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Corrosion of metals and alloys — Guidelines for applying statistics to analysis of corrosion data

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National foreword

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Corrosion des métaux et alliages — Lignes directrices pour l'application des statistiques à l'analyse des données de corrosion

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Foreword

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ISO 14802 was prepared by Technical Committee ISO/TC 156, *Corrosion of metals and alloys.*

Corrosion of metals and alloys — Guidelines for applying statistics to analysis of corrosion data

1 Scope

This International Standard gives guidance on some generally accepted methods of statistical analysis which are useful in the interpretation of corrosion test results. This International Standard does not cover detailed calculations and methods, but rather considers a range of approaches which have applications in corrosion testing. Only those statistical methods that have wide acceptance in corrosion testing have been considered in this International Standard.

2 Significance and use

Corrosion test results often show more scatter than many other types of tests because of a variety of factors, including the fact that minor impurities often play a decisive role in controlling corrosion rates. Statistical analysis can be very helpful in allowing investigators to interpret such results, especially in determining when test results differ from one another significantly. This can be a difficult task when a variety of materials are under test, but statistical methods provide a rational approach to this problem.

Modern data reduction programs in combination with computers have allowed sophisticated statistical analyses to be made on data sets with relative ease. This capability permits investigators to determine whether associations exist between different variables and, if so, to develop quantitative expressions relating the variables.

Statistical evaluation is a necessary step in the analysis of results from any procedure which provides quantitative information. This analysis allows confidence intervals to be estimated from the measured results.

3 Scatter of data

3.1 Distributions

When measuring values associated with the corrosion of metals, a variety of factors act to produce measured values that deviate from expected values for the conditions that are present. Usually the factors which contribute to the scatter of measured values act in a more or less random way so that the average of several values approximates the expected value better than a single measurement. The pattern in which data are scattered is called its distribution, and a variety of distributions such as the normal, log–normal, bi-nominal, Poisson distribution, and extreme-value distribution (including the Gumbel and Weibull distribution) are observed in corrosion work.

3.2 Histograms

A bar graph, called a histogram, may be used to display the scatter of data. A histogram is constructed by dividing the range of data values into equal intervals on the abscissa and then placing a bar over each interval of a height equal to the number of data points within that interval.

The number of intervals, k, can be calculated using the following equation:

$$k = 1 + (3{,}32)\log n \quad (1)$$

where

n is the total number of data.

3.3 Normal distribution

Many statistical techniques are based on the normal distribution. This distribution is bell-shaped and symmetrical. Use of analysis techniques developed for the normal distribution on data distributed in another manner can lead to grossly erroneous conclusions. Thus, before attempting data analysis, the data should either be verified as being scattered like a normal distribution or a transformation should be used to obtain a data set which is approximately normally distributed. Transformed data may be analysed statistically and the results transformed back to give the desired results, although the process of transforming the data back can create problems in terms of not having symmetrical confidence intervals.

3.4 Normal probability paper

3.4.1 If the histogram is not confirmatory in terms of the shape of the distribution, the data may be examined further to see if it is normally distributed by constructing a normal probability plot as follows (see Reference [2]).

3.4.2 It is easiest to construct a normal probability plot if normal probability paper is available. This paper has one linear axis and one axis which is arranged to reflect the shape of the cumulative area under the normal distribution. In practice, the “probability” axis has 0,5 or 50 % at the centre, a number approaching 0 % at one end, and a number approaching 1,0 or 100 % at the other end. The scale divisions are spaced close in the centre and wider at both ends. A normal probability plot may be constructed as follows with normal probability paper.

NOTE Data that plot approximately on a straight line on the probability plot may be considered to be normally distributed. Deviations from a normal distribution may be recognized by the presence of deviations from a straight line, usually most noticeable at the extreme ends of the data.

3.4.2.1 Rearrange the data in order of magnitude from the smallest to the largest and number them as 1,2, … i, … n, which are called the rank of the points.

3.4.2.2 In order to plot the ith ranked data on the normal probability paper, calculate the ”midpoint” plotting position, $F(x_i)$, defined by the following equation:

$$F\left(x_i\right) = \frac{100\left(i - ½\right)}{n} \qquad (2)$$

3.4.2.3 The data points [x_i, $F(x_i)$] can be plotted on the normal probability paper.

NOTE Occasionally, two or more identical values are obtained in a set of results. In this case, each point may be plotted, or a composite point may be located at the average of the plotting positions for all identical values.

It is recommended that probability plotting be used because it is a powerful tool for providing a better understanding of the population than traditional statements made only about the mean and standard deviation.

3.5 Other probability paper

If the histogram is not symmetrical and bell-shaped, or if the probability plot shows non-linearity, a transformation may be used to obtain a new, transformed data set that may be normally distributed. Although it is sometimes possible to guess the type of distribution by looking at the histogram, and thus determine the exact transformation to be used, it is usually just as easy to use a computer to calculate a number of different transformations and to check each for the normality of the transformed data. Some transformations based on known non-normal distributions, or that have been found to work in some situations, are listed as follows:

$y = \log x$ $y = \exp x$

$y = x^{0,5}$ $y = x^2$

$y = 1/x$ $y = \sin^{-1}(x/n)^{0,5}$

where

y is the transformed datum;

x is the original datum;

n is the number of data points.

Time to failure in stress corrosion cracking is often fitted with a log x transformation (see References [3][4]).

Once a set of transformed data is found that yields an approximately straight line on a probability plot, the statistical procedures of interest can be carried out on the transformed data. It is essential that results, such as predicted data values or confidence intervals, be transformed back using the reverse transformation.

3.6 Unknown distribution

3.6.1 General

If there are insufficient data points or if, for any other reason, the distribution type of the data cannot be determined, then two possibilities exist for analysis.

3.6.1.1 A distribution type may be hypothesized, based on the behaviour of similar types of data. If this distribution is not normal, a transformation may be sought which will normalize that particular distribution. See 3.5 for suggestions. Analysis may then be conducted on the transformed data.

3.6.1.2 Statistical analysis procedures that do not require any specific data distribution type, known as non-parametric methods, may be used to analyse the data. Non-parametric tests do not use the data as efficiently.

3.7 Extreme value analysis

If determining the probability of perforation by a pitting or cracking mechanism, the usual descriptive statistics for the normal distribution are not the most useful. Extreme value statistics should be used instead (see Reference [5]).

3.8 Significant digits

The proper number of significant digits should be used when reporting numerical results.

3.9 Propagation of variance

If a calculated value is a function of several independent variables and those variables have errors associated with them, the error of the calculated value can be estimated by a propagation of variance technique. See References [6][7] for details.

3.10 Mistakes

Mistakes when carrying out an experiment or in the calculations are not a characteristic of the population and can preclude statistical treatment of data or lead to erroneous conclusions if included in the analysis. Sometimes mistakes can be identified by statistical methods by recognizing that the probability of obtaining a particular result is very low. In this way, outlying observations can be identified and dealt with.

4 Central measures

4.1 Average

It is accepted practice to employ several independent (replicate) measurements of any experimental quantity to improve the estimate of precision and to reduce the variance of the average value. If it is assumed that the processes operating to create error in the measurement are random in nature and are as likely to overestimate the true unknown value as to underestimate it, then the average value is the best estimate of the unknown value in question. The average value is usually indicated by placing a bar over the symbol representing the measured variable and calculated by

$$\bar{x} = \sum \frac{x_i}{n} \tag{3}$$

NOTE In this International Standard, the term "mean" is reserved to describe a central measure of a population, while "average" refers to a sample.

4.2 Median

If processes operate to exaggerate the magnitude of the error, either in overestimating or underestimating the correct measurement, then the median value is usually a better estimate. The median value, x_m, is defined as the value in the middle of all data and can be determined from the m-th ranked data.

$$x_m = \begin{cases} x_{n/2} & \text{for an even number, } n\text{, of data points} \\ x_{(n+1)/2} & \text{for an odd number, } n\text{, of data points} \end{cases} \tag{4}$$

4.3 Which to use

If the processes operating to create error affect both the probability and magnitude of the error, then other approaches are required to find the best estimation procedure. A qualified statistician should be consulted in this case.

In corrosion testing, it is generally observed that average values are useful in characterizing corrosion rates. In cases of penetration from pitting and cracking, failure is often defined as the first through-penetration and average penetration rates or times are of little value. Extreme value analysis has been used in these instances.

When the average value is calculated and reported as the only result in experiments where several replicate runs were made, information on the scatter of data is lost.

5 Variability measures

5.1 General

Several measures of distribution variability are available, which can be useful in estimating confidence intervals and making predictions from the observed data. In the case of normal distribution, a number of procedures are available and can be handled by computer programs. These measures include the following: variance, standard deviation, and coefficient of variation. The range is a useful non-parametric estimate of variability and can be used with both normal and other distributions.

5.2 Variance

Variance, σ^2, may be estimated for an experimental data set of n observations by computing the sample estimated variance, S^2, assuming that all observations are subject to the same errors:

$$S^2 = \frac{\sum d^2}{(n-1)} = \frac{\sum (\bar{x} - x_i)^2}{(n-1)} \tag{5}$$

where

d is the difference between the average and the measured value;

$n - 1$ is the number of degrees of freedom available.

Variance is a useful measure because it is additive in systems that can be described by a normal distribution, but the dimensions of variance are the square of units. A procedure known as analysis of variance (ANOVA) has been developed for data sets involving several factors at different levels in order to estimate the effects of these factors.

5.3 Standard deviation

Standard deviation, σ, is defined as the square root of the variance. It has the property of having the same dimensions as the average value and the original measurements from which it was calculated, and is generally used to describe the scatter of the observations.

The standard deviation of an average is different from the standard deviation of a single measured value, but the two standard deviations are related as in the following equation:

$$S_{\overline{x}} = \frac{S}{\sqrt{n}} \tag{6}$$

where

n is the total number of measurements which were used to calculate the average value.

When reporting standard deviation calculations, it is important to note clearly whether the value reported is the standard deviation of the average or of a single value. In either case, the number of measurements should also be reported. The sample estimate of the standard deviation is S.

5.4 Coefficient of variation

The population coefficient of variation is defined as the standard deviation divided by the mean. The sample coefficient of variation may be calculated as S/x and is usually reported as a percentage. This measure of variability is particularly useful in cases where the size of the errors is proportional to the magnitude of the measured value, so that the coefficient of variation is approximately constant over a wide range of values.

5.5 Range

The range, w, is defined as the difference between the maximum, x_{max}, and minimum, x_{min}, values in a set of replicate data values. The range is non-parametric in nature, i.e. its calculation makes no assumption about the distribution of error.

$$w = x_{\text{max}} - x_{\text{min}} \tag{7}$$

In cases when small numbers of replicate values are involved and the data are normally distributed, the range can be used to estimate the standard deviation by the relationship:

$$S = \frac{w}{\sqrt{n}} \tag{8}$$

where

S is the estimated sample standard deviation;

w is the range;

n is the number of observations.

The range has the same dimensions as the standard deviation.

5.6 Precision

5.6.1 General

Precision is the closeness of agreement between randomly selected individual measurements or test results. The standard deviation of the error of measurement may be used as a measure of imprecision.

5.6.1.1 One aspect of precision concerns the ability of one investigator or laboratory to reproduce a measurement previously made at the same location with the same method. This aspect is sometimes called repeatability.

5.6.2.1 Another aspect of precision concerns the ability of different investigators and laboratories to reproduce a measurement. This aspect is sometimes called reproducibility.

5.7 Bias

5.7.1 General

Bias is the closeness of agreement between an observed value and an accepted reference value. When applied to individual observations, bias includes a combination of a random component and a component due to systematic error. Under these circumstances, accuracy contains elements of both precision and bias. Bias refers to the tendency of a measurement technique to consistently underestimate or overestimate. In cases where a specific quantity such as corrosion rate is being estimated, a quantitative bias may be determined.

5.7.1.1 Corrosion test methods which are intended to simulate service conditions, for example natural environments, often produce a different severity of corrosion and relative ranking of performance of materials, as compared to severity and ranking under the conditions which the test is simulating. This is particularly true for test procedures which produce damage rapidly as compared to the service experience. In such cases, it is important to establish the correspondence between results from the service environment and test results for the class of material in question. Bias in this case refers to the variation in the acceleration of corrosion for different materials.

5.7.1.2 Another type of corrosion test method measures a characteristic that is related to the tendency of a material to suffer a form of corrosion damage, for example pitting potential. Bias in this type of test refers to the inability of the test to properly rank the materials to which the test applies as compared to service results.

6 Statistical tests

6.1 Null hypothesis

Null-hypothesis statistical tests are usually carried out by postulating a hypothesis of the form: the distribution of data under test is not significantly different from some postulated distribution. It is necessary to establish a probability that will be acceptable for rejecting the null hypothesis. In experimental work, it is conventional to use probabilities of 0,05 or 0,01 to reject the null hypothesis.

6.1.1 Type I errors occur when the null hypothesis is rejected falsely. The probability of rejecting the null hypothesis falsely is described as the significance level and is often designated as α.

6.1.2 Type II errors occur when the null hypothesis is accepted falsely. If the significance level is set too low, the probability of a Type II error, β, becomes larger. When a value of α is set, the value of β is also set. With a fixed value of β, it is possible to decrease β only by increasing the sample size, assuming that no other factors can be changed to improve the test.

6.2 Degrees of freedom

The number of degrees of freedom of a statistical test refers to the number of independent measurements that are available for the calculation.

6.3 t-Test

The t-statistic may be written in the form:

$$t = \frac{|\bar{x} - \mu|}{S(\bar{x})} \tag{9}$$

where

$\bar{x}$ is the sample average;

μ is the population mean;

$S(\bar{x})$ is the estimated standard deviation of the sample average.

The t-distribution is usually tabulated in terms of significance levels and degrees of freedom.

6.3.1 The t-test may be used to test the null hypothesis:

$$m = \mu$$

For example, the value m is not significantly different from μ, the population mean. The t-test is then:

$$t = \frac{|\bar{x} - m|}{S(x)\sqrt{\frac{1}{n}}} \tag{10}$$

The calculated value of t may be compared to the value of t for the number of degrees of freedom, n, and the significance level.

6.3.2 The t-statistic may be used to obtain a confidence interval for an unknown value, for example a corrosion-rate value calculated from several independent measurements:

$$\left(\bar{x} - tS(\bar{x})\right) < \mu < \left(\bar{x} + tS(\bar{x})\right) \tag{11}$$

where $tS(\bar{x})$ represents the one-half width confidence interval associated with the significance level chosen.

6.3.3 The t-test is often used to test whether there is a significant difference between two sample averages. In this case, the expression becomes:

$$t = \frac{\left|\overline{x_1} - \overline{x_2}\right|}{S(x)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \tag{12}$$

where

$\overline{x_1}$ and $\overline{x_2}$ are the sample averages;

n_1 and n_2 are the number of measurements used in calculating x_1 and x_2;

$S(x)$ is the pooled estimate of the standard deviation from both sets of data.

i.e.

$$S(x)=\sqrt{\frac{(n_1-1)S^2(x_1)+(n_2-1)S^2(x_2)}{n_1+n_2-2}} \quad (13)$$

6.3.4 One-sided t-test. The t-function is symmetrical and can have negative as well as positive values. In the above examples, only absolute values of the differences were discussed. In some cases, a null hypothesis of the form:

$$\mu > m$$

or

$$\mu < m$$

may be desired. This is known as a one-sided t-test and the significance level associated with this t-value is half of that for a two-sided t.

6.4 F-test

The F-test is used to test whether the variance associated with a variable, x_1, is significantly different from a variance associated with a variable x_2. The F-statistic is then:

$$Fx_1x_2 = S^2(x_2) \quad (14)$$

The F-test is an important component in the analysis of variance used in experimental designs. Values of F are tabulated for significance levels and degrees of freedom for both variables. In cases where the data are not normally distributed, the F-test approach may falsely show a significant effect because of the non-normal distribution rather than an actual difference in variances being compared.

6.5 Correlation coefficient

The correlation coefficient, r, is a measure of a linear association between two random variables. Correlation coefficients vary between –1 and +1 and the closer they are to either –1 or +1, the better the correlation. The sign of the correlation coefficient simply indicates whether the correlation is positive (y increases with x) or negative (y decreases as x increases). The correlation coefficient, r, is given by:

$$r=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\left[\sum(x_i-\bar{x})\sum(y_i-\bar{y})\right]^{½}}=\frac{\sum x_i y_i - n\overline{xy}}{\left[\left(\sum x_i^2-n\bar{x}^2\right)\sum(y_i^2-n\bar{y}^2)\right]^{½}} \quad (15)$$

where

x_i are the observed values of random variable x;

y_i are the observed values of random variable y;

$\bar{x}$ is the average value of x;

$\bar{y}$ is the average value of y;

n is the number of observations.

Generally, r^2 values are preferred because they avoid the problem of signs and relate directly to variance. Values of r or r^2 have been tabulated for different significance levels and degrees of freedom. In general, it is desirable to report values of r or r^2 presenting correlations and regression analyses.

NOTE The procedure for calculating correlation coefficients does not require that the x and y variables be random and, consequently, some investigators have used the correlation coefficient as an indication of the goodness of fit of data in a regression analysis.

However, the significance test using the correlation coefficient requires that the x and y values be independent variables of a population measured on randomly selected samples.

6.6 Sign test

The sign test is a non-parametric test used in sets or in paired data to determine if one component of the pair is consistently larger than the other (see Reference [9]). In this test method, the values of the data pairs are compared, and, if the first entry is larger than the second, a plus sign is recorded. If the second term is larger, then a minus sign is recorded. If both are equal, then no sign is recorded. The total number of plus signs, P, and minus signs, N, is computed. Significance is determined by the following test:

$$|P - N| > k\sqrt{P + N} \quad (16)$$

where

k is a function of significance level as follows:

k	**Significance level**
1,6	0,10
2,0	0,05
2,6	0,01

The sign test does not depend on the magnitude of the difference and so can be used in cases where normal statistics would be inappropriate or impossible to apply.

6.7 Outside count

The outside count test is a useful non-parametric technique to evaluate whether the magnitude of one of two data sets of approximately the same number of values is significantly larger than the other. The details of the procedure may be found elsewhere (see Reference [9]).

7 Curve fitting — Method of least squares

7.1 Minimizing variance

It is often desirable to determine the best algebraic expression to fit a data set with the assumption that a normally distributed random error is operating. In this case, the best fit will be obtained when the condition of minimum variance between the measured value and the calculated value is obtained for the data set. The procedures used to determine equations of best fit are based on this concept. Software is available for computer calculation of regression equations, including linear, polynomial and multiple-variable regression equations.

7.2 Linear regression — 2 variables

Linear regression is used to fit data to a linear relationship of the following form:

$$y = mx + b \quad (17)$$

In this case, the best fit is given by:

$$m = \left(n\sum xy - \sum x \sum y\right) / \left[n\sum x^2 - (\sum x)^2\right] \quad (18)$$

$$b = \frac{1}{n}\left[\sum x - m\sum y\right] \tag{19}$$

where

y is the dependent variable;

x is the independent variable;

m is the slope of the estimated line;

b is the y intercept of the estimated line;

Σx is the sum of x values, etc.;

n is the number of observations of x and y.

This standard deviation of m and the standard error of the expression are often of interest and can be calculated easily (see References [6][8][10]). One problem with linear regression is that all the errors are assumed to be associated with the dependent variable, y, and this might not be a reasonable assumption. A variation of the linear regression approach is available, assuming that the fitting equation passes through the origin. In this case, only one adjustable parameter will result from the fit. It is possible to use statistical tests, such as the F-test, to compare the goodness of fit between this approach and the two adjustable parameter fits described above.

7.3 Polynomial regression

Polynomial regression analysis is used to fit data to a polynomial equation of the following form:

$$y = a + bx + cx^2 + dx^3 + \ldots \tag{20}$$

where

a, b, c, d	are the adjustable constants used to fit the data set;
x	is the observed independent variable;
y	is the observed dependent variable.

The equations required to carry out the calculation of the best-fit constants are complex and best handled by a computer. It is usually desirable to run a series of expressions and compute the residual variance for each expression to find the simplest expression fitting the data.

7.4 Multiple regression

Multiple-regression analysis is used when data sets involving more than one independent variable are encountered. An expression of the following form is desired in a multiple linear regression:

$$y = a + b_1x_1 + b_2x_2 + b_4x_4 + \ldots \tag{21}$$

where

a, b_1, b_2, b_4, ...	are the adjustable constants used to obtain the best fit of the data set;
x_1, x_2, x_4, ...	are the observed independent variables;
y	is the observed dependent variable.

Because of the complexity of this problem, it is generally handled with the help of a computer. One strategy is to compute the value of all the “b’s,” together with standard deviation for each “b”. It is usually necessary to run several regression analyses, dropping variables, to establish the relative importance of the independent variables under consideration.

8 Analysis of variance

8.1 Comparison of effects

Analysis of variance is useful to determine the effect of a number of variables on a measured value when a small number of discrete levels of each independent variable is studied (see References [6], [8], [10], [11] and [12]). This is best handled by using a factorial or similar experimental design to establish the magnitude of the effects associated with each variable and the magnitude of the interactions between the variables.

8.2 The two-level factorial design

8.2.1 The two-level factorial design experiment is an excellent method for determining which variables have an effect on the outcome.

8.2.2 Each time an additional variable is to be studied, twice as many experiments are required to complete the two-level factorial design. When many variables are involved, the number of experiments becomes prohibitive.

8.2.3 Fractional replication can be used to reduce the amount of testing. When this is done, the amount of information that can be obtained from the experiment is also reduced.

9 Extreme value statistics

9.1 Scope of this clause

9.1.1 Extreme-value statistics provide a powerful method for analysing localized corrosion data, and especially for estimating pit depth. The maximum pit depth is more important than the average pit depth because perforation is caused by the deepest pit (see References [5][13][16][18]).

9.1.2 The normal (Gausian), Poisson, binomial, exponential and log–normal distributions are often observed in engineering data. The largest or the smallest values from these original distributions make another group of distributions, called extreme-value distributions. There are three types of extreme value distributions which are asymptotic limiting forms of the original distribution for large samples. Type I for the largest value is called the Gumbel, or doubly exponential, distribution and is often observed for the distribution of the deepest pits. Type III for the smallest value is called the Weibull distribution which is widely used for analysing failure life data in the field of reliability engineering. The procedure for estimating the parameters of the Gumbel distribution presented here can also be applied to the Weibull distribution.

9.1.3 This method allows for the estimation of the parameters of the Gumbel distribution. The maximum pit-depth perforation probability for a large area of given thickness can be estimated from the Gumbel distribution observed for small areas.

9.2 Gumbel distribution and its probability paper

When the pit depths obey an exponential type of distribution, as is typical, the maximum pit depths obey the Gumbel distribution (see Reference [13]). The cumulative distribution function, $F(x)$, for a random variable x with a Gumbel distribution is given by a doubly exponential function:

$$F(x) = \exp\left\{-\exp\left[-(x-\lambda)/\alpha\right]\right\} \tag{22}$$

where

λ is the location parameter;

α is the scale parameter.

