BS ISO 12107:2003

Metallic materials — Fatigue testing — Statistical planning and analysis of data

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British Standards

National foreword

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The UK participation in its preparation was entrusted by Technical Committee ISE/NFE/4, Mechanical testing of metals, to Subcommittee ISE/NFE/4/6, Fatigue testing of metals, which has the responsibility to:

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— monitor related international and European developments and promulgate them in the UK.

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INTERNATIONAL STANDARD

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Metallic materials — Fatigue testing — Statistical planning and analysis of data

Matériaux métalliques — Essais de fatigue — Programmation et analyse statistique de données

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Foreword

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ISO 12107 was prepared by Technical Committee ISO/TC 164, *Mechanical testing of metals*, Subcommittee SC 5, *Fatigue testing*.

Introduction

It is known that the results of fatigue tests display significant variations even when the test is controlled very accurately. In part, these variations are attributable to non-uniformity of test specimens. Examples of such non-uniformity include slight differences in chemical composition, heat treatment, surface finish, etc. The remaining part is related to the stochastic process of fatigue failure itself that is intrinsic to metallic engineering materials.

Adequate quantification of this inherent variation is necessary to evaluate the fatigue property of a material for the design of machines and structures. It is also necessary for test laboratories to compare materials in fatigue behaviour, including its variation. Statistical methods are necessary to perform these tasks. They include both the experimental planning and procedure to develop fatigue data and the analysis of the results.

Metallic materials — Fatigue testing — Statistical planning and analysis of data

1 Scope

1.1 Objectives

This International Standard presents methods for the experimental planning of fatigue testing and the statistical analysis of the resulting data. The purpose is to determine the fatigue properties of metallic materials with both a high degree of confidence and a practical number of specimens.

1.2 Fatigue properties to be analysed

This International Standard provides a method for the analysis of fatigue life properties at a variety of stress levels using a relationship that can linearly approximate the material's response in appropriate coordinates.

Specifically, it addresses:

a) the fatigue life for a given stress, and

b) the fatigue strength for a given fatigue life.

The term “stress” in this International Standard can be replaced by “strain”, as the methods described are also valid for the analysis of life properties as a function of strain. Fatigue strength in the case of strain-controlled tests is considered in terms of strain, as it is ordinarily understood in terms of stress in stress-controlled tests.

1.3 Limit of application

This International Standard is limited to the analysis of fatigue data for materials exhibiting homogeneous behaviour due to a single mechanism of fatigue failure. This refers to the statistical properties of test results that are closely related to material behaviour under the test conditions.

In fact, specimens of a given material tested under different conditions may reveal variations in failure mechanisms. For ordinary cases, the statistical property of resulting data represents one failure mechanism and may permit direct analysis. Conversely, situations are encountered where the statistical behaviour is not homogeneous. It is necessary for all such cases to be modelled by two or more individual distributions.

An example of such behaviour is often observed when failure can initiate from either a surface or internal site at the same level of stress. Under these conditions, the data will have mixed statistical characteristics corresponding to the different mechanisms of failure. These types of results are not considered in this International Standard because a much higher complexity of analysis is required.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534 (all parts), *Statistics — Vocabulary and symbols*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534 and the following apply.

3.1 Terms related to statistics

3.1.1
confidence level
value $1 - \alpha$ of the probability associated with an interval of statistical tolerance

3.1.2
degree of freedom
number calculated by subtracting from total number of items of test data the number of parameters estimated from the data

3.1.3
distribution function
function giving, for every value x, the probability that the random variable X is less than or equal to x

3.1.4
estimation
operation made for the purpose of assigning, from the values observed in a sample, numerical values to the parameters of a distribution from which this sample has been taken

3.1.5
population
totality of individual materials or items under consideration

3.1.6
random variable
variable that may take any value of a specified set of values

3.1.7
sample
one or more items taken from a population and intended to provide information on the population

3.1.8
size
n
number of items in a population, lot, sample, etc.

3.1.9
standard deviation
σ
positive square root of the mean squared deviation from the arithmetic mean

3.2 Terms related to fatigue

3.2.1
fatigue life
N
number of stress cycles applied to a specimen, at an indicated stress level, before it attains a failure criterion defined for the test

3.2.2
fatigue limit
fatigue strength at infinite life

3.2.3
fatigue strength
value of stress level S, expressed in megapascals, at which a specimen would fail at a given fatigue life

3.2.4
specimen
portion or piece of material to be used for a single test determination and normally prepared in a predetermined shape and in predetermined dimensions

3.2.5
stress level
S
intensity of the stress under the conditions of control in the test

EXAMPLES Amplitude, maximum, range.

3.2.6
stress step
d
difference between neighbouring stress levels, expressed in megapascals, when conducting the test by the staircase method

4 Statistical distributions in fatigue properties

4.1 Concept of distributions in fatigue

The fatigue properties of metallic engineering materials are determined by testing a set of specimens at various stress levels to generate a fatigue life relationship as a function of stress. The results are usually expressed as an S-N curve that fits the experimental data plotted in appropriate coordinates. These are generally either log-log or semi-log plots, with the life values always plotted on the abscissa on a logarithmic scale.

Fatigue test results usually display significant scatter even when the tests are carefully conducted to minimize experimental error. A component of this variation is due to inequalities, related to chemical composition or heat treatment, among the specimens, but another component is related to the fatigue process, an example being the initiation and growth of small cracks under test environments.

The variation in fatigue data is expressed in two ways: the distribution of fatigue life at a given stress and the distribution of strength at a given fatigue life (see [1] to [5]).

4.2 Distribution of fatigue life

Fatigue life, N, at a given test stress, S, is considered as a random variable. It is expressed as the normal distribution of the logarithm of the fatigue life. This relationship is:

$$P(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2} \left(\frac{x - \mu_x}{\sigma_x} \right)^2 \right] \mathrm{d}x \qquad (1)$$

where $x = \log N$ and μ_x and σ_x are, respectively, the mean and the standard deviation of x.

Equation (1) gives the cumulative probability of failure for x. This is the proportion of the population failing at lives less than or equal to x.

Equation (1) does not relate to the probability of failure for specimens at or near the fatigue limit. In this region, some specimens may fail, while others may not. The shape of the distribution is often skewed, displaying even greater scatter on the longer-life side. It also may be truncated to represent the longest failure life observed in the data set.

This International Standard does not address situations in which a certain number of specimens may fail, but the remaining ones do not.

Other statistical distributions can also be used to express variations in fatigue life. The Weibull [4] distribution is one of the statistical models often used to represent skewed distributions.

Figure 1 shows an example of data from a fatigue test conducted with a statistically based experimental plan using a large number of specimens (see [5]). The shape of the fatigue life distributions is demonstrated for explanatory purposes.

4.3 Distribution of fatigue strength

Fatigue strength at a given fatigue life, N, is considered as a random variable. It is expressed as the normal distribution:

$$P(y) = \frac{1}{\sigma_y \sqrt{2\pi}} \int_{-\infty}^{y} \exp\left[-\frac{1}{2} \left(\frac{y - \mu_y}{\sigma_y} \right)^2 \right] \mathrm{d}y \qquad (2)$$

where $y = S$ (the fatigue strength at N), and μ_y and σ_y are, respectively, the mean and the standard deviation of y.

Equation (2) gives the cumulative probability of failure for y. It defines the proportion of the population presenting fatigue strengths less than or equal to y.

Other statistical distributions can also be used to express variations in fatigue strength. When a linear relationship is assumed between stress and fatigue life using log-log coordinates, the distribution of $y = \log S$ is assumed to be normal as long as $x = \log N$ is normal.

