BS ISO 7870-3:2012

BSI Standards Publication

Control charts

Part 3: Acceptance control charts

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National foreword

This British Standard is the UK implementation of ISO 7870-3:2012. It supersedes BS 7783:1994 which is withdrawn.

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A list of organizations represented on this committee can be obtained on request to its secretary.

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Part 3: **Acceptance control charts**

Cartes de contrôle —

Partie 3: Cartes de contrôle pour acceptation

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7870-3 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This first edition of ISO 7870-3 cancels and replaces ISO 7966:1993.

ISO 7870 consists of the following parts, under the general title *Control charts*:

— *Part 1: General guidelines*

— *Part 2: Shewhart control charts*

— *Part 3: Acceptance control charts*

— *Part 4: Cumulative sum charts*

Additional parts on specialized control charts and on the application of statistical process control (SPC) charts are planned.

Introduction

An acceptance control chart combines consideration of control implications with elements of acceptance sampling. It is an appropriate tool for helping to make decisions with respect to process acceptance. The bases for the decisions may be defined in terms of

a) whether or not a designated percentage of units of a product or service derived from that process will satisfy specification requirements;

b) whether or not a process has shifted beyond some allowable zone of process level locations.

A difference from most acceptance sampling approaches is the emphasis on process acceptability rather than on product disposition decisions.

A difference from usual control chart approaches is that the concept of process acceptance is introduced in the process control. The process usually does not need to be in control about a single standard process level; as long as the within-subgroup variability remains in control and is much smaller than the tolerance spread, it can (for the purpose of acceptance) run at any level or levels within a zone of process levels which would be acceptable in terms of tolerance requirements. Thus, it is assumed that some assignable causes will create shifts in the process levels which are small enough in relation to requirements that it would be uneconomical to attempt to control them too tightly for the purpose of mere acceptance.

The use of an acceptance control chart does not, however, rule out the possibility of identifying and removing assignable causes for the purpose of continuing process improvement.

A check on the inherent stability of the process is required. Therefore, variables are monitored using Shewhart-type range or sample standard deviation control charts to confirm that the variability inherent within rational subgroups remains in a steady state. Supplementary examinations of the distribution of the encountered process levels form an additional source of control information. A preliminary Shewhart control chart study should be conducted to verify the validity of using an acceptance control chart.

Control charts —

Part 3: Acceptance control charts

1 Scope

This part of ISO 7870 gives guidance on the uses of acceptance control charts and establishes general procedures for determining sample sizes, action limits and decision criteria. An acceptance control chart should be used only when:

a) the within subgroup variation is in-control and the variation is estimated efficiently;

b) a high level of process capability has been achieved.

An acceptance control chart is typically used when the process variable under study is normally distributed; however, it can be applied to a non-normal distribution. The examples provided in this part of ISO 7870 illustrate a variety of circumstances in which this technique has advantages; these examples provide details of the determination of the sample size, the action limits and the decision criteria.

2 Normative references

The following standards, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the refferenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of document, the terms and definitions given in ISO 3534-1 and ISO 3534-2 apply.

3.1
acceptable process
process which is represented by a Shewhart control chart with a central line within the acceptable process zone

NOTE 1 Ideally, the average value $\overline{X}$ of such a control chart would be at the target value.

NOTE 2 The acceptable process zone is shown in Figure 1. Information on the Stewhart control chart can be found in ISO 7870-2.

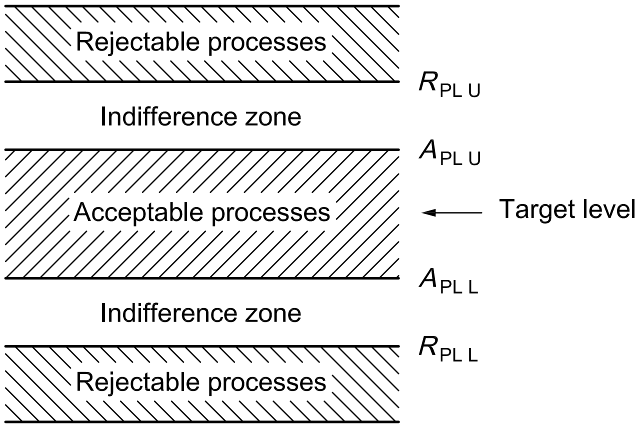


Figure 1 — Two-sided specification limits: Upper and lower APL and RPL lines in relation to processes of acceptable, rejectable, and indifference (borderline) quality

4 Symbols and abbreviated terms

NOTE The ISO/IEC Directives makes it necessary to depart from common SPC usage in respect to the differentiation between abbreviated terms and symbols. An abbreviated term and its symbol can differ in appearance in two ways: by font and by layout. To distinguish between abbreviated terms and symbols, abbreviated terms are given in Arial upright and symbols in Times New Roman or Greek italics, as applicable. Whereas abbreviated terms can contain multiple letters, symbols consist only of a single letter. For example, the conventional abbreviation of acceptable process limit, APL, is valid but its symbol in equations becomes A_{PL}. The reason for this is to avoid misinterpretation of compound letters as an indication of multiplication.

4.1 Symbols

A_{CL} acceptance control limits

A_{PL} acceptable process level

L lower specification limit

n subgroup sample size

p_0 acceptable proportion nonconforming items

p_1 rejectable proportion nonconforming items

P_{a} probability of acceptance

R_{PL} rejectable process level or non-acceptable process zone

T target value, i.e. the optimum value of the characteristic

U upper specification limit

$\bar{X}$ average value of the variable X plotted on a control chart

z variable that has a normal distribution with zero mean and unit standard deviation

$z_{p'}$ normal deviate that is exceeded by 100p' % of the deviate in a specified direction (similarly for z_α, z_β, etc.)

α risk of not accepting a process centred at the APL

β risk of not rejecting a process centred at the RPL

μ process mean

σ_W within-subgroup standard deviation corresponding to the inherent process variability

$\sigma_{\bar{X}}$ standard deviation of the subgroup average corresponding to the inherent process variability: $\sigma_{\bar{X}} = \sigma_W / \sqrt{n}$

4.2 Abbreviated terms

ACL acceptance control limits

APL acceptable process level

L lower specification limit (used as a subscript)

OC operating characteristic

RPL rejectable process level or non-acceptable process zone

U upper specification limit (used as a subscript)

5 Description of acceptance control chart practice

In the pursuit of an acceptable product or service, there often is room for some latitude in the ability to centre a process around its target level. The contribution to overall variation of such location factors is additional to the inherent random variability of individual elements around a given process level. In most cases, some shifts in process level must be expected and can be tolerated. These shifts usually result from an assignable cause that cannot be eliminated because of engineering or economic considerations. They often enter the system at infrequent or irregular intervals, but can rarely be treated as random components of variance.

There are several seemingly different approaches to treating these location factors contributing variation beyond that of inherent variability. At one extreme is the approach in which all variability that results in deviations from the target value must be minimized. Supporters of such an approach seek to improve the capability to maintain a process within tighter tolerance limits so that there is greater potential for process or product quality improvement.

At the other extreme is the approach that if a high level of process capability has been achieved, it is not only uneconomic and wasteful of resources, but it can also be counterproductive to try to improve the capability of the process. This often is the result of the introduction of pressures which encourage “tampering” with the process (over-control) by people qualified to work on control aspects but not product or process quality improvement programmes.

