text{ with } C_{\mathrm{H}\alpha\mathrm{I}} = -C_{\mathrm{H}\alpha\mathrm{II}}$$

See 6.2.2.

Twist of flank profile, S_β:

$$\left|S_\beta\right| = \left|C_{\mathrm{H}\beta\mathrm{Na}} - C_{\mathrm{H}\beta\mathrm{Nf}}\right| \text{ with } C_{\mathrm{H}\beta\mathrm{Na}} = -C_{\mathrm{H}\beta\mathrm{Nf}}$$

See 6.3.2.

Figure 33 — Flank twist

6.5 Descriptions of modifications by functions

Modifications of the profile can be given as functions of the diameter, d_y, or the corresponding roll distances or angles, and modifications of the flank lines as functions of the axial distance from the start of the usable facewidth in the direction of the non-datum face. The combination of both functional relationships describes the modification of the whole flank surface.

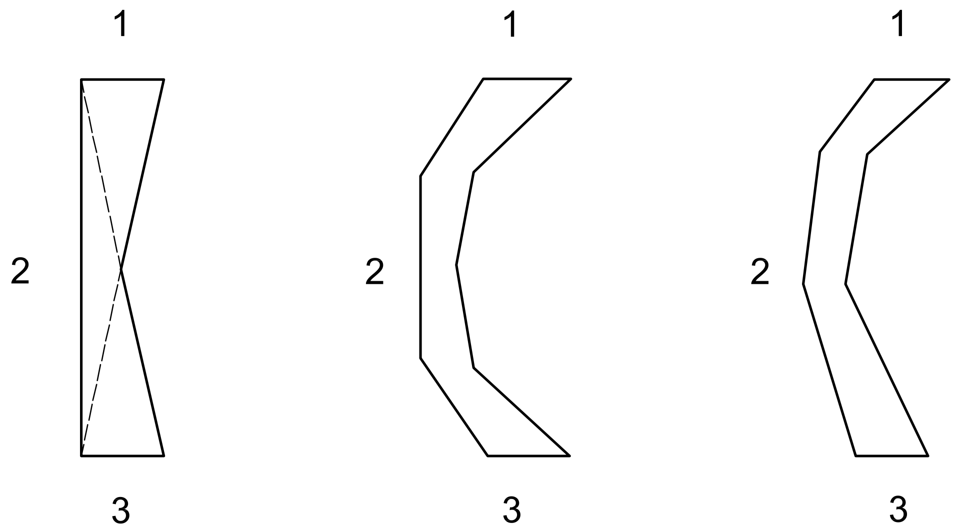
— Modification of the profile: $C_{\mathrm{ay}} = f\left(d_\mathrm{y}\right)$; alternatively, $C_{\mathrm{ay}} = f\left(L_\mathrm{y}\right)$ or $C_{\mathrm{ay}} = f\left(\xi_\mathrm{y}\right)$.

— Modification of the flank line: $C_{\beta\mathrm{y}} = f\left(b_{\mathrm{Fy}}\right)$.

— Modification of the flank surface: $C_{\Sigma\mathrm{y}} = f\left(d_\mathrm{y}, b_{\mathrm{Fy}}\right)$; alternatively, $C_{\Sigma\mathrm{y}} = f\left(L_\mathrm{y}, b_{\mathrm{Fy}}\right)$ or $C_{\Sigma\mathrm{y}} = f\left(\xi_\mathrm{y}, b_{\mathrm{Fy}}\right)$.

— Definition of tooth flank modification by tolerance fields.

Graphically, it is usual to show tooth surface modifications as deviations from the exact involute helicoid with respect to roll length for radial deviations (as in Figure 25) and position across the tooth width for axial deviations (as in Figure 28). For intentional deviations which vary in the radial/axial direction from the exact involute geometry in any defined transverse plane, these deviations can be defined by tolerance fields, generally called "K" diagrams, which show the range of acceptable measured values, see Figure 34. Such diagrams are not necessarily bounded by straight lines.



Key

1 tip or datum face

2 non-material side

3 root or non-datum face

Figure 34 — *K*-diagram (examples)

7 Geometrical limits

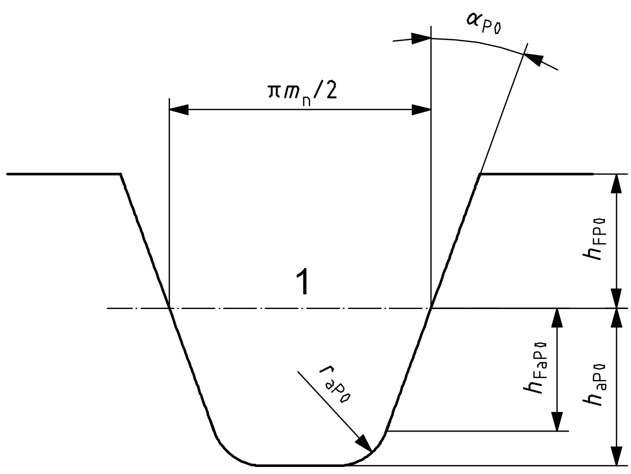
In this clause, the finished state at the conclusion of all manufacturing operations is examined. The basic concepts presented in Clause 4 are expanded to include the effect of such items as tooth thinning for backlash and manufacturing tolerances. The effects of manufacturing tolerances are both direct, such as tooth thickness tolerance, and indirect, such as the change in functional tooth thickness as a result of runout or profile slope deviation. The classification of tolerances is not covered in this International Standard (for this, see ISO 1328).

In manufacturing a cylindrical gear with involute teeth using generating methods, the tool (e.g. hob, pinion-type cutter, rack-shaped cutter, grinding wheel, grinding worm) and the gear form a generating process. The same concepts and the corresponding equations which apply to a cylindrical gear pair shall apply to the paired work piece and generating tool (if $\alpha_{P0} = \alpha_P$, see Clause 5). When producing a cylindrical gear with involute teeth by means of forming (non-generating) methods, the enveloping surface produced by the geometry of the tool and its motions are mapped directly onto the work piece.

If bottom land, root rounding and involute helicoid are machine-finished using the same tool, only the working cycle using this tool is of importance for the dimensions of the tooth parameters. Otherwise, the teeth produced (and their reference rack) are the sum of separate processes which produce the final dimensions of root circle, root rounding and usable flank surface including modifications in each case. The total result of the working cycles can be represented by a single hypothetical tool, the counterpart rack (see Figure 36).

7.1 Counterpart rack tooth profile

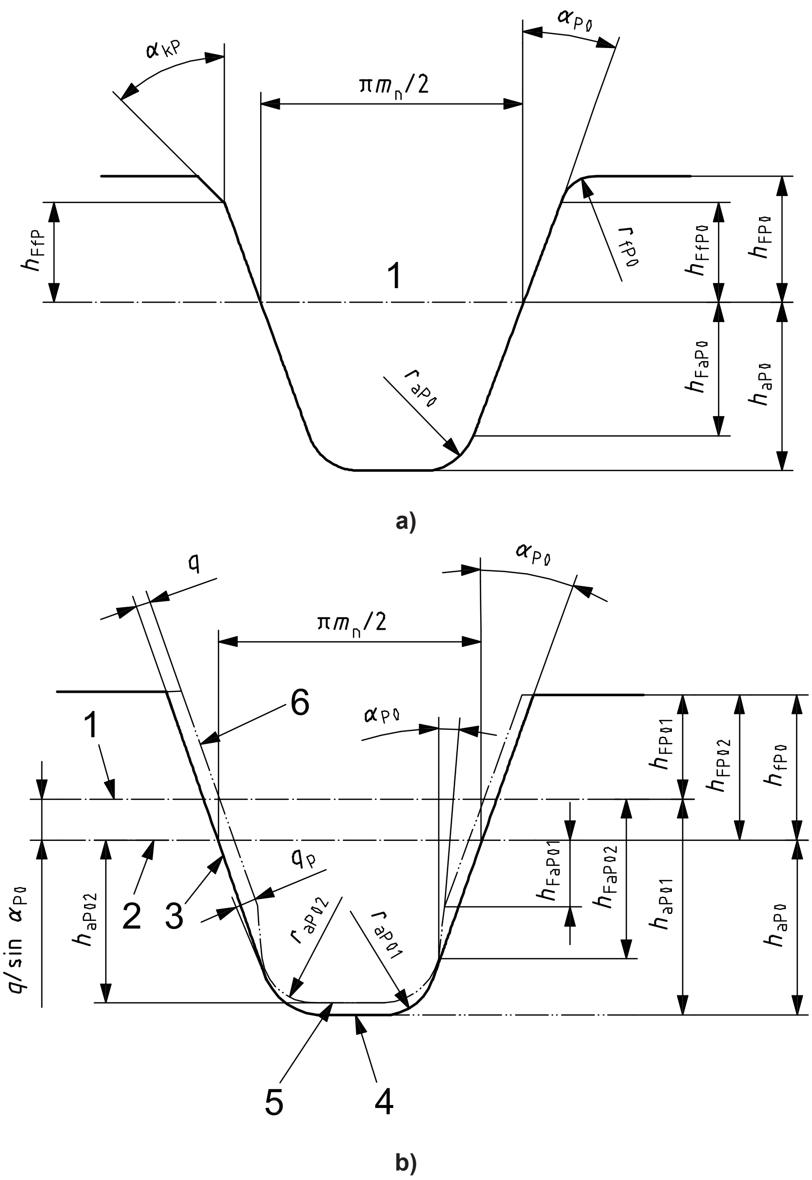
The counterpart rack tooth profile is the complement to the standard basic rack tooth profile enclosing the bottom land, see Figures 35 and 36.



Key

1 datum line of tool

Figure 35 — Counterpart rack tooth profile



Key

1 datum line of roughing tool

2 datum line of finishing tool

3 finishing tool rack

4 tip of roughing tool

5 tip of finishing tool

6 roughing tool rack

NOTE Items 3 and 4 together make up the hypothetical tool.

Figure 36 — Modified counterpart rack tooth profiles

7.2 Machining allowance

A roughing gear-cutting tool leaves the machining allowance, q, for the subsequent finish gear cutting on the flank of the cylindrical gear. The machining allowance is defined normal to the tooth surface. The tooth thickness produced by the roughing gear-cutting tool on the cylindrical gear is thus $2q/\cos\alpha_n$ greater than the tooth thickness, s_n, produced by the finish gear-cutting tool. In practice, the machining allowance ranges from q_{min} to q_{max}. See Figure 37.

7.3 Deviations in tooth thickness

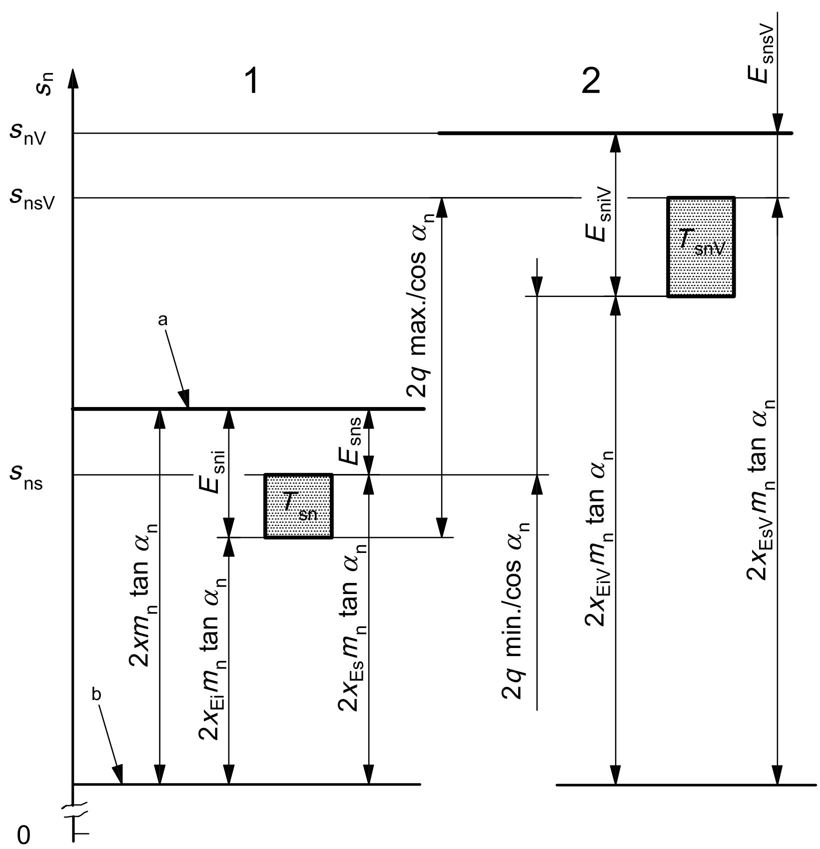
The corresponding maximum and minimum limits of tooth thickness (s_{ns} and s_{ni}) to be required on the generated teeth are obtained by determining (upper and lower) tooth thickness allowances (E_{sns} and E_{sni}).

$$s_{ns} = s_n + E_{sns} \tag{118}$$

$$s_{ni} = s_n + E_{sni} \tag{119}$$

A negative tooth thickness allowance reduces the tooth thickness and increases the space width compared to the nominal dimensions, determining the contribution of the tooth thickness to the backlash, j_{bn} (see 5.5).

