BS ISO 3951-2:2013

BSI Standards Publication

Sampling procedures for inspection by variables

Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

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National foreword

This British Standard is the UK implementation of ISO 3951-2:2013. It supersedes BS 6002-2:2007 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee SS/5, Acceptance sampling schemes.

A list of organizations represented on this committee can be obtained on request to its secretary.

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INTERNATIONAL STANDARD

ISO 3951-2

Second edition
2013-09-01

Sampling procedures for inspection by variables —

Part 2:
General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

Règles d'échantillonnage pour les contrôles par mesures —

Partie 2: Spécification générale pour les plans d'échantillonnage simples indexés par une limite de qualité acceptable (LQA) pour le contrôle lot par lot de caractéristiques-qualité indépendantes

Reference number
ISO 3951-2:2013(E)

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2. www.iso.org/directives

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The committee responsible for this document is ISO/TC 69, *Application of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: http://www.iso.org/iso/home/standards_development/resources-for-technical-work/foreword.htm

This second edition cancels and replaces the first edition (ISO 3951-2:2006), of which it constitutes a minor revision with the following changes:

— procedures have been introduced to accommodate measurement uncertainty;

— many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in ISO 2859-1.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

— *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

— *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

— *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

— *Part 4: Procedures for assessment of declared quality levels*

— *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL) and is of a technical nature, aimed at users who are already familiar with sampling by variables or who have complicated requirements. (A more introductory treatment is given in ISO 3951-1.)

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

a) automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;

b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is either implicitly or explicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is complementary to ISO 2859-1. When specified by the responsible authority, both this part of ISO 3951 and ISO 2859-1 may be referenced in a product specification, contract, inspection instructions, or other documents, and the provisions set forth therein govern. The responsible authority shall be designated in one of the above documents.

Caution — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

Inspection by variables for percent nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

— unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;

— a single specification limit, or double specification limits with combined, separate, or complex control;

— univariate or multivariate cases;

— three inspection severities, namely normal inspection, tightened inspection, or reduced inspection.

Table 1 is intended to facilitate the use of this part of ISO 3951 by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. Table 1 only deals with Clauses 15, 16, 17, 18, 19, 23, 24, and 25; in every case, it is necessary first of all to have read all the preceding clauses.

Table 1 — Summary table

	Single specification limit				Double specification limits with combined control			
	s-method		*σ*-method		*s*-method		*σ*-method	
	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables/ Annexes
Normal inspection	16.1, 16.2, 16.3, 17.1, 17.2, 20 24.1	A.1, B.1	18.1, 18.2, 19, 20, 24.1	A.1, G.3	16.1, 16.3, 17.1,17.2, 20, 24.1 Annex L	A.1, D.1, Annex F (for n = 3), G.1	18.1, 18.3, 19, 20, 24.1	A.1, C.1, E.1
Switching between normal and tightened inspection	24.2, 24.3	B.1, B.2	24.2, 24.3	C.1, C.2	24.2, 24.3	D.1, D.2 F.1, F.2	24.2, 24.3	E.1 G.1, G.2
Switching between normal and reduced inspection	24.4, 24.5	B.1, B.3 J.1	24.4, 24.5	C.1, J.1	24.4, 24.5	D.1, D.3 F.1, F.3 J.1	24.4, 24.5	E.1 G.1, G.3 J.1
Switching between tightened and discontinued inspection	22, 25	B.2	25	C.2	22, 25	D.2 F.2	25	E.1 G.2
Switching between the *s*-method and *σ*-method	26	I.1	26	K.2, I.1	26 L.2.1 L.3, L.4, L.5	I.1	26 L.2.2	K.2, I.1