The acceptance control chart is a useful tool for covering this wide range of approaches in a logical and simple manner. It distinguishes between the inherent variability components randomly occurring throughout the process and the additional location factors which contribute at less frequent intervals.

When shifts appear, the process may then stabilize at a new level until the next such event occurs. Between such disturbances, the process runs in control with respect to inherent variability.

An illustration of this situation is a process using large uniform batches of raw material. The within-batch variability could be considered to be the inherent variability. When a new batch of material is introduced, its deviation from the target may differ from that of the previous batch. The between-batch variation component enters the system at discrete intervals.

An example of this within- and between-batch variation might very well occur in a situation where a blanking die is blanking a machine part. The purpose of the chart is to determine when the die has worn to a point where it must be repaired or reworked. The rate of wear is dependent upon the hardness of the successive batches of material and is therefore not readily predictable. It will be seen that the use of an acceptance control chart makes it possible to judge the appropriate time to service the blanking die.

The acceptance control chart is based on the Shewhart control chart (i.e. $\bar{X} - R$ chart or $\bar{X} - s$ chart) but is set up so that the process mean can shift outside of control limits of the Shewhart control chart if the specifications are sufficiently wide, or be confined to narrower limits if the inherent variability of the process is comparatively large or a large fraction of the total tolerance spread.

What is required is protection against a process that has shifted so far from the target value that it will yield some predetermined undesirable percentage of items falling outside the specification limits, or exhibits an excessive degree of process level shift.

When a chart of the average value of data sets from a process is plotted, in sequence of the production, one notices a continual variation in average values. In a central zone (acceptable process, Figure 1), there is a product that is indisputably acceptable. Data in the outer zones (Figure 1) represent a process that is producing product that is indisputably not acceptable.

Between the inner and the outer zones are zones where the product is acceptable but there is an indication that the process should be watched and, as the outer zone is approached, corrective action may be taken. These criteria are the basic concepts for the acceptance control chart. The description in this part of ISO 7870 is designed to provide practices for the establishment of appropriate action lines for one- and two-sided specification situations.

Since it is impossible to have a single dividing line that can sharply distinguish a good from an unsatisfactory quality level, one must define a process level that represents a process that should be accepted almost always ($1 - \alpha$). This is called the acceptable process level (APL), and it marks the outer boundary of the acceptable process zone located about the target value (see Figure 1).

Any process centred closer to the target value than the APL will have a risk smaller than α of not being accepted. So the closer the process is to the target, the smaller the likelihood that a satisfactory process will not be accepted.

It is also necessary to define the process level that represents processes that should almost never be accepted ($1 - \beta$). This undesirable process level is labelled the rejectable process level (RPL). Any process located further away from the target value than the RPL will have a risk of acceptance smaller than β.

The process levels lying between the APL and RPL would yield a product of borderline quality. That is, process levels falling between the APL and RPL would represent quality which is not so good that it would be a waste of time, or represent over-control, if the process were adjusted, and not so bad that the product could not be used if no shift in level were made. This region is often called the "indifference zone". The width of this zone is a function of the requirements for a particular process and the risks one is willing to take in connection with it. The narrower the zone, i.e. the closer the APL and RPL are to each other, the larger the sample size will have to be. This approach will permit a realistic appraisal of the effectiveness of any acceptance control system, and will provide a descriptive method for showing just what any given control system is intended to do.

As with any acceptance sampling system, the four elements required for the definition of an acceptance control chart are:

a) an acceptable process level (APL) associated with a one-sided α-risk;

b) a rejectable process level (RPL) associated with a one-sided β-risk;

c) an action criterion or acceptance control limit (ACL);

d) the sample size (n).

NOTE Generally, the defined risks are one-sided in this part of ISO 7870. In the case of two-sided specifications, the risks are either a 5 % risk to go above an upper limit or a 5 % risk to go below a lower limit. This results in a 5 % (not 10 %) total risk.

Simplicity of operation is of critical importance to the use of a procedure such as an acceptance control chart. Only the acceptance control limits and the sampling instructions (such as sample size, frequency, or method of selection) need to be known to the operator who uses the chart, although training him to understand the derivation is not difficult and can be helpful. It is thus no more complicated to use than the Shewhart chart. The supervisor, quality expert, or trained operator will derive these limits without much effort from the above considerations and will obtain a more meaningful insight into the process acceptance procedure, and a better understanding of the control implications.

6 Acceptance control of a process

6.1 Plotting the chart

The sample average value of the quality characteristic is plotted on acceptance control charts in the following way. A point is plotted on the chart for each sample with an identification number (numerical order, time order, etc.) on the horizontal scale, and the corresponding sample average on the vertical scale.

6.2 Interpreting the chart

When the plotted point falls above the upper acceptance control limit ACL_U or below the lower acceptance control limit ACL_L, the process shall be considered non-acceptable.

If a plotted point is close to the control line, the numerical values shall be used to make the decision.

7 Specifications

Theoretically, the specification of the values of any two of the defining elements APL (with α–risk), RPL (with β–risk), acceptance control limit (ACL) or sample size (n) of an acceptance control chart system determines the remaining two values; however, in practice, it is essential that APL (with α-risk) be defined first. In addition, the within-rational subgroup value of σ_W must be known or have been estimated by the usual control chart techniques such as using $\hat{\sigma}_w = \bar{R} / d_2$ or $\bar{s} / c_4$. It is essential that the inherent random variability be in a state of statistical control in order for the risk computations to be meaningful. This can be monitored through the use of a Shewhart-type control chart for ranges or standard deviations. (See ISO 7870-2.)

Two selections of pairs of defining elements may be chosen.

a) Definition of the APL and RPL along with their respective α- and β-risks, and determination of the sample size (n) and the acceptance control limit (ACL).

   Often, $\alpha = 0{,}05$ is chosen in acceptance control chart applications since there are few instances where a process continuously runs at the APL. This means that the risk of rejection on each side of the target value, T, should always be smaller than α.

   This option is generally used when

   1) acceptable processes are defined either for economic or other practical reasons in terms of process capabilities that include allowance for small discrete shifts in process level in addition to inherent random variation, or in terms of an acceptable quality level described by the percentage of items exceeding specification limits, and

   2) when rejectable processes are defined either for practical reasons in terms of unnecessarily large shifts in process level, or in terms of a process level yielding an unsatisfactory percentage of items exceeding specification limits.

b) Definition of the APL (with α) and the sample size n, and determination of the RPL for a given β-risk and the ACL.

   This option is used when acceptable processes are defined as in 1) above, and when there is a restriction determining the allowable sample size. The option a) is preferable in most cases.

The examples in this part of ISO 7870 deal with variables data and are described in terms of two-sided specifications with limits and levels defined both above and below the target value. However, the method is equally valid for one-sided specification limits. In addition, there is no requirement that the values selected above and below the target value be symmetrical should more latitude be desired on either side. If different values are selected above and below the target, the sample size required for the more stringent situation (i.e. smaller distance between the APL and RPL) shall be used (see 8.1.1).

