BRITISH STANDARD

Measurement of fluid flow — Procedures for the evaluation of uncertainties

ICS 17.120.10

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British Standards

National foreword

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The UK participation in its preparation was entrusted by Technical Committee CPI/30, Measurement of fluid flow in closed conduits, to Subcommittee CPI/30/9, General topics, which has the responsibility to:

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Summary of pages

This document comprises a front cover, an inside front cover, the ISO title page, pages ii to v, a blank page, pages 1 to 65 and a back cover.

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Amendments issued since publication

Amd. No.	Date	Comments

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 10 August 2005

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ISBN 0 580 46279 X

INTERNATIONAL STANDARD

ISO 5168

Second edition
2005-06-15

Measurement of fluid flow — Procedures for the evaluation of uncertainties

Mesure de débit des fluides — Procédures pour le calcul de l'incertitude

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Foreword

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ISO 5168 was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, Subcommittee SC 9, *General topics*.

This second edition of ISO 5168 cancels and replaces ISO/TR 5168:1998, which has been technically revised (see Annex I).

Introduction

Whenever a measurement of fluid flow (discharge) is made, the value obtained is simply the best estimate that can be obtained of the flow-rate or quantity. In practice, the flow-rate or quantity could be slightly greater or less than this value, the uncertainty characterizing the range of values within which the flow-rate or quantity is expected to lie, with a specified confidence level.

GUM is the authoritative document on all aspects of terminology and evaluation of uncertainty and should be referred to in any situation where this International Standard does not provide enough depth or detail. In particular, GUM (1995), Annex F, gives guidance on evaluating uncertainty components.

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Measurement of fluid flow — Procedures for the evaluation of uncertainties

1   Scope

This International Standard establishes general principles and describes procedures for evaluating the uncertainty of a fluid flow-rate or quantity.

A step-by-step procedure for calculating uncertainty is given in Annex A.

2   Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 9300, *Measurement of gas flow by means of critical flow Venturi nozzles*

ISO Guide to the expression of uncertainty in measurement (*GUM*), 1995

International vocabulary of basic and general terms in metrology (*VIM*), 1993

3   Terms and definitions

For the purposes of this document, the terms and definitions given in VIM (1993), GUM (1995) and the following apply.

3.1
uncertainty
parameter, associated with the results of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

NOTE   Uncertainties are expressed as an absolute value and do not take a positive or negative sign.

3.2
standard uncertainty
$u(x)$
uncertainty of the result of a measurement expressed as a standard deviation

3.3
relative uncertainty
$u^*(x)$
standard uncertainty divided by the best estimate

NOTE 1 $u^*(x) = u(x)/x$.

NOTE 2 $u^*(x)$ can be expressed either as a percentage or in parts per million.

NOTE 3   Relative uncertainty is sometimes referred to as dimensionless uncertainty.

NOTE 4   The best estimate is in most cases the arithmetic mean of the related uncertainty interval.

3.4
combined standard uncertainty
$u_c(y)$
standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities

3.5
relative combined uncertainty
$u_c^*(y)$
combined standard uncertainty divided by the best estimate

NOTE 1 $u_c^*(y)$ can be expressed as a percentage or parts per million.

NOTE 2 $u_c^*(y) = u_c(y)/y$.

NOTE 3   Relative combined uncertainty is sometimes referred to as dimensionless combined uncertainty.

NOTE 4   The best estimate is in most cases the arithmetic mean of the related uncertainty interval.

3.6
expanded uncertainty
U
quantity defining an interval about the result of a measurement that can be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

NOTE 1   The fraction can be viewed as the coverage probability or the confidence level of the interval.

NOTE 2 $U = ku_c(y)$

3.7
relative expanded uncertainty
U^*
expanded uncertainty divided by the best estimate

NOTE 1 U^* can be expressed as a percentage or in parts per million.

NOTE 2 $U^* = ku_c^*(y)$.

NOTE 3   Relative expanded uncertainty is sometimes referred to as dimensionless expanded uncertainty.

NOTE 4   The best estimate is in most cases the arithmetic mean of the related uncertainty interval.

3.8
coverage factor
k
numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty

NOTE   A coverage factor is typically in the range 2 to 3.

3.9
Type A evaluation
⟨uncertainty⟩ method of evaluation of uncertainty by the statistical analysis of a series of observations

3.10
Type B evaluation
⟨uncertainty⟩ method of evaluation of uncertainty by means other than the statistical analysis of a series of observations

3.11
sensitivity coefficient
c_i
change in the output estimate, y, divided by the corresponding change in the input estimate, x_i

3.12
relative sensitivity coefficient
c_i^*
relative change in the output estimate, y, divided by the corresponding relative change in the input estimate, x_i

4   Symbols and abbreviated terms

4.1   Symbols

a_i	estimated semi-range of a component of uncertainty associated with input estimate, x_i, as defined in Annex B
A_t	area of the throat
b_i	breadth associated with a vertical i
b'_i	upper bound of an asymmetric uncertainty distribution as defined in Annex B
c_i	sensitivity coefficient used to multiply the uncertainty in the input estimate, x_i, to obtain the effect of a change in the input quantity on the uncertainty of the output estimate, y
c_i^*	relative sensitivity coefficient used to multiply the relative uncertainty in input estimate, x_i, to obtain the effect of a relative change in the input quantity on the relative uncertainty of the output estimate, y
C_c	calibration coefficient
C	discharge coefficient
C_V	coefficient of variation
d_i	depth associated with a vertical i
d_o	orifice diameter
$d_{o,0}$	orifice diameter measured at temperature $T_{0,x}$
d_p	pipe diameter
$d_{p,0}$	pipe diameter measured at temperature $T_{0,x}$
$\overline{E}$	mean meter error, expressed as a fraction

E_j	jth meter error, expressed as a fraction
f	functional relationship between estimates of the measurand, y, and the input estimates, x_i, on which y depends
$\dfrac{\partial f}{\partial x_i}$	partial derivative with respect to input quantity, x_i, of the functional relationship, f, between the measurand and the input quantities
F	flow factor, equal to $\dfrac{q}{\sqrt{\Delta p_r}}$
F_{exp}	flow factor for a new design
F_{Redp}	$\left(19\,000 \cdot \beta/\mathrm{Re}_{dp}\right)^{0,8}$
F_{ref}	reference flow factor
F_s	factor, assumed to be unity, that relates the discrete sum over the finite number of verticals to the integral of the continuous function over the cross-section
k	coverage factor used to calculate the expanded uncertainty, U
k_t	coverage factor derived from a table; see D.12
K	meter factor
$\overline{K}$	mean meter factor
K_j	jth K-factor;
l_b	length of crest
l_h	gauged head
l_1	distance from the upstream tapping to the upstream face
L_1	l_1 divided by the pipe diameter, d_p
l'_2	distance from the downstream tapping to the downstream face
L'_2	l'_2 divided by the pipe diameter, d_p
m	particular item in a set of data
m'	number of data sets to be pooled
m''	number of verticals
M'_2	$2L'_2/(1-\beta)$
n	number of repeat readings or observations
n'	exponent of l_h, usually 1,5 for a rectangular weir and 2,5 for a V-notch

n''	number of depths in a vertical at which velocity measurements are made
N	number of input estimates, x_i, on which the measurand depends
p_0	upstream pressure
Δp_{mt}	pressure difference across the orifice meter
Δp_r	pressure difference across the radiator
$P(a_i)$	probability that an input estimate, x_i, has a value of a_i
q	volume flow-rate
q_{ma}	mass flow;
Q	flow, expressed in cubic metres per second, at flowing conditions
R	specific gas constant
Re_{dp}	Reynolds number related to d_p by the expression $V d_p \rho / \mu$
$s_{mt,po}$	pooled experimental standard deviation of the orifice plate readings
s_{pe}	standard deviation of a larger set of data used with a smaller data set
s_{po}	standard deviation pooled from several sets of data
$s_{r,po}$	pooled experimental standard deviation for the radiator readings
$s(x)$	experimental standard deviation of a random variable, x, determined from n repeated observations
$s(\bar{x})$	experimental standard deviation of the arithmetic mean, $\bar{x}$
t	Student's statistic
T_0	upstream absolute temperature
$T_{0,x}$	temperature at which measurement x is made
T_{op}	operating temperature
$u_{c,corr}(y)$	combined uncertainty for those components for multiple meters that are correlated
$u_{c,uncorr}(y)$	combined uncertainty for those components for multiple meters that are uncorrelated
u^{*}_{cal}	instrument calibration uncertainty from all sources, formerly called systematic errors or biases
u^{*}_{cri}	relative uncertainty in point velocity at a particular depth in vertical i due to the variable responsiveness of the current meter
u^{*}_{d}	relative standard uncertainty in the coefficient of discharge

u_{ei}^*	relative uncertainty in point velocity at a particular depth in vertical i due to velocity fluctuations (pulsations) in the stream
u_{lb}^*	relative standard uncertainty in the measurement of the crest length
u_{lh}^*	relative standard uncertainty in the measurement of the gauged head
$u_{m''}^*$	relative uncertainty due to the limited number of verticals
u_{pi}^*	relative uncertainty in mean velocity, V_i, due to the limited number of depths at which velocity measurements are made at vertical, i
$u^*(Q)$	combined relative standard uncertainty in the discharge;
u_{sm}	standard uncertainty of a single value based on past experience
$u(x_{i,corr})$	correlated components of uncertainty in a single meter
$u(x_{i,uncorr})$	uncorrelated components of uncertainty in a single meter
$u^*(x_i)$	standard uncertainty associated with the input estimate, x_i
$u_c^*(y)$	combined standard uncertainty associated with the output estimate, y
$u^*(x_i)$	relative standard uncertainty associated with the input estimate x_i
$u_c^*(y)$	combined relative standard uncertainty associated with the output estimate, y
$U^*(y)$	relative expanded uncertainty associated with the output estimate
$U(y)$	expanded uncertainty associated with the output estimate, y
U_{CMC}	combined uncertainty of the calibration rig
$U_{AS\text{-}overall\text{-}E}$	type A uncertainty in meter error
$U_{AS\text{-}overall\text{-}K}^*$	type A uncertainty in the K-factor
V	mean velocity in the pipe
V_i	mean velocity associated with a vertical i
x_i	estimate of the input quantity, X_i
x_m	mth observation of random quantity, x
x_0	dimension at temperature T_0, x
$\bar{x}$	arithmetic mean or average of n repeated observations, x_m, of randomly varying quantity, x
y	estimate of the measurand, Y
Δx_i	increment in x_i used for numerical determination of sensitivity coefficient

Δy	increment in y found in numerical determination of sensitivity coefficient
Z_n	Grubbs test statistic for outliers
β	orifice plate diameter ratio, equal to d_o/d_p
φ_{cf}	critical flow function
Φ_F	ratio of the factor F for a new design compared to the old design
λ	expansion coefficient
μ	dynamic fluid viscosity
ρ	fluid density
v	degrees of freedom
v_{eff}	effective degrees of freedom
v_{po}	degrees of freedom associated with a pooled standard deviation

4.2 Subscripts

c	combined
corr	correlated
do	orifice diameter
dp	pipe diameter, effective
ex	external
i	of the ith input
j	of the jth set
$k = 2$	obtained with a coverage factor of 2
m	of the mth observation
n	of the nth observation
N	of the Nth input
nom	nominal value of
op	operating temperature
pe	from past experience
po	pooled
sm	based on a single measurement
t	tolerance interval

uncorr	uncorrelated
x	of x
$\bar{x}$	of the mean value of x
95	with a 95 % confidence level

5 Evaluation of the uncertainty in a measurement process

The first stage in an uncertainty evaluation is to define the measurement process. For the measurement of flow-rate, it will normally be necessary to combine the values of a number of input quantities to obtain a value for the output. The definition of the process should include the enumeration of all the relevant input quantities.

Annex E enumerates a number of categories of sources of uncertainty. This categorization can be of value when defining all of the sources of uncertainty in the process. It is assumed in the following sections that the sources of uncertainty are uncorrelated; correlated sources require different treatment (see Annex F).

Consideration should also be given to the time over which the measurement is to be made, taking into account that flow-rate will vary over any period of time and that the calibration can also change with time.

If the functional relationship between the input quantities X_1, X_2, ..., X_N, and output quantity Y in a flow measurement process is specified in Equation (1):

$$Y = f\left(X_1, X_2, ..., X_N\right)$$ (1)

then an estimate of Y, denoted by y, is obtained from Equation (1) using input estimates x_1, x_2, ... x_N, as shown in Equation (2):

$$y = f\left(x_1, x_2, ..., x_N\right)$$ (2)

Provided the input quantities, X_i, are uncorrelated, the total uncertainty of the process can be found by calculating and combining the uncertainty of each of the contributing factors in accordance with Equation (3):

$$u_c\left(y\right) = \sqrt{\sum_{i=1}^{N}\left[c_i u\left(x_i\right)\right]^2}$$ (3)

Where the extent of interdependence is known to be small, Equation (3) may be applied even though some of the input quantities are correlated; ISO 5167-1:2003 [1] provides an example of this.

Each of the individual components of uncertainty, $u(x_i)$, is evaluated using one of the following methods:

— Type A evaluation: calculated from a series of readings using statistical methods, as described in Clause 6;

— Type B evaluation: calculated using other methods, such as engineering judgement, as described in Clause 7.

Uncertainty sources are sometimes classified as "random" or "systematic" and the relationship between these categorizations and Type A and Type B evaluations is given in Annex I.

The sensitivity coefficients, c_i, provide the links between uncertainty in each input and the resulting uncertainty in the output. The methods of calculating the individual sensitivity coefficients, c_i, are described in detail in Clause 8.

6 Type A evaluations of uncertainty

6.1 General considerations

Type A evaluations of uncertainty are those using statistical methods, specifically, those that use the spread of a number of measurements.

Whilst no correction can be made to remove random components of uncertainty, their associated uncertainty becomes progressively less as the number of measurements increases. In taking a series of measurements, it should be recognized that, as the purpose is to define the random fluctuations in the process, the timescale for the data collection should reflect the anticipated timescale for the fluctuations. Collecting readings at millisecond intervals for a process that fluctuates over several minutes will not characterize those fluctuations adequately.

In many measurement situations, it is not practical to make a large number of measurements. In this case, this component of uncertainty may have to be assigned on the basis of an earlier Type A evaluation, based on a larger number of readings carried out under similar conditions. Caution should be exercised in making these estimates (see Annex D), as there will always be some uncertainty associated with the assumption that the earlier measurements were taken under truly similar conditions.

The methods of calculating the uncertainty in a mean and in a single value reflect the reduction in uncertainty obtained by averaging several readings [Equations (4) to (8)] and are explained in more detail in D.4 to D.6.

6.2 Calculation procedure

Further explanation of the equations given below can be found in Annex D.

The standard uncertainty of a measured value, x_i, is calculated from a sample of measurements, $x_{i,m}$, in accordance with Equations (4) to (8):

a) Calculate the average value of the measurements in accordance with Equation (4); see D.1:

$$\overline{x}_i = \frac{1}{n}\sum_{m=1}^{n} x_{i,m} \tag{4}$$

b) Calculate the standard deviation of the sample in accordance with Equation (5); see D.2:

$$s(x_i) = \sqrt{\frac{1}{(n-1)}\sum_{m=1}^{n}\left(x_{i,m} - \overline{x}_i\right)^2} \tag{5}$$

The standard uncertainty of a single sample is the same as its standard deviation and is given by Equation (6):

$$u(x_i) = s(x_i) \tag{6}$$

c) Calculate the standard deviation of the mean value in accordance with Equation (7); see D.4:

$$s(\overline{x}_i) = \frac{s(x_i)}{\sqrt{n}} \tag{7}$$

The standard uncertainty of the mean value is then given by Equations (8):

$$u(\overline{x}_i) = s(\overline{x}_i) \tag{8}$$

The use of the mean of several readings is a key technique for reducing uncertainty in readings subject to random variations. For the derivation of Equation (7) see Dietrich [2].

NOTE   The approach outlined here represents a simplification and, when the functional relationship defined by Equation (1) is highly non-linear and uncertainties are large, the more rigorous approach described in the GUM (1995), 4.1.4, could yield a more robust answer.

7   Type B evaluation of uncertainties

7.1   General considerations

Type B evaluations of uncertainty are those carried out by means other than the statistical analysis of series of observations.

As explained in D.9, Type A uncertainties result in a bandwidth of 1 standard deviation that would encompass 68 % of the possible values of the measured quantity. In making Type B assessments, it is necessary to ensure that a similar confidence level is obtained such that the uncertainties obtained by different methods can be compared and combined.

Type B assessments are not necessarily governed by the normal distribution and the limits assigned can represent varying confidence levels. Thus, a calibration certificate could give the meter factor for a turbine meter with 95 % confidence while an instrument resolution uncertainty defines with 100 % confidence the range of values that will be represented by that number rather than the next higher or lower. The equations for obtaining the standard uncertainty for various common distributions are given in 7.3 to 7.8.

7.2   Calculation procedure

Type B evaluations of uncertainty require a knowledge of the probability distribution associated with the uncertainty. The most common probability distributions are presented in 7.3 to 7.8; the shapes of the distributions are shown in Annex B.

7.3   Rectangular probability distribution

Typical examples of rectangular probability distributions include

— maximum instrument drift between calibrations,

— error due to limited resolution of an instrument's display,

— manufacturers' tolerance limits.

The standard uncertainty of a measured value, x_i, is calculated from Equation (9):

$$u\left(x_i\right) = \frac{a_i}{\sqrt{3}}$$

(9)

where the range of measured values lies between $x_i - a_i$ and $x_i + a_i$. The derivation of Equation (9) is given by Dietrich [2].

