

BS ISO 8855:2011



BSI Standards Publication

# Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary

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**National foreword**

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The UK participation in its preparation was entrusted to Technical Committee AUE/15, Safety related to vehicles.

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**8855**

Second edition  
2011-12-15

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## Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary

*Véhicules routiers — Dynamique des véhicules et tenue de route —  
Vocabulaire*



Reference number  
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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 8855 was prepared by Technical Committee ISO/TC 22, *Road vehicles*, Subcommittee SC 9, *Vehicle dynamics and road-holding ability*.

This second edition cancels and replaces the first edition (ISO 8855:1991), which has been technically revised. It also incorporates the Addendum ISO 8855:1991/Add.1:1992.

## Introduction

This International Standard defines terms appertaining to road vehicle dynamics, principally for use by design, simulation and development engineers in the automotive industries. This second edition has been prepared in response to a requirement to update the first, and to harmonize its contents with that of the comparable standard published by SAE International (SAE J670:JAN2008). This revision extends the scope to include provision for separate tyre and wheel axis systems, inclined and non-uniform road surfaces, tyre forces and moments, multiple unit commercial vehicles, and two-axle vehicles possessed of four-wheel steer geometry.

The vocabulary contained in this International Standard has been developed from the previous edition, and SAE J670, in order to facilitate accurate and unambiguous communication of the terms and definitions employed in the test, analysis and general description of the lateral, longitudinal, vertical and rotational dynamics of road vehicles.





# Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary

## 1 Scope

This International Standard defines the principal terms used for road vehicle dynamics. The terms apply to passenger cars, buses and commercial vehicles with one or more steered axles, and to multi-unit vehicle combinations.

## 2 Axis system

### 2.1 reference frame

geometric environment in which all points remain fixed with respect to each other at all times

### 2.2 inertial reference frame

Newtonian reference frame

**reference frame** (2.1) that is assumed to have zero linear and angular acceleration and zero angular velocity

NOTE In Newtonian physics, the Earth is assumed to be an inertial reference frame.

### 2.3 axis system

set of three orthogonal directions associated with  $X$ ,  $Y$  and  $Z$  axes

NOTE A right-handed axis system is assumed throughout this International Standard, where:  $\vec{Z} = \vec{X} \times \vec{Y}$ .

### 2.4 coordinate system

numbering convention used to assign a unique ordered trio  $(x, y, z)$  of values to each point in a **reference frame** (2.1), and which consists of an **axis system** (2.3) plus an origin point

### 2.5 ground plane

horizontal plane in the **inertial reference frame** (2.2), normal to the gravitational vector

### 2.6 road surface

surface supporting the tyre and providing friction necessary to generate shear forces in the **road plane** (2.7)

NOTE The surface may be flat, curved, undulated or of other shape.

### 2.7 road plane

plane representing the **road surface** (2.6) within the tyre contact patch

NOTE 1 For an uneven road, a different road plane may exist at each tyre contact patch.

NOTE 2 For a planar road surface, the road plane is coincident with the road surface. For road surfaces with surface contours having a wavelength similar to or less than the size of the tyre contact patch, as in the case of many ride events,

it is intended that an equivalent road plane be determined. Determination of the equivalent road plane is dependent on the requirements of the analysis being performed. The equivalent road plane may not be coincident with the actual road surface at the **contact centre** (4.1.4).

### 2.7.1 road plane elevation angle

$\lambda$   
angle from the normal projection of the  $X_T$  axis on to the **ground plane** (2.5) to the  $X_T$  axis

### 2.7.2 road plane camber angle

$\eta$   
angle from the normal projection of the  $Y_T$  axis on to the **ground plane** (2.5) to the  $Y_T$  axis

### 2.8 earth-fixed axis system

$(X_E, Y_E, Z_E)$   
**axis system** (2.3) fixed in the **inertial reference frame** (2.2), in which the  $X_E$  and  $Y_E$  axes are parallel to the **ground plane** (2.5), and the  $Z_E$  axis points upward and is aligned with the gravitational vector

NOTE The orientation of the  $X_E$  and  $Y_E$  axes is arbitrary and is intended to be based on the needs of the analysis or test.

### 2.9 earth-fixed coordinate system

$(x_E, y_E, z_E)$   
**coordinate system** (2.4) based on the **earth-fixed axis system** (2.8) with an origin that is fixed in the **ground plane** (2.5)

NOTE The location of the origin is generally an arbitrary point defined by the user.

### 2.10 vehicle axis system

$(X_V, Y_V, Z_V)$   
**axis system** (2.3) fixed in the **reference frame** (2.1) of the vehicle **sprung mass** (4.12), so that the  $X_V$  axis is substantially horizontal and forwards (with the vehicle at rest), and is parallel to the vehicle's longitudinal plane of symmetry, and the  $Y_V$  axis is perpendicular to the vehicle's longitudinal plane of symmetry and points to the left with the  $Z_V$  axis pointing upward

See Figure 1.

NOTE 1 For multi-unit combinations a separate vehicle axis system may be defined for each **vehicle unit** (3.1) (see Figure 2).

NOTE 2 The symbolic notation  $(X_{V,1}, Y_{V,1}, Z_{V,1}), (X_{V,2}, Y_{V,2}, Z_{V,2}), \dots, (X_{V,n}, Y_{V,n}, Z_{V,n})$  may be assigned to the vehicle axis systems of a multi-unit combination with  $n$  **vehicle units** (3.1).

### 2.11 vehicle coordinate system

$(x_V, y_V, z_V)$   
**coordinate system** (2.4) based on the **vehicle axis system** (2.10) with the origin located at the **vehicle reference point** (2.12)

### 2.12 vehicle reference point

point fixed in the vehicle **sprung mass** (4.12)

NOTE The vehicle reference point may be defined in a variety of locations, based on the needs of the analysis or test. Commonly used locations include the total vehicle centre of gravity, the sprung mass centre of gravity, the mid-**wheelbase** (4.2) point at the height of the centre of gravity, and the centre of the front axle. For multi-unit combinations, a vehicle reference point may be defined for each **vehicle unit** (3.1).

**2.13  
 intermediate axis system**

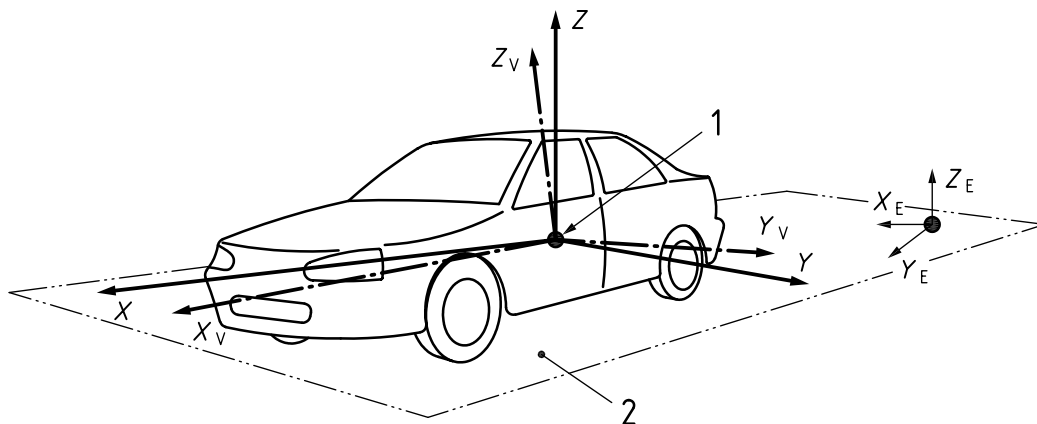
( $X, Y, Z$ )

**axis system** (2.3) whose  $X$  and  $Y$  axes are parallel to the **ground plane** (2.5), with the  $X$  axis aligned with the vertical projection of the  $X_V$  axis on to the **ground plane** (2.5)

See Figure 1.

NOTE 1 For multi-unit combinations, a separate intermediate axis system may be defined for each **vehicle unit** (3.1).

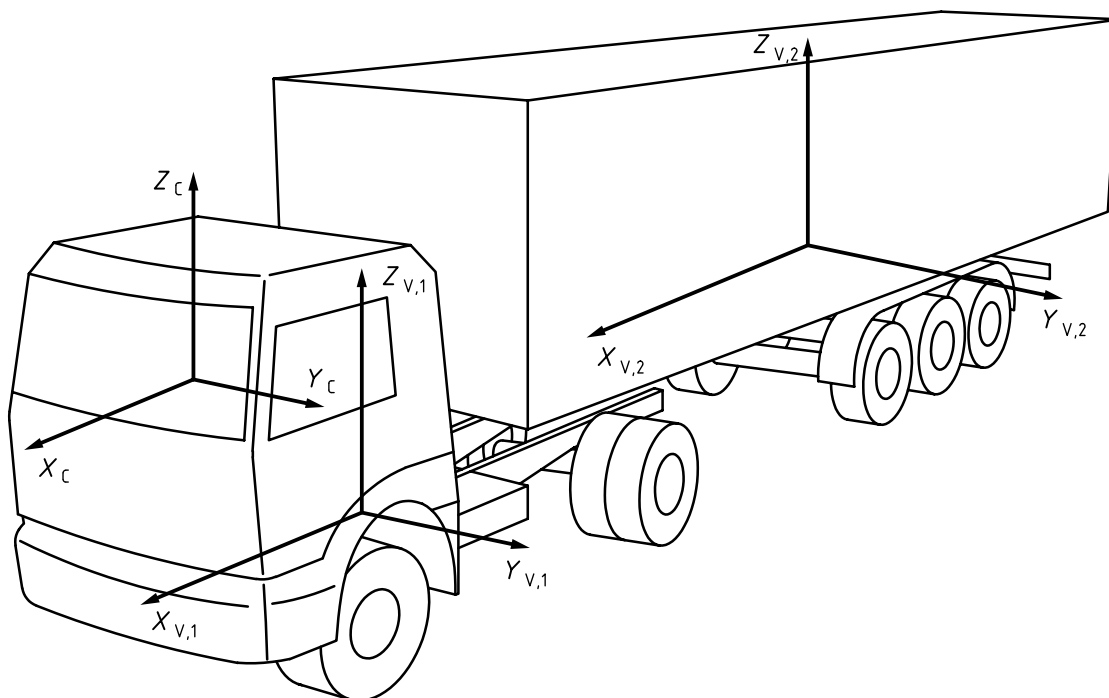
NOTE 2 The intermediate axis system is used to facilitate the definition of angular orientation terms and the components of force, moment, and motion vectors. An intermediate coordinate system is not defined herein.



**Key**

- 1 vehicle reference point
- 2 ground plane

**Figure 1 — Vehicle and intermediate axis systems**



**Figure 2 — Multi-unit axis systems**

**2.14**  
**tyre axis system**

$(X_T, Y_T, Z_T)$

**axis system** (2.3) whose  $X_T$  and  $Y_T$  axes are parallel to the local **road plane** (2.7), with the  $Z_T$  axis normal to the local road plane, where the orientation of the  $X_T$  axis is defined by the intersection of the **wheel plane** (4.1) and the road plane, and the positive  $Z_T$  axis points upward

NOTE A local tyre axis system may be defined at each wheel (see Figure 3).