8 Calculation procedures

8.1 Selection of pairs of elements

8.1.1 Defining elements APL and RPL

In the case of variables ($\bar{X}$), the APL may be selected in several ways. If the specification limits are known, as well as the underlying distribution of the individual population items, the APL may be defined in terms of an acceptable proportion (or percentage) p_0 of nonconforming items which would occur when the process is centred at the APL. See Figure 2. If the underlying distribution is normal (Gaussian), a one-tailed table of standard normal deviate z values can be used where,

$$p = \int_{z_p}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

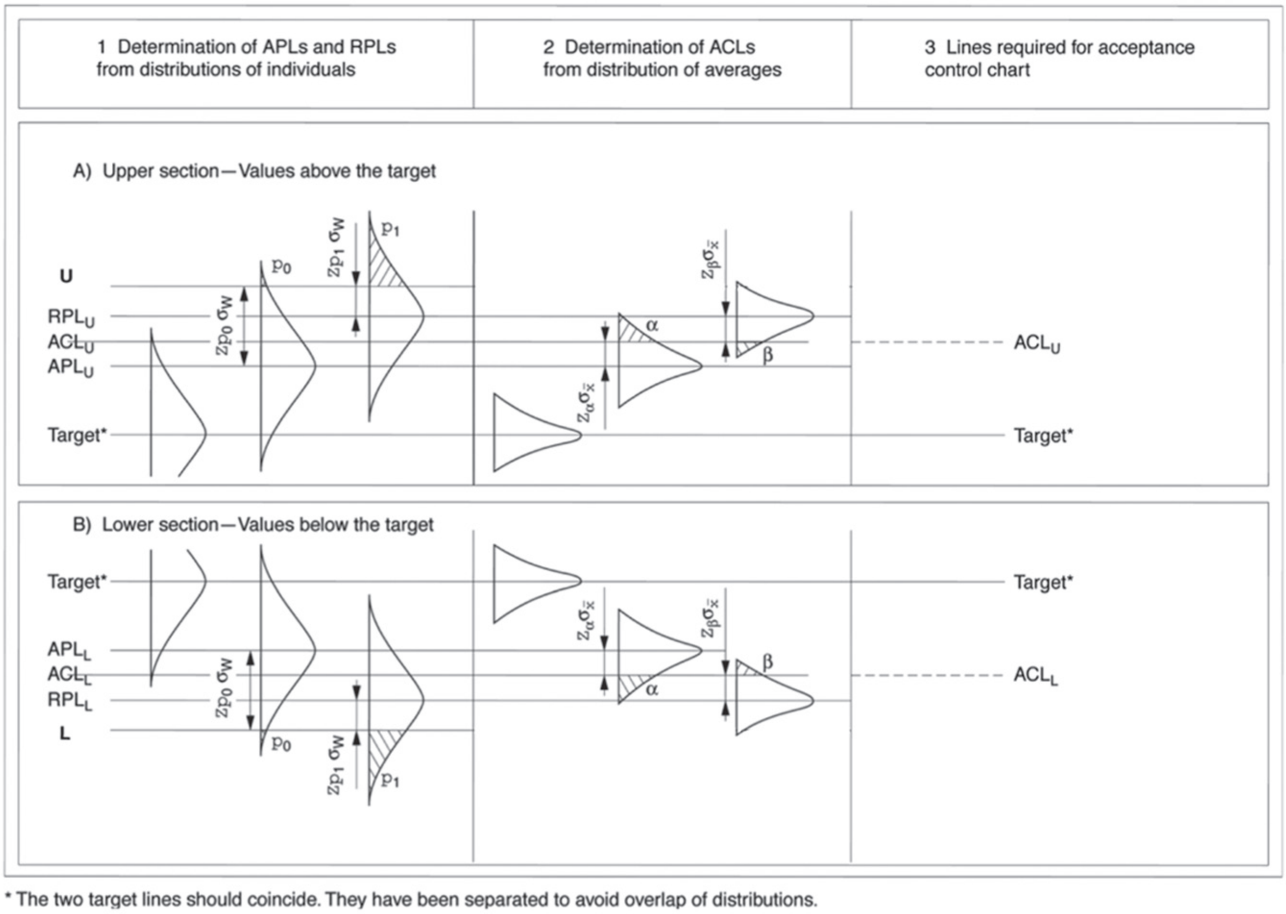


Figure 2 — Limits and defining elements of acceptance control charts

For samples of four or more, the assumption of a normal distribution for control purposes is generally valid for $\bar{X}$ charting. However, the interpretation of the proportion (percentage) of nonconforming items associated with the APL and RPL levels is dependent on the underlying distribution. Thus, for other distributions, appropriate tables should be followed and the standard normal deviate values z_{p_t} replaced accordingly. The advantage of the z approach in this application is that the limits and defining elements fall above and below the centre, so that it is convenient to have identical α and β values on both sides of the target rather than having to deal with α and $1 - \alpha$ or β and $1 - \beta$, depending on which side of the centre is involved. This also aids in a geometric interpretation such as

$$z_\alpha \sigma_{\bar{X}} + z_\beta \sigma_{\bar{X}} = R_{\text{PL}} - A_{\text{PL}}$$

Upper APL $\left(A_{\text{PL U}}\right) = U - z_{p_0}\sigma_{\text{w}}$

Lower APL $\left(A_{\text{PL L}}\right) = L + z_{p_0}\sigma_{\text{w}}$

See example 1 in 9.1 where $\bar{X}$ charts with the APL and RPL are defined in terms of the percentage of nonconforming items. A flowchart for the calculation procedure is shown in Figure 3.

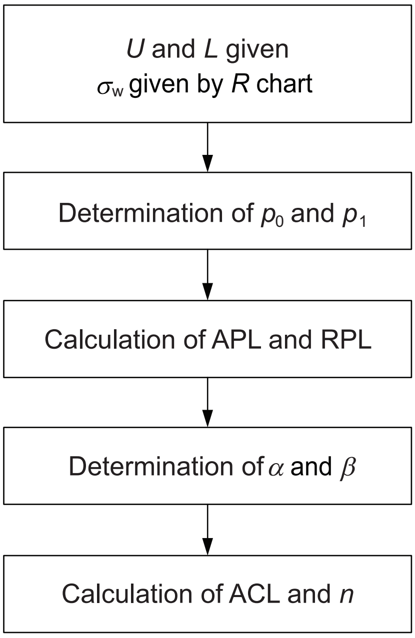


Figure 3 — Flowchart for calculation procedure (defining elements APL and RPL)

In some cases, the selection of an APL value may not be directly related to the specification limits, but may be chosen on an arbitrary basis. Experience may show that the “uneconomic” or “not readily adjustable” causes for shifts in process level correspond to a narrow band. The edge of this band may be arbitrarily designated as the APL (see example 2 in 9.2). In this case, the normal distribution assumption is not invoked since the APL is not directly related to the specification limits.

In a similar fashion, the RPL may be selected in several ways. It can be related to the specification limits by defining an unacceptable proportion (percentage) p_1 of nonconforming items which would occur when the process is centred at the RPL.

Upper RPL $(R_{\text{PL U}}) = U - z_{p_1}\sigma_{\text{w}}$

Lower RPL $(R_{\text{PL L}}) = L + z_{p_1}\sigma_{\text{w}}$

Once the APL and α, and RPL and β, values are defined, the upper acceptance control limit ($A_{\text{CL U}}$) is located at

$$A_{\text{CL U}} = A_{\text{PL U}} + \left(\frac{z_\alpha}{z_\alpha + z_\beta}\right)\left(R_{\text{PL U}} - A_{\text{PL U}}\right)$$

where z_α and z_β are the cut-off points for a proportion α and β respectively.