In addition to the finish machining tolerances, the backlash is also affected by elemental tolerances such as profile, helix and runout. The elemental tolerances will increase the tolerance band of functional tooth thickness; this will reduce the minimum backlash and increase the maximum backlash. (See Figure 37.) Therefore, a complete analysis of the tooth thickness for the purpose of determining the backlash must include all the elemental tolerances that affect the functional tooth thickness. The relative importance of the different tolerances depends not only on those tolerances but also on the measuring methods used. See Annex A, ISO/TR 10064-2, AGMA 2002 and DIN 3967 for additional information.



Key

1 finishing

2 pre-finish

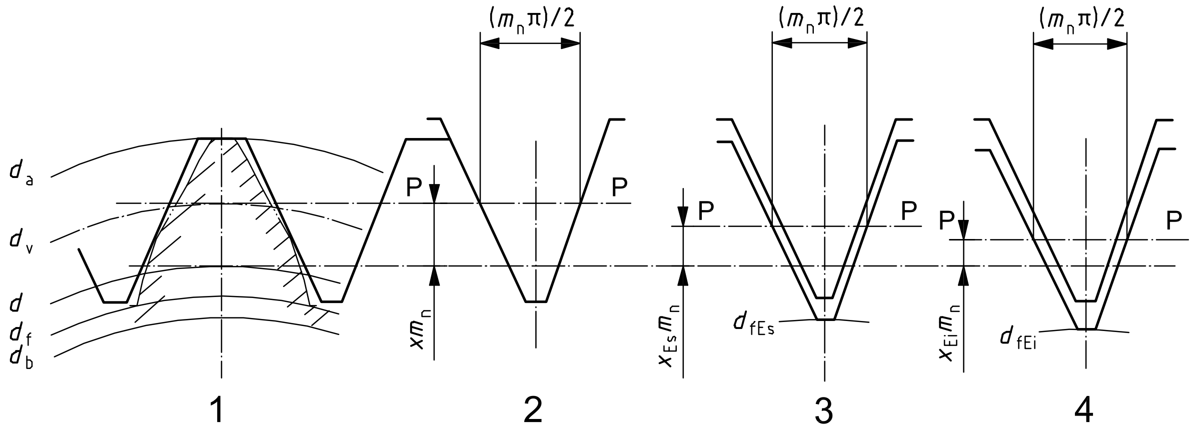
[a] s_n where $x > 0$ and $j_{bn} = 0$.

[b] s_n where $x = 0$.

Figure 37 — Diagram of dimensioning of tooth thickness with positive profile shift

7.4 Generating profile shift, generating profile shift coefficient

The generating profile shift, $x_E m_n$, of a gear with involute teeth is the distance between the datum line of the counterpart rack tooth profile and the reference cylinder of the gear; see Figure 38.



Key

P-P datum line of basic rack

1 reference rack

2 counterpart rack

3 counterpart rack at upper allowance

4 counterpart rack at lower allowance

Figure 38 — Generating profile shift, $x_E m_n$ — Example: external gear, $x > 0$

The generating profile shift takes account of the predetermined upper and lower tooth thickness allowances, E_{sns} and E_{sni} (see 7.3) and, if necessary, the machining allowances, q_{max} and q_{min}, provided for the finish-machining of a gear are included.

The values of x_E are determined with Equations (120) and (121). The permissible maximum and minimum tooth thickness inspection dimensions can be determined by calculation using x_E in the equations in Annex A. The cases of application are to be identified by corresponding additional subscripts.

Taking account of the relations shown in Figure 38, it follows for roughing with tooth thickness allowances (E_{snsV} and E_{sniV}) and a machining allowance, q, that:

$$x_{EsV} m_n = x_{Ei} m_n + \frac{q_{max}}{\sin\alpha_n} \tag{120}$$

$$x_{EiV} m_n = x_{Es} m_n + \frac{q_{min}}{\sin\alpha_n} \tag{121}$$

$$q_{max} = q_{min} + \left(T_{sn} + T_{snv}\right)\frac{\cos\alpha_n}{2} \tag{122}$$

The following applies to finish gear cutting (q = 0):

$$x_{Es} m_n = x m_n + \frac{E_{sns}}{2\tan\alpha_n} \tag{123}$$

$$x_{Ei} m_n = x m_n + \frac{E_{sni}}{2\tan\alpha_n} \tag{124}$$

7.5 Generated root diameter

The root diameter generated by a rack gear cutter (e.g. hob, rack-shaped cutter or grinding wheel) with the addendum h_{aP0} is

$$d_{fE} = d + 2x_E m_n - 2h_{aP0} \tag{125}$$

The root diameter produced by a pinion-type cutter is

$$d_{fE} = 2a_0 - \frac{z}{|z|} d_{a0} \tag{126}$$

For x_E, h_{aP0}, a_0 and d_{a0}, it is necessary to use the values for the process that produces the finished tooth.

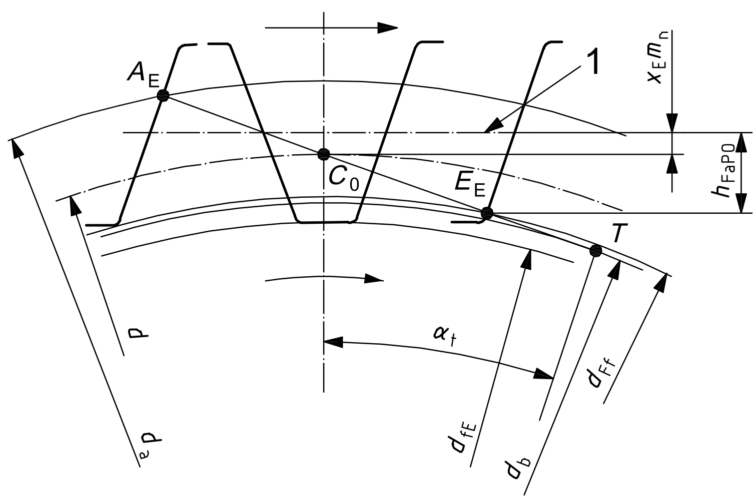
7.6 Usable area of the tooth flank, tip and root form diameter

The maximum usable area of the tooth flank of a cylindrical gear is enclosed by the tip form circle (diameter d_{Fa}) and the root form circle (diameter d_{Ff}), see Figure 24. These circles arise during the generation of the cylindrical gear. They are determined by the starting point A_E and the finishing point E_E of the generating path of contact (see Figure 39) and limit the involute section of the tooth profile. With direct transition between the nominal involute helicoid and the top land of the tooth, the tip form diameter is equal to the tip diameter ($d_{Fa} = d_a$). In the case of tip radiusing or tip chamfering, tip form diameter and tip diameter differ by double the radius h_K:

$$d_{Fa} = d_a - 2\frac{z}{|z|} h_K \tag{127}$$

(See Figure 24.)

The root form diameter, d_{Ff}, follows from the relevant working cycle during gear cutting.



$\alpha_{P0} = \alpha_P$

Key

C_0	pitch point of the generating gear	T	contact point between generating line of action and base circle of gear
A_E	starting point of meshing	h_{FaP0}	straight part of tip flank of tool-generating profile
E_E	end point of meshing	1	datum line of basic rack

NOTE Figure 39 also covers the possible case of different pressure angles α_{P0} and α_P at the tool and the cylindrical gear standard basic rack tooth profile — for example, with single-tooth and single-flank tools (e.g. part generating grinding). The generating gear then has the normal pressure angle, α_{wt0}, and the generating pitch circle diameter $d_{wE} = d_b / \cos \alpha_{wt0}$ instead of α_t and d in the case of $\alpha_{P0} = \alpha_P$.

Figure 39 — Meshing of involute transverse profile of left flank of cylindrical gear during generation with straight flank part of tooth flank of basic generating profile

In the case of tooth systems which are finish-machined using the generating method and tools whose cutter tips lie parallel to the datum line (hob, rack-shaped cutter) and have no undercut or pre-finish root relief, the following applies to an external gear:

$$d_{\mathrm{Ff}}=\sqrt{\left\{d\sin\alpha_{\mathrm{t}}-\frac{2\left[h_{\mathrm{aP0}}-x_{\mathrm{E}}m_{\mathrm{n}}-\rho_{\mathrm{aP0}}\left(1-\sin\alpha_{\mathrm{t}}\right)\right]}{\sin\alpha_{\mathrm{t}}}\right\}^{2}+d_{\mathrm{b}}{}^{2}}$$
$$=\sqrt{\left\{d-2\left[h_{\mathrm{aP0}}-x_{\mathrm{E}}m_{\mathrm{n}}-\rho_{\mathrm{aP0}}\left(1-\sin\alpha_{\mathrm{t}}\right)\right]\right\}^{2}+4\left[h_{\mathrm{aP0}}-x_{\mathrm{E}}m_{\mathrm{n}}-\rho_{\mathrm{aP0}}\left(1-\sin\alpha_{\mathrm{t}}\right)\right]^{2}\cot^{2}\alpha_{\mathrm{t}}} \qquad (128)$$

or, using the roll angle $\tan\alpha_{\mathrm{Ff}}=\xi_{\mathrm{Ff}}$, the following is produced:

$$d_{\mathrm{Ff}}=\frac{d_{\mathrm{b}}}{\cos\alpha_{\mathrm{Ff}}} \qquad (129)$$

where α_{Ff} follows from

$$\tan\alpha_{\mathrm{Ff}}=\xi_{\mathrm{Ff}}=\xi_{\mathrm{t}}-\frac{4\left[h_{\mathrm{aP0}}-\rho_{\mathrm{aP0}}\left(1-\sin\alpha_{\mathrm{t}}\right)/m_{\mathrm{n}}-x_{\mathrm{E}}\right]\cos\beta}{z\sin2\alpha_{\mathrm{t}}} \qquad (130)$$

In the case of external and internal gears, which are generated by means of the generating method using a pinion-type cutter (number of teeth z_0, base diameter d_{b0}, tip form diameter d_{Fa0}, generating centre distance a_0, and generating working transverse pressure angle α_{wt0}) and have no undercut or pre-finish root relief, the following applies:

$$d_{\mathrm{Ff}}=\sqrt{\left(2a_0\sin\alpha_{\mathrm{wt0}}-\frac{z}{|z|}\sqrt{d_{\mathrm{Fa0}}^{2}-d_{\mathrm{b0}}^{2}}\right)^{2}+d_{\mathrm{b}}^{2}} \qquad (131)$$

or, using the roll angle $\tan\alpha_{\mathrm{Ff}}=\xi_{\mathrm{Ff}}$:

$$d_{\mathrm{Ff}}=\frac{d_{\mathrm{b}}}{\cos\alpha_{\mathrm{Ff}}} \qquad (132)$$

with

$$\xi_{\mathrm{Ff}}=\frac{z_0}{z}(\xi_{\mathrm{wt0}}-\xi_{\mathrm{Fa0}})+\xi_{\mathrm{wt0}} \qquad (133)$$

$$\xi_{\mathrm{Fa0}}=\tan\left(\arccos\frac{d_{\mathrm{b0}}}{d_{\mathrm{Fa0}}}\right) \qquad (134)$$

In the case of gears with undercut, the root form diameter arises from the intersection between the involute part of the tooth flank and the root curve.

7.7 Undercut

Undercut is the removal of material in the dedendum flank on external cylindrical gears. Undercutting occurs when the relative path of the tool tip corner rounding cuts into the involute portion of the tooth flank during the rolling action in the generating gear unit. This undercutting can be avoided or minimized by positive profile shift.

For a cylindrical gear produced using a non-protuberance rack-shaped cutter or hob, the minimum value of the generating profile shift coefficient for zero-undercut teeth arises from

$$x_{\mathrm{E\,min}} = \frac{h_{\mathrm{Fa}P0}}{m_{\mathrm{n}}} - \frac{z \sin^2 \alpha_{\mathrm{t}}}{2 \cos \beta} \quad (135)$$

When using a pinion-type cutter, $x_{\mathrm{E\,min}}$ arises from the mating conditions of the generating gear unit.

7.8 Overcut

Overcut is the removal of material from the flank of an internal gear. Overcutting occurs when the relative path of the tool tip corner rounding cuts into the involute portion of the tooth flank near the tooth tip during the rolling action in the generating gear unit.

Overcutting will not occur if the radius of curvature of the flank of an involute cutter at the tip is less than the radius of curvature of the flank of the internal gear at the tip.