Figure 2 is based on the same experimental data as Figure 1. The variation in the fatigue property is expressed here in terms of strength at typical fatigue lives (see [5]).

5 Statistical planning of fatigue tests

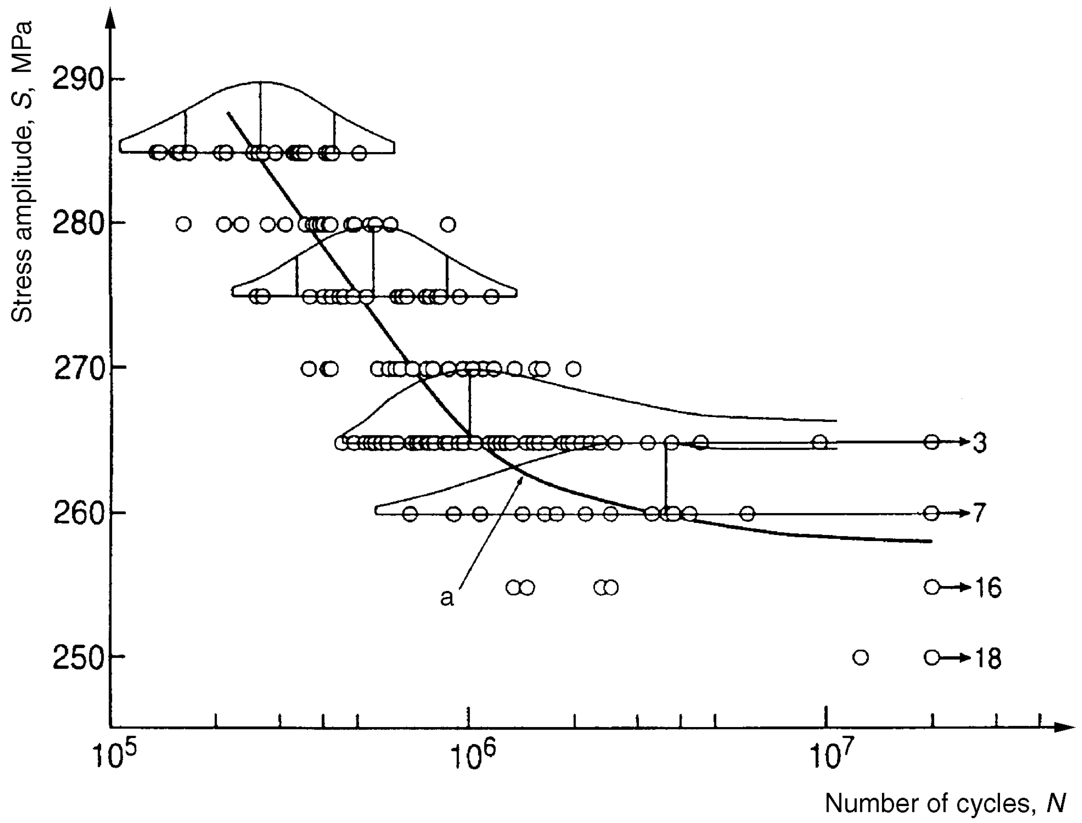
5.1 Sampling

It is necessary to define clearly the population of the material for which the statistical distribution of fatigue properties is to be estimated. Specimen selection from the population shall be performed in a random fashion. It is also important that the specimens be selected so that they accurately represent the population they are intended to describe.

If the population consists of several lots or batches of material, the test specimens shall be selected randomly from each group in a number proportional to the size of each lot or batch. The total number of specimens taken shall be equal to the required sample size, n.

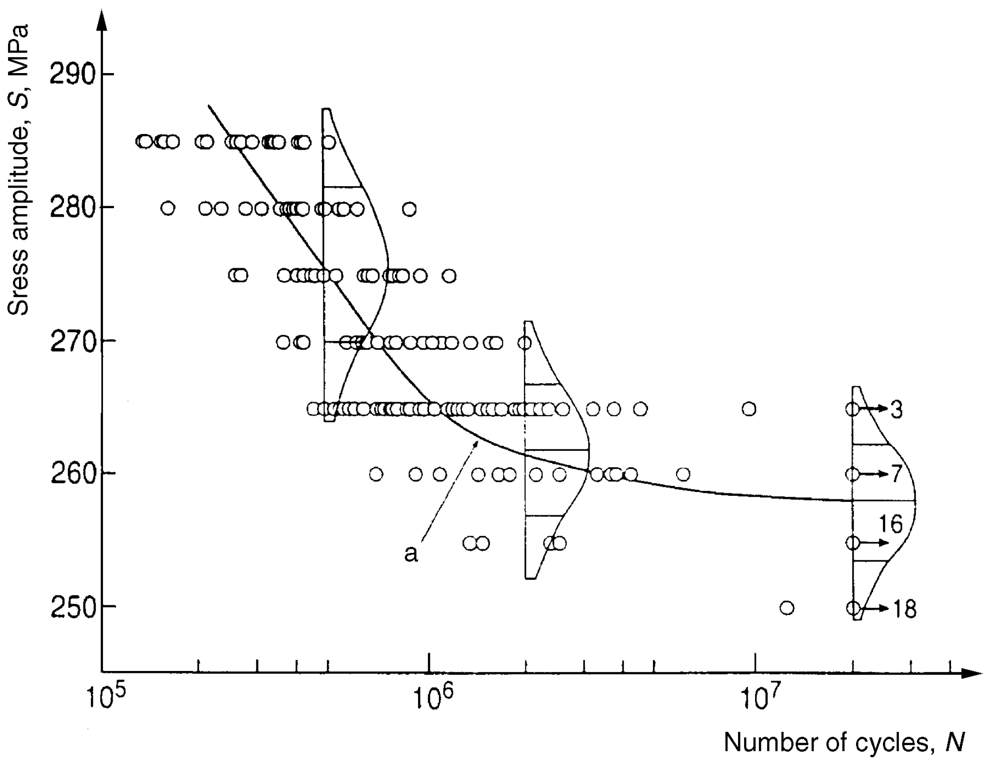
If the population displays any serial nature, e.g. if the properties are related to the date of fabrication, the population shall be divided into groups related to time. Random samples shall be selected from each group in numbers proportional to the group size.

The specimens taken from a particular batch of material will reveal a variability specific to the batch. This within-batch variation can sometimes be of the same order of importance as the between-batch variation. When the relative importance of different kinds of variation is known from experience, sampling shall be performed taking this into consideration.



[a] median curve

Figure 1 — Concept of variation in a fatigue property — Distribution of fatigue life at given stresses for a 0,25 % C carbon steel tested in the rotating-bending mode



[a] median curve

Figure 2 — Concept of variation in a fatigue property — Distribution of fatigue strength at typical fatigue lives for a 0,25 % C carbon steel tested in the rotating-bending mode

Hardness measurement is recommended for some materials, when possible, to divide the population of the material into distinct groups for sampling. The groups should be of as equal size as possible. Specimens may be extracted randomly in equal numbers from each group to compose a test sample of size n. This procedure will generate samples uniformly representing the population, based upon hardness.

5.2 Number of specimens to be tested

The reliability of test results is primarily dependent on the number of specimens tested. It increases with the number of tests, n.

For a random variable, x, taking values always less than or equal to $x_{(P)}$ at a probability, P, in a population, define x_1 as a minimum observed value in a set of n specimens extracted from the population. The probability that $x_1 \geqslant x_{(P)}$ is less than or equal to α, i.e. $(1 - P)^n$. Therefore, it can be expected that $x_{(P)}$ is greater than x_1 with a probability of at least $1 - \alpha$, i.e. at least $1 - (1 - P)^n$. This gives:

$$n = \frac{\ln \alpha}{\ln(1 - P)} \tag{3}$$

In the case of fatigue life tests, Equation (3) indicates at a confidence level of $1 - \alpha$ that the true fatigue life at probability of failure P of the population can be expected to be greater than the minimum life observed from n specimens.