The lower limit is located at

$$A_{\text{CL L}} = A_{\text{PL L}} + \left(\frac{z_\alpha}{z_\alpha + z_\beta}\right)\left(A_{\text{PL L}} - R_{\text{PL L}}\right)$$

When the α- and β-risks are selected to be equal, the acceptance control limit lies halfway between the APL and RPL.

The sample size can be calculated as

$$n = \left[\frac{(z_\alpha + z_\beta)\sigma_{\text{w}}}{R_{\text{PL}} - A_{\text{PL}}}\right]^2$$

For asymmetrical limits, as at the end of Clause 7:

$$n = \max\left\{\left[\frac{\left(z_{\alpha,\text{U}} + z_{\beta,\text{U}}\right)\sigma_{\text{w}}}{R_{\text{PL U}} - A_{\text{PL U}}}\right]^2 \text{ or } \left[\frac{\left(z_{\alpha,\text{L}} + z_{\beta,\text{L}}\right)\sigma_{\text{w}}}{R_{\text{PL L}} - A_{\text{PL L}}}\right]^2\right\}$$

A nomograph, which also provides an OC (operating characteristic) curve, can be used instead of these calculations. Both the calculation and nomograph methods are easy to use (see Annex A).

8.1.2 Defining elements APL, α, β and n

The APL may be selected as specified in 8.1.1. The sample size may be specified as a matter of convenience in the operation, or it may be entered as a trial proposal to discover what kind of RPL and β values will result. If these are unsatisfactory, the process can be iterated or one of the other combinations used so that n is calculated. Given the APL, α and n values:

$$A_{\text{CL U}} = A_{\text{PL U}} + z_\alpha \sigma_{\text{w}} / \sqrt{n}$$

$$A_{\text{CL L}} = A_{\text{PL L}} - z_\alpha \sigma_{\text{w}} / \sqrt{n}$$

$$R_{\text{PL U}} = A_{\text{CL U}} + z_\beta \sigma_{\text{w}} / \sqrt{n}$$

$$R_{\text{PL L}} = A_{\text{CL L}} - z_\beta \sigma_{\text{w}} / \sqrt{n}$$

See example 2 in 9.2. A flowchart for calculation procedure is shown in Figure 4.

8.2 Frequency of sampling

The relationship between sample size and the α- and β-risks has been discussed above. The determination of frequency of sampling will not be treated in this part of ISO 7870. If the history of a process is one of well-behaved inherent variability and of level shifts usually within the zone of acceptable processes, the sampling frequency may be relatively low when compared to that for processes exhibiting less stability. The costs of erroneous decisions are to some extent considered in the selection of the α and β values, but are clearly related to the frequency of sampling as well.

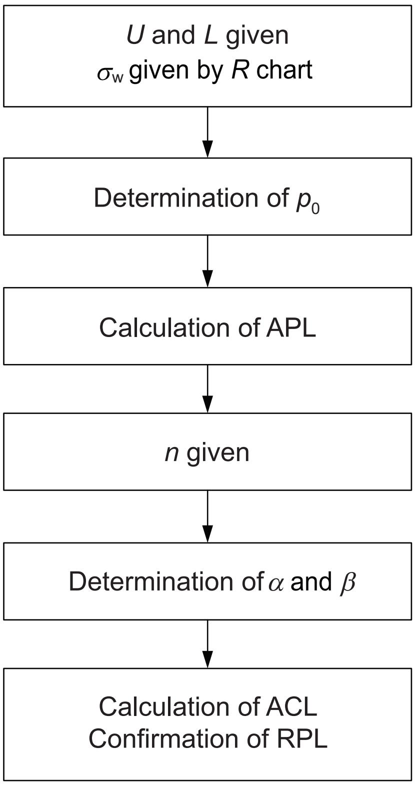


Figure 4 — Flowchart for calculation procedure (defining elements APL, α, β and n)

9 Examples

9.1 Example 1 (see also Figures A.3 and A.4)

Operation: Filling bottles with 10,0 cm^3 ± 0,5 cm^3 of solution.

Measurement: Amount of solution; nominal value 10 cm^3.

Variability: The inherent variability due to random causes is known to have a normal distribution. Past experience shows $\sigma_W = 0{,}1$ cm^3.

Objective: It is desired to accept the set-up by an operator if less than 0,1 % of the bottles filled are above and/or below the range 10,0 cm^3 ± 0,5 cm^3. It is desired to reject the set-up by an operator if more than 2,5 % of the bottles are above and/or below 10,0 cm^3 ± 0,5 cm^3.

The following data are used to calculate the APL and RPL:

Upper specification limit: $U = 10{,}5$ cm^3

Lower specification limit: $L = 9{,}5$ cm^3

Process standard deviation: $\sigma_W = 0{,}1$ cm^3

The critical value of z of the normal distribution (cutting off a tail area equal to the specified fraction exceeding specification limits):

$z_{p_0} = 3{,}090$ for $p_0 = 0{,}001$

$z_{p_1} = 1{,}960$ for $p_1 = 0{,}025$

The evaluation yields:

$$A_{\text{PL}} \begin{cases} U - z_{0{,}001}\sigma_{\text{w}} = 10{,}5 - 3{,}090 \times 0{,}1 = 10{,}191 \\ L + z_{0{,}001}\sigma_{\text{w}} = 9{,}5 + 3{,}090 \times 0{,}1 = 9{,}809 \end{cases}$$

$$R_{\text{PL}} \begin{cases} U - z_{0{,}025}\sigma_{\text{w}} = 10{,}5 - 1{,}960 \times 0{,}1 = 10{,}304 \\ L + z_{0{,}025}\sigma_{\text{w}} = 9{,}5 + 1{,}960 \times 0{,}1 = 9{,}696 \end{cases}$$

It has been decided to take an α-risk of 5 % and a β-risk of 5 % so that $z_\alpha = z_\beta = 1{,}645$. Therefore:

$$\begin{aligned} A_{\text{CL U}} &= A_{\text{PL U}} + \left(\frac{z_\alpha}{z_\alpha + z_\beta} \right) \left(R_{\text{PL U}} - A_{\text{PL U}} \right) \\ &= 10{,}191 + 0{,}5(10{,}304 - 10{,}191) \\ &= 10{,}245 \end{aligned}$$

and

$$\begin{aligned} A_{\text{CL L}} &= A_{\text{PL L}} - \left(\frac{z_\alpha}{z_\alpha + z_\beta} \right) \left(A_{\text{PL L}} - R_{\text{PL L}} \right) \\ &= 9{,}809 - 0{,}5(9{,}809 - 9{,}696) \\ &= 9{,}755 \end{aligned}$$

The sample size is:

$$\begin{aligned} n &= \left[\frac{(z_\alpha + z_\beta)\sigma}{R_{\text{PL}} - A_{\text{PL}}} \right]^2 \\ &= \left[\frac{(1{,}645 + 1{,}645) \times 0{,}1}{0{,}113} \right]^2 \\ &= (2{,}912)^2 = 8{,}48 \end{aligned}$$

The sample size is rounded up to $n = 9$ to ensure that the risks do not exceed the specified values of α and β.

