BS ISO 7870-5:2014

BSI Standards Publication

Control charts

Part 5: Specialized control charts

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National foreword

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Part 5: **Specialized control charts**

Cartes de contrôle —

Partie 5: Cartes de contrôle particulières

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Foreword

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 7870 consists of the following parts, under the general title *Control charts*:

— *Part 1: General guidelines*

— *Part 2: Shewhart control charts*

— *Part 3: Acceptance control charts*

— *Part 4: Cumulative sum control charts*

— *Part 5: Specialized control charts*

— *Part 6: EWMA control charts*

Introduction

The Shewhart control charts as given in ISO 7870-2 aid in detection of unnatural patterns of variations in data from repetitive processes and provide criteria for detecting a lack of statistical control.

However, there may be several special situations for variables data where Shewhart control charts may be inadequate, insufficient or less efficient in detecting the unnatural patterns of variation of the process, particularly where:

a) it takes considerable time to produce an item and as such sample results are available at large intervals;

b) there are several subgroup sources that have approximately the same production rate, process average and process capability;

c) process average is changing systematically;

d) sample size is large and sequence of production is irrelevant;

e) process does not have a constant target value.

In such situations, specialized control charts are to be used.

Similarly, special situations may be encountered in dealing with attributes data. There may be situations when criticality of an incidence in a subgroup (nonconformity) is a matter of concern, but different nonconformities are having different criticality. As such, all types of nonconformities cannot be treated alike. Depending upon criticality, different ratings (weights) are required to be given to each class of nonconformity, and accordingly demerit scores are calculated. The control limits are calculated based on such demerit scores and accordingly control charts are plotted to exercise process control.

There may be situations when inspection by attributes is preferred to that by variables, from practical considerations, for controlling both the location and the variability parameters of a measureable characteristic of a process (for example, inspection by gauging). The information is also available on the number of items less than the lower specification limits (no-go gauge) as well as the number of items above upper specification limit (go gauge) in assembly operations. In such situation, a specialized pair of control charts may be used.

There may also be situations when data do not follow normal distribution. Such situations of non-normal data are quite often encountered in service industry, besides in special processes of manufacturing. In such a situation specialized control chart is to be used.

This part of ISO 7870 has been prepared to provide guidance on the use of specialized control charts to address above typical, unusual situations.

Control charts —

Part 5:
Specialized control charts

1 Scope

This part of ISO 7870 establishes a guide to the use and understanding of specialized control charts in situations where commonly used Shewhart control chart approach to the methods of statistical control of a process may either be not applicable or less efficient in detecting unnatural patterns of variation of the process.

The specialized control charts included in this part of ISO 7870 for variables data are:

a) moving average and moving range charts;

b) *z*-charts;

c) group control charts;

d) high–low control charts;

e) trend control charts;

f) control charts for coefficient of variation;

g) control charts for non-normal data.

For attributes data, specialized control charts included in this part of ISO 7870 are:

a) standardized p-charts;

b) demerit control charts;

c) control charts for inspection by gauging.

This part of ISO 7870 also provides guidance as to when each of the above control charts should be used, their control limits, advantages and limitations. Each control chart is illustrated with an example.

2 Normative references

The following referenced documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-2 and the following apply.

3.1
control chart
chart on which some statistical measure of a series of samples is plotted in a particular order to steer the process with respect to that measure and to control and reduce variation

Note 1 to entry: The particular order is usually based on time or sample number order.

Note 2 to entry: The control chart operates most effectively when the measure is a process variable which is correlated with an ultimate product or service characteristic.

[SOURCE: ISO 3534-2:2006, 2.3.1]

3.2
Shewhart control chart
control chart with Shewhart control limits intended primarily to distinguish between the variation in the plotted measure due to random causes and that due to special causes

[SOURCE: ISO 3534-2:2006, 2.3.2]

3.3
variables control chart
Shewhart control chart in which the measure plotted represents data on a continuous scale

[SOURCE: ISO 3534-2:2006, 2.3.6]

3.4
attributes control chart
Shewhart control chart in which the measure plotted represents countable or categorized data

[SOURCE: ISO 3534-2:2006, 2.3.7]

3.5
Xbar control chart
average control chart
variables control chart for evaluating the process level in terms of subgroup averages

[SOURCE: ISO 3534-2:2006, 2.3.12]

3.6
R chart
range control chart
variables control chart for evaluating variation in terms of subgroup ranges

Note 1 to entry: The value of the subgroup range, given by the symbol *R*, is the difference between the largest and smallest observation of a subgroup.

Note 2 to entry: The average of the range values for all subgroups is denoted by the symbol $\overline{R}$.

[SOURCE: ISO 3534-2:2006, 2.3.18]

3.7
moving average control chart
control chart for evaluating the process level in terms of the arithmetic average of each successive *n* observations

Note 1 to entry: This chart is particularly useful when only one observation per subgroup is available. Examples are process characteristics such as temperature, pressure and time.

Note 2 to entry: The current observation replaces the oldest of the latest n + 1 observations.

Note 3 to entry: It has the disadvantage of an unweighted carry-over effect lasting *n* points.

[SOURCE: ISO 3534-2:2006, 2.3.14]

3.8
moving range control chart
variables control chart for evaluating variation in terms of the range of each successive *n* observations

Note 1 to entry: The current observation replaces the oldest of the latest n +1 observations.

[SOURCE: ISO 3534-2:2006, 2.3.20]

3.9
z-chart
variables control chart for evaluating the process in terms of subgroup standardized normal variates

3.10
group control chart for averages
variables control chart for evaluating the process level in terms of subgroup (with several sources) highest and lowest averages with corresponding source identification

3.11
group control chart for ranges
variables control chart for evaluating the process variation in terms of subgroup (with several sources) highest ranges with corresponding source identification

3.12
high – low control chart
variables control chart for evaluating the process level in terms of subgroup largest and smallest values

3.13
trend control chart
control chart for evaluating the process level with respect to the deviation of the subgroup averages from an expected change in the process level

Note 1 to entry: The trend may be determined empirically or by regression techniques.

Note 2 to entry: A trend is an upward or downward tendency, after exclusion of the random variation and cyclical effects, when observed values are plotted in the time order of the observations.

[SOURCE: ISO 3534-2:2006, 2.3.17]

3.14
control chart for coefficient of variation
variables control chart for evaluating variation in terms of subgroup coefficient of variation

3.15
p chart
proportion or percent categorized units control chart
attributes control chart for number of units of a given classification per total number of units in the sample expressed either as a proportion or percent

Note 1 to entry: In the quality field, the classification usually takes the form of “nonconforming unit”.

Note 2 to entry: The “p” chart is applied particularly when the sample size is variable.

Note 3 to entry: The plotted measure can be expressed as a proportion or as a percentage.

[SOURCE: ISO 3534-2:2006, 2.3.11]

3.16
standardized p-chart
attributes control chart where proportions of given classification are expressed as standardized normal variates

Note 1 to entry: In this chart, the centre line is zero, upper control limit is +3 and lower control limit is −3.

3.17
demerit control chart
quality score chart
multiple characteristic control chart for evaluating the process level where different weights are apportioned to events depending on their perceived significance

[SOURCE: ISO 3534-2:2006, 2.3.23]

3.18
control chart for inspection by gauging
attributes control chart when the inspection is done by gauging and the information is available on the number of units above upper gauge limit and below lower gauge limit

4 Symbols and abbreviated terms

4.1 Symbols

n	subgroup sample size
k	number of subgroups
x	individual measured value
$\bar{x}_i$	average value of *i*-th subgroup
$\bar{\bar{x}}$	average of the subgroup average values
μ	true process mean value
σ	true process standard deviation value
R	range
$\bar{R}$	average range
s	sample standard deviation
$\bar{s}$	average of subgroup sample standard deviations
p	proportion or fraction of units
$\bar{p}$	average value of the proportion or fraction of units
$\mathrm{C_L}$	centre line
$\mathrm{U_{CL}}$	upper control limit
$\mathrm{L_{CL}}$	lower control limit
$\bar{X}$	average value of the variable X plotted on a control chart
x_H	largest observation in a subgroup
x_L	smallest observation in a subgroup

$\bar{x}_H$ average of largest observations for all subgroups

$\bar{x}_L$ average of smallest observations for all subgroups

z variable that has a normal distribution with zero mean and unit standard deviation

v coefficient of variation

$\bar{v}$ average of coefficient of variation values

4.2 Abbreviated terms

BPO business process outsourcing

CV coefficient of variation

L_{GL} lower gauge limit

U_{GL} upper gauge limit

5 Specialized control charts

The following specialized control charts for variables have been included:

a) moving average and moving range control charts;

b) z-charts;

c) group control charts;

d) high–low control charts;

e) trend control charts;

f) control charts for coefficient of variation;

g) control charts for non-normal data.

The following specialized control charts for attributes have been included:

a) standardized p-chart;

b) demerit control chart;

c) control chart for inspection by gauging.

6 Moving average and moving range control charts

In certain cases of industrial production it takes considerable time to produce a new item or the tests are destructive in nature. As a result, it is inconvenient to sample frequently to accumulate sample of size $n > 1$. In the meantime process average or dispersion may have changed and this may incur some appreciable loss. Under such situations subgroups, each consisting of individual observations, are used for process monitoring.

In these situations, use of moving averages and moving ranges instead of Shewhart control charts has been suggested. Moving averages of k subgroups (each of size one) are obtained as follows. Initially, the values of first k subgroups are averaged. Then in the second step the value for the first subgroup is dropped in favour of the value for $(k+1)^{th}$ subgroup and an average obtained. Next, the value for the second subgroup is dropped and the value for $(k+2)^{th}$ subgroup is included and these values are averaged, and so on. In a similar manner moving ranges are obtained. The rate of production helps to decide the number of subgroups to be considered at a time for moving average and moving range. Additionally, the lesser the magnitude of shift in process average and variation one wishes to detect, the higher will be the value of *k*.

6.1 Control limits

6.1.1 Moving range chart

$$C_L = \overline{R}$$

$$U_{CL} = D_4\overline{R}$$

$$L_{CL} = D_3\overline{R}$$

6.1.2 Moving average chart

$$C_L = \overline{\overline{x}}$$

$$U_{CL} = \overline{\overline{x}} + A_2\overline{R}$$

$$L_{CL} = \overline{\overline{x}} - A_2\overline{R}$$

where, $\overline{R}$ is the homogenized range. The values of A_2, D_3 and D_4 are given in Annex A for various sample sizes (n) = k.

6.2 Interpretation

Unlike the case of Shewhart control charts, here successive moving averages and moving ranges are not independent. Hence, in moving average and moving range control charts, runs on either side of the centre line do not have the same interpretation as is given by Shewhart control chart. However, a point beyond control limits here has the same significance as in case of Shewhart control chart. Cyclic pattern and/or increasing or decreasing trend in the moving range chart is indicative of potential for improvement. However, the assignable causes for the moving average chart and those for moving range chart may be different.

6.3 Advantages

In some situations a control chart for moving average and moving range is more efficient. It gives a warning signal earlier than with usual $(\overline{X}, R)$ charts. It is not necessary to wait until an entire new sample is accumulated. This may be important if the product is either expensive or the rate of output is small.

6.4 Limitations

Successive points are not independent. Since the probability of obtaining a run of any kind is much larger with control chart for moving average or moving range as compared to the Shewhart control charts, the traditional interpretation of runs is not valid for these control charts.

6.5 Example

The crown of the watchcase is used to adjust the time. The pin of the crown is fitted through a hole in the watch case. The diameter of the hole has to be maintained at 0,005 ± 0,001mm. Table 1 gives the data in order of production, where reaming operation is done to make the hole for the pin of crown to fit in the watchcase. It is decided to plot control charts for the moving average and moving range by averaging diameter values from 3 consecutive subgroups.

Table 1 — Subgroup results from diameter of the hole for the pin of crown

Sub group number	Hole diameter	Sum of 3 moving observations	Moving average	Moving range	Remarks
1	0,003				
2	0,005				
3	0,001	0,009	0,0030	0,004	
4	0,003	0,009	0,0030	0,004	
5	0,002	0,006	0,0020	0,002	
6	0,005	0,010	0,0033	0,003	
7	0,006	0,013	0,0043	0,004	Shift change
8	0,003	0,014	0,0047	0,003	
9	0,004	0,013	0,0043	0,003	
10	0,005	0,012	0,0040	0,002	
11	0,005	0,014	0,0047	0,001	
12	0,006	0,016	0,0053	0,001	
13	0,001	0,012	0,0040	0,005	
14	0,002	0,009	0,0030	0,005	Tool changed
15	0,007	0,010	0,0033	0,006	
16	0,001	0,010	0,0033	0,006	
17	0,003	0,011	0,0037	0,006	
18	0,004	0,008	0,0027	0,003	
19	0,003	0,010	0,0033	0,001	
20	0,001	0,008	0,0027	0,003	
21	0,006	0,010	0,0033	0,005	
22	0,005	0,012	0,0040	0,005	
23	0,004	0,015	0,0050	0,002	
24	0,002	0,011	0,0037	0,003	
25	0,001	0,007	0,0023	0,003	
Total			**0,0829**	**0,080**	

6.5.1 Control limits for moving range control chart

$$C_L = \overline{R} = \frac{0{,}080}{23} = 0{,}0035$$

$$U_{CL} = D_4 \overline{R} = 2{,}575 \times 0{,}0035 = 0{,}0090$$

$$L_{CL} = D_3\overline{R} = 0 \times 0{,}0035 = 0$$

The above values of D_3 and D_4 are taken from Annex A for n = 3. As all range values are less than U_{CL}, the value of average homogenized range is taken as 0,0035 for computation of control limits for moving average control chart.

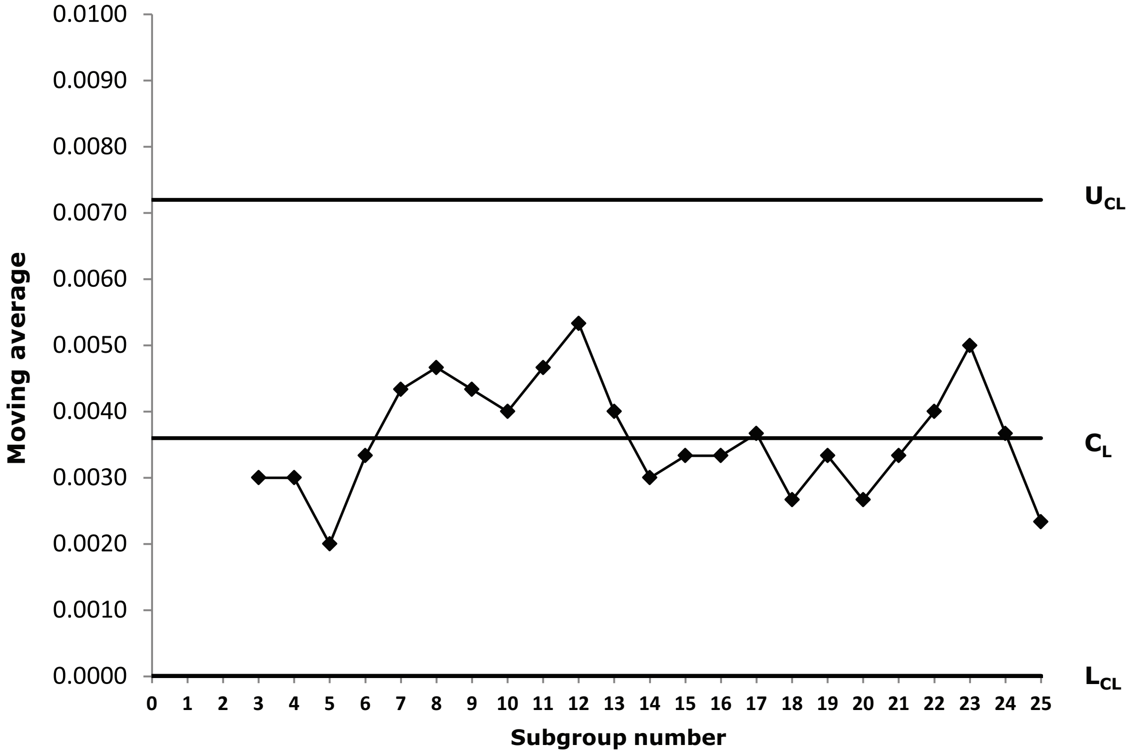
6.5.2 Control limits for moving average control chart

$$C_L = \overline{\overline{x}} = \frac{0{,}0829}{23} = 0{,}0036$$

$$U_{CL} = \overline{\overline{x}} + A_2\overline{R} = 0{,}0036 + 1{,}023 \times 0{,}0035 = 0{,}0072$$

$$L_{CL} = \overline{\overline{x}} - A_2\overline{R} = 0{,}0036 - 1{,}023 \times 0{,}0035 = 0$$

The value of A_2 is taken as 1,023 from Annex A for n = 3. The control chart is plotted in Figure 1.