**2.15**  
**tyre coordinate system**

$(x_T, y_T, z_T)$

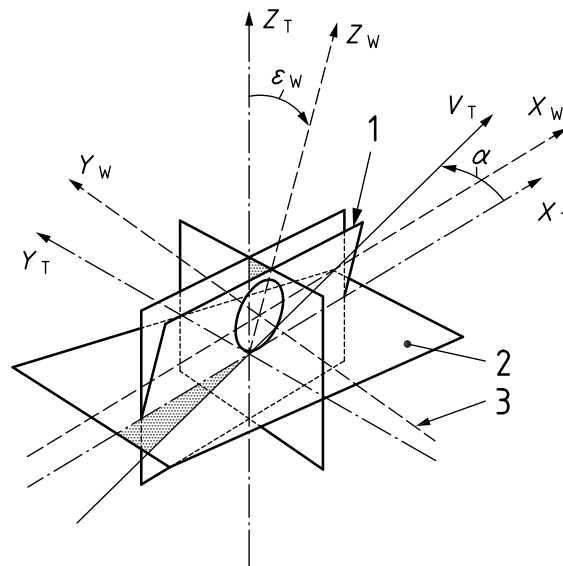
**coordinate system** (2.4) based on the **tyre axis system** (2.14) with the origin fixed at the **contact centre** (4.1.4)

**2.16**  
**wheel axis system**

$(X_W, Y_W, Z_W)$

**axis system** (2.3) whose  $X_W$  and  $Z_W$  axes are parallel to the **wheel plane** (4.1), whose  $Y_W$  axis is parallel to the **wheel-spin axis** (4.1.1), and whose  $X_W$  axis is parallel to the local **road plane** (2.7), and where the positive  $Z_W$  axis points upward

NOTE A local wheel axis system may be defined for each wheel (see Figure 3).



**Key**

- 1 wheel plane
- 2 road plane
- 3 wheel-spin axis

**Figure 3 — Tyre and wheel axis system**

**2.17**  
**wheel coordinate system**

$(x_W, y_W, z_W)$

**coordinate system** (2.4) based on the **wheel axis system** (2.16) with the origin fixed at the **wheel centre** (4.1.2)

## 2.18

### **cab axis system**

$(X_C, Y_C, Z_C)$

**axis system** (2.3) fixed in the **reference frame** (2.1) of the cab sprung mass, so that the  $X_C$  axis is substantially horizontal and forwards (with the vehicle at rest), and is parallel to the vehicle's longitudinal plane of symmetry, and where the  $Y_C$  axis is perpendicular to the cab's longitudinal plane of symmetry and points to the left with the  $Z_C$  axis pointing upward

NOTE A cab axis system applies only to vehicles with a suspended cab only.

## 2.19

### **cab coordinate system**

$(x_C, y_C, z_C)$

**coordinate system** (2.4) based on the **cab axis system** (2.18) with the origin fixed at an arbitrary point defined by the user

## 3 Vehicle unit

### 3.1

#### **vehicle unit**

rigid (i.e. non-articulating) vehicle element operating alone or in combination with one or more other rigid elements joined at yaw-articulation joints

NOTE Tractor, **semi trailer** (3.2.2) and **dolly** (3.2.4) are examples of vehicle units. A drawbar **trailer** (3.2) may consist of more than one vehicle unit.

### 3.2

#### **trailer**

**vehicle unit** (3.1) or combination of multiple vehicle units that is towed by another vehicle unit and can be disconnected from its towing vehicle unit

NOTE A trailer may have a single axle or multiple axles positioned along its length.

#### 3.2.1

##### **full trailer**

**trailer** (3.2) that has both front and rear running gear and, hence, provides fully its own vertical support

#### 3.2.2

##### **semi trailer**

**trailer** (3.2) that has only rear running gear and hence depends on its towing **vehicle unit** (3.1) for a substantial part of its vertical support

NOTE A semi trailer is typically coupled to the towing vehicle unit using a **fifth-wheel coupling** (3.2.6).

#### 3.2.3

##### **centre-axle trailer**

**trailer** (3.2) with only rear running gear located only slightly aft of the nominal position of the centre of gravity of the unit

NOTE A centre-axle trailer is typically coupled to the towing unit with a **hitch coupling** (3.2.7).

#### 3.2.4

##### **dolly**

portion of a **full trailer** (3.2.1) that includes the steerable front running gear and tow bar

#### 3.2.5

##### **converter dolly**

**dolly** (3.2.4) unit that couples to a **semi trailer** (3.2.2) with a **fifth-wheel coupling** (3.2.6) and thereby "converts" the semi trailer to a **full trailer** (3.2.1)

### 3.2.6

#### **fifth-wheel coupling**

device used to connect a **semi trailer** (3.2.2) to its towing **vehicle unit** (3.1) that is designed to bear the very substantial vertical load imposed by the front of the semi trailer

NOTE A fifth-wheel coupling provides rotational degrees of freedom in the  $Y_V$  and  $Z_V$  directions, but transmits moments about the  $X_V$  axis (all axes are in the towing vehicle unit).

### 3.2.7

#### **hitch coupling**

device used to connect a **trailer** (3.2) or **converter dolly** (3.2.5) tow bar to its towing **vehicle unit** (3.1), which approximates a spherical joint by providing three rotational degrees of freedom within the normal operating range

NOTE Typical examples of hitch couplings include ball hitches and pintle hitches.

## 4 Vehicle geometry and masses

### 4.1

#### **wheel plane**

plane normal to the **wheel-spin axis** (4.1.1), which is located halfway between the rim flanges

#### 4.1.1

##### **wheel-spin axis**

axis of wheel rotation

NOTE This axis is coincident with the  $Y_W$  axis.

#### 4.1.2

##### **wheel centre**

point at which the **wheel-spin axis** (4.1.1) intersects the **wheel plane** (4.1)

NOTE This point is the origin of the **wheel coordinate system** (2.17).

#### 4.1.3

##### **contact line**

intersection of the **wheel plane** (4.1) and the **road plane** (2.7)

#### 4.1.4

##### **contact centre**

intersection of the **contact line** (4.1.3) and the normal projection of the **wheel-spin axis** (4.1.1) on to the **road plane** (2.7)

NOTE This point is the origin of the **tyre coordinate system** (2.15). The contact centre may not be the geometric centre of the tyre **contact patch** (4.1.5) due to distortion of the tyre produced by external forces.

#### 4.1.5

##### **contact patch**

footprint

portion of the tyre touching the **road surface** (2.6)

### 4.2

#### **wheelbase**

/

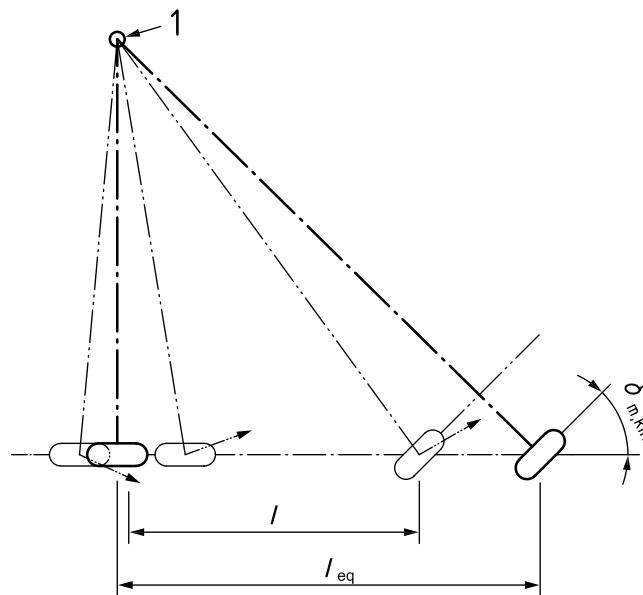
distance between the **contact centres** (4.1.4) on the same side of the vehicle, measured parallel to the  $X$  axis, with the vehicle at rest on a horizontal surface, with zero **steer angle** (7.1.1)

NOTE 1 A vehicle may have a different wheelbase on the left and right sides by design. It is common practice to average the left and right wheelbases; however, the difference may need to be taken into account in performing some analyses. The wheelbase typically changes as the suspension trim height changes.

NOTE 2 This applies to two-axle vehicles only. ISO 21308-2:2006, 6.1, defines the “configuration wheelbase”, for multi-axle vehicles, as the distance between the centre of the first front axle to the centre of the first driven rear axle. This term is a dimensional description and is not used in dynamic analysis.

### 4.3 equivalent wheelbase

$l_{eq}$   
**wheelbase** (4.2) of a conventional two-axle vehicle (i.e. a vehicle with one steering front axle and one non-steering rear axle) which, given similar front and rear cornering compliance properties, would exhibit the same **steady state** (12.2.1) turning behaviour as is exhibited by the multi-axle vehicle



#### Key

1 turn centre

Figure 4 — Equivalent wheelbase

### 4.4 track

$b$   
 distance between the **contact centres** (4.1.4), on a single wheel axle, measured parallel to the  $Y$  axis, with the vehicle at rest on a horizontal surface

NOTE For dual wheel axles, it is the distance between the points centrally located between the contact centres of the inner and outer dual wheels.

### 4.5 articulation point

instant centre of rotation of two **vehicle units** (3.1) established by the mechanical coupling device joining those two units, typically on the plane of symmetry of both units

NOTE 1 An articulation point may establish one, two, or three degrees of rotational freedom between the two coupled units.

NOTE 2 For **semi trailers** (3.2.2), the longitudinal ( $X$ ) coordinate of the articulation point is equal to the **fifth-wheel position** (4.9). For **full trailers** (3.2.1), the longitudinal ( $X$ ) coordinate of the articulation point is equal to the **hitch position** (4.10). For vehicle combinations with more than one **trailer** (3.2), several articulation points may exist.

#### 4.6

##### **axle distance**

average longitudinal distance between the **contact centres** (4.1.4), on two consecutive axles, measured parallel to the  $X$  axis, with the vehicle at rest on a horizontal surface, at zero **steer angle** (7.1.1)

#### 4.7

##### **axle position**

average longitudinal distance between the **vehicle reference point** (2.12) and the **contact centres** (4.1.4) of the axle, measured parallel to the  $X$  axis, with the vehicle combination at rest on a horizontal surface at zero **steer angle** (7.1.1)

#### 4.8

##### **trailer-axle position**

average longitudinal distance between the **contact centres** (4.1.4) of the **trailer** (3.2) axle and the vertical projection of the **articulation point** (4.5) (of the first trailer) on to the **ground plane** (2.5), with the vehicle combination at rest on a horizontal surface in a straight-ahead condition

NOTE Trailers may consist of more than one axle and/or articulation points.

#### 4.9

##### **fifth-wheel position**

kingpin position

average longitudinal distance between the **contact centres** (4.1.4) of the first driven rear axle of the towing **vehicle unit** (3.1) and the projection of the **articulation point** (4.5) on to the **ground plane** (2.5), with the vehicle combination at rest on a horizontal surface in a straight-ahead condition

NOTE Applicable to **semi trailers** (3.2.2) only.

#### 4.10

##### **hitch position**

average longitudinal distance between the **contact centres** (4.1.4) of the first driven rear axle of the towing **vehicle unit** (3.1) and the projection of the **articulation point** (4.5) on to the **ground plane** (2.5), with the vehicle combination at rest on a horizontal surface in a straight-ahead condition

NOTE Applicable to **full trailers** (3.2.1) only.

#### 4.11

##### **unsprung mass**

mass that is not carried by the suspension, but is supported directly by the tyres

#### 4.12

##### **sprung mass**

mass that is supported by the suspension, i.e. the total vehicle mass less the **unsprung mass** (4.11)

NOTE It is common practice to allocate a portion of the mass of the suspension linkage, driveshafts and springs in the sprung mass and the remainder in the unsprung mass.

## 5 Vehicle motion variables

### 5.1 Linear motion variables

For the definitions in this subclause, velocity and acceleration are relative to the **earth-fixed axis system** (2.8) ( $X_E, Y_E, Z_E$ ). They are resolved into components in the **intermediate axis system** (2.13) ( $X, Y, Z$ ).

NOTE It is also possible to resolve velocity and acceleration vectors into components in other **axis systems** (2.3). For example, velocity and acceleration vectors may be resolved in the **vehicle axis system** (2.10) ( $X_V, Y_V, Z_V$ ) to produce  $v_{X_V}, v_{Y_V}, v_{Z_V}$  and  $a_{X_V}, a_{Y_V}, a_{Z_V}$ .