7.9 Minimum tooth thickness at the tip circle of a gear

In calculating minimum tooth thickness at the tip circle, the upper limit (not theoretical) tip diameter must be used, and tip chamfering should be accounted for. Minimum tooth thickness should be limited based on material, its heat treatment and the application.

Equation (38) may be used to calculate the tooth thickness at the tip.

The minimum tooth thickness, as well as the undercut, sets a lower limit on the number of teeth that it is practicable to cut in an external gear.

Annex A
(informative)

Calculations related to tooth thickness

A.1 Purpose

Tooth thickness is defined as a circular or helical arc, and is difficult to measure directly. Therefore, indirect measuring methods, such as measurements over balls, span measurement, and chordal measurement, are used. This annex presents two systems for calculating the values related to tooth thickness and the corresponding backlash of a gear set. They are known as the *nominal tooth thickness system* and the *functional tooth thickness system*. Information specific to coordinate-measuring methods is not included, although such methods are becoming quite common.

A.1.1 Symbols

The following symbols are used in this annex.

$d_{\mathrm{y+F}}$	diameter at which the maximum acceptable chordal tooth thickness is calculated
$d_{\mathrm{y-F}}$	diameter at which the minimum acceptable chordal tooth thickness is calculated
F_{rc}	runout tolerance correction to Y-cylinder diameter to account for manufacturing tolerances
h_{cy}	height above the chord to the outside diameter
h_{ccy}	height above the chord to the outside diameter, used for both maximum and minimum chordal tooth thickness measurements (chordal addendum)
s_{ny}	tooth thickness, normal, on the Y-cylinder
s_{c}	tooth thickness, chordal
s_{cy}	tooth thickness, chordal, at Y-cylinder
$s_{\mathrm{(y+F)n\,max}}$	tooth thickness, normal, maximum effective, at Y + F cylinder,
$s_{\mathrm{(y-F)n\,min}}$	tooth thickness, normal, minimum effective, at Y – F cylinder,
$\beta_{\mathrm{y+F}}$	helix angle at Y + F cylinder
$\beta_{\mathrm{y-F}}$	helix angle at Y – F cylinder
Δs	correction for the difference in arc heights of the maximum and minimum chords

A.1.2 Tooth thickness relationships

The relationship between measured tooth thickness and the operating backlash depends on both the tooth thickness measuring method and the accuracy grade. This is because the effective (functional) tooth thickness of a gear will be different to the measured tooth thickness by an amount equal to the combined effects of deviations in the mounting of the gear and all the tooth element deviations. In the nominal system, this difference is applied to the backlash. In the functional system, this difference is accounted for in the allowable measured tooth thickness. Both systems have been used with equal success.

The nominal system uses a direct calculation to go from the specified tooth thickness tolerance to the allowable range of measured tooth thickness (planned test dimensions). The range of predicted backlash for the gear pair is then calculated based on the tolerances of the operating centre distance, the tooth thickness test dimensions for each of the gears, the measuring methods used and the elemental tolerances of the gears. The predicted backlash range must be verified for suitability and, if necessary, changes made to the nominal tooth thickness tolerance, the measuring method, the centre distance or the accuracy grade (the elemental tolerances, also known as the allowable elemental deviations, are a function of the specified accuracy grade). This annex includes the technique for calculation of the tooth thickness test dimensions using the nominal system, but does not include calculations for the predicted backlash range. See DIN 3967 for such calculations.

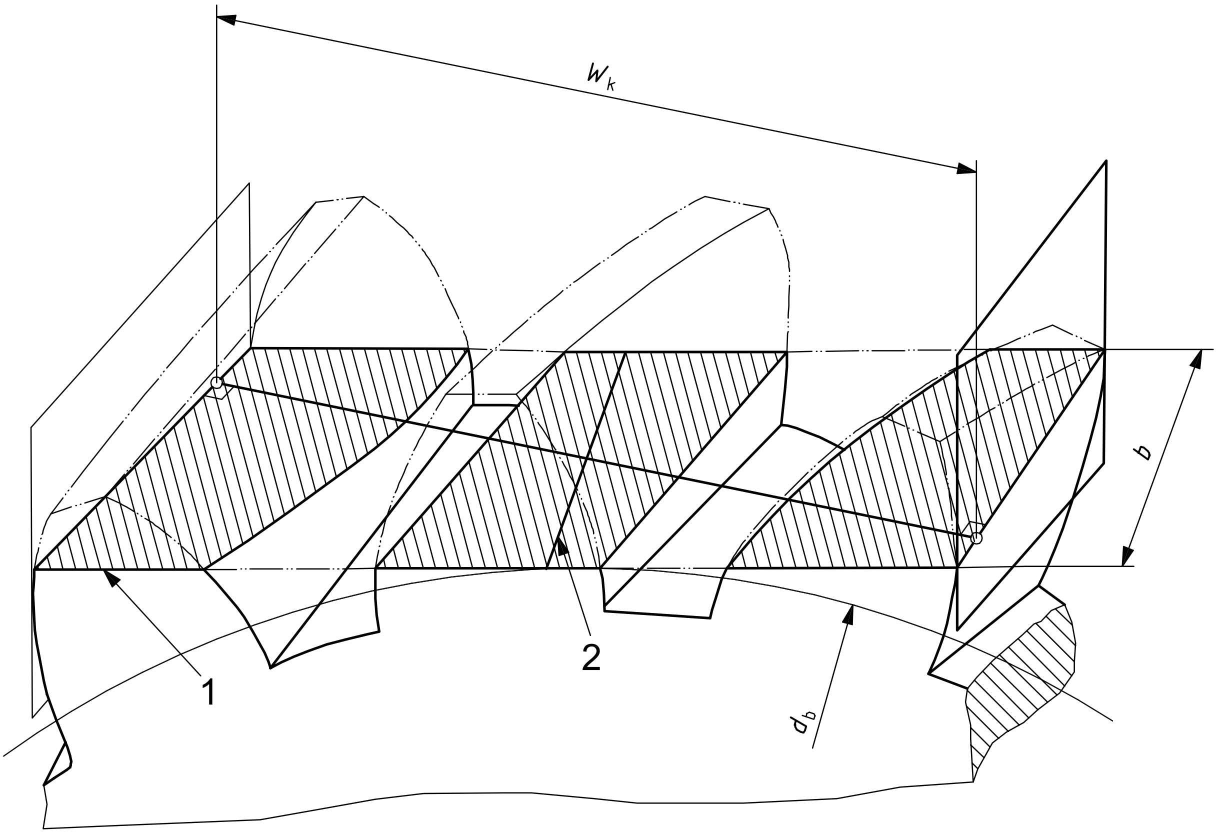
In the functional system there is a direct correlation between the specified functional tooth thickness and the backlash. The tooth thickness test dimensions are calculated as a function of both the measuring method used and the allowable values of the elemental deviations (tolerances) of the gears. The resulting allowable measured tooth thickness range must be verified for suitability and, if necessary, changes must be made to the functional tooth thickness tolerance, the measuring method, the centre distance or the accuracy grade.

Within the nominal tooth thickness system, the equations in 4.7.1 to 4.7.5 produce the nominal test dimensions if the nominal (zero-backlash) profile shift coefficient x is used in these and in the equations for the tooth thickness half angle ψ (4.7.2) or space width half angle η (4.7.4). The planned test dimensions (the permissible maximum and minimum dimensions or their arithmetic means) for the finish or rough gear cutting can be determined if, instead of x, the generating profile shift coefficient, x_E, is used according to Equations (120) and (121) for $q \geqslant 0$ and for the half angles.

Within the functional tooth thickness system, the generating profile shift coefficient x_E is always used.

A.2 Span measurement

The span measurement, W_k, is the distance measured between two parallel planes normal to the base tangent plane which contact a left and right flank over k teeth on an external helical or spur gear, or measured over k tooth spaces in an internal spur gear. The contact points lie in a base tangent plane. Internal helical gears cannot be measured with this technique. The two parallel planes must contact a right flank and a left flank respectively in the involute portion of those tooth flanks. (See Figure A.1.) For internal spur gears, measuring cylinders or measuring balls must be used instead of flat measuring surfaces. See ISO/TR 10064-2 for additional information.



Key

1 base cylinder tangential plan

2 projection of the gear axis

Figure A.1 — Span measurement, W_k, on helical cylindrical gear

The span measurement is a flank-related measurement quantity and therefore independent of (not a function of) deviations of position between the gear axis and the axis of the teeth.

In many cases, on the same gear, span can be measured over several different numbers of teeth (or tooth spaces). Flank modifications, undercut, tip diameter variations and changed parameters of the standard basic rack tooth profile (e.g. alignment teeth with involute tooth profile according to ISO 4156) can lead to the reduction of the usable area of the tooth flank for measuring the span. This restricts the possible number of teeth spanned (measured number of tooth spaces), k. In some cases, particularly for helical gears with a low face to diameter ratio, span measurement cannot be used.

In the following equations, the integer function (INT) signifies that k is the closest whole number less than or equal to the decimal number of the value in brackets.

A.2.1 External gears, number of teeth spanned

The number of teeth spanned (measured number of tooth spaces) may be chosen from either:

$$k = \mathrm{INT}\left[\frac{z}{\pi}\left(\frac{\tan\alpha_{\mathrm{vt}}}{\cos^2\beta_{\mathrm{b}}} - \mathrm{inv}\,\alpha_{\mathrm{t}} - \frac{2x}{z}\tan\alpha_{\mathrm{n}}\right) + 1\right] \quad \text{(A.1)}$$

or, alternatively,

$$k = \mathrm{INT}\left(\frac{\dfrac{\sqrt{d_{\mathrm{v}}{}^2 - d_{\mathrm{b}}^2}}{\cos\beta_{\mathrm{b}}} - s_{\mathrm{bn}}}{p_{\mathrm{bn}}} + 1\right) \tag{A.2}$$

or by means of an approximation, which is sufficient in many cases as

$$k = \mathrm{INT}\left(z\frac{\mathrm{inv}\alpha_{\mathrm{t}}}{\mathrm{inv}\alpha_{\mathrm{n}}}\frac{\alpha_{\mathrm{vn}}}{\pi} + 1\right) \tag{A.3}$$

α_{vn} may be calculated according to Equation (15) on the V circle.

The usable range of number of teeth spanned (measured number of tooth spaces), k, on flanks without modification as limited by the diameter of the root form circle, d_{Ff}, and the tip form circle, d_{Fa}, can be obtained from

$$k_{\mathrm{min}} = \mathrm{INT}\left[\frac{z}{\pi}\left(\frac{\tan\alpha_{\mathrm{Ff}}}{\cos^2\beta_{\mathrm{b}}} - \mathrm{inv}\alpha_{\mathrm{t}} - \frac{2x}{z}\tan\alpha_{\mathrm{n}}\right) + 1{,}5\right] = \mathrm{INT}\left(\frac{\dfrac{\sqrt{d_{\mathrm{Ff}}^2 - d_{\mathrm{b}}^2}}{\cos\beta_{\mathrm{b}}} - s_{\mathrm{bn}}}{p_{\mathrm{bn}}} + 1{,}5\right) \tag{A.4}$$

$$k_{\mathrm{max}} = \mathrm{INT}\left[\frac{z}{\pi}\left(\frac{\tan\alpha_{\mathrm{Fa}}}{\cos^2\beta_{\mathrm{b}}} - \mathrm{inv}\alpha_{\mathrm{t}} - \frac{2x}{z}\tan\alpha_{\mathrm{n}}\right) + 0{,}5\right] = \mathrm{INT}\left(\frac{\dfrac{\sqrt{d_{\mathrm{Fa}}^2 - d_{\mathrm{b}}^2}}{\cos\beta_{\mathrm{b}}} - s_{\mathrm{bn}}}{p_{\mathrm{bn}}} + 0{,}5\right) \tag{A.5}$$

If the flanks are modified, use the unmodified flank limit diameters instead of the root and tip form diameters. With the user-chosen integer value for k ($k_{\mathrm{min}} \leqslant k \leqslant k_{\mathrm{max}}$) the base tangent length is given by:

$$\begin{aligned} W_{\mathrm{k}} &= m_{\mathrm{n}}\cos\alpha_{\mathrm{n}}\left[\pi(k-1) + z\,\mathrm{inv}\alpha_{\mathrm{t}} + z\psi\right] \\ &= m_{\mathrm{n}}\cos\alpha_{\mathrm{n}}\left[\pi(k-0{,}5) + z\,\mathrm{inv}\alpha_{\mathrm{t}}\right] + 2xm_{\mathrm{n}}\sin\alpha_{\mathrm{n}} \\ &= (k-1)\,p_{\mathrm{bn}} + s_{\mathrm{bn}} \end{aligned} \tag{A.6}$$