7.4 Normal probability distribution

Typical examples include calibration certificates quoting a confidence level or coverage factor with the expanded uncertainty. Here, the standard uncertainty is calculated from Equation (10):

$$u(x_i) = \frac{U}{k} \tag{10}$$

where

U is the expanded uncertainty;

k is the quoted coverage factor; see Annex C.

Where a coverage factor has been applied to a quoted expanded uncertainty, care should be exercised to ensure that the appropriate value of k is used to recover the underlying standard uncertainty. However, if the coverage factor is not given and the 95 % confidence level is quoted, then k should be assumed to be 2.

7.5 Triangular probability distribution

Some uncertainties are given simply as maximum bounds within which all values of the quantity are assumed to lie. There is often reason to believe that values close to the bounds are less likely than those near the centre of the bounds, in which case the assumption of rectangular distribution could be too pessimistic. In this case, the triangular distribution, as given by Equation (11), may be assumed as a prudent compromise between the assumptions of a normal and a rectangular distribution.

$$u(x_i) = \frac{a_i}{\sqrt{6}} \tag{11}$$

7.6 Bimodal probability distribution

When the error is always at the extreme value, then a bimodal probability distribution is applicable and the standard uncertainty is given by Equation (12):

$$u(x_i) = a_i \tag{12}$$

Examples of this type of distribution are rare in flow measurement.

7.7 Assigning a probability distribution

When the source of the uncertainty information is well defined, such as a calibration certificate or a manufacturer's tolerance, the choice of probability distribution will be clear-cut. However, when the information is less well defined, for example when assessing the impact of a difference between the conditions of calibration and use, the choice of a distribution becomes a matter of the professional judgement of the instrument engineer.

7.8 Asymmetric probability distributions

The above cases are for symmetrical distributions, however, it is sometimes the case that the upper and lower bounds for an input quantity, X_i, are not symmetrical with respect to the best estimate, x_i. In the absence of information on the distribution, GUM recommends the assumption of a rectangular distribution with a full range equal to the range from the upper to the lower bound. The standard uncertainty is then given by Equation (13):

$$u(x_i) = \frac{a_i + b_i'}{\sqrt{12}} \tag{13}$$

where $(x_i - a_i) < X_i < (x_i + b_i')$.

A more conservative approach would be to take a rectangular distribution based on the larger of two asymmetric bounds.

$$u(x_i) = \text{the greater of } \frac{a_i}{\sqrt{3}} \text{ or } \frac{b_i'}{\sqrt{3}} \tag{14}$$

If the asymmetric element of uncertainty represents a very significant proportion of the overall uncertainty, it would be more appropriate to consider an alternative approach to the analysis such as a Monte Carlo analysis; see Annex K.

A common example of an asymmetric distribution is seen in the drift of instruments due to mechanical changes, for example, increasing friction in the bearings of a turbine meter or erosion of the edge of an orifice plate.

8   Sensitivity coefficients

8.1   General

Before considering methods of combining uncertainties, it is essential to appreciate that it is insufficient to consider only the magnitudes of component uncertainties in input quantities, it is also necessary to consider the effect each input quantity has on the final result. For example, an uncertainty of 50 μm in a diameter or 5 % in a thermal expansion coefficient is meaningless in terms of the flow through an orifice plate without knowledge of how the diameter or thermal expansion impact the measurement of flow-rate. It is, therefore, convenient to introduce the concept of the sensitivity of an output quantity to an input quantity, i.e. the sensitivity coefficient, sometimes referred to as the influence coefficient.

The sensitivity coefficient of each input quantity is obtained in one of two ways:

— analytically; or

— numerically.

8.2   Analytical solution

When the functional relationship is specified as in Equation (1), the sensitivity coefficient is defined as the rate of change of the output quantity, y, with respect to the input quantity, x_i, and the value is obtained by partial differentiation in accordance with Equation (15):

$$c_i = \frac{\partial y}{\partial x_i} \tag{15}$$

However, when non-dimensional uncertainties (for example percentage uncertainty) are used, non-dimensional sensitivity coefficients shall also be used in accordance with Equation (16):

$$c_i^\star = \frac{\partial y}{\partial x_i} \frac{x_i}{y} \tag{16}$$

In certain special cases where, for example, a calibration experiment has made the functional relationship between the input and output simple, the value of c_i or $c_i^\star$ can be unity. Example 1 in Annex G gives an example for a calibrated nozzle.

8.3   Numerical solution

Where no mathematical relationship is available, or the functional relationship is complex, it is easier to obtain the sensitivity coefficients numerically, by calculating the effect of a small change in the input variable, x_i, on the output value, y.

First calculate y using x_i, and then recalculate using $(x_i + \Delta x_i)$, where Δx_i is a small increment in x_i. The result of the recalculation can be expressed as $y + \Delta y$, where Δy is the increment in y caused by Δx_i.

The sensitivity coefficients are then calculated in accordance with Equation (17):

$$c_i \approx \frac{\Delta y}{\Delta x_i} \tag{17}$$

They are calculated in non-dimensional, or relative, form in accordance with Equation (18):

$$c_i^* \approx \frac{\Delta y}{\Delta x_i}\frac{x_i}{y} \tag{18}$$

Table 1 shows how a typical spreadsheet could be set up to calculate a specific sensitivity coefficient for any function where $y = f(x_1, x_2, .. , x_N)$.

Table 1 — Spreadsheet set-up for calculating sensitivity coefficients

Sensitivity coefficient	Increment	x_1	x_2	...	x_i	x_N	y	c	c^*
—	—	x_1	x_2	...	x_i	x_N	$y = f(x_1, x_2,...., x_N) = y_{nom}$	—	—
c_1	$\Delta x_i \approx 10^{-6} \cdot x_1$	$x_i + \Delta x_i$	x_2	...	x_i	x_N	$y_i = f(x_1 + \Delta x_i, x_2,...., x)$	$\dfrac{(y_1 - y_{nom})}{\Delta x_1}$	$c_1 \cdot \dfrac{x_1}{y_{nom}}$

The analytical solution calculates the gradient of y with respect to x_i at the nominal value, x_i, whereas the numerical solution obtains the average gradient over the interval x_i to $(x_i + \Delta x_i)$. The increment used (Δx_i) should therefore be as small as practical and certainly no larger than the uncertainty in the parameter x_i. However, a complication can arise if the increment is so small as to result in changes in the calculated result, y, that are comparable with the resolution of the calculator or computer spreadsheet. In these circumstances the calculation of c_i can become unstable. The problem can be avoided by starting with a value of Δx_i equal to the uncertainty in x_i and progressively reducing Δx_i until the value of c_i agrees with the previous result within a suitable tolerance. This iteration process can, of course, be automated with a computer spreadsheet.

9   Combination of uncertainties

Once the standard uncertainties of the input quantities and their associated sensitivity coefficients have been determined from either Type A or Type B evaluations, the overall uncertainty of the output quantity can be determined in accordance with Equation (19):

$$u_c(y) \ = \ \sqrt{\sum_{i=1}^{N}\left[c_i\, u(x_i)\right]^2} \tag{19}$$

Where relative uncertainties have been used, relative sensitivity coefficients shall also be used, in accordance with Equation (20):

$$u_c^*(y) \ = \ \sqrt{\sum_{i=1}^{N}\left[c_i^*\, u^*(x_i)\right]^2} \tag{20}$$

Equations (19) and (20) assume that the individual input quantities are uncorrelated; the treatment of correlated uncertainties is discussed in C.6. Correlation arises where the same instrument is used to make several measurements or where instruments are calibrated against the same reference.

In general, the choice of absolute or relative uncertainties is of little consequence. However, once the decision has been made, care is needed to ensure that all uncertainties are expressed in the same terms. Measurements with arbitrary zero points give rise to problems if uncertainties are expressed in relative terms. For example, an uncertainty of 1 mm in a diameter of 500 mm gives a relative uncertainty of 0,2 % and, if expressed in inches, the uncertainty becomes 0,039 4 in out of 19,69 in, leaving the relative uncertainty unchanged. However, if the uncertainty in a temperature of 20 °C is 0,5 °C, the relative uncertainty is 2,5 %, but by expressing the values in degrees Fahrenheit, the temperature becomes 68 °F and the uncertainty becomes 0,9 °F, giving a relative uncertainty of 1,3 %. Relative uncertainties cannot be used in these circumstances and absolute uncertainties should be used. A relative uncertainty can only be used when it is based on a measurement that is used to calculate the end result.

10 Expression of results

10.1 Expanded uncertainty

In Equations (19) and (20), the overall result is obtained from a summation of the contributions of the standard uncertainty of each input source to the uncertainty of the result. The resulting combined uncertainty is, therefore, a standard uncertainty; by referring to Figure 1, it can be seen that, with an effective k factor of 1, the bandwidth defined by a standard uncertainty will only have a confidence level of about 68 % associated with it. There is, therefore, a 2:1 chance that the true value will lie within the band, or a 1 in 3 chance that it will lie outside the band. Such odds are of little value in engineering terms and the normal requirement is to provide an uncertainty statement with 90 % or 95 % confidence level; in some extreme cases, 99 % or higher might be required. To obtain the desired confidence level, an expanded uncertainty, U, is used in accordance with Equation (21):

$$U = k u_c (y)$$

(21)

or, where relative uncertainties are being used, in accordance with Equation (22):

$$U^* = k u_c^* (y)$$

(22)
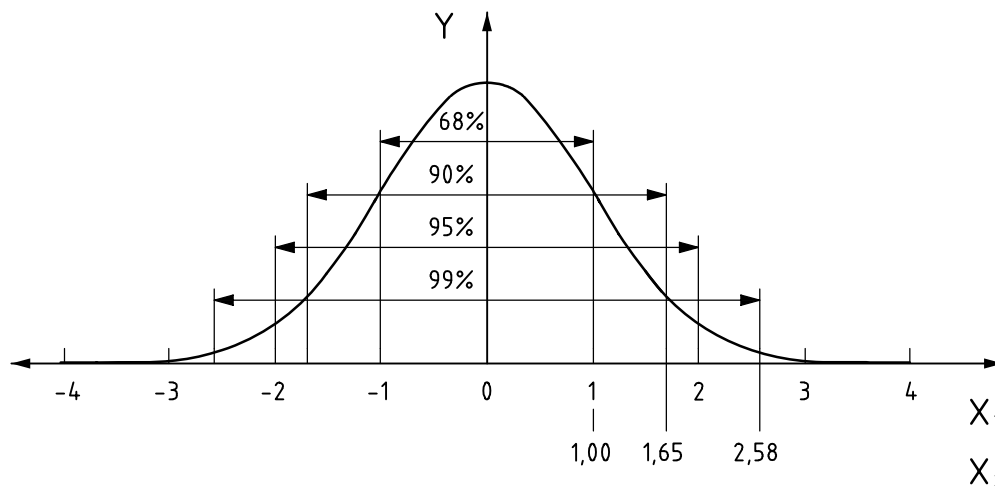
Key

X_1 standard deviation
X_2 coverage factor
Y percent of readings in bandwidth

Figure 1 — Coverage factors for different levels of confidence with the normal, or Gaussian, distribution

It is recommended that for most applications a coverage factor, $k = 2$, be utilized to provide a confidence level of approximately 95 %; the choice of coverage factor will depend on the requirement of the application. Values of k for various levels of confidence are given in Table 2.

Table 2 — Coverage factors for different levels of confidence with the normal, or Gaussian, distribution

Confidence level, %	68,27	90,00	95,00	95,45	99,00	99,73
Coverage factor, k	1,000	1,645	1,960	2,000	2,576	3,000

If the random contribution to uncertainty is large compared with the other contributions and the number of readings is small, the above method provides an optimistic coverage factor. In this case, the procedure outlined in Annex C should be used to estimate the actual coverage factor. A criterion that can be used to determine whether the procedure described in Annex C should be applied is as follows.

Generally, if an uncertainty evaluation involves only one Type A evaluation and that Type A standard uncertainty is less than half the combined standard uncertainty, there is no need to use the method described in Annex C to determine a value for the coverage factor, provided that the number of observations used in the Type A evaluation is greater than 2.

The uncertainty associated with an expanded uncertainty can be denoted using subscripts.

EXAMPLE U_{95} or $U_{k\,=\,2}$.

10.2 Uncertainty budget

In reports providing an uncertainty estimate, an uncertainty budget table should be presented, (or referenced) providing at least the information set out in Table 3.

Table 3 — Uncertainty budget

Symbol	Source of uncertainty	Input uncertainty	Probability distribution	Divisor [see Equations (9) to (14)]	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Contribution to overall uncertainty $[c_i\,u(x_i)]^2$
$u(x_1)$	e.g. calibration	5	Normal	2	2,5	0,5	1,56
$u(x_2)$	e.g. output resolution	1	Rectangular	$\sqrt{3}$	0,58	2,0	1,35
…							
$u(x_i)$							
$u(x_N)$							
u_c	Combined uncertainty	—	—	—	$u_c(y) = \sqrt{\Sigma}$	←a	$= \sum [c_i\,u(x_i)]^2$
U	Expanded uncertainty	$= k\,u_c(y)$	←a	k	↵ⅼa	—	—

a   The arrows in the last two rows of the table indicate that, whereas in the upper rows the calculation proceeds from left to right, in these rows the calculation of the final expanded uncertainty proceeds from right to left.

Table 3 is presented here in absolute terms and each input and corresponding standard uncertainty will have the units of the appropriate input parameter. The table may, equally validly, be presented in relative terms, in which case all input and resulting standard uncertainties will be in percentages or parts per million. Where the inputs are all standard uncertainties, the columns headed "Input uncertainty," "Probability distribution" and "Divisor" may be omitted.

If the computation of a combined uncertainty is in response to a requirement for a test result to have a specified level of uncertainty and the analysis shows that level to be exceeded, the budget table can be of particular value in identifying the largest sources of uncertainty as an indicator of the problem areas which should be addressed.

After the expanded uncertainty has been calculated for a minimum confidence level of 95 %, the measurement result should be stated as follows.

— "The result of the measurement is [value]."

— "The uncertainty of the result is [value] (expressed in absolute or relative terms as appropriate)."

— "The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor $k = 2$, providing a confidence level of approximately 95 %."

In cases where the procedure of Annex C has been followed, the actual value of the coverage factor should be substituted for $k = 2$. In cases where a confidence level greater than 95 % has been used, the appropriate k factor and confidence level should be substituted.

In reporting the result of any uncertainty analysis, it is important to make a clear statement of whether the reported uncertainty is that of a single value, of a mean of a specified number of values, or of a curve fit based on a specified number of values.

Annex A
(normative)

Step-by-step procedure for calculating uncertainty

A.1 Dimensional and non-dimensional uncertainty

Decide whether dimensional or non-dimensional uncertainty estimates (for example parts per million or per cent) will be used to prevent any confusion. In making this decision, the guidance of Clause 9 concerning parameters with arbitrary zeroes should be borne in mind.

A.2 Mathematical relationship

Determine the mathematical relationship between the input quantities and the output quantity in accordance with Equation (1):

$$Y = f\left(X_1, \ X_2, \ ..., X_N\right)$$

NOTE   The equation numbers referred to in this annex correspond with the equation numbers in the body of the text.

A.3 Standard uncertainty

A.3.1 General

Identify the sources of uncertainty in each of the input quantities; see Annex E. Estimate the standard uncertainty for each source. The calculation method for each component is dependent upon the uncertainty estimates provided and associated probability distributions. The data available usually allow the standard uncertainty to be calculated using one of the following methods.

A.3.2 Type A evaluations — Standard deviation of the mean of repeated measurements

$$u\left(\overline{x}_i\right) = s\left(\overline{x}_i\right)$$

See Equation (8).

A.3.3 Type B evaluations — Based on subjective assessment and experience

A.3.3.1   Rectangular probability distribution

$$u\left(x_i\right) = \frac{a_i}{\sqrt{3}}$$

See Figure B.1 and Equation (9).

A.3.3.2   Normal probability distribution

$$u\left(x_i\right) = \frac{U}{k}$$

See Figure B.2 and Equation (10).

A.4 Sensitivity coefficients

A.4.1 General

The sensitivity coefficients can be calculated either dimensionally or non-dimensionally using either analytical or numerical methods. The choice of dimensional or non-dimensional sensitivity coefficients will be determined by the choice made in A.1.

A.4.2 Dimensional

$$c_i = \frac{\partial y}{\partial x_i} \approx \frac{\Delta y}{\Delta x_i}$$

See Equations (15) and (17).

A.4.3 Non-dimensional

$$c_i^\star = \frac{\partial y}{\partial x_i}\frac{x_i}{y} \approx \frac{\Delta y}{\Delta x_i}\frac{x_i}{y}$$

See Equations (16) and (18).

A.5 Combined uncertainty

A.5.1 General

Decide whether any inputs are correlated. If there are no correlations, calculate the combined standard uncertainty of the measurement from A.5.2 or A.5.3. If there are correlations, follow the guidance given in Annex F.

A.5.2 Dimensional

$$u_c(y) = \sqrt{\sum_{i=1}^{N}\left[c_i\,u(x_i)\right]^2}$$

See Equation (19).

A.5.3 Non-dimensional

$$u_c^*(y) = \sqrt{\sum_{i=1}^{N}\left[c_i^*\,u^*(x_i)\right]^2}$$

See Equation (20).

A.6 Unreliable input quantities

Where unreliable input quantities are used, for example small sample sizes, the procedure of Annex C should be used to obtain the coverage factor for the calculation of expanded uncertainty in A.7.

A.7 Expanded uncertainty

Calculate the expanded uncertainty.

$$U = k\,u_c\left(y\right)$$

See Equation (21);

or

$$U^* = k\,u_c^*\left(y\right)$$

See Equation (22).

A.8 Expression of results

The results calculated in accordance with Annex A shall be reported as described in Clause 10.