Table 1 — *(continued)*

	Double specification limits with separate control				Double specification limits with complex control			
	s-method		*σ*-method		*s*-method		*σ*-method	
	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables/ Annexes	Clauses or sub-clauses	Tables/ Annexes
Normal inspection	16.1, 17.1, 17.2, 20, 24.1, Annex L	A.1, D.1, Annex F (for $n = 3$), G.1	18.1, 18.2, 18.3, 19, 20, 24.1	Annex A, C.1, E.1	16.1, 16.3.4, 17.1, 17.2, 20, 24.1, Annex L	A.1, D.1, Annex F (for $n = 3$), G.1	18.1, 18.3, 19, 20, 24.1	A.1, C.1, E.1
Switching between normal and tightened inspection	24.2, 24.3	D.1, D.2, F.1, F.2	24.2, 24.3	E.1, E.2, G.2	24.2, 24.3	D.1, D.2, F.1, F.2	24.2, 24.3	E.1, E.2, G.3
Switching between normal and reduced inspection	24.4, 24.5	D.1, D.3 F.1, F.3 J.1	24.4, 24.5	E.1, E.3, G.2, J.1	24.4, 24.5	D.1, D.3 F.1, F.3 J.1	24.4, 24.5	E.1, E.3, G.3, J.1
Switching between tightened and dis-continued inspection	22, 25	D.2 F.2	25	E.2 G.2	22, 25	D.2 F.2	25	E.2 G.3
Switching between the *s*-method and *σ*-method	26 L.2.1 L.3, L.4, L.5	I.1	26 L.2.2	I.1, K.2	26 L.2.1 L.3, L.4, L.5	I.1	26 L.2.2	I.1, K.2

16 annexes are provided. Annexes A to J provide the tables needed to support the procedures. Annex K indicates how the sample standard deviation, *s*, and the presumed known value of the process standard deviation, *σ*, should be determined. Annex L provides formulae for the estimation of the process fraction nonconforming, together with a highly accurate approximation for use when the process standard deviation is unknown. Annex M provides formulae for the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection under the *s*-method and *σ*-method. Annex N provides similar information for the producer's risks. Annex O gives the general formula for the operating characteristic of the *σ*-method. Annex P provides procedures for accommodating measurement uncertainty.

Sampling procedures for inspection by variables —

Part 2:
General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

1 Scope

This part of ISO 3951 is primarily designed for use under the following conditions:

a) where the inspection procedure is to be applied to a **continuing series of lots** of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 shall be applied to each one separately;

b) where the **quality characteristics** of the items of product are **measurable on a continuous scale**;

c) where the measurement error is negligible (i.e. with a standard deviation no more than 10 % of the corresponding process standard deviation). However, procedures are also provided in Clause 9 and Annex P for accommodating measurement error when it has a non-negligible standard deviation;

d) where production is stable (under statistical control) and the quality characteristics are distributed, at least to a close approximation, according to **normal distributions**;

e) where, in the case of multiple quality characteristics, the characteristics are independent, or almost independent, of one another;

f) where a contract or standard defines **a lower specification limit**, L, **an upper specification limit**, U, **or both** on each of the quality characteristics. If there is only one quality characteristic, an item is qualified as conforming if its measured quality characteristic x satisfies the appropriate one of the following inequalities:

   1) $x \geq L$ (i.e. the lower specification limit is not violated);

   2) $x \geq U$ (i.e. the upper specification limit is not violated);

   3) $x \geq L$ and $x \geq U$ (i.e. neither the lower nor the upper specification limit is violated).

   If there are two or more, say m, quality characteristics, then, designating the lower and upper limits for the i^{th} quality characteristic by L_i and U_i respectively, an item of product is qualified as nonconforming if one or more of its m measured quality characteristics, x_i, fails to satisfy the appropriate one of the following inequalities:

   4) $x_i \geq L_i$;

   5) $x_i \leq U_i$;

   6) $x_i \geq L_i$ and $x_i \leq U_i$.

Inequalities 1), 2), 4), and 5) are called cases with a **single specification limit** while 3) and 6) are called cases with **double specification limits**. For double specification limits, a further distinction is made between combined control, separate control, and complex control. If there is only one quality characteristic, then

— combined control is where a single AQL applies to nonconformity beyond both limits,

— separate control is where separate AQLs apply to nonconformity beyond each of the limits, and

— complex control is where one AQL applies to nonconformity beyond the limit that is of greater seriousness and a larger AQL applies to the total nonconformity beyond both limits.