The same concept can be applied to the case of S-N data items, because the deviations in individual log-life data from the mean S-N curve are considered to be randomly distributed. Further, the variance is assumed to be constant for different stresses, as a model S-N curve is fitted by an ordinary least-squares method in many cases.

Table 1 gives some typical figures for the number of specimens. The numbers in the column corresponding to a confidence level of 95 % are used for reliability design purposes, those at the 50 % confidence level for exploratory tests and the others for general engineering applications.

Table 1 — Number of specimens required so that the minimum value of test data can be expected to fall below the true value for the population at a given level of probability of failure at various confidence levels

Probability of failure P (%)	**Confidence level,** $1 - \alpha$ (%)		
	50	90	95
	Number of specimens, n [a]		
50	1	3	4
10	7	22	28
5	13	45	58
1	69	229	298

[a] The values of n are rounded to the nearest whole number.

5.3 Allocation of specimens for testing

Specimens taken from the test materials shall be allocated to individual fatigue tests in principle in a random way, in order to minimize unexpected statistical bias. The order of testing of the specimens shall also be randomized in a series of fatigue tests.

When several test machines are used in parallel, specimens shall be tested on each machine in equal or nearly equal numbers and in a random order. The equivalence of the machines in terms of their performance shall be verified prior to testing.

When the test programme includes several independent test series, e.g. tests at different stress levels or on different materials for comparison purposes, each test series shall be carried out at equal or nearly equal rates of progress, so that all testing can be completed at approximately the same time.

For a given number of specimens tested at several stresses, the number of repeated tests at each stress affects the statistical confidence of the estimate of the variability of the results. Recommended ratios for the number of stress levels to the total number of specimens can be found in [6].

6 Statistical estimation of fatigue life at a given stress

6.1 Testing to obtain fatigue life data

Conduct fatigue tests a given stress, S, on a set of carefully prepared specimens to determine the fatigue life values for each. The number of specimens, n, required may be determined by reference to the typical values given in Table 1. The number selected will be dependent upon the purpose of the test and the availability of test material.

A set of seven specimens is recommended in this International Standard for exploratory tests. For reliability purposes, however, at least 28 specimens are recommended.

6.2 Plotting data on probability paper

Prepare fatigue life data, $x = \log N$, for n specimens for probability plotting by ranking the data from minimum to maximum values. Label each data item with an order number, i, as $x_1 \leqslant x_2 \leqslant \ldots \leqslant x_n$. The probability of failure for the ith data item is approximated by:

$$P_i = \frac{i - 0{,}3}{n + 0{,}4} \tag{4}$$

Plot the data pairs thus obtained, (x_1, P_1), (x_2, P_2), ... , (x_n, P_n), on normal probability paper. If a straight line fits all the data points reasonably well, it can be concluded the data follow a log-log distribution.

If the data pairs do not give a straight line, it is recommended that other types of plot be attempted. Plotting on Weibull [4] probability paper is helpful in such situations.

When log-log probability paper is used, the fatigue lives, N_i, can be plotted directly without converting them into logarithms.

Figure 3 shows an example of log-log probability paper.

A worked example of the preparation of a data plot is given in Clause A.1.

The probability of failure for the ith data item can be approximated by other equations. One of the frequently used alternatives that generates almost identical results when n is sufficiently large is:

$$P_i = \frac{i}{n + 1}$$

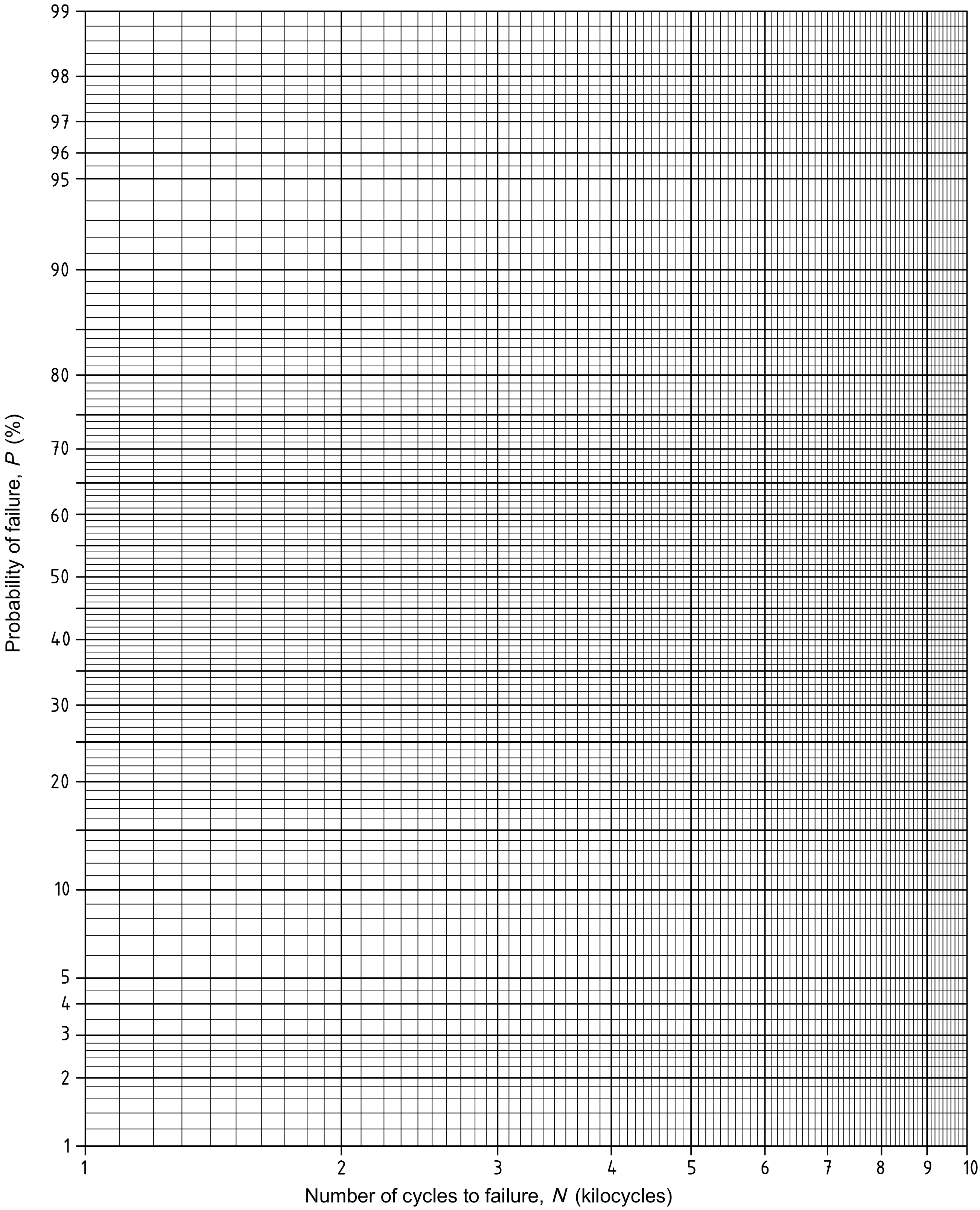


Figure 3 — Example of log-log probability paper

6.3 Estimating distribution parameters

Estimate parameters defining the statistical distribution of the fatigue life, Equation (1), from the linear curve that best fits the experimental data.