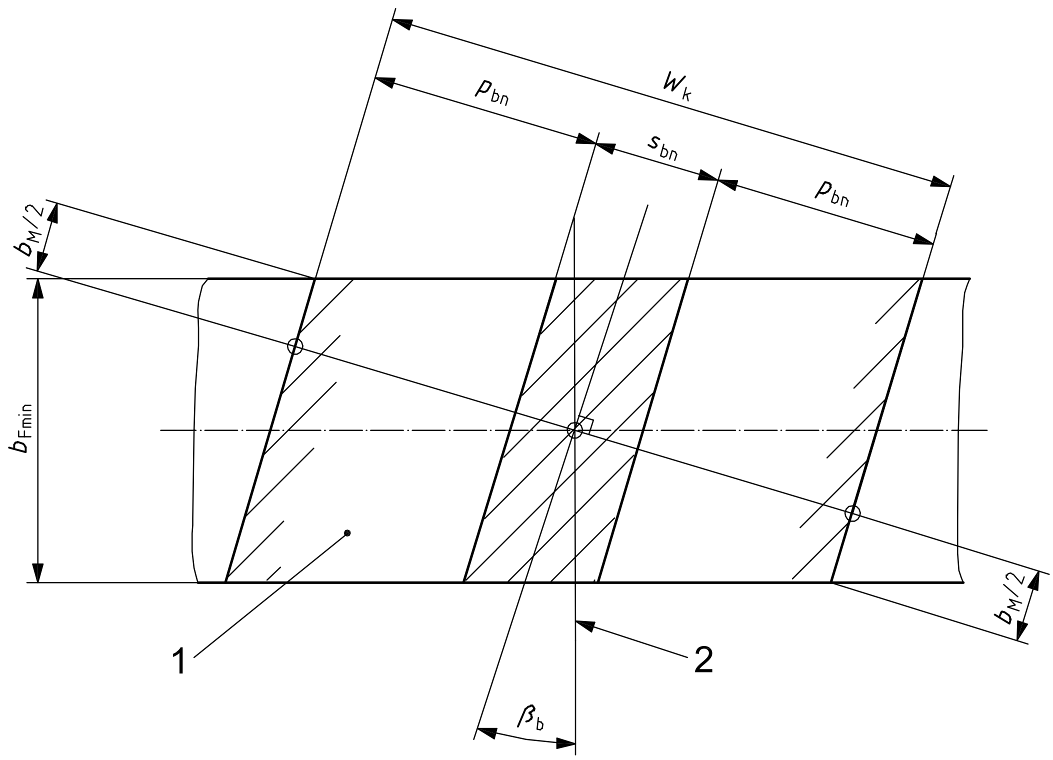
where tooth thickness half angle ψ is determined using Equation (41) with the generating profile shift coefficient.

In the case of external helical gears, it is always necessary to check the practicality of the span measurement for the calculated or selected number, k, of teeth to be spanned. In order to ensure a sufficiently reliable measurement of W_{k} for a gear with usable facewidth b_{F} (b reduced by the chamfers or curves at the tooth ends, see 4.2.8), b_{F} must equal or exceed a minimum value of $b_{\mathrm{F\,min}}$, which is required to ensure that the straight contact lines between the measuring surfaces and the two tooth flanks (involute helicoids) are satisfactorily long. Thereby, a secure measuring surface contact is guaranteed and the imaginary axis of the measuring device (shown by the points on the straight contact lines indicated by dimension W_{k} in Figures A.1 to A.3) is positioned perpendicular to the flank generators. Usable facewidth b_{F} (see Figure A.2) should not be less than value $b_{\mathrm{F\,min}}$ determined using:

$$b_F \geqslant b_{Fmin} = W_k \sin \beta_b + b_M \cos \beta_b \tag{A.7}$$

with

$$b_M = 1,2 + 0,018 W_k \tag{A.8}$$



Key

1 base cylinder tangential plane

2 projection of the gear axis

Figure A.2 — Diagram showing facewidth required to permit adequate span measurement

In the case of spur gears, with the chosen integer number of teeth spanned (measured number of tooth spaces) k, the measuring planes will contact the tooth flanks (with symmetrical positioning of the measuring planes, see Figure A.3) at the measuring circle diameter d_M:

$$d_M = \sqrt{d_b^2 + W_k^2} \tag{A.9}$$

The possibilities for rocking the measuring device in relation to the symmetrical positioning of the measuring surfaces (angle of rock, δ_W) within the tooth flank surface limited by tip form diameter d_{Fa} and root form diameter d_{Ff} are determined by the tooth parameters.

Where $W_k - \frac{d_b}{2} \tan \alpha_{Fa} > \frac{d_b}{2} \tan \alpha_{Ff}$, it follows that

$$\delta_W = 2\left(\tan \alpha_{Fa} - \frac{W_k}{d_b}\right) \tag{A.10}$$

Where $W_k - \frac{d_b}{2}\tan\alpha_{Fa} \leqslant \frac{d_b}{2}\tan\alpha_{Ff}$ (see Figure A.3), it follows that

$$\delta_w = 2\left(\frac{W_k}{d_b} - \tan\alpha_{Ff}\right) \quad \text{(A.11)}$$

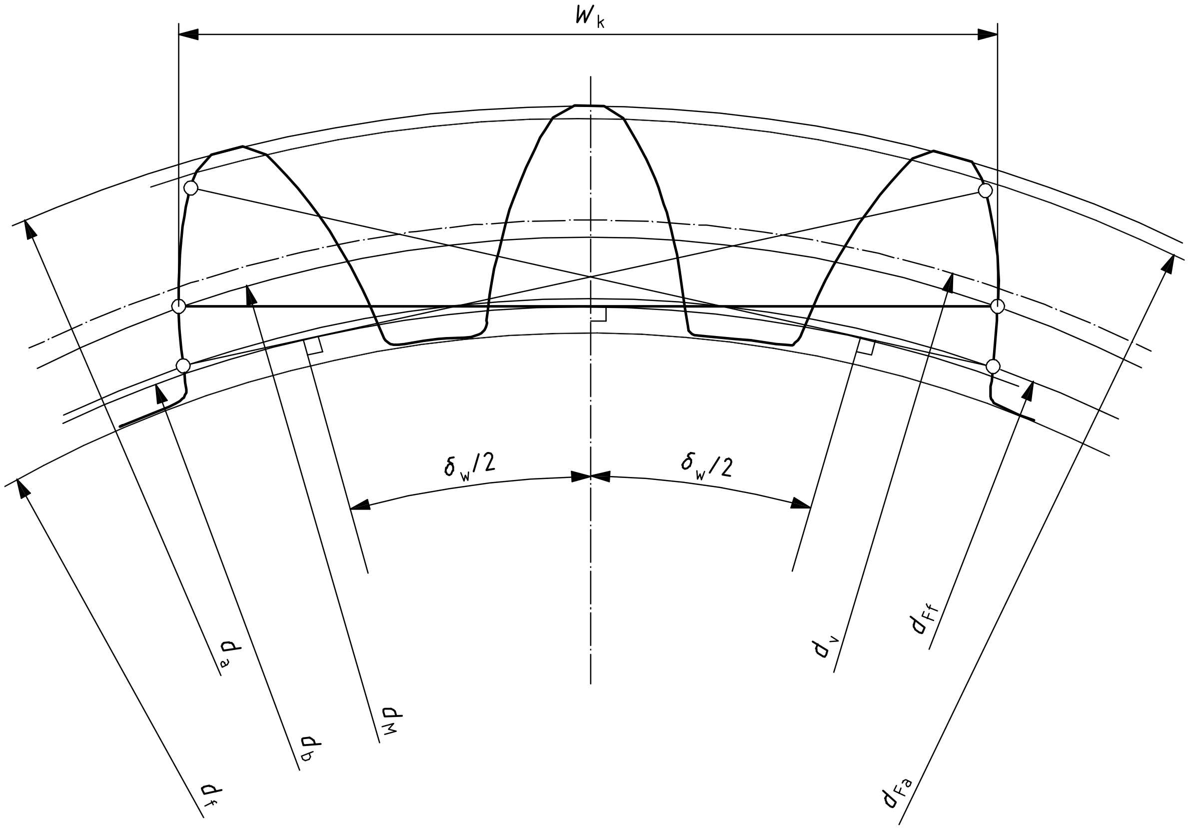


Figure A.3 — Diagram of portion of transverse section available for span measurement on external spur gear with number of teeth spanned k = 3 for a secure measuring surface contact

A.2.2 Internal spur gears, number of tooth spaces spanned

Number of tooth spaces spanned, k, may be chosen from

$$k = \mathrm{INT}\left[\frac{|z|}{\pi}\left(\tan\alpha_{\mathrm{v}} - \mathrm{inv}\,\alpha - \frac{2x}{z}\cdot\tan\alpha_{\mathrm{n}}\right) - 1\right] \tag{A.12}$$

or, alternatively,

$$k = \mathrm{INT}\left(\frac{\sqrt{d_{\mathrm{v}}{}^{2} - d_{\mathrm{b}}^{2}} + s_{\mathrm{b}}}{p_{\mathrm{b}}} - 1\right) \tag{A.13}$$

or by means of approximation, which is sufficient in many cases as

$$k = \mathrm{INT}\left(|z|\frac{\alpha_{\mathrm{v}}}{\pi} - 1\right) \tag{A.14}$$

α_{vn} may be calculated according to Equation (16) on the V, circle.

The root form diameter limits the maximum number of tooth spaces that can be spanned, while the internal tooth tip diameter limits the minimum number of tooth spaces that can be spanned:

$$k_{\mathrm{max}} = \mathrm{INT}\left(\frac{\sqrt{d_{\mathrm{Ff}}^{2} - d_{\mathrm{b}}^{2}} + s_{\mathrm{bn}}}{p_{\mathrm{bn}}} - 0{,}5\right) \tag{A.15}$$

$$k_{\mathrm{min}} = \mathrm{INT}\left(\frac{\sqrt{d_{\mathrm{a}}{}^{2} - d_{\mathrm{b}}^{2}} + s_{\mathrm{bn}}}{p_{\mathrm{bn}}} + 0{,}5\right) \tag{A.16}$$

If the flanks are modified, use the unmodified flank limit diameters instead of the root and tip form diameters.

With the user-chosen integer value for k $\left(k_{\mathrm{min}} \leqslant k \leqslant k_{\mathrm{max}}\right)$, the base tangent length is given by

$$\begin{aligned} W_{\mathrm{k}} &= m_{\mathrm{n}} \cos\alpha_{\mathrm{n}} \left(\pi k + |z|\,\mathrm{inv}\,\alpha_{\mathrm{t}} + z\psi\right) \\ &= m_{\mathrm{n}} \cos\alpha_{\mathrm{n}} \left[\pi\left(k - 0{,}5\right) + |z|\,\mathrm{inv}\,\alpha_{\mathrm{t}}\right] - 2x m_{\mathrm{n}} \sin\alpha_{\mathrm{n}} \\ &= k\, p_{\mathrm{bn}} - s_{\mathrm{bn}} \end{aligned} \tag{A.17}$$

where tooth thickness half angle ψ is determined using Equation (41) with the generating profile shift coefficient.

A.2.3 Functional system — Span adjustment for allowable tolerances

The number of teeth to be spanned in the functional system is the same as that in the nominal system (see A.2.1).

The span measurement gives the tooth thickness in relation to the base cylinder. Deviations in base pitch, accumulated pitch over k teeth, tooth profile, and lead can all affect the span measurement, and runout (between the base cylinder and the bearing journals) can affect the functional tooth thickness. The allowable span measurement must therefore be adjusted if the functional tooth thickness is to be limited.

The adjustment is made by decreasing the maximum base tooth thickness when calculating the maximum span measurement and increasing the minimum base tooth thickness when calculating the minimum span measurement. The effects of lead and profile deviations can usually be ignored since they are usually much smaller than the runout and pitch deviations:

$$s_{\text{bn.amax}} = s_{\text{bn.max}} - \cos\beta_{\text{b}} \left(F_{\text{r}} \tan\alpha_{\text{tM}} + F_{\text{pk}} \cos\alpha_{\text{tM}} \right) \tag{A.18}$$

$$s_{\text{bn.amin}} = s_{\text{bn.min}} + \cos\beta_{\text{b}} \left(F_{\text{r}} \tan\alpha_{\text{tM}} + F_{\text{pk}} \cos\alpha_{\text{tM}} \right) \tag{A.19}$$

The deviations F_{r} and F_{pk} are found in ISO 1328.

The transverse pressure angle at the measuring diameter may be calculated at the diameter found from Equation (A.9).