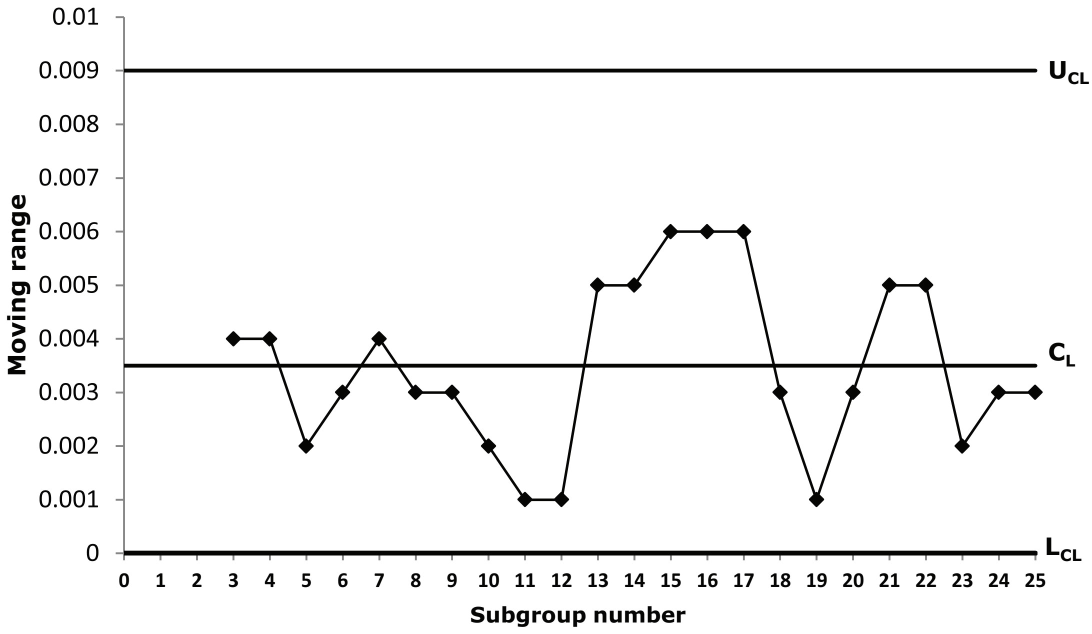


Figure 1 — Moving average and moving range control charts

6.5.3 Interpretation

Process appears to be in state of statistical control.

7 *z*- chart

There are situations where processes are needed to be controlled when there are large varieties of products with different specifications, production runs are small and varying sample/lot sizes. When there are substantial differences in the variance of these products, using the deviation from the process target becomes problematic. There may also be situations when the process does not have a constant target value; instead, the target value keeps on changing with time.

In such cases the commonly used charts like $\left(\bar{X},R\right)$ or $\left(\bar{X},s\right)$ fail to provide a basis for viewing and validly interpreting control chart and appropriate decision making. A suitable chart to see the pattern and take decisions is the z-chart. The idea is to standardize the data to compensate for the different product parameters in terms of averages and variability and transform each point to a standard normal variate by using the transformation $z=(x-\mu)/\sigma$, provided the expected value of the standard deviation is known at that time point. If process is under control then standard normal variates will lie between +3 and −3. These types of charts are referred to as z-charts.

7.1 Control limits

$$\mathrm{C_L}=0$$

$$\mathrm{U_{CL}}=+3$$

$$\mathrm{L_{CL}}=-3$$

7.2 Advantages

The z-chart has the advantage of simplified calculation, presentation and most important it facilitates usual interpretation of Shewhart control charts to control the processes and decision-making.

7.3 Limitations

When there is no historical data, this chart may be difficult to apply because it requires past data for estimating variability.

7.4 Example

Graphite rods used in steel manufacturing are baked in furnaces. At different time points (subgroups), the temperature inside the furnace differs. The target values of temperature and the estimates of the inherent standard deviation of the furnace temperature at different times are also given in Table 2. Standard normal variate (z) values for different subgroups have been calculated and are given in Table 2. It is then seen whether they lie between +3 and −3. If it is above +3 or below −3, then it is out of control situation. The control chart is plotted in Figure 2.

8 Group control chart

In industrial production it may happen that the data presented for the purpose of controlling the quality comes from a number of sources, say from multi-spindle machine with same standard output, or several workers or several machines. In such cases, unless proper steps are taken in choosing the sample, it is difficult for the quality engineer to single out the problem when the control chart shows lack of control. One obvious way is to maintain a separate chart for each possible source of variation, which is rather uneconomical and time-consuming. A group control chart, first devised with a view to controlling the dimensions on multiple-spindle automatics, having wide applicability, provides an answer to the problem.

The group control charts are valid only when there are enough reasons to presume that the averages of each source of data, and also the variability of each source are uniform. Instead of maintaining a pair of average and range charts for each possible source, (such as machine or worker) only one pair of average and range charts is maintained. In the average chart, the highest and lowest average values are plotted along with suitable source identifications (such as serial number of spindles/machines/workers) and the largest range is plotted on the range chart. In the average chart, the highest values are connected by a line, so also, the lowest values in order to avoid confusion. The underlying idea is that if corresponding to a particular sample the highest value is below the Upper Control Limit (U_{CL}), the others are necessarily so. Similarly, if the lowest value is above the Lower Control Limit (L_{CL}), others are necessarily so. The identifying number attached to the highest value that is beyond the U_{CL} or to a lowest value that is below the L_{CL} at once detects the trouble-yielding source. It calls for an attention if a particular identifying number is appearing more frequently either in the high value or in low value. If high and the low values together show a cyclic pattern for the same identifying number, this provides vital clue for attention.

Table 2 — Subgroup results from the temperature in side furnace

Subgroup number	Time (h)	Target value(μ)	Standard deviation from past data (σ)	Observed value (x)	$z = \frac{(x-\mu)}{\sigma}$	Remarks
1	2	205	2,12	200	−2,36	
2	4	210	7,07	200	−1,41	
3	6	210	8,48	210	0,00	
4	8	220	6,36	215	−0,79	
5	10	220	7,07	215	−0,71	
6	12	230	7,07	220	−1,41	

Table 2 *(continued)*

Subgroup number	Time (h)	Target value(μ)	Standard deviation from past data (σ)	Observed value (x)	$z=\frac{(x-\mu)}{\sigma}$	Remarks
7	14	230	6,36	225	−0,79	
8	16	230	17,68	240	0,57	
9	18	240	11,31	245	0,44	
10	20	240	10,61	260	1,89	
11	22	240	7,07	265	3,54	Heating system malfunctioned
12	24	240	3,53	245	1,42	
13	26	240	5,53	255	2,71	
14	28	250	8,08	260	1,24	
15	30	250	12,65	270	1,58	
16	32	250	13,62	285	2,57	
17	34	260	10,5	285	2,38	
18	36	260	10,07	285	2,48	
19	38	270	8,48	285	1,77	
20	40	270	6,36	285	2,36	
21	42	270	7,07	285	2,12	
22	44	270	7,07	285	2,12	
23	46	280	6,36	300	3,14	
24	48	280	7,67	300	2,61	
25	50	320	4,95	330	2,02	
26	52	380	4,95	350	−6,06	
27	54	460	5,15	430	−5,83	
28	56	480	6,7	460	−2,99	
29	58	550	8,1	530	−2,47	
30	60	550	5,1	545	−0,98	
31	62	550	4,8	555	1,04	
32	64	550	5,25	550	0,00	
33	66	550	4,5	545	−1,11	
34	68	550	6,02	540	−1,66	
35	70	550	8,07	530	−2,48	
36	72	460	7,8	450	−1,28	
37	74	340	10,2	350	0,98	
38	76	300	8,76	310	1,14	

NOTE Events such as change in raw material, shift, operator, etc. may be recorded in “Remarks” to facilitate traceability of assignable cause at that stage.

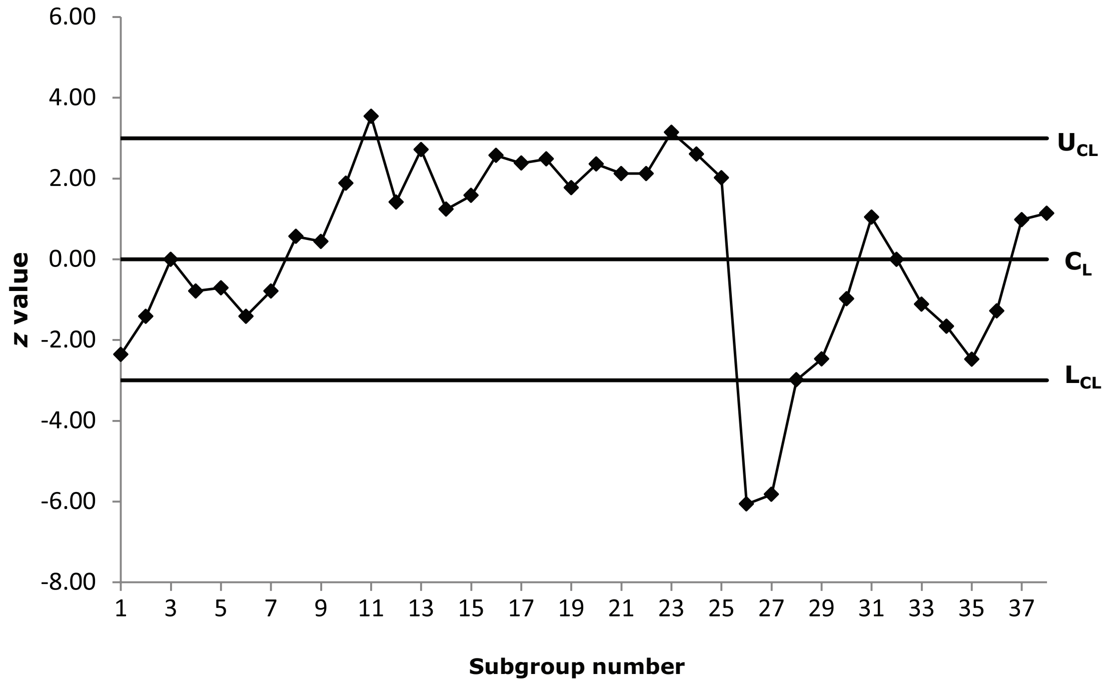


Figure 2 — *z* chart

8.1 Control limits

8.1.1 Group control chart for range

$C_L = \overline{R}$

$U_{CL} = D_4\overline{R}$

$L_{CL} = D_3\overline{R}$

8.1.2 For group control chart for averages

$C_L = \overline{\overline{x}} = \sum_{i=1}^{k} \overline{x}_i$

$U_{CL} = \overline{\overline{x}} + A_2\overline{R}$

$L_{CL} = \overline{\overline{x}} - A_2\overline{R}$

The values of the factors A_2, D_3, and D_4 are given in Annex A for various sample sizes

8.2 Advantages

The advantages are:

a) It involves less work in plotting.

b) A compact presentation of all information from a group of machines on a single chart makes the interpretation easier.

c) It is easier to find out whether a particular source is giving consistently high or low values on average or range chart. If there is no real difference among the sources, the numbers corresponding to the various sources should occur on the charts almost equally in the long run.

8.3 Limitations

The limitations are:

a) The group control charts requires that there should be several subgroup sources that yield approximately equal number of subgroups at approximately the same rate, such as different spindles on one automatic machine, several identical machines, or several operators each doing the same operation. There should be consistency among the averages or dispersions of various subgroups which can be sustained. For example, if there are 10 machines on the same job but two of the machines have different process capability, then the group control chart cannot be applied for all the 10 machines and should not include these two machines. The two machines, which give different process capability, should be treated separately.

b) Experience and skill are needed for interpretation.

c) Conventional interpretation of runs above or below the centre line is not applicable.

8.4 Example

Table 3 gives two measurements of the diameters of two pieces produced on each of six spindles of an automatic screw machine. The values given are in units of 0,001 mm in excess of 12 mm. The highest and lowest average values are indicated in Table 3 as H and L respectively. The highest ranges are also indicated by H in Table 3. It is decided to plot group control chart.

8.4.1 Control limits for group control chart for range

$$\mathrm{C_L} = \bar{R} = \frac{35}{36} = 0{,}97 \text{ microns} = 0{,}00097 \text{ mm}$$

$$\mathrm{U_{CL}} = \mathrm{D_4}\bar{R} = 3{,}267 \times 0{,}97 = 3{,}17 \text{ microns} = 0{,}00317 \text{ mm}$$

$$\mathrm{L_{CL}} = \mathrm{D_3}\bar{R} = 0 \times 0{,}97 = 0 \text{ microns}$$

The values of D_3 and D_4 for sample size (n) = 2 from Annex A are 0 and 3,267 respectively. Since all range values are less than U_{CL}, the ranges are homogeneous. The average range can therefore be used for computing control limits for group control chart for average.

8.4.2 Control limits for group control chart for average

$$\mathrm{C_L} = \bar{\bar{x}} = \frac{195{,}5}{36} = 5{,}43 \text{ microns} = 0{,}00543 \text{ mm}$$

$$\mathrm{U_{CL}} = \bar{\bar{x}} + \mathrm{A_2}\bar{R} = 5{,}43 + 1{,}88 \times 0{,}97 = 7{,}25 \text{ microns} = 0{,}00725 \text{ mm}$$

$$\mathrm{L_{CL}} = \bar{\bar{x}} - \mathrm{A_2}\bar{R} = 5{,}43 - 1{,}88 \times 0{,}97 = 3{,}61 \text{ microns} = 0{,}00361 \text{ mm}$$

The value of A_2 from Annex A for sample size 2 is 1,880.