The interpretation of the results leads to the following conclusions.

a) Operator set-ups that deviate from the nominal level by $\pm 0{,}191$ cm^3 or less (which would mean that fewer than 0,1 % of the individual bottles will exceed specifications) are reasonably sure (95 % or higher) of being accepted.

b) Operator set-ups that deviate from the nominal level by $\pm 0{,}304$ cm^3 or more (which would mean that more than 2,5 % of the individual bottles would exceed specifications) are reasonably sure (95 % or higher) of being rejected.

c) Operator set-ups that deviate from the nominal level by more than $\pm 0{,}191$ cm^3 but less than $\pm 0{,}304$ cm^3 may or may not be rejected for readjustment. These are considered not bad enough to be sure of rejecting but not good enough to be sure of accepting. They represent borderline or “indifferent” quality with respect to the accuracy of their set-up.

9.2 **Example 2** (see also Figure A.5)

Operation: Coating process.

Measurement: Thickness of the coating.

Variability: The inherent variability of narrow lengthwise strips measured across the coating can be characterized by the standard deviation within strips along the coating; $\sigma_{\text{w}} = 0{,}005$.

Objective: Since uniformity from strip to strip is more important than actual level, it is decided that strips having mean values deviating from the grand average of all strips by less than ±0,008 mm should be accepted with a risk of rejection of less than $\alpha = 5$ %. For operating convenience, a sample size of $n = 4$ has been established. Thus, the given parameters are $\sigma_W = 0{,}005$ and

$A_{PL\ L} = -0{,}008$ associated with $\alpha = 0{,}05$ and $z_\alpha = 1{,}645$

$A_{PL\ U} = +0{,}008$ associated with $\alpha = 0{,}05$ and $z_\alpha = 1{,}645$

The lower acceptance control limit is

$$\begin{aligned} A_{CL\ L} &= A_{PL\ L} - z_\alpha \sigma_{\bar{X}} \\ &= -0{,}008 - 1{,}645 \times \frac{0{,}005}{\sqrt{4}} \\ &= -0{,}012 \end{aligned}$$

The lower rejectable process level associated with a β-risk of 5 % is

$$\begin{aligned} R_{PL\ L} &= A_{CL\ L} - z_\beta \sigma_{\bar{X}} \\ &= -0{,}012 - 1{,}645 \times \frac{0{,}005}{\sqrt{4}} \\ &= -0{,}016 \end{aligned}$$

Similarly,

$$\begin{aligned} A_{CL\ U} &= A_{PL\ U} + z_\alpha \sigma_{\bar{X}} \\ &= 0{,}012 \\ R_{PL\ U} &= A_{CL\ U} + z_\beta \sigma_{\bar{X}} \\ &= 0{,}016 \end{aligned}$$

Interpretation of the results leads to the following conclusions.

a) Strips across the coating which have an average thickness deviating from the average thickness of the entire coating by ±0,008 mm or less will be reasonably sure (95 % or higher) of being accepted for uniformity.

b) Strips across the coating which have an average thickness deviating from the average thickness of the entire coating by ±0,016 mm or more will be reasonably sure (95 % or higher) of being rejected for lack of uniformity.

c) Strips across the coating which have an average thickness deviating from the average thickness of the entire coating by more than ±0,008 mm but less than ±0,016 mm may or may not be rejected for lack of uniformity. These represent thickness deviations which are not so small that they should definitely be accepted nor so large that they should definitely be rejected.

Note that if this "indifference zone" of 0,008 mm to 0,016 mm is considered too wide, it can be reduced by the use of a larger sample size. If $n = 16$ instead of $n = 4$, the acceptance control limits become ±0,010 mm and the RPL values become ±0,012 mm. Or, if instead of demanding a smaller indifference zone, it is decided to demand that a better job be done in obtaining uniform coatings, the APL can be shifted closer to the nominal value. For example, if it is decided that a deviation of ±0,004 mm is as far as the 95 % acceptance protection is to go, then for a sample size of 4, the new acceptance control limits would be ±0,008 mm and the RPL values ±0,012 mm.

10 Factors for acceptance control limits

Acceptance control limit factors are based on one-tail normal distribution probabilities unless the APLs lie within $0{,}85\sigma_W / \sqrt{n}$ of the target level, for $\alpha = 0{,}05$, or within $0{,}67\sigma_W / \sqrt{n}$ for $\alpha = 0{,}01$. These values are the outer bound for situations representing "tight" specification requirements for which the α-risk must be divided appropriately on either side of the target value. The columns in Table 1 give:

a) the multiple of $\sigma_W / \sqrt{n}$; the APL distance from the target level;

b) the multiple of $\sigma_w / \sqrt{n}$; the ACL distance from the target level which is the sum of the APL distance and the corresponding z component for varying degrees of two-sided α-risks;

c) the P_a value for the APL using nomographs similar to Figures A.1 to Figure A.5.

It should be noted that when the difference between APL and the target value is small in units of σ_w, that is, for the "tight" specification situations, the acceptance control chart is not appropriate.

Table 1 — Acceptance control limit factors

$\alpha = 0{,}05$				$\alpha = 0{,}01$			
Difference between APL and target	z	**Difference between ACL and target**	P_a	**Difference between APL and target**	z	**Difference between ACL and target**	P_a
col.1	col. 2	col. 3 (col. 1 + col. 2)	col. 4	col. 5	col. 6	col.7 (col. 5 + col. 6)	col. 8
≥0,85	1,65	≥2,50	0,950	≥0,67	2,33	≥3,00	0,990
0,80	1,65	2,45	0,951	0,60	2,33	2,93	0,990
0,70	1,66	2,36	0,952	0,50	2,33	2,83	0,990
0,60	1,67	2,27	0,953	0,40	2,37	2,77	0,991
0,50	1,68	2,18	0,954	0,30	2,37	2,67	0,991
0,40	1,71	2,11	0,956	0,20	2,41	2,61	0,992
0,30	1,75	2,05	0,960	0,10	2,52	2,62	0,994
0,20	1,80	2,00	0,964	0,00	2,58	2,58	0,995
0,10	1,87	1,97	0,969				
0,00	1,96	1,96	0,975				

NOTE The control limit factors given in this table are for use in locating acceptance and control limit lines:

APL = target value ± (factor[a]) $\left(\sigma_w / \sqrt{n}\right)$

ACL = target value ± (factor[b]) $\left(\sigma_w / \sqrt{n}\right)$

[a] Use the appropriate factor from column 1 or 5.

[b] Use the appropriate factor from column 3 or 7.

11 Modified acceptance control charts

A modified acceptance control chart is a special case of a process acceptance control chart in which its acceptance control limits can be determined in terms of its specification limits as shown in the following equations:

$$A_{PL\ U} = U - z_{p_0} \sigma_w$$

$$A_{PL\ L} = L + z_{p_0} \sigma_w$$

$$A_{CL\ U} = A_{PL\ U} + z_\alpha \sigma_w / \sqrt{n}$$

$$A_{\text{CL L}} = A_{\text{PL L}} - z_{\alpha}\sigma_{\text{w}} / \sqrt{n}$$

The acceptance control limits determined above are located inside the specification limits. The determination procedure is similar to Example 1 shown in 9.1; however, it does not define the β-risk for specified rejectable process levels nor does it provide rules for determining the sample size.