Thus the allowable range of span measurement for external gears is

$$W_{\text{k.amax}} = (k-1)\, p_{\text{bn}} + s_{\text{bn.amax}} \tag{A.20}$$

$$W_{\text{k.amin}} = (k-1)\, p_{\text{bn}} + s_{\text{bn.amin}} \tag{A.21}$$

The allowable range of span measurement for internal gears is

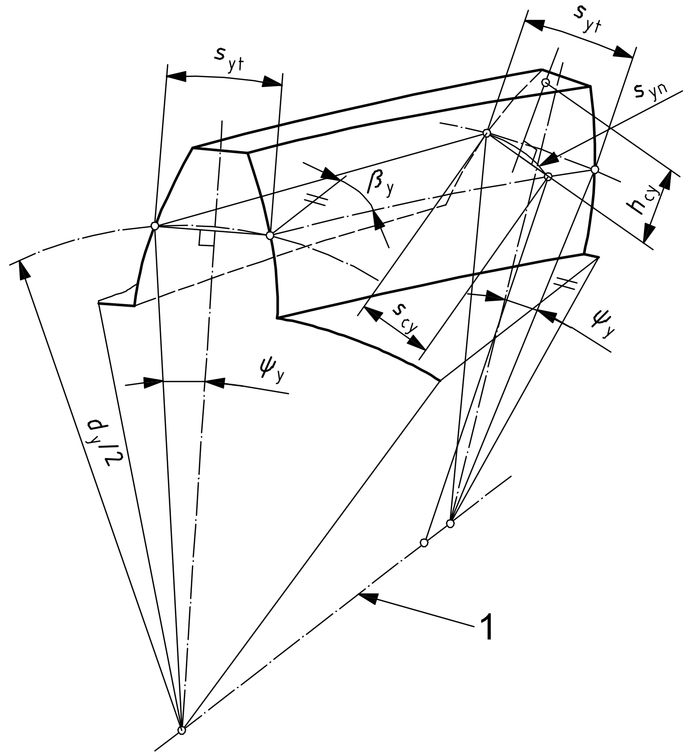
$$W_{\text{k.amax}} = k\, p_{\text{bn}} + s_{\text{bn.amax}} \tag{A.22}$$

$$W_{\text{k.amin}} = k\, p_{\text{bn}} + s_{\text{bn.amin}} \tag{A.23}$$

A.3 Normal chordal tooth thicknesses and heights above the chords

A.3.1 General

The normal chordal tooth thickness, s_{cy}, is the shortest straight-line distance between tooth traces of a tooth on any pitch cylinder (Y-cylinder). (See Figure A.4.) In calculating and measuring the chordal tooth thickness, $d_a - 2m_n$ is often used as the Y-cylinder diameter.



Key

1 gear axis

Figure A.4 — Tooth thicknesses, normal chordal tooth thicknesses and height above normal chordal tooth thickness on tooth at Y-cylinder of right-hand helical cylindrical gear

On the Y-cylinder, the following applies:

$$s_{cy} = d_y \sqrt{\left(\psi_y \cos\beta_y \sin\beta_y\right)^2 + \sin^2\left(\psi_y \cos^2\beta_y\right)} \tag{A.24}$$

or

$$s_{cy} = \sqrt{\left(s_{yn} \sin\beta_y\right)^2 + \left[d_y \sin\left(\frac{s_{yn}\cos\beta_y}{d_y}\right)\right]^2} \tag{A.25}$$

The normal chordal tooth thickness on the reference cylinder is

$$s_c = d\sqrt{\left(\psi \cos\beta \sin\beta\right)^2 + \sin^2\left(\psi \cos^2\beta\right)} \tag{A.26}$$

or

$$s_{\mathrm{c}} = \sqrt{\left(s_{\mathrm{n}} \sin\beta\right)^2 + \left(d \sin\left(\frac{s_{\mathrm{n}} \cos\beta}{d}\right)\right)^2} \tag{A.27}$$

The height, h_{cy}, above the chord, s_{cy}, to outside diameter d_{a} is

$$h_{\mathrm{cy}} = \left|\frac{d_{\mathrm{a}}}{2} - \frac{d_{\mathrm{y}}}{2}\cos\left(\frac{s_{\mathrm{yn}} \cos\beta_{\mathrm{y}}}{d_{\mathrm{y}}}\right)\right| \tag{A.28}$$

NOTE h_{cy} is also known as the chordal addendum. It is calculated in the transverse plane. The absolute value is used for internal gears.

For the reference chordal addendum, h_{c}, above the reference chordal measurement, s_{c}:

$$h_{\mathrm{c}} = \left|\frac{d_{\mathrm{a}}}{2} - \frac{d}{2}\cos\left(\frac{s_{\mathrm{n}} \cos\beta}{d}\right)\right| \tag{A.29}$$

A.3.2 Functional tooth thickness system — Chordal tooth thickness

The effective tooth thickness is influenced by runout. Therefore, runout should be included in the specified (manufacturing or measured) chordal tooth thickness tolerance calculation. Although large amounts of involute and helix form and slope deviations can affect the chordal thickness measurement, they can normally be neglected. To adjust the maximum effective tooth thickness, one half of the runout tolerance is added to the radius at which calculations are made. However, the calculation of the addendum bar (height slide) setting is made based on diameter d_{y}. Thus the maximum effective tooth thickness will be acceptable when, due to runout, that point on the tooth is effectively at a larger diameter than d_{y}. Therefore, the diameter to be used in calculating the maximum allowable chordal tooth thickness is

$$d_{\mathrm{y+F}} = d_{\mathrm{y}} + F_{\mathrm{rc}} \tag{A.30}$$

where F_{rc} is the correction made to the chordal addendum to account for manufacturing tolerances.

The value used for this runout allowance should be a combination of the outside diameter runout and the gear tooth runout. If the actual outside diameter and its runout at the point of measurement are known, they should be used. If the outside diameter runout is not known, it can be assumed to be equal to the allowable runout of the gear teeth. The runout of the gear teeth can be assumed to be 80 % of the total cumulative pitch deviation. [See ISO 1328-2:1997, Equation (B.1).]

The corrected values for maximum measured tooth thickness thus become:

$$s_{\mathrm{cy.max}} = \sqrt{\left(s_{\mathrm{(y+F)n.max}} \sin(\beta_{\mathrm{y+F}})\right)^2 + \left\{d_{\mathrm{y+F}} \sin\left[\frac{s_{\mathrm{(y+F)n.max}} \cos\left(\beta_{\mathrm{y+F}}\right)}{d_{\mathrm{y+F}}}\right]\right\}^2} \tag{A.31}$$

$$h_{\mathrm{ccy}} = \frac{\left|d_{\mathrm{a.max}} - d_{\mathrm{y+F}} \cos\left(\frac{s_{\mathrm{(y+F)n.max}} \cos(\beta_{\mathrm{y+F}})}{d_{\mathrm{y+F}}}\right)\right| + F_{\mathrm{rc}}}{2} \tag{A.32}$$

Thus $s_{\mathrm{cy.max}}$ is based on diameter $d_{\mathrm{y+F}}$, but the runout corrected chordal addendum, h_{ccy}, is essentially set at diameter d_{y}, since although it starts with $d_{\mathrm{y+F}}$, it is corrected not only for the difference in height between the arc and the chord, but also corrected by F_{rc}.

If the backlash is to be tightly controlled, then the minimum tooth thickness is important. In this case, the effective runout must be subtracted from diameter d_y. In addition, the measurement should be referenced to the minimum tip diameter. Thus the diameter for calculation becomes:

$$d_{y-F} = d_y - F_{rc} - \left(d_{a.max} - d_{a.min}\right) \tag{A.33}$$

To allow the same chordal addendum to be used for the acceptable minimum measured chordal tooth thickness, the equation for $s_{cy.min}$ also contains the term Δs to correct for the difference in arc heights of the maximum and minimum chords. Thus the corrected value for minimum measured chordal tooth thickness is

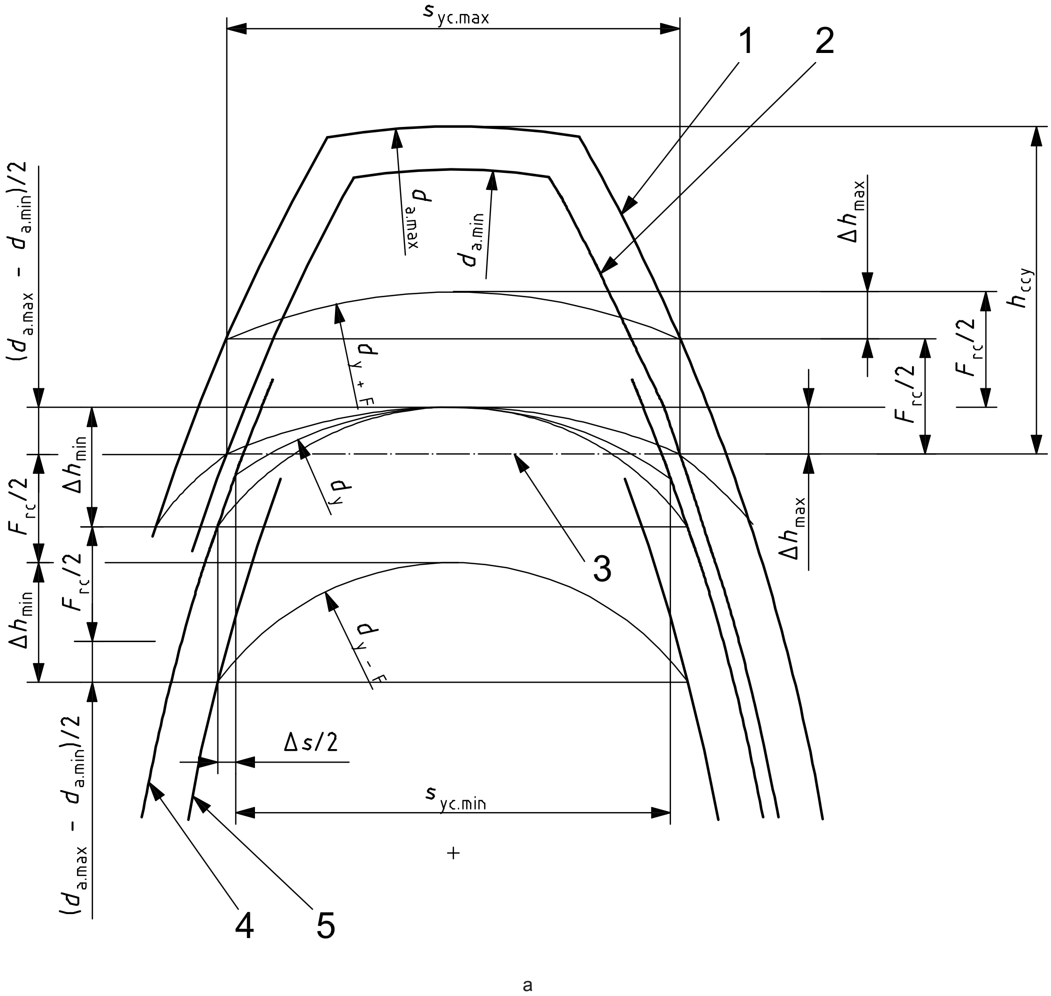
$$s_{cy.min} = \sqrt{\left(s_{(y-F)n.min} \sin(\beta_{y-F})\right)^2 + \left\{d_{y-F} \sin\left[\frac{s_{(y-F)n.min} \cos\left(\beta_{y-F}\right)}{d_{y-F}}\right]\right\}^2} - \Delta s \tag{A.34}$$

$$\Delta s \approx 2\left(\Delta h_{min} - \Delta h_{max}\right)\tan\alpha_{yn} \quad \Delta h = \frac{d_y}{2}\left[1 - \cos\left(\frac{s_{yn}\cos\beta_y}{d_y}\right)\right]$$

$$\Delta s \approx \left\{d_{y-F}\left[1-\cos\left(\frac{s_{(y-F)nmin}\cos\beta_{y-F}}{d_{y-F}}\right)\right] - d_{y+F}\left[1-\cos\left(\frac{s_{(y+F)nmax}\cos\beta_{y+F}}{d_{y+F}}\right)\right]\right\}\tan\alpha_{yn} \tag{A.35}$$

The measured chordal tooth thickness can then be specified with the limits $s_{cy.max}$ and $s_{cy.min}$, both measured at h_{ccy}. As can be seen in Figure A.5, the allowable variation in measured tooth thickness is less than the variation in effective tooth thickness due to manufacturing tolerances. In Figure A.5, the curvature of the arcs is highly exaggerated so they can be easily seen, and the runout is exaggerated so that the arcs are well separated. What is not exaggerated is the fact that the effective tooth thickness can be significantly larger or smaller than the apparent measured tooth thickness.

See Figures A.6 and A.7 for the maximum and minimum allowable chordal tooth thickness measurement corrections, respectively.