If there are two or more quality characteristics, this generalizes as follows:

— combined control is where nonconformity beyond both limits on a variable belongs to the same class, to which a single AQL applies;

— separate control is where nonconformity beyond the two limits on a variable belongs to separate classes, to each of which a single AQL applies;

— complex control is where nonconformity beyond the limit that is of greater seriousness belongs to one class to which a single AQL applies, and the total nonconformity beyond both limits belongs to another class to which a larger AQL applies.

Note that, in the case of two or more quality characteristics, nonconformity on more than one quality characteristic may belong to the same class.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-1:2005, *Sampling procedures for inspection by variables — Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1, and ISO 3534-2 and the following apply.

3.1
inspection by variables
inspection by measuring the magnitude of a characteristic of an item

[SOURCE: ISO 3534-2]

3.2
sampling inspection
inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2]

3.3
acceptance sampling inspection
acceptance sampling
sampling inspection (3.2) to determine whether or not to accept a lot or other amount of product, material, or service

[SOURCE: ISO 3534-2]

3.4
acceptance sampling inspection by variables
acceptance sampling inspection (3.3) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

3.5
process fraction nonconforming
rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

3.6
acceptance quality limit
AQL
worst tolerable *process fraction nonconforming* (3.5) when a continuing series of lots is submitted for *acceptance sampling* (3.3)

Note 1 to entry: See Clause 5.

3.7
quality level
quality expressed as a rate of occurrence of nonconforming items

3.8
consumer's risk quality
CRQ
quality level (3.7) of a process which, in the acceptance sampling plan, corresponds to a specified consumer's risk

Note 1 to entry: In this part of ISO 3951, the *quality level* (3.7) is the process fraction nonconforming.

Note 2 to entry: In this part of ISO 3951, the consumer's risk quality corresponds to a consumer's risk of 10 %.

3.9
producer's risk
PR
probability of non-acceptance when the quality level has a value stated by the plan as acceptable

Note 1 to entry: Quality level relates to the *process fraction nonconforming* (3.5) and acceptable relates to the *acceptance quality limit* (3.6).

3.10
nonconformity
non-fulfilment of a requirement

Note 1 to entry: Nonconformity will generally be classified by its degree of seriousness such as the following:

Class A. Nonconformity of a type considered to be of the highest concern for the product or service. Such types of nonconformity will typically be assigned very small AQL values;

Class B. Nonconformity of a type considered to have the next lower degree of concern; this is typically assigned a larger AQL value than that in class A and smaller than that in class C if a third class exists and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

3.11
nonconforming unit
unit with one or more nonconformities

[SOURCE: ISO 3534-2]

3.12
***s*-method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the sample standard deviation(s)

[SOURCE: ISO 3534-2]

Note 1 to entry: See Clause 15.

3.13
***σ*-method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the presumed value(s) of the process standard deviation(s)

[SOURCE: ISO 3534-2]

Note 1 to entry: See Clause 16.

3.14
specification limit
conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2]

3.15
lower specification limit
L
specification limit (3.14) that defines the lower conformance boundary

[SOURCE: ISO 3534-2]

3.16
upper specification limit
U
specification limit (3.14) that defines the upper conformance boundary

[SOURCE: ISO 3534-2]

3.17
combined control
requirement when nonconformance beyond both the *lower specification limit* (3.15) and the *upper specification limit* (3.16) of a quality characteristic belongs to the same class, to which a single *AQL* (3.6) applies

Note 1 to entry: See 5.3, 16.3.2, 18.3.