Annex A
(normative)

Nomographs for acceptance control chart design

A.1 General

Instead of a computation procedure, a nomograph procedure can be used for designing an acceptance control chart. This approach includes the advantage of gaining easy access to any information on the accompanying OC curve.

A.2 Acceptance control charts for process mean, μ

The nomograph paper used with (approximately) normally distributed processes is given in Figure A.1. Assigning a linear scale to the horizontal axis, any OC curve (probability of acceptance P_a as a function of the process mean μ) may be represented as a straight line by appropriate choice of probability scale for the vertical axis.

The principle of the one-sided procedure is presented in Figure A.2. The OC curve is represented by a straight line. For $\alpha = \beta$, the acceptance control limit is equal to the value of μ that yields a probability of acceptance P_a = 0,5 or 50 %. The slope of the OC curve depends upon the scale chosen for the horizontal axis and upon the process standard deviation σ and is related to the sample size n. The interrelation between these parameters is represented by the dashed line parallel to the OC curve. This dashed line is needed for the control chart design. Besides the process standard deviation, σ, there are four parameters in the design:

a) the acceptable process level at probability of acceptance $P_a = 1 - \alpha$, $\mu_{APL} = A_{PL}$;

b) the rejectable process level at probability of acceptance $P_a = \beta$; $\mu_{RPL} = R_{PL}$;

c) the acceptance control limit, $\mu_{ACL} = A_{CL}$;

d) the sample size, n.

If any two of these four parameters are given, the remaining two parameters can be deduced. The following examples illustrate the procedures in detail:

EXAMPLE 1 (see Figure A.3 and Figure A.4)

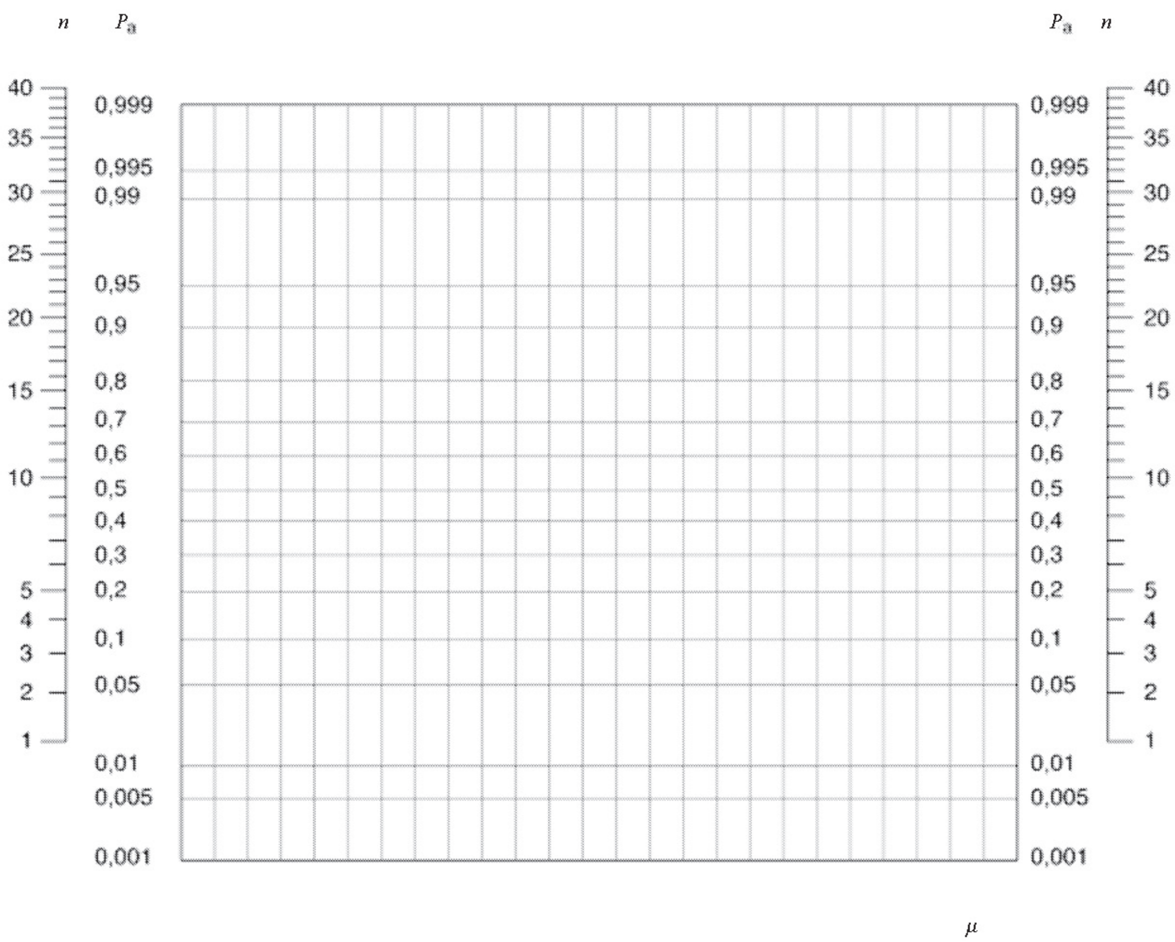
Given: APL with $P_a = 1 - \alpha$

RPL with $P_a = \beta$

EXAMPLE 2 (see Figure A.5)