Define the value $x_{(P)}$ corresponding to a probability of failure P (%) that can be read from the curve. Estimate the mean, μ_x, and the standard deviation, σ_x, from the two values $x_{(10)}$ and $x_{(90)}$ as follows:

$$\hat{\mu}_x = \frac{x_{(10)} + x_{(90)}}{2} \tag{5}$$

$$\hat{\sigma}_x = \frac{x_{(90)} - x_{(10)}}{2{,}56} \tag{6}$$

where the caret sign "ˆ" is used to indicate that they are estimates.

The number of degrees of freedom for the standard deviation are considered to be $n - 1$, where n is the number of data items.

The coefficient of variation of the fatigue life, η_N, can be estimated as follows:

$$\hat{\eta}_N = \sqrt{\exp\left[\left(\ln 10\right)^2 \hat{\sigma}_x^2 - 1\right]} \tag{7}$$

where ln10 is the natural logarithm of 10, the square of which is 5,302.

The mean and the standard deviation thus estimated may differ from computed values calculated from the following equations:

$$\hat{\mu} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\hat{\sigma} = \frac{\sqrt{\sum_{i=1}^{n} \left(x_i - \hat{\mu}\right)^2}}{n - 1}$$

This International Standard recommends use of the graphical method, since it can adjust the data to fit the normal distribution represented on the probability plot.

6.4 Estimating the lower limit of the fatigue life

Estimate the lower limit of the fatigue life at a given probability of failure, assuming a normal distribution, at the confidence level $1 - \alpha$ from the equation:

$$\hat{x}_{(P, 1-\alpha)} = \hat{\mu}_x - k_{(P, 1-\alpha, \nu)}\,\hat{\sigma}_x \tag{8}$$

The coefficient $k_{(P,\,1-\alpha,\,\nu)}$ is the one-sided tolerance limit for a normal distribution, as given in Table B.1. Take as the number of degrees of freedom, ν, the number which was used in estimating the standard deviation.

7 Statistical estimation of fatigue strength at a given fatigue life

7.1 Testing to obtain fatigue strength data

Conduct fatigue tests to generate strength data for a set of specimens in a sequential way using the method known as the staircase method (see [7]).

It is necessary to have rough estimates of the mean and the standard deviation of the fatigue strength for the materials to be tested. Start the test at a first stress level preferably close to the estimated mean strength. Also select a stress step, preferably close to the standard deviation, by which to vary the stress level during the test.

If no information is available about the standard deviation, a step of about 5 % of the estimated mean fatigue strength may be used as the stress step.

Test a first specimen, randomly chosen, at the first stress level to find if it fails before the given number of cycles. For the next specimen, also randomly chosen, increase the stress level by a step if the preceding specimen did not fail, and decrease the stress by the same amount if it failed. Continue testing until all the specimens have been tested in this way.

Exploratory research requires a minimum of 15 specimens to estimate the mean and the standard deviation of the fatigue strength. Reliability data requires at least 30 specimens.

A worked example of the staircase method is given in A.2.1, together with worked examples of the analyses described in 7.2 and 7.3.

7.2 Statistical analysis of test data

Rearrange the test data in order to count the frequencies of failure and non failure of the specimens tested at different stress levels. Use statistical analysis only for the events “failure” and “non-failure”. Use the analysis for the group with the least number of observations.

Denote the stress levels arranged in ascending order by $S_0 \leqslant S_1 \leqslant \ldots \leqslant S_l$, where l is the number of stress levels, denote the number of events by f_i, and denote the stress step by d. Estimate the parameters for the statistical distribution of the fatigue strength, Equation (2), from:

$$\hat{\mu}_y = S_0 + d\left(\frac{A}{C} \pm \frac{1}{2}\right) \tag{9}$$

$$\hat{\sigma}_y = 1{,}62d\left(D + 0{,}029\right) \tag{10}$$

where

$$A = \sum_{i=1}^{l} i f_i$$

$$B = \sum_{i=1}^{l} i^2 f_i$$

$$C = \sum_{i=1}^{l} f_i$$

$$D = \frac{BC - A^2}{C^2}$$

In Equation (9), take the value of ± 1/2 as:

− 1/2 when the event analysed is failure;

+ 1/2 when the event analysed is non-failure.

In [7], it is stated that Equation (10) is valid only when $D > 0{,}3$. This condition is generally satisfied when d/σ_y is selected properly within the range 0,5 to 2.

7.3 Estimating the lower limit of the fatigue strength

Estimate the lower limit of the fatigue strength at a probability of failure P for the population at a confidence level of $1 - \alpha$, if the assumption of a normal distribution of the fatigue strength is correct, from the equation:

$$\hat{y}_{(P,1-\alpha)} = \hat{\mu}_y - k_{(P,1-\alpha,v)}\hat{\sigma}_y \tag{11}$$

where the coefficient $k_{(P,\,1-\alpha,\,v)}$ is the one-sided tolerance limit for a normal distribution, as given in Table B.1. Take as the number of degrees of freedom, v, the number which was used in estimating the standard deviation.

7.4 Modified method when standard deviation is known

A modified staircase method, with fewer specimens, is possible if the standard deviation is known and only the mean of the fatigue strength needs to be estimated (see [8]).

Conduct tests as in the staircase method described in 7.1, by decreasing or increasing the stress level by a fixed step depending whether the preceding event was a failure or non-failure, respectively. Choose the initial stress level close to the roughly estimated mean and the stress step approximately equal to the known standard deviation.

A minimum of six specimens is required for exploratory tests and at least 15 for reliability data.

If the test is conducted on n specimens at stress levels S_1, S_2, … , S_n in a sequential way, then the mean fatigue strength is determined by averaging the test stresses, S_2 to S_{n+1}, beyond the first, without regard to whether each event was a failure or a non-failure:

$$\hat{\mu}_y = \frac{\sum_{i=2}^{n+1} S_i}{n} \tag{12}$$

The test at S_{n+1} is not carried out, but the stress level itself is determined from the result of nth test.

Estimate the lower limit of the fatigue strength for the population from Equation (11). Take as the number of degrees of freedom that corresponding to the standard deviation used for the test or, if this number is unknown, take it as $n - 1$.

In the modified staircase method, it is necessary to know the standard deviation of the fatigue strength. It may be estimated from the S-N curve as described in Clause 8.

A worked example is given in A.2.2.

8 Statistical estimation of *S-N* curve

8.1 Fatigue testing to obtain *S-N* data

Conduct fatigue tests at various stress levels in order to determine the mean *S-N* curve giving a probability of failure of 50 %. It is assumed that the variation in the logarithm of the fatigue life follows a normal distribution with constant variance as a function of stress.

The total number of specimens required may be determined by reference to the typical values given in Table 1, taking into account the purpose of the test and the availability of test material.

Use a minimum of eight specimens for exploratory testing. It is recommended that two specimens be tested at each of four equally spaced stress levels. For reliability design purposes, however, at least 30 specimens are required. In this case, test six specimens at each of five equally spaced stress levels.

For ordinary high-cycle fatigue tests, the stress levels are generally chosen so that the resultant fatigue lives will have a spread of three decades of cycles, e.g. from 5×10^4 to 1×10^6 cycles.

Alternative methods can be found in [1] to [3] and in [5]. Continuous mathematical models are also used to fit curves to fatigue data extending from finite to infinite fatigue life regions (see [3] and [9] to [11]).

8.2 Statistical analysis of *S-N* data

To analyse the *S-N* relationship, use a linear mathematical model of the form:

$$x = b - ay$$

where $x = \log N$, and a and b are constants. For the variable y, either $y = S$ or $y = \log S$ may be used, whichever gives better plot linearity.