**5.1.1  
vehicle velocity**

$\vec{v}$

vector quantity expressing the velocity of the **vehicle reference point** (2.12)

**5.1.2  
longitudinal velocity**

$\vec{v}_X$

component of the **vehicle velocity** (5.1.1) in the direction of the  $X$  axis

**5.1.3  
lateral velocity**

$\vec{v}_Y$

component of the **vehicle velocity** (5.1.1) in the direction of the  $Y$  axis

**5.1.4  
vertical velocity**

$\vec{v}_Z$

component of the **vehicle velocity** (5.1.1) in the direction of the  $Z$  axis

**5.1.5  
horizontal velocity**

$\vec{v}_h$

resultant of the **longitudinal velocity** (5.1.2) and the **lateral velocity** (5.1.3)

**5.1.6  
tyre trajectory velocity**

$\vec{v}_T$

vector quantity expressing the velocity of the **contact centre** (4.1.4)

**5.1.7  
tyre longitudinal velocity**

$\vec{v}_{XT}$

component of the **tyre trajectory velocity** (5.1.6) in the  $X_T$  direction

**5.1.8  
tyre lateral velocity**

$\vec{v}_{YT}$

component of the **tyre trajectory velocity** (5.1.6) in the  $Y_T$  direction

**5.1.9  
tyre vertical velocity**

$\vec{v}_{ZT}$

component of the **tyre trajectory velocity** (5.1.6) in the  $Z_T$  direction

**5.1.10  
vehicle acceleration**

$\vec{a}$

vector quantity expressing the acceleration of the **vehicle reference point** (2.12)

**5.1.11  
longitudinal acceleration**

$\vec{a}_X$

component of the **vehicle acceleration** (5.1.10) in the direction of the  $X$  axis

**5.1.12  
lateral acceleration**

$\vec{a}_Y$

component of the **vehicle acceleration** (5.1.10) in the direction of the  $Y$  axis

**5.1.13**  
**vertical acceleration**

$\bar{a}_z$   
component of the **vehicle acceleration** (5.1.10) in the direction of the  $Z$  axis

**5.1.14**  
**tangential acceleration**

$\bar{a}_t$   
component of the **vehicle acceleration** (5.1.10) in the direction of the **horizontal velocity** (5.1.5)

**5.1.15**  
**centripetal acceleration**

$\bar{a}_c$   
component of the **vehicle acceleration** (5.1.10) in the direction of the horizontal normal to the **horizontal velocity** (5.1.5)

**5.1.16**  
**horizontal acceleration**

$\bar{a}_h$   
resultant of the **longitudinal acceleration** (5.1.11) and the **lateral acceleration** (5.1.12), or the resultant of the **tangential acceleration** (5.1.14) and the **centripetal acceleration** (5.1.15)

**5.2 Angular motion variables**

For the definitions in this subclause, the sign of angles resulting from angular rotations is determined in accordance with the right-hand rule and angular velocities and accelerations are relative to the **earth-fixed axis system** (2.8) ( $X_E, Y_E, Z_E$ ); they are resolved into components in the **intermediate axis system** (2.13) ( $X, Y, Z$ ).

NOTE It is also possible to resolve angular velocity and acceleration vectors into components in other **axis systems** (2.3). For example, angular velocity and acceleration vectors may be resolved in the **vehicle axis system** (2.10) ( $X_V, Y_V, Z_V$ ) to produce  $\omega_{X_V}, \omega_{Y_V}, \omega_{Z_V}$  and  $\dot{\omega}_{X_V}, \dot{\omega}_{Y_V}, \dot{\omega}_{Z_V}$ .

**5.2.1**  
**yaw angle**

$\psi$   
angle from the  $X_E$  axis to the  $X$  axis, about the  $Z_E$  axis

**5.2.2**  
**pitch angle**

$\theta$   
angle from the  $X$  axis to the  $X_V$  axis, about the  $Y$  axis

NOTE The pitch angle is not measured relative to the **road surface** (2.6), thus a vehicle at rest on an inclined road surface has a non-zero pitch angle.

**5.2.3**  
**roll angle**

$\varphi$   
angle from the  $Y$  axis to the  $Y_V$  axis, about the  $X_V$  axis

NOTE These three angles;  $\psi, \theta, \varphi$  are called the Vehicle Euler Angles. They define the orientation of the **vehicle axis system** (2.10) ( $X_V, Y_V, Z_V$ ) with respect to the **earth-fixed axis system** (2.8) ( $X_E, Y_E, Z_E$ ), as a sequence of consecutive angular rotations as defined in Table 1.

Table 1 — Euler rotations

Rotation order	Angle produced by rotation	Rotation nature
First rotation	Yaw, $\psi$	$X_E$ axis to the $X$ axis about the $Z_E$ axis
Second rotation	Pitch, $\theta$	$X$ axis to the $X_V$ axis about the $Y$ axis
Third rotation	Roll, $\varphi$	$Y$ axis to the $Y_V$ axis about the $X_V$ axis

#### 5.2.4 vehicle roll angle

$\varphi_V$   
angle from the  $Y$  axis to the  $Y_V$  axis, about the  $X$  axis

NOTE This angle is different from **roll angle** (5.2.3),  $\varphi$ , if the **pitch angle** (5.2.2),  $\theta$ , is not zero. The vehicle roll angle may be computed using the equation  $\sin\varphi_V = \sin\varphi \cos\theta$ . The vehicle roll angle is not measured relative to the **road surface** (2.6); thus, a vehicle at rest on an inclined road surface has a non-zero vehicle roll angle.

#### 5.2.5 suspension roll angle

$\varphi_K$   
angle from a line joining the **wheel centres** (4.1.2) of an axle to the  $X_V$ - $Y_V$  plane

#### 5.2.6 cab pitch angle

$\theta_C$   
angle from the  $X$  axis to the  $X_C$  axis, about the  $Y_C$  axis

#### 5.2.7 cab roll angle

$\varphi_C$   
angle from the  $Y$  axis to the  $Y_C$  axis, about the  $X_C$  axis

#### 5.2.8 sideslip angle

angle from the  $X$  axis to the vertical projection of the velocity vector of the point on to the **ground plane** (2.5), about the  $Z$  axis, at a given point on the vehicle

#### 5.2.9 vehicle sideslip angle

$\beta$   
angle from the  $X$  axis to the vertical projection of the **vehicle velocity** (5.1.1) on to the **ground plane** (2.5), about the  $Z$ -axis

NOTE Vehicle sideslip angle may be calculated from the **longitudinal velocity** (5.1.2)  $v_X$  and the **lateral velocity** (5.1.3)  $v_Y$  as given by:

$$\beta = \arctan \frac{v_X}{v_Y} \quad (1)$$

#### 5.2.10 tangent point

point on the  $X$  axis whose **sideslip angle** (5.2.8) is zero

**5.2.11  
 axle slip angle**

$\alpha_f, \alpha_r$

**sideslip angle** (5.2.8) at the centre of an axle minus the **mean kinematic steer angle** (7.1.4) for the axle

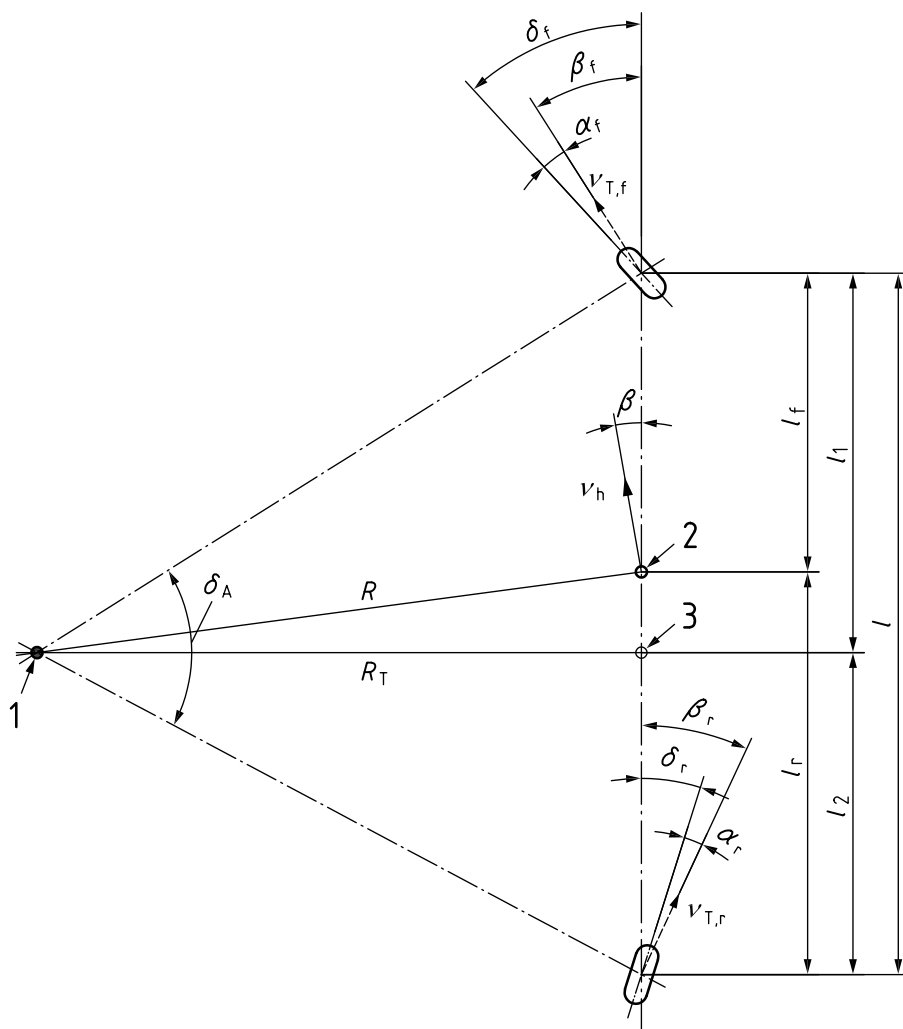
NOTE The symbolic notation  $\alpha_f, \alpha_r$  applies to two-axle vehicles. For multi-axle vehicles the notation  $\alpha_1, \alpha_2, \dots, \alpha_K$  may be assigned to a vehicle with  $K$  axles.

**5.2.12  
 slip angle**

tyre slip angle

$\alpha$

angle from the  $X_T$  axis to the normal projection of the **tyre trajectory velocity** (5.1.6) vector on to the  $X_T$ - $Y_T$  plane



**Key**

- 1 instantaneous centre of rotation
- 2 centre of gravity
- 3 tangent point

**Figure 5 — Slip angles for a single track two-axle model**

NOTE 1 The following angles are shown positive: **vehicle sideslip angle** (5.2.9),  $\beta$ , **front steer angle** (7.1.1),  $\delta_f$ , and **sideslip angle** (5.2.8) at the front axle,  $\beta_f$ ; the following angles are shown negative: **front axle slip angle** (5.2.11),  $\alpha_f$ , **rear steer angle**,  $\delta_r$ , **sideslip angle at the rear axle**,  $\beta_r$ , and **rear axle slip angle**,  $\alpha_r$ .

NOTE 2 In the single track model, the axle slip angle,  $\alpha_f$ ,  $\alpha_r$ , is equivalent to the tyre slip angle,  $\alpha$ . Strict adherence to the ISO coordinate system dictates that these angles are negative for positive **lateral forces** (6.1.3); however, the association of positive lateral forces with positive tyre slip angles is in common use.

**5.2.13**  
**yaw articulation angle**

$\Delta\psi_n$   
angle from the  $X_V$  axis of the trailing **vehicle unit** (3.1) (with index  $n+1$ ) to the normal projection of the  $X_V$  axis of the towing vehicle unit (with index  $n$ ) on to the trailing unit's  $X_V$ - $Y_V$  plane, about the  $Z_V$  axis of the trailing vehicle unit

NOTE The polarity is determined in the trailing vehicle unit axis system.