Annex B
(normative)

Probability distributions

Figures B.1 to B.5 illustrate the types of probability distributions.

$$P(x_i)$$

$$u(x_i) = a_i/\sqrt{3}$$

$$-a_i \qquad 0 \qquad +a_i$$
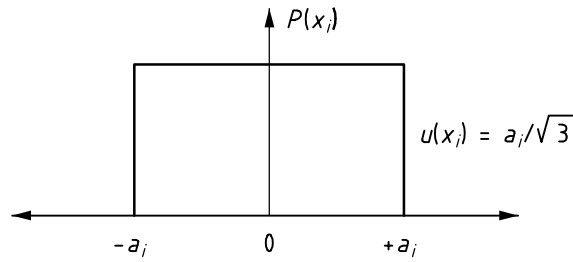
Figure B.1 — Rectangular probability distribution

$$P(x_i)$$

$$u(x_i) = a_i/k$$

$$-a_i \qquad 0 \qquad +a_i$$
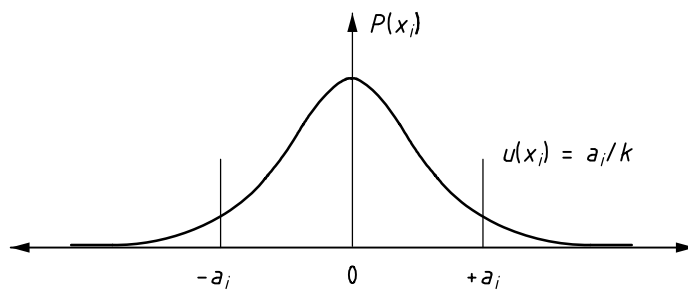
k is the coverage factor appropriate to the range, $\pm a_i$.

Figure B.2 — Normal probability distribution

$$P(x_i)$$

$$u(x_i) = a_i/\sqrt{6}$$

$$-a_i \qquad 0 \qquad +a_i$$
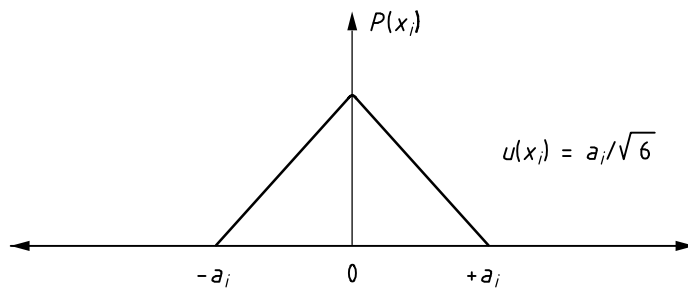
Figure B.3 — Triangular probability distribution

$P(x_i)$

$u(x_i) = a_i$

$-a_i$ 0 $+a_i$
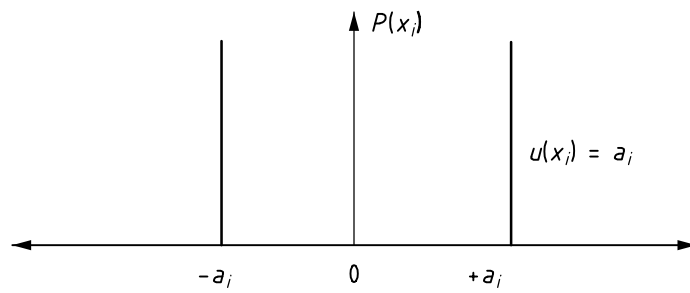
Figure B.4 — Bimodal probability distribution

$P(x)$

$u(x_i) = (a_i + b_i)/\sqrt{12}$

$-a$ $+b$
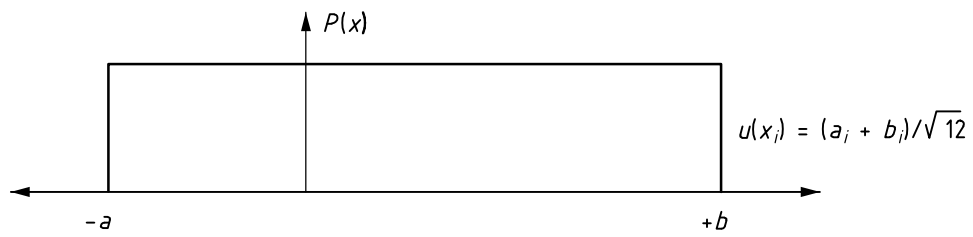
Figure B.5 — Asymmetric probability distribution

Annex C
(normative)

Coverage factors

For a full discussion of this topic, see GUM (1995), Annex G.

Ideally, uncertainty estimates are based upon reliable Type B evaluations and Type A evaluations with a sufficient number of observations such that using a coverage factor $k = 2$ will mean that the expanded uncertainty will provide a confidence level close to 95 %. However, where either of these assumptions is invalid, a revised coverage factor and expanded uncertainty have to be determined using the following four-step procedure.

a)   Calculate the output value, y, the combined standard uncertainty, $u_c(y)$, and the individual components of uncertainty, $u_i(y) = c_i\, u(x_i)$.

b)   Calculate the effective degrees of freedom, v_{eff}, of the combined standard uncertainty $u_c(y)$ using Equation (C.1), the Welch-Satterthwaite equation:

$$v_{eff} = \frac{u_c^4(y)}{\displaystyle\sum_{i=1}^{N} \frac{u_i^4(y)}{v_i}} \tag{C.1}$$

where the degrees of freedom for Type A evaluations is equal to the number of observations minus 1, as given by Equation (C.2):

$$v_i = n - 1 \tag{C.2}$$

and for Type B evaluations, by Equation (C.3):

$$v_i \approx \frac{1}{2}\left[\frac{\Delta u_i(y)}{u_i(y)}\right]^{-2} \tag{C.3}$$

where the relative uncertainty of $u_i(y)$ is given by $\Delta u_i(y)/u_i(y)$. Its value is estimated subjectively by scientific judgement based on the pool of information available.

However where upper and lower limits are used in Type B evaluations and the probability of the quantity lying outside these values is negligible, the degrees of freedom are infinite, as given by Equation (C.4):

$$v_i \to \infty \tag{C.4}$$

c)   Having obtained a value for v_{eff}, determine the value of Student's t from Table C.1. The values quoted give a confidence level of approximately 95 %. It is conventional to use the values for 95,45 % to ensure that the coverage factor, k = 2 is applicable for $v_{eff} \to \infty$.

Table C.1 — Student's distribution, t — 2-sided test, 95,45 % confidence level [a,b]

v_{eff}	1	2	3	4	5	6	7	8	10	12	14	16
t_{95}	13,97	4,53	3,31	2,87	2,65	2,52	2,43	2,37	2,28	2,23	2,20	2,17
v_{eff}	18	20	25	30	35	40	45	50	60	80	100	∞
t_{95}	2,15	2,13	2,11	2,09	2,07	2,06	2,06	2,05	2,04	2,03	2,02	2,00

[a]   Values of t for other degrees of freedom can be obtained with sufficient accuracy by linear interpolation between the values shown.

[b]   Values of t for other confidence levels can be obtained from the statistical tables given in, for example, Dietrich [2].

d)   Calculate the expanded uncertainty from Equation (C.5):

$$U_{95} = k_{95}\, u_c(y) = t_{95}\, u_c(y)$$

(C.5)

NOTE   If $k = 2$ is assumed for any v_{eff} less than ∞, U_{95} is always underestimated; for $v_{eff} = 10$ the underestimation amounts to 14 %.

Annex D
(informative)

Basic statistical concepts for use in Type A assessments of uncertainty

D.1 Mean of a set of data, $\bar{x}$

The sample mean, $\bar{x}$, of a set of data is defined as the arithmetic average of all the values in the sample in accordance with Equation (D.1):

$$\bar{x} = \frac{1}{n}\left(x_1 + x_2 + x_3 + \dots\dots + x_n\right) = \frac{1}{n}\sum_{m=1}^{n} x_m \tag{D.1}$$

where

x_m is the mth value in the sample;

n is the number of values in the sample.

D.2 Experimental standard deviation, s, of a set of data

Within any sample of experimental data, there will always be variation between values. In general, it is of more interest to estimate the variability of the entire population of values from which the sample is drawn and this estimate is given by the standard deviation, s, of the data in the sample. It is defined in accordance with Equation (D.2):

$$s(x) = \sqrt{\frac{1}{n-1}\sum_{m=1}^{n}\left(x_m - \bar{x}\right)^2} \tag{D.2}$$

Care needs to be exercised when using a calculator or spreadsheet to calculate $s(x)$ as these devices sometimes use the value n in place of $n-1$ in the equation and strictly treat the sample as if it were the entire population. This has the effect of underestimating the standard deviation. With large ($n \geqslant 200$) data samples, the difference is small ($< 0,25\ \%$).

In many statistical applications, the square of the standard deviation is required. This is referred to as the variance and is normally denoted by the symbol s^2, rather than being given a specific symbol of its own.

It is sometimes useful to express the variability as a proportion of the mean and this can be done using the coefficient of variation, C_V, defined in accordance with Equation (D.3):

$$C_V = \frac{s}{\bar{x}} \tag{D.3}$$

NOTE   The coefficient of variation can be expressed as a pure number, as a percentage or in parts per million.

The use of C_V is restricted to measurements that have a true zero and C_V is meaningless for measurements with an arbitrary zero.

D.3 Degrees of freedom, ν, associated with a sample variance or standard deviation

The degrees of freedom, ν, is the number of independent observations under a given constraint. When calculating the standard deviation, the constraint imposed is that the deviations sum to zero (as they are the deviations from the mean). Thus the first $n-1$ deviations can take any value but the last has to be such that the sum of the deviations is zero. There are thus $n-1$ independent observations and therefore $n-1$ degrees of freedom.

D.4 Standard uncertainty, $u_{\bar{x}}$, of a sample mean based on the sample standard deviation

The mean, $\bar{x}$, of a sample of data provides only an estimate of the mean of the entire population, since if another sample were taken, a new estimate of the mean would be obtained. Clearly, the greater the variability of the data, the greater will be the uncertainty about the true mean value, and the greater the number of values used, the better the estimate of the mean will be. The measure of the uncertainty in the sample mean is called the standard uncertainty of the mean and is defined in accordance with Equation (D.4):

$$u_{\bar{x}} = \frac{s}{\sqrt{n}} \qquad\qquad (D.4)$$

For the derivation of Equation (D.4), see Dietrich [2].

D.5 Standard uncertainty, $u_{\bar{x}}$, of the sample mean based on a standard deviation derived from past experience

It is often the case that the sample of data is small and that more information about the variability is available from past experience with a larger set of data. In this case, it is permissible to base the standard uncertainty of the mean on the standard deviation, s_{pe}, of the larger set of data. The mean, $\bar{x}$, and the number of readings, n, remain those of the current set of data, but the degrees of freedom, ν, are those associated with the standard deviation, s_{pe}. This will be seen in D.10 to be important to selecting a coverage factor. Thus, $u_{\bar{x}}$ is calculated in accordance with Equation (D.5):

$$u_{\bar{x}} = \frac{s_{pe}}{\sqrt{n}} \qquad\qquad (D.5)$$

D.6 Standard uncertainty, u_{sm}, of a single value based on past experience

The use of an external standard deviation derived from past data allows an uncertainty value to be estimated for a single measurement; this is of particular value in such flow measurement situations as custody transfer where repeat measurements are not possible. In this case, the mean, $\bar{x}$, becomes the single measurement and the number of readings $n = 1$; however the degrees of freedom, ν, is again that associated with the external standard deviation, s_p. Thus, u_{sm} is defined in accordance with Equation (D.6):

$$u_{sm} = s_{pe} \qquad\qquad (D.6)$$

Comparing Equation (D.7) with Equation (D.6), the value of obtaining a mean from two or more readings, where possible, can readily be seen, since the standard uncertainty for a single reading is $\sqrt{2}$ times, or 41 %, greater than that from the mean of two readings and $\sqrt{3}$ times, or 73 %, greater than that from the mean of three readings. Whenever possible, a mean based on multiple readings should be used rather than a single value.

D.7 Pooled standard deviation, s_{po}, from several sets of data

Data from past measurements do not always form one continuous set of data but can be drawn from several sets taken at different times under somewhat different conditions. Provided that the differences in test conditions are not likely to have affected the variability, the data from the various sets can be combined to provide a pooled standard deviation based on many more degrees of freedom. It is important to note that it is the standard deviations (or as will be seen the variances) that are being pooled and not the data sets themselves. It is the variability of the sets about their own means that is being combined to provide a better estimate of the variability of the measurement technique, and variations between the means of the sets are not of interest. The pooled standard deviation, s_{po}, is calculated in accordance with Equation (D.7):

$$s_{po} = \sqrt{\sum_{j=1}^{m'} v_j s_j^2 \left/ \sum_{j=1}^{m'} v_j \right.}$$

(D.7)

where

s_j is the standard deviation of the jth set of data;

v_j is the degrees of freedom associated with s_j;

m' is the number of data sets to be pooled.

s_{po} is therefore derived from a weighted average of the variances, s_j^2, of the sets of data to be pooled and the weighting factors are the degrees of freedom, v_j, in each set.

The standard uncertainty of the sample mean then is calculated in accordance with Equation (D.8):

$$u_{\bar{x}} = \frac{s_{po}}{\sqrt{n}}$$

(D.8)

and that of a single value, in accordance with Equation (D.9):

$$u_{sm} = s_{po}$$

(D.9)

D.8 Degrees of freedom, v_{po}, associated with a pooled standard deviation

The pooled standard deviation is a better estimate of the population standard deviation than any of the individual standard deviations because it has more degrees of freedom associated with it. The combined degrees of freedom is obtained simply by adding the degrees of freedom associated with each of the contributing standard deviations in accordance with Equation (D.10):

$$v_{po} = \sum_{j=1}^{m'} v_j$$

(D.10)

D.9 Expanded uncertainty, $U_{\bar{x}}$, of a sample mean based on the sample standard deviation

While the standard uncertainty of a mean provides a measure of the bandwidth within which the mean might lie, the band is narrow and there is a considerable risk that the mean could actually lie outside the band. With a standard deviation, and therefore a standard uncertainty, based on two degrees of freedom, there is a 42 % chance that the mean will lie outside the band defined by the standard uncertainty and even with 100 degrees of freedom, there remains a 32 % chance. It is therefore normal practice to extend the bandwidth to provide a

greater level of confidence that the true mean will lie within the expanded band. Bandwidths can be calculated to give confidence levels of 90 %, 95 % or 99 %, but in measurement uncertainty analysis a level of 95 % is normally selected. This is accomplished by applying a coverage factor, k, to the standard uncertainty in accordance with Equation (D.11):

$$U_{\bar{x}} = k\,u_{\bar{x}} \qquad\qquad\qquad\qquad\qquad\qquad (D.11)$$

The value of the coverage factor depends on the degrees of freedom associated with the standard uncertainty, in the case of a standard uncertainty based on the standard deviation of the current sample of data $\nu = n - 1$. A range of values is given in Table C.1. Strictly speaking, the values listed are for a confidence level of 95,45 %, this level having been being selected in preference to 95 % to give a coverage factor of two as $\nu \rightarrow \infty$.

D.10   Expanded uncertainty, $U_{\bar{x}}$, of a sample mean based on a standard deviation derived from past experience

The equation for the expanded uncertainty is equally applicable when the standard uncertainty is obtained from a standard deviation based on past experience, whether from a single set of data or from the pooling of several sets. However, in this case, the coverage factor has to be selected for the degrees of freedom associated with the standard deviation from past experience.

D.11   Expanded uncertainty, U_{sm}, of a single value

The equation for the expanded uncertainty is also applicable in the case of a single value and again the coverage factor has to be selected for the degrees of freedom associated with the standard deviation used.

D.12   Tolerance interval for individual measurements

The expanded uncertainty of a mean defines, for a given confidence level, a range within which the true mean of a measurand can be expected to lie. However, individual values of the measurand will lie in a much wider range and there is often a need to define the range within which a given proportion of the values will lie. For a known standard deviation, the normal distribution defines the limits within which a given percentage of readings will lie. However, when based on a limited sample, the standard deviation is itself subject to uncertainty and confidence limits have therefore to be placed on the interval containing the required percentage of readings. These limits are provided by the tolerance interval.

The tolerance interval is defined in accordance with Equation (D.12):

$$\bar{x} \pm k_{\mathrm{t}}\,s \qquad\qquad\qquad\qquad\qquad\qquad (D.12)$$

where

$\bar{x}$ is the sample mean;

s is the sample standard deviation;

k_{t} is taken from Table D.1.

It should be noted that the values of k_{t} in Table D.1 are presented for different sample sizes, n, and not for the degrees of freedom associated with the standard deviation. The values in Table D.1 are based on the assumption that the sample is drawn from a normal, or Gaussian, distribution.

Table D.1 — Tolerance intervals (values of k_t) [2]

Sample size	Confidence level					
	95 %			99 %		
	Percent of items within the tolerance interval			Percent of items within the tolerance interval		
	90 %	95 %	99 %	90 %	95 %	99 %
3	8,38	9,92	12,86	18,93	22,40	29,06
4	5,37	6,37	8,30	9,40	11,15	14,53
5	4,28	5,08	6,63	6,61	7,85	10,26
6	3,71	4,41	5,78	5,34	6,35	8,30
7	3,31	4,01	5,25	4,61	5,49	7,19
8	3,14	3,73	4,89	4,15	4,94	6,47
9	2,97	3,53	4,63	3,82	4,55	5,97
10	2,84	3,38	4,43	3,58	4,27	5,59
12	2,66	3,16	4,15	3,25	3,87	5,08
14	2,53	3,01	3,96	3,03	3,61	4,74
16	2,44	2,90	3,81	2,87	3,42	4,49
18	2,37	2,82	3,70	2,75	3,28	4,31
20	2,31	2,75	3,62	2,66	3,17	4,16
30	2,14	2,55	3,35	2,39	2,84	3,73
40	2,05	2,45	3,21	2,25	2,68	3,52
50	2,00	2,38	3,13	2,16	2,58	3,39

D.13 Detection of outliers

Occasionally, when a set of measurements is taken, one value appears to be substantially larger or smaller than all the others and there is then a temptation to reject the outlying value as being wrong. There could be obvious reasons for the outlier, but frequently the reasons will not be apparent and the metrologist will be left to decide for himself whether the value is wrong or is simply an extreme value from the same distribution as all the others.