The most probable estimate of the mean *S-N* curve for the population is given by:

$$\hat{\mu}_x = \hat{b} - \hat{a}y \tag{13}$$

$$\hat{a}_x = -\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)}{\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2} \tag{14}$$

$$\hat{b} = \overline{x} + \hat{a}\overline{y} \tag{15}$$

where

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

and n is the number of data items.

Estimate the standard deviation of the logarithm of the fatigue life from the mean *S-N* curve for the population from the equation:

$$\hat{\sigma}_x = \sqrt{\frac{\sum_{i=1}^{n} \left[x_i - \left(\hat{b} - \hat{a} y_i \right) \right]^2}{n-2}} \tag{16}$$

with $n - 2$ degrees of freedom.

Then estimate the standard deviation of the fatigue strength for the population from the equation:

$$\hat{\sigma}_y = \frac{\hat{\sigma}_x}{\hat{a}} \tag{17}$$

The standard deviation of the fatigue strength thus obtained may be used in fatigue tests conducted by the modified staircase method described in 7.4.

8.3 Estimating the lower limit of the *S-N* curve

Estimate the lower limit to the *S-N* curve corresponding to a probability of failure P for the population at a confidence level $1 - \alpha$ and for a number of degrees of freedom v using the equation:

$$\hat{x}_{(P, 1-\alpha, v)} = \hat{b} - \hat{a}y - k_{(P, 1-\alpha, v)} \hat{\sigma}_x \sqrt{1 + \frac{1}{n} + \frac{\left(y - \overline{y} \right)^2}{\sum_{i=1}^{n} \left(y_i - \overline{y} \right)^2}} \tag{18}$$

where the coefficient $k_{(P,\ 1-\alpha,\ v)}$ is the one-sided tolerance limit for a normal distribution, as given in Table B.1. Take as the number of degrees of freedom, v, the number which was used in estimating the standard deviation.

The term inside the root sign in Equation (18) is a correction to the estimated standard deviation for the population, this correction depending on the number of tests and the range covered by the tests. When the number and range of the tests are large enough, the correction term is close to 1 and may be neglected.

An worked example is given in Clause A.3.

8.4 Verifying the adequacy of the linear model

It is possible to verify statistically the adequacy of the linear model if more than one specimen is tested at each of three or more stress levels.

When m_i specimens are tested at a stress level S_i, the data obtained for the jth specimen can be written as x_{ij}. The hypothesis of linearity is rejected if:

$$\frac{\sum_{i=1}^{l} m_i \left[\left(\hat{b} - \hat{a} y_i \right) - \overline{x}_i \right]^2 (l-2)}{\sum_{i=1}^{l} \sum_{j=1}^{m_i} \left(x_{ij} - \overline{x}_i \right)^2 (n-l)} > F_{(1-\alpha, v_1, v_2)} \tag{19}$$

where

$$\bar{x}_i = \frac{1}{m_i} \sum_{i=1}^{m_i} x_i$$

$$n = \sum_{i=1}^{l} m_i$$

and l is the number of stress levels.

Table B.2 gives the values of $F_{(1-\alpha, \nu_1, \nu_2)}$ corresponding to a confidence level, $1-\alpha$, of 95 %. The two numbers of degrees of freedom are defined as: $\nu_1 = l - 2$ for the numerator and $\nu_2 = n - 1$ for the denominator of Equation (19).

It is useful to look at the differences between observed values and estimated values, $x_i - \hat{x}_i$, by plotting them against $\hat{x}_i$ (see [12]). The fit is regarded as satisfactory if the plot appears approximately uniform.

9 Test report

9.1 Presentation of test results

9.1.1 General

The test report shall include the following information as appropriate to the type of test:

9.1.2 Fatigue life at a given stress

a) The test stress level and the estimated mean fatigue life, plus the estimated standard deviation of the logarithm of the fatigue life or the coefficient of variation of the fatigue life. The number of test specimens shall be indicated. The method of estimating the parameters (graphical or by calculation) shall also be reported.

b) A compilation of the experimental fatigue life data obtained for each specimen, with observations such as the mode of failure or non-failure, and indicating the test stress.

c) A plot of the experimental data on probability paper showing the curve which fits the data. No excessive extrapolation of the curve is allowed beyond the range covered by the observations.

d) The estimated lower limit of the fatigue life at the selected probability of failure, when necessary. No excessive extrapolation of the probability curve is allowed beyond the range covered by the observations.

9.1.3 Fatigue strength at a given fatigue life

a) The estimated mean fatigue strength and the estimated standard deviation, indicating the number of specimens tested. Report the method used to estimate these parameters, such as the staircase method.

b) A list of the experimental data on the stress level and the number of cycles to which each specimen was subjected, with observations on failure or non-failure, in the order of the test.

c) The estimated lower limit of the fatigue strength at the selected probability of failure, when necessary. No excessive extrapolation of the probability curve is allowed beyond the range covered by the observations.

9.1.4 *S-N* curve

a) The estimated mean *S-N* curve, showing plots of the experimental data. No excessive extrapolation of the curve is allowed beyond the range covered by the observations.

b) List of experimental data including the stress level and the number of cycles applied to each specimen with observations such as failure or non-failure.

c) The estimated lower limit of the *S-N* curve at the selected probability of failure, when necessary. No excessive extrapolation of the probability curve is allowed beyond the range covered by the observations.

9.2 Related information

9.2.1 Material tested

The report shall include information about the material tested, such as a standard designation or equivalent, process of fabrication, chemical composition, heat treatment, microstructure, mechanical properties.

9.2.2 Specimens tested

The report shall include information about the specimens tested, such as a standard designation or equivalent, dimensions, orientation of specimens with respect to the material from which they were taken, surface finish.

9.2.3 Conditions of fatigue test

The report shall include information about the conditions of the fatigue test, such as the type of stress (or strain in the case of a strain-controlled test), stress ratio or other parameter characterizing the test series, stress wave form, test frequency or equivalent, definition of failure, test environment.

Annex A
(informative)

Examples of applications

A.1 Example of statistical estimation of fatigue life

A set of seven data items is given in Table A.1 as an example. The data are arranged in order of magnitude, so that the corresponding probability, P_i, can be calculated by Equation (4).

The values of x_i = log N_i are then plotted on normal probability paper and, by visual inspection, a linear curve fitted, as shown in Figure A.1. The values of $x_{(90)}$ and $x_{(10)}$, corresponding to a probability of failure of 90 % and 10 %, respectively, are read from the curve:

$x_{(90)} = 5{,}06$

$x_{(10)} = 4{,}75$

The parameters for the distribution are calculated from Equations (5) and (6), as follows:

$\hat{\mu}_x = 4{,}905$

$\hat{\sigma}_x = 0{,}121$

or

$\hat{N}_{(50)} = 8{,}04 \times 10^4$ cycles

$\hat{\eta}_N = 0{,}63$

The lower limit of the fatigue life for a 10 % probability of failure, at a confidence level of 95 %, is estimated from Equation (8), taking $k_{(0{,}1;\ 0{,}95;\ 6)}$ as 2,755 as given in Table B.1:

$$\hat{x}_{(10)} = 4{,}905 - (2{,}755 \times 0{,}121) = 4{,}572$$

or

$$\hat{N}_{(10)} = 10^{4{,}572} = 3{,}73 \times 10^4 \text{ cycles}$$

Table A.1 — Example of fatigue life data

Specimen number i	Fatigue life N_i cycles	Log of fatigue life $x_i = \log N_i$	Probability P_i %
1	$6,05 \times 10^4$	4,782	9,43
2	$6,31 \times 10^4$	4,800	22,8
3	$7,39 \times 10^4$	4,869	36,4
4	$8,46 \times 10^4$	4,927	50,0
5	$9,11 \times 10^4$	4,960	63,6
6	$9,37 \times 10^4$	4,972	77,2
7	$1,25 \times 10^5$	5,098	90,6