As the values are in excess of 12 mm, the actual control limits for the group control chart for average are:

C_L = 12,0054 mm

U_{CL} = 12,0072 mm

L_{CL} = 12,0036 mm

Table 3 — Subgroup results from diameter measurements (microns in excess of 12 mm)

Subgroup number	Spindle number	Diameter		Average $\bar{x}$		Range R		Remarks
		Piece 1	Piece 2					
1	1	6	7	6,5	H	1		
	2	4	6	5,0		2	H	
	3	6	4	5,0		2	H	
	4	5	4	4,5	L	1		
	5	6	5	5,5		1		
	6	4	5	4,5	L	1		
2	1	6	6	6,0	H	0		
	2	6	6	6,0	H	0		
	3	5	6	5,5		1		
	4	5	5	5,0	L	0		
	5	5	6	5,5		1		
	6	7	5	6,0	H	2	H	
3	1	5	6	5,5		1	H	
	2	6	6	6,0	H	0		
	3	5	5	5,0	L	0		
	4	6	5	5,5		1	H	
	5	5	5	5,0	L	0		
	6	6	6	6,0	H	0		
4	1	5	6	5,5		1		
	2	6	5	5,5		1		
	3	5	5	5,0		0		
	4	4	4	4,0	L	0		
	5	5	7	6,0	H	2	H	
	6	6	4	5,0		2	H	
5	1	5	6	5,5		1		
	2	5	4	4,5	L	1		
	3	6	5	5,5		1		
	4	7	4	5,5		3	H	
	5	7	6	6,5	H	1		
	6	5	7	6,0		2		

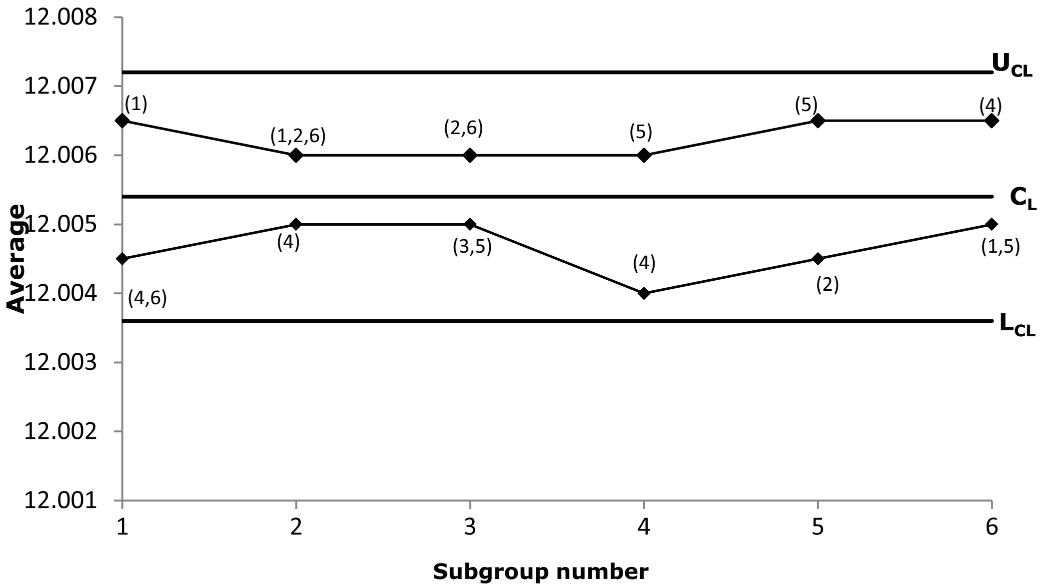
Table 3 *(continued)*

Subgroup number	Spindle number	Diameter		Average $\overline{X}$		Range R		Remarks
		Piece 1	Piece 2					
6	1	5	5	5,0	L	0		
	2	6	5	5,5		1		
	3	4	7	5,5		3	H	
	4	7	6	6,5	H	1		
	5	5	5	5,0	L	0		
	6	6	5	5,5		1		
		Total		**195,5**		**35**		

The group control chart for average and range are plotted in Figure 3. In this figure, in the group control chart for average, the highest and lowest average values along with suitable source identifying indications (spindle number) are plotted. Similarly in the group control chart for range, the highest range along with suitable source identifying indications of spindle number is plotted.

8.4.3 Interpretation

There is no evidence of any out of control situation



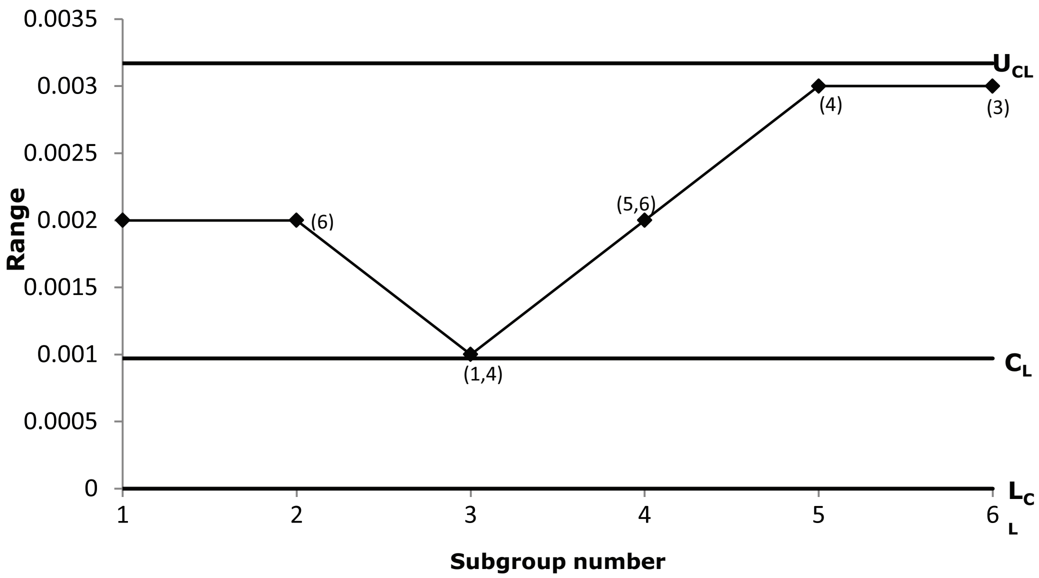


Figure 3 — Group control chart for averages and ranges

9 High-low control chart

There may be situations when sample size is large and the sequence of production is not traceable. For example, in batch production (e.g. zinc coating, heat treatment for annealing) the sequence of production is lost. Also, since several batches get mixed up, the systematic variation becomes an inherent part of further processing. In such situations, it is desirable to use a control chart for largest and smallest values or the high-low control chart, as it is popularly called, in place of the conventional Shewhart control charts.

9.1 Control limits

9.1.1 Average and standard deviation not known

When the values of the process average and dispersion are not known from the past data, they are estimated with the help of the initial data collected and the control limits are computed as follows:

$$C_L = \frac{(\bar{x}_H + \bar{x}_L)}{2} = \bar{M}$$

$$U_{CL} = \bar{M} + H_2\,\bar{R}$$

$$L_{CL} = \bar{M} - H_2\,\bar{R}$$

where $\bar{R} = \bar{x}_H - \bar{x}_L$ where, x_H and x_L denote the largest (high) and smallest (low) values respectively in each subgroup, and $\bar{x}_H$ and $\bar{x}_L$ are the average of largest and smallest values respectively for all subgroups. The values of H_2 are given in Annex A.

9.1.2 Average and standard deviation known

If the values of process mean and standard deviation are known as μ and σ respectively, the control limits are:

$C_L = \mu$

$U_{CL} = \mu + H\sigma$

$L_{CL} = \mu - H\sigma$

The values of H are given in Annex A.

9.2 Interpretation

Although an upper control limit for x_H and a lower control limit for x_L are usually drawn, it is possible to have an upper and a lower control limit for each x_H and x_L separately. The control limits for x_H and x_L are given by $\bar{x}_H \pm \left(H_2 - \frac{1}{2}\right)\bar{R}$ and $\bar{x}_L \pm \left(H_2 - \frac{1}{2}\right)\bar{R}$ respectively. In such a case, shift in the process is indicated if both x_H and x_L are above the respective upper control limits or below the lower control limits. On the other hand, if x_H is above the relevant upper control limit and x_L is below the relevant lower control limit, this is enough evidence to conclude the increase in process variability.

A run of 6 or 7 points of high as well as low points closer to the centre line shows improvement in the process. Control limits for the subsequent process may be consequentially changed. If there is a trend in both x_H and x_L, dip or rise simultaneously, then it shows a shift in the average. Likewise, any increasing or decreasing trends and cyclic patterns should be investigated for special causes. If x_H and x_L points are very close to centre line, then either sampling method is not appropriate or the data are not authentic.

9.3 Advantages

This type of chart is extremely simple since no calculations are needed for plotting the points on control chart. Besides, only one chart needs to be maintained in this case in place of the two conventional charts since information concerning both process level and variation are provided on the single chart.

It has been found that under most conditions, high-low control charts are nearly as good as the $(\bar{X}, R)$ charts for detecting lack of control and ease of interpretation.

9.4 Limitations

This type of chart is not very useful when the process is erratic. The systematic variation within the batch may go unnoticed.

9.5 Example

There are 25 subgroups of 'bolts' machined on a Turret lathe, collected at regular intervals of production and recorded in order of production. Each subgroup is of size 5. Table 4 gives the largest (x_H) and smallest (x_L) values of head diameter for all 25 subgroups in columns 2 and 3 respectively. The control limits for the high-low chart are calculated as follows:

$$\bar{x}_H = \frac{\sum_{i=1}^{k} x_{Hi}}{k} = \frac{99{,}59}{25} = 3{,}984$$

$$\bar{x}_L = \frac{\sum_{i=1}^{k} x_{Li}}{k} = \frac{98,98}{25} = 3,959$$

$$\bar{R} = \bar{x}_H - \bar{x}_L = 3,984 - 3,959 = 0,025$$

Control limits are:

$$C_L = \frac{\bar{x}_H + \bar{x}_L}{2} = \frac{3,984 + 3,959}{2} = 3,972 = \bar{M}$$

$$U_{CL} = \bar{M} + H_2\,\bar{R} = 3,972 + 1,363 \times 0,025 = 4,006$$

$$L_{CL} = \bar{M} - H_2\,\bar{R} = 3,972 - 1,363 \times 0,025 = 3,938$$

The value of H_2 for sample size 5 from Annex A is 1,363.

Data of Table 4 have been plotted on a High-low control chart both for high and low values in Figure 4.

Table 4 — Subgroup results from high and low values of head diameter

Subgroup number	High value (x_H) mm	Low value (x_L) mm	Remarks
1	4,00	3,96	
2	3,99	3,95	
3	3,99	3,97	
4	4,00	3,97	
5	3,99	3,97	
6	4,00	3,97	
7	3,98	3,96	
8	3,99	3,98	
9	4,00	3,98	
10	3,99	3,97	
11	4,00	3,98	
12	4,01	3,98	
13	3,98	3,97	
14	4,00	3,98	Change of material
15	3,98	3,97	
16	3,96	3,95	
17	3,96	3,94	
18	3,96	3,94	
19	3,98	3,93	
20	3,98	3,93	
21	3,97	3,94	Tool Broken, changed
22	3,97	3,95	
23	3,97	3,94	

Table 4 *(continued)*

Subgroup number	High value (x_H) mm	Low value (x_L) mm	Remarks
24	3,97	3,95	
25	3,97	3,95	
TOTAL	**99,59**	**98,98**	

NOTE Events such as change in raw material, shift, operator, etc. may be recorded in "Remarks" to facilitate traceability of assignable causes at that stage.

9.5.1 Interpretation

Points 19 and 20 on Table 4 are indicative of increase in variability linked to play in the tool. The process average is not steady. It is different for the earlier period (first 7 points); middle period (8 to 15 points) and the later period. It is possible to achieve stability by monitoring associated independent process parameters.

10 Trend control chart

In certain industries, the process level changes systematically during the course of production. For example, tool wear in a machine shop, pressure falls as tank empties, and chemical reactions slow down as the concentration of chemicals in the batch becomes weaker with passage of time. In case of tool wear, it may be desirable to adjust or re-sharpen the tooling to avoid production of nonconforming items. On the other hand, it may not be advisable to unduly interrupt production for replacing or re-sharpening the tooling or adjusting tool settings. The goal is to economize the combined costs of nonconforming items and replacing, re-sharpening or adjusting tooling.

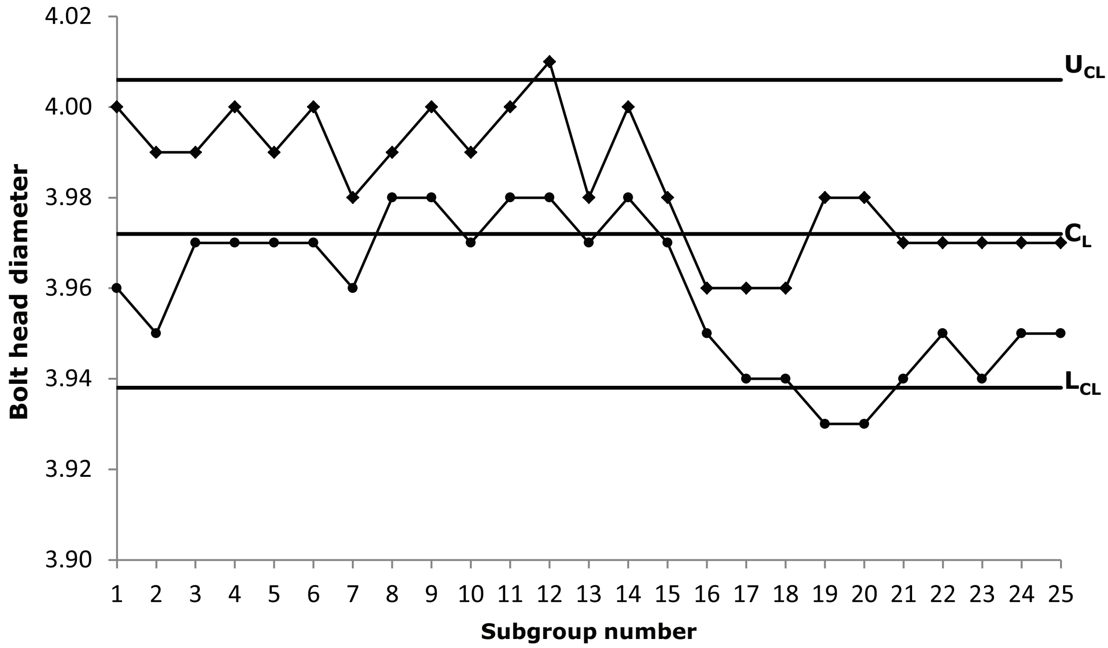


Figure 4 — High low control chart

In such cases Shewhart $(\bar{X} - R)$ control charts are not sufficient for economic control of the process because the variation in the process is not only due to chance causes alone but also due to assignable causes as discussed above. In this situation a trend control chart is useful. For this chart, samples are collected in such a manner that the process level between two successive subgroups is more or less constant; and individual items of a subgroup are consecutive items from the production so that the trend will have minimum effect on the range of the subgroup.

10.1 Control limits

10.1.1 Control limits for range chart

$$\mathrm{C_L} = \bar{R}$$

$$\mathrm{U_{CL}} = \mathrm{D_4}\bar{R}$$

$$\mathrm{L_{CL}} = \mathrm{D_3}\bar{R}$$

where $\bar{R}$ is the average of homogenized ranges.