Note 2 to entry: The use of a combined *acceptance quality limit* (3.6) requirement implies that nonconformance beyond either *specification limit* (3.14) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.18
separate control
requirement when nonconformance beyond the *lower specification limit* (3.15) and the *upper specification limit* (3.16) of a quality characteristic belong to different classes, to which separate *acceptance quality limits* (3.6) are applied

Note 1 to entry: See 5.3, 16.3.3, 17.2.

3.19
complex control
requirement when nonconformance beyond the *lower specification limit* (3.15) and the *upper specification limit* (3.16) of a quality characteristic belongs to one class and nonconformance beyond either the *upper specification limit* (3.16) or the *lower specification limit* (3.15) belongs to a different class, with separate *acceptance quality limits* (3.6) being applied to the two classes

Note 1 to entry: See 5.3, 16.3.4, 18.3.

3.20
acceptability constant
k, *p**
constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2]

Note 1 to entry: See 16.2 and 16.3.

3.21
quality statistic
Q
function of the *specification limit* (3.14), the sample mean, and the sample or process standard deviation used in assessing the acceptability of a lot

[SOURCE: ISO 3534-2]

Note 1 to entry: For the case of a single *specification limit* (3.14), the lot may be sentenced on the result of comparing *Q* with the *acceptability constant* (3.20)*k*.

Note 2 to entry: See 16.2 and 16.3.

3.22
lower quality statistic
Q_L
function of the *lower specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *lower specification limit* (3.15), the lot is sentenced on the result of comparing Q_L with the *acceptability constant* (3.20)*k*.

Note 2 to entry: [SOURCE: ISO 3534-2].

Note 3 to entry: See Clause 4, 16.2 and 16.3.

3.23
upper quality statistic
Q_U
function of the *upper specification limit* (3.16), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single *upper specification limit* (3.16), the lot is sentenced on the result of comparing Q_U with the *acceptability constant* (3.20)*k*.

Note 2 to entry: [SOURCE: ISO 3534-2].

Note 3 to entry: See Clause 4, 16.2, and 18.3.

3.24
maximum sample standard deviation
MSSD
s_{max}
largest sample standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double *specification limits* (3.14) with a combined *acceptance quality limit* (3.6) requirement and unknown process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See 16.3.2.1 and Annex F.

3.25
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample size code letter and *acceptance quality limit* (3.6) for which it is possible to satisfy the acceptance criterion for double *specification limits* (3.14) with a combined *acceptance quality limit* (3.6) requirement under tightened inspection with known process variability

[SOURCE: ISO 3534-2]

Note 1 to entry: See 17.2, 17.3.

3.26
switching rule
instruction within an *acceptance sampling* (3.3) scheme for changing from one *acceptance sampling* (3.3) plan to another of greater or lesser severity based on demonstrated quality history

[SOURCE: ISO 3534-2]

Note 1 to entry: See Clause 23.

Note 2 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of greater or lesser severity.

3.27
measurement
set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2]

4 Symbols

4.1 Univariate symbols

The symbols used when there is only one quality characteristic in the class are as follows:

c_U factor for determining the upper control limit for the sample standard deviation (See Annex I.)

f_s factor that relates the maximum sample standard deviation (MSSD) to the difference between U and L (See Annex F.)

f_σ factor that relates the maximum process standard deviation (MPSD) to the difference between U and L (See Annex G.)

k Form k acceptability constant, for use with a single specification limit and a single quality characteristic (See Annexes B and C.)

L lower specification limit (As a subscript to a variable, denotes its value at L.)

μ process mean

N lot size (number of items in a lot)

n sample size (number of items in a sample)

$\hat{p}$ estimate of the process fraction nonconforming (See Annex L.)

p_L process fraction nonconforming below the lower specification limit

$\hat{p}_L$ estimate of the process fraction nonconforming below the lower specification limit

p_U process fraction nonconforming above the upper specification limit

$\hat{p}_U$ estimate of the process fraction nonconforming above the upper specification limit

p^* Form p^* acceptability constant, the maximum acceptable value of the estimate of the process fraction nonconforming (See Annexes D and E.)