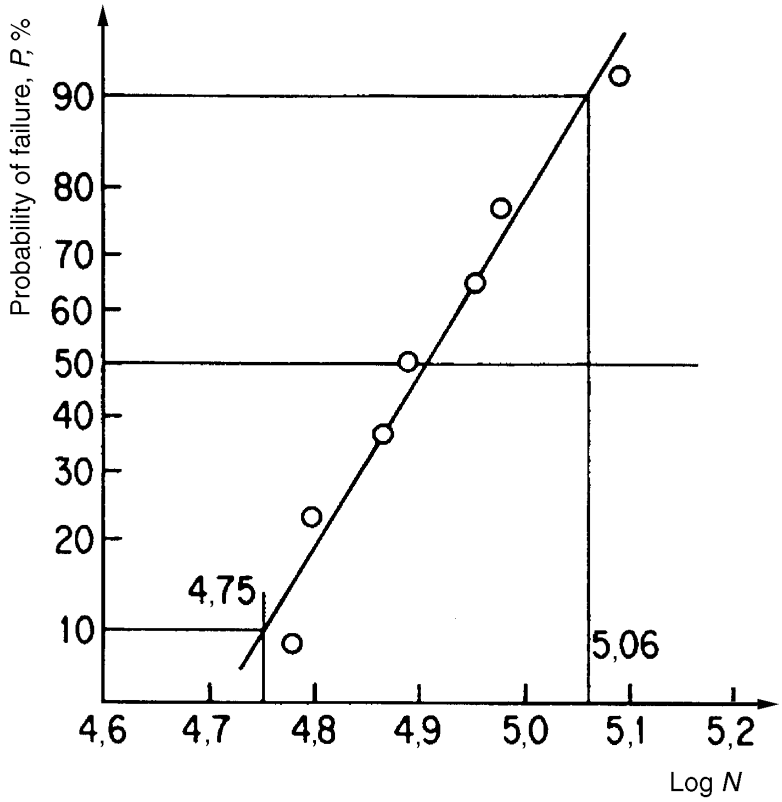


Figure A.1 — Example of plot of fatigue life data on normal probability paper

A.2 Examples of statistical estimation of fatigue strength

A.2.1 Staircase method

When using the staircase method, specimens are tested sequentially under increasing stresses until a failure occurs. An example of a set of data is given in Table A.2. From the beginning, the last non-failure in terms of stress is the first valid data which is 500 MPa in Table A.2. In this test, there are seven failures and eight non-failures. The failure event is therefore the one considered in the analysis.

Only three stress levels are considered in the analysis, as shown in Table A.3, with S_0 = 500 MPa and stress step d = 20 MPa. The number of the relevant event, f_i, is given in the third column of the table. The values of A, B, C and D are as follows:

$A = 7$

$B = 11$

$C = 7$

$D = 0{,}571$

The mean and the standard deviation of the fatigue strength are calculated from Equations (9) and (10), as follows:

$$\hat{\mu}_y = 500 + 20\left(7/7 - 1/2\right) = 510 \text{ MPa}$$

$$\hat{\sigma}_y = 1{,}62 \times 20\left(0{,}571 + 0{,}029\right) = 19{,}4 \text{ MPa}$$

and

$$\hat{\eta}_S = 19{,}4/510 = 0{,}038$$

NOTE By analogy with Equation (7), the coefficient of variation for the strength can be estimated from the equation:

$$\hat{\eta}_S = \sqrt{\exp\left[\left(\ln 10\right)^2 \hat{\sigma}_y^2 - 1\right]}$$

Table A.2 — Example of staircase test data

Stress S_i MPa	**Sequence number of specimen**																
			1				**5**					**10**					**15**
540									X								X
520				X				O		X		X				O	
500			O		X		O				O		X		O		
480		O*				O								O			
460	O*																
X for failure																	
O for non-failure																	
* not counted																	

Table A.3 — Analysis of the data in Table A.2

Stress S_i MPa	Level i	Values f_i	 if_i	 i^2f_i
540	2	2	4	8
520	1	3	3	3
500	0	2	0	0
Sum	—	7	7	11

The lower limit of the fatigue strength for a probability of failure of 10 % is calculated from Equation (11) at a confidence level of 95 %. The value of the appropriate coefficient, $k_{(0,1;\ 0,95;\ 6)}$, taken from Table B.1, is 2,755.

$$\begin{aligned}\hat{y}_{(10)} &= 510 - (2{,}755 \times 19{,}4) \\ &= 456 \text{ MPa}\end{aligned}$$

In this example, the stress step d is close enough to the estimated standard deviation and D is greater than 0,3.

A.2.2 Modified staircase method

This example is based on the same fatigue test data as in A.2.1, but only for sequence numbers 1 to 6. The set of data used is given in Table A.4. The standard deviation of the fatigue strength is 19,4 MPa with a number of degrees of freedom of 6, as calculated above. The test was conducted with a stress step of 20 MPa which is close enough to the standard deviation.

The mean fatigue strength is calculated from the data, using Equation (12), as follows:

$$\begin{aligned}\hat{\mu}_y &= (520 + 500 + 480 + 500 + 520 + 540)/6 \\ &= 510 \text{ MPa}\end{aligned}$$

The lower limit of the fatigue strength for a probability of failure of 10 % is calculated from Equation (11), at a confidence level of 95 % and taking a value for $k_{(0,1;\ 0,95;\ 6)}$ of 2,755 from Table B.1, as follows:

$$\begin{aligned}\hat{y}_{(10)} &= 510 - (2{,}755 \times 19{,}4) \\ &= 456 \text{ MPa}\end{aligned}$$

Table A.4 — Example of modified staircase test data

Parameter	Test sequence						
i	1	2	3	4	5	6	7
S_i, MPa	500	520	500	480	500	520	540
Event	O	X	X	O	O	O	[a]

[a] Test not actually carried out (stress level calculated from previous value).

A.3 Statistical estimation of *S-N* curve

A set of test data obtained from eight specimens is given as an example in Table A.5. Statistical analysis is performed here for $y = S$ and $x = \log N$, using a linear model and semi-logarithmic coordinates.

The mean values of y and x are easily obtained from the table as:

$$\bar{y} = 405 \text{ MPa}$$

$$\bar{x} = 5{,}311$$

The following quantities are then calculated:

$$\sum (x - \bar{x})^2 = 2{,}196$$

$$\sum (y - \bar{y})^2 = 9\,000$$

$$\sum (x - \bar{x})(y - \bar{y}) = -138{,}1$$

The coefficients for the most probable estimate of the mean *S-N* curve are given by Equations (14) and (15), as follows:

$$\hat{a} = 0{,}015\,3$$

$$\hat{b} = 11{,}527$$

The estimated standard deviations for the logarithm of the fatigue life and for the fatigue strength are then calculated from Equations (16) and (17), respectively:

$$\hat{\sigma}_x = 0{,}114$$

$$\hat{\sigma}_y = 7{,}5 \text{ MPa}$$

The lower limit of the *S-N* curve, corresponding to a probability of failure, P, of 10 %, is calculated at a confidence level of 95 % from Equation (18) and taking a value for $k_{(0,1;\ 0,95;\ 6)}$ of 2,755 from Table B.1, as follows:

$$\hat{x}_{(10)} = 11{,}527 - 0{,}015\,3y - 0{,}314\sqrt{1{,}166\,7 + \frac{(y - 405)^2}{9\,000}}$$

This curve is shown in Figure A.2.