Key

1 maximum tooth thickness, effective

2 maximum tooth thickness, manufacturing (measured)

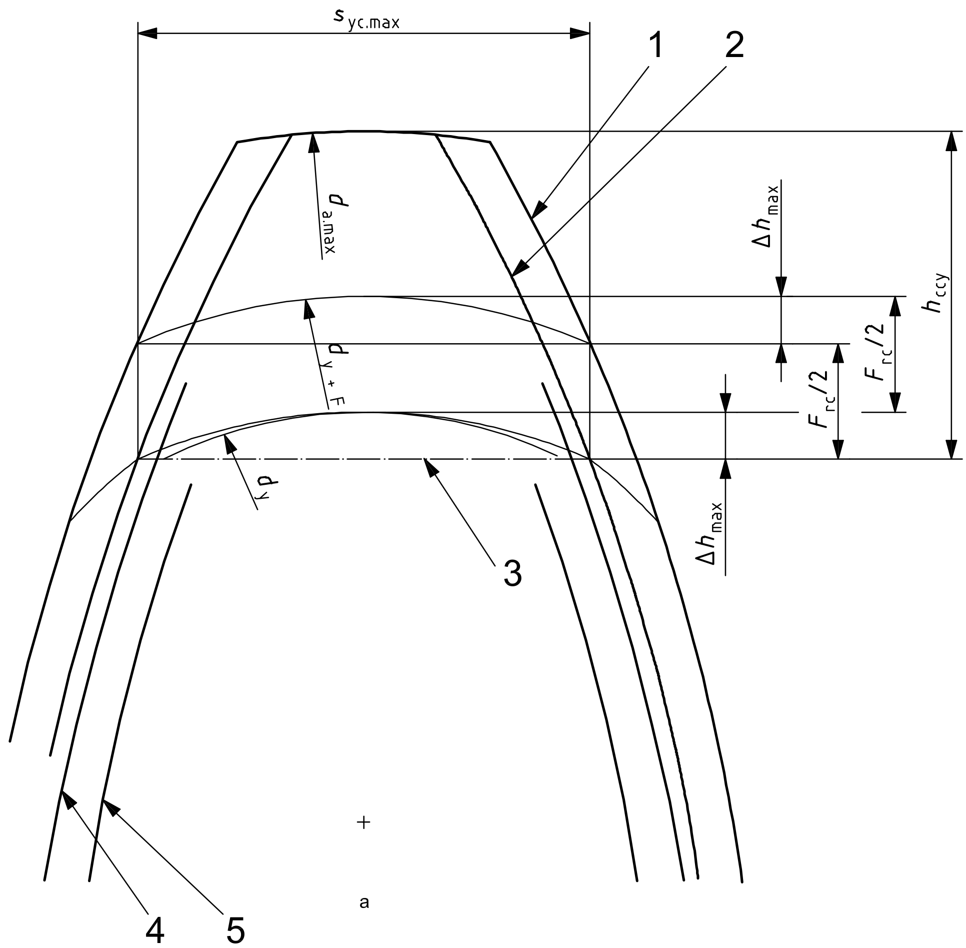
3 measurement line

4 minimum tooth thickness, manufacturing (measured)

5 minimum tooth thickness, effective

[a] Transverse plane.

Figure A.5 — Correction of allowable chordal tooth thickness measurement to account for runout and outside diameter deviations



Key

1 maximum tooth thickness, effective

2 maximum tooth thickness, manufacturing (measured)

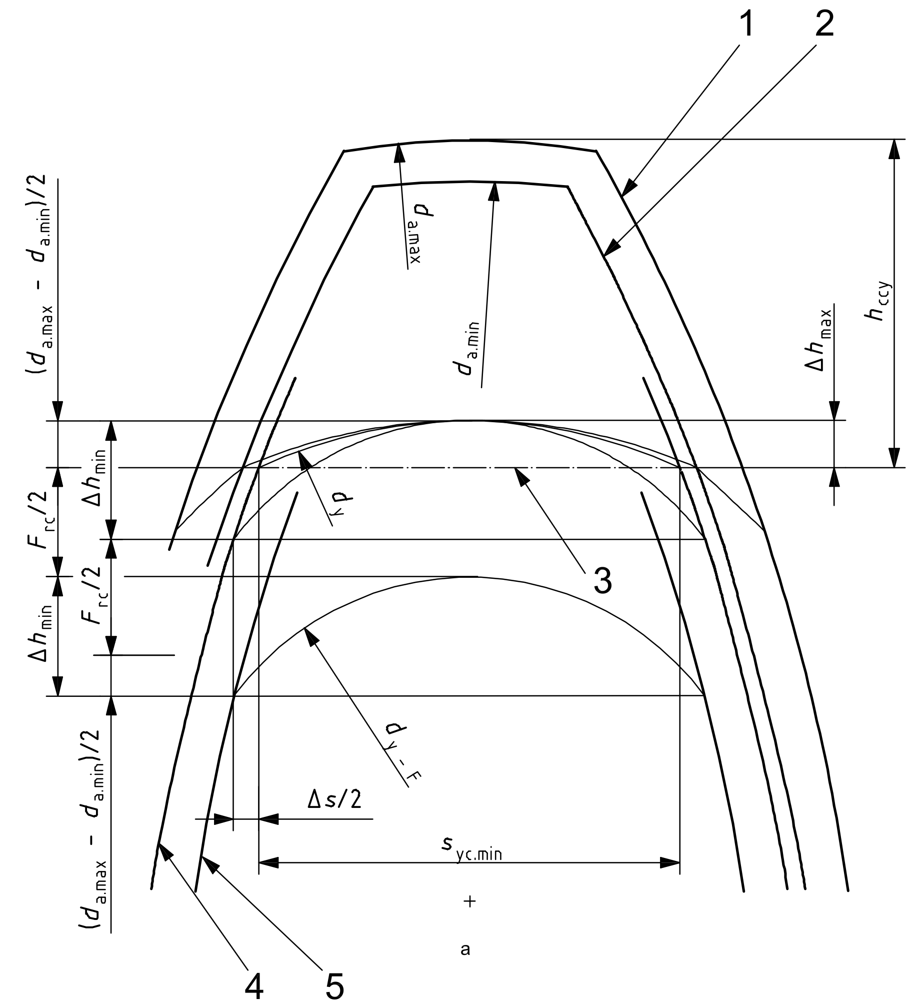
3 measurement line

4 minimum tooth thickness, manufacturing (measured)

5 minimum tooth thickness, effective

a Transverse plane.

Figure A.6 — Correction of maximum allowable chordal tooth thickness measurement to account for runout



Key

1 maximum tooth thickness, effective

2 maximum tooth thickness, manufacturing (measured)

3 measurement line

4 minimum tooth thickness, manufacturing (measured)

5 minimum tooth thickness, effective

[a] Transverse plane.

Figure A.7 — Correction of minimum allowable chordal tooth thickness measurement to account for runout and outside diameter deviations

A.4 Constant chord

The constant chordal tooth thickness, s_{cc}, is the length of the straight lines produced between the flank points when two tangents forming an apex angle of $2\alpha_t$ at both profiles of a tooth in the transverse section are positioned symmetrically, see Figure A.8.

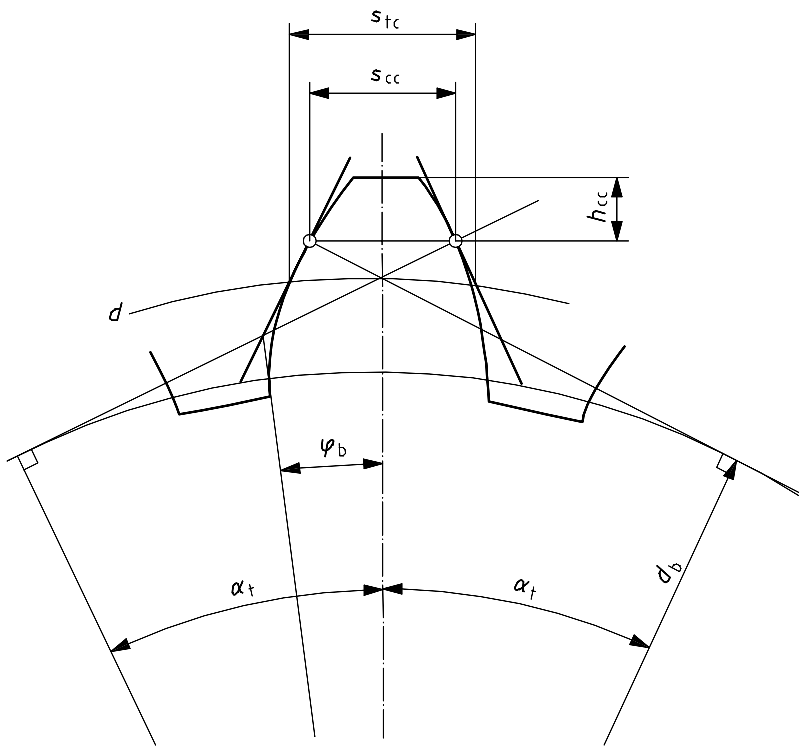


Figure A.8 — Constant chord s_{cc} and height h_{cc} above constant chord in transverse section of external helical gear

Constant tooth thickness chord s_{cc} and height h_{cc} above the constant chord are expressed as

$$s_{cc} = s_n \frac{\cos^2 \alpha_t}{\cos \beta} = m_n \left(\frac{\pi}{2} + 2x \tan \alpha_n \right) \frac{\cos^2 \alpha_t}{\cos \beta} \tag{A.36}$$

$$h_{cc} = h_a - \frac{s_t}{2} \sin \alpha_t \cos \alpha_t = h_a - \frac{m_n}{2} \left(\frac{\pi}{2} + 2x \tan \alpha_n \right) \frac{\sin \alpha_t \cos \alpha_t}{\cos \beta} \tag{A.37}$$

Spur-toothed cylindrical gears of the same standard basic rack tooth profile and with the same profile shift have the same (constant) chordal tooth thickness, s_{cc}, independent of the number of teeth. For this reason, s_{cc} is termed the *constant chord*.

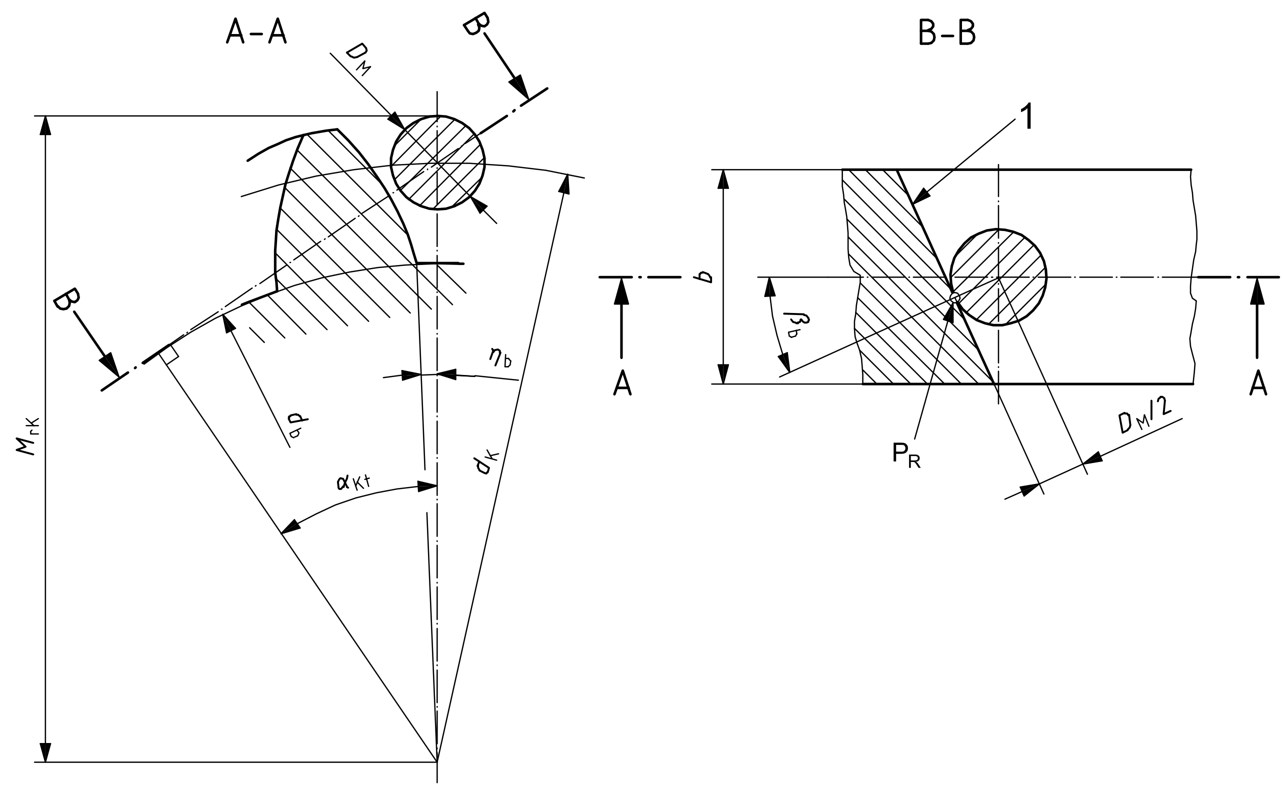
A.5 Radial single-ball test dimension

The radial single-ball dimension, M_{rK}, is the distance between the gear axis and

— in the case of an external gear, the outermost point, and

— in the case of an internal gear, the innermost point

of a measuring ball of diameter, D_M, which lies in a tooth space on both tooth flanks; see Figures A.9 and A.10.