An extreme value will distort both the mean and the standard deviation of the set and these values could be more representative of normal operation if the outlier is rejected from the analysis. However, such rejection should not be done lightly, as there is always a risk of rejecting valid data.

Many statistical tests have been developed to assist in deciding the significance of outliers, some testing for single outliers, others testing for multiple outliers either at the same or at opposite ends of the range. One such test is Grubbs' test, which compares the distance between the outlier and the mean with the standard deviation of the whole set of data.

Consider a set of data ($x_1, x_2, ... x_n$) with mean $\bar{x}$, standard deviation, s, and the reading, x_m, suspected of being an outlier. The Grubbs' test statistic, Z_n, is defined in accordance with Equation (D.13):

$$Z_n = \frac{|x_m - \bar{x}|}{s}$$

(D.13)

Z_n is then compared with the value given in Table D.2 for the appropriate confidence level and number of samples. If Z_n exceeds the tabulated value, the measurement, x_m, can be classed as an outlier with the stated confidence level.

Although Grubbs' test can be automated within a data collection system to flag outliers, the rejection of data requires judgement and should not be based purely on the statistical result.

Table D.2 — Grubbs' outlier test based on mean and standard deviation

Number of observations	Confidence level	
	95 %	99 %
4	1,48	1,50
5	1,71	1,76
6	1,89	1,97
7	2,02	2,14
8	2,13	2,27
9	2,21	2,39
10	2,29	2,48
12	2,41	2,64
14	2,51	2,76
16	2,59	2,85
18	2,65	2,93
20	2,71	3,00
30	2,91	3,24
40	3,04	3,38
50	3,13	3,48
100	3,38	3,75

D.14   Worked examples

D.14.1   Mean, variance, standard deviation, degrees of freedom and coefficient of variation

D.14.1.1   General

Toluene is being used as a feedstock in a petrochemical plant and the flow-rate is measured using a turbine meter. To reduce Type A uncertainties in the flow-rate measurement, each "reading" used for control purposes is derived from five individual readings. A typical set of values is given in Table D.3. Calculate the mean, standard deviation and coefficient of variation.

Table D.3 — Typical set of flow-rate readings

Reading number	1	2	3	4	5
Flow-rate, litres per second	122,7	123,2	122,3	122,8	123,0

D.14.1.2   Mean

The mean, expressed in litres per second, is calculated as

$$\bar{x} = \frac{1}{n}\sum_{m=1}^{n} x_m$$
$$= \left(122{,}7 + 123{,}2 + 122{,}3 + 122{,}8 + 123{,}0\right)/5$$
$$= 122{,}8$$

D.14.1.3   Variance

The variance, expressed in litres per second quantity squared, is calculated as

$$s^2 = \frac{1}{(n-1)}\sum_{m=1}^{n}\left(x_m - \bar{x}\right)^2$$
$$= \frac{\left[\left(122{,}7 - 122{,}8\right)^2 + \ldots + \left(123{,}0 - 122{,}8\right)^2\right]}{(5-1)}$$
$$= 0{,}115\ 0$$

D.14.1.4   Standard deviation

The standard deviation, expressed in litres per second, is calculated as

$$s = \sqrt{s^2}$$
$$= \sqrt{0{,}115\ 0}$$
$$= 0{,}339$$

D.14.1.5   Degrees of freedom

The degrees of freedom is calculated as

$$v = n - 1$$
$$= 5 - 1$$
$$= 4$$

D.14.1.6   Coefficient of variation

The coefficient of variation is calculated as

$$C_V = \frac{s}{\bar{x}}$$
$$= 0{,}339/122{,}8$$
$$= 0{,}002\ 76$$

D.14.2   Standard and expanded uncertainties of a mean using sample standard deviation

D.14.2.1   General

For the data of example D.14.1, calculate the standard uncertainty of the mean and the expanded uncertainty at the 95 % confidence level.

D.14.2.2   Standard uncertainty of the mean

The standard uncertainty of the mean, expressed in litres per second, is calculated as

$$u_{\overline{x}} = \frac{s}{\sqrt{n}}$$
$$= \frac{0,339}{\sqrt{5}}$$
$$= 0,152$$

D.14.2.3   Expanded uncertainty of the mean at the 95 % confidence level

For four degrees of freedom, Table E.1 gives a coverage factor, k, of 2,87, thus the expanded uncertainty, expressed in litres per second, is calculated as

$$U_{\overline{x}} = k u_{\overline{x}}$$
$$= 2,87 \cdot 0,152$$
$$= 0,436$$

D.14.3   Standard and expanded uncertainties of a single value

D.14.3.1 General

If the control of the flow in example D.14.1 is now based on a single reading of the flow-rate, calculate the standard uncertainty and expanded uncertainty at the 95 % confidence level.

The data of example D.14.1 provide the necessary information on the variability of the flow-rate in question and the standard deviation derived from those data can be used as an external standard deviation to calculate the required uncertainties for a single reading.

D.14.3.2   Standard uncertainty

The standard uncertainty, expressed in litres per second, is calculated as

$$u_{\mathrm{sm}} = s_{\mathrm{ex}}$$
$$= 0,339$$

D.14.3.3   Expanded uncertainty

As the external standard deviation on which the standard uncertainty is based was obtained from a set of five data points, it has four degrees of freedom associated with it and the value of k remains equal to 2,87 (from Table C.1). Thus, the expanded uncertainty can be calculated as

$$U_{\mathrm{sm}} = k u_{\mathrm{sm}}$$
$$= 2,87 \cdot 0,339$$
$$= 0,973$$

These can be seen to be very much larger than the values obtained for the uncertainties of the mean of five readings and this demonstrates the consequences of making single measurements.

D.14.4   Pooled standard deviation from several sets of data

D.14.4.1   General

In an effort to get a better estimate of the variability of the flow-rate due to Type A uncertainties, the plant engineer consults past records of flow-rate and identifies six sets of data obtained at similar flow-rates. These data are shown in Table D.4, together with the mean of each set, the standard deviation of each set about its own mean and the degrees of freedom associated with each standard deviation. Calculate the pooled standard deviation, and its associated degrees of freedom from all the data.

Table D.4 — Flow-rate data for the example in D.14.4

		Flow-rate [a] per day per data-set					
Set	**Statistical parameter**	**Day**					
		1	2	3	4	5	6
1	—	120,2	123,0	124,3	127,3	118,3	122,7
2	—	120,8	122,6	124,9	126,7	118,5	123,1
3	—	121,0	122,7	124,9	127,2	118,2	123,0
4	—	121,1	122,9	125,1	126,5	118,6	122,7
5	—	120,4	122,4	124,5	—	118,8	122,2
6	—	—	—	—	—	118,3	122,4
7	—	—	—	—	—	119,1	—
—	—	—	—	—	—	—	—
—	$\bar{x}$ [a]	120,70	122,72	124,74	126,93	118,54	122,68
—	s [a]	0,387	0,239	0,329	0,386	0,321	0,343
—	ν	4	4	4	3	6	5
[a] Flow rates given in litres per second.							

D.14.4.2   Pooled standard deviation

The pooled standard deviation, expressed in litres per second, is calculated as

$$s_{\text{po}} = \sqrt{\left.\sum_{j=1}^{m'} \nu_j \, s_j^2 \middle/ \sum_{j=1}^{m'} \nu_j \right.}$$

$$= \sqrt{\frac{\left(4 \cdot 0{,}387^2 + 4 \cdot 0{,}239^2 + \ldots + 6 \cdot 0{,}321^2 + 5 \cdot 0{,}343^2\right)^2}{4+4+4+3+6+5}}$$

$$= 0{,}335$$

D.14.4.3   Pooled degrees of freedom

The pooled degrees of freedom is calculated as

$$v_{po} = \sum_{j=1}^{m'} v_j$$
$$= 4 + 4 + 4 + 3 + 6 + 5$$
$$= 26$$

Although in this example the pooling of past data has had little effect on the standard deviation, it has greatly increased the degrees of freedom associated with the pooled standard deviation. The benefits are shown in the example in D.14.5.

D.14.5   Expanded uncertainty of a sample mean based on a standard deviation from past experience

D.14.5.1   General

Using the pooled data of example D.14.4, recalculate the standard and expanded uncertainties of a mean based on five readings.

D.14.5.2   Standard uncertainty

The standard deviation used to calculate the standard uncertainty is now the pooled value but, as the sample from which the mean is derived is still limited to five values, the divisor in the standard uncertainty formula remains $\sqrt{5}$, thus the equation becomes

$$u_{\overline{x}} = \frac{s_{pe}}{\sqrt{n}}$$
$$= \frac{s_{po}}{\sqrt{n}}$$
$$= \frac{0{,}335}{\sqrt{5}}$$
$$= 0{,}150$$

As the pooled standard deviation was very close to the original sample value, the standard uncertainty is, in this example, largely unaffected by the pooling process.

D.14.5.3   Expanded uncertainty

In obtaining the coverage factor from Table C.1 to calculate the expanded uncertainty, it is important to remember that the degrees of freedom associated with the standard uncertainty is now that associated with the pooled standard deviation. The coverage factor is therefore obtained for 26 degrees of freedom, $k = 2{,}11$ and the uncertainty, expressed in litres per second is calculated as

$$U_{\overline{x}} = k u_{\overline{x}} = 2{,}11 \cdot 0{,}150 = 0{,}317$$

This is substantially smaller than the value of 0,436 obtained using only the data of the original set in example D.14.2 and illustrates the value of pooling past data to obtain a better estimate of variability, the improvement coming, in this case, from the increased degrees of freedom associated with the pooled standard deviation.

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D.14.6   Tolerance interval for individual values

Whisky bottles are marked with a minimum content of 700 ml. Recognizing that there are variations in the filling process, the bottling plant manager has to set the average fill volume above 700 ml to minimize the chances of bottles containing a short measure. Measurement of the contents of 10 bottles selected at random gives a standard deviation of 4 ml. At what value should the plant manager set the mean fill to be 95 % confident that 99,5 % of bottles will meet the minimum requirement?

Since the distribution can be assumed to be symmetrical, 99,5 % of bottles above the minimum implies 0,5 % below the minimum, 99 % within the tolerance interval and 0,5 % above the upper bound of the interval. Selecting the value in Table D.1 to yield a 95 % confidence that 99 % of items are in the interval gives $k_t = 4{,}43$.

The interval is therefore $\pm\, 4{,}43 \times 4$ ml $= \pm\, 17{,}72$ ml.

So, for the lower bound of the interval to be 700 ml, the mean has to be set at 717,72 ml.

The plant manager recognizes that this mean represents whisky given away with almost every bottle and he is keen to tighten up on these losses. He decides that he can accept a mean of 705 ml and at the same time wants to improve his confidence level to 99 % that 99,5 % of bottles will conform to the minimum requirement. In trying to reduce the uncertainties in the filling process, what standard deviation should he be looking for in a sample of 30 bottles?

For 99 % confidence that 99 % of bottles are in the range (0,5 % below the lower limit) and a sample size of 30, Table D.1 gives $k_t = 3{,}73$. So for a tolerance interval of $\pm\, 5$ ml, the sample standard deviation needs to be reduced to 5 ml divided by 3,73, or 1,34 ml.

D.14.7   Rejection of outliers

The flow of water to a cooling tower is measured using a Venturi meter. An estimate of the average daily consumption is required and the following data are collected over a period of 20 days.

Table D.5 — Volume data for example D.14.7

Day	1	2	3	4	5
Volume, m^3	7,80	7,66	7,87	8,02	8,01

Day	6	7	8	9	10
Volume, m^3	8,08	7,18	7,81	7,99	7,69

Day	11	12	13	14	15
Volume, m^3	7,74	7,60	7,58	7,70	7,73

Day	16	17	18	19	20
Volume, m^3	7,54	7,76	7,78	7,86	7,79

A calculation of the mean and standard deviation using Equations (D.1) and (D.2) gives a mean of 7,76 m^3 and a standard deviation of 0,202 m^3. As the mean is based on 20 readings, the standard uncertainty, expressed in cubic metres, of the mean is given by

$$u_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{0{,}202}{\sqrt{20}} = 0{,}045$$

The coverage factor for 20 values and therefore 19 degrees of freedom from Table C.1 is 2,14 (by interpolation) and the expanded uncertainty, expressed in cubic metres, is

$$U_{\overline{x}} = k\,u_{\overline{x}} = 2,14 \cdot 0,045 = 0,096$$

However, the value of 7,18 recorded on the 7th day appears substantially lower than the others and it is tested as an outlier using the Grubbs' test.

$$Z_n = \frac{|x_m - \overline{x}|}{s} = \frac{|7,18 - 7,76|}{0,202} = 2,87$$

As the value of Z_n exceeds the tabulated value (Table D.2) for 20 observations at the 95 % confidence level, the value of 7,18 can be regarded as an outlier with 95 % confidence. However, Z_n does not exceed the tabulated value at the 99 % confidence level and the value of 7,18 cannot be regarded as an outlier at the higher confidence level. An examination of the plant records reveals a problem with feedstock concentrations on day 7 and this could have affected cooling requirements. It is therefore decided that the low value can be rejected.

Having rejected the outlier, the mean and standard deviation can be recalculated as 7,79 m^3 and 0,153 m^3 respectively. There are now 19 observations, so the standard uncertainty, expressed in cubic metres, of the mean is

$$u_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{0,153}{\sqrt{19}} = 0,035$$

The coverage factor for 19 observations and 18 degrees of freedom from Table C.1 is 2,15 and the expanded uncertainty, expressed in cubic metres, of the mean at the 95 % confidence level is therefore

$$U_{\overline{x}} = k\,u_{\overline{x}} = 2,15 \cdot 0,035 = 0,075$$

Annex E
(informative)

Measurement uncertainty sources

E.1 Categories of uncertainty sources

Sources introducing uncertainty in a measurement process can be divided arbitrarily into the following categories:

a) calibration uncertainty;

b) data acquisition uncertainty;

c) data processing uncertainty;

d) uncertainty due to methods;

e) others.

While often helpful, dividing uncertainty sources by category is not necessary for a correct uncertainty analysis.

E.2 Calibration uncertainty

Each measurement instrument can introduce uncertainties. The main purpose of the calibration is to reduce the measurement uncertainty to an acceptable level. The calibration process achieves that goal by replacing the large uncertainty of an uncalibrated instrument by the smaller combination of uncertainties of the standard instrument and the comparison between it and the measurement instrument.

Calibrations are also used to provide traceability to known reference standards and/or physical constants. In some countries, there is a hierarchy of laboratories that are concerned with calibration, with the national standards laboratory at the apex of the hierarchy, providing the ultimate reference for every standards laboratory. Each level in the calibration hierarchy is traceable to the level above and so carries the uncertainty of the higher laboratory as its calibration uncertainty, to which is added its own instrumentation and usage uncertainties. In this way each level adds uncertainty to the measurement process and, when a particular level of uncertainty is sought, it is therefore important to enter the calibration chain at the correct level. Thus, if an overall uncertainty of 0,5 % is required and the usage and instrumentation uncertainties of the application contribute 0,4 %, the calibration hierarchy should be entered at a level where the calibration uncertainty is 0,3 %, to yield a combined uncertainty, expressed in percent, of $\sqrt{0,4^2 + 0,3^2}$, or 0,5 %, the required value.

E.3 Data acquisition uncertainty

Uncertainty in data acquisition systems can arise from the signal conditioning, the sensors, the recording devices, etc. The best method to minimize the effects of many of these uncertainty sources is to perform overall system calibrations. By comparing known input values with their measured results, estimates of the data acquisition uncertainty can be obtained. However, it is not always possible to do this. In these cases, it is necessary to evaluate each element of uncertainty and combine them to predict the overall uncertainty.

E.4 Data processing uncertainty

Typical uncertainty sources in this category stem from curve fits and computational resolution; the latter are normally negligible. Curve fits can be used to allow for non-linearities in, for example, a meter factor. However, while the equation obtained from a regression analysis of the calibration data represents the best fit to those data, the scatter about the curve indicates that, with more data, a slightly different equation would be obtained in the same way as the mean of a set of data will change as more values are obtained. Thus, each coefficient in a regression equation will have an uncertainty associated with it, just as the mean of a set of values does. For details of the methods of evaluating the uncertainties resulting from fitting straight lines or curves to data, see ISO/TR 7066-1 [3] and ISO 7066-2 [4], respectively.

Meter performance characteristics such as non-repeatability are included in the curve-fit uncertainty because the curve is necessarily based on multiple readings. In addition, careful design of the calibration experiment allows sources of uncertainty such as hysteresis to be included.