Table A.5 — Example of *S*-*N* data

Specimen number i	Stress $y_i = S_i$ MPa	Fatigue life N_i cycles	Log of fatigue life $x_i = \log N_i$
1	450	$3{,}41 \times 10^4$	4,533
2	450	$5{,}23 \times 10^4$	4,719
3	420	$9{,}66 \times 10^4$	4,985
4	420	$1{,}50 \times 10^5$	5,176
5	390	$2{,}73 \times 10^5$	5,436
6	390	$4{,}12 \times 10^5$	5,615
7	360	$8{,}01 \times 10^5$	5,904
8	360	$1{,}32 \times 10^6$	6,121

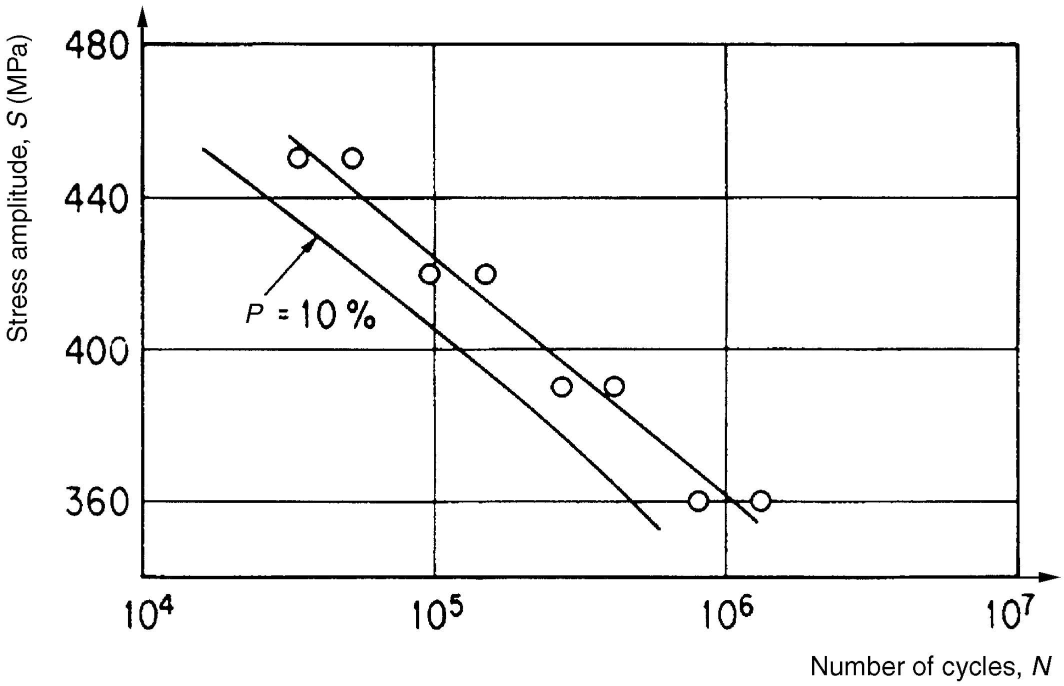


Figure A.2 — Example of *S*-*N* data analysis

Annex B
(informative)

Statistical tables

Table B.1 — Coefficient $k_{(P,\ 1-\alpha,\ v)}$ for the one-sided tolerance limit for a normal distribution for point P %

Number of degrees of freedom v	Probability, P (%)							
	10		5		1		0,1	
	Confidence level, $1-\alpha$ (%)							
	90	95	90	95	90	95	90	95
2	4,258	6,158	5,310	7,655	7,340	10,55	9,651	13,86
3	3,187	4,163	3,957	5,145	5,437	7,042	7,128	9,215
4	2,742	3,407	3,400	4,202	4,666	5,741	6,112	7,501
5	2,494	3,006	3,091	3,707	4,242	5,062	5,556	6,612
6	2,333	2,755	2,894	3,399	3,972	4,641	5,301	6,061
7	2,219	2,582	2,755	3,188	3,783	4,353	4,955	5,686
8	2,133	2,454	2,649	3,031	3,641	4,143	4,772	5,414
9	2,065	2,355	2,568	2,911	3,532	3,981	4,629	5,203
10	2,012	2,275	2,503	2,815	3,444	3,852	4,515	5,036
11	1,966	2,210	2,448	2,736	3,370	3,747	4,420	4,900
12	1,928	2,155	2,403	2,670	3,310	3,659	4,341	4,787
13	1,895	2,108	2,363	2,614	3,257	3,585	4,274	4,690
14	1,866	2,068	2,329	2,566	3,212	3,520	4,215	4,607
15	1,842	2,032	2,299	2,523	3,172	3,463	4,164	4,534
16	1,820	2,001	2,272	2,486	3,136	3,415	4,118	4,471
17	1,800	1,974	2,249	2,453	3,106	3,370	4,078	4,415
18	1,781	1,949	2,228	2,423	3,078	3,331	4,041	4,364
19	1,765	1,926	2,208	2,396	3,052	3,295	4,009	4,319
20	1,750	1,905	2,190	2,371	3,028	3,262	3,979	4,276
21	1,736	1,887	2,174	2,350	3,007	3,233	3,952	4,238
22	1,724	1,869	2,159	2,329	2,987	3,206	3,927	4,204
23	1,712	1,853	2,145	2,309	2,969	3,181	3,904	4,171
24	1,702	1,838	2,132	2,292	2,952	3,158	3,882	4,143
25	1,657	1,778	2,080	2,220	2,884	3,064	3,794	4,022

Table B.2 — Values of $F_{(1-\alpha, \nu 1, \nu 2)}$ at a confidence level, $1-\alpha$, of 95 %

Number of degrees of freedom						
ν_2	ν_1					
	1	**2**	**3**	**4**	**5**	**6**
1	161	200	216	225	230	234
2	18,5	19,0	19,2	19,2	19,3	19,3
3	10,1	9,55	9,28	9,12	9,01	8,94
4	7,71	6,94	6,59	6,39	6,26	6,16
5	6,61	5,79	5,41	5,19	5,05	4,95
6	5,99	5,14	4,76	4,53	4,39	4,28
7	5,59	4,74	4,35	4,12	3,97	3,87
8	5,32	4,46	4,07	3,84	3,69	3,58
9	5,12	4,26	3,86	3,63	3,48	3,37
10	4,96	4,10	3,71	3,48	3,33	3,22
11	4,84	3,98	3,59	3,36	3,20	3,09
12	4,75	3,89	3,49	3,26	3,11	3,00
13	4,67	3,81	3,41	3,18	3,03	2,92
14	4,60	3,74	3,34	3,11	2,96	2,85
15	4,54	3,68	3,29	3,06	2,90	2,79
16	4,49	3,63	3,24	3,01	2,85	2,74
17	4,45	3,59	3,20	2,96	2,81	2,70
18	4,41	3,55	3,16	2,93	2,77	2,66
19	4,38	3,52	3,13	2,90	2,74	2,63
20	4,35	3,49	3,10	2,87	2,71	2,60
21	4,32	3,47	3,07	2,84	2,68	2,57
22	4,30	3,44	3,05	2,82	2,66	2,55
23	4,28	3,42	3,03	2,80	2,64	2,53
24	4,26	3,40	3,01	2,78	2,62	2,51
25	4,24	3,39	2,99	2,76	2,60	2,49
26	4,23	3,37	2,98	2,74	2,59	2,47
27	4,21	3,35	2,96	2,73	2,57	2,46
28	4,20	3,34	2,95	2,71	2,56	2,45
29	4,18	3,33	2,93	2,70	2,55	2,43
30	4,17	3,32	2,92	2,69	2,53	2,42

Annex C
(informative)

Combined method for statistical estimation of a full *S-N* curve

C.1 Scope

This annex presents a method of estimating statistically a full *S-N* curve, including both finite and infinite fatigue life ranges, using a practical number of specimens. It is assumed that the *S-N* curve consists of an inclined straight line in the finite fatigue life range and a horizontal straight line in the infinite fatigue life regime. This is often realistic for many engineering materials, when the data are represented using appropriate coordinates, generally on semi-log or log-log paper.