Given: APL with $P_a = 1 - \alpha$

n; σ



Key

P_a probability of acceptance

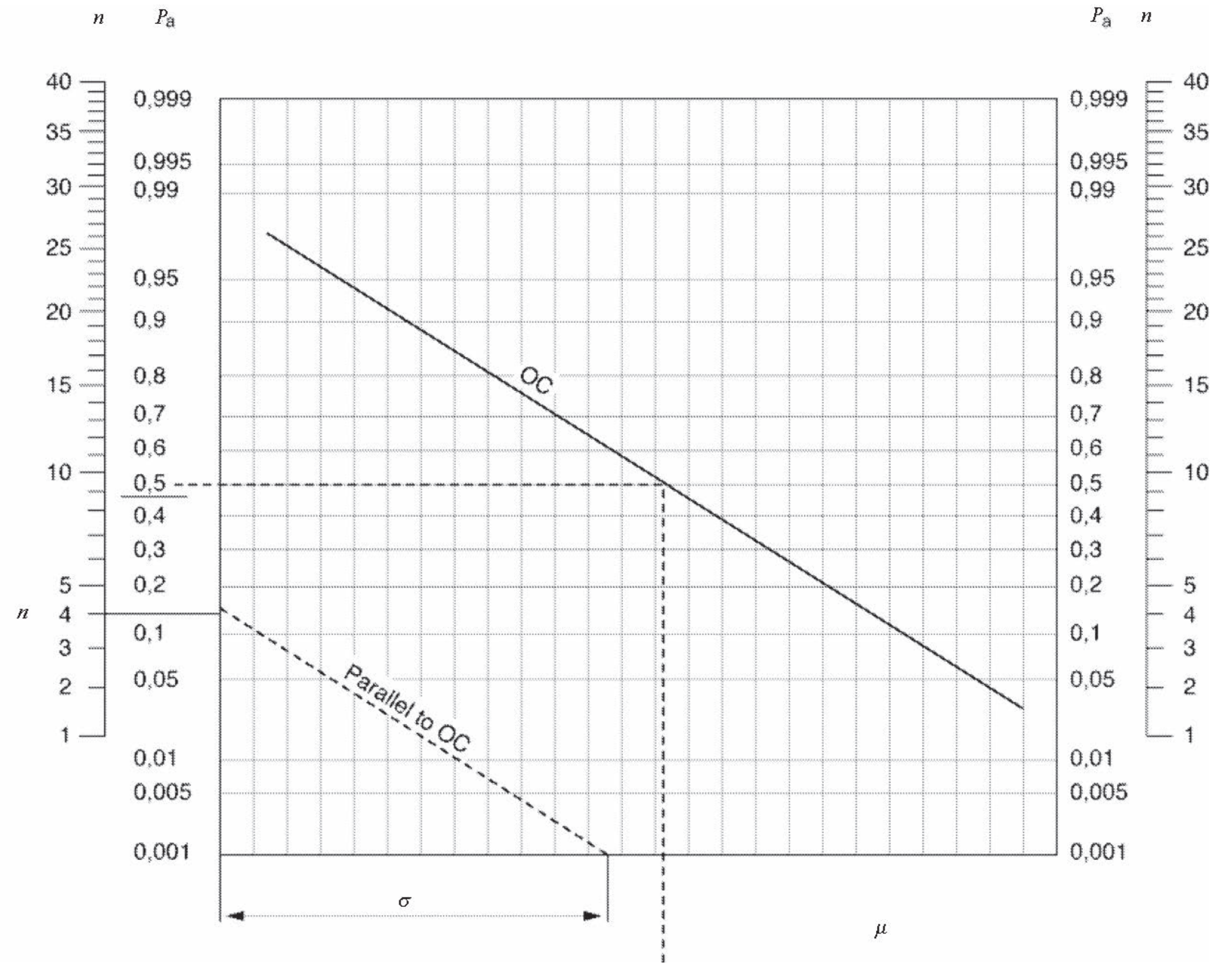
μ process mean

n sample size

σ standard deviation (inherent variability)

$P_a = P_a(\mu)$

Figure A.1 — Nomograph paper for acceptance control chart design



Key

P_a probability of acceptance

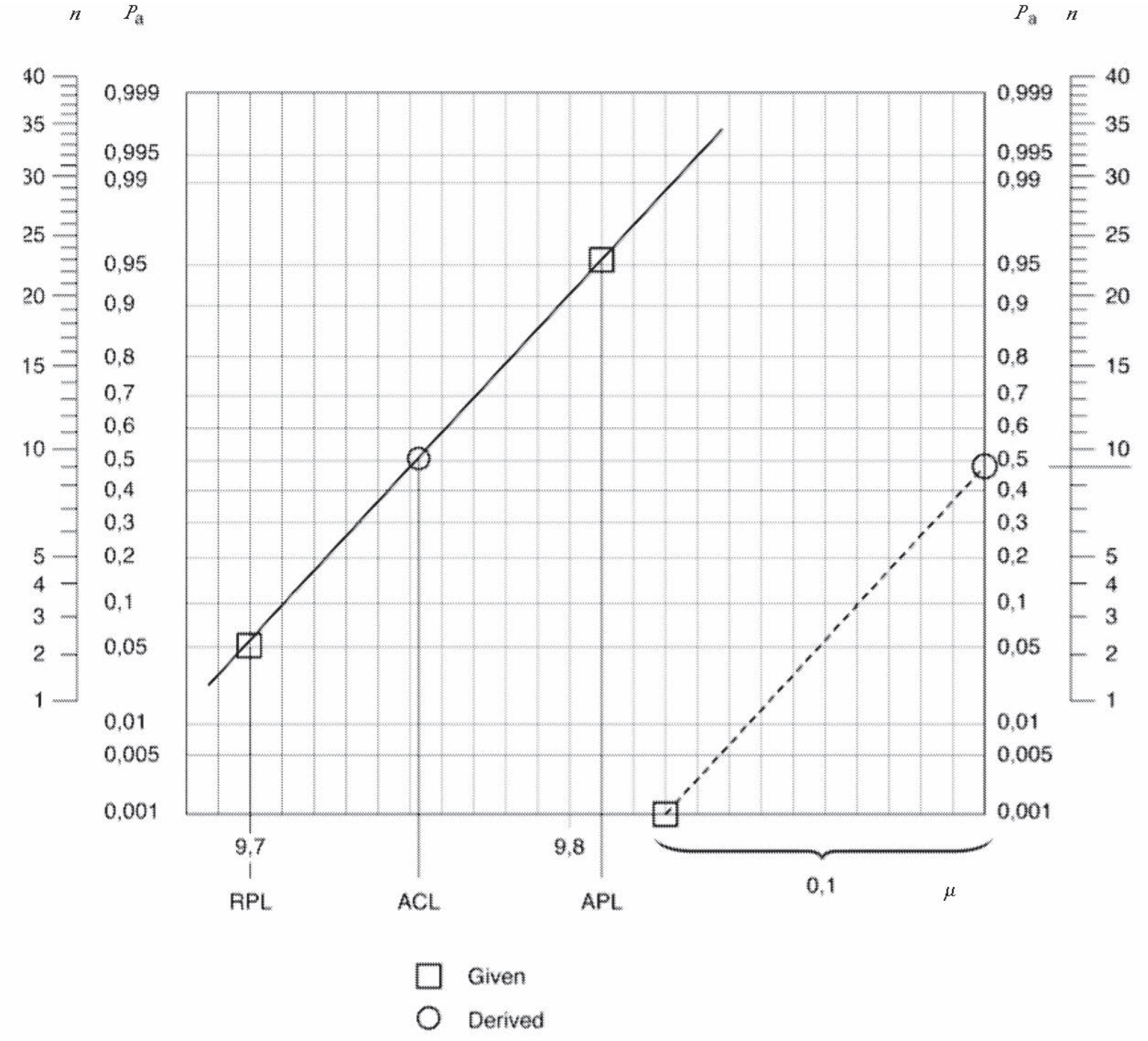
μ process mean

n sample size

σ standard deviation (inherent variability)