**5.2.14**  
**pitch articulation angle**

$\Delta\theta_n$   
angle from the  $X_V$  axis of the trailing **vehicle unit** (3.1) (with index  $n+1$ ) to the normal projection of the  $X_V$  axis of the towing vehicle unit (with index  $n$ ) on to the trailing unit's  $X_V$ - $Z_V$  plane, about the  $Y_V$  axis of the trailing vehicle unit

NOTE The polarity is determined in the trailing vehicle unit axis system.

**5.2.15**  
**roll articulation angle**

$\Delta\varphi_n$   
angle from the  $Y_V$  axis of the trailing **vehicle unit** (3.1) (with index  $n+1$ ) to the normal projection of the  $Y_V$  axis of the towing vehicle unit (with index  $n$ ) on to the trailing unit's  $Y_V$ - $Z_V$  plane, about the  $X_V$  axis of the trailing vehicle unit

NOTE The polarity is determined in the trailing vehicle unit axis system.

**5.2.16**  
**vehicle angular velocity**

$\vec{\omega}$   
vector quantity expressing the angular velocity of the **vehicle axis system** (2.10)

**5.2.17**  
**roll velocity**

$\vec{\omega}_X$   
 $X$  component of the **vehicle angular velocity** (5.2.16)

**5.2.18**  
**pitch velocity**

$\vec{\omega}_Y$   
 $Y$  component of the **vehicle angular velocity** (5.2.16)

**5.2.19**  
**yaw velocity**

$\vec{\omega}_Z$   
 $Z$  component of the **vehicle angular velocity** (5.2.16)

**5.2.20**  
**wheel-spin velocity**

$\vec{\omega}_W$   
angular velocity of the wheel about the  $Y_W$  axis

**5.2.21**  
**reference wheel-spin velocity**

$\vec{\omega}_{W0}$   
**wheel-spin velocity** (5.2.20) of the **straight free-rolling tyre** (10.1.1) at a given set of operating conditions

### 5.2.22

#### vehicle angular acceleration

$\vec{\omega}$

vector quantity expressing the angular acceleration of the **vehicle axis system** (2.10)

### 5.2.23

#### roll acceleration

$\vec{\omega}_X$

X component of the **vehicle angular acceleration** (5.2.22)

### 5.2.24

#### pitch acceleration

$\vec{\omega}_Y$

Y component of the **vehicle angular acceleration** (5.2.22)

### 5.2.25

#### yaw acceleration

$\vec{\omega}_Z$

Z component of the **vehicle angular acceleration** (5.2.22)

## 5.3 Terms relating to vehicle trajectory measures

### 5.3.1

#### vehicle trajectory

path of a selected point on the vehicle in the **earth-fixed coordinate system** (2.9)

NOTE This point is usually the **vehicle reference point** (2.12).

### 5.3.2

#### vehicle path

vertical projection of the **vehicle trajectory** (5.3.1) on to the **ground plane** (2.5)

### 5.3.3

#### path radius

$R_P$

instantaneous radius of curvature of the **vehicle path** (5.3.2)

NOTE The path radius is calculated using Formula (2):

$$R_P = \frac{v_h^2}{a_c} \quad (2)$$

### 5.3.4

#### vehicle motion radius

$R$

distance between the projection of the centre of gravity on to the **ground plane** (2.5) and the instantaneous centre of rotation

NOTE 1 The vehicle motion radius is calculated using Formula (3):

$$R = \frac{v_h}{\omega_Z} \quad (3)$$

NOTE 2 For **steady state** (12.2.1) cornering conditions,  $R = R_P$

### 5.3.5 path curvature

curvature of trajectory

$\kappa$

inverse of **path radius** (5.3.3)

NOTE The path curvature is calculated using Formula (4):

$$\kappa = \frac{1}{R_p} \quad (4)$$

### 5.3.6 course angle

$\nu$

angle between the projection of the **vehicle velocity** (5.1.1) on to the **ground plane** (2.5) and the  $X_E$  axis

NOTE The course angle can be computed from the **yaw angle** (5.2.1),  $\psi$ , and the **vehicle sideslip angle** (5.2.9),  $\beta$ , using Formula (5):

$$\nu = \psi + \beta \quad (5)$$

## 6 Forces and moments

For the definitions in this clause, forces and moments are resolved into components in the **intermediate axis system** (2.13) ( $X, Y, Z$ ).

NOTE It is also possible to resolve force and moment vectors into components in other **axis systems** (2.3). For example, force and moment vectors may be resolved in the **vehicle axis system** (2.10) ( $X_V, Y_V, Z_V$ ) to produce  $F_{X_V}, F_{Y_V}, F_{Z_V}$  and  $M_{X_V}, M_{Y_V}, M_{Z_V}$ .

### 6.1 Forces

#### 6.1.1 vehicle force

$\vec{F}$

vector quantity expressing the sum of the external forces acting on the vehicle at any instant, with its line of action passing through the **vehicle reference point** (2.12)

#### 6.1.2 longitudinal force

$\vec{F}_X$

component of the **vehicle force** (6.1.1) in the direction of the  $X$  axis

#### 6.1.3 lateral force

$\vec{F}_Y$

component of the **vehicle force** (6.1.1) in the direction of the  $Y$  axis

#### 6.1.4 vertical force

$\vec{F}_Z$

component of the **vehicle force** (6.1.1) in the direction of the  $Z$  axis

## 6.2 Moments

### 6.2.1 vehicle moment

$\vec{M}$   
vector quantity expressing the sum of the external moments acting on the vehicle at any instant, consistent with the **vehicle force** (6.1.1)

### 6.2.2 roll moment

$\vec{M}_X$   
component of the **vehicle moment** (6.2.1) about the  $X$  axis

### 6.2.3 pitch moment

$\vec{M}_Y$   
component of the **vehicle moment** (6.2.1) about the  $Y$  axis

### 6.2.4 yaw moment

$\vec{M}_Z$   
component of the **vehicle moment** (6.2.1) about the  $Z$  axis

### 6.2.5 steering-wheel torque

hand-wheel torque  
 $\vec{M}_H$   
moment applied to the steering-wheel, usually by the driver, about its axis of rotation

NOTE Usually the measured torque does not contain any torques attributable to steering-wheel inertia effects.

### 6.2.6 steering-axis torque

kingpin torque  
 $\vec{M}_S$   
moment about the **steering axis** (7.2.1) of a steerable wheel due to the sum of tyre and suspension forces and moments

## 7 Suspension and steering geometry

### 7.1 Steer and camber angles

#### 7.1.1 steer angle

$\delta$   
angle from  $X_V$  axis to the **wheel plane** (4.1), about the  $Z_V$  axis, for each road wheel

NOTE The sign of steer angle is determined using the right-hand rule. The symbolic notation  $\delta_{1L}, \delta_{2L}, \dots, \delta_{1R}, \delta_{2R}, \dots$  may be used for axle 1, axle 2, ... and left, right.

#### 7.1.2 kinematic steer angle

$\delta_{kin}$   
**steer angle(s)** (7.1.1) corresponding to a given **steering-wheel angle** (7.1.8), defined by the kinematics of the steering system in the absence of tyre forces and moments, and vertical wheel-to-body travel, but including static tyre vertical load



NOTE 1 The kinematic steer angle may be defined for any steerable wheel. It is intended that the wheel position be specified.

NOTE 2 For practical measurement purposes, the vertical wheel-to-body travel need not be constrained.

NOTE 3 Some vehicles may utilize steered axles whose steer angle is a function of a separate input, for example **trailer** (3.2) **yaw articulation angle** (5.2.13). In these cases, it is intended that the kinematic steer angle of the axle concerned be referenced to the corresponding input.

### 7.1.3

#### mean steer angle

$\delta_m$

mean road wheel steer angle

average of the left and right hand **steer angles** (7.1.1) on the same axle

### 7.1.4

#### mean kinematic steer angle

$\delta_{m,kin}$

**mean steer angle** (7.1.3), corresponding to a given **steering-wheel angle** (7.1.8), defined by the kinematics of the steering system in the absence of tyre forces and moments, and vertical wheel-to-body travel, but including static tyre vertical load

NOTE The mean kinematic steer angle may be defined for any steerable axle. It is intended that the position of the axle be specified.

### 7.1.5

#### included kinematic steer angle

$\delta_{inc,kin}$

**mean kinematic steer angle** (7.1.4) of the front axle minus the mean kinematic steer angle of the rear axle

### 7.1.6

#### static toe angle

angle between the  $X_V$  axis and the **wheel plane** (4.1), about the  $Z_V$  axis, with the vehicle at rest and the steering in the straight ahead position, where the wheel is “toed-in” if the forward portion of the wheel is closer to the vehicle centreline than the **wheel centre** (4.1.2) and “toed-out” if it is farther away

NOTE By convention, toe-in is considered a positive angle, and toe-out is a negative angle.

### 7.1.7

#### total static toe angle

sum of the **static toe angles** (7.1.6) on the same axle

### 7.1.8

#### steering-wheel angle

hand-wheel angle

$\delta_H$

angular displacement of the steering-wheel measured from the straight ahead position, which is positive for a left turn of the vehicle

NOTE The straight ahead position may be determined dynamically, as the steering-wheel angle which produces zero **yaw velocity** (5.2.19) under a given set of initial conditions or statically, as the steering-wheel angle corresponding to zero **mean steer angle** (7.1.3) ( $\delta_m = 0$ ).

### 7.1.9

#### Ackermann geometry

steering system kinematics where the  $Y_W$  axes of the steerable wheels intersect the centre of rotation, for non-zero **steering-wheel angles** (7.1.8) at negligible **lateral accelerations** (5.1.12)

NOTE Ackermann geometry may be achieved at one or more steering-wheel angles during steering articulation. For a front steer only vehicle, the centre of rotation is on the line defined by the rear **wheel centres** (4.1.2). For a four-wheel steer vehicle, and vehicles with more than two axles, the centre of rotation is generally not on this line. A steering system

with Ackermann geometry is said to be 100 % Ackermann, and one with equal **steer angles** (7.1.1) (parallel steer) on the steerable axle is said to be 0 % Ackermann.

### 7.1.10

#### Ackermann wheel steer angle

$\delta_{AW}$

**steer angle** (7.1.1) necessary to cause the local  $Y_W$  axis to pass through the vehicle centre of rotation

NOTE 1 An Ackermann wheel steer angle is defined for each individual wheel.

NOTE 2 This definition is more general than the definition in the previous edition in order to accommodate four-wheel steering. For a front steer vehicle with the centre of rotation lying on a projection of the rear axle centreline, this definition accommodates the former definition and defines an Ackermann steer angle for both front wheels. The symbolic notation  $\delta_{AW,1L}, \delta_{AW,2L}, \dots, \delta_{AW,1R}, \delta_{AW,2R}, \dots$  may be used for left, right, axle 1, axle 2, .....

### 7.1.11

#### included Ackermann steer angle

Ackermann steer angle

$\delta_A$

included angle between the line from the centre of rotation to the centre of the front axle and the line from the centre of rotation to the centre of the rear axle

NOTE 1 It is calculated using Formula (6):

$$\delta_A = \arctan \frac{l_1}{R_T} - \arctan \frac{l_2}{R_T} \quad (6)$$

where

$l_1$  is the signed distance from the **tangent point** (5.2.10) to the front axle;

$l_2$  is the signed distance from the tangent point to the rear axle;

$R_T$  is the **path radius** (5.3.3) of the tangent point (see Figure 5).

NOTE 2 This definition can also be expressed as the difference between the **sideslip angle** (5.2.8) of the centre of the front axle and the sideslip angle of the centre of the rear axle.

For a tangent point located between the axles,  $l_1$  is positive and  $l_2$  is negative.