E.5 Uncertainty due to methods

Uncertainty due to methods is defined as those additional uncertainty sources that originate from the techniques or methods inherent in the measurement process. These uncertainty sources can significantly affect the uncertainty of the final results and, in a modern measurement system, are likely to be more significant than those contained in calibration, data acquisition and data processing. Some common examples include the following.

a) Uncertainty in the assumptions or constants in the calculations. For example, the constant π can be taken as 3,14 or 3,141 593 and the gravitational acceleration, g, can be taken as 9,81 m/s^2 or can be calculated for the particular location using the International Union of Geodesy and Geophysics equation.

b) Uncertainty due to intrusive disturbance effects caused by the installed instrumentation. For example, a pitot tube will cause blockage and increase the flow velocity being measured.

c) Spatial or profile uncertainty in the conversion from discrete point measurements of a velocity profile to station average flow-rate.

d) Environmental effects on measurement transducers, such as conduction, convection and radiation. Heat transfer effects on a temperature probe are particularly important when dealing with very hot or very cold fluids.

e) Uncertainty due to instability, non-repeatability and hysteresis of the measurement process.

f) Uncertainty due to drift of an instrument between successive calibrations.

g) Electrical interference with electronic components such as by magnetic fields, electric fields and mains spikes.

h) Variation between the calibration and usage conditions. Meters calibrated at room temperature in a laboratory can have increased uncertainty when used in the field at a wide range of ambient temperatures or when used in process plant with high or low temperature fluids. Upstream pipework configurations can also have a strong influence on some types of flow meter.

Annex F
(informative)

Correlated input variables

When listing all sources of uncertainty from different categories, the sources should be defined where possible so that the uncertainties in the various sources are independent of each other. The input variables and their associated uncertainties are then said to be uncorrelated. Where the input variables or the uncertainties in those variables are not independent of each other, they are said to be correlated. That correlation can be either positive or negative and can be either a 100 % or a partial correlation.

Correlation will arise where the same instrument is used to make several measurements or where instruments are calibrated against the same reference. The latter practice is common in flow measurement laboratories where several flowmeters are calibrated against the same reference while connected in series and are then used in parallel to meter a larger flow. Correlation also arises when an external influence such as pressure, temperature or humidity impacts on several instruments within the measurement system.

Positive correlation tends to increase the overall uncertainty, as it is no longer possible to assume that the uncertainties will be distributed throughout their possible range and so derive a most likely value based on the root sum square technique. Instead, the combined value has to reflect the fact that the uncertainties are linked and so will act in the same direction on any one measurement. As an example, a temperature correction applied to the pipe diameter and meter bore of an orifice plate installation will apply equally to both the diameter and the bore.

Negative correlation occurs when, for example, the same instrument is used to make two measurements and the difference or ratio of these measurements is the final measurand. In the former case, a zero offset in both readings will have no effect on the final result; while in the latter case, an error in the slope of the calibration line will not affect the ratio. Negative correlation can therefore be seen to reduce uncertainty.

The handling of correlated uncertainties can be difficult, particularly for partial correlation; a detailed description is contained in GUM (1995), 5.2. The method described in the GUM is mathematically complex and it is recommended that, for most practical applications, the simpler techniques described below be carried out to assess the importance of the correlated elements to determine whether or not the complexity of the GUM technique is required.

The best approach to the analysis is to redefine the mathematical relationship to eliminate the correlations. For example, as already mentioned, where a correction is made for thermal expansion of the pipe and bore in an orifice plate installation, the uncertainties in the corrections will be positively correlated through the uncertainty in the temperature and, if the materials are the same, in the thermal expansion coefficient. By redefining the mathematical relationship to include equations for the bore and pipe diameter in terms of the dimensions at a reference temperature, the operating temperature and the thermal expansion coefficient, the correlating variables are introduced into the analysis as independent variables and their contribution to uncertainty is fully accounted for through the sensitivity coefficient analysis of Clause 8. Annex G, example 3, illustrates the procedure for an orifice plate. Negative correlations can be addressed by redefining the measurand to eliminate the correlated variables; Annex G, example 2, where a flow ratio is required, illustrates the procedure.

An alternative approach to the assessment of positively correlated uncertainties is to assume that the correlations are all 100 % as it can be shown that, in this case [see, for example, GUM (1995), 5.2], the combined uncertainty, u_c, is given by Equation (F.1):

$$u_c = c_1 u(x_1) + c_2 u(x_2) + ... + c_N u(x_N)$$ (F.1)

or, in relative terms, by Equation (F.2):

$$u_c^* = c_1^* u^*(x_1) + c_2^* u^*(x_2) + ... + c_N^* u^*(x_N)$$ (F.2)

The analysis technique is then to divide the sources of uncertainty into correlated and uncorrelated and carry out parallel analyses adding contributions linearly for the correlated sources and by root sum square for the uncorrelated. As a final step, the total correlated and uncorrelated uncertainties are added by a root sum of the squares to obtain the overall uncertainty. This approach will overestimate the effect of any elements of uncertainty that are only partially correlated, thus adhering to the principle of erring on the side of pessimism in assessing uncertainty.

In dealing with negative correlations, it should be remembered that 100 % negative correlations result in the source being eliminated from the analysis and so making no contribution to the overall uncertainty. The principle of erring on the side of pessimism therefore requires that partial negative correlations are treated as uncorrelated and retained in the analysis.

Where the rigorous approach of redefining the mathematical relationship cannot be adopted, it is wise to compare the contribution of the potentially correlated sources with that of the uncorrelated sources to decide whether the correlated effects are worthy of a more detailed analysis.

Annex G
(informative)

Examples

G.1   Example 1 — A critical flow nozzle is used to measure the mass flow of air in a calibration rig

G.1.1   Mathematical model

The mass flow is given by Equation (G.1):

$$q_{ma} = A_t\, C\, \varphi_{cf}\, p_0 \sqrt{\frac{1}{RT_0}} \tag{G.1}$$

where

q_{ma} is the mass flow;

A_t is the area of the throat;

C is the discharge coefficient;

φ_{cf} is the critical flow function;

p_0 is the upstream pressure;

R is the specific gas constant;

T_0 is the upstream absolute temperature.

As the nozzle is calibrated in air against a reference standard, this equation reduces to Equation (G.2):

$$q_{ma} = C_c\, p_0 \sqrt{\frac{1}{T_0}} \tag{G.2}$$

where C_c is the calibration coefficient.

G.1.2   Contributory variances

The application of Equation (19) to Equation (G.2) yields Equation (G.3):

$$u_c^2(q_{ma}) = c_{C_c}^2\, u^2(C_c) + c_{p_0}^2\, u^2(p_0) + c_{T_0}^2\, u^2(T_0) \tag{G.3}$$

The sensitivity coefficients in Equation (G.4) can be obtained by differentiation of Equation (G.2):

$$c_{C_c} = p_0 \sqrt{\frac{1}{T_0}}\,,\ c_{p_0} = C_c \sqrt{\frac{1}{T_0}}\ \text{and}\ c_{T_0} = -\tfrac{1}{2} C_c\, p_0\, T_0^{-3/2} \tag{G.4}$$

Thus, Equation (G.3) can be rewritten as Equation (G.5):

$$u_c^2\left(q_{ma}\right) \;=\; \frac{p_0^{\,2}}{T_0}\,u^2(C_c) \;+\; \frac{C_c^{\,2}}{T_0}\,u^2(p_0) \;+\; \frac{C_c^{\,2}\,p_0^{\,2}}{4T_0^{\,3}}\,u^2(T_0) \tag{G.5}$$

and dividing by q_{ma}^2 results in Equation (G.6):

$$\frac{u_c^2\left(q_{ma}\right)}{q_{ma}^{\,2}} \;=\; \frac{u^2(C_c)}{C_c^{\,2}} \;+\; \frac{u^2(p_0)}{p_0^{\,2}} \;+\; \frac{u^2(T_0)}{4T_0} \tag{G.6}$$

Thus the relative sensitivity coefficients, c^*, are as given in Equation (G.7):

$$c_{\varphi_C}^{*} = 1, \quad c_{p_0}^{*} = 1, \quad \text{and} \quad c_{T_0}^{*} = -\tfrac{1}{2} \tag{G.7}$$

G.1.2.1 Uncertainty in the calibration coefficient, φ_C

The calibration certificate gives as the expanded uncertainty of the calibration coefficient C_c, $U(C_c) = 0{,}25\ \%$ at the 95 % confidence level (or with a coverage factor of 2); $k = 2$ is therefore used to recover the standard uncertainty. The calibration was carried out at an outside laboratory. The instrumentation used in the calibration was that of the independent laboratory and, in consequence, there are no correlations with the instrumentation employed in the use of the nozzle. However, had the calibration experiment been carried out using the pressure or temperature instrumentation used in normal operation, the correlation would have had to be taken into account.

G.1.2.2 Uncertainty in the measurement of upstream pressure, p_0

The gauge used to measure the upstream pressure has an acceptance criterion of 0,5 % of the full-scale reading. The instrument has a full-scale reading of 2 MPa (20 bar) and the line pressure is normally run at 1,5 MPa (15 bar). As no calibration correction is applied to the gauge readings provided that they fall within the acceptance limit, the maximum uncertainty is 0,5 % of 2 MPa (20 bar) or 0,010 MPa (0,1 bar). Nothing is known of the distribution of calibration values within the acceptance range and it is, therefore, prudent to take the pessimistic view and assume that all values are equally likely, i.e. a rectangular distribution. The standard uncertainty is therefore 0,010 MPa (0,1 bar) divided by $\sqrt{3}$, or 0,005 8 MPa (0,058 bar). In use, the instrument is read via a 10-bit computer data-acquisition card giving a resolution of 1 part in 1 024. The full range of the card is set to the full-scale reading of the pressure gauge [2 MPa (20 bar)], 1 bit on the computer card therefore represents 2 MPa (20 bar) divided by 1 024, or 0,002 MPa (0,02 bar). The expanded uncertainty is therefore 0,001 MPa (0,01 bar) and, as the digital value represents all values in the range with equal probability, a rectangular distribution is assumed, yielding a standard uncertainty of 0,001 MPa (0,01 bar) divided by $\sqrt{3}$, or 0,000 58 MPa (0,005 8 bar). This is added in quadrature to the calibration uncertainty to obtain the overall standard uncertainty, expressed in pressure units, as given in Equation (G.8). Thus $u^2(P_0)$, expressed in square pressure units, is equal to $(0{,}005\,8^2 + 0{,}000\,58^2)$ MPa^2 [$(0{,}058^2 + 0{,}005\,8^2)$ bar^2] and $u(P_0)$ is equal to 0,005 8 MPa (0,058 bar). With an operating pressure of 1,5 MPa (15 bar), the overall relative standard uncertainty in the pressure measurement is 0,005 8 / 1,5 (expressed in MPa) [0,058 /15 (expressed in bar)], or 0,39 %.

$$u(P_0) = \sqrt{(0{,}005\,8^2 + 0{,}000\,58^2)} = 0{,}005\,8 \text{ MPa}$$

$$= \sqrt{(0{,}058^2 + 0{,}005\,8^2)} = 0{,}058 \text{ bar} \tag{G.8}$$

G.1.2.3 Uncertainty in the measurement of upstream temperature, T_0

The upstream temperature is measured with a Type J thermocouple with a stated uncertainty at the 95 % confidence level of 1 K. This is an expanded uncertainty and as the confidence level is stated to be 95 %, it is assumed that $k = 2$ when deriving the standard uncertainty. Thus, the standard uncertainty is 1 K divided by 2,

or 0,5 K. The scale division on the temperature readout is 0,1 K, giving an expanded uncertainty of 0,05 K. This has a rectangular distribution and the standard uncertainty is 0,05 K divided by $\sqrt{3}$, or 0,029 K. An additional uncertainty arises from the use of the thermocouple and how accurately it measures the mean temperature of the flowing gas. The probe is mounted in accordance with the recommendations of ISO 9300 and compressible flow effects are therefore small. At 313 K the gas temperature is close to ambient and conduction effects of the probe are also small. An expanded uncertainty of 0,1 K is therefore assumed and this is judged to have a rectangular distribution giving a standard uncertainty of 0,1 K divided by $\sqrt{3}$, or 0,058 K. The standard uncertainties from the various sources are independent of each other and are added in quadrature to obtain the overall standard uncertainty in the temperature measurement as given in Equations (G.9), with $u^2(T_0)$ expressed in square kelvins and $u(T_0)$ expressed in kelvins:

$$u(T_0) = \sqrt{(0,5^2 + 0,028^2 + 0,058^2}$$

(G.9)

$$= 0,5$$

With an operating temperature of 313 K, the relative standard uncertainty, $u^*(T_0)$, expressed in kelvins, is equal to 0,5 divided by 313, or 0,16 %.

G.1.2.4   Combined uncertainty

The overall uncertainty budget is set out in Table G.1

Table G.1 — Uncertainty budget

Symbol	Source of uncertainty	Relative expanded uncertainty $U^*(x_i)$ %	Probability distribution	Divisor	Relative sensitivity coefficient c^*_i	Relative standard uncertainty $u^*(x_i)$ %	Contribution to overall uncertainty $[c_i\,u(x_i)]^2$ 10^{-4}
$u^*(\varphi_C)$	Calibration	0,25	normal	2,00	1,00	0,13	0,02
$u^*(p_0)$	Pressure	0,67	rectangular	1,73	1,00	0,39	0,15
$u^*(T_0)$	Temperature	0,32	normal	2,00	0,50	0,16	0,01
				Multiplier			0,18
	Combined	0,84	←	2,00	←	0,42	↵

The combined standard uncertainty, u_c^*, is therefore 0,42 % and the overall expanded uncertainty $U^*_{95} = 0,84$ %. From Table G.1 it can be seen that most of the overall uncertainty in flow-rate comes from the uncertainties in measuring upstream pressure. As a general rule, when an uncertainty contribution $[c_i\,u(x_i)]$ is less than 20 % of the largest contribution, the smaller source can be ignored. In the final column of Table G.1, the contributions are shown as $[c_i\,u(x_i)]^2$ and therefore only those contributing $(0,2)^2$ or 4 % of the largest contribution can be safely ignored. On this basis, although the contribution from the temperature measurement is small at 7 % of the pressure contribution, it cannot be ignored.

G.2   Example 2 — Comparing two flow-rates measured with the same meter

G.2.1   General

In many engineering situations, the interest lies not in the true flow-rate but in a comparison of two flow-rates measured with the same meter. The uncertainty of the comparison is then independent of many of the uncertainties in the measured flow-rate. This example presents an analysis of such a comparison.

A company manufacturing automotive radiators uses an orifice plate to compare coolant flows through new radiator designs with those through a reference design.

G.2.2 Mathematical model

The flow performance of the radiator is expressed in terms of a flow factor, F, that is defined as in Equation (G.10):

$$F = \frac{q}{\sqrt{\Delta p_r}}$$

(G.10)

where

q is the volume flow-rate of coolant;

Δp_r the pressure difference across the radiator.

In the development of a new radiator, the interest is in the ratio, Φ_F, of the factor F_{exp} for the new design to F_{ref}, for that for a standard design. The measurand is thus calculated in accordance with Equations (G.11):

$$\Phi_F = \frac{F_{exp}}{F_{ref}} \text{ or } \Phi_F = \frac{\left(q_{exp}\big/\sqrt{\Delta p_{r,exp}}\right)}{\left(q_{ref}\big/\sqrt{\Delta p_{r,ref}}\right)}$$

(G.11)

$$= \frac{q_{exp} \cdot \sqrt{\Delta p_{r,ref}}}{q_{ref} \cdot \sqrt{\Delta p_{r,exp}}}$$

where the subscripts "exp" and "ref" refer to the experimental and reference radiators, respectively.

The flow-rate, q, is measured with an orifice plate and q is therefore given by Equation (G.12):

$$q = \left(\frac{C}{\sqrt{1-\beta^4}}\right) \left(\frac{\pi d_o^2}{4}\right) \sqrt{\frac{2\,\Delta p_{mt}}{\rho}}$$

(G.12)

where

C is the discharge coefficient;

d_o is the orifice diameter;

β is the ratio of d_o to the diameter of the pipe, d_p;

ρ is the fluid density;

Δp_{mt} is the pressure difference across the orifice meter.