Key

A-A transverse section through the centre of the measuring ball

B-B section of base cylinder tangential plane of the right flank on a left-hand cylindrical gear

P_R contact point of the measuring ball on the right flank

1 generator

Figure A.9 — Radial single-ball dimension, M_{rK}, on external helical gear

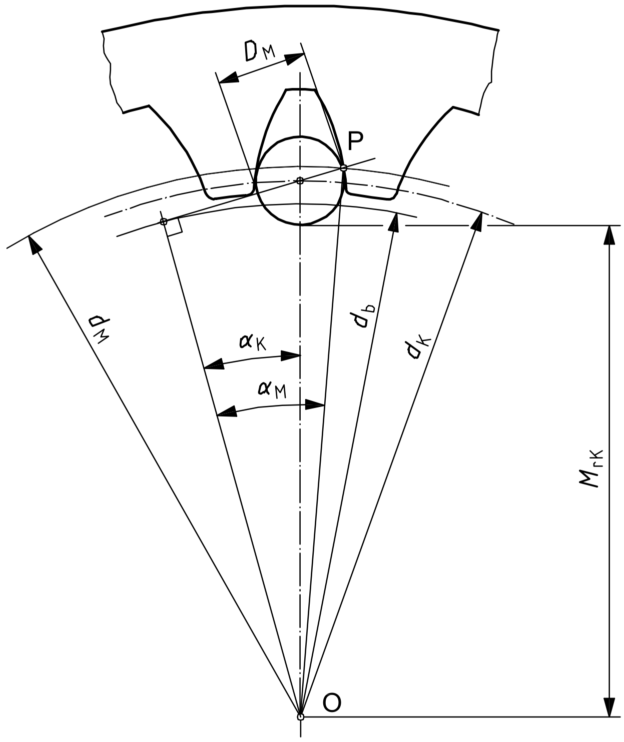


Figure A.10 — Radial single-ball dimension M_{rK} in transverse section of internal spur gear

The contact points, P_R and P_L, between the measuring ball and the right and left flank shall lie on or near to the V-cylinder. In order for the contact points to lie on the V-cylinder, D_M must have the following value in the case of helical cylindrical gears:

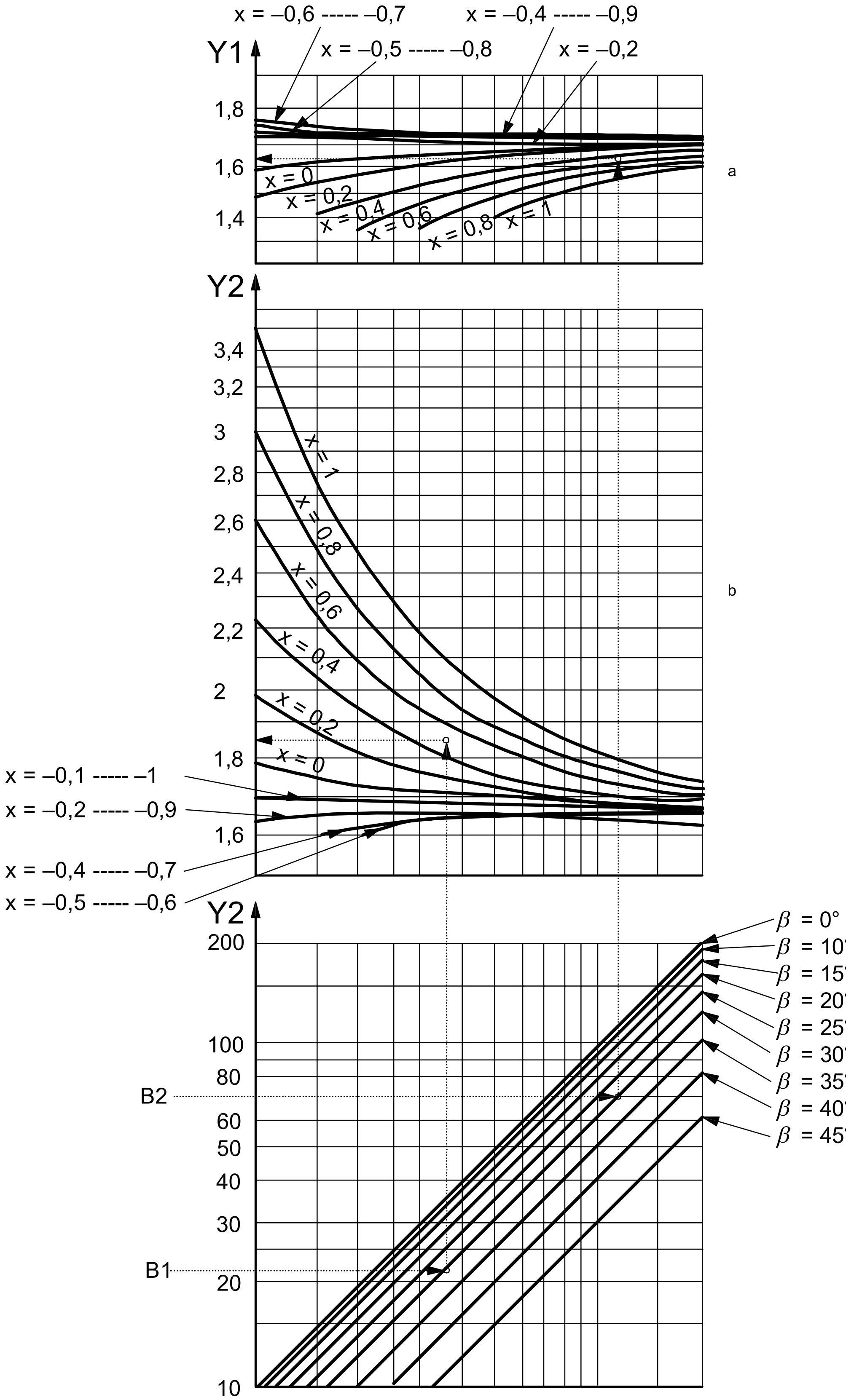
$$D_M = |z| m_n \cos\alpha_n \frac{\tan\alpha_{Kt} - \tan\alpha_{vt}}{\cos^2\beta_b} \tag{A.38}$$

with α_{vt} according to 4.3.5 and α_{Kt} according to Equation (A.39), which is not explicitly resolvable:

$$\alpha_{Kt} + \mathrm{inv}\,\alpha_{Kt} \sin^2\beta_b = \tan\alpha_{vt} + \frac{z}{|z|}\eta_b \cos^2\beta_b \tag{A.39}$$

With sufficient approximation, for tooth systems with $\alpha_n = 20°$, the measuring ball diameter D_M required for the contact on a V-cylinder can also be determined using the nomogram shown in Figure A.11. The measuring ball diameter is calculated from the coefficient $D_M{}^*$:

$$D_M = m_n D_M{}^* \tag{A.40}$$



Key

Y1 measuring ball diameter coefficient, D_m

Y2 amount of teeth, z

[a] Internal gear.

[b] External gear.

In the case of cylindrical gears with manufactured or actual tooth thickness dimensions then use x_E instead of x; example B1 (external gear): z = 22; β = 30°; x = 0,5; example B2 (internal gear): z = 70; β = 30°; x = 0,5.

Figure A.11 — Nomogram for determining measuring ball diameter coefficient D_M for radial single-ball dimension or dimension over balls for $\alpha_n = 20°$

In the case of spur-type cylindrical gears, α_K can be calculated exactly from the explicit equation of calculation:

$$\alpha_K = \tan\alpha_v + \eta_b \tag{A.41}$$

D_M is to be calculated according to Equation (A.38). As the measuring balls only need to contact the tooth flanks in the vicinity of the V-cylinder, measuring balls having diameters slightly different from the calculated values may be used.

If measuring ball diameter D_M is known, then the pressure angle at a point, α_{Kt}, in the transverse section on the circle through the centre of the ball is found from

$$\operatorname{inv}\alpha_{Kt} = \frac{D_M}{z m_n \cos\alpha_n} - \frac{z}{|z|}\eta + \operatorname{inv}\alpha_t = \frac{D_M}{d_b \cos\beta_b} - \frac{z}{|z|}\eta_b \tag{A.42}$$

Diameter d_K of the circle on which the centre of the measuring ball lies is found as

$$d_K = d\frac{\cos\alpha_t}{\cos\alpha_{Kt}} = \frac{d_b}{\cos\alpha_{Kt}} \tag{A.43}$$

The radial single-ball dimension is

$$M_{rK} = \frac{1}{2}\left(d_K + \frac{z}{|z|}D_M\right) \tag{A.44}$$

When calculating the test dimensions and determining d_M in order to check whether measuring balls or measuring cylinders contact the tooths flanks in the usable area, the actual values for the selected measuring ball and measuring cylinder diameters are to be used. Diameter d_M of the cylinder on which contact points P_L and P_R between the measuring ball and the two tooth flanks lie is given as

$$d_M = \frac{d_b}{\cos\alpha_{Mt}} = \frac{z m_n \cos\alpha_t}{\cos\beta\cos\alpha_{Mt}} \tag{A.45}$$

where the pressure angle at a point, α_{Mt}, on the circle of diameter d_M is produced by

$$\tan\alpha_{Mt} = \tan\alpha_{Kt} - \frac{D_M}{d_b}\cos\beta_b \tag{A.46}$$

A.6 Radial single-cylinder dimension

In the case of external gears and internal spur gears, measuring cylinders of diameter D_M can also be used instead of measuring balls. Equations (A.38) to (A.46) also apply to the radial single-cylinder dimension, M_{rZ}.

A.7 Diametral test dimension over balls

In the case of an external gear, the dimension over balls, M_{dK}, is the largest external dimension over two balls; while in the case of an internal gear it is the smallest internal dimension between two balls of diameter D_M and in contact with the flanks in two tooth spaces at the maximum possible separation from each other on the gear. The centres of the two measuring balls must be located in the same transverse section of the gear. For selection and calculation of measuring ball diameter D_M and diameter d_K of the ball centre circle, see A.5.

For an even number of teeth, see Figure A.12; test dimension M_{dK} is calculated using

$$M_{dK} = d_K + \frac{z}{|z|} D_M \tag{A.47}$$

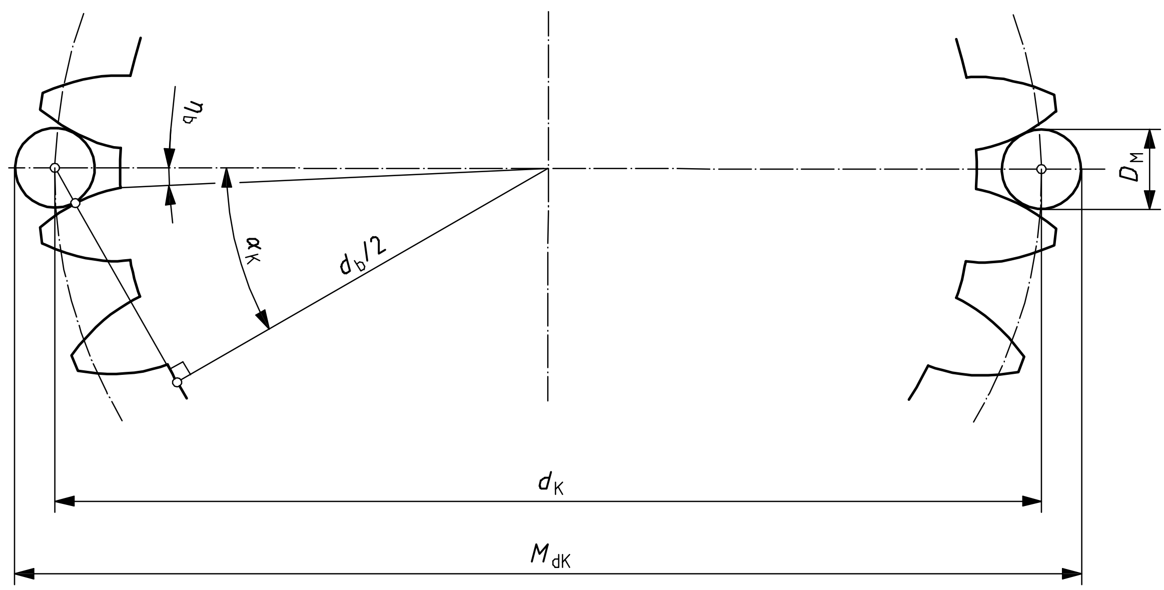


Figure A.12 — Dimension over balls M_{dK} in the case of external spur gear with even number of teeth