A normalized variable, y, is defined as:

$$y = (x-\lambda)/\alpha \tag{23}$$

Then $F(y)$ becomes:

$$F(y) = \exp\left[-\exp(-y)\right] \tag{24}$$

Figure 1 is a Gumbel probability plot showing x and y scales along with $F(y)$. The value of x corresponding to $y = 0$ or $F(y) = 0{,}368$ corresponds to λ, while the slope corresponds to α. The scale T on the right is the return period defined by:

$$F(y) = 1 - 1/T \tag{25}$$

If $T \geq 18$, T can be expressed as:

$$y = \ln(T) \tag{26}$$

When one evaluates the maximum pit depth over the total area, A, from sampling blocks of small area, a, T is given by:

$$T = A/a \tag{27}$$

T is then an index to indicate the area effect.

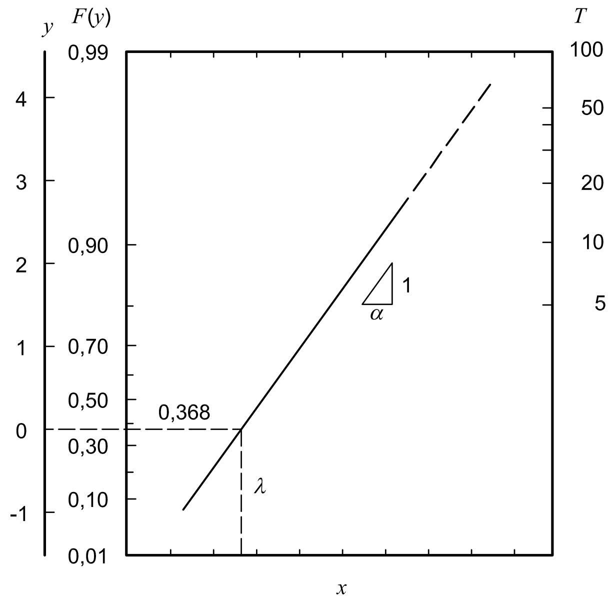


Figure 1 — Gumbel probability paper and its coordinates

9.3 Estimation of distribution parameters

9.3.1 Data collection

The surface area A is the total surface area in the equipment in question. It is essential that the corrosion environment over A be uniform in order that random samples within a are statistically homogeneous. If there are any doubts, the area should be split up into suitable blocks to provide homogeneity within each. With A defined as above, N blocks, each with area a, are selected at random for sampling. Appropriate measurements are made on the N blocks to determine the maximum pit depth for each block. Often, the maximum depths cannot be determined for all blocks, for example the depths in certain blocks may be less than the limit of measurement. The actual number of data sets n can thus be less than N. The measured maximum depths x_1, x_2, …, x_n are ranked in the order from the largest, x_1, to the smallest, x_n. The cumulative probability, $F(y)$, is derived from the average-rank method by

$$F\left(y\right)=1-\frac{i}{\left(N+1\right)} \tag{28}$$

A linearity test is then applied to the x and $F(y)$ or y plots.

9.3.2 Distribution parameter estimation

9.3.2.1 Probability paper evaluation

The best-fit line is drawn and the intercept on the x-axis is taken as λ and the slope as α.

9.3.2.2 Use of a linear unbiased estimator

Such a linear unbiased estimator may be of the type proposed by Lieblein[14] and White[15] to define the best-fit line in such plots:

$$\alpha=\sum b_i\left(N,n\right)x_i \tag{29}$$

$$\alpha = \sum a_i\left(N,n\right)x_i \tag{30}$$

One can derive $a_i(N,n)$ and b_i (N,n) here from the MVLUE coefficient tables[16], a part of which is presented in Table 1. One can next estimate λ and α from the equations shown above.

The extreme value, x_{max}, is the maximum pit depth expected for the total area, A, and may be determined from the intercept between the extrapolated best fit line and the return period, T:

$$x_{max} = \lambda + \alpha \ln\left(T\right) \tag{31}$$

9.3.3 Probability distribution of x_{max} and perforation probability

The variable x_{max} is a random variable, and its associated cumulative distribution function, $F_{max}(x)$, is determined from T specimens, in which a Gumbel distribution $F(x)$ applies for x:

$$F_{max}\left(x\right) = \exp\left\{-\exp\left[-\left(x-\left[\lambda+\alpha\ln T\right]\right)/\alpha\right]\right\} \tag{32}$$

The Gumbel distribution for x_{max} is identical to that of $F(x)$, while the location parameter is greater than the λ in $F(x)$ by α ln$T$. If the wall thickness is d, the perforation probability P is:

$$P = 1-\exp\left\{-\exp\left[d-\left(\lambda+\alpha\ln T\right)/\alpha\right]\right\} \tag{33}$$

Figure 2 shows the relationship between $F(x)$ and F_{max} $(x)\cdot x_{max}$ for the total can be calculated as:

$$x_{max} = \lambda + \alpha \ln T \tag{34}$$

This equation can be used to obtain P.

9.3.4 Estimating the deviation of x_{max} from the distribution

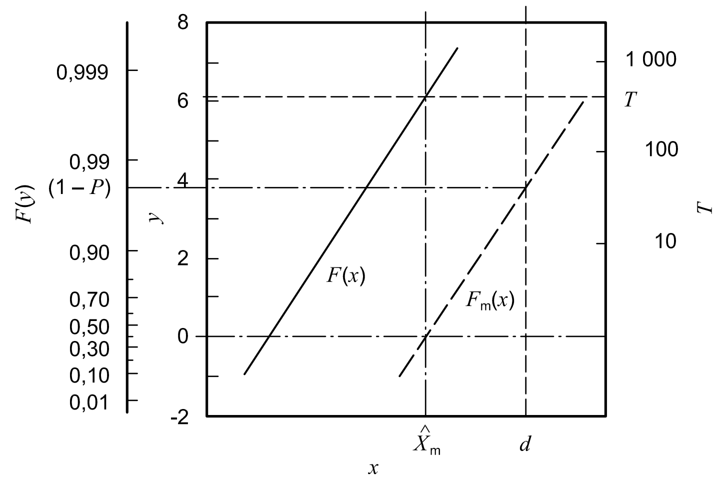


Figure 2 — Estimation of perforation probability, P, from Gumbel plot

The parameters in $F(x)$ are determined from the fitted line for the sample data, but there is no guarantee that those data give a distribution representing that for the population, so it is essential that the probable error be estimated. In the linear unbiased estimator method, the error variance, $V(x)$, is given by:

$$V\left(x\right) = \alpha^2\left[A\left(N,n\right)y^2 + B\left(N,n\right)y + C\left(N,n\right)\right] \tag{35}$$

in which $A(N,n)$, $B(N,n)$ and $C(N,n)$ are derived from the MVLUE tables(Reference [16]), as shown in Table 2.

9.4 Report

The report should contain the following:

— equipment service period and conditions, alloy chemistry, specifications, heat treatment, wall thickness, and other material specifications;

— type of localized corrosion;

— method of measurement of corrosion depth, measurement positions, total area, A, number of sampling blocks, N, area of each block, a, and return period, $T = A/a$;

— Gumbel probability plot;

— estimated λ ,α , and x_{max};

— estimated perforation probability, P.

9.5 Other topics

9.5.1 Sample size

It is essential to select a and N carefully. While small a and N make measurement easy, the error margin is increased. The equations shows that y increases with T, while Table 2 shows that the coefficients $A(N,n)$, $B(N,n)$, and $C(N,n)$ tend to decrease as N increases. To improve the accuracy in estimating x_{max}, the variables, N or a, should be increased in order to reduce $y = \ln T$. However, in practice it is best to make N small and y large on the premise that the extreme value method enables x_{max} to be estimated from small samples. The optimum sample size is thus a compromise determined by the accuracy of x_{max}. There are no clear-cut guidelines for optimizing N and T. The following may be applicable: the standard error deviation, σ , for x_{max}, is the square root of $V(x)$ defined by the equation above. It is a function of α, so it can not be evaluated unless α or α/λ are known. Given that $\alpha/\lambda = 0{,}3$ is typical of pitting corrosion for mild steel in fresh water and soil (see Reference [17]), a relationship between N and T can be derived which gives $\lambda = m\sigma$, for $m = 1$, 2, and 3 from the above equations with the coefficients taken from the MVLUE tables. Figure 3 shows the relationship between N and T. To maintain $\lambda = 3\sigma$. $N = 30$ is required for $T = 1\ 000$.

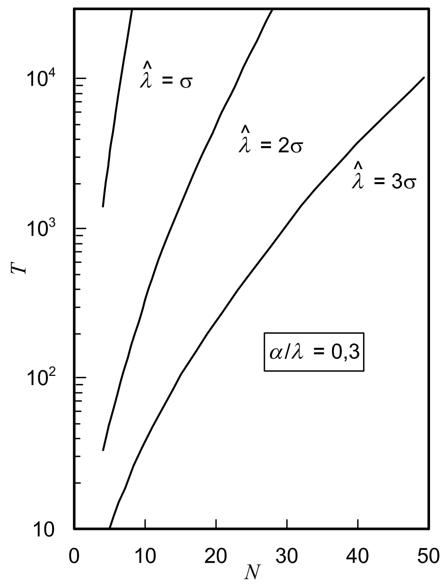


Figure 3 — Selection of optimum N and T

Figure 3 indicates that increasing N rather than reducing T is more effective at minimizing the error for a given total sampling error aN. Reducing a can improve the precision in estimating x_{max}. However, an excessively small a causes a deviation from the Gumbel distribution, which applies for the maximum value distribution only if the number of pits in a given block, k, is fairly large. There are no guidelines for the optimum k to give a Gumbel distribution for the maximum values. While mathematical rigour would require a fairly large k, $k = 20$ is sufficient to give results of practical validity.

9.5.2 Truncated sample

The maximum pit depths cannot necessarily be determined for all the N blocks. These truncated data can be analysed in the same manner as the case where $N = n$.

9.5.3 Other methods for estimating distribution parameters

Various methods have been proposed to estimate the distribution parameters, but the linear unbiased estimator method appears to be best. The MVLUE coefficient tables as devised by Lieblein (Reference [14]) and White (Reference [15]) can be used, as coefficients up to $N = 45$ have been published (see Reference [16]). For data greater than $N = 30$, the maximum likelihood method can be used.

Table 1 — Part of the MVLUE coefficient table from $N = 2$ to $N = 23$ (Reference [16])

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
2	2	1	0,083 63	0,721 35	6	6	1	0,048 87	0,145 81
2	2	2	0,916 37	−0,721 35	6	6	2	0,083 52	0,149 53
3	2	1	−0,377 70	0,822 10	6	6	3	0,121 05	0,126 72
3	2	2	1,377 70	−0,822 10	6	6	4	0,165 62	0,073 20
3	3	1	0,087 97	0,374 73	6	6	5	0,225 49	−0,035 99
3	3	2	0,255 71	0,255 82	6	6	6	0,355 45	−0,459 27
3	3	3	0,656 32	−0,630 54	7	2	1	−1,338 27	0,926 74
4	2	1	−0,706 32	0,869 02	7	2	2	2,338 27	−0,926 74
4	2	2	1,706 32	−0,869 02	7	3	1	−0,403 61	0,454 96
4	3	1	−0,080 11	0,414 40	7	3	2	−0,301 21	0,405 57
4	3	2	0,060 43	0,325 86	7	3	3	1,704 82	−0,860 53
4	3	3	1,019 67	−0,740 26	7	4	1	−0,146 33	0,294 04
4	4	1	0,071 38	0,248 80	7	4	2	−0,094 07	0,276 02
4	4	2	0,153 68	0,223 92	7	4	3	−0,007 10	0,210 16
4	4	3	0,263 94	0,085 90	7	4	4	1,247 49	−0,780 22
4	4	4	0,511 00	−0,558 62	7	5	1	−0,039 26	0,211 02
5	2	1	−0,959 86	0,896 28	7	5	2	−0,004 36	0,206 46
5	2	2	1,959 86	−0,896 28	7	5	3	0,045 83	0,169 12
5	3	1	−0,210 12	0,434 34	7	5	4	0,113 42	0,099 18
5	3	2	−0,086 02	0,364 25	7	5	5	0,884 37	−0,685 78
5	3	3	1,296 14	−0,798 59	7	6	1	0,013 73	0,158 69
5	4	1	−0,015 38	0,273 03	7	6	2	0,041 80	0,160 87
5	4	2	0,051 96	0,249 94	7	6	3	0,075 68	0,139 64
5	4	3	0,152 08	0,149 11	7	6	4	0,117 59	0,095 07
5	4	4	0,811 34	−0,672 09	7	6	5	0,172 12	0,017 65
5	5	1	0,058 35	0,184 48	7	6	6	0,579 09	−0,571 91
5	5	2	0,108 82	0,181 66	7	7	1	0,041 84	0,120 14
5	5	3	0,167 61	0,130 46	7	7	2	0,067 33	0,125 86
5	5	4	0,246 28	0,006 53	7	7	3	0,093 75	0,114 87
5	5	5	0,418 93	−0,503 13	7	7	4	0,123 22	0,087 34
6	2	1	−1,165 57	0,914 14	7	7	5	0,158 59	0,036 19
6	2	2	2,165 57	−0,914 14	7	7	6	0,206 26	−0,060 70
6	3	1	−0,315 40	0,446 60	7	7	7	0,309 01	−0,423 70
6	3	2	−0,203 43	0,388 65	8	2	1	−1,486 92	0,936 11
6	3	3	1,518 83	−0,835 25	8	2	2	2,486 92	−0,936 11
6	4	1	−0,086 54	0,285 87	8	3	1	−0,479 40	0,461 03
6	4	2	−0,028 06	0,265 48	8	3	2	−0,384 82	0,418 02
6	4	3	0,064 95	0,185 87	8	3	3	1,864 22	−0,879 04
6	4	4	1,049 65	−0,737 21	8	4	1	−0,197 72	0,299 76
6	5	1	0,005 73	0,201 54	8	4	2	−0,150 20	0,283 69
6	5	2	0,046 58	0,197 27	8	4	3	−0,068 49	0,227 49
6	5	3	0,100 24	0,153 61	8	4	4	1,416 41	−0,810 93
6	5	4	0,172 29	0,064 59	8	5	1	−0,078 14	0,217 25
6	5	5	0,675 16	−0,617 01	8	5	2	−0,047 42	0,212 76

Table 1 (*continued*)

N	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$	*N*	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$
8	5	3	−0,000 09	0,180 28	9	7	1	−0,005 77	0,136 37
8	5	4	0,063 71	0,122 52	9	7	2	0,011 79	0,140 04
8	5	5	1,061 93	−0,732 80	9	7	3	0,033 59	0,129 72
8	6	1	−0,017 24	0,166 13	9	7	4	0,059 97	0,107 65
8	6	2	0,006 53	0,167 49	9	7	5	0,092 19	0,072 32
8	6	3	0,038 02	0,148 30	9	7	6	0,132 52	0,019 37
8	6	4	0,077 99	0,110 53	9	7	7	0,675 71	−0,605 47
8	6	5	0,129 20	0,049 99	9	8	1	0,017 80	0,110 19
8	6	6	0,765 49	−0,642 44	9	8	2	0,033 97	0,115 41
8	7	1	0,016 81	0,130 29	9	8	3	0,051 58	0,109 74
8	7	2	0,037 59	0,134 79	9	8	4	0,071 36	0,095 00
8	7	3	0,061 23	0,123 87	9	8	5	0,094 27	0,070 01
8	7	4	0,088 87	0,099 08	9	8	6	0,121 84	0,031 23
8	7	5	0,122 43	0,057 12	9	8	7	0,156 91	−0,029 22
8	7	6	0,165 46	−0,010 87	9	8	8	0,452 28	−0,502 36
8	7	7	0,507 60	−0,534 29	9	9	1	0,032 29	0,088 39
8	8	1	0,036 49	0,101 94	9	9	2	0,047 96	0,094 37
8	8	2	0,056 13	0,108 07	9	9	3	0,063 40	0,091 97
8	8	3	0,075 90	0,102 73	9	9	4	0,079 57	0,082 65
8	8	4	0,097 14	0,087 16	9	9	5	0,097 22	0,065 57
8	8	5	0,121 18	0,058 93	9	9	6	0,117 36	0,037 98
8	8	6	0,150 20	0,011 12	9	9	7	0,141 79	−0,006 49
8	8	7	0,189 43	−0,075 77	9	9	8	0,174 88	−0,085 20
8	8	8	0,273 54	−0,394 19	9	9	9	0,245 54	−0,369 24
9	2	1	−1,617 28	0,943 35	10	2	1	−1,733 28	0,949 12
9	2	2	2,617 28	−0,943 35	10	2	2	2,733 28	−0,949 12
9	3	1	−0,545 76	0,465 65	10	3	1	−0,604 74	0,469 29
9	3	2	−0,457 73	0,427 55	10	3	2	−0,522 29	0,435 09
9	3	3	2,003 49	−0,893 20	10	3	3	2,127 03	−0,904 38
9	4	1	−0,242 71	0,304 00	10	4	1	−0,282 68	0,307 28
9	4	2	−0,198 98	0,289 53	10	4	2	−0,242 07	0,294 14
9	4	3	−0,121 90	0,240 49	10	4	3	−0,169 08	0,250 63
9	4	4	1,563 59	−0,834 03	10	4	4	1,693 83	−0,852 05
9	5	1	−0,112 27	0,221 70	10	5	1	−0,142 61	0,225 07
9	5	2	−0,084 66	0,217 41	10	5	2	−0,117 46	0,221 00
9	5	3	−0,039 83	0,188 72	10	5	3	−0,074 84	0,195 32
9	5	4	0,020 61	0,139 44	10	5	4	−0,017 40	0,152 31
9	5	5	1,216 15	−0,767 26	10	5	5	1,352 32	−0,793 70
9	6	1	−0,044 61	0,171 17	10	6	1	−0,069 03	0,174 85
9	6	2	−0,023 91	0,172 03	10	6	2	−0,050 64	0,175 39
9	6	3	0,005 73	0,154 69	10	6	3	−0,022 57	0,159 64
9	6	4	0,043 96	0,121 99	10	6	4	0,014 07	0,130 83
9	6	5	0,092 46	0,072 08	10	6	5	0,060 19	0,088 27
9	6	6	0,926 37	−0,691 96	10	6	6	1,067 99	−0,728 98

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
10	7	1	−0,026 08	0,140 58	11	5	2	−0,146 74	0,223 86
10	7	2	−0,010 88	0,143 66	11	5	3	−0,106 11	0,200 64
10	7	3	0,009 52	0,134 03	11	5	4	−0,051 37	0,162 46
10	7	4	0,034 84	0,114 26	11	5	5	1,474 11	−0,814 67
10	7	5	0,065 84	0,083 76	11	6	1	−0,091 03	0,177 67
10	7	6	0,104 05	0,040 19	11	6	2	−0,074 46	0,178 00
10	7	7	0,822 71	−0,656 48	11	6	3	−0,047 74	0,163 60
10	8	1	0,000 63	0,115 27	11	6	4	−0,012 58	0,137 85
10	8	2	0,014 32	0,119 79	11	6	5	0,031 42	0,100 72
10	8	3	0,030 46	0,114 20	11	6	6	1,194 40	−0,757 84
10	8	4	0,049 26	0,100 60	11	7	1	−0,044 45	0,143 70
10	8	5	0,071 37	0,078 52	11	7	2	−0,031 06	0,146 35
10	8	6	0,097 90	0,046 03	11	7	3	−0,011 80	0,137 39
10	8	7	0,130 65	−0,000 88	11	7	4	0,012 56	0,119 52
10	8	8	0,605 42	−0,573 52	11	7	5	0,042 42	0,092 70
10	9	1	0,017 83	0,095 25	11	7	6	0,078 82	0,055 58
10	9	2	0,030 86	0,100 55	11	7	7	0,953 51	−0,695 25
10	9	3	0,044 59	0,097 76	11	8	1	−0,015 02	0,118 86
10	9	4	0,059 57	0,088 60	11	8	2	−0,003 20	0,122 84
10	9	5	0,076 34	0,072 75	11	8	3	0,011 81	0,117 46
10	9	6	0,095 68	0,048 61	11	8	4	0,029 82	0,104 95
10	9	7	0,118 84	0,012 86	11	8	5	0,051 23	0,085 26
10	9	8	0,148 14	−0,041 52	11	8	6	0,076 81	0,057 28
10	9	9	0,408 16	−0,474 86	11	8	7	0,107 83	0,018 74
10	10	1	0,028 93	0,077 94	11	8	8	0,740 72	−0,625 38
10	10	2	0,041 75	0,083 55	11	9	1	0,004 37	0,099 58
10	10	3	0,054 19	0,082 77	11	9	2	0,015 44	0,104 31
10	10	4	0,066 99	0,077 02	11	9	3	0,027 96	0,101 40
10	10	5	0,080 62	0,066 06	11	9	4	0,042 12	0,092 72
10	10	6	0,095 64	0,048 67	11	9	5	0,058 29	0,078 23
10	10	7	0,112 87	0,022 18	11	9	6	0,077 06	0,057 03
10	10	8	0,133 85	−0,019 21	11	9	7	0,099 33	0,027 19
10	10	9	0,162 31	−0,091 16	11	9	8	0,126 60	−0,014 87
10	10	10	0,222 87	−0,347 83	11	9	9	0,548 82	−0,545 59
11	2	1	−1,837 73	0,953 82	11	10	1	0,017 43	0,083 75
11	2	2	2,837 73	−0,953 82	11	10	2	0,028 19	0,088 85
11	3	1	−0,657 79	0,472 22	11	10	3	0,039 26	0,087 71
11	3	2	−0,580 16	0,441 21	11	10	4	0,051 07	0,081 88
11	3	3	2,237 95	−0,913 43	11	10	5	0,063 96	0,071 37
11	4	1	−0,318 61	0,309 89	11	10	6	0,078 39	0,055 42
11	4	2	−0,280 64	0,297 86	11	10	7	0,094 99	0,032 46
11	4	3	−0,211 28	0,258 76	11	10	8	0,114 80	−0,000 58
11	4	4	1,810 53	−0,866 51	11	10	9	0,139 78	−0,049 98
11	5	1	−0,169 89	0,227 71	11	10	10	0,372 13	−0,450 88