For the condition of low speed running, with zero **slip angles** (5.2.12), on a radius of turn,  $R$ , which is large in comparison to the vehicle **wheelbase** (4.2),  $l$ , the following approximation can be used:

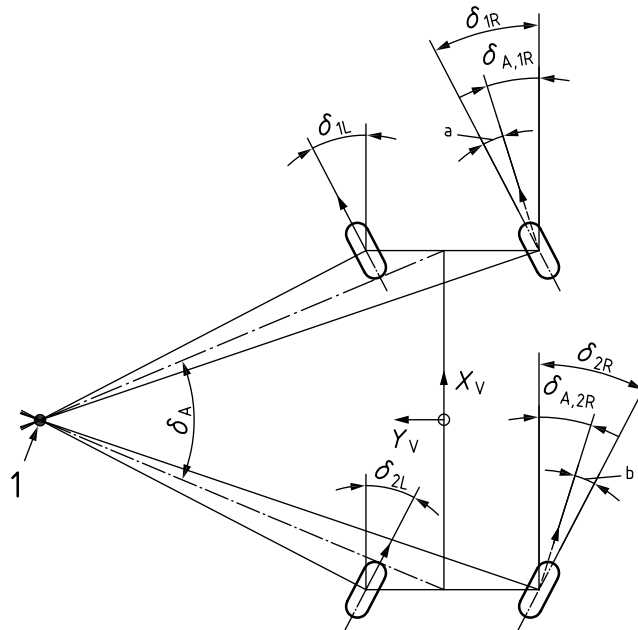
$$\delta_A = \frac{l}{R} \quad (7)$$

### 7.1.12

#### Ackermann error

difference between the **steer angle** (7.1.1) and the **Ackermann wheel steer angle** (7.1.10)

NOTE For a front steer only vehicle, Ackermann error exists if the  $Y_W$  axes of the steerable wheels do not intersect on the line defined by the rear **wheel centres** (4.1.2). For a four-wheel steer vehicle, Ackermann error exists if the  $Y_W$  axes of the steerable wheels do not intersect at a point. If Ackermann error exists, an ideal centre of rotation may be defined by the intersection of the  $Y_W$  axes of the inner wheels. In this case, Ackermann error is the difference between the steer angle and the Ackermann wheel steer angle of the outer wheels, for which the  $Y_W$  axis passes through the ideal centre of rotation. Ackermann error is defined only at the front axle for a front only steer vehicle and at both the front and rear axles for a four-wheel steer vehicle. For vehicles with more than two axles, an ideal centre of rotation cannot be fixed by geometry.



**Key**

- 1 turn centre
- a Ackermann error, front.
- b Ackermann error, rear.

**Figure 6 — Ackermann geometry**

**7.1.13  
steering ratio**

rate of change of **steering-wheel angle** (7.1.8) with respect to the **mean kinematic steer angle** (7.1.4) of a pair of steered wheels at a given steering-wheel position

NOTE For a two-axle vehicle, the symbolic notations,  $i_{S,f}$  and  $i_{S,r}$ , may be used for the front and rear axle steering ratios, respectively.

**7.1.14  
trailer-axle steering ratio**

rate of change of **mean kinematic steer angle** (7.1.4) of a pair of steered wheels with respect to the **yaw articulation angle** (5.2.13) at a given yaw articulation angle

**7.1.15  
overall steering ratio**

$i_S$   
rate of change of **steering-wheel angle** (7.1.8) with respect to the **included kinematic steer angle** (7.1.5) at a given steering-wheel position

NOTE 1 This definition is only applicable to two-axle vehicles. For vehicles with three or more axles, a combined steering ratio for all front axles and/or all rear axles has to be determined by virtually merging the front axles and/or rear axles together.

NOTE 2 The overall steering ratio can be expressed by:  $1/i_S = (1/i_{S,f} - 1/i_{S,r})$ .

**7.1.16  
inclination angle**

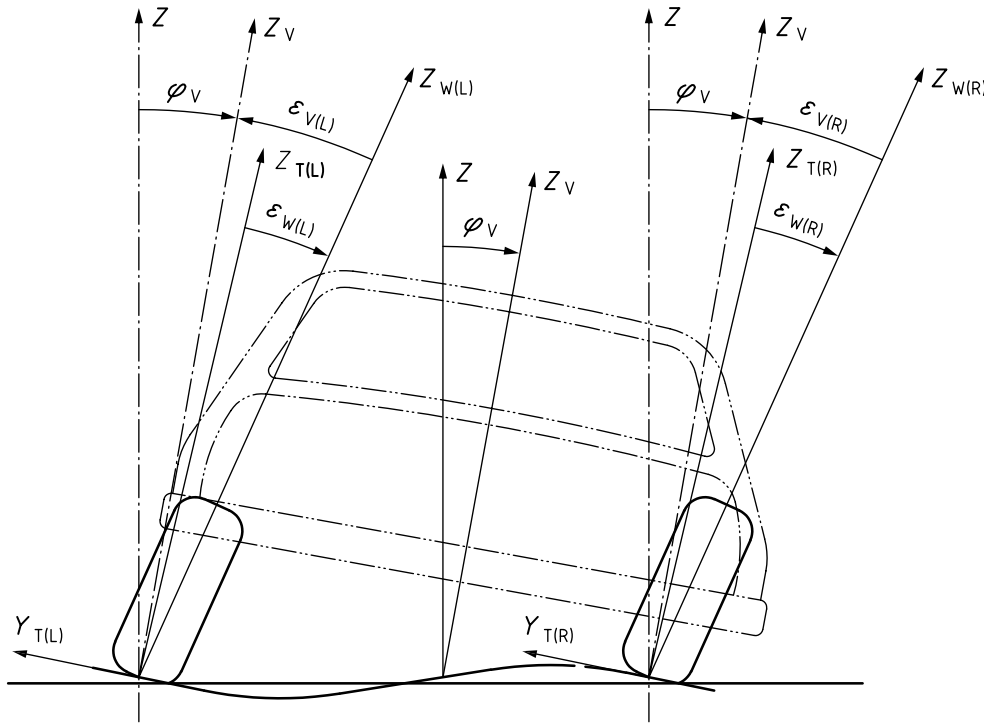
$\varepsilon_W$   
angle from the  $Z_T$  axis to the  $Z_W$  axis

NOTE Also commonly referred to as the camber angle with respect to the road.

**7.1.17  
 camber angle**

$\varepsilon_V$   
 angle between the  $Z_V$  axis and the **wheel plane** (4.1), about the  $X_V$  axis

NOTE It is considered positive when the wheel leans outward at the top, relative to the vehicle body, and negative when it leans inward.



For  $\theta = \delta_L = \delta_R = 0$ .

**Figure 7 — Camber, inclination and vehicle roll angles**

**7.2 Steering-axis geometry**

**7.2.1  
 steering axis**

kingpin axis

axis about which the wheel and hub assembly rotates relative to the vehicle structure when steered, in the absence of tyre forces and moments, except the tyre vertical load

NOTE This axis can shift, due to suspension kinematics, as the **steer angle** (7.1.1) changes.

**7.2.2  
 castor angle**

$\tau$   
 angle between the  $Z_V$  axis and the normal projection of the **steering axis** (7.2.1) on to the  $X_V$ - $Z_V$  plane

NOTE The angle is positive when the top of the steering axis is inclined rearward.

### 7.2.3

#### castor offset at ground

castor trail  
kinematic trail

$n_k$

distance in the  $X_T$  direction from the  $Y_T$ - $Z_T$  plane to the point where the **steering axis** (7.2.1) intersects the  $X_T$ - $Y_T$  plane

### 7.2.4

#### castor offset at wheel centre

spindle trail

$n_\tau$

distance between the projection of the **wheel centre** (4.1.2) and the projection of the **steering axis** (7.2.1) on to a plane which is normal to the  $X_V$ - $Y_V$  plane and parallel to the intersection of the **wheel plane** (4.1) with the  $X_V$ - $Y_V$  plane

NOTE This distance is measured parallel to the  $X_V$ - $Y_V$  plane and is positive if the projection of the steering axis is forward of the wheel centre.

### 7.2.5

#### steering-axis inclination angle

kingpin inclination angle

$\sigma$

angle between the  $Z_V$  axis and the normal projection of the **steering axis** (7.2.1) on to the  $Y_V$ - $Z_V$  plane

NOTE The angle is positive when the top of the steering axis is inclined inward.

### 7.2.6

#### steering-axis offset at ground

kingpin offset at ground

$r_k$

distance in the  $Y_T$  direction between the **wheel plane** (4.1) and the point where the **steering axis** (7.2.1) intersects the  $X_T$ - $Y_T$  plane

NOTE This distance is positive if the steering axis intersection point is inboard of the wheel plane.

### 7.2.7

#### steering-axis offset at wheel centre

kingpin offset at wheel centre

$r_\sigma$

distance between the projection of the **wheel centre** (4.1.2) and the projection of the **steering axis** (7.2.1) on to a plane which is normal to the  $X_V$ - $Y_V$  plane and normal to the intersection of the **wheel plane** (4.1) with the  $X_V$ - $Y_V$  plane

NOTE This distance is measured parallel to the  $X_V$ - $Y_V$  plane and is positive if the projection of the steering axis is inboard of the wheel centre.

### 7.2.8

#### normal steering-axis offset at wheel centre

$q_W$

distance between the projection of the **wheel centre** (4.1.2) and the projection of the **steering axis** (7.2.1) on to a plane which is normal to the  $X_V$ - $Y_V$  plane and normal to the intersection of the **wheel plane** (4.1) with the  $X_V$ - $Y_V$  plane

NOTE This distance is measured normal to the projected steering axis and is positive if the projection of the steering axis is inboard of the wheel centre.

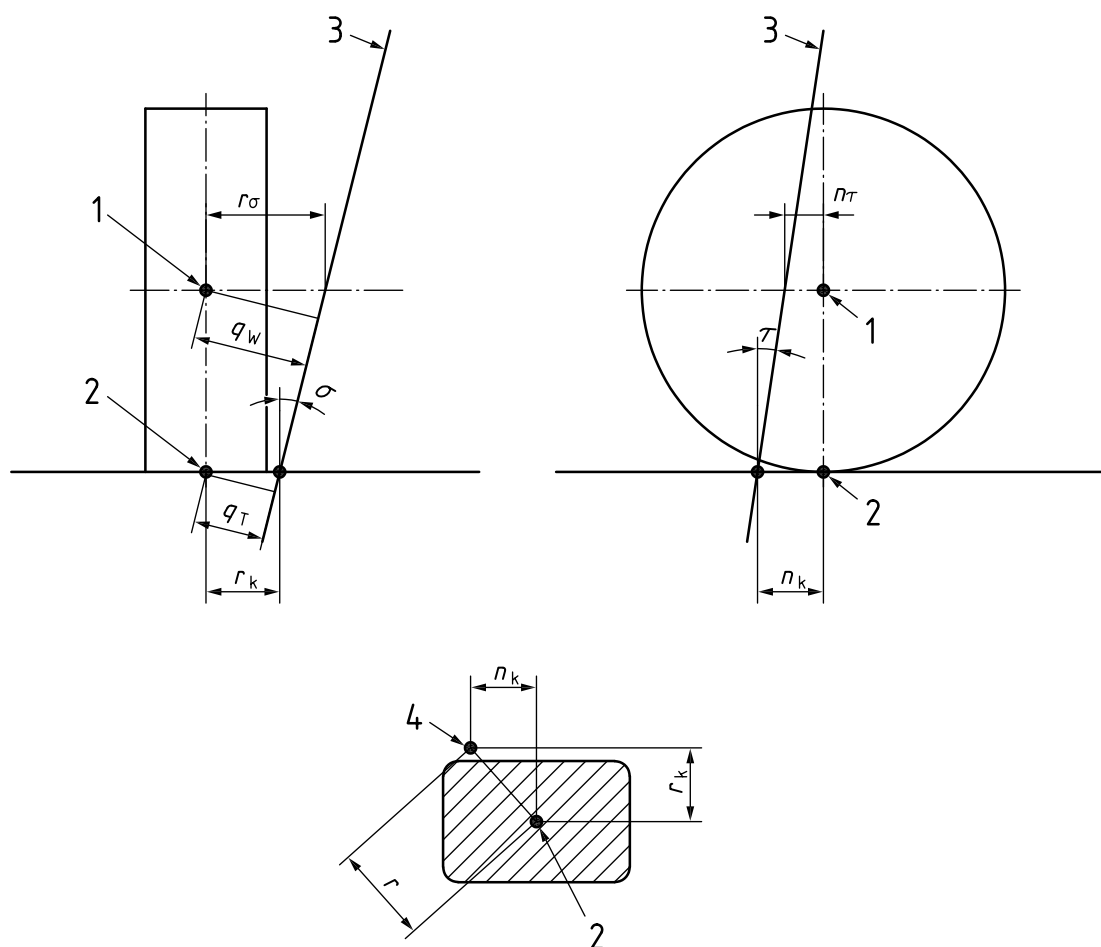
**7.2.9**  
**normal steering-axis offset at ground**

$q_T$   
 distance from the projection of the **contact centre** (4.1.4) to the projection of the **steering axis** (7.2.1) on to a plane which is normal to the  $X_V-Y_V$  plane and normal to the intersection of the **wheel plane** (4.1) with the  $X_V-Y_V$  plane

NOTE This distance is measured normal to the projected steering axis and is positive if the projection of the steering axis is inboard of the **wheel centre** (4.1.2).