10.1.2 Control limits for average chart

The average of the k^{th} subgroup ($\bar{x}_k$) may be expressed as $\bar{x}_k = a + bk$, where a and b are constants and are determined as follows:

$$b = \frac{\sum_{i=1}^{k}\left(\bar{x}_i - \bar{\bar{x}}\right)\left(i - \bar{k}\right)}{\sum_{i=1}^{k}\left(i - \bar{k}\right)^2} = \frac{12\sum_{i=1}^{k}\bar{x}_i\left(i - \bar{k}\right)}{k\left(k^2 - 1\right)}$$

and

$$a = \bar{\bar{x}} - b\bar{k}$$

The control limits for average are:

$$\mathrm{C_L} = a + bk$$

$$\mathrm{U_{CL}} = a + bk + \mathrm{A_2}\bar{R}$$

$$\mathrm{L_{CL}} = a + bk - \mathrm{A_2}\bar{R}$$

The values of A_2, D_3 and D_4 are given in Annex A for various sample sizes.

10.2 Advantages

The control chart minimizes the frequency of process adjustments resulting in a corresponding cost reduction. Further, the criteria for out of control signals are the same as for the Shewhart control chart.

10.3 Limitations

The trend control chart is:

a) useful only when process capability is very high;

b) applicable only when no further assembly is involved;

c) less efficient than average chart.

10.4 Example

Table 5 gives data on "Head thickness" of "Starter" for 25 subgroups (each sample of size 5) machined with new tools, collected at regular intervals of production and recorded in order of production. The average and range for each subgroup have been calculated and are also given in Table 5. It is decided to install a trend control chart.

10.4.1 Control limits for range chart

$$C_L = \overline{R} = \frac{0,40}{25} = 0,016$$

$$U_{CL} = D_4\overline{R} = 2,115 \times 0,016 = 0,034$$

$$L_{CL} = D_3\overline{R} = 0 \times 0,016 = 0$$

All the range values are less than $D_4\overline{R}$. Hence, the above are taken as the control limits.

10.4.2 Control limits for average chart

$$\overline{\overline{x}} = \frac{\sum_{i=1}^{25} x_i}{25} = \frac{49,440}{25} = 1,9776$$

$$\overline{k} = \frac{k+1}{2} = \frac{25+1}{2} = 13$$

$$b = \frac{12\sum_{i=1}^{k} \overline{x}_i\left(i-\overline{k}\right)}{k\left(k^2-1\right)} = \frac{12 \times 1,534}{25\left(25^2-1\right)} = 0,00118$$

$$a = \overline{\overline{x}} - b\overline{k} = 1,9776 - 0,00118 \times 13 = 1,9623$$

$$C_L = a + bk = 1,9623 + 0,00118k$$

$$U_{CL} = a + bk + A_2\overline{R} = 1,9623 + 0,00118k + 0,577 \times 0,016 = 1,9715 + 0,00118k$$

$$L_{CL} = a + bk - A_2\overline{R} = 1,9623 + 0,00118k - 0,577 \times 0,016 = 1,9530 + 0,00118k$$

Table 5 — Subgroup results from head thickness of starters

Subgroup number (i)	Average $(\overline{x}_i)$	Range (mm)	$(i-\overline{k})$	$x_i(i-\overline{k})$	U_{CL}	L_{CL}	C_L	Remarks
1	1,962	0,03	−12	−23,544	1,9727	1,9542	1,9635	
2	1,964	0,00	−11	−21,604	1,9739	1,9554	1,9647	
3	1,960	0,03	−10	−19,600	1,9750	1,9565	1,9658	
4	1,966	0,02	−9	−17,694	1,9762	1,9577	1,9670	Lot change
5	1,968	0,03	−8	−15,744	1,9774	1,9589	1,9682	
6	1,968	0,02	−7	−13,776	1,9786	1,9601	1,9694	
7	1,970	0,01	−6	−11,820	1,9798	1,9613	1,9706	
8	1,974	0,02	−5	−9,870	1,9809	1,9624	1,9717	
9	1,972	0,00	−4	−7,888	1,9821	1,9636	1,9729	Shift change
10	1,976	0,01	−3	−5,928	1,9833	1,9648	1,9741	
11	1,976	0,01	−2	−3,952	1,9845	1,9660	1,9753	
12	1,980	0,01	−1	−1,980	1,9857	1,9672	1,9765	
13	1,978	0,02	0	0,000	1,9868	1,9683	1,9776	Tool change
14	1,982	0,03	1	1,982	1,9880	1,9695	1,9788	
15	1,984	0,01	2	3,968	1,9892	1,9707	1,9800	
16	1,980	0,03	3	5,940	1,9904	1,9719	1,9812	
17	1,984	0,02	4	7,936	1,9916	1,9731	1,9824	
18	1,986	0,00	5	9,930	1,9927	1,9742	1,9835	
19	1,986	0,03	6	11,916	1,9939	1,9754	1,9847	
20	1,988	0,00	7	13,916	1,9951	1,9766	1,9859	
21	1,988	0,01	8	15,904	1,9963	1,9778	1,9871	
22	1,990	0,02	9	17,910	1,9975	1,9790	1,9883	
23	1,992	0,01	10	19,920	1,9986	1,9801	1,9894	
24	1,980	0,02	11	21,780	1,9998	1,9813	1,9906	
25	1,986	0,01	12	23,832	2,0010	1,9825	1,9918	
TOTAL	49,440	0,40	0	1,5340				

NOTE Events such as change in raw material, shift, operator, etc. may be recorded in “Remarks” to facilitate traceability of assignable causes at that stage.

The data given in Table 5 have been plotted on trend control chart for average and range chart in Figure 5.

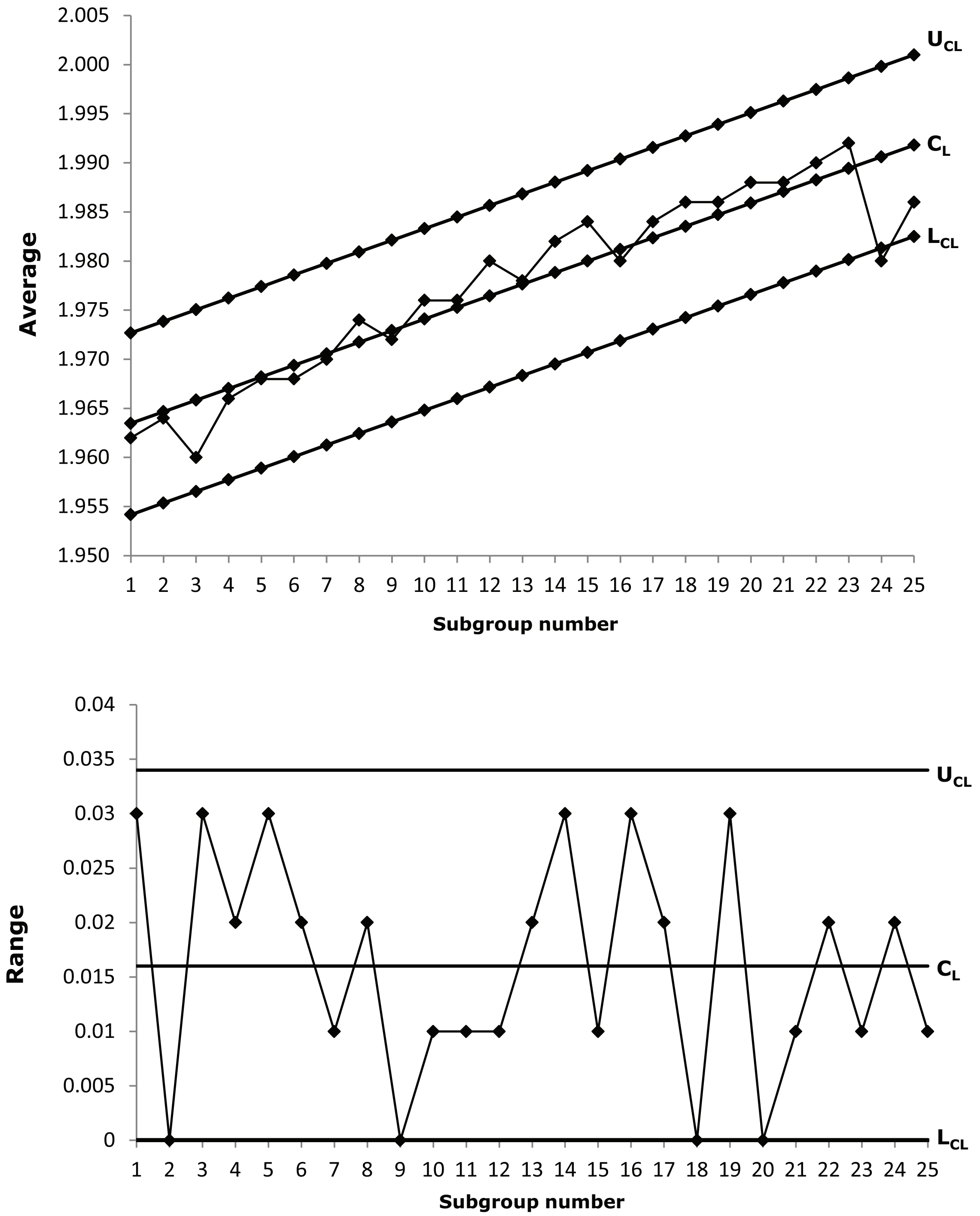


Figure 5 — Trend control chart

10.4.3 Interpretation

The process variation is under control as is evident on range chart. The average (trend chart) shows extreme uniformity with most of the points clustering around the centre line. Process pattern and sampling method should be examined for hidden systematic effect to assess the potential for improvement.

11 Control chart for coefficient of variation

Coefficient of variation (v) may give a useful characterization of variability in cases where samples are drawn from populations with different averages and standard deviations but their ratio is the same, e.g. the strength of concrete. In fact, the average compressive strength of cement mortar cubes increases with the curing time, and the standard deviation increases in the same proportion, but their ratio remains practically constant. Other examples include crushing strength of bricks, the sliver thickness of jute-fibres at different process stages (e.g. carding, drawing, etc.).

The relative variability may then be controlled by computing the sample coefficient of variation and plotting these on a control chart. Higher values of coefficient of variation will result either from high variability or lower average or both. These are taken to be indicative of an unsatisfactory process. Corresponding low values of the coefficient of variation are considered favourable. Therefore, in control charts for coefficient of variation, it is only necessary to red-flag the advent of a manufacturing trouble causing a high value of the sample coefficient of variation. Thus, one is concerned with the upper limit mainly, such as in range or standard deviation charts.

11.1 Control limits

11.1.1 Process average and variation are not known

$$\mathrm{C_L} = \bar{v} = \frac{\sum_{i=1}^{k} v_i}{k}$$

$$\mathrm{U_{CL}} = \mathrm{B_4}\bar{v}$$

$$\mathrm{L_{CL}} = \mathrm{B_3}\bar{v}$$

The values of factors $\mathrm{B_3}$ and $\mathrm{B_4}$ are given in Annex A for various sample sizes.

11.1.2 Process average and variation are known

If the value of the process average (μ) and process standard deviation (σ) are known either from past experience or records, then:

$$\mathrm{C_L} = \mathrm{c_2}\sigma / \mu$$

$$\mathrm{U_{CL}} = \mathrm{B_2}\sigma / \mu$$

$$\mathrm{L_{CL}} = \mathrm{B_1}\sigma / \mu$$

The values of factors $\mathrm{c_2}$, $\mathrm{B_1}$ and $\mathrm{B_2}$ are given in Annex A for various sample sizes.

11.2 Advantage

Only one control chart is to be plotted instead of two.

11.3 Limitation

Care needs to be taken that reduced variability is not accompanied by a high average. Since for a specific product, coefficient of variation needs to be sustained at desired levels by conforming to specified average and variability, a focus on coefficient of variation alone may occasionally be misleading. In case any value of coefficient of variation goes beyond control limits on a coefficient of variation chart, the contribution from the average or standard deviation should be assessed before reaching any conclusion. This will also pave the way to corrective actions.

11.4 Example

In the jute industry, uniformity of linear density of sliver is an important criterion which affects the subsequent operations of spinning and weaving. It is, therefore, desirable to control this property properly. With a view to installing a control chart for coefficient of variation, weights of five 10 m lengths of sliver are collected daily at the finisher card stage. The records of 25 days at a particular reference moisture regain are given in Table 6 along with the average and coefficient of variation (the observations in a particular sample correspond to one machine only at a particular time).

Table 6 — Subgroup results from weights of 10 m lengths of silvers

Subgroup number	Weights/10-m lengths(g) percent moisture					Average	Standard deviation	CV	Remarks
1	751	681	708	748	704	718,4	30,22	4,21	
2	808	794	839	887	811	827,8	36,90	4,46	
3	760	731	774	771	765	760,2	17,20	2,26	
4	794	794	777	774	811	790,0	14,98	1,90	
5	714	712	695	697	703	704,2	8,58	1,22	
6	735	735	760	705	764	739,8	23,72	3,21	
7	730	735	780	715	705	733,0	28,85	3,94	
8	735	820	700	765	790	762,0	46,72	6,13	
9	740	745	705	765	715	734,0	24,08	3,28	
10	695	725	745	730	770	733,0	27,52	3,75	
11	645	640	685	660	657	657,4	17,50	2,66	
12	655	690	605	618	655	644,6	33,74	5,23	
13	662	682	655	705	670	674,8	19,64	2,91	
14	620	610	630	610	695	633,0	35,64	5,63	
15	760	710	740	690	745	729,0	28,37	3,89	
16	632	703	688	655	740	683,6	42,00	6,14	
17	720	700	681	777	792	734,0	48,41	6,60	
18	600	612	697	775	780	692,8	85,91	12,40	
19	717	690	753	686	681	705,4	30,04	4,26	Product changed (accordingly material and humidity adjusted
20	795	822	707	725	774	764,6	47,94	6,27	
21	605	715	764	655	660	679,8	61,10	8,99	
22	810	740	825	733	796	780,8	41,79	5,35	
23	650	600	693	651	666	652,0	33,86	5,19	
24	700	767	720	783	710	736,0	36,74	4,99	
25	665	640	700	653	704	672,4	28,47	4,23	
Total								**119,10**	

NOTE Events like change in raw material, shift, operator, etc. may be recorded in "Remarks" to facilitate traceability of assignable causes of that stage.

11.4.1 Control limits

$$C_L = \overline{v} = \frac{\sum_{i=1}^{k} v_i}{k} = \frac{119,10}{25} = 4,76$$

$$U_{CL} = B_4\overline{v} = 2,089 \times 4,76 = 9,94$$

$$L_{CL} = B_3\overline{v} = 0 \times 4,76 = 0$$

The values of B_3 and B_4 for sample size 5, as given in Annex A, are 0 and 2,089 respectively. The data along with control limits have been plotted on a control chart for coefficient of variation in Figure 6.