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
11	11	1	0,026 18	0,069 64	12	8	7	0,087 47	0,033 34
11	11	2	0,036 89	0,074 83	12	8	8	0,862 18	−0,665 30
11	11	3	0,047 16	0,074 98	12	9	1	−0,008 06	0,102 70
11	11	4	0,057 58	0,071 38	12	9	2	0,001 51	0,106 95
11	11	5	0,068 49	0,064 07	12	9	3	0,013 11	0,104 05
11	11	6	0,080 22	0,052 46	12	9	4	0,026 63	0,095 89
11	11	7	0,093 23	0,035 28	12	9	5	0,042 29	0,082 64
11	11	8	0,108 23	0,010 03	12	9	6	0,060 53	0,063 79
11	11	9	0,126 52	−0,028 60	12	9	7	0,082 03	0,038 16
11	11	10	0,151 39	−0,094 87	12	9	8	0,107 82	0,003 66
11	11	11	0,204 12	−0,329 21	12	9	9	0,674 16	−0,597 84
12	2	1	−1,932 68	0,957 73	12	10	1	0,006 60	0,087 49
12	2	2	2,932 68	−0,957 73	12	10	2	0,015 79	0,092 13
12	3	1	−0,705 97	0,474 65	12	10	3	0,025 87	0,090 82
12	3	2	−0,632 56	0,446 27	12	10	4	0,036 98	0,085 15
12	3	3	2,338 53	−0,920 92	12	10	5	0,049 37	0,075 30
12	4	1	−0,351 22	0,312 02	12	10	6	0,063 38	0,060 83
12	4	2	−0,315 51	0,300 94	12	10	7	0,079 52	0,040 76
12	4	3	−0,249 42	0,265 43	12	10	8	0,098 55	0,013 28
12	4	4	1,916 15	−0,878 38	12	10	9	0,121 70	−0,024 83
12	5	1	−0,194 66	0,229 83	12	10	10	0,502 24	−0,520 92
12	5	2	−0,173 17	0,226 21	12	11	1	0,016 83	0,074 64
12	5	3	−0,134 33	0,205 01	12	11	2	0,025 90	0,079 44
12	5	4	−0,082 02	0,170 68	12	11	3	0,035 06	0,079 27
12	5	5	1,584 19	−0,831 72	12	11	4	0,044 65	0,075 52
12	6	1	−0,111 02	0,179 91	12	11	5	0,054 92	0,068 32
12	6	2	−0,095 92	0,180 10	12	11	6	0,066 17	0,057 34
12	6	3	−0,070 41	0,166 85	12	11	7	0,078 74	0,041 74
12	6	4	−0,036 61	0,143 57	12	11	8	0,093 19	0,020 01
12	6	5	0,005 47	0,110 61	12	11	9	0,110 40	−0,010 64
12	6	6	1,308 49	−0,781 03	12	11	10	0,132 03	−0,055 91
12	7	1	−0,061 18	0,146 14	12	11	11	0,342 13	−0,429 74
12	7	2	−0,049 22	0,148 45	12	12	1	0,023 89	0,062 91
12	7	3	−0,030 93	0,140 10	12	12	2	0,032 98	0,067 67
12	7	4	−0,007 46	0,123 82	12	12	3	0,041 63	0,068 36
12	7	5	0,021 33	0,099 86	12	12	4	0,050 30	0,066 12
12	7	6	0,056 13	0,067 49	12	12	5	0,059 27	0,061 11
12	7	7	1,071 33	−0,725 86	12	12	6	0,068 75	0,053 05
12	8	1	−0,029 35	0,121 57	12	12	7	0,079 02	0,041 28
12	8	2	−0,018 98	0,125 12	12	12	8	0,090 46	0,024 55
12	8	3	−0,004 87	0,119 99	12	12	9	0,103 67	0,000 53
12	8	4	0,012 46	0,108 45	12	12	10	0,119 84	−0,035 66
12	8	5	0,033 20	0,090 71	12	12	11	0,141 83	−0,097 09
12	8	6	0,057 90	0,066 13	12	12	12	0,188 36	−0,312 84

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
13	2	1	−2,019 70	0,961 03	13	10	1	−0,003 50	0,090 22
13	2	2	3,019 70	−0,961 03	13	10	2	0,004 47	0,094 47
13	3	1	−0,750 08	0,476 68	13	10	3	0,013 76	0,093 07
13	3	2	−0,680 40	0,450 53	13	10	4	0,024 33	0,087 64
13	3	3	2,430 49	−0,927 21	13	10	5	0,036 32	0,078 46
13	4	1	−0,381 06	0,313 80	13	10	6	0,049 97	0,065 29
13	4	2	−0,347 33	0,303 52	13	10	7	0,065 69	0,047 48
13	4	3	−0,284 17	0,270 99	13	10	8	0,084 05	0,023 92
13	4	4	2,012 56	−0,888 31	13	10	9	0,105 91	−0,007 28
13	5	1	−0,217 33	0,231 58	13	10	10	0,618 99	−0,573 26
13	5	2	−0,197 25	0,228 16	13	11	1	0,007 93	0,077 91
13	5	3	−0,160 03	0,208 66	13	11	2	0,015 73	0,082 34
13	5	4	−0,109 93	0,177 47	13	11	3	0,024 05	0,081 99
13	5	5	1,684 54	−0,845 87	13	11	4	0,033 05	0,078 26
13	6	1	−0,129 33	0,181 73	13	11	5	0,042 89	0,071 38
13	6	2	−0,115 45	0,181 82	13	11	6	0,053 79	0,061 17
13	6	3	−0,091 00	0,169 56	13	11	7	0,066 06	0,047 09
13	6	4	−0,058 48	0,148 32	13	11	8	0,080 11	0,028 16
13	6	5	−0,018 12	0,118 67	13	11	9	0,096 61	0,002 74
13	6	6	1,412 38	−0,800 10	13	11	10	0,116 58	−0,032 09
13	7	1	−0,076 52	0,148 09	13	11	11	0,463 21	−0,498 94
13	7	2	−0,065 72	0,150 14	13	12	1	0,016 15	0,067 25
13	7	3	−0,048 27	0,142 34	13	12	2	0,023 91	0,071 72
13	7	4	−0,025 62	0,127 39	13	12	3	0,031 64	0,072 15
13	7	5	0,002 16	0,105 75	13	12	4	0,039 62	0,069 73
13	7	6	0,035 52	0,077 02	13	12	5	0,048 04	0,064 70
13	7	7	1,178 47	−0,750 73	13	12	6	0,057 09	0,056 89
13	8	1	−0,042 52	0,123 71	13	12	7	0,067 00	0,045 86
13	8	2	−0,033 31	0,126 90	13	12	8	0,078 09	0,030 79
13	8	3	−0,019 96	0,122 04	13	12	9	0,090 82	0,010 25
13	8	4	−0,003 25	0,111 35	13	12	10	0,105 96	−0,018 31
13	8	5	0,016 86	0,095 21	13	12	11	0,124 94	−0,060 09
13	8	6	0,040 73	0,073 28	13	12	12	0,316 75	−0,410 94
13	8	7	0,069 03	0,044 69	13	13	1	0,021 97	0,057 33
13	8	8	0,972 42	−0,697 19	13	13	2	0,029 79	0,061 70
13	9	1	−0,019 54	0,105 08	13	13	3	0,037 18	0,062 70
13	9	2	−0,011 16	0,108 95	13	13	4	0,044 53	0,061 36
13	9	3	−0,000 31	0,106 10	13	13	5	0,052 05	0,057 86
13	9	4	0,012 67	0,098 44	13	13	6	0,059 90	0,052 10
13	9	5	0,027 87	0,086 28	13	13	7	0,068 26	0,043 71
13	9	6	0,045 61	0,069 33	13	13	8	0,077 37	0,032 01
13	9	7	0,066 39	0,046 84	13	13	9	0,087 54	0,015 84
13	9	8	0,090 97	0,017 51	13	13	10	0,099 32	−0,007 00
13	9	9	0,787 50	−0,638 53	13	13	11	0,113 76	−0,041 01

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
13	13	12	0,133 42	−0,098 28	14	9	8	0,075 64	0,028 33
13	13	13	0,174 92	−0,298 31	14	9	9	0,891 04	−0,671 32
14	2	1	−2,099 99	0,963 85	14	10	1	−0,012 90	0,092 34
14	2	2	3,099 99	−0,963 85	14	10	2	−0,005 92	0,096 25
14	3	1	−0,790 76	0,478 41	14	10	3	0,002 74	0,094 82
14	3	2	−0,724 40	0,454 16	14	10	4	0,012 86	0,089 65
14	3	3	2,515 16	−0,932 57	14	10	5	0,024 49	0,081 08
14	4	1	−0,408 57	0,315 30	14	10	6	0,037 81	0,069 01
14	4	2	−0,376 57	0,305 72	14	10	7	0,053 12	0,053 02
14	4	3	−0,316 06	0,275 71	14	10	8	0,070 87	0,032 37
14	4	4	2,101 19	−0,896 73	14	10	9	0,091 70	0,005 87
14	5	1	−0,238 22	0,233 05	14	10	10	0,725 24	−0,614 41
14	5	2	−0,219 35	0,229 81	14	11	1	−0,000 43	0,080 33
14	5	3	−0,183 61	0,211 77	14	11	2	0,006 35	0,084 43
14	5	4	−0,135 51	0,183 18	14	11	3	0,014 01	0,083 97
14	5	5	1,776 68	−0,857 81	14	11	4	0,022 54	0,080 32
14	6	1	−0,146 20	0,183 25	14	11	5	0,032 04	0,073 81
14	6	2	−0,133 34	0,183 26	14	11	6	0,042 66	0,064 34
14	6	3	−0,109 87	0,171 85	14	11	7	0,054 65	0,051 55
14	6	4	−0,078 52	0,152 33	14	11	8	0,068 35	0,034 80
14	6	5	−0,039 75	0,125 38	14	11	9	0,084 25	0,013 04
14	6	6	1,507 68	−0,816 07	14	11	10	0,103 08	−0,015 43
14	7	1	−0,090 68	0,149 70	14	11	11	0,572 50	−0,551 16
14	7	2	−0,080 83	0,151 53	14	12	1	0,008 72	0,070 14
14	7	3	−0,064 13	0,144 22	14	12	2	0,015 43	0,074 31
14	7	4	−0,042 25	0,130 41	14	12	3	0,022 45	0,074 56
14	7	5	−0,015 41	0,110 68	14	12	4	0,029 92	0,072 11
14	7	6	0,016 65	0,084 83	14	12	5	0,037 96	0,067 22
14	7	7	1,276 65	−0,771 37	14	12	6	0,046 71	0,059 82
14	8	1	−0,054 68	0,125 44	14	12	7	0,056 38	0,049 62
14	8	2	−0,046 43	0,128 35	14	12	8	0,067 22	0,036 06
14	8	3	−0,033 73	0,123 73	14	12	9	0,079 60	0,018 23
14	8	4	−0,017 58	0,113 79	14	12	10	0,094 07	−0,005 40
14	8	5	0,001 92	0,099 00	14	12	11	0,111 53	−0,037 47
14	8	6	0,025 01	0,079 20	14	12	12	0,430 00	−0,479 20
14	8	7	0,052 19	0,053 82	14	13	1	0,015 45	0,061 15
14	8	8	1,073 31	−0,723 33	14	13	2	0,022 19	0,065 29
14	9	1	−0,030 18	0,106 98	14	13	3	0,028 81	0,066 07
14	9	2	−0,022 77	0,110 52	14	13	4	0,035 57	0,064 56
14	9	3	−0,012 53	0,107 76	14	13	5	0,042 62	0,060 99
14	9	4	−0,000 03	0,100 57	14	13	6	0,050 09	0,055 30
14	9	5	0,014 74	0,089 34	14	13	7	0,058 15	0,047 26
14	9	6	0,031 99	0,073 94	14	13	8	0,066 98	0,036 38
14	9	7	0,052 10	0,053 89	14	13	9	0,076 85	0,021 90

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
14	13	10	0,088 18	0,002 47	15	7	7	1,367 22	−0,788 79
14	13	11	0,101 64	−0,024 25	15	8	1	−0,065 99	0,126 88
14	13	12	0,118 49	−0,063 05	15	8	2	−0,058 52	0,129 55
14	13	13	0,294 99	−0,394 08	15	8	3	−0,046 39	0,125 16
14	14	1	0,020 32	0,052 64	15	8	4	−0,030 76	0,115 88
14	14	2	0,027 13	0,056 65	15	8	5	−0,011 84	0,102 22
14	14	3	0,033 53	0,057 83	15	8	6	0,010 53	0,084 16
14	14	4	0,039 85	0,057 08	15	8	7	0,036 68	0,061 34
14	14	5	0,046 26	0,054 62	15	8	8	1,166 29	−0,745 19
14	14	6	0,052 89	0,050 42	15	9	1	−0,040 07	0,108 53
14	14	7	0,059 87	0,044 26	15	9	2	−0,033 46	0,111 80
14	14	8	0,067 33	0,035 77	15	9	3	−0,023 76	0,109 13
14	14	9	0,075 48	0,024 28	15	9	4	−0,011 68	0,102 37
14	14	10	0,084 62	0,008 69	15	9	5	0,002 68	0,091 94
14	14	11	0,095 22	−0,013 04	15	9	6	0,019 46	0,077 84
14	14	12	0,108 23	−0,045 12	15	9	7	0,038 94	0,059 74
14	14	13	0,125 97	−0,098 77	15	9	8	0,061 57	0,037 07
14	14	14	0,163 31	−0,285 32	15	9	9	0,986 32	−0,698 42
15	2	1	−2,174 50	0,966 28	15	10	1	−0,021 67	0,094 04
15	2	2	3,174 50	−0,966 28	15	10	2	−0,015 51	0,097 66
15	3	1	−0,828 49	0,479 90	15	10	3	−0,007 37	0,096 22
15	3	2	−0,765 10	0,457 30	15	10	4	0,002 36	0,091 31
15	3	3	2,593 59	−0,937 20	15	10	5	0,013 66	0,083 30
15	4	1	−0,434 06	0,316 58	15	10	6	0,026 65	0,072 17
15	4	2	−0,403 60	0,307 62	15	10	7	0,041 58	0,057 67
15	4	3	−0,345 51	0,279 77	15	10	8	0,058 77	0,039 27
15	4	4	2,183 17	−0,903 97	15	10	9	0,078 72	0,016 19
15	5	1	−0,257 57	0,234 30	15	10	10	0,822 81	−0,647 84
15	5	2	−0,239 76	0,231 23	15	11	1	−0,008 26	0,082 23
15	5	3	−0,205 38	0,214 44	15	11	2	−0,002 32	0,086 04
15	5	4	−0,159 11	0,188 05	15	11	3	0,004 81	0,085 50
15	5	5	1,861 83	−0,868 02	15	11	4	0,012 95	0,081 98
15	6	1	−0,161 84	0,184 53	15	11	5	0,022 14	0,075 82
15	6	2	−0,149 86	0,184 49	15	11	6	0,032 51	0,067 01
15	6	3	−0,127 27	0,173 83	15	11	7	0,044 24	0,055 32
15	6	4	−0,097 01	0,155 76	15	11	8	0,057 60	0,040 30
15	6	5	−0,059 68	0,131 05	15	11	9	0,072 97	0,021 26
15	6	6	1,595 66	−0,829 65	15	11	10	0,090 89	−0,002 91
15	7	1	−0,103 81	0,151 04	15	11	11	0,672 47	−0,592 56
15	7	2	−0,094 77	0,152 71	15	12	1	0,001 69	0,072 31
15	7	3	−0,078 74	0,145 83	15	12	2	0,007 55	0,076 20
15	7	4	−0,057 57	0,133 00	15	12	3	0,014 01	0,076 33
15	7	5	−0,031 61	0,114 86	15	12	4	0,021 06	0,073 89
15	7	6	−0,000 73	0,091 36	15	12	5	0,028 80	0,069 19

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
15	12	6	0,037 32	0,062 22	15	15	11	0,081 77	0,002 77
15	12	7	0,046 78	0,052 79	15	15	12	0,091 38	−0,017 93
15	12	8	0,057 39	0,040 51	15	15	13	0,103 20	−0,048 29
15	12	9	0,069 46	0,024 76	15	15	14	0,119 31	−0,098 77
15	12	10	0,083 39	0,004 57	15	15	15	0,153 18	−0,273 61
15	12	11	0,099 82	−0,021 61	16	2	1	−2,244 00	0,968 41
15	12	12	0,532 75	−0,531 15	16	2	2	3,244 00	−0,968 41
15	13	1	0,009 15	0,063 73	16	3	1	−0,863 66	0,481 20
15	13	2	0,015 02	0,067 62	16	3	2	−0,802 96	0,460 03
15	13	3	0,021 04	0,068 25	16	3	3	2,666 61	−0,941 23
15	13	4	0,027 35	0,066 66	16	4	1	−0,457 82	0,317 70
15	13	5	0,034 06	0,063 14	16	4	2	−0,428 74	0,309 27
15	13	6	0,041 28	0,057 67	16	4	3	−0,372 85	0,283 29
15	13	7	0,049 12	0,050 10	16	4	4	2,259 41	−0,910 25
15	13	8	0,057 76	0,040 08	16	5	1	−0,275 60	0,235 38
15	13	9	0,067 42	0,027 09	16	5	2	−0,258 73	0,232 46
15	13	10	0,078 43	0,010 27	16	5	3	−0,225 59	0,216 76
15	13	11	0,091 26	−0,011 78	16	5	4	−0,181 00	0,192 25
15	13	12	0,106 70	−0,041 50	16	5	5	1,940 92	−0,876 85
15	13	13	0,401 41	−0,461 34	16	6	1	−0,176 41	0,185 62
15	14	1	0,014 76	0,056 03	16	6	2	−0,165 18	0,185 54
15	14	2	0,020 67	0,059 86	16	6	3	−0,143 42	0,175 54
15	14	3	0,026 42	0,060 86	16	6	4	−0,114 16	0,158 72
15	14	4	0,032 24	0,059 96	16	6	5	−0,078 15	0,135 91
15	14	5	0,038 24	0,057 40	16	6	6	1,677 32	−0,841 34
15	14	6	0,044 54	0,053 19	16	7	1	−0,116 04	0,152 19
15	14	7	0,051 25	0,047 18	16	7	2	−0,107 69	0,153 70
15	14	8	0,058 48	0,039 10	16	7	3	−0,092 27	0,147 22
15	14	9	0,066 41	0,028 49	16	7	4	−0,071 76	0,135 24
15	14	10	0,075 28	0,014 60	16	7	5	−0,046 63	0,118 45
15	14	11	0,085 45	−0,003 80	16	7	6	−0,016 84	0,096 90
15	14	12	0,097 53	−0,028 90	16	7	7	1,451 23	−0,803 71
15	14	13	0,112 62	−0,065 12	16	8	1	−0,076 53	0,128 09
15	14	14	0,276 12	−0,378 85	16	8	2	−0,069 73	0,130 55
15	15	1	0,018 89	0,048 65	16	8	3	−0,058 10	0,126 38
15	15	2	0,024 89	0,052 33	16	8	4	−0,042 96	0,117 68
15	15	3	0,030 48	0,053 60	16	8	5	−0,024 58	0,105 00
15	15	4	0,035 98	0,053 27	16	8	6	−0,002 90	0,088 40
15	15	5	0,041 53	0,051 53	16	8	7	0,022 32	0,067 66
15	15	6	0,047 22	0,048 41	16	8	8	1,252 47	−0,763 77
15	15	7	0,053 14	0,043 80	16	9	1	−0,049 32	0,109 83
15	15	8	0,059 40	0,037 45	16	9	2	−0,043 37	0,112 86
15	15	9	0,066 13	0,028 99	16	9	3	−0,034 13	0,110 29
15	15	10	0,073 49	0,017 78	16	9	4	−0,022 44	0,103 91

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
16	9	5	−0,008 47	0,094 19	16	13	7	0,040 91	0,052 49
16	9	6	0,007 86	0,081 18	16	13	8	0,049 39	0,043 25
16	9	7	0,026 76	0,064 68	16	13	9	0,058 86	0,031 51
16	9	8	0,048 55	0,044 29	16	13	10	0,069 58	0,016 68
16	9	9	1,074 55	−0,721 24	16	13	11	0,081 92	−0,002 13
16	10	1	−0,029 88	0,095 45	16	13	12	0,096 42	−0,026 37
16	10	2	−0,024 40	0,098 82	16	13	13	0,498 36	−0,512 93
16	10	3	−0,016 70	0,097 39	16	14	1	0,009 36	0,058 34
16	10	4	−0,007 32	0,092 72	16	14	2	0,014 54	0,061 97
16	10	5	0,003 68	0,085 20	16	14	3	0,019 77	0,062 83
16	10	6	0,016 36	0,074 89	16	14	4	0,025 20	0,061 85
16	10	7	0,030 90	0,061 62	16	14	5	0,030 90	0,059 29
16	10	8	0,047 57	0,045 02	16	14	6	0,036 97	0,055 19
16	10	9	0,066 75	0,024 55	16	14	7	0,043 48	0,049 45
16	10	10	0,913 04	−0,675 66	16	14	8	0,050 55	0,041 88
16	11	1	−0,015 62	0,083 77	16	14	9	0,058 33	0,032 14
16	11	2	−0,010 37	0,087 33	16	14	10	0,067 01	0,019 72
16	11	3	−0,003 69	0,086 74	16	14	11	0,076 88	0,003 82
16	11	4	0,004 12	0,083 35	16	14	12	0,088 36	−0,016 85
16	11	5	0,013 05	0,077 53	16	14	13	0,102 15	−0,044 54
16	11	6	0,023 17	0,069 31	16	14	14	0,376 51	−0,445 10
16	11	7	0,034 64	0,058 55	16	15	1	0,014 10	0,051 68
16	11	8	0,047 67	0,044 93	16	15	2	0,019 34	0,055 22
16	11	9	0,062 57	0,027 98	16	15	3	0,024 38	0,056 34
16	11	10	0,079 73	0,006 94	16	15	4	0,029 45	0,055 87
16	11	11	0,764 73	−0,626 43	16	15	5	0,034 64	0,054 03
16	12	1	−0,004 94	0,074 02	16	15	6	0,040 04	0,050 87
16	12	2	0,000 21	0,077 67	16	15	7	0,045 72	0,046 30
16	12	3	0,006 20	0,077 70	16	15	8	0,051 77	0,040 16
16	12	4	0,012 92	0,075 31	16	15	9	0,058 31	0,032 16
16	12	5	0,020 40	0,070 82	16	15	10	0,065 49	0,021 86
16	12	6	0,028 71	0,064 26	16	15	11	0,073 52	0,008 55
16	12	7	0,037 97	0,055 51	16	15	12	0,082 72	−0,008 91
16	12	8	0,048 37	0,044 30	16	15	13	0,093 64	−0,032 58
16	12	9	0,060 14	0,030 19	16	15	14	0,107 28	−0,066 55
16	12	10	0,073 61	0,012 54	16	15	15	0,259 60	−0,365 01
16	12	11	0,089 24	−0,009 67	16	16	1	0,017 65	0,045 21
16	12	12	0,627 16	−0,572 64	16	16	2	0,022 97	0,048 60
16	13	1	0,003 16	0,065 67	16	16	3	0,027 91	0,049 91
16	13	2	0,008 30	0,069 34	16	16	4	0,032 75	0,049 86
16	13	3	0,013 84	0,069 84	16	16	5	0,037 60	0,048 65
16	13	4	0,019 79	0,068 24	16	16	6	0,042 54	0,046 31
16	13	5	0,026 23	0,064 81	16	16	7	0,047 65	0,042 79
16	13	6	0,033 24	0,059 60	16	16	8	0,052 99	0,037 94

Table 1 (*continued*)