Substituting from Equation (G.12) into Equation (G.11) yields Equation (G.13):

$$\Phi_F = \frac{\sqrt{\Delta p_{r,ref}} \cdot \left(\dfrac{C_{exp}}{\sqrt{1-\beta^4}}\right) \cdot \left(\dfrac{\pi d_o^2}{4}\right) \cdot \sqrt{\dfrac{2\,\Delta \rho_{mt,exp}}{\rho_{exp}}}}{\sqrt{\Delta p_{r,exp}} \left(\dfrac{C_{ref}}{\sqrt{1-\beta^4}}\right) \left(\dfrac{\pi d_o^2}{4}\right) \sqrt{\dfrac{2\,\Delta p_{mt,ref}}{\rho_{ref}}}}$$

(G.13)

Since the dimensions of the orifice plate remain constant, terms involving d_o and β cancel out, yielding Equation (G.14):

$$\Phi_F = \frac{\sqrt{\Delta p_{r,ref}} \cdot C_{exp} \cdot \sqrt{\rho_{ref} \; \Delta p_{mt,exp}}}{\sqrt{\Delta p_{r,exp}} \cdot C_{ref} \cdot \sqrt{\rho_{exp} \; \Delta p_{mt,ref}}}$$

(G.14)

The measurand, Φ_F, is therefore independent of the meter dimensions and of any uncertainty in those dimensions. Similarly, any uncertainties in C due to the positioning of the tappings or the sharpness of the orifice edge are frozen and do not affect the measurand, Φ_F. Then C will depend solely on the Reynolds number and if the tests are carried out with similar flow-rates, C_{exp} will equal C_{ref}, since C is only very weakly dependent on the Reynolds number. Thus, Equation (G.14) reduces to Equation (G.15):

$$\Phi_F = \sqrt{\frac{\Delta p_{r,ref} \cdot \rho_{ref} \cdot \Delta p_{mt,exp}}{\Delta p_{r,exp} \cdot \rho_{exp} \cdot \Delta p_{mt,ref}}}$$

(G.15)

G.2.3   Contributory variances

Substituting Equation (G.15) into Equation (19) yields Equation (G.16):

$$u_c^2\left(\Phi_F\right) = c_{\rho,exp}^2 \cdot u^2\left(\rho_{exp}\right) + c_{\Delta p,mt,exp}^2 \cdot u^2\left(\Delta p_{mt,exp}\right) + c_{\Delta p,r,ref}^2 \cdot u^2\left(\Delta p_{r,ref}\right) + c_{\Delta p,r,exp}^2 \cdot u^2\left(\Delta p_{r,exp}\right) + \ldots\ldots$$

$$\ldots + c_{\rho,ref}^2 \cdot u^2\left(\rho_{ref}\right) + c_{\Delta p,mt,ref}^2 \cdot u^2\left(\Delta p_{mt,ref}\right)$$

(G.16)

The relative sensitivity coefficients can be obtained by partial differentiation of Equation (G.15) to yield Equation (G.17):

$$\frac{u_c^2\left(\Phi_F\right)}{\Phi_F^2} = \frac{1}{4}\frac{u^2\left(\rho_{exp}\right)}{\rho_{exp}^2} + \frac{1}{4}\frac{u^2\left(\Delta p_{mt,exp}\right)}{\Delta p_{mt,exp}^2} + \frac{1}{4}\frac{u^2\left(\Delta p_{r,ref}\right)}{\Delta p_{r,ref}^2} + \frac{1}{4}\frac{u^2\left(\Delta p_{r,exp}\right)}{\Delta p_{r,exp}^2} + \ldots$$

$$\ldots\ldots + \frac{1}{4}\frac{u^2\left(\rho_{ref}\right)}{\rho_{ref}^2} + \frac{1}{4}\frac{u^2\left(\Delta p_{mt,ref}\right)}{\Delta p_{mt,ref}^2}$$

(G.17)

The relative sensitivity coefficients can then be defined as in Equations (G.18):

$$c_{\rho_{ref}}^* = c_{\Delta p_{mt,exp}}^* = c_{\Delta p_{r,ref}}^* = 0{,}5; \quad c_{\rho_{exp}}^* = c_{\Delta p_{mt,ref}}^* = c_{\Delta p_{r,exp}}^* = -0{,}5;$$

(G.18)

G.2.4   Uncertainty in density measurement

The density depends on the composition of the coolant (a water‑ethylene-glycol mixture) and on the temperature. Samples are drawn from the rig during each test and the density is estimated as the mean of four readings taken with a hydrometer. In the reference design test, the mean density is 1,070 kg/m³ and in the test with the experimental design, it is 1,065 kg/m³. The uncertainty in these values could be obtained from the standard deviation of each set of four readings but is more accurately determined from the pooled experimental standard deviations of a large number of earlier tests. Five previous tests are used to obtain a pooled standard deviation based on 10 sets each of four readings and the value obtained is 1,60 kg/m³. The standard uncertainty, expressed in kilograms per cubic metre, of the mean of four readings is then given by Equation (G.19):

$$u\left(\rho_{mt}\right) = s\left(\rho_{mt}\right) = 1{,}60 / \sqrt{4} = 0{,}80$$

(G.19)

The stated "uncertainty" of the hydrometer is 1 kg/m³ and this is taken to be an expanded uncertainty with a normal distribution ($k = 2$), yielding a standard uncertainty of 1 kg/m³ divided by 2, or 0,5 kg/m³. This uncertainty is correlated between the two density measurements. Since the densities are used as a ratio, the densities are negatively correlated, as indicated by the signs of the relative sensitivities calculated in G.2.3.

The uncertainties in the calibration of the hydrometer tend to cancel out, although they would only cancel completely if the densities were in fact equal. In the testing of the reference design, the relative uncertainty in the density due to the calibration of the hydrometer, $u^*(\rho_{ref})_{calib}$, is equal to 0,5 kg/m^3 divided by 1,070 kg/m^3, or 0,046 7 %. In the testing of the reference design, the relative uncertainty in the density due to the calibration of the hydrometer, $u^*(\rho_{exp})_{calib}$, is equal to 0,5 kg/m^3 divided by 1,065 kg/m^3, or 0,046 9 %. Substituting these values and those of the relative sensitivity coefficients calculated in G.2.3 into Equation (F.2), the combined uncertainty due the correlation between the two densities arising from the common calibration can be calculated in accordance with the Equations (G.20):

$$u_c^* = c_1^* u^*\left(x_1\right) + c_2^* u^*\left(x_2\right) + ... + c_N^* u^*\left(x_N\right)$$
$$= 0,5 \cdot 0,000\ 469 - 0,5 \cdot 0,000\ 467$$
$$= 0,000\ 001 \quad \text{or } 0,000\ 1\%$$

(G.20)

This confirms that in this case the densities are so nearly equal that the residual calibration uncertainty can be ignored.

The use of the standard deviation of multiple readings of the hydrometer to obtain the density makes it unnecessary to consider the effect of the resolution to which the hydrometer can be read. This source of uncertainty has already been accounted for as a contributor to the scatter of the values obtained, and to make any further allowance would result in it being counted twice.

The percentage uncertainty in each of the two densities is then 0,8 % divided by 1,070, or 0,075 %.

G.2.5   Uncertainty in manometer readings

All pressures in the test rig are measured using mercury-in-glass, U-tube manometers. As the pressures are used only in calculating pressure ratios, the manometer readings are used directly without the need to convert them to pressure units. In each case, four readings are taken and the mean is calculated, yielding the values given in Table G.2.

Table G.2 — Manometer readings

Manometer location	Mean mm Hg	Standard deviation mm Hg
Across orifice plate with reference radiator	264	1,7
Across orifice plate with experimental radiator	249	1,9
Across reference radiator	637	2,8
Across experimental radiator	632	2,6

As with the density measurements, the uncertainties can be obtained from the individual sets of readings or from a pooling of past test sets. However, there is a third option and that is to pool the experimental standard deviations of the two sets of orifice plate readings to obtain a standard deviation for that range of pressure differences and similarly to pool the radiator data for the larger pressure differences. The pooled standard deviation, s_{po}, is calculated in accordance with Equation (G.21):

$$s_{po} = \sqrt{\frac{\sum s_j^2 v_j}{\sum v_j}}$$

(G.21)

where

s_j is the standard deviation of set j;

v_j is the number of degrees of freedom in the standard deviation of set j, equal to the number of readings in set j minus 1.

Thus, the pooled experimental standard deviation, $s_{mt,po}$, expressed in millimetres of mercury, of the orifice plate readings is calculated in accordance with Equation (G.22):

$$s_{mt,po} = \sqrt{\frac{\left[(4-1)\cdot 1,7^2 + (4-1)\cdot 1,9^2\right]}{\left[(4-1)+(4-1)\right]}} = 1,8 \qquad (G.22)$$

and the pooled experimental standard deviation, $s_{r,po}$, expressed in millimetres of mercury, for the radiator readings is calculated in accordance with Equation (G.23):

$$s_{r,po} = \sqrt{\frac{\left[(4-1)\cdot 2,8^2 + (4-1)\cdot 2,6^2\right]}{\left[(4-1)+(4-1)\right]}} = 2,7 \qquad (G.23)$$

As the mean of the readings in each set is obtained from four repeated readings, the standard uncertainties of the means are 1,8 divided by $\sqrt{4}$, or 0,9 mm Hg for the orifice plate values and 2,7 divided by $\sqrt{4}$, or 1,35 mm Hg for the radiator.

As with the manometer readings, the resolution of the manometer scales has already been covered by the use of multiple readings and double accounting is avoided by making no further allowance for this source of uncertainty. Additional uncertainties will arise from imperfections in the manometer rulers but these are judged to be small compared with the standard uncertainties derived from the spread of readings and, following the guidance given in G.1.2.4, they are ignored.

G.2.6 Combined uncertainty in the flow ratio, Φ_F

The combined uncertainty in the measurand, Φ_F, is obtained from the uncertainty budget set out in Table G.3.

Table G.3 — Uncertainty budget for flow ratio, Φ_F

Source	Units	Value	Standard uncertainty	Relative standard uncertainty, $u^*(x_i)$ %	Relative sensitivity coefficient, c_i^*	Contribution to overall uncertainty $[c_i\,u(x_i)]^2$ 10^{-4}
Reference density	kg/m³	1 070	0,8	0,074 8	0,5	0,001 4
Experimental density	kg/m³	1 065	0,8	0,075 1	⁻0,5	0,001 4
Radiator Δp reference	mm Hg	637	1,35	0,211 9	0.5	0,011 2
Radiator Δp experimental	mm Hg	632	1,35	0,213 6	⁻0,5	0,011 4
Orifice plate Δp reference	mm Hg	264	0,9	0,340 9	⁻0,5	0,029 1
Orifice plate Δp experimental	mm Hg	249	0,9	0,361 4	0,5	0,032 7
Combined relative standard uncertainty, expressed in percent			$\sqrt{\sum\left[c_x^*\,u^*(x)\right]^2}$	0,295 2	$\sum\left[c_x^*\,u^*(x)\right]^2$	0,087 2

Table G.3 shows that the density measurements make only very small contributions to the overall uncertainty and can be ignored. The pressure differentials make almost equal contributions and should all be considered.

To obtain the expanded uncertainty at the 95 % confidence level, it is necessary to estimate the number of degrees of freedom in the standard uncertainty and this is done using Equation (C.1), the Welch-Satterthwaite equation.

The uncertainties in the two density values were obtained from a pooled experimental standard deviation derived from 10 sets of data with four readings in each set. There are therefore three degrees of freedom in each set and $3 \times 10 = 30$ in the pooled standard deviation.

The uncertainties in the four pressure-difference values were obtained from pooled standard deviations derived from two sets of data with four readings in each set. There are therefore three degrees of freedom in each set and $3 \times 2 = 6$ in each of the two pooled standard deviations.

The application of Equation (C.1) is set out in Table G.4.

Table G.4 — Calculation of effective degrees of freedom in combined standard uncertainty

Source	Degrees of freedom v_x	Relative standard uncertainty $u^*(x_i)$ %	Relative sensitivity coefficient c_i^*	Contribution to uncertainty $c_i^* u(x_i)$ %	$\dfrac{\left[c_i^* u^*(x_i) \right]^4}{v_i}$ 10^{-8}
Reference density	30	0,074 8	0,5	0,037 4	$0,652 \times 10^{-7}$
Experimental density	30	0,075 1	−0,5	−0,037 6	$0,663 \times 10^{-7}$
Radiator Δp reference	6	0,211 9	0,5	0,211 9	$0,210 \times 10^{-4}$
Radiator Δp experimental	6	0,213 6	−0,5	−0,213 6	$0,217 \times 10^{-4}$
Orifice plate Δp reference	6	0,340 9	−0,5	−0,170 5	$0,141 \times 10^{-3}$
Orifice plate Δp experimental	6	0,361 4	0,5	0,180 7	$0,178 \times 10^{-3}$
$\displaystyle\sum \frac{\left[c_i^* u^*(x_i) \right]^4}{v_i}$					0,000 361
Relative combined standard uncertainty					0,295 2 %
Combined effective degrees of freedom					21

With 21 degrees of freedom, Table C.1 gives a coverage factor of 2,13 at the 95 % confidence level and the expanded uncertainty U_{95} of the flow ratio is thus equal to 2,13 times 0,295 %, or 0,63 %. Had the experimental data for the manometer readings not been pooled, the degrees of freedom for each differential pressure in Table G.4 would have been three and the analysis of that table would have resulted in an overall effective degrees of freedom of 10. This would have given a value of 2,28 for k at the 95 % confidence level and an expanded uncertainty U_{95} of the flow ratio of 2,28 times 0,295 %, or 0,67 %.

G.3 Example 3 — Computation of the uncertainty of flow measurement made by an orifice plate

G.3.1 General

An orifice plate has been manufactured to the requirements of ISO 5167-2 [5]. Its dimensions are measured in the workshop inspection department at 20 °C and the device is then used with D and D/2 tappings to meter the flow of an industrial liquid at a process temperature of 170 °C.

The practical computational method given in ISO 5167-1:2003 [1] is fully compliant with the method given in this International Standard as second-order effects and issues of correlation are assessed prior to the application of Equation (3) to the key parameters. However, a more rigorous approach is followed here to illustrate such points as the handling of correlation. The approach given here goes beyond what is necessary in most practical applications of an orifice plate, where the method of ISO 5167-1:2003 [1] would be adequate.

G.3.2   The mathematical model

The mathematical model is given by Equation (24):

$$q_{ma} = \frac{C}{\sqrt{1-\beta^4}} \frac{\pi d_o{}^2}{4} \sqrt{2\rho\Delta p_{mt}} \tag{G.24}$$

and C is given by Equation (25), the Reader-Harris/Gallagher (1998) equation [8]:

$$C = 0{,}596\ 1 + 0{,}026\ 1\,\beta^2 - 0{,}216\,\beta^8 + 0{,}000\ 521\cdot\left(\frac{10^6\beta}{Re_{dp}}\right)^{0{,}7} + \ldots$$

$$\ldots + \left(0{,}018\ 8 + 0{,}006\ 3 F_{Redp}\right)\cdot\beta^{3{,}5}\cdot\left(\frac{10^6}{Re_{dp}}\right)^{0{,}3} + \ldots \tag{G.25}$$

$$\ldots + \left(0{,}043 + 0{,}080e^{-10L_1} - 0{,}123e^{-7L_1}\right)\cdot\left(1 - 0{,}11F_{Redp}\right)\cdot\left(\frac{\beta^4}{1-\beta^4}\right) - 0{,}031\left(M'_2 - 0{,}8M'_2{}^{1{,}1}\right)\beta^{1{,}3}$$

where

β is the orifice plate diameter ratio, equal to d_o/d_p;

d_o is the diameter of the orifice plate bore;

d_p is the pipe diameter;

ρ is the fluid density;

Δp_{mt} is the pressure difference across the orifice plate;

Re_{dp} is the Reynolds number related to d_p by the expression $Vd_p\rho/\mu$;

V is the mean velocity in the pipe;

μ is the fluid viscosity;

L_1 is the distance, l_1, from the upstream tapping to the upstream face divided by the pipe diameter, d_p;

> NOTE 1   As the meter is designed and installed in accordance with the requirements of ISO 5167-2, L_1 can be equated to 1 and the dependence on l_1 can be dropped from the analysis (ISO 5167-2:2003, 5.3.2.1) [5];

L'_2 is the distance, l'_2, from the downstream tapping to the downstream face divided by the pipe diameter, d_p;

> NOTE 2   As the meter designed and installed in accordance with the requirements of ISO 5167-2, L'_2 can be equated to 0,47 and the dependence on l'_2 can be dropped from the analysis (ISO 5167-2:2003, 5.3.2.1) [5];

M'_2 is equal to $2L'_2/(1-\beta)$;

F_{Redp} is equal to $\left(19\ 000\cdot\beta/Re_{dp}\right)^{0{,}8}$.

As the plate and pipe dimensions are measured at a temperature different from the operating condition, the expansion of the plate and the pipe should be taken into account. All the components are made of duralumin with an expansion coefficient, λ equal to $27 \times 10^{-6}/°C$. A typical linear dimension, x, is then given by Equation (G.26):

$$x = x_0 \cdot \left[1 + \lambda\left(T_{op} - T_{0,x}\right)\right]$$ (G.26)

where

x_0 is the dimension at temperature $T_{0,x}$;

T_{op} is the operating temperature.

All length-dependent parameters, such as β and M'_2 can then be coded into the model in terms of their dimension at $T_{0,x}$ and their expansion. For example, β is replaced in Equation (G.24) by the expression given in Equation (G.27):

$$\beta = \left\{d_{o,0}\left[1 + \lambda_{do}\left(T_{op} - T_{0,x,do}\right)\right]\right\} \Big/ \left\{d_{p,0}\left[1 + \lambda_{dp}\left(T_{op} - T_{0,x,dp}\right)\right]\right\}$$ (G.27)

In this way, all correlations due to temperature are eliminated, at the expense of making Equation (G.25) more complex.

G.3.3 Contributory variances

It is clear from Equations (G.24) and (G.25) that the measured flow-rate will depend on a number of measurements in a very complex way. The basic measurements fall into two groups: those relating to the basic geometry of the meter and those relating to the operating conditions. Uncertainties in the first group will be fixed for all measurements made with the orifice plate while uncertainties in the second group will be different for each measurement.

The functional relationship between q_{ma} and the input variables is too complex for an analytical approach and the numerical approach to the calculation of sensitivity coefficients is the only practical method available. Nevertheless Equation (19) can be applied in the form of Equation (G.28):

$$u_c^2\left(q_{ma}\right) = c_1^2 u^2\left(1\right) + c_2^2 u^2\left(2\right) + \ldots + c_n^2 u^2\left(n\right)$$ (G.28)

where

c_i is the sensitivity coefficient for input variable i;

$u(i)$ is the uncertainty in input variable, i.

The n input variables and their nominal values are as follows:

— $d_{o,0}$ 60 mm;

— $d_{p,0}$ 100 mm;

— $T_{0,x}$ 20 °C;

— T_{op} (actual operating temperature);

— $T_{op\ nominal}$ 170 °C;

— Δp 5 500 Pa;

— λ $27 \times 10^{-6}/°C$;

— ρ $937,5 \times [1 - 0,006\,0 \times (T_{op} - T_{op\ nominal})]$ kg/m³

— μ $604,0 \times [1 - 0,014\,1 \times (T_{op} - T_{op\ nominal})] \times 10^{-6}$ Pa·s

Several parameters (d_o, d_p, ρ and μ) can be seen to be dependent on temperature and the uncertainties in these values arising from the uncertainty in the determination of the process temperature will all be correlated. This complicates the calculation of overall uncertainty but the difficulty can be overcome by coding each of the temperature dependencies into the spreadsheet calculation of the sensitivities. In this way, the second order effects of temperature on C through the changes in Reynolds number, etc are taken into account.

The Reader-Harris/Gallagher (1998) equation is a best fit to the available data and is therefore subject to some uncertainty; a sensitivity coefficient for the basic value of C is therefore required.