C.2 Fatigue testing to obtain a full *S-N* curve

The test requires at least 14 specimens, eight of these being used for estimating the *S-N* curve in the finite fatigue life range (inclined line) and six for the fatigue strength at the infinite fatigue life regime (horizontal line). Figure C.1 displays this concept graphically.

The number of specimens allocated to each line is determined in a way that permits the fatigue strengths predicted by each, at their point of intersection, to have equal statistical confidence. It is recommended that the following relationship be satisfied (see [13]):

$$\frac{n_2}{n_1} = \frac{l+1}{2l-1} \tag{C.1}$$

where n_1 and n_2 are the number of tests for the inclined line and the horizontal line, respectively, and l is the number of stress levels for testing along the inclined line.

A few extra specimens should be kept in reserve, as tests may not always take place as expected. Having extra specimens available may help to resolve such unexpected problems.

C.3 Fatigue tests in the finite fatigue life range

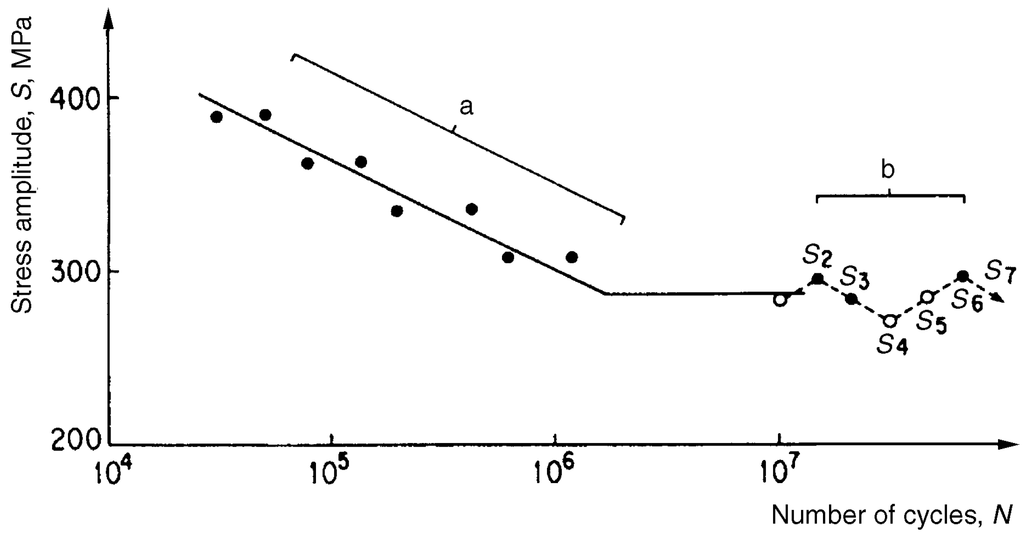
The mean *S-N* curve in the finite fatigue life range (inclined part) is estimated using eight specimens tested by the procedure described in 8.1.

The equation of the inclined part of the *S-N* curve is determined using Equations (13) to (15). The standard deviation of the fatigue strength is calculated using Equations (16) and (17).

C.4 Fatigue tests in the infinite fatigue life range

The fatigue strength in the infinite fatigue life range (horizontal part) is estimated using six specimens tested by the modified staircase method described in 7.4. The stress step for the test is selected so that it is close enough to the standard deviation calculated in Clause C.3.

The mean fatigue strength in the horizontal part of the curve is calculated using Equation (12).



- • failure
- o non-failure after 1×10^7 cycles

a Eight specimens

b Six specimens by staircase method

Figure C.1 — Model of combined method for the *S-N* curve with 14 specimens

C.5 Estimating the full *S-N* curve

The full *S-N* curve, including both the finite and infinite fatigue life regions, is obtained by combining Equations (13) and (11):

$$\begin{cases} x = \hat{b} - \hat{a}y & \text{when } y > \hat{\mu}_y \\ y = \hat{\mu}_y \end{cases} \qquad \text{(C.2)}$$

C.6 Lower limit of the full *S-N* curve

The lower limit of the full *S-N* curve is obtained by combining Equations (18) and (11):

$$\begin{cases} \hat{x}_{(P,1-a)} = \hat{b} - \hat{a}y - k_{(P,1-a,v)}\hat{\sigma}_x \sqrt{1 + \dfrac{1}{n} + \dfrac{(y - \bar{y})^2}{\sum\limits_{i=1}^{x}(y_i - \bar{y})^2}} & \text{when } y > \hat{y}_{(P,1-a)} \\ \hat{y}_{(P,1-a)} = \hat{\mu}_y - k_{(P,1-a,v)}\hat{\sigma}_y \end{cases} \qquad \text{(C.3)}$$

The number of degrees of freedom, v, is $n_1 - 2$ for both the finite and the infinite fatigue life regimes. This is because the statistical uncertainty in the fatigue limit is dependent on that of the standard deviation which is derived from the analysis of finite fatigue life data.

Bibliography

[1] BS 3518-5:1966, *Methods of fatigue testing — Guide to the application of statistics*

[2] JSME S 002:1981, *Standard Method of Statistical Fatigue Testing*

[3] NF A 03-405:1991, *Metallic products — Fatigue tests — Statistical treatment of data*

[4] WEIBULL, W., *Fatigue Testing and the Analysis of Results*, (1961), Pergamon Press

[5] NISHIJIMA, S., *Statistical Fatigue Properties of Some Heat-Treated Steels for Machine Structural Use*, STP 744 (1981), pp. 75-88, ASTM

[6] ASTM E 739-91, *Standard Practice for Statistical Analysis of Linear or Linearized Stress-Life (S-N) and Strain-Life (ε-N) Fatigue Data*

[7] DIXSON, W.J., and MOOD, A.M., A Method for Obtaining and Analyzing Sensitivity Data, *Journal of the American Statistical Association*, Vol. 43 (1948), pp. 109-126

[8] BROWNLEE, K.A., HODGES, J.L., Jr., and ROSENBLATT, Murray, *The Up-and-Down Method with Small Samples*, Journal of the American Statistical Association, Vol. 48 (1953), pp. 262-277

[9] BASTENAIRE, F., POMEY, G., and RABBE, P., Etude statistique des durées de vie en fatigue et des courbes de Wöhler de cinq nuances d’acier, *Mémoires scientifiques de la revue de métallurgie*, Vol. 68 (1971), pp. 645-664

[10] BASTENAIRE, F., *New Method for the Statistical Evaluation of Constant Stress Amplitude Fatigue Test Results*, STP 511 (1972), ASTM

[11] SPINDEL, J.E., and HEIBACH, E., The method of maximum likelihood applied to the statistical analysis of fatigue data, *International Journal of Fatigue*, Vol. 1 (1979), pp. 81-88

[12] NETER, J., WASSERMAN, W., and KUTNER, M.H., *Applied Linear Statistical Models*, (1985), pp. 123-132, Irwin, Homewood, Il, USA

[13] NAKAZAWA, H., and KODAMA, S., Statistical Fatigue Testing Method with 14 Specimens — JSME Standard Method for Determination of S-N Curves, *Current Japanese Materials Research*, Vol. 2 (1987), pp. 59-69, Elsevier

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