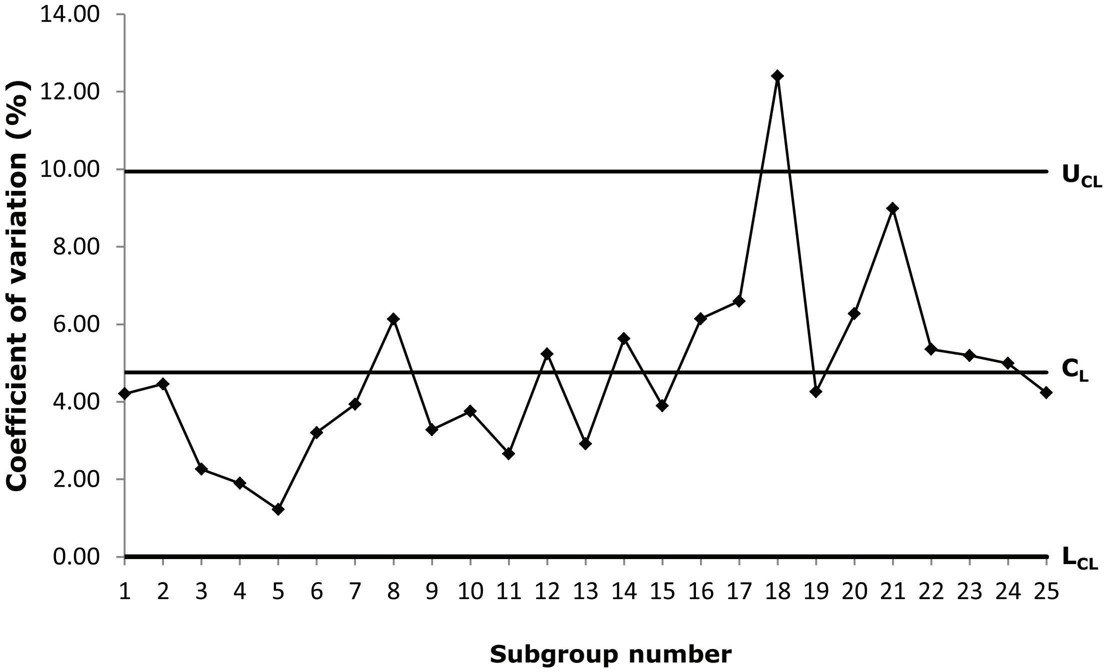


Figure 6 — Control chart for coefficient of variation

11.4.2 Interpretation

a) There is run of points 1 to 7 indicating low average of coefficient of variation which may be due to low variation or high average or both. This should be investigated.

b) Subsequently, from points 8 to 17, there is no evidence of any lack of control. Point 18 was out of control due to either increase in variability or decrease in average, for which action is to be taken.

12 Control chart for non-normal data

The fundamental assumption of Shewhart control charts is that the underlying distribution of the quality characteristic is normal. The sensitivity of Shewhart control charts decreases with increase in departure from the normal distribution. There are several situations where the processes exhibit non-normal behaviour, such as the service industry (hospitals, business processing houses, etc.), special processes (e.g. welding), chemical processes, etc.

Therefore, before applying Shewhart control charts, it may be necessary to first check for normality, and if data do not follow a normal distribution, the following methods may be used:

a) Use large subgroups. By the central limit theorem, as the subgroup size increases the distribution of subgroup averages tends to more closely follow a normal distribution.

b) Transform the data in such a way that the transformed data follow normal distribution. For this purpose, the Johnson family of transformations and/or a Box-Cox transformation may be used.

c) If it is not feasible to have large subgroups and/or it is difficult to transform the data, the percentile points may be computed using a suitable probability paper or computer software. They may also be computed by using standardized Pearson curves necessitating estimation of skewness and kurtosis in addition to the mean and standard deviation for the data. For further details reference may be made to ISO/TR 22514-4.

12.1 Control limits

12.1.1 Control limits for range chart

$$\mathrm{C_L} = \overline{R}$$

$$\mathrm{U_{CL}} = \mathrm{D_4}\overline{R}$$

$$\mathrm{L_{CL}} = \mathrm{D_3}\overline{R}$$

12.1.2 Control limits for standard deviation chart

$$\mathrm{C_L} = \overline{s} = \left[\frac{\sum_{i=1}^{k} s_i^2}{k} \right]^{1/2}$$

$$\mathrm{U_{CL}} = \mathrm{B_4}\overline{s}$$

$$\mathrm{L_{CL}} = \mathrm{B_3}\overline{s}$$

The value of factors D_3, D_4, B_3 and B_4 are given in Annex A.

12.1.3 Control limits for average chart

12.1.3.1 When the average values follow normal distribution and subgroup size is less than 10

$$\mathrm{C_L} = \overline{\overline{x}}$$

$$\mathrm{U_{CL}} = \overline{\overline{x}} + \mathrm{A_2}\overline{R}$$

$$\mathrm{L_{CL}} = \overline{\overline{x}} - \mathrm{A_2}\overline{R}$$

12.1.3.2 When the average values follow normal distribution and subgroup size is 10 or more

$$\mathrm{C_L} = \overline{\overline{x}}$$

$$U_{CL} = \overline{\overline{x}} + A_3\overline{s}$$

$$L_{CL} = \overline{\overline{x}} - A_3\overline{s}$$

12.1.3.3 When average values do not follow normal distribution

$$C_L = \overline{\overline{x}}$$

$$U_{CL} = \overline{\overline{x}} + 99{,}865 percentile \times s_{average}$$

$$L_{CL} = \overline{\overline{x}} + 0{,}135 percentile \times s_{average}$$

Where $s_{average}$ is the standard deviation of subgroup averages. The values of the factors A_2, A_3, B_3, B_4 are given in Annex A.

12.2 Example

The data regarding time taken in minutes for sending the blood samples to reach the phlebotomy laboratory are given in Table 7. As the subgroup size is 10, the standard deviation chart is used for controlling variation. Applying test for normality using normal probability paper, it is observed that the sample data are not normal. However, the averages of the 28 subgroups are following normal distribution.

Table 7 — Subgroup results from time taken (minutes) for samples to reach phlebotomy laboratory

Subgroup number	Time taken (minutes) for sample units									$\overline{x}_i$	s_i
1	0,12	0,06	0,06	0,19	0,14	0,04	0,14	0,04	0,09	0,10	0,098	0,050
2	0,06	0,12	0,16	0,17	0,20	0,19	0,16	0,17	0,06	0,14	0,143	0,049
3	0,15	0,14	0,07	0,13	0,17	0,08	0,16	0,13	0,16	0,15	0,134	0,034
4	0,14	0,13	0,17	0,19	0,13	0,10	0,21	0,08	0,17	0,18	0,150	0,041
5	0,17	0,10	0,10	0,18	0,18	0,17	0,15	0,20	0,18	0,16	0,159	0,034
6	0,08	0,16	0,16	0,17	0,17	0,11	0,11	0,15	0,14	0,17	0,142	0,032
7	0,14	0,19	0,15	0,16	0,14	0,17	0,16	0,16	0,16	0,07	0,150	0,032
8	0,18	0,09	0,06	0,13	0,13	0,08	0,14	0,07	0,10	0,11	0,109	0,037
9	0,08	0,08	0,13	0,10	0,07	0,07	0,06	0,07	0,08	0,08	0,082	0,020
10	0,08	0,06	0,05	0,08	0,09	0,08	0,08	0,07	0,08	0,09	0,076	0,013
11	0,08	0,09	0,08	0,09	0,12	0,08	0,08	0,06	0,08	0,09	0,085	0,015
12	0,10	0,08	0,10	0,07	0,14	0,12	0,12	0,08	0,10	0,11	0,102	0,021
13	0,09	0,10	0,10	0,09	0,11	0,10	0,08	0,08	0,09	0,10	0,094	0,010
14	0,09	0,08	0,18	0,11	0,08	0,10	0,10	0,13	0,12	0,12	0,111	0,030
15	0,08	0,08	0,08	0,15	0,16	0,19	0,19	0,16	0,13	0,09	0,131	0,045
16	0,11	0,06	0,07	0,14	0,11	0,18	0,15	0,09	0,13	0,14	0,118	0,037
17	0,13	0,17	0,12	0,14	0,13	0,16	0,08	0,16	0,12	0,11	0,132	0,027
18	0,09	0,13	0,07	0,12	0,11	0,07	0,12	0,16	0,07	0,16	0,110	0,035
19	0,12	0,13	0,15	0,22	0,17	0,17	0,14	0,17	0,12	0,16	0,155	0,030

Table 7 *(continued)*

Subgroup number	Time taken (minutes) for sample units										$\overline{x}_i$	s_i
20	0,07	0,14	0,19	0,11	0,18	0,09	0,08	0,08	0,09	0,11	0,114	0,042
21	0,08	0,14	0,13	0,05	0,06	0,07	0,05	0,14	0,12	0,11	0,095	0,037
22	0,10	0,07	0,09	0,14	0,13	0,11	0,10	0,05	0,08	0,06	0,093	0,029
23	0,05	0,09	0,07	0,13	0,07	0,05	0,08	0,08	0,10	0,07	0,079	0,024
24	0,05	0,08	0,05	0,06	0,03	0,07	0,09	0,10	0,05	0,07	0,065	0,021
25	0,06	0,09	0,06	0,08	0,07	0,08	0,05	0,04	0,08	0,08	0,069	0,016
26	0,07	0,08	0,08	0,05	0,08	0,08	0,05	0,07	0,05	0,10	0,071	0,017
27	0,07	0,04	0,06	0,05	0,06	0,08	0,05	0,07	0,07	0,05	0,060	0,012
28	0,04	0,07	0,09	0,05	0,07	0,08	0,07	0,05	0,06	0,14	0,072	0,028
										sum	2,999	0,818

12.2.1 Control limits

12.2.1.1 Control limits for standard deviation chart

$$C_L = \overline{s} = \left[\frac{\sum_{i=1}^{k} s_i^2}{k} \right]^{1/2} = \left(\frac{0,0273}{28} \right)^{1/2} = 0,0312$$

$$U_{CL} = 1,716 \times 0,0312 = 0,0535$$

$$L_{CL} = 0,284 \times 0,0312 = 0,0089$$

The values of B_3 = 0,0284 and B_4 = 1,716 are taken from Annex A. Since all standard deviation values are within control limits, $\overline{s}$ is the homogenized average standard deviation value.

12.2.1.2 Control limits for average chart

$$C_L = \overline{\overline{x}} = \frac{2,999}{28} = 0,1071$$

$$U_{CL} = \overline{\overline{x}} + A_3\overline{s} = 0,107 + 0,975 \times 0,0312 = 0,1375$$

$$L_{CL} = \overline{\overline{x}} - A_3\overline{s} = 0,107 - 0,975 \times 0,0312 = 0,0767$$

The control charts are plotted in Figure 7.

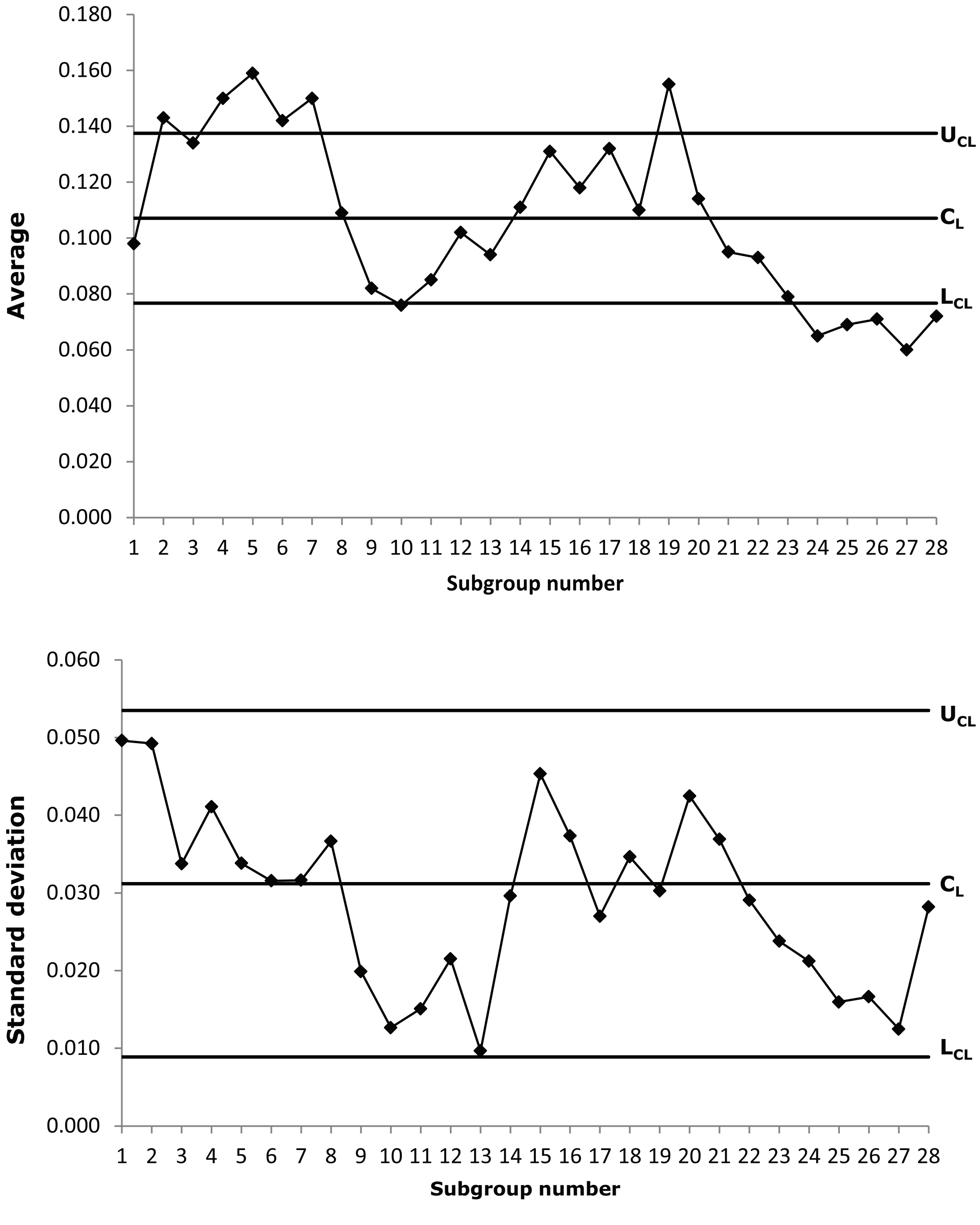


Figure 7 — Control chart for subgroup averages and standard deviations

12.2.2 Interpretation

a) The average time taken by blood samples to the laboratory is not under control on the average chart. In fact, 12 out of 28 points lie beyond the control limits.

b) The standard deviations values are well under control, i.e. within a day the dispersion is within limits, but with a high value. However, there are signs of trends which mean there is systematic variation over a period of 7 to 8 days. It implies that there exists potential to reduce variation. It should be explored, verified and suitable changes incorporated.

12.2.3 Systematic factors and corrective actions

During discussions about out of control points on the average chart, many systematic factors were noted including:

a) age of patients;

b) size of needles used in the syringe;

c) condition of patient;

d) time of the day;

e) availability of General Duty Assistant;

f) skill of phlebotomist;

g) lab person documenting the data;

h) amount of blood to be collected;

i) placement of the patient with respect to laboratory;

j) disease of the patient.

The following corrective actions were taken:

a) standard operating procedure was defined and frozen;

b) phlebotomists were trained to follow standard operating procedure; and

c) at any point in time, two Phlebotomists or General Duty Assistant would be made available.

After implementing the above corrective actions, fresh data are collected as given in Table 8.