**7.2.10**  
**scrub radius**

$r$   
 distance from the **contact centre** (4.1.4) to the point where the **steering axis** (7.2.1) intersects the  $X_T-Y_T$  plane



**Key**

- 1 wheel centre
- 2 contact centre
- 3 steering axis
- 4 intersection of steering axis with the road plane

**Figure 8 — Steering-axis geometry**

## 8 Kinematics and compliances

### 8.1 Kinematics

NOTE The terms in this subclause describe the changes in wheel geometry caused by vertical wheel-to-body travel and/or steering in the absence of tyre forces and moments. For practical use, these terms can be determined in the presence of vertical loads. The term “wheel-to-body displacement” refers to the **wheel centre** (4.1.2) and does not include tyre deflection.

#### 8.1.1

##### **ride track change**

change of **track** (4.4) resulting from symmetric wheel-to-body displacement

#### 8.1.2

##### **ride track change gradient**

rate of change of **track** (4.4) with respect to symmetric wheel-to-body displacement

#### 8.1.3

##### **ride wheel precession/recession**

change of longitudinal displacement of the **wheel centre** (4.1.2) resulting from symmetric wheel-to-body displacement

NOTE Precession is  $X_V$  positive, recession is  $X_V$  negative.

#### 8.1.4

##### **ride wheel precession/recession gradient**

rate of change of longitudinal displacement of the **wheel centre** (4.1.2) with respect to symmetric wheel-to-body displacement

#### 8.1.5

##### **ride steer**

change of **steer angle** (7.1.1) of a wheel resulting from symmetric wheel-to-body displacement

#### 8.1.6

##### **ride steer gradient**

rate of change of **steer angle** (7.1.1) with respect to symmetric wheel-to-body displacement

#### 8.1.7

##### **ride camber**

change of **camber angle** (7.1.17) of a wheel resulting from symmetric wheel-to-body displacement

#### 8.1.8

##### **ride camber gradient**

rate of change of **camber angle** (7.1.17) with respect to symmetric wheel-to-body displacement

#### 8.1.9

##### **ride castor**

change of **castor angle** (7.2.2) resulting from symmetric wheel-to-body displacement

#### 8.1.10

##### **ride castor gradient**

rate of change of **castor angle** (7.2.2) with respect to symmetric wheel-to-body displacement

NOTE The terms “bump” and “jounce” are also in common use to describe symmetric wheel-to-body displacement.

#### 8.1.11

##### **roll steer**

change of **steer angle** (7.1.1) of a wheel resulting from suspension roll displacement

#### **8.1.12**

##### **roll steer gradient**

rate of change of **steer angle** (7.1.1) with respect to suspension roll displacement

#### **8.1.13**

##### **roll camber**

change of **camber angle** (7.1.17) of a wheel resulting from suspension roll displacement

#### **8.1.14**

##### **roll camber gradient**

rate of change of **camber angle** (7.1.17) with respect to suspension roll displacement

#### **8.1.15**

##### **roll castor**

change of **castor angle** (7.2.2) resulting from suspension roll displacement

#### **8.1.16**

##### **roll castor gradient**

rate of change of **castor angle** (7.2.2) with respect to suspension roll displacement

NOTE 1 The definitions from 8.1.1 to 8.1.16 for roll kinematics can refer to “wheel-to-body displacement” or to “**suspension roll angle** (5.2.5)”.

NOTE 2 Suspension roll displacement may be defined based on the needs of the analysis or test. Commonly used definitions include anti-symmetric wheel-to-body displacement and roll displacement whereby a constant axle load is maintained.

#### **8.1.17**

##### **roll centre**

point in the transverse vertical plane through the **wheel centres** (4.1.2) on an axle at which **lateral forces** (6.1.3) may be applied to the **sprung mass** (4.12) without producing suspension roll

NOTE The roll centre constitutes an idealized kinematic concept and does not necessarily represent a true instantaneous centre of rotation of the sprung mass.

#### **8.1.18**

##### **roll centre height**

height of the **roll centre** (8.1.17) above a line connecting the **contact centres** (4.1.4) for an axle

#### **8.1.19**

##### **roll axis**

line joining the front and rear **roll centres** (8.1.17)

NOTE This definition is only applicable to two-axle vehicles.

#### **8.1.20**

##### **steer camber**

change of **camber angle** (7.1.17) of a wheel resulting from **steer angle** (7.1.1) displacement

#### **8.1.21**

##### **steer camber gradient**

rate of change of **camber angle** (7.1.17) with respect to **steer angle** (7.1.1) displacement

#### **8.1.22**

##### **steer castor**

change of **castor angle** (7.2.2) resulting from **steer angle** (7.1.1) displacement

#### **8.1.23**

##### **steer castor gradient**

rate of change of **castor angle** (7.2.2) with respect to **steer angle** (7.1.1) displacement



## 8.2 Compliances

NOTE The terms in this subclause describe the changes in wheel geometry caused by tyre forces and moments.

### 8.2.1

#### **camber compliance**

rate of change of **camber angle** (7.1.17) with respect to a tyre force or moment

### 8.2.2

#### **steer compliance**

rate of change of **steer angle** (7.1.1) with respect to a tyre force or moment

### 8.2.3

#### **lateral compliance at the wheel centre**

rate of change of the lateral displacement of the **wheel centre** (4.1.2) with respect to a tyre force or moment

### 8.2.4

#### **lateral compliance at the contact centre**

rate of change of the lateral displacement of the **contact centre** (4.1.4) with respect to a tyre force or moment

NOTE The lateral compliance at the contact centre may consist of several compound suspension compliances (camber, lateral, steer, etc.), but excludes tyre compliances.

### 8.2.5

#### **longitudinal compliance**

rate of change of the longitudinal displacement of the **wheel centre** (4.1.2) with respect to a tyre force or moment

### 8.2.6

#### **windup compliance**

rate of change of the angular displacement of the wheel about the **wheel-spin axis** (4.1.1) with respect to a tyre force or moment

### 8.2.7

#### **axle windup compliance**

rate of change of the angular displacement of the axle housing of a solid axle suspension about the **wheel-spin axis** (4.1.1) with respect to a tyre force or moment

## 9 Ride and roll stiffness

### 9.1

#### **ride rate**

rate of change of the **tyre normal force** (10.2.1) with respect to a displacement of the **contact centre** (4.1.4) in the  $Z_V$  direction

### 9.2

#### **suspension ride rate**

wheel rate

rate of change of the **tyre normal force** (10.2.1) with respect to a displacement of the **wheel centre** (4.1.2) in the  $Z_V$  direction

### 9.3

#### **roll stiffness**

rate of change of the restoring couple exerted by an axle on the **sprung mass** (4.12) with respect to the angular change about the  $X_V$  axis of a line joining the **contact centres** (4.1.4) of the axle

## 9.4

### **suspension roll stiffness**

rate of change of the restoring couple exerted by an axle on the **sprung mass** (4.12) with respect to the **suspension roll angle** (5.2.5) of the axle

## 9.5

### **vehicle roll stiffness**

sum of the individual **roll stiffnesses** (9.3)

## 9.6

### **roll stiffness distribution**

distribution of the **vehicle roll stiffness** (9.5) between the individual axles expressed as a percentage of the vehicle roll stiffness

NOTE The terms “vehicle roll stiffness” and “roll stiffness distribution” both reference the definition of **roll stiffness** (9.3), and, therefore, include the compliances of both the tyres and the suspensions. Similar terms that would exclude the influence of the tyres, and depend only on the compliances of the suspensions, can be defined with reference to the **suspension roll stiffness** (9.4).

## 10 Tyres

### 10.1 Tyre geometry

#### 10.1.1

##### **straight free-rolling tyre**

loaded rolling tyre, at zero **slip** (5.2.12) and **inclination** (7.1.16) angles, moving along a linear path in the absence of braking or driving torques

#### 10.1.2

##### **loaded radius**

distance from the **wheel centre** (4.1.2) to the **contact centre** (4.1.4) at a specified operating condition

#### 10.1.3

##### **static loaded radius**

**loaded radius** (10.1.2) at zero speed

#### 10.1.4

##### **dynamic rolling circumference**

$C_R$   
distance travelled on the ground during one rotation of the wheel at a specified operating condition

#### 10.1.5

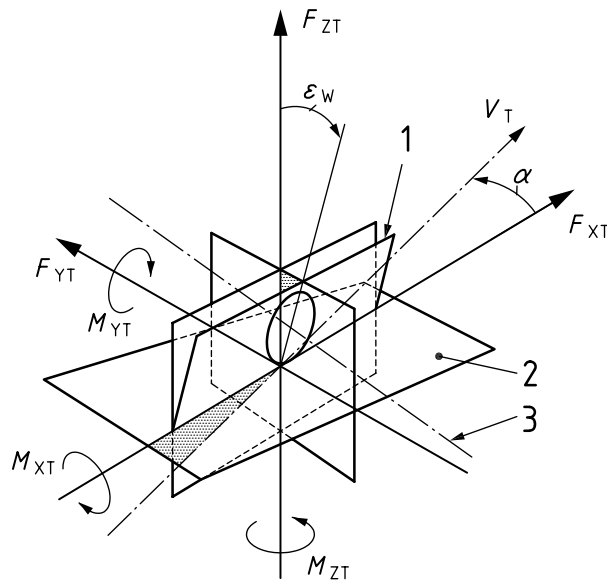
##### **dynamic rolling radius**

$r_{\text{dyn}}$   
radius derived from the **dynamic rolling circumference** (10.1.4)

NOTE The dynamic rolling radius is expressed as:

$$r_{\text{dyn}} = \frac{C_R}{2\pi} \quad (8)$$

## 10.2 Tyre forces and moments



### Key

- 1 wheel plane
- 2 road plane
- 3 wheel-spin axis

Figure 9 — Tyre force and moment nomenclature

### 10.2.1

#### tyre normal force

$\bar{F}_{ZT}$   
component of the total force exerted on the tyre by the road in the direction of the  $Z_T$  axis

### 10.2.2

#### tyre vertical force

tyre vertical load  
component of the total force exerted on the tyre by the road in the direction of the  $Z_E$  axis

### 10.2.3

#### static tyre load

tyre vertical force (10.2.2) with the vehicle at rest on a horizontal surface

### 10.2.4

#### tyre shear force

$\bar{F}_{XYT}$   
component of the total force exerted on the tyre by the road in the  $X_T$ - $Y_T$  plane

### 10.2.5

#### tyre longitudinal force

$\bar{F}_{XT}$   
component of the tyre shear force (10.2.4) vector in the direction of the  $X_T$  axis

### 10.2.6

#### tyre lateral force

$\bar{F}_{YT}$   
component of the tyre shear force (10.2.4) vector in the direction of the  $Y_T$  axis

### 10.2.7

#### tyre lateral load transfer

tyre vertical force (10.2.2) transfer from the tyre(s) on one side of an axle to the tyre(s) on the other side of the axle

NOTE It can be caused by, for example, lateral acceleration (5.1.12), road camber and drive-train torque.