N	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$	*N*	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$
16	16	9	0,058 66	0,031 53	17	9	2	−0,052 60	0,113 76
16	16	10	0,064 77	0,023 17	17	9	3	−0,043 77	0,111 29
16	16	11	0,071 48	0,012 27	17	9	4	−0,032 44	0,105 25
16	16	12	0,079 02	−0,002 17	17	9	5	−0,018 83	0,096 15
16	16	13	0,087 80	−0,021 93	17	9	6	−0,002 93	0,084 08
16	16	14	0,098 60	−0,050 73	17	9	7	0,015 41	0,068 91
16	16	15	0,113 35	−0,098 41	17	9	8	0,036 45	0,050 38
16	16	16	0,144 27	−0,262 99	17	9	9	1,156 69	−0,740 75
17	2	1	−2,309 11	0,970 29	17	10	1	−0,037 59	0,096 63
17	2	2	3,309 11	−0,970 29	17	10	2	−0,032 68	0,099 78
17	3	1	−0,896 59	0,482 34	17	10	3	−0,025 37	0,098 38
17	3	2	−0,838 33	0,462 44	17	10	4	−0,016 29	0,093 93
17	3	3	2,734 92	−0,944 77	17	10	5	−0,005 59	0,086 86
17	4	1	−0,480 06	0,318 67	17	10	6	0,006 79	0,077 26
17	4	2	−0,452 22	0,310 72	17	10	7	0,020 97	0,065 01
17	4	3	−0,398 35	0,286 37	17	10	8	0,037 15	0,049 89
17	4	4	2,330 63	−0,915 77	17	10	9	0,055 65	0,031 48
17	5	1	−0,292 48	0,236 32	17	10	10	0,996 96	−0,699 23
17	5	2	−0,276 43	0,233 54	17	11	1	−0,022 54	0,085 05
17	5	3	−0,244 44	0,218 79	17	11	2	−0,017 87	0,088 38
17	5	4	−0,201 40	0,195 92	17	11	3	−0,011 58	0,087 76
17	5	5	2,014 74	−0,884 57	17	11	4	−0,004 06	0,084 51
17	6	1	−0,190 05	0,186 57	17	11	5	0,004 62	0,079 00
17	6	2	−0,179 48	0,186 46	17	11	6	0,014 52	0,071 31
17	6	3	−0,158 46	0,177 04	17	11	7	0,025 74	0,061 34
17	6	4	−0,130 14	0,161 32	17	11	8	0,038 45	0,048 89
17	6	5	−0,095 36	0,140 13	17	11	9	0,052 90	0,033 59
17	6	6	1,753 49	−0,851 52	17	11	10	0,069 41	0,014 95
17	7	1	−0,127 49	0,153 18	17	11	11	0,850 40	−0,654 78
17	7	2	−0,119 74	0,154 57	17	12	1	−0,011 20	0,075 41
17	7	3	−0,104 87	0,148 43	17	12	2	−0,006 64	0,078 85
17	7	4	−0,084 98	0,137 21	17	12	3	−0,001 04	0,078 81
17	7	5	−0,060 62	0,121 57	17	12	4	0,005 39	0,076 49
17	7	6	−0,031 83	0,101 68	17	12	5	0,012 64	0,072 20
17	7	7	1,529 53	−0,816 63	17	12	6	0,020 76	0,066 01
17	8	1	−0,086 40	0,129 13	17	12	7	0,029 84	0,057 86
17	8	2	−0,080 17	0,131 42	17	12	8	0,040 02	0,047 55
17	8	3	−0,069 00	0,127 44	17	12	9	0,051 51	0,034 78
17	8	4	−0,054 30	0,119 26	17	12	10	0,064 55	0,019 07
17	8	5	−0,036 44	0,107 43	17	12	11	0,079 52	−0,000 24
17	8	6	−0,015 40	0,092 07	17	12	12	0,714 65	−0,606 79
17	8	7	0,008 96	0,073 04	17	13	1	−0,002 52	0,067 22
17	8	8	1,332 75	−0,779 78	17	13	2	0,002 02	0,070 68
17	9	1	−0,057 98	0,110 94	17	13	3	0,007 15	0,071 09

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
17	13	4	0,012 81	0,069 49	17	16	6	0,036 31	0,048 52
17	13	5	0,019 02	0,066 18	17	16	7	0,041 20	0,045 00
17	13	6	0,025 84	0,061 22	17	16	8	0,046 35	0,040 24
17	13	7	0,033 35	0,054 55	17	16	9	0,051 86	0,034 08
17	13	8	0,041 68	0,045 99	17	16	10	0,057 82	0,026 22
17	13	9	0,050 96	0,035 29	17	16	11	0,064 36	0,016 24
17	13	10	0,061 42	0,022 03	17	16	12	0,071 67	0,003 49
17	13	11	0,073 34	0,005 58	17	16	13	0,080 06	−0,013 12
17	13	12	0,087 12	−0,014 95	17	16	14	0,090 00	−0,035 50
17	13	13	0,587 81	−0,554 38	17	16	15	0,102 40	−0,067 50
17	14	1	0,004 20	0,060 11	17	16	16	0,245 01	−0,352 37
17	14	2	0,008 76	0,063 54	17	17	1	0,016 56	0,042 21
17	14	3	0,013 57	0,064 29	17	17	2	0,021 31	0,045 35
17	14	4	0,018 68	0,063 27	17	17	3	0,025 71	0,046 66
17	14	5	0,024 14	0,060 75	17	17	4	0,030 00	0,046 81
17	14	6	0,030 02	0,056 79	17	17	5	0,034 29	0,045 98
17	14	7	0,036 38	0,051 34	17	17	6	0,038 63	0,044 21
17	14	8	0,043 32	0,044 26	17	17	7	0,043 08	0,041 49
17	14	9	0,050 96	0,035 30	17	17	8	0,047 71	0,037 72
17	14	10	0,059 47	0,024 09	17	17	9	0,052 57	0,032 76
17	14	11	0,069 08	0,010 10	17	17	10	0,057 74	0,026 35
17	14	12	0,080 11	−0,007 52	17	17	11	0,063 33	0,018 15
17	14	13	0,093 03	−0,030 07	17	17	12	0,069 48	0,007 57
17	14	14	0,468 30	−0,496 25	17	17	13	0,076 40	−0,006 32
17	15	1	0,009 43	0,053 77	17	17	14	0,084 47	−0,025 22
17	15	2	0,014 03	0,057 14	17	17	15	0,094 39	−0,052 62
17	15	3	0,018 63	0,058 14	17	17	16	0,107 96	−0,097 79
17	15	4	0,023 36	0,057 59	17	17	17	0,136 36	−0,253 31
17	15	5	0,028 28	0,055 73	18	2	1	−2,370 35	0,971 96
17	15	6	0,033 46	0,052 61	18	2	2	3,370 35	−0,971 96
17	15	7	0,038 97	0,048 19	18	3	1	−0,927 55	0,483 35
17	15	8	0,044 88	0,042 36	18	3	2	−0,871 52	0,464 57
17	15	9	0,051 30	0,034 88	18	3	3	2,799 07	−0,947 91
17	15	10	0,058 35	0,025 46	18	4	1	−0,500 95	0,319 53
17	15	11	0,066 20	0,013 59	18	4	2	−0,474 25	0,312 01
17	15	12	0,075 12	−0,001 46	18	4	3	−0,422 24	0,289 10
17	15	13	0,085 48	−0,020 91	18	4	4	2,397 44	−0,920 64
17	15	14	0,097 89	−0,046 84	18	5	1	−0,308 33	0,237 14
17	15	15	0,354 62	−0,430 24	18	5	2	−0,293 03	0,234 50
17	16	1	0,013 48	0,047 93	18	5	3	−0,262 09	0,220 60
17	16	2	0,018 15	0,051 22	18	5	4	−0,220 49	0,199 15
17	16	3	0,022 63	0,052 40	18	5	5	2,083 93	−0,891 38
17	16	4	0,027 09	0,052 23	18	6	1	−0,202 86	0,187 40
17	16	5	0,031 62	0,050 92	18	6	2	−0,192 87	0,187 27

Table 1 (*continued*)

N	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$	*N*	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$
18	6	3	−0,172 55	0,178 37	18	11	7	0,017 43	0,063 78
18	6	4	−0,145 10	0,163 60	18	11	8	0,029 84	0,052 30
18	6	5	−0,111 45	0,143 81	18	11	9	0,043 88	0,038 37
18	6	6	1,824 82	−0,860 46	18	11	10	0,059 80	0,021 60
18	7	1	−0,138 25	0,154 03	18	11	11	0,930 38	−0,678 94
18	7	2	−0,131 02	0,155 32	18	12	1	−0,017 11	0,076 58
18	7	3	−0,116 65	0,149 50	18	12	2	−0,013 05	0,079 82
18	7	4	−0,097 35	0,138 94	18	12	3	−0,007 79	0,079 74
18	7	5	−0,073 70	0,124 32	18	12	4	−0,001 61	0,077 48
18	7	6	−0,045 85	0,105 83	18	12	5	0,005 43	0,073 38
18	7	7	1,602 82	−0,827 93	18	12	6	0,013 36	0,067 54
18	8	1	−0,095 68	0,130 03	18	12	7	0,022 26	0,059 92
18	8	2	−0,089 94	0,132 16	18	12	8	0,032 24	0,050 38
18	8	3	−0,079 18	0,128 37	18	12	9	0,043 46	0,038 70
18	8	4	−0,064 91	0,120 65	18	12	10	0,056 12	0,024 55
18	8	5	−0,047 53	0,109 56	18	12	11	0,070 51	0,007 43
18	8	6	−0,027 10	0,095 26	18	12	12	0,796 19	−0,635 54
18	8	7	−0,003 52	0,077 69	18	13	1	−0,007 90	0,068 50
18	8	8	1,407 85	−0,793 72	18	13	2	−0,003 88	0,071 77
18	9	1	−0,066 13	0,111 89	18	13	3	0,000 91	0,072 11
18	9	2	−0,061 24	0,114 53	18	13	4	0,006 31	0,070 53
18	9	3	−0,052 77	0,112 15	18	13	5	0,012 32	0,067 34
18	9	4	−0,041 77	0,106 43	18	13	6	0,018 98	0,062 62
18	9	5	−0,028 51	0,097 88	18	13	7	0,026 35	0,056 34
18	9	6	−0,013 02	0,086 61	18	13	8	0,034 52	0,048 39
18	9	7	0,004 80	0,072 58	18	13	9	0,043 62	0,038 56
18	9	8	0,025 16	0,055 58	18	13	10	0,053 83	0,026 55
18	9	9	1,233 48	−0,757 65	18	13	11	0,065 38	0,011 93
18	10	1	−0,044 85	0,097 64	18	13	12	0,078 57	−0,005 92
18	10	2	−0,040 42	0,100 60	18	13	13	0,670 99	−0,588 70
18	10	3	−0,033 45	0,099 23	18	14	1	−0,000 72	0,061 52
18	10	4	−0,024 66	0,094 99	18	14	2	0,003 32	0,064 78
18	10	5	−0,014 22	0,088 32	18	14	3	0,007 77	0,065 43
18	10	6	−0,002 14	0,079 33	18	14	4	0,012 62	0,064 40
18	10	7	0,011 69	0,067 97	18	14	5	0,017 88	0,061 94
18	10	8	0,027 42	0,054 07	18	14	6	0,023 59	0,058 14
18	10	9	0,045 28	0,037 34	18	14	7	0,029 81	0,052 97
18	10	10	1,075 36	−0,719 50	18	14	8	0,036 63	0,046 33
18	11	1	−0,029 07	0,086 13	18	14	9	0,044 15	0,038 05
18	11	2	−0,024 89	0,089 27	18	14	10	0,052 50	0,027 85
18	11	3	−0,018 94	0,088 63	18	14	11	0,061 87	0,015 34
18	11	4	−0,011 68	0,085 51	18	14	12	0,072 51	−0,000 04
18	11	5	−0,003 22	0,080 29	18	14	13	0,084 78	−0,019 13
18	11	6	0,006 46	0,073 06	18	14	14	0,553 30	−0,537 58

Table 1 (*continued*)

N	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$	*N*	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$
18	15	1	0,004 93	0,055 37	18	17	14	0,077 49	−0,016 61
18	15	2	0,009 01	0,058 59	18	17	15	0,086 59	−0,037 84
18	15	3	0,013 24	0,059 48	18	17	16	0,097 94	−0,068 08
18	15	4	0,017 69	0,058 88	18	17	17	0,232 02	−0,340 77
18	15	5	0,022 39	0,057 03	18	18	1	0,015 60	0,039 58
18	15	6	0,027 40	0,053 99	18	18	2	0,019 87	0,042 49
18	15	7	0,032 77	0,049 76	18	18	3	0,023 82	0,043 79
18	15	8	0,038 57	0,044 23	18	18	4	0,027 65	0,044 07
18	15	9	0,044 88	0,037 26	18	18	5	0,031 46	0,043 52
18	15	10	0,051 81	0,028 60	18	18	6	0,035 31	0,042 19
18	15	11	0,059 51	0,017 91	18	18	7	0,039 24	0,040 06
18	15	12	0,068 18	0,004 67	18	18	8	0,043 29	0,037 09
18	15	13	0,078 11	−0,011 88	18	18	9	0,047 52	0,033 18
18	15	14	0,089 72	−0,032 98	18	18	10	0,051 97	0,028 17
18	15	15	0,441 80	−0,480 90	18	18	11	0,056 72	0,021 82
18	16	1	0,009 39	0,049 83	18	18	12	0,061 86	0,013 79
18	16	2	0,013 52	0,052 97	18	18	13	0,067 52	0,003 54
18	16	3	0,017 61	0,054 05	18	18	14	0,073 91	−0,009 84
18	16	4	0,021 77	0,053 81	18	18	15	0,081 36	−0,027 94
18	16	5	0,026 07	0,052 46	18	18	16	0,090 53	−0,054 07
18	16	6	0,030 56	0,050 07	18	18	17	0,103 08	−0,097 00
18	16	7	0,035 29	0,046 62	18	18	18	0,129 30	−0,244 45
18	16	8	0,040 32	0,042 05	19	2	1	−2,428 14	0,973 45
18	16	9	0,045 71	0,036 21	19	2	2	3,428 14	−0,973 45
18	16	10	0,051 57	0,028 90	19	3	1	−0,956 75	0,484 25
18	16	11	0,057 99	0,019 79	19	3	2	−0,902 77	0,466 47
18	16	12	0,065 15	0,008 45	19	3	3	2,859 52	−0,950 72
18	16	13	0,073 25	−0,005 84	19	4	1	−0,520 66	0,320 30
18	16	14	0,082 66	−0,024 21	19	4	2	−0,494 98	0,313 16
18	16	15	0,093 91	−0,048 59	19	4	3	−0,444 70	0,291 52
18	16	16	0,335 23	−0,416 58	19	4	4	2,460 34	−0,924 98
18	17	1	0,012 90	0,044 68	19	5	1	−0,323 28	0,237 87
18	17	2	0,017 10	0,047 72	19	5	2	−0,308 64	0,235 34
18	17	3	0,021 09	0,048 94	19	5	3	−0,278 69	0,222 20
18	17	4	0,025 05	0,048 98	19	5	4	−0,238 41	0,202 01
18	17	5	0,029 06	0,048 06	19	5	5	2,149 02	−0,897 42
18	17	6	0,033 17	0,046 23	19	6	1	−0,214 93	0,188 14
18	17	7	0,037 43	0,043 48	19	6	2	−0,205 47	0,187 99
18	17	8	0,041 89	0,039 74	19	6	3	−0,185 78	0,179 55
18	17	9	0,046 61	0,034 90	19	6	4	−0,159 15	0,165 63
18	17	10	0,051 65	0,028 78	19	6	5	−0,126 56	0,147 07
18	17	11	0,057 10	0,021 10	19	6	6	1,891 89	−0,868 38
18	17	12	0,063 10	0,011 46	19	7	1	−0,148 40	0,154 79
18	17	13	0,069 80	−0,000 77	19	7	2	−0,141 62	0,155 98

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
19	7	3	−0,127 72	0,150 45	19	12	2	−0,019 08	0,080 64
19	7	4	−0,108 96	0,140 47	19	12	3	−0,014 11	0,080 52
19	7	5	−0,085 99	0,126 74	19	12	4	−0,008 16	0,078 33
19	7	6	−0,059 00	0,109 48	19	12	5	−0,001 30	0,074 42
19	7	7	1,671 69	−0,837 91	19	12	6	0,006 46	0,068 89
19	8	1	−0,104 43	0,130 82	19	12	7	0,015 18	0,061 74
19	8	2	−0,099 12	0,132 82	19	12	8	0,024 95	0,052 87
19	8	3	−0,088 74	0,129 20	19	12	9	0,035 91	0,042 11
19	8	4	−0,074 86	0,121 88	19	12	10	0,048 21	0,029 21
19	8	5	−0,057 94	0,111 45	19	12	11	0,062 10	0,013 83
19	8	6	−0,038 08	0,098 07	19	12	12	0,872 56	−0,660 14
19	8	7	−0,015 23	0,081 74	19	13	1	−0,013 01	0,069 58
19	8	8	1,478 38	−0,805 97	19	13	2	−0,009 43	0,072 68
19	9	1	−0,073 83	0,112 71	19	13	3	−0,004 94	0,072 95
19	9	2	−0,069 35	0,115 20	19	13	4	0,000 24	0,071 40
19	9	3	−0,061 20	0,112 91	19	13	5	0,006 08	0,068 33
19	9	4	−0,050 51	0,107 48	19	13	6	0,012 58	0,063 84
19	9	5	−0,037 59	0,099 41	19	13	7	0,019 81	0,057 92
19	9	6	−0,022 49	0,088 85	19	13	8	0,027 83	0,050 49
19	9	7	−0,005 16	0,075 79	19	13	9	0,036 76	0,041 40
19	9	8	0,014 56	0,060 09	19	13	10	0,046 73	0,030 43
19	9	9	1,305 57	−0,772 43	19	13	11	0,057 94	0,017 26
19	10	1	−0,051 70	0,098 52	19	13	12	0,070 62	0,001 44
19	10	2	−0,047 69	0,101 31	19	13	13	0,748 78	−0,617 73
19	10	3	−0,041 03	0,099 96	19	14	1	−0,005 40	0,062 69
19	10	4	−0,032 50	0,095 92	19	14	2	−0,001 81	0,065 79
19	10	5	−0,022 32	0,089 61	19	14	3	0,002 34	0,066 37
19	10	6	−0,010 51	0,081 16	19	14	4	0,006 96	0,065 33
19	10	7	0,002 98	0,070 57	19	14	5	0,012 04	0,062 94
19	10	8	0,018 28	0,057 71	19	14	6	0,017 61	0,059 30
19	10	9	0,035 58	0,042 37	19	14	7	0,023 71	0,054 39
19	10	10	1,148 92	−0,737 12	19	14	8	0,030 41	0,048 16
19	11	1	−0,035 24	0,087 06	19	14	9	0,037 80	0,040 46
19	11	2	−0,031 49	0,090 03	19	14	10	0,045 99	0,031 10
19	11	3	−0,025 82	0,089 38	19	14	11	0,055 14	0,019 79
19	11	4	−0,018 80	0,086 38	19	14	12	0,065 44	0,006 13
19	11	5	−0,010 55	0,081 42	19	14	13	0,077 18	−0,010 47
19	11	6	−0,001 09	0,074 60	19	14	14	0,632 58	−0,571 97
19	11	7	0,009 65	0,065 92	19	15	1	0,000 64	0,056 67
19	11	8	0,021 76	0,055 28	19	15	2	0,004 25	0,059 73
19	11	9	0,035 41	0,042 48	19	15	3	0,008 17	0,060 54
19	11	10	0,050 81	0,027 24	19	15	4	0,012 38	0,059 91
19	11	11	1,005 35	−0,699 80	19	15	5	0,016 91	0,058 08
19	12	1	−0,022 71	0,077 58	19	15	6	0,021 77	0,055 14

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
19	15	7	0,027 02	0,051 09	19	18	3	0,019 74	0,045 88
19	15	8	0,032 71	0,045 86	19	18	4	0,023 29	0,046 07
19	15	9	0,038 92	0,039 34	19	18	5	0,026 86	0,045 44
19	15	10	0,045 74	0,031 35	19	18	6	0,030 50	0,044 04
19	15	11	0,053 30	0,021 63	19	18	7	0,034 26	0,041 87
19	15	12	0,061 75	0,009 81	19	18	8	0,038 16	0,038 90
19	15	13	0,071 33	−0,004 63	19	18	9	0,042 25	0,035 05
19	15	14	0,082 34	−0,022 48	19	18	10	0,046 58	0,030 19
19	15	15	0,522 77	−0,522 06	19	18	11	0,051 22	0,024 15
19	16	1	0,005 45	0,051 30	19	18	12	0,056 25	0,016 66
19	16	2	0,009 12	0,054 31	19	18	13	0,061 77	0,007 35
19	16	3	0,012 88	0,055 29	19	18	14	0,067 95	−0,004 39
19	16	4	0,016 79	0,054 99	19	18	15	0,075 02	−0,019 51
19	16	5	0,020 89	0,053 63	19	18	16	0,083 41	−0,039 71
19	16	6	0,025 23	0,051 29	19	18	17	0,093 85	−0,068 38
19	16	7	0,029 83	0,047 96	19	18	18	0,220 38	−0,330 07
19	16	8	0,034 75	0,043 58	19	19	1	0,014 73	0,037 26
19	16	9	0,040 06	0,038 07	19	19	2	0,018 60	0,039 95
19	16	10	0,045 83	0,031 26	19	19	3	0,022 16	0,041 23
19	16	11	0,052 15	0,022 91	19	19	4	0,025 61	0,041 61
19	16	12	0,059 16	0,012 71	19	19	5	0,029 03	0,041 27
19	16	13	0,067 04	0,000 16	19	19	6	0,032 47	0,040 26
19	16	14	0,076 04	−0,015 45	19	19	7	0,035 96	0,038 59
19	16	15	0,086 55	−0,035 27	19	19	8	0,039 55	0,036 22
19	16	16	0,418 25	−0,466 73	19	19	9	0,043 26	0,033 10
19	17	1	0,009 29	0,046 41	19	19	10	0,047 15	0,029 11
19	17	2	0,013 02	0,049 34	19	19	11	0,051 25	0,024 10
19	17	3	0,016 68	0,050 46	19	19	12	0,055 64	0,017 84
19	17	4	0,020 38	0,050 44	19	19	13	0,060 39	0,010 00
19	17	5	0,024 17	0,049 47	19	19	14	0,065 63	0,000 06
19	17	6	0,028 11	0,047 63	19	19	15	0,071 55	−0,012 83
19	17	7	0,032 22	0,044 91	19	19	16	0,078 46	−0,030 20
19	17	8	0,036 56	0,041 29	19	19	17	0,086 98	−0,055 17
19	17	9	0,041 18	0,036 65	19	19	18	0,098 63	−0,096 08
19	17	10	0,046 13	0,030 87	19	19	19	0,122 95	−0,236 29
19	17	11	0,051 50	0,023 74	20	2	1	−2,482 86	0,974 79
19	17	12	0,057 39	0,014 96	20	2	2	3,482 86	−0,974 79
19	17	13	0,063 94	0,004 10	20	3	1	−0,984 39	0,485 06
19	17	14	0,071 35	−0,009 50	20	3	2	−0,932 29	0,468 18
19	17	15	0,079 95	−0,026 89	20	3	3	2,916 68	−0,953 23
19	17	16	0,090 21	−0,049 90	20	4	1	−0,539 30	0,320 98
19	17	17	0,317 93	−0,403 98	20	4	2	−0,514 57	0,314 19
19	18	1	0,012 35	0,041 83	20	4	3	−0,465 88	0,293 70
19	18	2	0,016 15	0,044 66	20	4	4	2,519 75	−0,928 87