The results of the sensitivity analysis are set out in Table G.5.

Table G.5 — Calculation of sensitivity coefficients

Parameter	Increment	Parameter											
		$d_{p,0}$ m	$d_{o,0}$ m	$T_{0,x}$ °C	T_{op} °C	ρ kg/m³	Δp Pa	$\lambda \times 10^6$ per °C	$\mu \times 10^6$ Pa·s	C -	q_{ma} kg/s	c	c^*
		Increment											
		0,100 0	0,060 0	20,0	170,0	937,5	5 500	27,0	604,0	0,600	5,994 0	—	—
$d_{p,0}$	0,000 1	**0,100 1**	0,060 0	20,0	170,0	937,5	5 500	27,0	604,0	0,600	5,992 0	−20,59	−0,344
$d_{o,0}$	0,000 1	0,100 0	**0,060 1**	20,0	170,0	937,5	5 500	27,0	604,0	0,600	6,017 6	235,3	2,352
$T_{0,x}$	0,2	0,100 0	0,060 0	**20,2**	170,0	937,5	5 500	27,0	604,0	0,600	5,994 0	0,000 3[a]	−0,001[a]
T_{op}	0,2	0,100 0	0,060 0	20,0	**170,2**	937,5	5 500	27,0	604,0	0,600	5,990 4	−0,018 1	−0,514
ρ	1	0,100 0	0,060 0	20,0	170,0	**938,5**	5 500	27,0	604,0	0,600	5,997 2	0,003 2	0,500
Δp	5	0,100 0	0,060 0	20,0	170,0	937,5	**5 505**	27,0	604,0	0,600	5,996 8	0,000 5	0,500
λ	1	0,100 0	0,060 0	20,0	170,0	937,5	5 500	**28,0**	604,0	0,600	5,995 8	1795,6	0,008
μ	1	0,100 0	0,060 0	20,0	170,0	937,5	5 500	27,0	**605,0**	0,600	5,994 1	49,98	0,005
C	0,001	0,100 0	0,060 0	20,0	170,0	937,5	5 500	27,0	604,0	**0,601**	6,004 0	9,990	1,000

[a] Values for c and c^* in this row result from changes in q_{ma} that are too small to be displayed in the table.

G.3.4   Uncertainty in measured pipe diameter, $d_{p,0}$

The pipe diameter is measured by internal micrometer across four diameters of the pipe and the mean of these values is taken as the value of d_p. The micrometer is calibrated with a stated expanded uncertainty ($k = 2$) of 0,01 mm, giving a standard uncertainty of 0,005 mm. The micrometer has a resolution of 0,01 mm, this is treated as a rectangular distribution with equal probability for all values ($k = \sqrt{3} = 1,73$); the standard uncertainty is therefore 0,01 mm divided by 2 then divided by $\sqrt{3}$, or 0,002 9 mm. The usage of the micrometer introduces a further uncertainty and this is assessed as a rectangular distribution ($k = 1,73$) with a range of 0,04 mm, giving a standard uncertainty of 0,011 5 mm. The use of the mean of four readings will reduce the impact of the uncertainty due to the resolution and usage of the micrometer as the uncertainties in successive readings are unrelated but the averaging process will not affect the uncertainty due to the calibration, which is correlated across all readings and affects all readings equally. The resolution and usage uncertainties are therefore summed in quadrature, and divided by $\sqrt{n} = \sqrt{4} = 2$, before being added in quadrature to the calibration uncertainty.

Thus, the combined standard uncorrelated uncertainty, expressed in millimetres, in a single reading is given by Equation (G.29):

$$u(d_{\mathrm{p},0})_{\mathrm{sm}} = \sqrt{(0,002\ 9^2 + 0,011\ 5^2)}$$
$$= 0,011\ 9$$

(G.29)

The combined standard uncorrelated uncertainty in the mean of four readings is then 0,011 9 divided by the square root of n, where $n = 4$, or 0,005 9 mm.

The total combined standard uncertainty, expressed in millimetres, in the measurement of diameter is given by Equation (G.30):

$$u(d_{\mathrm{p},0}) = \sqrt{(0,005\ 9^2 + 0,005^2)}$$
$$= 0,007\ 8$$

(G.30)

The expanded value ($k = 2$) is 0,015 5 mm. For a nominal value of $d_{\mathrm{p}} = 100$ mm, this gives a relative uncertainty of 0,016 %.

G.3.5   Uncertainty in measured orifice diameter, $d_{\mathrm{o},0}$

The bore of the orifice plate is measured, using the same procedure, with a micrometer of smaller size. The analysis is identical to that for d_{p} and the resulting expanded ($k = 2$) uncertainty is 0,015 5 mm. The nominal value of $d_{\mathrm{o}} = 60$ mm and the relative uncertainty is therefore 0,026 %.

G.3.6   Uncertainty in temperature, $T_{0,x}$

The workshop inspection department is maintained at a temperature of 20 °C $\pm$ 2 °C. This is taken to be a rectangular distribution giving a standard uncertainty of 2 °C divided by $\sqrt{3}$, or 1,15 °C. With a sensitivity coefficient of 0,001, no further analysis of, for example, the calibration of the thermometer is considered necessary.

G.3.7   Uncertainty in the fluid temperature, T_{op}

The fluid temperature is measured using a platinum resistance thermometer with a stated calibration uncertainty of 0,2 °C ($k = 2$), giving a standard uncertainty of 0,1 °C. The indicating device has a scale interval of 0,2 °C giving a standard uncertainty of 0,058 °C. The usage uncertainty is assessed on the basis that the thermometer is installed in a temperature pocket that is in good condition but the fluid has low thermal conductivity and an uncertainty value of 1 °C is assumed. This is taken as having a rectangular distribution giving a standard uncertainty of 0,58 °C. The flow-rate is calculated from a single measurement of temperature and the combined uncertainty of the temperature, expressed in degrees Celsius, is therefore given by Equation (G.31):

$$u\left(T_{\mathrm{op}}\right) = \sqrt{\left(0,1^2 + 0,058^2 + 0,58^2\right)^2} = 0,59$$

(G.31)

This gives an expanded uncertainty of 1,18 °C ($k = 2$).

G.3.8   Uncertainty in density, ρ

The equation used to represent the temperature dependence of the fluid density is known to fit the data with an expanded uncertainty of 2 % ($k = 2$) and the standard uncertainty is therefore 1 % or 9,4 kg/m³. The usage uncertainty arising from uncertainties in the measurement of the fluid temperature has been accounted for in the analysis of the impact of uncertainty in T_{op} and need not be considered again.

G.3.9   Uncertainty in pressure difference, Δp

The pressure difference across the orifice plate is measured using a differential pressure transmitter with a calibration uncertainty of 0,5 % ($k = 2$) giving a standard uncertainty of 0,25 % or 13,75 Pa. The readout has a

resolution of 10 Pa giving a standard uncertainty of 2,9 Pa. To allow for factors such as the operating environment a usage uncertainty of 1 % of reading is assumed and this is taken as a rectangular distribution giving a standard uncertainty, expressed in percent, of 1 divided by $\sqrt{3}$, or 0,58 % of reading or 31,75 Pa. As the flow rate is derived from a single reading of the pressure differential, the combined uncertainty in the pressure differential, expressed in pascals, is calculated in accordance with Equation (G.32):

$$u\left(\Delta p\right) = \sqrt{\left(13{,}75^2 + 2{,}9^2 + 31{,}75^2\right)^2} = 35 \tag{G.32}$$

The expanded uncertainty ($k = 2$) is therefore 70 Pa or 1,27 %.

G.3.10   Uncertainty in thermal expansion coefficient, λ

The thermal expansion coefficient has a quoted uncertainty of 5 %, and it is assumed that all values in this range are equally likely, giving a standard uncertainty of 2,89 % or $7{,}8 \times 10^{-7}$/°C.

G.3.11   Uncertainty in fluid viscosity, μ

The equation used to represent the temperature dependence of the fluid viscosity is known to fit the data with an expanded uncertainty of 3 % ($k = 2$) and the standard uncertainty is therefore 1,5 % or $9{,}1 \times 10^{-6}$ Pa·s. The usage uncertainty arising from uncertainties in the measurement of the fluid temperature has been accounted for in the analysis of the impact of uncertainty in T_{op} (G.3.8) and need not be considered again.

G.3.12   Uncertainty in the Reader-Harris/Gallagher (1998) equation

The Reader-Harris/Gallagher (1998) equation is known to fit the data with an expanded uncertainty of 0,5 % ($k = 2$) and the standard uncertainty is therefore 0,25 %. The nominal value of the discharge coefficient is 0,6 giving an absolute standard uncertainty of 0,001 5.

G.3.13   Combined uncertainty in the flow-rate

Although relative sensitivity coefficients have been calculated in Table G.5, the temperature inputs have arbitrary zeros and the use of relative values is therefore inappropriate. The overall combined uncertainty is, therefore, calculated in absolute terms as defined in Table G.6.

Table G.6 — Uncertainty budget for orifice plate example

Source of uncertainty	Unit	Nominal value	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	Contribution to overall uncertainty $[c_i u(x_i)]^2$
Pipe diameter, d_p	metre	0,1	0,000 008	−20,59	$27{,}1 \times 10^{-9}$
Orifice bore, d_o	metre	0,06	0,000 008	235,3	$3{,}54 \times 10^{-6}$
Inspection temperature, $T_{0,x}$	degrees Celsius	20	1,15	−0,000 3	$0{,}119 \times 10^{-6}$
Fluid temperature, T_{op}	degrees Celsius	170	0,59	−0,018 1	0,000 114
Fluid density, ρ	kilograms per cubic metre	937,5	9,4	0,003 2	0,000 905
Pressure differential, Δp	pascals	5 500	35	0,000 5	0,000 306
Thermal expansion coefficient, λ	per degree Celsius	27×10^{-6}	$0{,}78 \times 10^{-6}$	1 795,6	$1{,}96 \times 10^{-6}$
Fluid viscosity, μ	pascal·seconds	604×10^{-6}	$9{,}1 \times 10^{-6}$	49,98	$0{,}207 \times 10^{-6}$
Discharge coefficient, C	—		0,001 5	9,990	0,000 225
—	—	—	$u(q_{ma})$ 0,039 4	$\Sigma[c_i u(x_i)]^2$	0,001 55

The standard uncertainty in the flow rate $u(q_{ma})$ is therefore 0,039 4 kg/s and the expanded uncertainty ($k = 2$) $U_{95}(q_{ma})$ is 0,078 9 kg/s. The best estimate of the flow rate is 5,994 kg/s giving a relative expanded uncertainty of 1,31 %. Table G.6 shows that the only significant contributors to uncertainty in the flow-rate are the fluid temperature, fluid density, pressure differential and the basic correlation of the Reader-Harris/Gallagher (1998) equation.

G.4 Example 4 — Computation of uncertainty in a flow (discharge) measurement made by a velocity-area survey using a current meter

G.4.1 Mathematical model

The measurement method, known as current-meter gauging, consists of dividing the channel cross-section under consideration into segments by m'' verticals and measuring the breadth, depth and mean velocity associated with each vertical i. The mean velocity, V_i, at each vertical is computed from point velocity measurements made at each of several depths on the vertical. The flow is computed in accordance with Equation (G.33):

$$Q = F_s \sum b_i d_i V_i \tag{G.33}$$

where

Q is the flow, expressed in cubic metres per second;

F_s is a factor, assumed to be unity, that relates the discrete sum over the finite number of verticals to the integral of the continuous function over the cross-section;

b_i is the breadth associated with vertical i;

d_i is the depth associated with vertical i;

V_i is the mean velocity associated with vertical i.

G.4.2 Contributory variances

The relative combined standard uncertainty in the measurement is given by Equation (G.34) [6]:

$$u^*(Q)^2 = u_{m''}^{*\,2} + u_{cal}^{*\,2} + \sum_{i=1}^{m''}\left[\left(b_i d_i V_i\right)^2\left(u_{bi}^{*\,2} + u_{di}^{*\,2} + u_{Vi}^{*\,2}\right)\right] \bigg/ \left[\sum_{i=1}^{m''}\left(b_i d_i V_i\right)\right]^2 \tag{G.34}$$

where

$u^*(Q)$ is the relative combined standard uncertainty in discharge;

$u_{bi}^{*}, u_{di}^{*}, u_{Vi}^{*}$ are the relative standard uncertainties in the breadth, depth, and mean velocity measured at vertical i;

u_{cal}^{*} is the relative uncertainty due to calibration errors in the current meter, breadth measurement instrument, and depth-sounding instrument, and is equal to $\sqrt{u_{cm}^{*\,2} + u_{bm}^{*\,2} + u_{ds}^{*\,2}}$. An estimated practical value of 1 % may be taken for this expression;

u_{cm}^{*} is the relative uncertainty in the calibration of the current meter;

u^*_{bm} is the relative uncertainty in the calibration of the breadth measurement;

u^*_{ds} is the relative uncertainty in the calibration of the depth-sounding instrument;

$u^*_{m''}$ is the relative uncertainty due to the limited number of verticals;

m'' is the number of verticals.

The mean velocity, V_i, at vertical i is the average of point measurements of velocity made at several depths in the vertical. The uncertainty in V_i is computed in accordance with Equation (G.35):

$$u^*\left(V_i\right)^2 = u^{*\,2}_{\text{p}i} + \left(\frac{1}{n''}\right)\left(u^{*\,2}_{\text{cr}i} + u^{*\,2}_{\text{e}i}\right) \tag{G.35}$$

where

$u^*_{\text{p}i}$ is the relative uncertainty in mean velocity, V_i, due to the limited number of depths at which velocity measurements are made at vertical, i;

n'' is the number of depths in the vertical at which velocity measurements are made;

$u^*_{\text{cr}i}$ is the relative uncertainty in point velocity at a particular depth in vertical i due to the variable responsiveness of the current meter;

$u^*_{\text{e}i}$ is the relative uncertainty in point velocity at a particular depth in vertical i due to velocity fluctuations (pulsations) in the stream.

Combining Equations (G.34) and (G.35) yields Equation (G.36):

$$u^*(Q)^2 = u^{*\,2}_{m''} + u^{*\,2}_{\text{cal}} + \sum_{i=1}^{m''}\left\{\left(b_i d_i V_i\right)^2\left[u^{*\,2}_{\text{b}i} + u^{*\,2}_{\text{d}i} + u^{*\,2}_{\text{p}i} + \left(\frac{1}{n''}\right)\left(u^{*\,2}_{\text{cr}i} + u^{*\,2}_{\text{e}i}\right)\right]\right\} \Bigg/ \left[\sum_{i=1}^{m''}\left(b_i d_i V_i\right)\right]^2 \tag{G.36}$$

If the measurement verticals are placed so that the segment discharges $(b_i\, d_i\, V_i)$ are approximately equal and if the component uncertainties are equal from vertical to vertical, then Equation (G.36) simplifies to Equation (G.37):

$$u^*\left(Q\right) = \left\{u^{*\,2}_{m''} + u^{*\,2}_{\text{cal}} + \left(\frac{1}{m''}\right)\left[u^{*\,2}_{\text{b}} + u^{*\,2}_{\text{d}} + u^{*\,2}_{\text{p}} + \left(\frac{1}{n''}\right)\left(u^{*\,2}_{\text{cr}} + u^{*\,2}_{\text{e}}\right)\right]\right\}^{1/2} \tag{G.37}$$

It is required to calculate the uncertainty in a current-meter gauging from the following particulars:

— number of verticals used in the gauging: 20;

— number of points taken in the vertical (0,2 and 0,8):   2.

Component uncertainties (as percentages) can be obtained from ISO 748:1997 [6], Tables E.1 to E.6 as follows:

— $u_{m''}$ 2,5 %   (Table E.6);

— u_{cal} 1,0 %   (see above) ;

— u_{b} 0,5 %   (Table E.1);

— u_{d} 0,5 %   (Table E.2);

— u_{p} 3,5 %   (Table E.4);

— u_{cr} 1,0 %   (Table E.5);

— u_e 2,5 %   (at 0,2 depth) (Table E.3);

— u_e 2,5 %   (at 0,8 depth) (Table E.3).

NOTE   The values of the component uncertainties in ISO 748, which are expressed at the 95 % confidence level, have been halved and expressed at one standard deviation.

The entire uncertainty calculation then becomes a Type B evaluation of uncertainty since the component uncertainties quoted in ISO 748:1997, Annex E, are based on previous measurements and calibration data.

G.4.3   Combined uncertainty

The combined uncertainty can be calculated from Equation (G.37), yielding Equation (G.38):

$$u^*(Q) = \left\{ u_{m''}^{*\,2} + u_{cal}^{*\,2} + \left(\frac{1}{m''}\right)\left[u_b^{*\,2} + u_d^{*\,2} + u_p^{*\,2} + \left(\frac{1}{n''}\right)\left(u_{cr}^{*\,2} + u_e^{*\,2} \right) \right] \right\}^{\frac{1}{2}} \tag{G.38}$$

$$= \left\{ 2,5^2 + 1,0^2 + \left(\frac{1}{20}\right)\left[0,5^2 + 0,5^2 + 3,5^2 + \left(\frac{1}{2}\right)\left(1,0^2 + 2,5^2 \right) \right] \right\}^{\frac{1}{2}} \%$$

$$= 2,84 \%$$

The expanded uncertainty at the 95 % confidence level, U_{95}, is obtained by applying a coverage factor of $k = 2$ as given in Equation (G.39):

$$U_{95}^*(Q) = k\,u^*(Q) \tag{G.39}$$

$$= 2 \times 2,84 \%$$

$$= 5,68 \%$$

Therefore, $U_{95}^*(Q) \approx 6 \%$.