### 10.2.8

#### total tyre lateral load transfer

sum of the tyre lateral load transfers (10.2.7) for all axles

### 10.2.9

#### tyre lateral load transfer distribution

ratio of the tyre lateral load transfer (10.2.7) on one axle to the total tyre lateral load transfer (10.2.8)

NOTE For a two-axle vehicle, this is the ratio of the front axle tyre lateral load transfer to the total tyre lateral load transfer.

### 10.2.10

#### tyre longitudinal load transfer

total tyre vertical force (10.2.2) transfer from one axle to the others

NOTE It can be caused by, for example, longitudinal acceleration (5.1.11) and road elevation.

### 10.2.11

#### tyre overturning moment

$\bar{M}_{XT}$

component of the moment produced by the total force exerted on the tyre by the road, about the  $X_T$  axis

### 10.2.12

#### tyre rolling moment

tyre rolling resistance moment

$\bar{M}_{YT}$

component of the moment produced by the total force exerted on the tyre by the road, about the  $Y_T$  axis

### 10.2.13

#### tyre aligning moment

tyre aligning torque

$\bar{M}_{ZT}$

component of the moment produced by the total force exerted on the tyre by the road, about the  $Z_T$  axis

NOTE Typically, components of the tyre aligning moment may consist of the moments resulting from tyre lateral force (10.2.6) and pneumatic trail and tyre longitudinal force (10.2.5) and lateral offset to the contact centre (4.1.4).

## 10.3 Terms relating to tyre measures

### 10.3.1

#### tyre normal stiffness

tyre radial stiffness

tyre spring rate

first derivative of the tyre normal force (10.2.1) with respect to the change in loaded radius (10.1.2)

### 10.3.2

#### tyre rolling resistance

$F_R$

loss of energy (or energy consumed) in the tyre per unit distance

NOTE This is equivalent to a drag force.

### 10.3.3

#### rolling resistance coefficient

ratio of the **tyre rolling resistance** (10.3.2) to the **tyre normal force** (10.2.1)

NOTE Rolling resistance coefficient is expressed as:

$$\frac{F_R}{F_{ZT}} \quad (9)$$

### 10.3.4

#### tyre longitudinal force coefficient

$\mu_{XT}$

ratio of the **tyre longitudinal force** (10.2.5) to the **tyre normal force** (10.2.1)

NOTE Tyre longitudinal force coefficient is expressed as:

$$\frac{F_{XT}}{F_{ZT}} \quad (10)$$

### 10.3.5

#### maximum longitudinal force coefficient

maximum value of the **tyre longitudinal force coefficient** (10.3.4) attainable in **longitudinal slip** (10.3.9) on a given surface under a given set of operating conditions

### 10.3.6

#### sliding braking force coefficient

value of the **tyre longitudinal force coefficient** (10.3.4) attainable by a locked wheel on a given surface under a given set of operating conditions

### 10.3.7

#### longitudinal adhesion utilization coefficient

ratio of the **tyre longitudinal force coefficient** (10.3.4) to the **maximum longitudinal force coefficient** (10.3.5)

### 10.3.8

#### longitudinal slip angular velocity

difference between the **wheel-spin velocity** (5.2.20) and the **reference wheel-spin velocity** (5.2.21)

### 10.3.9

#### longitudinal slip

$S_X$

ratio of the **longitudinal slip angular velocity** (10.3.8) to the **reference wheel-spin velocity** (5.2.21)

NOTE Longitudinal slip is expressed as:

$$\mu_{XT} = \frac{F_{XT}}{F_{ZT}} \quad (11)$$

### 10.3.10

#### peak longitudinal slip

value of **longitudinal slip** (10.3.9) at which the **maximum longitudinal force coefficient** (10.3.5) occurs

### 10.3.11

#### tyre longitudinal stiffness

rate of change of **tyre longitudinal force** (10.2.5) with respect to **longitudinal slip** (10.3.9)

### 10.3.12

#### tyre lateral force coefficient

ratio of the **tyre lateral force** (10.2.6) to the **tyre normal force** (10.2.1)

NOTE Tyre lateral force coefficient is expressed as:

$$\frac{F_{YT}}{F_{ZT}} \quad (12)$$

### 10.3.13 maximum lateral force coefficient

maximum value of the **tyre lateral force coefficient** (10.3.12) attainable on a given surface under a given set of operating conditions

### 10.3.14 peak slip angle

value of the **slip angle** (5.2.12) at which the maximum **tyre lateral force** (10.2.6) occurs under given conditions

### 10.3.15 tyre cornering stiffness

negative of the first derivative of **tyre lateral force** (10.2.6) with respect to **slip angle** (5.2.12)

NOTE Tyre cornering stiffness is expressed as:

$$-\frac{\partial F_{YT}}{\partial \alpha} \quad (13)$$

### 10.3.16 tyre inclination stiffness

first derivative of **tyre lateral force** (10.2.6) with respect to **inclination angle** (7.1.16)

NOTE Tyre inclination stiffness is expressed as:

$$\frac{\partial F_{YT}}{\partial \varepsilon_W} \quad (14)$$

## 11 Input types and control modes

### 11.1 Input types

#### 11.1.1 control input

positioning of, or application of force to, an element of the vehicle (typically within the steering, braking or propulsion systems) for the purposes of maintaining, or inducing a change in, motion of the vehicle

#### 11.1.2 disturbance input

inputs, excluding **control inputs** (11.1.1), on the vehicle that induce a change in motion of the vehicle

NOTE For example road displacements, which generate vehicular ride motions, and wind gusts, which generate uncommanded vehicular lateral responses.

### 11.2 Control modes

#### 11.2.1 position control

mode of vehicle control wherein **control inputs** (11.1.1) or restraints in the form of displacements are placed at some control point in the system, independent of the force required

### 11.2.2

#### **fixed control**

mode of vehicle control wherein the position of some point in the control system is held fixed

NOTE This is a special case of **position control** (11.2.1).

### 11.2.3

#### **force control**

mode of vehicle control wherein **control inputs** (11.1.1) or restraints in the form of forces are placed upon some control point in the system, independent of the displacement required

### 11.2.4

#### **free control**

mode of vehicle control wherein no restraint is placed on the control system

NOTE This is a special limit case of **force control** (11.2.3).

### 11.2.5

#### **closed-loop control**

feedback control

mode of vehicle control wherein information about the **vehicle response** (12.1.1) is fed back to the input controller (driver or mechanical actuator) for comparison with the desired vehicle response, and the **control inputs** (11.1.1) are modified so as to reduce the error between the actual and desired vehicle response

### 11.2.6

#### **open-loop control**

mode of vehicle control wherein **control inputs** (11.1.1) are independent of the resulting **vehicle response** (12.1.1)

## 12 Responses

### 12.1 General response types

#### 12.1.1

##### **vehicle response**

vehicle motion resulting from **control inputs** (11.1.1) or **disturbance inputs** (11.1.2) to the vehicle

#### 12.1.2

##### **control response**

vehicle motion resulting from a **control input** (11.1.1)

#### 12.1.3

##### **disturbance response**

vehicle motion resulting from a **disturbance input** (11.1.2)

### 12.2 Equilibrium and stability

NOTE See Annex A.

#### 12.2.1

##### **steady state**

state of a vehicle where the sum of the applied external forces and moments and the inertial forces and moments which balance them form an unchanging force and moment system in the  $(X_V, Y_V, Z_V)$  and  $(X, Y, Z)$  frames of reference over an arbitrarily long time period

See A.1.

### 12.2.2

#### **transient state**

state of a vehicle wherein the applied external forces and moments, the control positions or the vehicle motion responses are varying with time

### 12.2.3

#### **non-oscillatory stability**

stability characteristic at a prescribed **steady state** (12.2.1) if, following any small temporary **disturbance input** (11.1.2) or **control input** (11.1.1), the vehicle returns to the steady state without oscillation

### 12.2.4

#### **non-oscillatory instability**

stability characteristic at a prescribed **steady state** (12.2.1) if a small temporary **disturbance input** (11.1.2) or **control input** (11.1.1) causes an ever-increasing **vehicle response** (12.1.1) without oscillation

See A.2.

### 12.2.5

#### **neutral stability**

stability characteristic at a prescribed **steady state** (12.2.1) if, as a result of any small temporary **disturbance input** (11.1.2) or **control input** (11.1.1), the vehicle attains a new steady state

### 12.2.6

#### **oscillatory stability**

stability characteristic at a prescribed **steady state** (12.2.1) if a small temporary **disturbance input** (11.1.2) or **control input** (11.1.1) causes an oscillatory **vehicle response** (12.1.1) of decreasing amplitude and a return to the original steady state

### 12.2.7

#### **oscillatory instability**

stability characteristic at a prescribed **steady state** (12.2.1) if a small temporary **disturbance input** (11.1.2) or **control input** (11.1.1) causes an oscillatory **vehicle response** (12.1.1) of ever-increasing amplitude about the initial steady state

See A.3.

## 12.3 Lateral response measures

### 12.3.1

#### **steering-wheel angle gradient**

hand-wheel angle gradient

rate of change of **steering-wheel angle** (7.1.8) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE 1 Steering-wheel angle gradient is expressed as:

$$\frac{\partial \delta_H}{\partial a_Y} \quad (15)$$

NOTE 2 This is the reciprocal of **lateral acceleration gain** (12.4.9).

### 12.3.2

#### **steering-wheel torque gradient**

hand-wheel torque gradient

rate of change of **steering-wheel torque** (6.2.5) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)



NOTE Steering-wheel torque gradient is expressed as:

$$\frac{\partial M_H}{\partial a_Y} \quad (16)$$

### 12.3.3

#### roll angle gradient

rate of change of **vehicle roll angle** (5.2.4) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE Roll angle gradient is expressed as:

$$\frac{\partial \varphi_V}{\partial a_Y} \quad (17)$$

### 12.3.4

#### suspension roll angle gradient

rate of change of **suspension roll angle** (5.2.5) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE Suspension roll angle gradient is expressed as:

$$\frac{\partial \varphi_K}{\partial a_Y} \quad (18)$$

### 12.3.5

#### sideslip angle gradient

rate of change of **vehicle sideslip angle** (5.2.9) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE Sideslip angle gradient is expressed as:

$$\frac{\partial \beta}{\partial a_Y} \quad (19)$$

### 12.3.6

#### yaw articulation angle gradient

articulation angle gradient

rate of change of **yaw articulation angle** (5.2.13) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE Yaw articulation angle gradient is expressed as:

$$\frac{\partial \Delta \psi_n}{\partial a_Y} \quad (20)$$

## 12.4 Understeer and oversteer measures

### 12.4.1

#### included Ackermann steer angle gradient

rate of change of **included Ackermann steer angle** (7.1.11) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE Included Ackermann steer angle gradient is expressed as:

$$\frac{\partial \delta_A}{\partial a_Y} \quad (21)$$

### 12.4.2

#### **included kinematic steer angle gradient**

rate of change of **included kinematic steer angle** (7.1.5) with respect to **lateral acceleration** (5.1.12) on a level road at a given **steady state** (12.2.1)

NOTE Included kinematic steer angle gradient is expressed as:

$$\frac{\partial \delta_{\text{inc,kin}}}{\partial a_Y} \quad (22)$$

### 12.4.3

#### **understeer/oversteer gradient**

*U*

quantity obtained by subtracting the **included Ackermann steer angle gradient** (12.4.1) from the **included kinematic steer angle gradient** (12.4.2), on a level road at a given **steady state** (12.2.1)

NOTE 1 Understeer/oversteer gradient is expressed as:

$$U = \frac{d \delta_{\text{inc,kin}}}{da_Y} - \frac{d \delta_A}{da_Y} \quad (23)$$

NOTE 2 Where the **overall steering ratio** (7.1.15) is constant, the understeer/oversteer gradient can be expressed as:

$$U = \frac{1}{i_S} \frac{d \delta_H}{da_Y} - \frac{d \delta_A}{da_Y} \quad (24)$$

### 12.4.4

#### **understeer**

steer property at a given **steady state** (12.2.1) where the **understeer/oversteer gradient** (12.4.3) is positive

See A.4.