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
20	5	1	−0,337 41	0,238 52	20	10	10	1,218 18	−0,752 61
20	5	2	−0,323 39	0,236 10	20	11	1	−0,041 09	0,087 87
20	5	3	−0,294 35	0,223 64	20	11	2	−0,037 70	0,090 68
20	5	4	−0,255 30	0,204 57	20	11	3	−0,032 30	0,090 03
20	5	5	2,210 45	−0,902 83	20	11	4	−0,025 49	0,087 15
20	6	1	−0,226 35	0,188 79	20	11	5	−0,017 44	0,082 43
20	6	2	−0,217 36	0,188 63	20	11	6	−0,008 18	0,075 98
20	6	3	−0,198 26	0,180 61	20	11	7	0,002 33	0,067 83
20	6	4	−0,172 40	0,167 45	20	11	8	0,014 16	0,057 90
20	6	5	−0,140 78	0,149 97	20	11	9	0,027 45	0,046 06
20	6	6	1,955 15	−0,875 45	20	11	10	0,042 36	0,032 09
20	7	1	−0,157 99	0,155 46	20	11	11	1,075 90	−0,718 02
20	7	2	−0,151 62	0,156 57	20	12	1	−0,028 03	0,078 44
20	7	3	−0,138 15	0,151 30	20	12	2	−0,024 77	0,081 35
20	7	4	−0,119 91	0,141 85	20	12	3	−0,020 06	0,081 20
20	7	5	−0,097 57	0,128 90	20	12	4	−0,014 30	0,079 08
20	7	6	−0,071 38	0,112 71	20	12	5	−0,007 62	0,075 34
20	7	7	1,736 62	−0,846 79	20	12	6	−0,000 02	0,070 09
20	8	1	−0,112 70	0,131 51	20	12	7	0,008 53	0,063 36
20	8	2	−0,107 77	0,133 39	20	12	8	0,018 10	0,055 06
20	8	3	−0,097 73	0,129 93	20	12	9	0,028 80	0,045 08
20	8	4	−0,084 23	0,122 98	20	12	10	0,040 78	0,033 23
20	8	5	−0,067 74	0,113 13	20	12	11	0,054 21	0,019 26
20	8	6	−0,048 42	0,100 57	20	12	12	0,944 38	−0,681 49
20	8	7	−0,026 24	0,085 32	20	13	1	−0,017 87	0,070 50
20	8	8	1,544 85	−0,816 83	20	13	2	−0,014 66	0,073 44
20	9	1	−0,081 10	0,113 44	20	13	3	−0,010 43	0,073 67
20	9	2	−0,076 99	0,115 79	20	13	4	−0,005 45	0,072 15
20	9	3	−0,069 15	0,113 58	20	13	5	0,000 23	0,069 20
20	9	4	−0,058 75	0,108 41	20	13	6	0,006 59	0,064 92
20	9	5	−0,046 13	0,100 77	20	13	7	0,013 68	0,059 32
20	9	6	−0,031 41	0,090 84	20	13	8	0,021 56	0,052 36
20	9	7	−0,014 54	0,078 62	20	13	9	0,030 31	0,043 90
20	9	8	0,004 60	0,064 03	20	13	10	0,040 06	0,033 80
20	9	9	1,373 47	−0,785 49	20	13	11	0,050 95	0,021 81
20	10	1	−0,058 19	0,099 29	20	13	12	0,063 19	0,007 60
20	10	2	−0,054 55	0,101 92	20	13	13	0,821 85	−0,642 68
20	10	3	−0,048 16	0,100 61	20	14	1	−0,009 86	0,063 69
20	10	4	−0,039 87	0,096 75	20	14	2	−0,006 66	0,066 64
20	10	5	−0,029 93	0,090 76	20	14	3	−0,002 77	0,067 15
20	10	6	−0,018 39	0,082 79	20	14	4	0,001 66	0,066 11
20	10	7	−0,005 22	0,072 87	20	14	5	0,006 58	0,063 80
20	10	8	0,009 67	0,060 90	20	14	6	0,012 02	0,060 31
20	10	9	0,026 45	0,046 73	20	14	7	0,018 00	0,055 65

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
20	14	8	0,024 59	0,049 78	20	17	7	0,027 38	0,046 08
20	14	9	0,031 85	0,042 59	20	17	8	0,031 63	0,042 58
20	14	10	0,039 89	0,033 94	20	17	9	0,036 16	0,038 16
20	14	11	0,048 82	0,023 61	20	17	10	0,041 04	0,032 71
20	14	12	0,058 82	0,011 31	20	17	11	0,046 33	0,026 08
20	14	13	0,070 10	−0,003 39	20	17	12	0,052 13	0,018 05
20	14	14	0,706 94	−0,601 19	20	17	13	0,058 54	0,008 29
20	15	1	−0,003 47	0,057 75	20	17	14	0,065 74	−0,003 63
20	15	2	−0,000 24	0,060 67	20	17	15	0,073 96	−0,018 40
20	15	3	0,003 41	0,061 41	20	17	16	0,083 52	−0,037 09
20	15	4	0,007 41	0,060 76	20	17	17	0,397 18	−0,453 58
20	15	5	0,011 78	0,058 97	20	18	1	0,009 15	0,043 42
20	15	6	0,016 51	0,056 13	20	18	2	0,012 54	0,046 15
20	15	7	0,021 65	0,052 26	20	18	3	0,015 84	0,047 29
20	15	8	0,027 25	0,047 31	20	18	4	0,019 15	0,047 42
20	15	9	0,033 36	0,041 19	20	18	5	0,022 53	0,046 74
20	15	10	0,040 07	0,033 77	20	18	6	0,026 01	0,045 32
20	15	11	0,047 49	0,024 85	20	18	7	0,029 63	0,043 16
20	15	12	0,055 74	0,014 17	20	18	8	0,033 42	0,040 25
20	15	13	0,065 01	0,001 35	20	18	9	0,037 42	0,036 53
20	15	14	0,075 53	−0,014 15	20	18	10	0,041 67	0,031 89
20	15	15	0,598 51	−0,556 45	20	18	11	0,046 24	0,026 20
20	16	1	0,001 66	0,052 49	20	18	12	0,051 19	0,019 27
20	16	2	0,004 93	0,055 37	20	18	13	0,056 61	0,010 80
20	16	3	0,008 42	0,056 28	20	18	14	0,062 64	0,000 40
20	16	4	0,012 12	0,055 94	20	18	15	0,069 46	−0,012 57
20	16	5	0,016 05	0,054 59	20	18	16	0,077 35	−0,029 09
20	16	6	0,020 25	0,052 30	20	18	17	0,086 77	−0,050 88
20	16	7	0,024 75	0,049 09	20	18	18	0,302 38	−0,392 30
20	16	8	0,029 58	0,044 92	20	19	1	0,011 85	0,039 31
20	16	9	0,034 80	0,039 71	20	19	2	0,015 30	0,041 95
20	16	10	0,040 49	0,033 35	20	19	3	0,018 55	0,043 15
20	16	11	0,046 71	0,025 65	20	19	4	0,021 75	0,043 46
20	16	12	0,053 59	0,016 38	20	19	5	0,024 95	0,043 04
20	16	13	0,061 26	0,005 19	20	19	6	0,028 20	0,041 97
20	16	14	0,069 94	−0,008 42	20	19	7	0,031 54	0,040 25
20	16	15	0,079 89	−0,025 18	20	19	8	0,034 99	0,037 86
20	16	16	0,495 56	−0,507 65	20	19	9	0,038 58	0,034 76
20	17	1	0,005 80	0,047 76	20	19	10	0,042 36	0,030 85
20	17	2	0,009 13	0,050 58	20	19	11	0,046 36	0,026 02
20	17	3	0,012 50	0,051 61	20	19	12	0,050 65	0,020 08
20	17	4	0,015 98	0,051 54	20	19	13	0,055 30	0,012 80
20	17	5	0,019 59	0,050 55	20	19	14	0,060 41	0,003 80
20	17	6	0,023 38	0,048 72	20	19	15	0,066 13	−0,007 48

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
20	19	16	0,072 68	−0,021 95	21	7	1	−0,167 09	0,156 06
20	19	17	0,080 43	−0,041 21	21	7	2	−0,161 08	0,157 10
20	19	18	0,090 09	−0,068 47	21	7	3	−0,148 01	0,152 07
20	19	19	0,209 90	−0,320 17	21	7	4	−0,130 25	0,143 09
20	20	1	0,013 96	0,035 18	21	7	5	−0,108 52	0,130 84
20	20	2	0,017 48	0,037 69	21	7	6	−0,083 07	0,115 59
20	20	3	0,020 71	0,038 93	21	7	7	1,798 01	−0,854 74
20	20	4	0,023 83	0,039 39	21	8	1	−0,120 56	0,132 13
20	20	5	0,026 92	0,039 20	21	8	2	−0,115 96	0,133 91
20	20	6	0,030 01	0,038 44	21	8	3	−0,106 24	0,130 59
20	20	7	0,033 14	0,037 12	21	8	4	−0,093 08	0,123 98
20	20	8	0,036 34	0,035 22	21	8	5	−0,077 01	0,114 64
20	20	9	0,039 64	0,032 69	21	8	6	−0,058 19	0,102 80
20	20	10	0,043 06	0,029 47	21	8	7	−0,036 64	0,088 49
20	20	11	0,046 65	0,025 45	21	8	8	1,607 67	−0,826 53
20	20	12	0,050 45	0,020 47	21	9	1	−0,088 01	0,114 09
20	20	13	0,054 52	0,014 32	21	9	2	−0,084 22	0,116 31
20	20	14	0,058 93	0,006 67	21	9	3	−0,076 64	0,114 18
20	20	15	0,063 81	−0,002 96	21	9	4	−0,066 51	0,109 25
20	20	16	0,069 32	−0,015 40	21	9	5	−0,054 20	0,102 00
20	20	17	0,075 76	−0,032 08	21	9	6	−0,039 83	0,092 62
20	20	18	0,083 70	−0,056 00	21	9	7	−0,023 39	0,081 14
20	20	19	0,094 57	−0,095 08	21	9	8	−0,004 81	0,067 52
20	20	20	0,117 21	−0,228 75	21	9	9	1,437 63	−0,797 10
21	2	1	−2,534 80	0,976 00	21	10	1	−0,064 35	0,099 96
21	2	2	3,534 81	−0,976 00	21	10	2	−0,061 03	0,102 46
21	3	1	−1,010 62	0,485 79	21	10	3	−0,054 88	0,101 19
21	3	2	−0,960 26	0,469 72	21	10	4	−0,046 82	0,097 49
21	3	3	2,970 88	−0,955 51	21	10	5	−0,037 11	0,091 79
21	4	1	−0,556 99	0,321 60	21	10	6	−0,025 83	0,084 26
21	4	2	−0,533 12	0,315 12	21	10	7	−0,012 96	0,074 92
21	4	3	−0,485 92	0,295 65	21	10	8	0,001 55	0,063 72
21	4	4	2,576 03	−0,932 37	21	10	9	0,017 84	0,050 55
21	5	1	−0,350 82	0,239 11	21	10	10	1,283 59	−0,766 34
21	5	2	−0,337 35	0,236 79	21	11	1	−0,046 64	0,088 58
21	5	3	−0,309 17	0,224 93	21	11	2	−0,043 58	0,091 25
21	5	4	−0,271 27	0,206 86	21	11	3	−0,038 41	0,090 60
21	5	5	2,268 60	−0,907 69	21	11	4	−0,031 80	0,087 84
21	6	1	−0,237 18	0,189 38	21	11	5	−0,023 93	0,083 33
21	6	2	−0,228 61	0,189 20	21	11	6	−0,014 87	0,077 22
21	6	3	−0,210 07	0,181 57	21	11	7	−0,004 58	0,069 53
21	6	4	−0,184 92	0,169 08	21	11	8	0,006 98	0,060 23
21	6	5	−0,154 21	0,152 57	21	11	9	0,019 93	0,049 21
21	6	6	2,014 99	−0,881 79	21	11	10	0,034 40	0,036 31

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
21	11	11	1,142 51	−0,734 10	21	15	5	0,006 96	0,059 73
21	12	1	−0,033 08	0,079 20	21	15	6	0,011 58	0,056 99
21	12	2	−0,030 15	0,081 96	21	15	7	0,016 62	0,053 29
21	12	3	−0,025 66	0,081 78	21	15	8	0,022 12	0,048 59
21	12	4	−0,020 09	0,079 74	21	15	9	0,028 14	0,042 83
21	12	5	−0,013 57	0,076 16	21	15	10	0,034 74	0,035 91
21	12	6	−0,006 13	0,071 17	21	15	11	0,042 02	0,027 68
21	12	7	0,002 26	0,064 80	21	15	12	0,050 08	0,017 93
21	12	8	0,011 64	0,057 01	21	15	13	0,059 07	0,006 39
21	12	9	0,022 10	0,047 71	21	15	14	0,069 19	−0,007 34
21	12	10	0,033 76	0,036 75	21	15	15	0,669 74	−0,585 77
21	12	11	0,046 78	0,023 93	21	16	1	−0,001 97	0,053 50
21	12	12	1,012 14	−0,700 20	21	16	2	0,000 96	0,056 25
21	13	1	−0,022 49	0,071 30	21	16	3	0,004 20	0,057 09
21	13	2	−0,019 62	0,074 10	21	16	4	0,007 72	0,056 73
21	13	3	−0,015 61	0,074 29	21	16	5	0,011 51	0,055 39
21	13	4	−0,010 80	0,072 81	21	16	6	0,015 59	0,053 16
21	13	5	−0,005 27	0,069 97	21	16	7	0,019 99	0,050 07
21	13	6	0,000 96	0,065 89	21	16	8	0,024 74	0,046 09
21	13	7	0,007 91	0,060 58	21	16	9	0,029 89	0,041 17
21	13	8	0,015 65	0,054 02	21	16	10	0,035 49	0,035 21
21	13	9	0,024 23	0,046 12	21	16	11	0,041 61	0,028 07
21	13	10	0,033 75	0,036 75	21	16	12	0,048 36	0,019 57
21	13	11	0,044 36	0,025 74	21	16	13	0,055 85	0,009 46
21	13	12	0,056 20	0,012 83	21	16	14	0,064 24	−0,002 62
21	13	13	0,890 75	−0,664 40	21	16	15	0,073 75	−0,017 16
21	14	1	−0,014 11	0,064 54	21	16	16	0,568 07	−0,541 98
21	14	2	−0,011 24	0,067 35	21	17	1	0,002 44	0,048 87
21	14	3	−0,007 58	0,067 82	21	17	2	0,005 42	0,051 57
21	14	4	−0,003 33	0,066 79	21	17	3	0,008 54	0,052 53
21	14	5	0,001 45	0,064 55	21	17	4	0,011 83	0,052 42
21	14	6	0,006 77	0,061 20	21	17	5	0,015 29	0,051 42
21	14	7	0,012 64	0,056 77	21	17	6	0,018 96	0,049 63
21	14	8	0,019 11	0,051 22	21	17	7	0,022 86	0,047 06
21	14	9	0,026 25	0,044 49	21	17	8	0,027 02	0,043 70
21	14	10	0,034 13	0,036 45	21	17	9	0,031 48	0,039 49
21	14	11	0,042 87	0,026 94	21	17	10	0,036 29	0,034 36
21	14	12	0,052 59	0,015 74	21	17	11	0,041 51	0,028 17
21	14	13	0,063 48	0,002 54	21	17	12	0,047 22	0,020 77
21	14	14	0,776 98	−0,626 40	21	17	13	0,053 51	0,011 91
21	15	1	−0,007 39	0,058 67	21	17	14	0,060 52	0,001 29
21	15	2	−0,004 51	0,061 46	21	17	15	0,068 43	−0,011 58
21	15	3	−0,001 09	0,062 14	21	17	16	0,077 48	−0,027 37
21	15	4	0,002 74	0,061 48	21	17	17	0,471 16	−0,494 23

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
21	18	1	0,006 05	0,044 66	21	20	8	0,032 27	0,036 73
21	18	2	0,009 08	0,047 30	21	20	9	0,035 45	0,034 20
21	18	3	0,012 12	0,048 36	21	20	10	0,038 78	0,031 02
21	18	4	0,015 23	0,048 44	21	20	11	0,042 28	0,027 09
21	18	5	0,018 45	0,047 74	21	20	12	0,045 99	0,022 31
21	18	6	0,021 80	0,046 31	21	20	13	0,049 98	0,016 50
21	18	7	0,025 31	0,044 20	21	20	14	0,054 30	0,009 42
21	18	8	0,029 02	0,041 37	21	20	15	0,059 04	0,000 72
21	18	9	0,032 94	0,037 79	21	20	16	0,064 36	−0,010 14
21	18	10	0,037 13	0,033 38	21	20	17	0,070 45	−0,024 01
21	18	11	0,041 63	0,028 03	21	20	18	0,077 65	−0,042 41
21	18	12	0,046 51	0,021 59	21	20	19	0,086 62	−0,068 40
21	18	13	0,051 85	0,013 86	21	20	20	0,200 40	−0,310 97
21	18	14	0,057 75	0,004 52	21	21	1	0,013 26	0,033 33
21	18	15	0,064 36	−0,006 83	21	21	2	0,016 48	0,035 67
21	18	16	0,071 90	−0,020 85	21	21	3	0,019 42	0,036 87
21	18	17	0,080 66	−0,038 53	21	21	4	0,022 27	0,037 37
21	18	18	0,378 21	−0,441 35	21	21	5	0,025 07	0,037 31
21	19	1	0,008 99	0,040 77	21	21	6	0,027 87	0,036 74
21	19	2	0,012 08	0,043 33	21	21	7	0,030 69	0,035 69
21	19	3	0,015 07	0,044 46	21	21	8	0,033 57	0,034 15
21	19	4	0,018 05	0,044 71	21	21	9	0,036 51	0,032 09
21	19	5	0,021 09	0,044 25	21	21	10	0,039 56	0,029 46
21	19	6	0,024 20	0,043 15	21	21	11	0,042 74	0,026 19
21	19	7	0,027 41	0,041 43	21	21	12	0,046 07	0,022 16
21	19	8	0,030 75	0,039 07	21	21	13	0,049 61	0,017 23
21	19	9	0,034 26	0,036 05	21	21	14	0,053 39	0,011 20
21	19	10	0,037 96	0,036 05	21	21	15	0,057 51	0,003 75
21	19	11	0,041 90	0,027 68	21	21	16	0,062 06	−0,005 59
21	19	12	0,046 13	0,022 10	21	21	17	0,067 21	−0,017 60
21	19	13	0,050 71	0,015 37	21	21	18	0,073 23	−0,033 65
21	19	14	0,055 72	0,007 20	21	21	19	0,080 66	−0,056 60
21	19	15	0,061 30	−0,002 78	21	21	20	0,090 83	−0,094 01
21	19	16	0,067 60	−0,015 16	21	21	21	0,111 99	−0,221 76
21	19	17	0,074 88	−0,030 89	22	2	1	−2,584 24	0,977 10
21	19	18	0,083 57	−0,051 59	22	2	2	3,584 24	−0,977 10
21	19	19	0,288 35	−0,381 44	22	3	1	−1,035 58	0,486 45
21	20	1	0,011 37	0,037 07	22	3	2	−0,986 83	0,471 12
21	20	2	0,014 53	0,039 53	22	3	3	3,022 41	−0,957 57
21	20	3	0,017 48	0,040 71	22	4	1	−0,573 81	0,322 15
21	20	4	0,020 38	0,041 10	22	4	2	−0,550 74	0,315 97
21	20	5	0,023 28	0,040 85	22	4	3	−0,504 93	0,297 43
21	20	6	0,026 21	0,040 03	22	4	4	2,629 48	−0,935 54
21	20	7	0,029 19	0,038 66	22	5	1	−0,363 57	0,239 63

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
22	5	2	−0,350 60	0,237 41	22	11	1	−0,051 93	0,089 22
22	5	3	−0,323 22	0,226 11	22	11	2	−0,049 15	0,091 75
22	5	4	−0,286 40	0,208 94	22	11	3	−0,044 19	0,091 11
22	5	5	2,323 80	−0,912 09	22	11	4	−0,037 76	0,088 45
22	6	1	−0,247 48	0,189 90	22	11	5	−0,030 07	0,084 14
22	6	2	−0,239 28	0,189 72	22	11	6	−0,021 20	0,078 33
22	6	3	−0,221 27	0,182 43	22	11	7	−0,011 13	0,071 06
22	6	4	−0,196 80	0,170 55	22	11	8	0,000 18	0,062 31
22	6	5	−0,166 94	0,154 90	22	11	9	0,012 80	0,052 00
22	6	6	2,071 76	−0,887 51	22	11	10	0,026 87	0,040 01
22	7	1	−0,175 74	0,156 59	22	11	11	1,205 58	−0,748 40
22	7	2	−0,170 05	0,157 58	22	12	1	−0,037 89	0,079 86
22	7	3	−0,157 36	0,152 76	22	12	2	−0,035 24	0,082 49
22	7	4	−0,140 06	0,144 21	22	12	3	−0,030 96	0,082 30
22	7	5	−0,118 88	0,132 59	22	12	4	−0,025 56	0,080 32
22	7	6	−0,094 13	0,118 17	22	12	5	−0,019 19	0,076 89
22	7	7	1,856 22	−0,861 90	22	12	6	−0,011 90	0,072 14
22	8	1	−0,128 02	0,132 69	22	12	7	−0,003 68	0,066 10
22	8	2	−0,123 72	0,134 37	22	12	8	0,005 52	0,058 76
22	8	3	−0,114 29	0,131 18	22	12	9	0,015 75	0,050 04
22	8	4	−0,101 47	0,124 87	22	12	10	0,027 12	0,039 85
22	8	5	−0,085 78	0,116 01	22	12	11	0,039 75	0,028 01
22	8	6	−0,067 45	0,104 80	22	12	12	1,076 27	−0,716 77
22	8	7	−0,046 49	0,091 33	22	13	1	−0,026 90	0,072 00
22	8	8	1,667 22	−0,835 24	22	13	2	−0,024 31	0,074 68
22	9	1	−0,094 58	0,114 66	22	13	3	−0,020 51	0,074 83
22	9	2	−0,091 07	0,116 77	22	13	4	−0,015 86	0,073 39
22	9	3	−0,083 75	0,114 72	22	13	5	−0,010 47	0,070 66
22	9	4	−0,073 87	0,110 00	22	13	6	−0,004 37	0,066 75
22	9	5	−0,061 85	0,103 11	22	13	7	0,002 46	0,061 71
22	9	6	−0,047 81	0,094 22	22	13	8	0,010 06	0,055 51
22	9	7	−0,031 78	0,083 40	22	13	9	0,018 47	0,048 10
22	9	8	−0,013 71	0,070 61	22	13	10	0,027 79	0,039 37
22	9	9	1,498 41	−0,807 50	22	13	11	0,038 12	0,029 18
22	10	1	−0,070 22	0,100 57	22	13	12	0,049 60	0,017 34
22	10	2	−0,067 17	0,102 94	22	13	13	0,955 92	−0,683 52
22	10	3	−0,061 25	0,101 70	22	14	1	−0,018 17	0,065 29
22	10	4	−0,053 39	0,098 15	22	14	2	−0,015 60	0,067 97
22	10	5	−0,043 90	0,092 72	22	14	3	−0,012 14	0,068 39
22	10	6	−0,032 87	0,085 58	22	14	4	−0,008 05	0,067 38
22	10	7	−0,020 30	0,076 76	22	14	5	−0,003 39	0,065 21
22	10	8	−0,006 14	0,066 24	22	14	6	0,001 81	0,062 00
22	10	9	0,009 69	0,053 94	22	14	7	0,007 58	0,057 78
22	10	10	1,345 55	−0,778 59	22	14	8	0,013 94	0,052 52