It is seen that data are not normal, but subgroup averages are following normal distribution. New control limits are calculated as under

12.2.4 Control limits

12.2.4.1 Control limit for standard deviation chart

$$C_L = \bar{s} = \left[\frac{\sum_{i=1}^{k} s_i^2}{k} \right]^{1/2} = \left(\frac{0,0106}{20} \right)^{1/2} = 0,0231$$

$$U_{CL} = 1,716 \times 0,0231 = 0,0396$$

$$L_{CL} = 0,284 \times 0,0231 = 0,0066$$

Table 8 — Subgroup results of time taken (minutes) for samples to reach phlebotomy laboratory after corrective actions

Subgroup number	Time taken (minutes) for sample units										Average	Standard deviation
1	0,06	0,04	0,08	0,08	0,08	0,08	0,04	0,10	0,11	0,13	0,080	0,0287
2	0,07	0,07	0,08	0,08	0,08	0,10	0,07	0,11	0,09	0,06	0,081	0,0152
3	0,04	0,06	0,13	0,05	0,11	0,11	0,10	0,10	0,10	0,09	0,089	0,0292
4	0,12	0,09	0,08	0,07	0,07	0,06	0,06	0,05	0,04	0,09	0,073	0,0231
5	0,07	0,08	0,06	0,04	0,04	0,12	0,11	0,06	0,04	0,03	0,065	0,0306
6	0,07	0,06	0,06	0,09	0,09	0,10	0,10	0,11	0,06	0,04	0,078	0,0230
7	0,05	0,05	0,11	0,05	0,06	0,06	0,05	0,06	0,06	0,11	0,066	0,0237
8	0,13	0,06	0,07	0,05	0,08	0,07	0,05	0,07	0,10	0,08	0,076	0,0241
9	0,06	0,04	0,06	0,09	0,07	0,08	0,07	0,08	0,10	0,07	0,072	0,0169
10	0,06	0,07	0,06	0,08	0,06	0,06	0,06	0,10	0,04	0,05	0,064	0,0165
11	0,08	0,08	0,08	0,11	0,11	0,06	0,06	0,07	0,06	0,09	0,080	0,0189
12	0,10	0,07	0,06	0,07	0,04	0,06	0,10	0,07	0,07	0,06	0,070	0,0183
13	0,07	0,07	0,02	0,02	0,07	0,07	0,09	0,05	0,06	0,05	0,057	0,0226
14	0,07	0,06	0,04	0,04	0,06	0,05	0,06	0,09	0,07	0,07	0,061	0,0152
15	0,07	0,06	0,06	0,04	0,07	0,03	0,08	0,05	0,06	0,06	0,058	0,0148
16	0,09	0,10	0,10	0,09	0,11	0,10	0,08	0,08	0,09	0,10	0,094	0,0097
17	0,12	0,13	0,05	0,12	0,10	0,09	0,07	0,07	0,05	0,06	0,086	0,0303
18	0,08	0,09	0,06	0,06	0,04	0,08	0,14	0,07	0,10	0,11	0,083	0,0287
19	0,07	0,04	0,06	0,05	0,06	0,08	0,05	0,07	0,07	0,05	0,060	0,0125
20	0,08	0,09	0,06	0,07	0,08	0,08	0,11	0,05	0,14	0,17	0,093	0,0371
Total											1,486	0,4390

12.2.4.2 Control limit for average chart

$$C_L = \overline{\overline{x}} = \frac{1,486}{20} = 0,0743$$

$$U_{CL} = \overline{\overline{x}} + A_3\overline{s} = 0,0743 + 0,975 \times 0,0231 = 0,0968$$

$$L_{CL} = \overline{\overline{x}} - A_3\overline{s} = 0,0743 - 0,975 \times 0,0231 = 0,0518$$

After taking corrective actions on the causes identified, the results subsequently showed improvement, both in the average and variation estimates. The control charts are plotted in Figure 8.

13 Standardized *p*- chart

This chart is recommended when sample size or lot size vary erratically. It improves visual appeal as only a single set of control limits is used and avoids confusion in cases where larger p for a subgroup is within its upper control limit and smaller p for another subgroup is beyond its upper control limit.

This chart consists of plotting sequence of production on *x*-axis and sample statistic $z = (p - \overline{p})\sqrt{n} \big/ \sqrt{\overline{p}(1-\overline{p})}$ on y-axis.

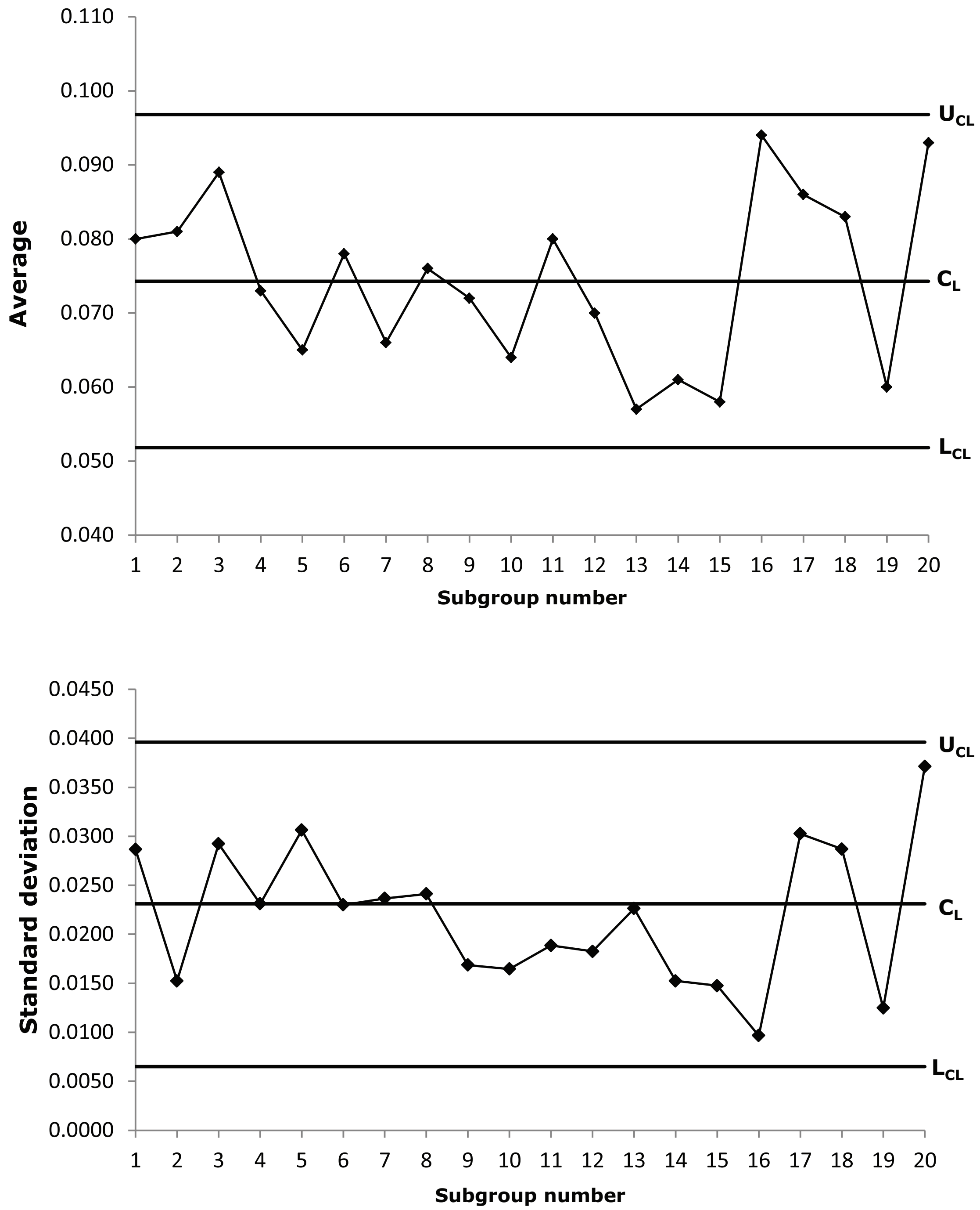


Figure 8 — Control chart for subgroup averages and standard deviations

13.1 Control limits

C_L = 0

U_{CL} = + 3

L_{CL} = - 3

13.2 Advantages and limitations

In standardized *p*-chart the control limit and centre line are fixed and interpretation is facilitated. However, it involves more calculations.

13.3 Example

The number of picture tubes manufactured and found nonconforming for a period of 25 consecutive days are given in Table 9. The standardized values of p for each day are obtained as follows:

$$\bar{p} = 1467 / 28474 = 0{,}0515$$

$$\left(\bar{p}(1-\bar{p})\right)^{1/2} = 0{,}2210$$

The values for standardized p, $z = \frac{(p-\bar{p})\sqrt{n}}{\sqrt{\bar{p}(1-\bar{p})}}$ for each day is calculated and given in Table 9. U_{CL} and L_{CL} are +3 and −3 respectively.

The control chart is plotted in Figure 9.

Table 9 — Subgroup results for data on non-conforming picture tubes

Subgroup number	Number processed	Non-conforming	*p*	*z*	Remarks
1	2417	143	0,059	1,705	
2	2334	105	0,045	−1,424	
3	954	63	0,066	2,032	
4	1104	55	0,050	−0,253	
5	1246	40	0,032	−3,098	New Inspector
6	792	54	0,068	2,124	
7	1298	73	0,056	0,773	
8	321	12	0,037	−1,144	
9	1204	51	0,042	−1,435	
10	576	15	0,026	−2,765	
11	1151	59	0,051	−0,037	
12	1256	53	0,042	−1,492	
13	1099	44	0,040	−1,720	
14	1811	98	0,054	0,503	
15	1073	65	0,061	1,346	
16	196	18	0,092	2,555	
17	1113	53	0,048	−0,586	

Table 9 *(continued)*

Subgroup number	Number processed	Non-conforming	*p*	*z*	Remarks
18	1066	59	0,055	0,568	
19	828	38	0,046	−0,730	
20	1083	63	0,058	0,993	
21	991	39	0,039	−1,730	
22	1102	57	0,052	0,034	
23	1226	60	0,049	−0,406	
24	1140	85	0,075	3,523	New Inspector
25	1093	65	0,059	1,192	
Total	28474	1467			

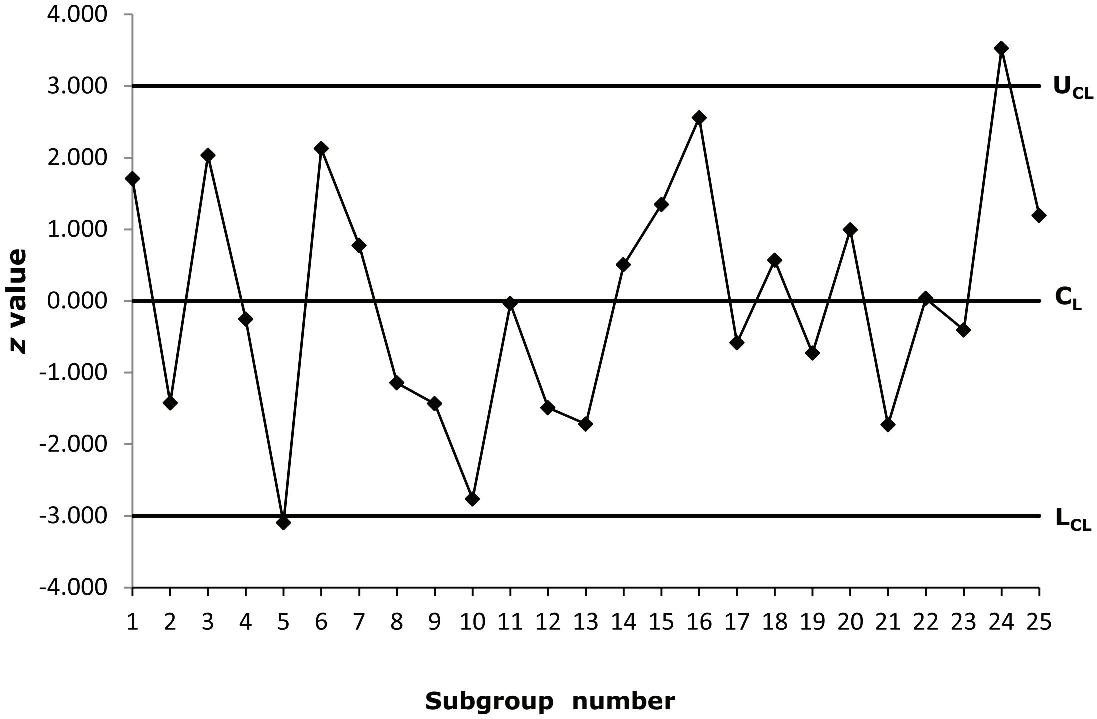


Figure 9 — Standardized *p* chart

13.3.1 Interpretation

It is observed that *p* for subgroup 16 is higher than *p* for subgroup 24, yet the former is within U_{CL} while the latter is beyond its U_{CL}. Again *p* for subgroup 10 is lower than *p* for subgroup 5, yet the former is within L_{CL} while the latter is beyond its L_{CL}. This is so because larger sample size provides stronger evidence/confidence for discrimination. This is seen on a *p*-chart, but not on a standardized *p*-chart.

For 2 out of 25 subgroups, the process is out of control. Further, there is a favourable run of 6 points, from subgroup 8 to 13. This provides evidence for potential recurring benefits. Therefore it is not rational to accept the present observed average proportion of nonconforming $\overline{p} = 0{,}0515$ as process standard. Instead the average of these 6 subgroups from 8 to 13 is 0,042 and may be accepted as the first iterative bench mark, showing an improvement over the present average of 0,0515.

14 Demerit control chart

In case of control chart for number of nonconformities, all the nonconformities on an item are counted and plotted on the control chart. This chart has a limitation as it gives equal weightage to each class of nonconformity. But the different nonconformities are unequal in their influence on costs or losses it can impart. Some may be corrected by simple inexpensive rework operation, others may require costly rework, still others may involve the scrapping of the items inspected. A practical solution to this problem is to classify the various nonconformities into some broad categories, like critical, major and minor nonconformities, and run separate control charts for each class of nonconformities. However, if the number of classes of nonconformities increase, it may be difficult to have so many control charts simultaneously. A simpler solution is to give the different demerit rating (weights) to each class of nonconformity and calculate the demerit score on number of items inspected. This demerit score may be plotted on the demerit control chart.

14.1 Criteria for selection of demerit weights

In case of demerit score, the underlying distribution is Poisson distribution. The nonconformities are given weights since all types of nonconformities cannot be treated alike. The criteria for selection of demerit weights for various nonconformities depend on the type of products under consideration. Broadly, there are two types of products. In the first instance, it is possible to rectify nonconformity by either replacing a component, by carrying out suitable rework, or by scrapping it. The products of engineering industry involving assembly of large number of components are an example of this type. In the second situation, it may not be possible to carry out the rectification of the production and the product is graded and sold as such. The textile products are examples of this category. Most of the nonconformities that arise in weaving, finishing, printing, etc, of fabric are graded and sold at varying prices.

In cases of the first type of product, the choice of demerit weights (w_i) depends on the criticality of the nonconformity, which is determined by taking into account various aspects of the product such as safety, performance, regulatory/statutory requirements and customer feedback. In general, nonconformity may be classified into three classes - minor, major and critical nonconformities. If need arises, this categorization may be further extended by sub-dividing these classes into two or more. A demerit weight is then assigned to each class taking into account the various aspects mentioned above. Another approach for selection of demerit weights is by estimating the financial loss in the form of scrap, rework or potential loss of resources due to diversion of production efforts. The financial loss due to passing of a nonconforming item having particular nonconformity to the assembly or final product can also be taken into account. The demerit weight may then be fixed in proportion to the financial loss due to presence of that nonconformity. Generally, few classes may then be worked out on the basis of the rationalization of these losses and the demerit weights assigned to each of these classes rather than treating each nonconformity separately. An approach where a combination of criticality of nonconformity and financial loss is taken into account for assigning the demerit weights may also be followed.