For an odd number of teeth, see Figures A.13 and A.14; test dimension M_{dK} is calculated using

$$M_{\mathrm{dK}} = d_{\mathrm{K}} \cos\frac{\pi}{2 * z} + \frac{z}{|z|} D_{\mathrm{M}} \tag{A.48}$$

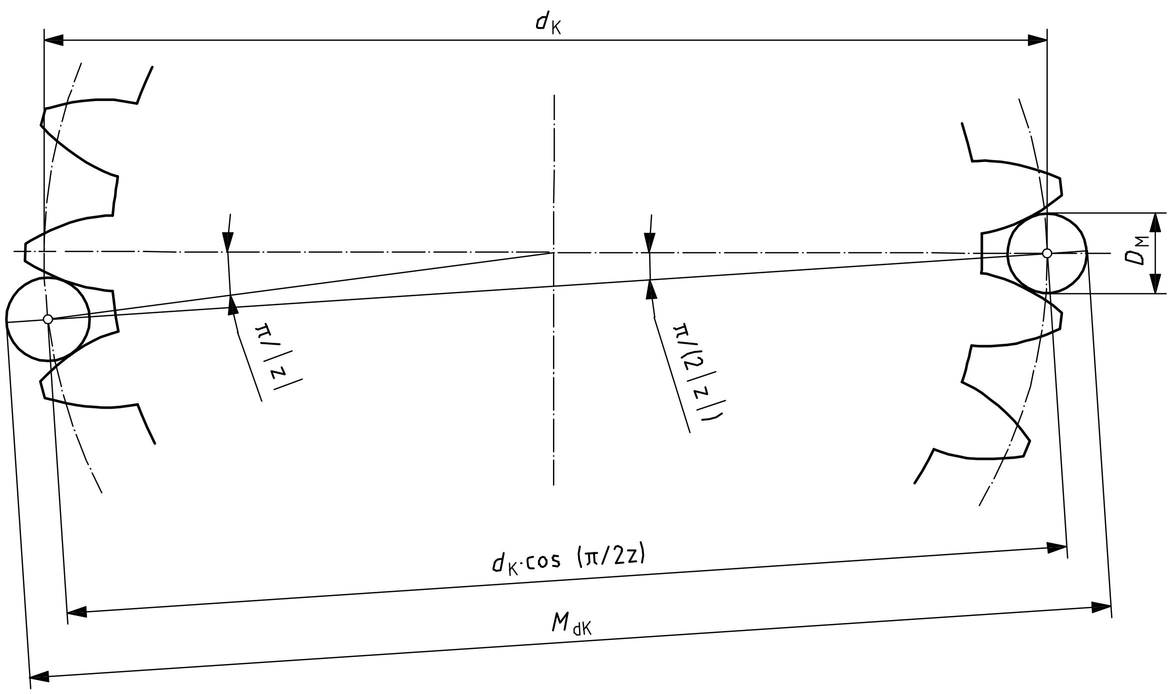


Figure A.13 — Dimension over balls M_{dK} in the case of external spur gear with odd number of teeth

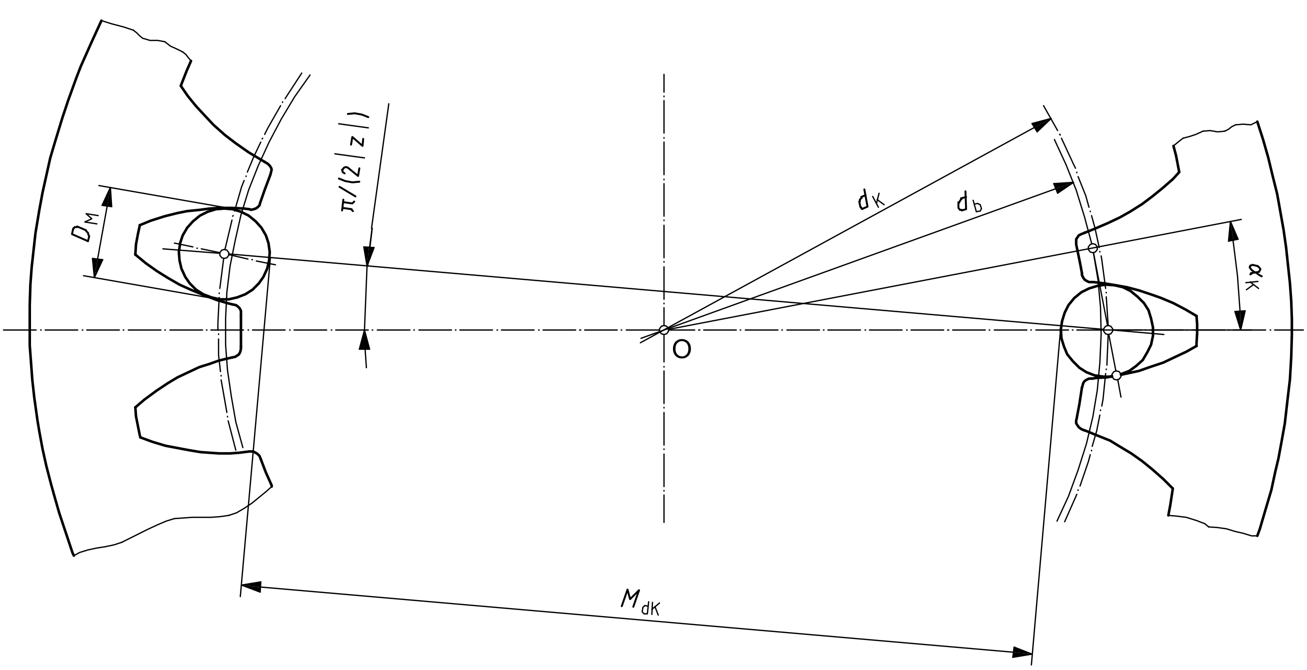


Figure A.14 — Dimension over balls M_{dK} in the case of internal spur gear with odd number of teeth

A.7.1 Dimension over cylinders

In the case of external gears and internal spur gears, measuring cylinders of diameter D_M can also be used instead of measuring balls. Equations (A.38) to (A.48) can be used when calculating the test dimension M_{dZ} for the dimension over cylinders in the case of an even number of teeth or in the case of spur gears. For helical gears with an odd number of teeth, special calculations are necessary.

A.7.2 Functional tooth thickness system — Measurement over balls — Correction for tooth deviations (adjustment for allowable tolerances)

If the pins or balls make contact near the mid-point of the active profile, the influence of flank deviations is minimized. The effect of allowable pitch deviation is usually much smaller than allowable runout, so it can be ignored except for gears with low numbers of teeth or other unusual cases.

If measurements are made from a ball or pin to the mounting diameter, the effect of runout is included and no correction is necessary. If the measurement is made over balls or pins on opposite sides of the gear, then the effect of runout should be calculated and the allowable measurements adjusted. This adjustment will decrease the allowable range of the measurement.

$$M_{\mathrm{dk.amax}} = M_{\mathrm{dk.max}} - \frac{F_\mathrm{r}}{2} \tag{A.49}$$

$$M_{\mathrm{dk.amin}} = M_{\mathrm{dk.min}} + \frac{F_\mathrm{r}}{2} \tag{A.50}$$

A.8 Double-flank centre distance

The double-flank centre distance, a_L, is the centre distance with zero-backlash mating of the cylindrical gear under test with a master gear. It can be used as a test dimension when checking the tooth thickness of the test object. For a test pair consisting of a gear with number of teeth z and a master gear with number of teeth z_L, profile shift coefficient x_L and known actual tooth thickness deviation E_snL, the values for the relevant test dimensions can be calculated using

$$a_\mathrm{L} = \left(|z| + \frac{z}{|z|} z_\mathrm{L}\right) \frac{m_\mathrm{n} \cos\alpha_\mathrm{t}}{2\cos\beta\cos\alpha_\mathrm{L}} \tag{A.51}$$

where the working transverse pressure angle a_L is found from

$$\mathrm{inv}\,\alpha_\mathrm{L} = \mathrm{inv}\,\alpha_\mathrm{t} + \frac{z}{|z|} \frac{2\tan\alpha_\mathrm{n}}{|z| + \frac{z}{|z|} z_\mathrm{L}} \left(x + x_\mathrm{L} + \frac{E_\mathrm{snL}}{2m_\mathrm{n}\tan\alpha_\mathrm{n}}\right) \tag{A.52}$$

A.9 Tooth thickness test dimensions relating to the tip circle

When producing external cylindrical gears using generating methods, if specially designed hobs and pinion-type cutters (topping gear-cutting tools) are used, it is possible to produce the bottom land, tooth flank and top land of the gear during the same working cycle. Nominally defined tip diameter d_a (see 4.5.3) will only be produced if its diameter is smaller than the pre-machined diameter and it happens to correspond to the tip diameter produced by the tool (root or edge chamfering). The diameter, d_aM, of the overcut tip cylinder is determined by the dedendum of tool rack tooth profile h_fP0. When using a hob, with x_Es according to Equations (120) and (121), d_aM is found as

$$d_\mathrm{aM} = d + 2x_\mathrm{Es} m_\mathrm{n} + 2h_\mathrm{fP0} \tag{A.53}$$

When using a pinion-type cutter, d_{aM} is determined by cutter root diameter d_{f0} as

$$d_{aM} = 2a_0 - d_{f0} \tag{A.54}$$

With these production variations, there is a relation between the dimension of overcut tip diameter d_{aM} and tooth thickness s_n. The deviation of overcut tip diameter E_{daM} can be converted to the actual tooth thickness deviation, E_{sn}:

$$E_{sn} = E_{daM} \tan\alpha_n \tag{A.55}$$

Bibliography

[1] ISO 54, *Cylindrical gears for general engineering and for heavy engineering — Modules*

[2] ISO 701:1998, *International gear notation — Symbols for geometrical data*

[3] ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

[4] ISO 4156, *Straight cylindrical involute splines — Metric module, side fit — Part 1: Generalities*

[5] ISO/TR 10064-1:1992, *Code of inspection practice — Part 1: Inspection of corresponding flanks of gear teeth*

[6] ISO/TR 10064-2:1996, *Code of inspection practice — Part 2: Inspection related to radial composite deviations, runout, tooth thickness and backlash*

[7] ISO/TR 10064-3:1996, *Code of inspection practice — Part 3: Recommendations relative to gear blanks, shaft centre distance and parallelism of axes*

[8] DIN 867:1986, *Basic rack tooth profiles for involute teeth of cylindrical gears for general engineering and heavy engineering*

[9] DIN 868:1976, *General definitions and specification factors for gears, gear pairs and gear trains*

[10] DIN 3967, *System of gear fits — Backlash, tooth thickness allowances, tooth thickness tolerances — Principles*

[11] ANSI/AGMA 2002-B88, *Tooth thickness specification and measurement*

BSI — British Standards Institution

BSI is the independent national body responsible for preparing British Standards. It presents the UK view on standards in Europe and at the international level. It is incorporated by Royal Charter.

Revisions

British Standards are updated by amendment or revision. Users of British Standards should make sure that they possess the latest amendments or editions.

It is the constant aim of BSI to improve the quality of our products and services. We would be grateful if anyone finding an inaccuracy or ambiguity while using this British Standard would inform the Secretary of the technical committee responsible, the identity of which can be found on the inside front cover. Tel: +44 (0)20 8996 9000. Fax: +44 (0)20 8996 7400.

BSI offers members an individual updating service called PLUS which ensures that subscribers automatically receive the latest editions of standards.

Buying standards

Orders for all BSI, international and foreign standards publications should be addressed to Customer Services. Tel: +44 (0)20 8996 9001. Fax: +44 (0)20 8996 7001. Email: orders@bsi-global.com. Standards are also available from the BSI website at http://www.bsi-global.com.

In response to orders for international standards, it is BSI policy to supply the BSI implementation of those that have been published as British Standards, unless otherwise requested.

Information on standards

BSI provides a wide range of information on national, European and international standards through its Library and its Technical Help to Exporters Service. Various BSI electronic information services are also available which give details on all its products and services. Contact the Information Centre. Tel: +44 (0)20 8996 7111. Fax: +44 (0)20 8996 7048. Email: info@bsi-global.com.

Subscribing members of BSI are kept up to date with standards developments and receive substantial discounts on the purchase price of standards. For details of these and other benefits contact Membership Administration. Tel: +44 (0)20 8996 7002. Fax: +44 (0)20 8996 7001. Email: membership@bsi-global.com.

Information regarding online access to British Standards via British Standards Online can be found at http://www.bsi-global.com/bsonline.

Further information about BSI is available on the BSI website at http://www.bsi-global.com.

Copyright

Details and advice can be obtained from the Copyright & Licensing Manager. Tel: +44 (0)20 8996 7070. Fax: +44 (0)20 8996 7553. Email: copyright@bsi-global.com.

BSI
389 Chiswick High Road
London
W4 4AL