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
22	14	9	0,020 96	0,046 18	22	17	8	0,022 70	0,044 69
22	14	10	0,028 69	0,038 67	22	17	9	0,027 10	0,040 68
22	14	11	0,037 23	0,029 86	22	17	10	0,031 84	0,035 83
22	14	12	0,046 70	0,019 57	22	17	11	0,036 99	0,030 04
22	14	13	0,057 23	0,007 58	22	17	12	0,042 61	0,023 18
22	14	14	0,843 19	−0,648 43	22	17	13	0,048 78	0,015 07
22	15	1	−0,011 15	0,059 47	22	17	14	0,055 62	0,005 48
22	15	2	−0,008 56	0,062 14	22	17	15	0,063 26	−0,005 94
22	15	3	−0,005 35	0,062 76	22	17	16	0,071 91	−0,019 64
22	15	4	−0,001 67	0,062 10	22	17	17	0,540 71	−0,528 46
22	15	5	0,002 42	0,060 39	22	18	1	0,003 04	0,045 69
22	15	6	0,006 93	0,057 76	22	18	2	0,005 77	0,048 22
22	15	7	0,011 88	0,054 21	22	18	3	0,008 59	0,049 22
22	15	8	0,017 29	0,049 75	22	18	4	0,011 53	0,049 26
22	15	9	0,023 22	0,044 31	22	18	5	0,014 61	0,048 54
22	15	10	0,029 71	0,037 83	22	18	6	0,017 85	0,047 14
22	15	11	0,036 85	0,030 18	22	18	7	0,021 27	0,045 07
22	15	12	0,044 73	0,021 21	22	18	8	0,024 89	0,042 34
22	15	13	0,053 48	0,010 70	22	18	9	0,028 75	0,038 91
22	15	14	0,063 24	−0,001 62	22	18	10	0,032 88	0,034 71
22	15	15	0,736 99	−0,611 17	22	18	11	0,037 33	0,029 68
22	16	1	−0,005 45	0,054 35	22	18	12	0,042 14	0,023 68
22	16	2	−0,002 83	0,056 99	22	18	13	0,047 40	0,016 56
22	16	3	0,000 21	0,057 77	22	18	14	0,053 18	0,008 09
22	16	4	0,003 57	0,057 39	22	18	15	0,059 62	−0,002 02
22	16	5	0,007 23	0,056 07	22	18	16	0,066 86	−0,014 22
22	16	6	0,011 21	0,053 92	22	18	17	0,075 14	−0,029 15
22	16	7	0,015 52	0,050 94	22	18	18	0,449 15	−0,481 70
22	16	8	0,020 19	0,047 15	22	19	1	0,006 20	0,041 93
22	16	9	0,025 26	0,042 48	22	19	2	0,008 98	0,044 40
22	16	10	0,030 78	0,036 87	22	19	3	0,011 74	0,045 46
22	16	11	0,036 81	0,030 22	22	19	4	0,014 55	0,045 67
22	16	12	0,043 43	0,022 38	22	19	5	0,017 44	0,045 18
22	16	13	0,050 75	0,013 15	22	19	6	0,020 43	0,044 07
22	16	14	0,058 89	0,002 28	22	19	7	0,023 54	0,042 37
22	16	15	0,068 02	−0,010 59	22	19	8	0,026 80	0,040 07
22	16	16	0,636 42	−0,571 37	22	19	9	0,030 24	0,037 14
22	17	1	−0,000 79	0,049 80	22	19	10	0,033 88	0,033 53
22	17	2	0,001 88	0,052 39	22	19	11	0,037 77	0,029 16
22	17	3	0,004 79	0,053 29	22	19	12	0,041 94	0,023 92
22	17	4	0,007 91	0,053 14	22	19	13	0,046 45	0,017 68
22	17	5	0,011 25	0,052 15	22	19	14	0,051 39	0,010 23
22	17	6	0,014 81	0,050 40	22	19	15	0,056 84	0,001 28
22	17	7	0,018 62	0,047 91	22	19	16	0,062 94	−0,009 56

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
22	19	17	0,069 89	−0,022 89	22	22	1	0,012 63	0,031 65
22	19	18	0,077 95	−0,039 67	22	22	2	0,015 58	0,033 84
22	19	19	0,361 04	−0,429 93	22	22	3	0,018 28	0,035 00
22	20	1	0,008 80	0,038 42	22	22	4	0,020 88	0,035 53
22	20	2	0,011 64	0,040 81	22	22	5	0,023 43	0,035 56
22	20	3	0,014 36	0,041 93	22	22	6	0,025 98	0,035 15
22	20	4	0,017 07	0,042 27	22	22	7	0,028 54	0,034 32
22	20	5	0,019 81	0,041 97	22	22	8	0,031 14	0,033 07
22	20	6	0,022 61	0,041 13	22	22	9	0,033 80	0,031 37
22	20	7	0,025 48	0,039 75	22	22	10	0,036 53	0,029 20
22	20	8	0,028 46	0,037 83	22	22	11	0,039 37	0,026 50
22	20	9	0,031 57	0,035 35	22	22	12	0,042 32	0,023 20
22	20	10	0,034 83	0,032 25	22	22	13	0,045 43	0,019 19
22	20	11	0,038 27	0,028 48	22	22	14	0,048 73	0,014 34
22	20	12	0,041 93	0,023 94	22	22	15	0,052 27	0,008 43
22	20	13	0,045 85	0,018 49	22	22	16	0,056 13	0,001 17
22	20	14	0,050 11	0,011 95	22	22	17	0,060 39	−0,007 89
22	20	15	0,054 76	0,004 08	22	22	18	0,065 22	−0,019 49
22	20	16	0,059 94	−0,005 51	22	22	19	0,070 86	−0,034 96
22	20	17	0,065 78	−0,017 37	22	22	20	0,077 84	−0,057 03
22	20	18	0,072 54	−0,032 38	22	22	21	0,087 39	−0,092 91
22	20	19	0,080 58	−0,052 09	22	22	22	0,107 23	−0,215 26
22	20	20	0,275 60	−0,371 30	23	2	1	−2,631 40	0,978 10
22	21	1	0,010 93	0,035 07	23	2	2	3,631 40	−0,978 10
22	21	2	0,013 82	0,037 37	23	3	1	−1,059 38	0,487 05
22	21	3	0,016 53	0,038 52	23	3	2	−1,012 13	0,472 40
22	21	4	0,019 17	0,038 96	23	3	3	3,071 51	−0,959 44
22	21	5	0,021 80	0,038 84	23	4	1	−0,589 84	0,322 66
22	21	6	0,024 45	0,038 22	23	4	2	−0,567 52	0,316 73
22	21	7	0,027 15	0,037 13	23	4	3	−0,523 01	0,299 04
22	21	8	0,029 91	0,035 55	23	4	4	2,680 37	−0,938 43
22	21	9	0,032 75	0,033 48	23	5	1	−0,375 72	0,240 11
22	21	10	0,035 71	0,030 86	23	5	2	−0,363 22	0,237 98
22	21	11	0,038 80	0,027 63	23	5	3	−0,336 59	0,227 18
22	21	12	0,042 06	0,023 72	23	5	4	−0,300 78	0,210 82
22	21	13	0,045 52	0,019 01	23	5	5	2,376 31	−0,916 09
22	21	14	0,049 24	0,013 32	23	6	1	−0,257 29	0,190 38
22	21	15	0,053 26	0,006 44	23	6	2	−0,249 44	0,190 19
22	21	16	0,057 69	−0,001 97	23	6	3	−0,231 92	0,183 22
22	21	17	0,062 65	−0,012 43	23	6	4	−0,208 08	0,171 89
22	21	18	0,068 33	−0,025 74	23	6	5	−0,179 02	0,157 02
22	21	19	0,075 05	−0,043 37	23	6	6	2,125 75	−0,892 71
22	21	20	0,083 41	−0,068 20	23	7	1	−0,183 99	0,157 08
22	21	21	0,191 75	−0,302 40	23	7	2	−0,178 59	0,158 00

Table 1 (*continued*)

N	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$	*N*	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$
23	7	3	−0,166 25	0,153 39	23	12	2	−0,040 09	0,082 96
23	7	4	−0,149 37	0,145 23	23	12	3	−0,035 99	0,082 76
23	7	5	−0,128 73	0,134 18	23	12	4	−0,030 74	0,080 85
23	7	6	−0,104 64	0,120 51	23	12	5	−0,024 51	0,077 55
23	7	7	1,911 56	−0,868 38	23	12	6	−0,017 36	0,073 01
23	8	1	−0,135 14	0,133 19	23	12	7	−0,009 30	0,067 28
23	8	2	−0,131 10	0,134 78	23	12	8	−0,000 29	0,060 34
23	8	3	−0,121 95	0,131 72	23	12	9	0,009 72	0,052 14
23	8	4	−0,109 43	0,125 69	23	12	10	0,020 82	0,042 60
23	8	5	−0,094 12	0,117 25	23	12	11	0,033 10	0,031 60
23	8	6	−0,076 23	0,106 62	23	12	12	1,137 13	−0,731 55
23	8	7	−0,055 83	0,093 88	23	13	1	−0,031 11	0,072 63
23	8	8	1,723 80	−0,843 12	23	13	2	−0,028 78	0,075 18
23	9	1	−0,100 84	0,115 18	23	13	3	−0,025 16	0,075 31
23	9	2	−0,097 59	0,117 19	23	13	4	−0,020 65	0,073 90
23	9	3	−0,090 49	0,115 21	23	13	5	−0,015 39	0,071 27
23	9	4	−0,080 86	0,110 69	23	13	6	−0,009 40	0,067 53
23	9	5	−0,069 10	0,104 11	23	13	7	−0,002 70	0,062 73
23	9	6	−0,055 39	0,095 67	23	13	8	0,004 76	0,056 86
23	9	7	−0,039 75	0,085 44	23	13	9	0,013 01	0,049 87
23	9	8	−0,022 15	0,073 39	23	13	10	0,022 13	0,041 70
23	9	9	1,556 16	−0,816 88	23	13	11	0,032 21	0,032 22
23	10	1	−0,075 80	0,101 11	23	13	12	0,043 35	0,021 29
23	10	2	−0,073 01	0,103 37	23	13	13	1,017 74	−0,700 50
23	10	3	−0,067 29	0,102 16	23	14	1	−0,022 05	0,065 95
23	10	4	−0,059 63	0,098 76	23	14	2	−0,019 74	0,068 51
23	10	5	−0,050 35	0,093 57	23	14	3	−0,016 46	0,068 89
23	10	6	−0,039 55	0,086 77	23	14	4	−0,012 51	0,067 90
23	10	7	−0,027 26	0,078 42	23	14	5	−0,007 97	0,065 80
23	10	8	−0,013 45	0,068 50	23	14	6	−0,002 87	0,062 72
23	10	9	0,001 97	0,056 96	23	14	7	0,002 79	0,058 69
23	10	10	1,404 38	−0,789 61	23	14	8	0,009 05	0,053 70
23	11	1	−0,056 98	0,089 78	23	14	9	0,015 94	0,047 71
23	11	2	−0,054 44	0,092 20	23	14	10	0,023 53	0,040 66
23	11	3	−0,049 67	0,091 57	23	14	11	0,031 89	0,032 45
23	11	4	−0,043 41	0,089 00	23	14	12	0,041 12	0,022 94
23	11	5	−0,035 90	0,084 88	23	14	13	0,051 33	0,011 95
23	11	6	−0,027 20	0,079 34	23	14	14	0,905 95	−0,667 87
23	11	7	−0,017 34	0,072 45	23	15	1	−0,014 74	0,060 17
23	11	8	−0,006 28	0,064 19	23	15	2	−0,012 42	0,062 72
23	11	9	0,006 05	0,054 50	23	15	3	−0,009 39	0,063 30
23	11	10	0,019 73	0,043 30	23	15	4	−0,005 85	0,062 63
23	11	11	1,265 45	−0,761 20	23	15	5	−0,001 87	0,060 98
23	12	1	−0,042 48	0,080 46	23	15	6	0,002 54	0,058 44

Table 1 (*continued*)

N	n	I	$a_i(N,n)$	$b_i(N,n)$	N	n	I	$a_i(N,n)$	$b_i(N,n)$
23	15	7	0,007 40	0,055 04	23	18	3	0,005 23	0,049 93
23	15	8	0,012 72	0,050 79	23	18	4	0,008 02	0,049 95
23	15	9	0,018 56	0,045 64	23	18	5	0,010 98	0,049 22
23	15	10	0,024 94	0,039 54	23	18	6	0,014 12	0,047 84
23	15	11	0,031 95	0,032 40	23	18	7	0,017 46	0,045 83
23	15	12	0,039 66	0,024 09	23	18	8	0,021 02	0,043 19
23	15	13	0,048 18	0,014 44	23	18	9	0,024 82	0,039 90
23	15	14	0,057 62	0,003 25	23	18	10	0,028 89	0,035 91
23	15	15	0,800 69	−0,633 44	23	18	11	0,033 28	0,031 16
23	16	1	−0,008 78	0,055 10	23	18	12	0,038 03	0,025 54
23	16	2	−0,006 43	0,057 63	23	18	13	0,043 21	0,018 95
23	16	3	−0,003 58	0,058 35	23	18	14	0,048 88	0,011 20
23	16	4	−0,000 36	0,057 96	23	18	15	0,055 15	0,002 08
23	16	5	0,003 19	0,056 67	23	18	16	0,062 15	−0,008 74
23	16	6	0,007 07	0,054 58	23	18	17	0,070 06	−0,021 69
23	16	7	0,011 30	0,051 72	23	18	18	0,515 96	−0,515 80
23	16	8	0,015 89	0,048 09	23	19	1	0,003 50	0,042 88
23	16	9	0,020 89	0,043 66	23	19	2	0,006 01	0,045 26
23	16	10	0,026 32	0,038 37	23	19	3	0,008 57	0,046 27
23	16	11	0,032 26	0,032 14	23	19	4	0,011 23	0,046 44
23	16	12	0,038 76	0,024 86	23	19	5	0,013 99	0,045 93
23	16	13	0,045 91	0,016 37	23	19	6	0,016 87	0,044 83
23	16	14	0,053 83	0,006 48	23	19	7	0,019 90	0,043 15
23	16	15	0,062 64	−0,005 07	23	19	8	0,023 09	0,040 92
23	16	16	0,701 09	−0,596 90	23	19	9	0,026 47	0,038 09
23	17	1	−0,003 89	0,050 60	23	19	10	0,030 06	0,034 63
23	17	2	−0,001 49	0,053 09	23	19	11	0,033 89	0,030 49
23	17	3	0,001 23	0,053 93	23	19	12	0,038 01	0,025 56
23	17	4	0,004 20	0,053 76	23	19	13	0,042 47	0,019 75
23	17	5	0,007 42	0,052 78	23	19	14	0,047 33	0,012 90
23	17	6	0,010 89	0,051 07	23	19	15	0,052 67	0,004 79
23	17	7	0,014 62	0,048 67	23	19	16	0,058 60	−0,004 85
23	17	8	0,018 63	0,045 57	23	19	17	0,065 27	−0,016 45
23	17	9	0,022 96	0,041 75	23	19	18	0,072 89	−0,030 61
23	17	10	0,027 64	0,037 15	23	19	19	0,429 18	−0,469 97
23	17	11	0,032 72	0,031 71	23	20	1	0,006 29	0,039 50
23	17	12	0,038 25	0,025 32	23	20	2	0,008 85	0,041 81
23	17	13	0,044 31	0,017 84	23	20	3	0,011 37	0,042 87
23	17	14	0,050 99	0,009 09	23	20	4	0,013 92	0,043 16
23	17	15	0,058 40	−0,001 17	23	20	5	0,016 53	0,042 84
23	17	16	0,066 71	−0,013 29	23	20	6	0,019 21	0,041 98
23	17	17	0,606 40	−0,557 86	23	20	7	0,022 00	0,040 61
23	18	1	0,000 15	0,046 55	23	20	8	0,024 90	0,038 72
23	18	2	0,002 60	0,048 99	23	20	9	0,027 94	0,036 30

Table 1 (*continued*)

N	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$	*N*	*n*	*I*	$a_i(N,n)$	$b_i(N,n)$
23	20	10	0,031 14	0,033 32	23	22	13	0,041 74	0,020 69
23	20	11	0,034 53	0,029 71	23	22	14	0,044 98	0,016 06
23	20	12	0,038 14	0,025 40	23	22	15	0,048 45	0,010 50
23	20	13	0,042 02	0,020 30	23	22	16	0,052 22	0,003 82
23	20	14	0,046 22	0,014 25	23	22	17	0,056 37	−0,004 33
23	20	15	0,050 80	0,007 06	23	22	18	0,061 01	−0,014 41
23	20	16	0,055 86	−0,001 52	23	22	19	0,066 32	−0,027 21
23	20	17	0,061 51	−0,011 89	23	22	20	0,072 61	−0,044 13
23	20	18	0,067 94	−0,024 61	23	22	21	0,080 43	−0,067 91
23	20	19	0,075 40	−0,040 57	23	22	22	0,183 84	−0,294 38
23	20	20	0,345 42	−0,419 24	23	23	1	0,012 06	0,030 14
23	21	1	0,008 61	0,036 32	23	23	2	0,014 77	0,032 19
23	21	2	0,011 22	0,038 56	23	23	3	0,017 26	0,033 31
23	21	3	0,013 72	0,039 66	23	23	4	0,019 64	0,033 85
23	21	4	0,016 19	0,040 05	23	23	5	0,021 98	0,033 96
23	21	5	0,018 68	0,039 89	23	23	6	0,024 31	0,033 67
23	21	6	0,021 21	0,039 25	23	23	7	0,026 65	0,033 01
23	21	7	0,023 80	0,038 13	23	23	8	0,029 02	0,031 99
23	21	8	0,026 47	0,036 56	23	23	9	0,031 42	0,030 58
23	21	9	0,029 25	0,034 51	23	23	10	0,033 89	0,028 78
23	21	10	0,032 14	0,031 95	23	23	11	0,036 44	0,026 53
23	21	11	0,035 17	0,028 83	23	23	12	0,039 08	0,023 80
23	21	12	0,038 38	0,025 08	23	23	13	0,041 84	0,020 49
23	21	13	0,041 80	0,020 61	23	23	14	0,044 75	0,016 52
23	21	14	0,045 46	0,015 29	23	23	15	0,047 85	0,011 74
23	21	15	0,049 42	0,008 95	23	23	16	0,051 17	0,005 96
23	21	16	0,053 77	0,001 34	23	23	17	0,054 79	−0,001 11
23	21	17	0,058 59	−0,007 88	23	23	18	0,058 80	−0,009 91
23	21	18	0,064 03	−0,019 25	23	23	19	0,063 33	−0,021 13
23	21	19	0,070 32	−0,033 61	23	23	20	0,068 65	−0,036 06
23	21	20	0,077 79	−0,052 42	23	23	21	0,075 21	−0,057 30
23	21	21	0,263 98	−0,361 82	23	23	22	0,084 21	−0,091 78
23	22	1	0,010 52	0,033 26	23	23	23	0,102 87	−0,209 20
23	22	2	0,013 18	0,035 42					
23	22	3	0,015 67	0,036 54					
23	22	4	0,018 09	0,037 01					
23	22	5	0,020 49	0,036 99					
23	22	6	0,022 90	0,036 54					
23	22	7	0,025 35	0,035 66					
23	22	8	0,027 84	0,034 37					
23	22	9	0,030 41	0,032 65					
23	22	10	0,033 06	0,030 48					
23	22	11	0,035 81	0,027 80					
23	22	12	0,038 70	0,024 57					

Table 2 — Extract of $A(N,n)$, $B(N,n)$ and $C(N,n)$ from MVLUE coefficient tables (Référence [16])