In cases of the second type of product, the approach followed involves the final grading of the product initially. This graded product is then inspected for various types of nonconformities present. The demerit weights to each type of nonconformity are assigned in such a way that a distinct total demerit score can be assigned to each grade of the product. The various combinations of nonconformities giving rise to a distinct grade of the product would result in a total demerit score which should not overlap for the different grades assigned.

14.2 Example of assigning demerit weights for fabrics

a) Initially a meeting of heads of various departments, namely, weaving, finishing, printing, folding and sales is organized. Classification of nonconformities is determined by placing them in two or more classes. The extent of a particular type of nonconformity on the fabric may also be taken into account when this classification is carried out (e.g. the broken or missing end can be classified differently depending on number and length of missing ends). During the meeting the final grade of the fabric is determined by considering different combinations of the nonconformities present on a cut or piece.

b) The next step is to select a large number of pieces from a production line and these are independently graded into final grades (e.g. fresh, second and sub-standard) by four or five experienced inspectors. The pieces which are found cutable are sub-divided into two groups and graded once again.

c) The pieces which give rise to different grading by various inspectors are re-examined and after discussions, properly re-graded. The pieces on which such a consensus of final grading cannot be achieved are not taken into account during further analysis of the data.

d) The various combinations of nonconformities under various grades are then listed from the pieces on which a consensus of the uniform grading is achieved. This allows a cross check of the definitions of grades and also enables completion of definitions by including additional combinations of nonconformities which might have escaped notice earlier. The next step is to assign different combinations of demerit weights for each category of nonconformity and the total demerit score for each piece is calculated. By trial and error, the demerit scores shall be so adjusted that there is no overlapping of total demerit scores on the pieces which are graded in different grades.

14.3 Control limits

c_{ij} is the number of nonconformities of *i*-th class in *j*-th subgroup, $i = 1, 2 \dots m,$ and $j = 1, 2 \dots k;$

$c_i = \sum_{j=1}^{m} c_{ij}$ is the total number of nonconformities of *i*-th class in all the subgroups;

w_i is the demerit weight for *i*-th class of nonconformity;

n_j is the number of items in *j*-th subgroup;

$N = \sum_{j=1}^{k} n_j$ is the total number of items in all the subgroups;

$D_j = \sum_{i=1}^{m} w_i c_{ij}$ is the total demerit score for *j*-th subgroup;

$d_j = \frac{D_j}{n_j} = \frac{\sum_{i=1}^{m} w_i c_{ij}}{n_j}$ is the demerit score per item for *j*-th subgroup;

$\bar{d} = \frac{\sum_{j=1}^{k} D_j}{N}$ is the demerit score per item for all the subgroups;

Alternatively

$$\bar{d} = \frac{\sum_{i=1}^{m} w_i c_i}{N}$$

The control limits are

$$C_L = \bar{d}$$

Upper Control Limit (U_{CL}) for the *j*-th subgroup

$$U_{CL} = \bar{d} + 3\left(\frac{\sum_{i=1}^{m} w_i^2 c_i}{N n_j}\right)^{1/2}$$

Lower Control Limit (L_{CL}) for the j-th subgroup

$$L_{CL} = \bar{d} - 3\left(\frac{\sum_{i=1}^{m} w_i^2 c_i}{N n_j}\right)^{1/2}$$

If calculated value of L_{CL} is negative, it is taken as zero.

If the demerit score per item for each subgroup is less than or equal to the corresponding upper control limit, the initial data collected shall be deemed to be homogeneous for the purpose of starting the control chart. If the demerit score per item for one or more subgroups exceeds the corresponding upper control limit, then the relevant subgroup(s) is discarded and a new average demerit score per item is computed for calculating the revised upper control limit. This process is continued until the demerit score per item for all the remaining subgroups are found to be less than their upper control limits.

14.4 Interpretation

In demerit control chart, a demerit score may be achieved in many ways. that is, it may have few critical nonconformities or many minor nonconformities. So even if there is a run, it shall not be interpreted in the usual sense that an assignable cause may be present.

14.5 Advantages

a) in addition to advantages of *c* or *u* chart, it takes into account the extent and criticality of nonconformity;

b) scoring system considers the extent and criticality of nonconformity.

14.6 Limitations

a) scoring system has to be worked out objectively and its usages explained suitably;

b) involves more calculations

14.7 Example

In the diesel engine industry, several nonconformities have been identified which affect the quality of the engine. From past experience, the demerit weights for each class of nonconformity are given in Table 10. The number of engines inspected and the number of nonconformities obtained for each class of nonconformities are also given in Table 10. Compute the control limits for the demerit control chart.

Since the number of engines inspected on each day do not differ much, average number of engines ($\bar{n}$) inspected on each day may be used for calculating upper control limit for all the subgroups. From Table 10:

$$N = \sum_{j=1}^{24} n_j = 4250$$

$$\bar{n} = \frac{N}{24} = \frac{4250}{24} = 177$$

$$\bar{d} = \frac{\sum_{i=1}^{m} w_i c_i}{N} = \frac{17840}{4250} = 4,20$$

$$\sum_{i=1}^{24} w_i^2 c_i = 881400$$

$$U_{CL} = \bar{d} + 3\left(\frac{\sum_{i=1}^{m} w_i^2 c_i}{N n_j}\right)^{1/2} = 4,20 + 3\left(\frac{881400}{4250 \times 177}\right)^{1/2} = 4,20 + 3,25 = 7,45$$

$$L_{CL} = 4,20 - 3,25 = 0,95$$

Since the demerit score per engine for each of the subgroups is less than U_{CL}, These are taken as control limits for demerit control chart.

With these control limits, the control chart for demerit score is maintained. The chart is illustrated in Figure 10.

Table 10 — Subgroup results for number of non-conformities

Non-conformity	Demerit score	Subgroup number																								c_i	w_ic_i	$w^2{}_ic_i$
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24			
Block	100	0	0	3	1	0	1	1	2	0	0	1	2	0	0	0	1	0	1	1	0	1	1	0	2	18	1800	180000
Housing	100	1	2	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	3	0	0	9	900	90000
Cylinder head	100	0	2	0	0	1	0	0	1	0	1	4	2	0	0	0	1	1	0	1	1	0	1	0	0	16	1600	160000
Filter body	50	0	0	1	0	0	0	1	0	2	0	2	3	3	0	1	0	2	0	1	0	1	3	3	3	26	1300	65000
Filter cover	50	0	0	1	1	3	4	3	2	1	2	1	0	0	1	1	2	2	1	0	0	0	0	5	7	37	1850	92500
Wheel faceout	50	5	7	3	5	2	2	2	2	0	2	1	1	4	2	2	2	2	1	1	2	1	1	2	4	56	2800	140000
Valve timing wrong	50	0	0	0	0	0	1	2	1	0	1	0	0	1	1	0	0	2	0	1	0	1	1	0	0	12	600	30000
Oil pipe	30	2	1	4	7	5	4	2	4	3	2	2	0	4	3	0	4	0	4	0	2	2	6	0	1	62	1860	55800
Oil seal	30	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	60	1800
Axial play (tight)	20	0	0	0	2	2	2	8	5	11	4	5	2	2	2	1	6	0	2	1	1	2	6	1	0	65	1300	26000
Valve tappet	20	0	0	0	0	1	0	0	0	0	0	1	1	3	1	1	2	1	1	0	1	0	0	0	0	13	260	5200
Housing leakage	10	22	20	16	18	15	23	32	29	28	26	14	5	3	7	6	4	1	5	3	6	14	6	2	2	307	3070	30700
Filter pipe	10	2	2	2	2	4	4	4	1	3	2	0	2	2	2	1	1	1	2	3	0	0	2	1	1	44	440	4400
Sum																											17840	881400
Number of engines inspected (n)		170	170	170	200	170	170	170	170	170	200	175	175	175	175	175	200	175	175	175	175	175	190	175	175			
Total Demerit score (D)		650	1000	850	850	750	880	1080	1170	770	770	1080	730	770	440	410	730	540	450	430	360	490	1130	550	960		17840	
Demerit score per engine (d)		3.8	5.9	5.0	4.3	4.4	5.2	6.4	6.9	4.5	3.9	6.2	4.2	4.4	2.5	2.3	3.7	3.1	2.6	2.5	2.1	2.8	5.9	3.1	5.5			

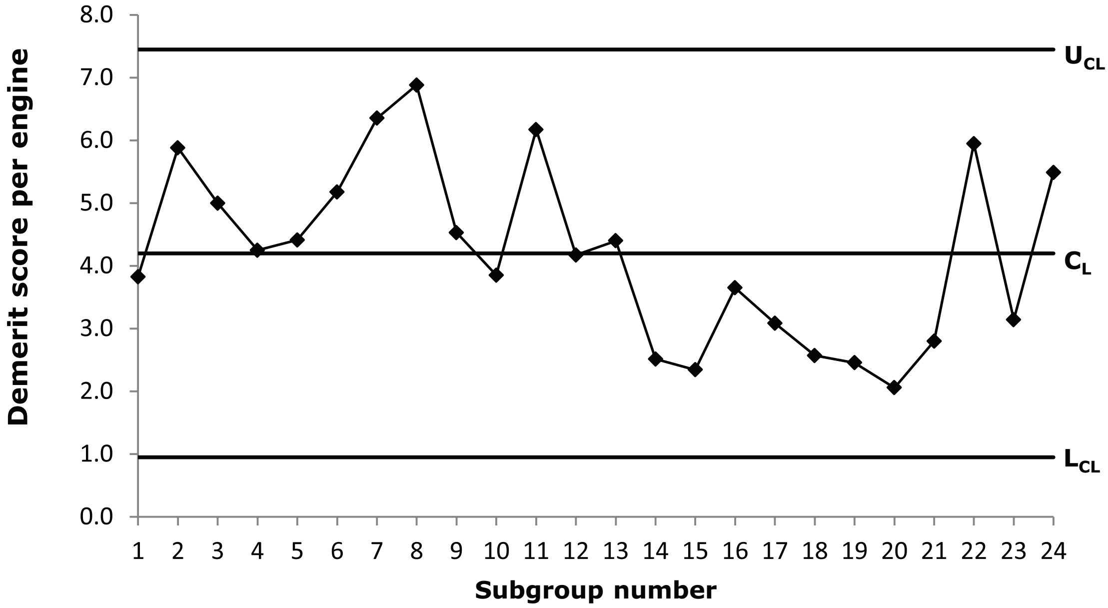


Figure 10 — Demerit control chart

14.7.1 Interpretation

The average demerit score of the first 13 subgroups (1 to 13) of about 5,0 and subsequent 8 subgroups (14 to 21) of about 2,7 provides strong evidence for improvement potential.

15 Control chart for inspection by gauging

Besides mechanical inspection of engineering items for dimensional requirements by using pair of go/no-go gauges, there are many other situations in which inspection results generate this type of data, for example, classifying the items by weight into underweight, normal, and over-weight; classifying fuse heads by their sensitivity into insensitive, normal and hypersensitive, etc.

Control charts based on go/no-go gauging can be conveniently used for controlling both the process level and the variation of a process having two-sided specifications.

These control charts are used in situations where:

a) the underlying distribution is normal or nearly normal;

b) shifts in the process level and the variation parameters are two-sided or one-sided (larger than the aimed value shifts) respectively;

c) the values of the process average and process variation (standard deviation) are known;

d) inspection by attributes is preferred to that by variables from practical considerations.

15.1 Lower and upper gauge limits

Inspection by gauging classifies items by using a pair of gauges, called lower gauge limit (L_{GL}) and upper gauge limit (U_{GL}), into the following three categories:

a) below the $L_{GL;}$

b) above the U_{GL};

c) lying between L_{GL} and $U_{GL.}$

A sample of size *n* is inspected by go/no-go gauges, and

a is the number of items below lower gauge limit (L_{GL});

b is the number of items above upper gauge limit (U_{GL}).

For symmetrical distribution, such as normal, when a sample of size *n* from each subgroup is gauged against such a pair of gauges, the numbers *a* (below L_{GL}) and *b* (above U_{GL}) are obtained. These values of *a* and *b* are used in calculating the upper and lower gauge limits.

Since U_{GL} and L_{GL} are usually different from upper specification limit (*U*) and lower specification limit (*L*) respectively, it is to be noted that the items not meeting the gauge limits may not necessarily be nonconforming.

15.2 Preliminary steps

15.2.1 Choice of probability of false alarm

Control limits for control charts based on inspection by gauging are set up to ensure a pre-assigned probability α of falsely rejecting a state of statistical control. While α = 0,005 or 0,01 is normally suitable for both the control charts and for installing an (*a*,*b*) chart, the value α = 0,005 is usually chosen, as it is nearest to the corresponding 3σ limits as used for variables control.

15.2.2 Choice of subgroup size

Control charts based on inspection by gauging are essentially attributes charts, similar to the conventional *np* charts. Thus, usually, a higher subgroup size, compared to control charts for variables, is needed to ensure reasonable protection against wrong decisions. However, through optimal determination of gauge-limits and control limits (using criteria for decision under risk and/or uncertainty), the error controlling properties of these charts may compare quite favourably with those of variables control charts even for identical sample size. In view of this, and also to keep subgroup size small, often a sample size between 7 to 10 should be adequate, contrary to the higher sample size for an attributes control chart.

15.2.3 Choice of control charts

The measures ($b-a$) and ($a+b$) are sensitive to changes in process average (μ) and process standard deviation (σ) respectively. Hence a pair of control charts for ($b-a$) and ($a+b$) should be looked upon as substitutes for the conventional ($\overline{X}-R$) charts.

However, it has been found that a single control chart for *a* and *b* (plotting two points corresponding to these measures against each subgroup number) is not only easier to use but also has better error controlling properties when compared to a pair of charts for ($b - a$) and ($a + b$). The pair of charts performs only marginally better on the line where σ = σ 0 (that is, when σ does not change). Thus, it is recommended that a single chart for (a,b) should be used.

15.3 Gauge limits and control limits

Having decided on the subgroup size *n* and the probability α of false alarm, find the values of the gauge factor (G_F) and the single control limit value r from Table 11.

15.3.1 Gauge-limits when process average (μ) and process variation σ are known

Lower gauge limit (U_{GL}) = $\mu - G_F\,\sigma$

Upper gauge limit (L_{GL}) = $\mu + G_F\,\sigma$

15.3.2 Control limit

A single control limit is drawn at $y = r$. Lower control limit is zero. There is no centre line.