### 12.4.5

#### **oversteer**

steer property at a given **steady state** (12.2.1) where the **understeer/oversteer gradient** (12.4.3) is negative

See A.4.

### 12.4.6

#### **neutral steer**

steer property at a given **steady state** (12.2.1) where the **understeer/oversteer gradient** (12.4.3) is zero

### 12.4.7

#### **stability factor**

*K*

**understeer/oversteer gradient** (12.4.3) divided by the **wheelbase** (4.2)

NOTE Stability factor is expressed as:

$$k = \frac{U}{l} \quad (25)$$

### 12.4.8

#### **yaw velocity gain**

rate of change of **yaw velocity** (5.2.19) with respect to **steering-wheel angle** (7.1.8) on a level road at a given **steady state** (12.2.1)

NOTE Common use is made of the approximation  $\dot{\psi} / \delta_H$  for the yaw velocity gain in the linear range and, where the **overall steering ratio** (7.1.15) is constant, it can be shown that:

$$\frac{\dot{\psi}}{\delta_H} = \frac{1}{i_S} \times \frac{v_h}{(l + v_h^2 U)} \quad (26)$$

#### 12.4.9 lateral acceleration gain

steering sensitivity

rate of change of **lateral acceleration** (5.1.12) with respect to **steering-wheel angle** (7.1.8) on a level road at a given **steady state** (12.2.1)

NOTE Lateral acceleration gain is expressed as:

$$-\frac{\partial a_Y}{\partial \delta_H} \quad (27)$$

#### 12.4.10 critical speed

$v_{crit}$

**horizontal velocity** (5.1.5) for an **oversteer** (12.4.5) vehicle at which the **yaw velocity gain** (12.4.8), at **steady state** (12.2.1), is infinite

NOTE 1 Critical speed is expressed as:

$$v_{crit} = \sqrt{\frac{l}{-U}}; \quad U < 0 \quad (28)$$

NOTE 2 Below this speed, the oversteer vehicle is stable. Above this speed the oversteer vehicle exhibits **non-oscillatory instability** (12.2.4). No critical speed exists for the **understeer** (12.4.4) and **neutral steer** (12.4.6) vehicles.

#### 12.4.11 characteristic speed

$v_{ch}$

**horizontal velocity** (5.1.5) for an **understeer** (12.4.4) vehicle at which the **yaw velocity gain** (12.4.8) is a maximum

NOTE 1 Characteristic speed is expressed as:

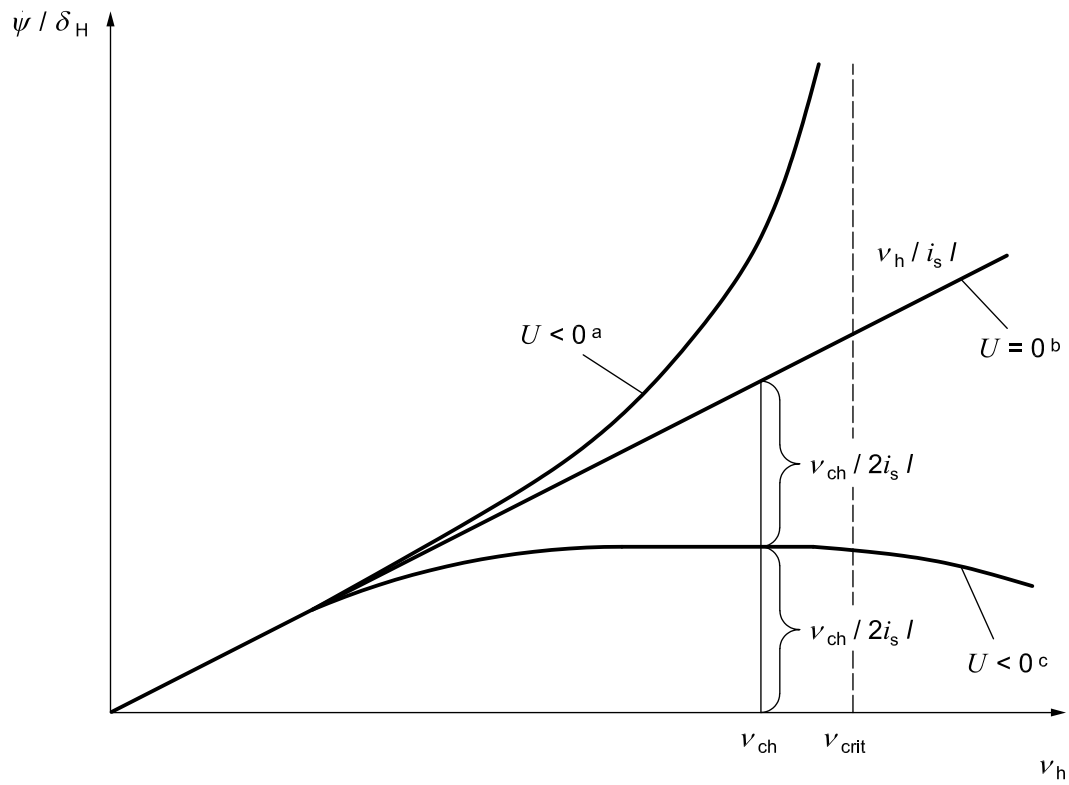
$$v_{ch} = \sqrt{\frac{l}{U}}; \quad U > 0 \quad (29)$$

NOTE 2 At the characteristic speed, the yaw velocity gain is half that of a **neutral steer** (12.4.6) vehicle.

#### 12.4.12 zero damping speed

$v_{zd}$

**longitudinal velocity** (5.1.2) at which the yaw damping of a tow vehicle/**trailer** (3.2) combination equals zero



- a Oversteer.
- b Neutral steer.
- c Understeer.

**Figure 10 — Critical and characteristic speeds**

## Annex A (informative)

### Comments on terms and definitions

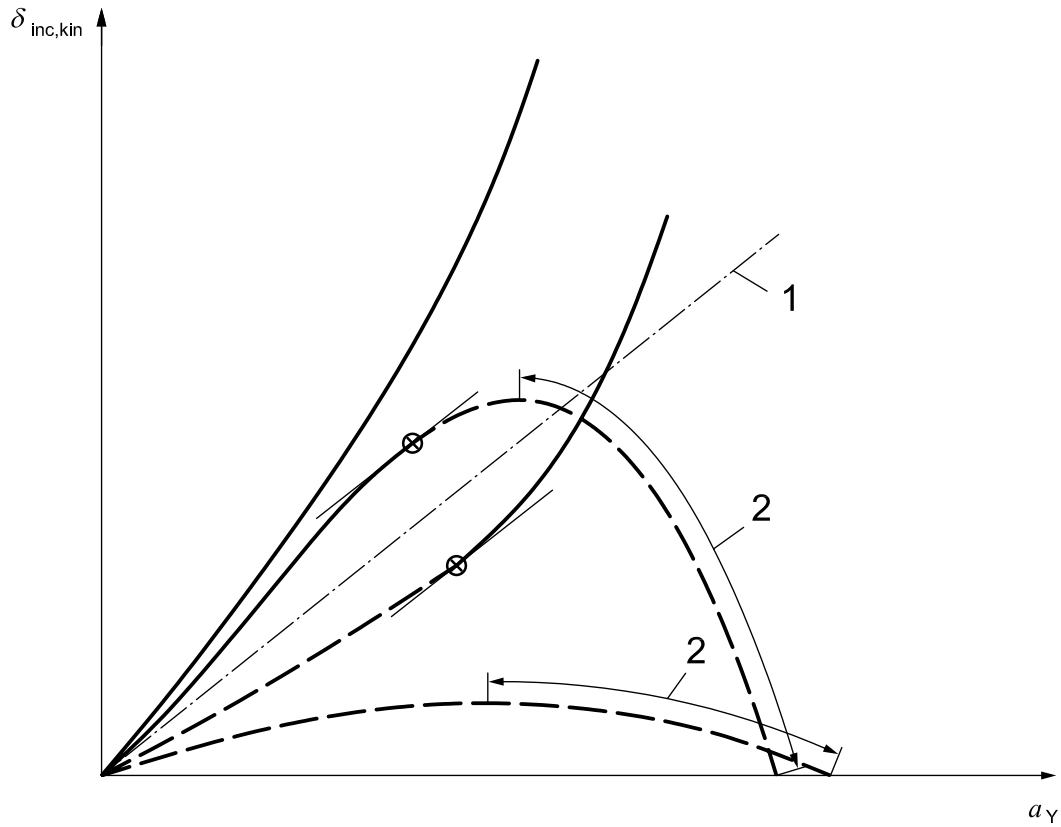
**A.1** Passenger vehicles exhibit varying characteristics depending upon test conditions and the particular steady state taken. Test conditions refer to vehicle conditions, such as wheel loads, front wheel alignment, tyre inflation pressure, and also atmospheric and road conditions, which affect vehicle parameters. For example, temperature can change damping characteristics and slippery road surfaces can change tyre cornering properties. Steady state has been previously defined as the vehicle operating conditions within a given environment and may be specified in part by steer angle, longitudinal velocity and lateral acceleration. Since all these factors change the vehicle behaviour, the vehicle stability needs to be examined separately for each environment and steady state.

For a given set of vehicle parameters and particular test conditions, the vehicle may be examined for each theoretically attainable steady state. The conditions that most affect stability are the steady-state values of longitudinal velocity and lateral acceleration. In practice, it is possible for a vehicle to be stable under one set of operating conditions and unstable under another.

**A.2** Non-oscillatory instability may be illustrated by operation above the critical speed of an oversteer vehicle. Any input to the steering-wheel places the vehicle in a turn of ever decreasing radius unless the driver makes compensations to the steering-wheel to maintain general equilibrium. This condition represents non-oscillatory instability (see Figure A.1). A linear mathematical model of a vehicle is unstable when its characteristic equation has any positive real root.

**A.3** Oscillatory instability may be illustrated by the free control response following a pulse of displacement or force to the steering-wheel. Some vehicles turn first in one direction and then to the other and so on, until the amplitude of the motion increases to the extent that the vehicle "spins out". In this event, the vehicle does not attempt to change its general direction of motion, but also does not achieve a steady-state condition and has an oscillatory motion. A linear mathematical model of a vehicle is oscillatorily unstable when its characteristic equation has any complex root with positive real parts.

**A.4** It is possible for a vehicle to show understeer for small inputs and oversteer for large inputs (or the opposite), since it is a non-linear system and does not have the same characteristics at all steady states (see Figure A.1). Consequently, it is necessary to specify the range of inputs and velocities when making a determination of the vehicle's steer characteristics.



- Key**
- 1 included Ackermann steer angle (neutral steer)
  - 2 non-oscillatory instability
  - $\delta_{inc,kin}$  included kinematic steer angle
  - $a_\gamma$  steady-state lateral acceleration
  - understeer
  - - - - - oversteer
  - ⊗ neutral steer

The steer properties are for constant horizontal speed.

**Figure A.1 — Steer properties**

## Bibliography

- [1] ISO 21308-2:2006, *Road vehicles — Product data exchange between chassis and bodywork manufacturers (BEP) — Part 2: Dimensional bodywork exchange parameters*
- [2] SAE J670, *Vehicle Dynamics Terminology*

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