$P_a = P_a(\mu)$

Figure A.2 — Acceptance control chart design — One-sided approach



Key

P_a probability of acceptance

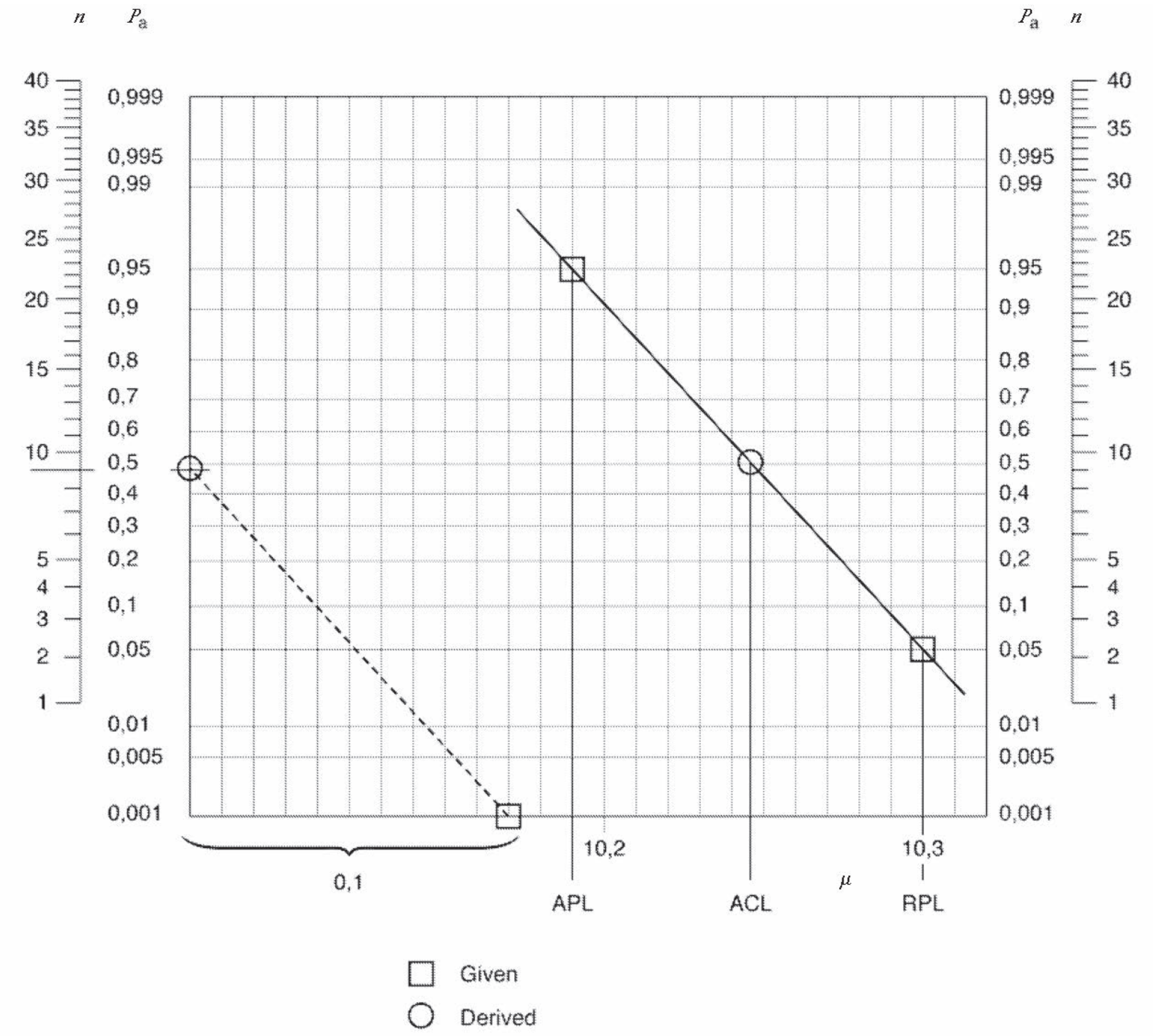
μ process mean

n sample size

σ standard deviation (inherent variability)

$P_a = P_a(\mu)$

Figure A.3 — Acceptance control chart design — Example 1



Key

P_a probability of acceptance

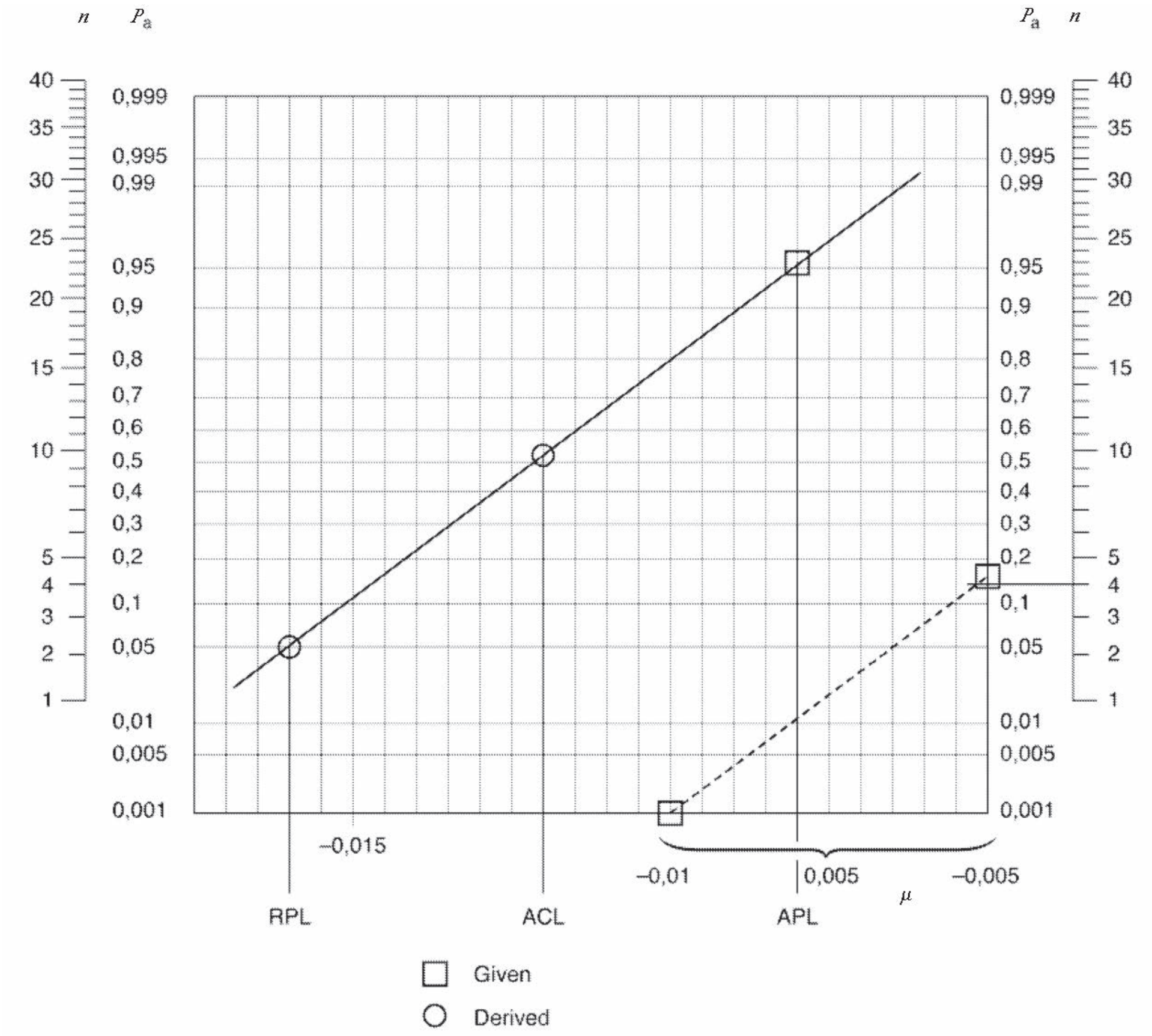
μ process mean

n sample size

σ standard deviation (inherent variability)

$P_a = P_a(\mu)$

Figure A.4 — Acceptance control chart design — Example 1



Key

P_a probability of acceptance

μ process mean

n sample size

σ standard deviation (inherent variability)

$P_a = P_a(\mu)$

Figure A.5 — Acceptance control chart design — Example 2

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