N	n	$a_i(N,n)$	$b_i(N,n)$	$c_i(N,n)$	N	n	$a_i(N,n)$	$b_i(N,n)$	$c_i(N,n)$
2	2	0,711 86	−0,128 64	0,659 55	10	10	0,071 57	0,043 95	0,112 97
3	2	0,818 37	−0,936 49	0,916 04	11	2	0,953 58	−3,721 18	4,275 42
3	3	0,344 71	0,049 54	0,402 86	11	3	0,464 47	−1,324 49	1,339 43
4	2	0,867 02	−1,544 06	1,334 02	11	4	0,301 03	−0,641 50	0,625 90
4	3	0,392 23	−0,236 06	0,433 16	11	5	0,218 96	−0,344 48	0,357 17
4	4	0,225 28	0,069 38	0,293 46	11	6	0,169 36	−0,188 15	0,233 98
5	2	0,895 05	−2,023 12	1,789 17	11	7	0,135 94	−0,096 46	0,171 11
5	3	0,416 82	−0,470 75	0,529 40	11	8	0,111 65	−0,038 93	0,137 04
5	4	0,253 79	−0,077 14	0,291 81	11	9	0,092 93	−0,001 28	0,118 10
5	5	0,166 65	0,067 98	0,231 40	11	10	0,077 67	0,023 92	0,107 70
6	2	0,913 29	−2,416 45	2,244 01	11	11	0,064 17	0,040 65	0,102 51
6	3	0,432 12	−0,666 50	0,652 94	12	2	0,957 53	−3,907 22	4,630 97
6	4	0,269 72	−0,204 05	0,323 72	12	3	0,467 55	−1,418 79	1,471 48
6	5	0,186 11	−0,021 07	0,223 61	12	4	0,303 90	−0,704 81	0,692 72
6	6	0,131 96	0,062 75	0,191 17	12	5	0,221 79	−0,392 02	0,394 83
7	2	0,926 13	−2,749 22	2,685 69	12	6	0,172 26	−0,226 04	0,255 80
7	3	0,442 61	−0,833 41	0,787 95	12	7	0,138 96	−0,127 74	0,183 26
7	4	0,280 16	−0,313 92	0,372 65	12	8	0,114 88	−0,065 34	0,142 82
7	5	0,197 57	−0,100 90	0,235 29	12	9	0,096 48	−0,023 84	0,119 43
7	6	0,146 27	0,002 98	0,182 70	12	10	0,081 75	0,004 56	0,105 74
7	7	0,109 10	0,057 21	0,162 93	12	11	0,069 37	0,024 27	0,097 89
8	2	0,935 65	−3,037 16	3,109 99	12	12	0,058 15	0,037 79	0,093 82
8	3	0,450 28	−0,978 48	0,927 03	13	2	0,960 85	−4,077 99	4,971 95
8	4	0,287 60	−0,410 18	0,430 72	13	3	0,470 14	−1,505 39	1,600 18
8	5	0,205 37	−0,171 87	0,258 05	13	4	0,306 30	−0,763 00	0,759 20
8	6	0,155 03	−0,051 91	0,186 58	13	5	0,224 15	−0,435 79	0,433 38
8	7	0,120 15	0,014 37	0,155 10	13	6	0,174 64	−0,261 00	0,279 11
8	8	0,092 92	0,052 17	0,141 98	13	7	0,141 42	−0,156 71	0,197 25
9	2	0,942 99	−3,290 66	3,516 00	13	8	0,117 47	−0,089 90	0,150 66
9	3	0,456 13	−1,106 54	1,066 44	13	9	0,099 27	−0,044 99	0,122 96
9	4	0,293 18	−0,495 57	0,493 73	13	10	0,084 82	−0,013 79	0,106 12
9	5	0,211 09	−0,235 32	0,287 48	13	11	0,072 91	0,008 33	0,095 85
9	6	0,161 17	−0,101 68	0,198 02	13	12	0,062 65	0,024 15	0,089 76
9	7	0,127 13	−0,025 69	0,155 62	13	13	0,053 15	0,035 28	0,086 49
9	8	0,101 76	0,019 98	0,135 06	14	2	0,963 70	−4,235 78	5,299 47
9	9	0,080 88	0,047 76	0,125 82	14	3	0,472 35	−1,585 43	1,725 46
10	2	0,948 83	−3,516 94	3,904 15	14	4	0,308 34	−0,816 82	0,824 97
10	3	0,460 74	−1,221 03	1,204 24	14	5	0,226 14	−0,476 33	0,472 36
10	4	0,297 54	−0,572 16	0,559 27	14	6	0,176 65	−0,293 44	0,303 42
10	5	0,215 47	−0,292 52	0,321 05	14	7	0,143 48	−0,183 65	0,212 56
10	6	0,165 77	−0,146 88	0,214 37	14	8	0,119 61	−0,112 82	0,160 01
10	7	0,132 13	−0,062 56	0,161 53	14	9	0,101 52	−0,064 81	0,128 15
10	8	0,107 48	−0,010 52	0,134 06	14	10	0,087 25	−0,031 11	0,108 26
10	9	0,088 14	0,022 72	0,119 78	14	11	0,075 59	−0,006 88	0,095 68

Table 2 (*continued*)

N	n	$a_i(N,n)$	$b_i(N,n)$	$c_i(N,n)$	N	n	$a_i(N,n)$	$b_i(N,n)$	$c_i(N,n)$
14	12	0,065 74	0,010 79	0,087 74	17	14	0,056 75	0,004 00	0,074 37
14	13	0,057 08	0,023 75	0,082 90	17	15	0,050 62	0,014 09	0,070 21
14	14	0,048 94	0,033 07	0,080 23	17	16	0,045 01	0,021 90	0,067 50
15	2	0,966 16	−4,382 39	5,614 55	17	17	0,039 52	0,027 81	0,065 91
15	3	0,474 25	−1,659 80	1,847 34	18	2	0,971 87	−4,768 48	6,494 15
15	4	0,310 09	−0,866 87	0,889 83	18	3	0,478 65	−1,855 67	2,193 58
15	5	0,227 84	−0,514 06	0,511 45	18	4	0,314 12	−0,998 75	1,077 82
15	6	0,178 35	−0,323 67	0,328 37	18	5	0,231 75	−0,613 60	0,627 60
15	7	0,145 21	−0,208 80	0,228 81	18	6	0,182 23	−0,403 57	0,404 89
15	8	0,121 40	−0,134 26	0,170 48	18	7	0,149 13	−0,275 42	0,280 85
15	9	0,103 40	−0,083 41	0,134 58	18	8	0,125 41	−0,191 26	0,206 21
15	10	0,089 24	−0,047 44	0,111 74	18	9	0,107 53	−0,133 06	0,158 83
15	11	0,077 73	−0,021 33	0,096 92	18	10	0,093 55	−0,091 25	0,127 58
15	12	0,068 11	−0,002 02	0,087 23	18	11	0,082 27	−0,060 34	0,106 41
15	13	0,059 82	0,012 40	0,080 96	18	12	0,072 95	−0,036 99	0,091 78
15	14	0,052 41	0,023 20	0,077 03	18	13	0,065 08	−0,019 04	0,081 55
15	15	0,045 34	0,031 11	0,074 81	18	14	0,058 30	−0,005 09	0,074 37
16	2	0,968 30	−4,519 28	5,918 14	18	15	0,052 34	0,005 86	0,069 34
16	3	0,475 91	−1,729 25	1,965 91	18	16	0,046 99	0,014 47	0,065 88
16	4	0,311 61	−0,913 62	0,953 63	18	17	0,042 03	0,021 23	0,063 58
16	5	0,229 32	−0,549 32	0,550 45	18	18	0,037 14	0,026 40	0,062 21
16	6	0,179 82	−0,351 96	0,353 71	19	2	0,973 37	−4,882 60	6,768 02
16	7	0,146 70	−0,232 36	0,245 73	19	3	0,479 80	−1,913 56	2,302 94
16	8	0,122 93	−0,154 40	0,181 81	19	4	0,315 17	−1,037 74	1,138 15
16	9	0,104 99	−0,100 92	0,141 97	19	5	0,232 76	−0,643 05	0,665 57
16	10	0,090 90	−0,062 86	0,116 26	19	6	0,183 23	−0,427 24	0,430 49
16	11	0,079 50	−0,035 02	0,099 26	19	7	0,150 14	−0,295 19	0,298 77
16	12	0,070 02	−0,014 25	0,087 89	19	8	0,126 43	−0,208 21	0,218 99
16	13	0,061 93	0,001 47	0,080 25	19	9	0,108 57	−0,147 86	0,167 99
16	14	0,054 85	0,013 45	0,075 19	19	10	0,094 62	−0,104 35	0,134 08
16	15	0,048 44	0,022 57	0,071 94	19	11	0,083 38	−0,072 07	0,110 89
16	16	0,042 24	0,029 37	0,070 08	19	12	0,074 11	−0,047 56	0,094 69
17	2	0,970 19	−4,647 65	6,211 09	19	13	0,066 30	−0,028 63	0,083 22
17	3	0,477 36	−1,794 38	2,081 28	19	14	0,059 60	−0,013 81	0,075 03
17	4	0,312 94	−0,957 47	1,016 31	19	15	0,053 75	−0,002 09	0,069 16
17	5	0,230 61	−0,582 42	0,589 20	19	16	0,048 55	0,007 23	0,064 98
17	6	0,181 10	−0,378 53	0,379 26	19	17	0,043 83	0,014 66	0,062 06
17	7	0,148 00	−0,254 52	0,263 13	19	18	0,039 41	0,020 56	0,060 09
17	8	0,124 25	−0,173 36	0,193 77	19	19	0,035 02	0,025 13	0,058 90
17	9	0,106 35	−0,117 44	0,150 11	20	2	0,974 72	−4,990 71	7,033 32
17	10	0,092 32	−0,077 44	0,121 59	20	3	0,480 84	−1,968 39	2,409 50
17	11	0,080 99	−0,048 01	0,102 48	20	4	0,316 11	−1,074 67	1,197 30
17	12	0,071 60	−0,025 89	0,089 46	20	5	0,233 66	−0,670 96	0,703 08
17	13	0,063 64	−0,009 02	0,080 51	20	6	0,184 12	−0,449 68	0,455 99

Table 2 (*continued*)

N	n	$a_i(N,n)$	$b_i(N,n)$	$c_i(N,n)$	N	n	$a_i(N,n)$	$b_i(N,n)$	$c_i(N,n)$
20	7	0,151 03	−0,313 94	0,316 80	22	12	0,076 81	−0,076 50	0,106 06
20	8	0,127 33	−0,224 30	0,232 03	22	13	0,069 11	−0,054 97	0,091 01
20	9	0,109 49	−0,161 93	0,177 50	22	14	0,062 54	−0,037 89	0,079 90
20	10	0,095 56	−0,116 82	0,141 00	22	15	0,056 86	−0,024 19	0,071 64
20	11	0,084 35	−0,083 24	0,115 83	22	16	0,051 88	−0,013 09	0,065 46
20	12	0,075 12	−0,057 65	0,098 10	22	17	0,047 45	−0,004 02	0,060 82
20	13	0,067 36	−0,037 79	0,085 41	22	18	0,043 46	0,003 41	0,057 35
20	14	0,060 72	−0,022 17	0,076 22	22	19	0,039 82	0,009 53	0,054 78
20	15	0,054 94	−0,009 75	0,069 54	22	20	0,036 43	0,014 56	0,052 92
20	16	0,049 84	0,000 22	0,064 68	22	21	0,033 19	0,018 67	0,051 62
20	17	0,045 25	0,008 24	0,061 16	22	22	0,029 90	0,021 94	0,050 80
20	18	0,041 06	0,014 71	0,058 67	23	2	0,978 05	−5,284 52	7,783 22
20	19	0,037 10	0,019 90	0,056 97	23	3	0,483 39	−2,117 36	2,713 65
20	20	0,033 13	0,023 97	0,055 93	23	4	0,318 42	−1,175 03	1,367 90
21	2	0,975 93	−5,093 41	7,290 62	23	5	0,235 88	−0,746 81	0,812 50
21	3	0,481 77	−2,020 47	2,513 39	23	6	0,186 31	−0,510 70	0,531 38
21	4	0,316 96	−1,109 76	1,255 29	23	7	0,153 21	−0,364 99	0,371 00
21	5	0,234 48	−0,697 47	0,740 08	23	8	0,129 52	−0,268 14	0,271 99
21	6	0,184 92	−0,471 00	0,481 33	23	9	0,111 72	−0,200 30	0,207 38
21	7	0,151 83	−0,331 77	0,334 89	23	10	0,097 83	−0,150 90	0,163 45
21	8	0,128 14	−0,239 61	0,245 25	23	11	0,086 68	−0,113 82	0,132 63
21	9	0,110 32	−0,175 32	0,187 27	23	12	0,077 52	−0,085 34	0,110 49
21	10	0,096 40	−0,128 71	0,148 24	23	13	0,069 84	−0,063 03	0,094 28
21	11	0,085 22	−0,093 89	0,121 15	23	14	0,063 30	−0,045 29	0,082 25
21	12	0,076 01	−0,067 28	0,101 92	23	15	0,057 65	−0,031 00	0,073 22
21	13	0,068 29	−0,046 56	0,088 03	23	16	0,052 70	−0,019 38	0,066 40
21	14	0,061 69	−0,030 19	0,077 87	23	17	0,048 32	−0,009 85	0,061 22
21	15	0,055 96	−0,017 11	0,070 39	23	18	0,044 39	−0,001 99	0,057 29
21	16	0,050 93	−0,006 56	0,064 86	23	19	0,040 83	0,004 52	0,054 31
21	17	0,046 44	0,002 01	0,060 78	23	20	0,037 55	0,009 92	0,052 09
21	18	0,042 37	0,008 99	0,057 79	23	21	0,034 48	0,014 40	0,050 45
21	19	0,038 61	0,014 67	0,055 64	23	22	0,031 52	0,018 09	0,049 30
21	20	0,035 04	0,019 27	0,054 16	23	23	0,028 51	0,021 05	0,048 57
21	21	0,031 44	0,022 91	0,053 24					
22	2	0,977 04	−5,191 19	7,540 44					
22	3	0,482 62	−2,070 05	2,614 74					
22	4	0,317 73	−1,143 16	1,312 15					
22	5	0,235 21	−0,722 72	0,776 56					
22	6	0,185 65	−0,491 31	0,506 47					
22	7	0,152 55	−0,348 76	0,352 97					
22	8	0,128 87	−0,254 20	0,258 59					
22	9	0,111 05	−0,188 09	0,197 25					
22	10	0,097 15	−0,140 05	0,155 74					
22	11	0,085 99	−0,104 08	0,126 77					

Annex A
(informative)

Sample calculations

A.1 Calculation of variance and standard deviation

A.1.1 Data

The 27 values shown in the second column of Table A.1 have been calculated as mass-loss-based corrosion rates, CR, for zinc panels in a 1 year rural atmospheric exposure.

Table A.1 — Zinc corrosion rate — 1 year exposure

Panel	A1-(a) CR µm/a	A1-(b) R µm/a	Rank	Plotting position %
121	2,16	1,70	1,0	1,85
122	2,21	1,86	2,0	5,56
123	2,15	1,88	3,5	11,11
131	2,05	1,88	3,5	11,11
132	2,06	1,90	5,0	16,67
133	2,04	1,92	6,5	22,22
141	1,90	1,92	6,5	22,22
142	2,03	1,93	8,0	27,78
143	2,06	1,99	9,5	33,33
151	2,02	1,99	9,5	33,33
152	2,06	2,01	11,0	38,89
153	1,92	2,02	12,5	44,44
161	2,08	2,02	12,5	44,44
162	2,05	2,03	14,0	50,00
163	1,88	2,04	15,0	53,70
211	1,99	2,05	16,5	59,26
212	2,01	2,05	16,5	59,26
213	1,86	2,06	19,0	68,52
411	1,70	2,06	19,0	68,52
412	1,88	2,06	19,0	68,53
413	1,99	2,08	21,0	78,93
X11	1,93	2,13	22,5	81,48
X12	2,20	2,13	22,5	81,48
X13	2,02	2,15	24,0	87,04
071	1,92	2,16	25,0	90,74
072	2,13	2,20	26,0	94,44
073	2,13	2,21	27,0	98,15

The same data are rearranged in ascending order from the minimum to the maximum value in the third column, A1-(b), and ranked in the fourth column. The fifth column is the plotting position of each datum which is calculated by $F(x_i) = 100(i - 1/2)/n$.

A.1.2 Calculation of statistics

Let x_i be the corrosion rate of the ith panel.

The average corrosion rate of 27 panels, $\bar{x}$;

$$\bar{x}_i = \frac{\sum x_i}{n} = \frac{54{,}43}{27} = 2{,}016 \quad (36)$$

The variance estimate based on this sample;

$$S^2(x) = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \frac{110{,}085 - 27 \times 2{,}016^2}{26} = \frac{0{,}350}{26} = 0{,}013\,5 \quad (37)$$

The standard deviation is:

$$S(x) = 0{,}013\,5^{½} = 0{,}116 \quad (38)$$

The coefficient of variation is 0,116/2,016 × 100 = 5,75 %.

The standard deviation of the average is:

$$S(\bar{x}) = \frac{S(x)}{n^{½}} = \frac{0{,}116}{27^{½}} = 0{,}022\,3 \quad (39)$$

The range, w, is the difference between the largest and smallest values:

$$w = x_{\text{max}} - x_{\text{min}} = 2{,}21 - 1{,}70 = 0{,}41 \quad (40)$$

The mid-range value is:

$$(2{,}21 + 1{,}70)/2 = 1{,}955$$

A.2 Calculation of rank and plotting points

A.2.1 The lowest corrosion rate value (1,70) is assigned a rank “1” of i and the remaining values are arranged in ascending order. Multiple values are assigned a rank of the average rank. For example, both the third and fourth panels have corrosion rates of 1,88 so that the rank is 3,5. See the second column in Table A.1.

A.2.2 The plotting positions are expressed as a percentage in the third column of Table A.1. They are derived from the rank by use of the formula $100(i - 1/2)/n$.

See Table A.1, fifth column, for plotting positions for this data set.

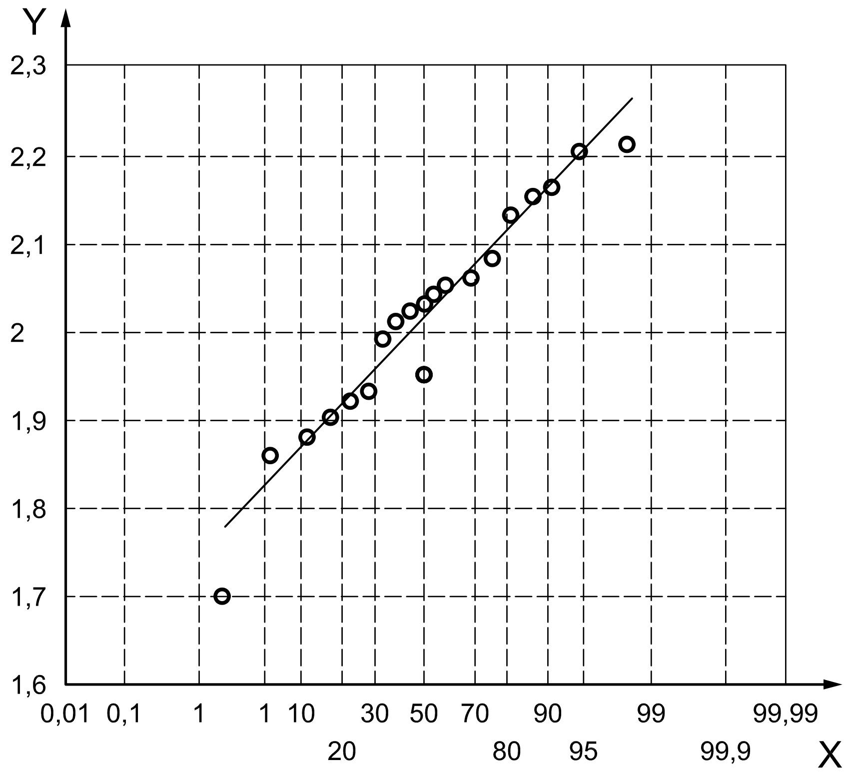
NOTE For extreme-value statistics, the plotting position formula is $100i/(n+1)$. Discussions on the plotting position is given in Reference [19].

The median is the corrosion rate at the 50 % plotting position and is 2,03 for panel 142.

A.3 Probability paper plot of data (See Table A.1)

A.3.1 Corrosion rate versus cumulative probability

The corrosion rate is plotted versus the plotting position on probability paper (see Figure A.1).



Key

X Cumulative probability (%)

Y Corrosion rate (μm/a)

Figure A.1 — Atmospheric corrosion rate of zinc versus cumulative normal distribution

A.3.2 Normal distribution plotting position reference

In order to compare the data points shown in Figure A.1 to what would be expected for a normal distribution, a straight line on the plot may be constructed to show a normal distribution.

— Plot the average value at 50 %, 2,016 at 50 %;

— Plot the average +1 standard deviation at 84,13 %, i.e. 2,016 + 0,116 = 2,136 at 84,13 %;

— Plot the average –1 standard deviation at 15,8 %, i.e. 2,016 – 0,116 = 1,900 at 15,87 %;

— Connect these three points with a straight line.

A.4 Evaluation of outlier

Data: see A.1, Table A.1 and Figure A.1.

— Is the 1,70 result (panel 411) an outlier? This point appears to be out of line in Figure A.1.

— Use Dixon's test.

— For this example, $\alpha = 0{,}05$ was chosen, i.e. the probability that this point is this far out of line based on normal probability is 5 % or less.

— The number of data points is 27.

$$r_{22} = \frac{x_3 - x_1}{x_{n-2} - x_1} = \frac{1{,}88 - 1{,}70}{2{,}16 - 1{,}70} = 0{,}391 \quad (41)$$

— The Dixon criterion at $\alpha = 0{,}05$, $n = 27$, is 0,393.

— The r_{22} value does not exceed the Dixon criterion for the value of n and the value of α chosen so that the 1,70 value is not an outliner by this test.

— A t-test is recommended as being the best test in this case:

$$t_1 = \frac{\bar{x} - x_1}{S} = \frac{2{,}016 - 1{,}70}{0{,}116} = 2{,}724 \quad (42)$$

Critical value t for $\alpha = 0{,}05$ and $n = 27$ is 2,698. Therefore, by this criterion, the 1,70 value is an outlier because the calculated t_1 value exceeds the critical t-value.

The 1,70 value for panel 411 does appear to be out of line as compared to the other values in this data set. The t-test confirms this conclusion if $\alpha = 0{,}05$ is chosen. The next step is to review the calculations that lead to the determination of a 1,70 value for this panel. The original and final mass values and panel size measurements require checking and comparison to the values obtained from the other panels.

If no errors are found, then the panel itself should be retrieved and examined to determine whether there is any evidence of corrosion products or other extraneous material that would cause its final mass to be greater than it should have been. If a reason can be found to explain the low mass-loss value, then the result can be excluded from the data set without reservation. If this point is excluded, the statistics for this distribution become:

$$\bar{x} = 2{,}028 \quad (43)$$

$$S^2(x) = 0{,}010\,2 \quad (44)$$

$$S(\bar{x}) = \frac{0{,}101}{26^{1/2}} = 0{,}019\,8 \quad (45)$$

The coefficient of variation is given by (0,101/2,028)(100) = 4,98 %.

$$S(x) = 0{,}101 \quad (46)$$

Median = 2,035

$$W = 2{,}21 - 1{,}86 = 0{,}35 \quad (47)$$

Mid-range = (2,21 + 1,86)/2 = 2,035

The average, median and mid-range are closer together after excluding the 1,70 value, as expected, although the changes are relatively small.

A.5 Confidence interval for corrosion rate

Data: see A.1, Table A.1 above and A.4, excluding the panel 411 result.

Significance level: $\alpha = 0{,}05$

Confidence interval calculation, CI:

$$\text{CI} = \bar{x} \pm tS(\bar{x})$$

For $\alpha = 0{,}05$ and $n = 25$, t is 2,060

95 % confidence interval for the average corrosion rate.

$$\bar{x} \pm (2{,}060) \times (0{,}019\,8) = \bar{x} \pm 0{,}041 \text{ or } 1{,}987 \text{ to } 2{,}069 \tag{48}$$

Note that this interval refers to the average corrosion rate. If one is interested in the interval in which 95 % of future measurements of corrosion rates of zinc panels exposed under those conditions will fall, it may be calculated as follows:

$$\bar{x} \pm tS(x) \tag{49}$$

$$\bar{x} \pm 2{,}060 \times 0{,}101 = \bar{x} \pm 0{,}208 = 1{,}820 - 2{,}236 \tag{50}$$

A.6 Difference between average values

A.6.1 Data

Triplicate zinc flat panels and wire helices were exposed for a one-year period at the 250 m lot at Kure Beach, North Carolina. The corrosion rates were calculated from the loss in mass after cleaning the specimens. The corrosion rate values are given in Table A.2.

Table A.2 — Corrosion rates, CR, of zinc alloy after one year of atmospheric exposure at the 250 m lot at Kure Beach, µm/year

Panel_ID	CR	Helix_ID	CR
13A111P	2,04	13A111H	2,49
13A112P	2,30	13A112H	2,54
13A113P	2,38	13A113H	2,62

Panel average: $\bar{x}_p = 2{,}24$ Helix average: $\bar{x}_h = 2{,}55$

Panel standard deviation = 0,18 Helix standard deviation = 0,066

A.6.2 Question

Are the helices corroding significantly faster than the panels? The null hypothesis is therefore that the panels and helices are corroding at the same rate. We will choose $\alpha = 0{,}05$, i.e. the probability of erroneously rejecting the null hypothesis is one chance in twenty.