If the best estimate of the measured flow, $\{Q\}$, is expressed in cubic metres per second, the result of the measurement is expressed as follows:

$Q = \{Q\}$ m³/s ± 0,06 $\{Q\}$ m³/s, (expanded uncertainty, coverage factor $k = 2$, approximate confidence level of 95 %).

G.5   Example 5 — Computation of uncertainty in a flow (discharge) measurement made using weirs and flumes

G.5.1   The mathematical model

The general equation for the determination of discharge through a weir or flume is given in Equation (G.40):

$$Q = C \cdot l_b \cdot l_h^{n'} \tag{G.40}$$

where

C is the coefficient of discharge;

l_b is the length of crest;

l_h is the gauged head;

n' is an exponent of l_h, usually 1,5 for a rectangular weir and 2,5 for a V-notch.

Details are given in a range of ISO standards for different types of weirs and flumes.

G.5.2 Contributory variances

The combined relative (percentage) standard uncertainty for a single determination of discharge can be obtained by substituting Equation (G.40) into Equation (19), the sensitivity coefficients being obtained by the partial differentiation of Equation (G.40) to yield Equation (G.41):

$$u^*(Q) = \left(u_d^{*2} + u_{lb}^{*2} + n'^2 u_{lh}^{*2} + u_{cal}^{*2} \right)^{1/2} \tag{G.41}$$

where

$u^*(Q)$ is the combined relative standard uncertainty in the discharge;

u^*_d is the relative standard uncertainty in the coefficient of discharge;

u^*_{lb} is the relative standard uncertainty in the measurement of the crest length;

u^*_{lh} is the relative standard uncertainty in the measurement of the gauged head;

u^*_{cal} is the instrument calibration uncertainty from all sources, formerly called systematic errors or biases.

The exponent, n', is assumed not to be subject to uncertainty.

Type A evaluations of the uncertainties in breadth and head can be made by repeated observations of those quantities by the user. Alternatively, recommended values (Type B evaluations) for the uncertainty in the discharge coefficient as well as for the uncertainties in the measurements of breadth and head are given in the ISO standards on weirs and flumes. The uncertainty values should include allowances for instrument calibration errors, denoted by u^*_{cal}, in Equation (G.41). These remain constant from observation to observation and are not reduced by averaging of repeated observations.

Typical values for relative standard uncertainties in discharge measurements made by use of thin-plate weirs are as follows (ISO 1438-1 [7]):

— u^*_d 1,0 %;

— u^*_{lb} 0,05 %;

— u^*_{lh} 0,5 %;

— u^*_{cal} 0,5 %.

The entire uncertainty calculation is then a Type B evaluation of uncertainty since the component uncertainties in ISO 1438-1 are based on previous measurements and calibration data.

Then, the combined relative standard uncertainty, $u^*(Q)$, expressed in percent, in the discharge is calculated in accordance with Equation (G.41):

$$u^*(Q) = \left[1,0^2 + 0,05^2 + \left(1,5^2 \cdot 0.5^2 \right) + 0,5^2 \right]^{1/2} \%$$

$$= 1,35 \%$$

The expanded uncertainty, with a coverage factor $k = 2$ and an approximate confidence level of 95 %, is calculated as shown in Equation (G.42):

$$U^*_{95}(Q) = ku^*(Q) \qquad\qquad\qquad (G.42)$$

$$= 2 \times 1,35 \ \%$$

$$= 2,70 \ \%$$

If the best estimate of the measured flow, Q, is expressed in cubic metres per second, the result of the measurement is presented as as follows:

$\{Q\}$ m^3/s $\pm$ 2,7 $\{Q\}$ m^3/s (expanded uncertainty, coverage factor $k = 2$, approximate confidence level of 95 %).

Annex H
(informative)

The calibration of a flow meter on a calibration rig

H.1 General

This annex describes the estimation of the uncertainty of a flowmeter calibrated on a calibration rig of known uncertainty. It also includes an estimation of the Type A uncertainty of a single measurement from the flowmeter under calibration.

H.2 Calibration rig uncertainty

When a flowmeter is calibrated on a calibration rig, the traceability and combined uncertainty of the calibration rig should be determined prior to the calibration. An assessment of the repeatability of the calibration rig should also be made for use when the meter under calibration is to be calibrated with only a single reading at each flow rate. The combined uncertainty of the calibration rig, U_{CMC}, ("calibration and measurement capability" or "calibration rig uncertainty") is derived from all uncertainty sources affecting the rig and is calculated in such a way that it represents the uncertainty of the quantity of fluid passing through the flowmeter under calibration. The uncertainty therefore contains contributions from the following:

a) the uncertainty of the reference device used (proving tank, bell prover, or weighing scale, etc);

b) the uncertainty of the temperature/pressure measurements in the reference device and near the flowmeter under calibration, including any equations applied to correct for expansion and compressibility;

c) the uncertainty of the transfer point when the "standing start and stop" method is used;

d) the uncertainty in the diverter (used with "flying start and stop" method);

e) the uncertainty in buoyancy when the weighing method is used.

U_{CMC} should also reflect variations in operating temperature and pressure during the calibration and any uncertainties arising from the calculation procedure used to derive the meter error or K-factor of the flowmeter under calibration.

In most cases, the U_{CMC} will be expressed as either a fraction or a percentage and will normally be for a confidence level of at least 95 %.

H.3 Use of the calibration rig

H.3.1 General

Before starting the calibration of a meter using the calibration rig, what is expected from this calibration should be clearly understood so that the calibration certificate can include an appropriate statement of the uncertainties taken into account.

a) If the uncertainty of each measurement has to be stated, the combined uncertainty in a single measurement (U_{CS}) should be stated in the results of the calibration; U_{CS} should also be stated when the meter is being assessed against acceptance limits.

b) If the stability of the meter over time is the subject of interest, the combined uncertainty in the mean (U_{CM}) should be quoted.

c) If the meter is to be used as a reference meter to calibrate other flowmeters (master-meter method), the combined uncertainty (U_{CM}) should again be quoted.

d) If the repeatability of the meter is the subject of interest, then the uncertainty of interest is the type A uncertainty in a single measurement (U_{AS}).

H.3.2 Calibration at a number of different flowrates with n measurements per flowrate

H.3.2.1 At each flow-rate, the mean meter error is given by Equation (H.1):

$$\overline{E} = \sum_{j=1}^{n} E_j \Big/ n \tag{H.1}$$

where

$\overline{E}$ is the mean meter error, expressed as a fraction;

E_j is the jth meter error, expressed as a fraction;

n is the number of measurements at this flow-rate.

The mean K-factor is given by Equation (H.2):

$$\overline{K} = \sum_{j=1}^{n} K_j \Big/ n \tag{H.2}$$

where

$\overline{K}$ is the mean K-factor;

K_j is the jth K-factor;

n is the number of measurements at this flow-rate.

H.3.2.2 At each flow-rate, the overall type A uncertainty in meter error or K-factor, with confidence level of at least 95 %, is calculated.

To demonstrate the procedure in both absolute and relative terms, Equation (H.3) gives the calculation in absolute terms for meter error and Equation (H.4), in relative terms for the K-factor.

$$U_{\text{AS-overall-}E} = k \sqrt{\frac{\sum_{j=1}^{n} (E_j - \overline{E})^2}{(n-1)}} \tag{H.3}$$

where

$U_{\text{AS-overall-}E}$ is the type A uncertainty in meter error;

$\overline{E}$ is the mean meter error as a fraction;

E_j is the jth meter error as a fraction;

n is the number of measurements at this flow-rate;

k is the coverage factor.

$$U^{*}_{\text{AS-overall-}K} = \frac{k}{\overline{\overline{K}}} \sqrt{\frac{\sum\limits_{j=1}^{n}(K_j - \overline{K})^2}{(n-1)}} \qquad \text{(H.4)}$$

where

$U^{*}_{\text{AS-overall-}K}$ is the type A uncertainty in the K-factor;

$\overline{K}$ is the mean K-factor;

K_j is the jth K-factor;

n is the number of measurements at this flow-rate;

k is the coverage factor.

If the purpose of the calibration is to assess the repeatability of the meter, the result is either $U_{\text{AS-}E}$ or $U_{\text{AS-}K}$ as appropriate.

H.3.2.3   At each flow-rate, the type A uncertainty in the mean meter error (in absolute terms) or mean K-factor (in relative terms) can then be calculated, from Equation (H.5) or (H.6), respectively:

$$U_{\text{AM-}E} = \frac{U_{\text{AS-overall-}E}}{\sqrt{n}} \qquad \text{(H.5)}$$

$$U^{*}_{\text{AM-}K} = \frac{U^{*}_{\text{AS-overall-}K}}{\sqrt{n}} \qquad \text{(H.6)}$$

H.3.2.4   At each flow-rate, the combined uncertainty for a single measurement, is given by Equation (H.7) (in absolute terms) or (H.8) (in relative terms):

$$U_{\text{CS-}E} = \sqrt{U_{\text{AS-overall-}E}{}^2 + U_{\text{CMC}}{}^2} \qquad \text{(H.7)}$$

$$U^{*}_{\text{CS-}K} = \sqrt{\left(\frac{U_{\text{AS-overall-}K}}{\overline{K}}\right)^2 + U^{*}_{\text{CMC}}{}^2} = \sqrt{U^{*}_{\text{AS-overall-}K}{}^2 + U^{*}_{\text{CMC}}{}^2} \qquad \text{(H.8)}$$

H.3.2.5   At each flow-rate, the combined uncertainty for the mean, is given by Equation (H.9) (in absolute terms) or (H.10) (in relative terms):

$$U_{\text{CM-}E} = \sqrt{U_{\text{AM-}E}{}^2 + U_{\text{CMC}}{}^2} \qquad \text{(H.9)}$$

$$U^{*}_{\text{CM-}K} = \sqrt{\left(\frac{U_{\text{AM-}K}}{\overline{K}}\right)^2 + U^{*}_{\text{CMC}}{}^2} = \sqrt{U^{*}_{\text{AM-}K}{}^2 + U^{*}_{\text{CMC}}{}^2} \qquad \text{(H.10)}$$

The uncertainties calculated can be different at different flow-rates; in this case the calibration certificate should state the values obtained at each flow-rate. However, if a single uncertainty is required, the certificate should state the largest value obtained.

Annex I
(informative)

Type A and Type B uncertainties in relation to contributions to uncertainty from "random" and "systematic" sources of uncertainty

In comparison with ISO/TR 5168:1998 [9], this International Standard contains significant changes in that the concepts and terminology of "random" and "systematic" components of uncertainty are no longer the preferred categories. There are two main reasons for this.

a) In conformance with GUM, components of uncertainty arising from random or systematic causes, after they have been evaluated, are treated identically.

b) The terms can be used in ways that are ambiguous or confusing.

The following two paragraphs are taken from GUM (1995), Annex E, 3.6 and 3.7:

"An uncertainty component is not either "random" or "systematic". Its nature is conditioned by the use made of the corresponding quantity, or more formally, by the context in which the quantity appears in the mathematical model that describes the measurement. Thus, when its corresponding quantity is used in a different context, a "random" component may become a "systematic" component, and vice-versa.

For the reason given above, Recommendation INC-1 (1980) [10] does not classify components of uncertainty as either "random" or "systematic". In fact, as far as the calculation of the combined standard uncertainty of a measurement result is concerned, there is no real need for any classificational scheme. Nonetheless, since convenient labels can sometimes be helpful in the communication and discussion of ideas, Recommendation INC-1 (1980) does provide a scheme for classifying the two distinct *methods* by which uncertainty components can be evaluated, as Type "A" and Type "B"."

When a series of measurements is made of a quantity that is varying randomly, an estimate of its value can be made from the mean of the measured values, and an estimate of the uncertainty resulting from random effects can be made from the spread of the measurements (see Clause 6). In this case, "random" corresponds with Type A.

However, in some circumstances components of uncertainty arising from random effects are evaluated using a Type B method, and conversely, a Type A method could have been used in the evaluation of a component of uncertainty that arises from a systematic effect, such as an error in the calibration of an intermediate measuring instrument.

As an example of the use of a Type B evaluation of a random uncertainty, consider the case of an instrument that displays the value it is measuring to just three digits, and is used to measure a quantity just once. This will introduce an error, defined by the limited resolution of the output, that is random in nature. The true value of the measurand can lie anywhere in the range $\pm 0,5 \times$ (value of the least significant digit) with equal probability, within this range the values will therefore have a rectangular distribution (see 7.3).

As an example of the use of a Type A evaluation of a systematic uncertainty, when a measuring instrument is calibrated against some standard, the calibration process normally involves taking a number of readings. The elements of the uncertainty associated with the calibration that result from random effects will then be evaluated statistically (Type A). When the calibrated measuring instrument is then used in the measurement of a flow-rate or quantity, the evaluation of uncertainty in the flow-rate measurement process has to include the uncertainty in the calibration, part of which will have arisen from random effects and will have been evaluated using a Type A method. However, in the evaluation of the uncertainty of the flow-rate measurement, errors in the calibration will contribute to errors in the flow-rate measurement in a systematic manner. The effect of the random errors in the calibration process will have become "fossilized" into an effect that is systematic.

Independent of the terminology, it is generally obvious which approach has to be used in evaluating the different components of uncertainty of a flow-rate measurement.

Annex J
(informative)

Special situations using two or more meters in parallel

When two or more meter runs are used in parallel on a meter skid, the total flow-rate value is derived by summing the values from each meter run. In this case, the uncertainty in the total flow-rate is evaluated as described in this annex.

Divide the sources of uncertainty into

— those that will produce the same effects in each meter run, and that are therefore correlated between the meters; and

— those that will produce different effects in each meter run, and that are therefore uncorrelated.

The uncertainties in each list are then combined to derive combined uncertainties for those sources that are correlated between meters, $u_{c,corr}(y)$ [see Equation (J.1)] and those sources that are uncorrelated, $u_{c,uncorr}(y)$ [see Equation (J.2)]. The contribution to uncertainty from each meter depends on the flow through that meter and the analysis is greatly simplified by considering absolute uncertainties. Thus

$$u_{c,corr}(y) = c_1 u(x_{1,corr}) + c_2 u(x_{2,corr}) + \ldots c_n u(x_{N,corr}) = \sum_{i=1}^{N} \left[c_i u(x_{i,corr}) \right] \tag{J.1}$$

Equation (J.1) assumes 100 % correlation.

$$u_{c,uncorr}(y) = \left\{ \left[c_1 u(x_{1,uncorr}) \right]^2 + \left[c_2 u(x_{2,uncorr}) \right]^2 + \ldots + \left[c_n u(x_{N,uncorr}) \right]^2 \right\}^{1/2}$$

$$= \left\{ \sum_{i=1}^{N} \left[c_i u(x_{i,uncorr}) \right]^2 \right\}^{1/2} \tag{J.2}$$

If the elements within either list are themselves correlated, then the method of combination will be in accordance with C.6. The combined uncertainties are then combined to obtain the overall combined uncertainty in the total flow.

As the total flow, Q, is given by

$$Q = q_1 + q_2 + \ldots + q_N$$

The sensitivity coefficients, c_i, in Equations (J.1) and (J.2) are all equal to 1.

$$u_c(Q) = \left[\left(u_{c,corr} \right)^2 + \left(u_{c,uncorr} \right)^2 \right]^{1/2} = \left\{ \left[\sum_{i=1}^{N} u(x_{i,corr}) \right]^2 + \sum_{i=1}^{N} \left[u(x_{i,uncorr}) \right]^2 \right\}^{1/2} \tag{J.3}$$

In the special case when the absolute uncertainties, u_i, are all equal, Equation (J.3) can be simplified. However, since some components of u_i will be proportional to the flow, the uncertainties in all meters are unlikely to be equal unless the meters are identical and the flows they are passing are equal. When these conditions are satisfied, Equation (J.3) simplifies to Equation (J.4):

$$u_{\mathrm{c}}\left(Q\right) = N\left\{\left[u(x_{i,\mathrm{corr}})\right]^2 + \frac{\left[u(x_{i,\mathrm{uncorr}})\right]^2}{N}\right\}^{1/2}$$
(J.4)

where $u(x_{i,\mathrm{corr}})$ and $u(x_{i,\mathrm{uncorr}})$ are the correlated and the uncorrelated components of uncertainty in a single meter.

In the case, for example, of a measurement based on parallel orifice plates, the following sources of uncertainty will contribute the same effects in each meter run and are therefore correlated between meters:

— discharge coefficient;

— expansion factor.

To the extent that the uncertainties of measurement in each of the parallel runs are independent of each other, the following sources of uncertainty will contribute different effects in each run and are therefore uncorrelated:

— pipeline diameter;

— orifice diameter;

— differential pressure;

— density;

— computation.

Uncertainties arising in any of these measurements that produce the same effect in each system, such as those arising from the use of the same instruments, have to be included in the first list.

Annex K
(informative)

Alternative technique for uncertainty analysis

The mathematical theory underlying the analysis of uncertainty is based on the assumption that the uncertainties involved are small compared with the measured values (the exception being when the measurements are close to zero). This is certainly true for the standards work for which the original theories were developed and can also be true for many industrial applications. However, it cannot be said to be true for all industrial situations; where the uncertainties are large compared with the measured values, the mathematical theory breaks down. In these situations, the technique known as Monte Carlo analysis can be of great value in assessing combined values of uncertainty. In this approach, many calculations of the flow-rate are made, in each of which different values are assigned to each of the input variables. Each input value is drawn at random from the assumed distribution for that parameter, and in this way the distribution of the output flow-rate is calculated.

To obtain a representative distribution for the output requires many thousands of calculations to be performed and it is only with the advent of cheap computer power that the Monte Carlo technique has become a viable method of assessing combined uncertainty. GUM does not deal specifically with large values of uncertainty and on this basis does not discuss the Monte Carlo technique; however those faced with large relative uncertainties could find the approach of considerable value.

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