Table 11 — Values of factors G_F and r for different (n,α) combinations

	α = 0,005		α = 0,010		α = 0,025		α = 0,05	
n	G_F	r	G_F	r	G_F	r	G_F	r
2	3,023	1	2,806	1	2,495	1	2,236	1
3	1,894	2	1,735	2	1,506	2	2,388	1
4	2,040	2	1,889	2	1,673	2	1,493	2
5	2,113	2	1,997	2	1,790	2	1,618	2
6	1,626	3	2,081	2	1,879	2	1,713	2
7	1,713	3	1,595	3	1,951	2	1,789	2
8	1,784	3	1,670	3	1,506	3	1,852	2
9	1,844	3	1,732	3	1,573	3	1,440	3
10	1,896	3	1,786	3	1,630	3	1,500	3
15	1,747	4	1,657	4	1,833	3	1,712	3
20	1,889	4	1,802	4	1,965	3	1,850	3
25	1,762	5	1,907	4	1,690	4	1,950	3
30	1,672	6	1,989	4	1,779	4	1,779	4
40	1,547	8	1,611	7	1,579	6	1,579	6
50	1,379	11	1,507	9	1,367	9	1,367	9

15.4 Plotting

Items in each subgroup are gauged one by one against L_{GL} and U_{GL} sequentially, and the numbers a (items below L_{GL}) and b (items above U_{GL}) are noted. Against each subgroup number, two points A and B are plotted corresponding to a and b respectively. Successive points corresponding to each of a and b may be connected by continuous and broken lines separately, preferably by using two different colours.

15.5 Interpretation

When the process is in a state of control, the two lines connecting a and b values are expected to frequently intersect each other. If they remain separated for a considerable period, even below the control limit, an ensuing shift in location may be suspected. The status of each subgroup, whether in state of control, will be indicated in the control chart as follows:

15.5.1 When both a and b lie below the control limit

This situation indicates that both process average and process variation are in state of control.

15.5.2 When a lies above the control limit and b lies on or below control limit

This situation indicates increase in process average.

15.5.3 When b lies above the control limit and a lies on or below control limit

This situation indicates decrease in process average.

15.5.4 When both a and b lie on or above control limit

This situation indicates increase in process variation.

15.6 Advantages

This type of chart has the following advantages:

a) Data on number of oversize and undersize items are available;

b) Measurement of individual values is difficult, uneconomic or impossible but easy to know whether it is below or above a certain value;

c) As substitute of $\overline{X}-R$ chart, it requires larger sample size, but still reduces overall inspection time/cost, record keeping. The analysis of results is much simpler and less time-consuming, and therefore more economical.

d) It is more efficient than p chart.

15.7 Limitations

This chart requires training and conviction that narrow gauge limits will not be harsh in application since artificial level of acceptance is created consistent with quality requirements.

15.8 Estimation of process average and process variation

If the gauge limits and the values of a and b for each of the k subgroups, taken from a process under statistical control, are known from the sample data, then process average and process variation can be estimated as follows:

Calculate

$$\overline{a}=\sum_{i=1}^{k}a_i \qquad \overline{b}=\sum_{i=1}^{k}b_i$$

$$p_{\overline{a}}=\frac{\overline{a}}{n} \qquad p_{\overline{b}}=\frac{\overline{b}}{n}$$

Where n is the constant subgroup size; and $z(p_{\overline{a}})$ and $z(1-p_{\overline{b}})$ are the values of standard normal variates corresponding to areas to the left of $p_{\overline{a}}$ and $1-p_{\overline{b}}$ respectively. $z(p_{\overline{a}})$ will be negative and $z(1-p_{\overline{b}})$ will be positive.

The estimates of process average (μ) and process standard deviation (σ) are obtained as follows:

$$\mu = \frac{L_{GL} \times z(1-p_{\bar{b}}) - U_{GL} \times z(p_{\bar{a}})}{z(1-p_{\bar{b}}) - z(p_{\bar{a}})}$$

and

$$\sigma = \frac{U_{GL} - L_{GL}}{z(1-p_{\bar{b}}) - z(p_{\bar{a}})}$$

Where U_{GL} and L_{GL} are upper and lower gauge limits.

15.9 Example

Table 12 gives the test results of ultimate tensile strength of each of 7 aluminium wires, which are stranded to make composite conductor of diameter 2,79 mm. From each cable drum of composite conductors, taken as subgroup, a sample is taken and 7 test results corresponding to the 7 wires of the composite conductor are given in Table 12 .From past data, the process average of this characteristic is found to be 19,5 kgf/mm^2 and the standard deviation as 1.00 kgf/mm^2. It is decided to install suitable (a,b) chart for ultimate tensile strength of the composite conductor, and also to compare its sensitivity with $\bar{X} - R$ chart.

For installing (a,b) chart, the value α = 0,005 is taken, as it is nearest to the corresponding 3 σ limits used for variables control charts.

For α = 0,005, n = 7, from Table 11, G_F = 1,713 and r = 3

Also μ_0 = 19,5 and σ_0 = 1,00, hence

$$U_{GL} = 19{,}5 + 1{,}713 \times 1 = 21{,}213$$

$$L_{GL} = 19{,}5 - 1{,}713 \times 1 = 17{,}787$$

Control limit is r = 3

For each subgroup, the items are gauged sequentially against U_{GL} and L_{GL} and the values of a and b are noted. These values are shown in Table 12. The control chart is shown in Figure 11.

It appears from the control chart that the value of b for subgroup 14 and 15 are found to have gone out of control limit. Suitable remedial actions are taken to stabilize the process in terms of location after which the process is found to be in a state of statistical control.

Table 12 — Subgroup results for Ultimate Tensile Strength (kgf/mm²) of Composite Conductor

Sub-group number	Sample unit number										
	1	2	3	4	5	6	7	*a*	*b*	average	range
1	19,63	19,30	18,81	18,98	19,30	18,98	19,80	0	0	19,26	0,99
2	19,47	19,14	19,96	18,00	18,49	18,32	19,80	0	0	19,03	1,96
3	19,14	18,98	18,32	19,63	19,30	19,30	19,30	0	0	19,14	1,31
4	18,49	18,81	18,65	18,98	18,81	18,81	19,47	0	0	18,86	0,98
5	18,32	18,80	18,81	18,98	18,32	18,02	18,81	0	0	18,58	0,96
6	18,81	18,00	18,00	18,32	19,96	19,30	19,80	0	0	18,88	1,96
7	19,14	18,65	18,00	18,32	18,49	17,51	18,65	0	1	18,39	1,63
8	18,98	18,32	19,63	19,80	18,00	22,74	19,14	1	0	19,52	4,74
9	19,80	18,32	20,78	19,63	19,43	19,63	20,94	0	0	19,79	2,62
10	19,11	18,00	21,74	19,27	18,65	20,75	19,96	1	0	19,64	3,74
11	18,32	20,92	19,96	19,96	20,78	21,08	20,78	0	0	20,26	2,76
12	18,16	18,00	17,34	18,65	19,14	18,32	18,00	0	1	18,23	1,80
13	18,12	18,61	18,28	18,61	18,32	18,32	17,62	0	1	18,27	0,99
14	22,90	22,90	20,94	21,60	19,96	21,27	21,90	5	0	21,64	2,94
15	22,09	19,47	22,90	23,39	18,32	22,90	22,90	5	0	21,71	5,07
16	18,00	18,32	19,63	18,12	18,81	19,63	20,12	0	0	18,95	2,12
17	17,83	19,14	18,32	19,60	17,79	19,30	18,28	0	1	18,61	1,81
18	19,96	18,80	21,76	19,27	21,25	20,94	20,29	2	0	20,47	2,49
19	16,80	17,79	20,42	18,32	19,76	17,29	18,28	0	2	18,38	3,62
20	18,00	18,32	19,63	18,12	18,81	19,63	20,12	0	0	18,95	2,12

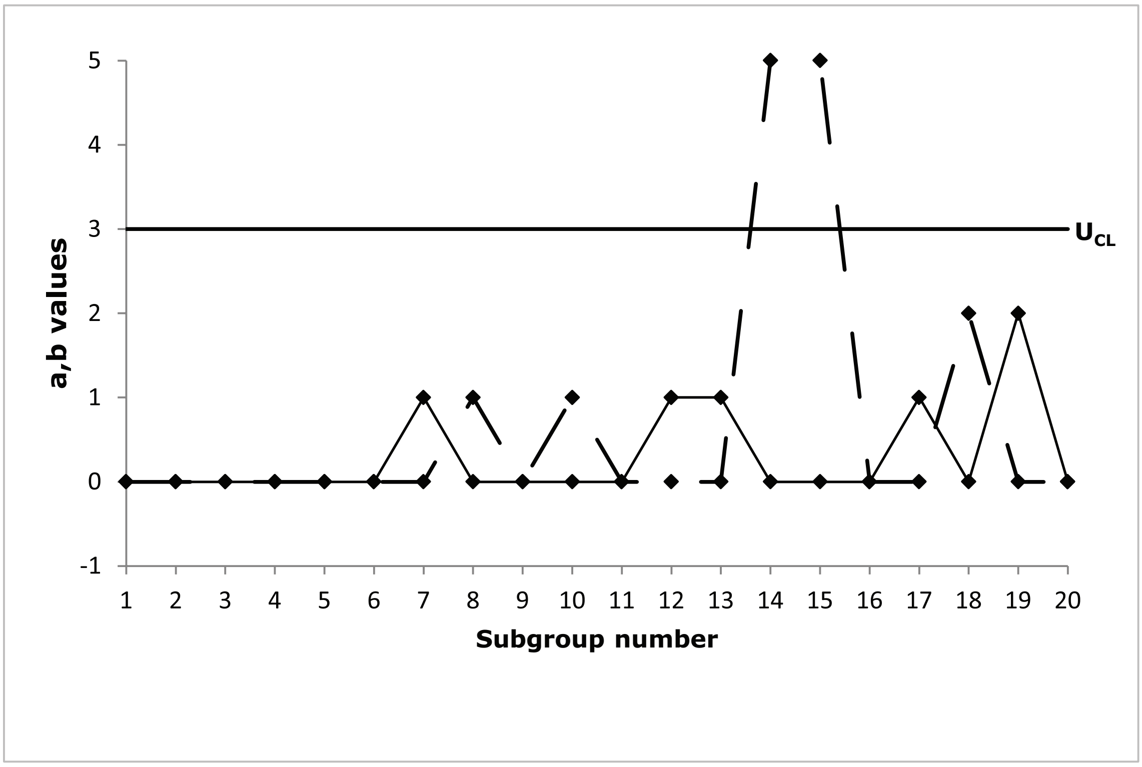


Figure 11 — (a, b chart)

15.9.1 Comparison with $\overline{X}-R$ chart

$\overline{X}-R$ chart has also been applied for data given in Table 12, and the control limits are given below:

Average Chart

$$U_{CL} = \mu + A\sigma = 19{,}5 + 1{,}134 \times 1 = 20{,}634$$

$$L_{CL} = \mu - A\sigma = 19{,}5 - 1{,}134 \times 1 = 18{,}366$$

Range chart

$$U_{CL} = D_2\sigma = 5{,}203 \times 1 = 5{,}203$$

$$L_{CL} = D_1\sigma = 0{,}205 \times 1 = 0{,}205$$

Values of A, D_1, and D_2, are given in Annex A for different sample sizes.

From the average and range values given in Table 12, it may be seen that in average chart also, the points corresponding to subgroups 14 and 15 are going out of upper control limit, and no point is going out of control on the range chart; the same way as shown by (a, b) chart Thus, for this example (a, b) chart is found to be as sensitive as $\left(\overline{X}-R\right)$ chart.

Annex A
(informative)

Factors for computing control limits

Table A.1 — Factors for computing control limits

	Using Standard Values of μ and σ								Using (s)			Using R			
	High Low chart		CV Chart			Range Chart			CV Chart		Average Chart	Extreme Value Chart	Average Chart	Range Chart	
C_L	μ	μ	$\frac{c_2\sigma}{\mu}$			$d_2\,\sigma$			$\overline{V}$		$\overline{\overline{X}}$	$M = \frac{\overline{x}_H + \overline{x}_L}{2}$	$\overline{\overline{X}}$	$\overline{R}$	
L_{CL}	μ - Hσ	μ - Aσ	$B_1\,\sigma/\mu$			$D_1\,\sigma$			$B_3\,\overline{V}$		$\overline{\overline{X}} - A_3\,\overline{S}$	$M - H_2\overline{R}$	$\overline{\overline{X}} - A_2\overline{R}$	$D_3\overline{R}$	
U_{CL}	μ + Hσ	μ + Aσ	$B_2\,\sigma/\mu$			$D_2\,\sigma$			$B_4\,\overline{V}$		$\overline{\overline{X}} + A_3\,\overline{S}$	$M + H_2\overline{R}$	$\overline{\overline{X}} + A_2\overline{R}$	$D_4\overline{R}$	
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
No. of observations in the sample	H	A	c_2	B_1	B_2	d_2	D_1	D_2	B_3	B_4	A_3	H_2	A_2	D_3	D_4
2	3,041	2,121	0,564	0	1,843	1,128	0	3,686	0	3,267	2,659	2,695	1,880	0	3,267
3	3,090	1,732	0,724	0	1,858	1,693	0	4,358	0	2,568	1,954	1,826	1,023	0	2,575
4	3,133	1,500	0,798	0	1,808	2,059	0	4,698	0	2,266	1,628	1,522	0,729	0	2,282
5	3,170	1,342	0,841	0	1,756	2,326	0	4,918	0	2,089	1,427	1,363	0,577	0	2,115
6	3,202	1,225	0,869	0,026	1,711	2,534	0	5,078	0,030	1,970	1,287	1,263	0,483	0	2,004
7	3,230	1,134	0,888	0,105	1,672	2,704	0,205	5,203	0,118	1,882	1,182	1,194	0,419	0,076	1,924
8	3,256	1,061	0,903	0,167	1,638	2,847	0,387	5,307	0,185	1,815	1,099	1,143	0,373	0,136	1,864
9	3,278	1,000	0,914	0,219	1,609	2,970	0,546	5,394	0,239	1,761	1,032	1,104	0,337	0,184	1,816
10	3,299	0,949	0,923	0,262	1,584	3,078	0,687	5,469	0,284	1,716	0,975	1,072	0,308	0,223	1,777
11			0,930	0,299	1,561	3,173	0,812	5,534	0,321	1,679	0,927				
12			0,936	0,331	1,541	3,258	0,924	5,592	0,354	1,646	0,886				
13			0,941	0,359	1,523	3,336	1,026	5,646	0,382	1,618	0,850				
14			0,945	0,384	1,507	3,407	1,121	5,693	0,406	1,594	0,817				
15			0,949	0,406	1,492	3472	1,207	5,737	0,428	1,572	0,789				
16			0,952	0,427	1,478	3,532	1,285	5,779	0,448	1,552	0,763				
17			0,955	0,445	1,465	3,588	1,359	5,817	0,466	1,534	0,739				
18			0,958	0,461	1,454	3,640	1,426	5,854	0,482	1,518	0,718				
19			0,960	0,477	1,443	3,689	1,490	5,888	0,497	1,503	0,698				
20			0,962	0,491	1,433	3,735	1,548	5,922	0,510	1,490	0,680				
21			0,964	0,504	1,424	3,778	1,606	5,950	0,523	1,477					

Table A.1 *(continued)*

	Using Standard Values of μ and σ								Using (s)			Using R			
	High Low chart		CV Chart			Range Chart			CV Chart		Average Chart	Extreme Value Chart	Average Chart	Range Chart	
22			0,966	0,516	1,415	3,819	1,659	5,979	0,534	1,466					
23			0,967	0,527	1,407	3,858	1,710	6,006	0,545	1,455					
24			0,968	0,538	1,399	3,895	1,759	6,031	0,555	1,445					
25			0,970	0,548	1,392	3,931	1,804	6,058	0,565	1,435					

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