A.6.3 Calculations

A.6.3.1 Note that the standard deviations for the panels and helices are different. If they are not significantly different, they may be pooled to yield a larger data set to test the hypothesis. The F-test may be used for this purpose.

$$F = \frac{S^2(x_p)}{S^2(x_h)} = \frac{0{,}18^2}{0{,}066^2} = 7{,}438 \tag{51}$$

The critical F for $\alpha = 0{,}05$ and both numerator and denominator degrees of freedom of 2 is 19,00. The calculated F is less than the critical F-value, so that the hypothesis that the two standard deviations are not significantly different may be accepted. As a consequence, the standard deviations may be pooled.

A.6.3.2 Calculation of pooled variance

$$t = \frac{\bar{x}_h - \bar{x}_p}{S_{p(x)}\left(\frac{1}{n_p} - \frac{1}{n_h}\right)^{1/2}} \tag{52}$$

and

$$S_p^2(x) = \frac{(n_p - 1)S^2(x_p) + (n_h - 1)S^2(x_h)}{(n_p - 1) + (n_h - 1)} \tag{53}$$

substituting:

$$S_p^2(x) = \frac{2(0{,}18)^2 + 2(0{,}066)^2}{2 + 2} = 0{,}018 \tag{54}$$

A.6.3.3 Calculation of t-statistic:

$$t = \frac{2{,}55 - 2{,}24}{\sqrt{0{,}018}\sqrt{\frac{1}{3} + \frac{1}{3}}} = \frac{0{,}31}{0{,}110} = 2{,}83 \tag{55}$$

$$n_{\mathrm{DF}} = 2 + 2 = 4$$

A.6.3.4 Conclusion — The critical value of t for $\alpha = 0{,}05$ and $n\mathrm{DF} = 4$ is 2,776. The calculated value for t exceeds the critical value and therefore the null hypothesis can be rejected, i.e. the helices are corroding at a significantly higher rate than the panels.

A.6.3.5 Discussion — Usually the level for the F-test shown in A.6.3.1 should be carried out at a more stringent significance level than in the t-test, e.g. 0,01 rather than 0,05. It is also desirable to consider the power of this test. In the event that the F-test did show a significant difference, it is essential to use a different procedure to carry out the t-test. Details on these procedures are beyond the scope of this annex but are covered in Reference [11].

A.7 Curve fitting — Regression analysis example

A.7.1 The mass loss per unit area of zinc is usually assumed to be linear with exposure time in atmospheric exposures. However, most other metals are better fitted with power function kinetics in atmospheric exposures. An exposure program was carried out with a commercial-purity rolled zinc alloy for 20 years in an industrial site. How can the mass loss results be converted to an expression that describes the results?

A.7.2 Forty panels of 16-gauge rolled zinc strips were cut to approximately 100 mm × 150 mm. The panels were cleaned, weighed and exposed at the same time. Five panels were removed after 0,5, 1, 2, 4, 6, 10, 15, and 20 years exposure. The panels were then cleaned and reweighed. The mass-loss values were calculated and converted to mass loss per unit area. The results are shown in Table A.3.

Table A.3 — Mass loss per unit area of zinc in the atmosphere, in mg/cm²

Exposure duration	**Panel designation**					**Average**	**Standard deviation**
Years	**1**	**2**	**3**	**4**	**5**		
0,5	0,387	0,392	0,362	0,423	0,319	0,376 6	0,038 8
1	0,829	0,759	0,801	0,738	0,780	0,781 4	0,035 5
2	1,793	1,667	1,585	1,727	1,642	1,682 8	0,080 0
4	3,688	3,406	3,297	3,280	3,297	3,393 6	0,172 0
6	5,825	5,257	5,391	5,280	5,333	5,417 2	0,233 8

A.7.3 Corrosion of zinc in the atmosphere is usually assumed to be a constant-rate process. This would imply that the mass loss per unit area, m, is related to exposure time, T, by:

$$m = k_1 T \tag{56}$$

where

k_1 is the corrosion rate constant.

Most other metals are better fitted by a power function such as:

$$m = kT^b \tag{57}$$

where

k is the mass-loss coefficient;

b is the time exponent.

The data in Table A.3 may be handled in several ways. Linear regression analysis can be applied to get a value for k_1 that minimizes the variance for the constant rate expression above, or any linear expression such as:

$$m = a + k_2 T \tag{58}$$

where a is a constant.

Alternatively, a non-linear regression analysis may be used that yields values for k and b that minimize the variance from the measured values to the calculated value for m at any time using the power function above. All of these approaches assume that the variance observed at short exposure times is comparable to variances at a long exposure time. However, the data in Table A.3 show standard deviations that are roughly proportional to the average value at each time, and so the assumption of comparable variance is not justified by the data at hand.

Another approach is to employ a logarithmic conversion of the data. A transformed data set is shown in Table A.4 where $x = \log T$ and $y = \log m$. These data may be handled in a linear-regression analysis. Such an analysis is equivalent to the power function fit with the k and b values minimizing the variance of the transformed variable, y.

The logarithmic conversion becomes:

$$\log m = \log k + \log T \tag{59}$$

or

$$y = a + bx \tag{60}$$

where $a = \log k$.

Note that the standard deviation values, $s(y_i)$, in Table A.4 are approximately constant for both short and long exposure times.

Table A.4 — Log of data from Table A.3

i	x_i	y_{i1}	y_{i2}	y_{i3}	y_{i4}	y_{i5}	**Average**	**Standard deviation**
1	−0,301 03	−0,412 29	−0,406 71	−0,441 29	−0,373 66	−0,496 21	−0,426 03	0,046 00
2	0,000 00	−0,081 44	−0,119 76	−0,096 37	−0,131 94	−0,107 90	−0,107 48	0,019 68
3	0,301 03	0,253 58	0,221 94	0,200 03	023729	0,215 37	0,225 64	0,020 56
4	0,602 06	0,566 79	0,532 24	0,518 12	0,515 87	0,518 12	0,530 23	0,021 44
5	0,778 15	0,765 30	0,720 74	0,731 67	0,722 63	0,726 97	0,733 46	0,018 29
6	1,000 00	1,031 77	0,989 98	0,984 66	0,998 52	0,992 77	0,999 54	0,018 70

The values in Table A.4 were used to calculate the following:

$$\sum x = 23{,}110\,56 \qquad \sum y = 20{,}922\,32 \qquad n = 39 \tag{61}$$

$$\sum x^2 = 24{,}742\,305 \qquad \sum y^2 = 24{,}159\,116 \qquad \sum xy = 24{,}341\,352 \tag{62}$$

$$\sum{}' x^2 = \sum x^2 - \left(\sum x\right)^2 / n \qquad \bar{x} = 0{,}592\,758 \qquad y = 0{,}536\,470 \tag{63}$$

$$\sum{}' x^2 = 24{,}742\,305 - (23{,}110\,56)^2 / 39 = 11{,}047\,485 \tag{64}$$

$$\sum{}' y^2 = 24{,}159\,116 - (20{,}922\,32)^2 / 39 = 12{,}934\,924 \tag{65}$$

$$\sum{}' xy = 24{,}341\,352 - (23{,}110\,56)^2 / 39 = 11{,}943\,236 \tag{66}$$

$$\sum{}' C^2 = \frac{\left(\sum{}' xy\right)^2}{\sum{}' x^2} = \frac{(11{,}943\,236)^2}{11{,}047\,485} = 12{,}911\,616 \tag{67}$$

$$\sum\sum{}' y^2 = 0{,}017\,810 \tag{68}$$

$$b = \frac{\sum{}' xy}{\sum{}' x^2} = \frac{11{,}943\,236}{11{,}047\,485} = 1{,}081\,08 \tag{69}$$

$$a = \bar{y} - b\bar{x} = 0{,}536\,470 - (1{,}081\,08)(0{,}592\,758) = -0{,}104\,16 \tag{70}$$

$$k = 0{,}786\,8 \tag{71}$$

A.7.4 Analysis of variance — One approach to test the adequacy of the analysis is to compare the residual variance from the regression to the error variance as estimated by the variance found in replication. The null hypothesis in this case is that the residual variance from the calculated regression expression is not significantly greater than the replication variance.

Table A.5 — Analysis of variance

Item	Expression	Symbol	SOS[a]	DF[b]	MS[c]
xy regression	$\left(\sum{}'xy\right)^2 / \sum{}'x^2$	$\sum{}'C^2$	12,911 616	1	12,911 616
Residual from xy line	$\sum{}'y^2 - \sum{}'C^2 - \sum\sum{}'y^2$	$\sum{}'\hat{y}^2$	0,005 498	6	0,000 916
Replication variance	—	$\sum\sum{}'y^2$	0,017 810	31	0,000 575

a SOS = Sum of squares value.

b DF = Degrees of freedom.

c MS = Mean squares value.

F-test on hypothesis in A.7.2:

$$F = 0{,}000\ 916 / 0{,}000\ 575 = 1{,}59 \tag{72}$$

The critical F-value at an α value of 0,05 and 6/31 degrees of freedom is 2,41. Therefore, the residual from the regression expression is not significantly greater than the replication error variance at the 95 % confidence limit. Thus, the expression derived is sufficient to describe the results for the time period.

A.7.5 The next question to be answered is whether the expression found is better than a linear expression. A linear kinetics function has a slope of one after the log conversion. Therefore, the reformulated question is whether the b value is significantly different from 1. The null hypothesis is that the b value is not different from 1,000 0 with $\alpha = 0{,}05$. A t-test will be used:

$$b = 1{,}081\,08 \tag{73}$$

$$S^2_{(b)} = \frac{\sum{}'\hat{y}^2}{(n-2)\sum{}'x^2} = 0{,}005\ 498 / (6)(11{,}047\ 49) = 0{,}000\ 082\ 9 \tag{74}$$

$$S_{(b)} = 0{,}009\,11 \tag{75}$$

$$t = (1{,}081\,08 - 1{,}000\ 00) / 0{,}009\,11 = 8{,}90 \tag{76}$$

The critical t at 6 degrees of freedom and $\alpha = 0{,}05$ is 2,45 which is smaller than the t calculated above. The null hypothesis may be rejected, and the slope is significantly different from 1.

A.7.6 Confidence interval for regression — A confidence interval calculated from the replicate information at each exposure time represents an interval in which the unknown mean mass-loss value will be located unless a 1 in 20 chance has occurred in the sampling of this experiment. On the other hand, a confidence interval calculated from the regression results represents an interval which will cover the unknown regression line unless a 1 in 20 chance has occurred in the sampling of this experiment.

The confidence interval for each exposure time is equally spaced around the average of the log values. This will also be true for the regression confidence interval. However, when these intervals are plotted on linear coordinates, the interval will appear to be unsymmetrical. An example of a confidence interval calculation is given hereafter:

Calculation of the confidence interval, CI, for the average value:

Exposure time $T = 6$ years, $\alpha = 0{,}05$, DF = 4, $t = 2{,}78$, $i = 5$

$$\bar{y}_5 = 0{,}733\ 46 \qquad s(y_5) = 0{,}018\ 29 \tag{77}$$

$$\mathrm{CI} = \bar{y} \pm \frac{ts}{\sqrt{n}} = \bar{y} \pm \frac{2,78 \times 0,018\ 29}{\sqrt{5}} = \bar{y} \pm 0,022\ 74 \quad (78)$$

CI = 0,710 72 to 0,756 20

Converting y to m: CI = 5,137 mg/cm^2 to 5,704 mg/cm^2

Calculation of the confidence interval for the regression expression exposure time $T = 6$, $\alpha = 0,05$, $n_{\mathrm{DF}} = 6$, $t = 2,45$, $i = 5$:

$$S^2\left(\hat{y}_1\right) = S^2\left(\bar{y}\right)\left[\frac{1}{n} + \frac{\left(\bar{x} - x_1\right)^2}{\sum{}' x_i^{\,2}}\right] \quad (79)$$

$$\sum{}'' x_i^2 = 11{,}047\ 5 \quad (80)$$

$$\bar{x} = 0,592\ 58 \qquad x_5 = \log 6 = 0,778\ 15 \quad (81)$$

$$s^2\left(\hat{y}_i\right) = \frac{\sum \hat{y}^2}{n-2} = \frac{0,005\ 498}{6} = 0,000\ 916 \quad (82)$$

$$s^2\left(\hat{y}_5\right) = 0,000\ 916\left[\frac{1}{6} + \frac{0,059\ 258 - 0,778\ 15^2}{11{,}047\ 5}\right] = 0,000\ 155\ 6 \quad (83)$$

$$s\left(\bar{y}_5\right) = 0,012\ 47 \quad (84)$$

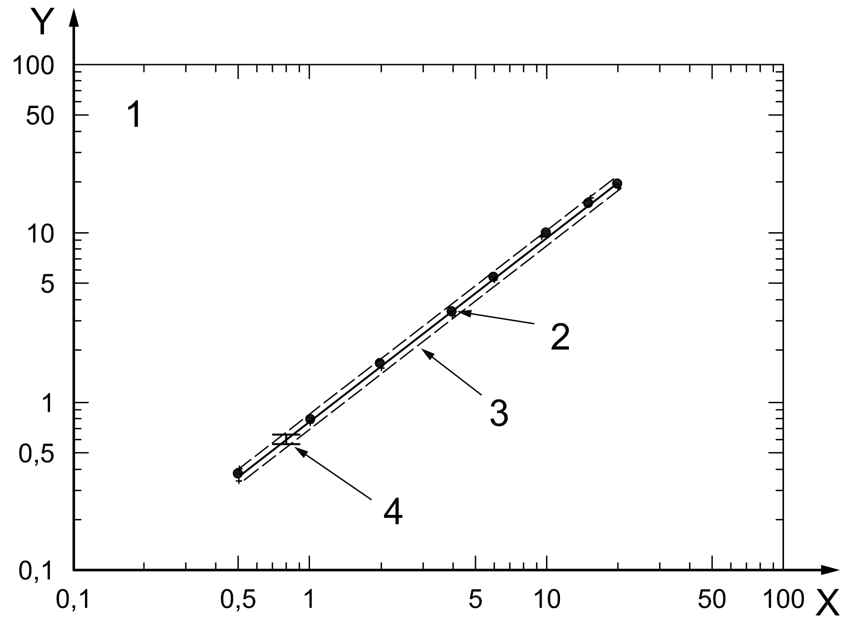
$$\hat{y}_5 = -0,104\ 16 + 1{,}081\ 08 x_5 = -0,104\ 16 + 1{,}081\ 08\left(0,778\ 15\right) = 0,737\ 08 \quad (85)$$

$$\mathrm{CI}_{yi} = \hat{y} \pm ts\left(\hat{y}_i\right) = 0,737\ 08 \pm 2,45\left(0,012\ 47\right) = 0,706\ 53 \text{ à } 0,767\ 63 \quad (86)$$

$$\mathrm{CI}_m = 5,088 \text{ to } 5,856 \quad (87)$$

Note that the confidence interval calculated from the regression expression is slightly larger than that calculated from the replicate values at that exposure time.

Figure A.2 is a log–log plot showing the regression equation with the 95 % confidence intervals for the regression shown as dashed lines, and the averages and corresponding confidence intervals shown as bars. Figure A.3 shows the same information on linear coordinates.



Key

X exposure time (years)

Y mass loss (mg/cm^2)

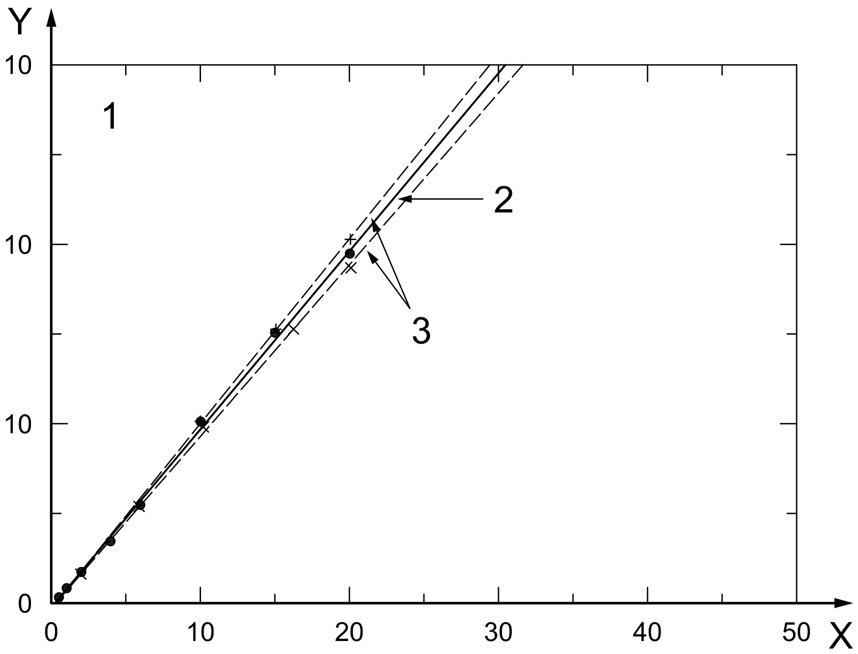
1 rolled zinc

2 regression

3 95 % confidence level of regression log average

4 95 % confidence interval average

Figure A.2 — Mass loss of zinc in the atmosphere with exposure time plotted on log–log paper



Key

X exposure time (years)

Y mass loss (mg/cm^2)

1 rolled zinc

2 regression

3 95 % confidence intervals for regression

Figure A.3 — Mass loss of zinc in the atmosphere with time in linear coordinates

A.7.7 Other statistics from the regression — Standard error of estimate, $S(\hat{y})$, for the logarithmic expression.

$$S(\hat{y}) = \sqrt{0{,}000\ 916} = 0{,}030\ 26 \quad (88)$$

Correlation coefficient, r, for the logarithmic expression.

$$r = 0{,}999\ 1 \quad (89)$$

r or r^2 are often quoted as a measure of the quality of fit of a regression expression. However, the correlation coefficient calculated for the logarithmic expression is not comparable to a correlation coefficient

$$r^2 = \frac{\left(\sum{'}xy\right)^2}{\sum{'}x^2 \sum{'}y^2} = \frac{(11{,}943\ 236)^2}{11{,}047\ 485 \times 12{,}934\ 924} = 0{,}998\ 198 \quad (90)$$

calculated for a non-transformed regression.

A.7.8 Discussion — The use of a log conversion to obtain a power function fit is convenient and simple, but has some limitations. The log conversion tends to bias the values on the low side of the linear average. It also produces a non-linear error function. In the example above, the use of a log conversion produces an almost constant standard deviation over the range of exposure times.

A linear regression analysis may also be used with these mass-loss results, and the corresponding expression may be a reasonable estimate of mass-loss performance for rolled zinc in this atmosphere. However, neither a linear nor a non-linear power-function regression analysis will yield a confidence interval that matches as closely the replicate data confidence intervals as the logarithmic transformation shown above.

The regression expression can be used to project future results by extrapolation of the results beyond the range of data available. This type of calculation is generally not advisable unless there is good information indicating that the procedure is valid, i.e. that no changes have occurred in any of the environmental and surface conditions that govern the kinetics of the corrosion reaction.

A.8 Extreme-value analysis of pitting

A.8.1 The maximum pit depths measured for nine 50 cm × 50 cm blocks in the bottom plate of a petroleum storage tank made of mild steel that had been in use for 20 years are:

1,6 mm, 2,0 mm, 1,8 mm, 2,5 mm, 1,3 mm, 0,8 mm, 2,3 mm, 1,0 mm, and 1,5 mm

The total area was 125 m^2 and the initial thickness of the bottom plate was 6,0 mm. The ranking of these data is shown in Table A.6.

A.8.2 Using the MVLUE coefficient table (Table 1) and using $N = n = 9$, λ is found to be 1,35 mm and α is found to be 0,544 mm. The results of these calculations are shown in Table A.6.

Table A.6 — Maximum pit-depth data analysis

i	x_i	a_i	b_i	$a_i x_i$	$b_i x_i$	$F(x_i)$
1	2,5	0,032 29	0,088 39	0,081	0,221	0,9
2	2,3	0,047 96	0,094 37	0,110	0,217	0,8
3	2,0	0,063 40	0,091 97	0,127	0,184	0,7
4	1,8	0,079 57	0,082 65	0,143	0,149	0,6
5	1,6	0,097 22	0,065 57	0,156	0,105	0,5
6	1,5	0,117 36	0,037 98	0,176	0,057	0,4

A.8.3 Calculating x_m and P for the total 125 m^2

$$\lambda = \sum a_i x_i = 1{,}348 \qquad \alpha = \sum b_i x_i = 0{,}545 \tag{91}$$

$$x_m = \lambda + \alpha \ln T = 1{,}348 + (0{,}545)\ln(125 / 0{,}25) = 4{,}73 \text{ mm} \tag{92}$$

$$P = 1 - \exp\left\{-\exp\left[-(d - x_m) / \alpha\right]\right\} = 1 - \exp\left\{-\exp\left[-(6{,}00 - 4{,}73) / 0{,}545\right]\right\} = 0{,}093 \tag{93}$$

The data points are plotted in Figure A.4.

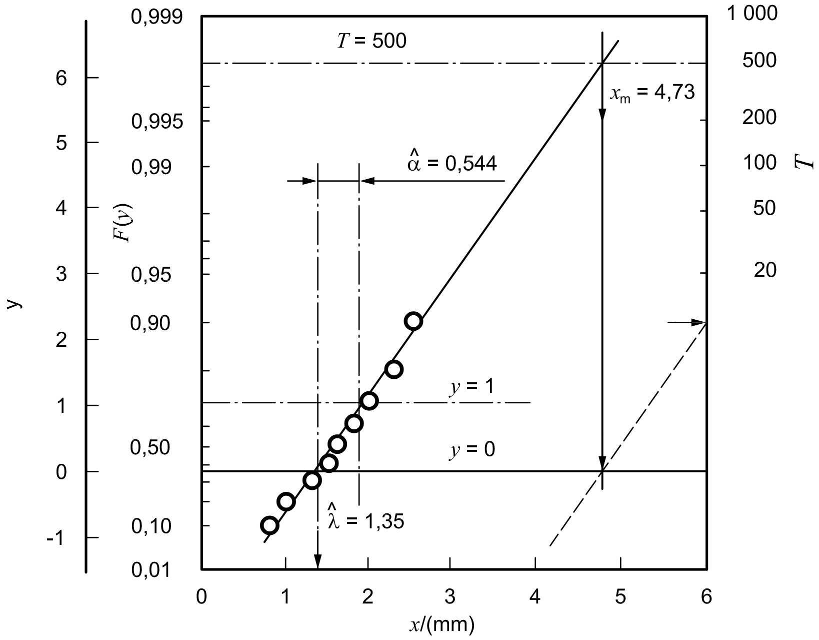


Figure A.4 — Gumbel plot for data in Table 1

A.8.4 The error variance for x_m is 1,02 mm on the basis of the MVLUE coefficients, which is not small compared with the value of x_m of 4,73 mm. However, this is all the accuracy that can be attained because $N = 9$ and $T = 125/0,25 = 500$.

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