P_a probability of acceptance

Q quality statistic

Q_L lower quality statistic

NOTE Q_L is defined as $(\bar{x}-L)/s$ when the process standard deviation is unknown, and as $(\bar{x}-L)/\sigma$ when it is presumed to be known.

Q_U upper quality statistic

NOTE Q_U is defined as $(U-\bar{x})/s$ when the process standard deviation is unknown, and as $(U-\bar{x})/\sigma$ when it is presumed to be known.

s sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s=\sqrt{\frac{\sum_{j=1}^{n}(x_j-\bar{x})^2}{n-1}}$$

(See Annex K.)

s_{max} maximum sample standard deviation (MSSD)

σ standard deviation of a process that is under statistical control

NOTE σ^2, the square of the process standard deviation, is known as the process variance.

σ_{max} maximum process standard deviation (MPSD)

U upper specification limit (As a suffix to a variable, denotes its value at U.)

x_j measured value of the quality characteristic for the j^{th} item of the sample

$\bar{x}$ the arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x}=\frac{\sum_{j=1}^{n}x_j}{n}$$

$\bar{x}_L$ lower acceptance value for $\bar{x}$

$\bar{x}_U$ upper acceptance value for $\bar{x}$

4.2 Multivariate symbols

Other symbols used when there are two or more quality characteristics in a class are as follows:

L_i lower specification limit for the i^{th} quality characteristic

y number of quality characteristics in the class

$\hat{p}_i$ estimate of the process fraction nonconforming for the i^{th} quality characteristic

$\hat{p}_{i,\text{L}}$ estimate of the process fraction nonconforming below the lower specification limit for the i^{th} quality characteristic

$\hat{p}_{i,\text{U}}$ estimate of the process fraction nonconforming above the upper specification limit for the i^{th} quality characteristic

s_i sample standard deviation for the i^{th} quality characteristic, i.e. $s_i = \sqrt{\frac{\sum_{j=1}^{n}(x_{ij} - \overline{x}_i)^2}{n-1}}$

σ_i process standard deviation for the i^{th} quality characteristic

U_i upper specification limit for the i^{th} quality characteristic

x_{ij} measured value of the i^{th} quality characteristic for the j^{th} item in the sample

$\overline{x}_i$ sample mean value of the i^{th} quality characteristic, i.e. $\overline{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n}$

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the AQL may be accepted with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, in the contract, or by the responsible authority. In all cases, one AQL shall be specified for each class of nonconformity. (See 3.10.)

Where both upper and lower specification limits are given for a quality characteristic, the following three cases may be identified:

a) *combined* control of double specification limits, where nonconformity beyond both limits belongs to the same class, to which a single AQL applies;

b) *separate* control, where nonconformity beyond both limits belongs to different classes, to which separate AQLs apply;

c) *complex* control, where nonconformity beyond the limit that is of greater seriousness belongs to one class to which one AQL applies, and nonconformity beyond both limits combined belongs to another class to which a larger AQL applies.

In other words, for a single quality characteristic that has a lower specification limit, L, an upper specification limit, U, an unknown process fraction nonconforming below L of p_L, and an unknown process fraction nonconforming above U of p_U, combined control seeks simply to control the sum $p_L + p_U$ within one class of nonconformity, to which a single AQL applies. Separate control seeks to control p_L within one class to which one AQL applies and to separately control p_U within another class to which a second AQL applies. Complex control seeks to control $p_L + p_U$ within one class, to which one AQL applies, and to separately control either p_L or p_U, whichever is relevant, within another class to which a lower AQL applies.

Including the control of single specification limits, there are therefore four types of control. A class may contain nonconformities under any number of these types of control.

An acceptance test shall be carried out according to the provisions of this part of ISO 3951 for each class of nonconformity. The lot shall only be accepted if all classes of nonconformity satisfy their respective acceptance tests.

5.4 Preferred AQLs

The 16 AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL other than a preferred AQL is designated, then this part of ISO 3951 is not applicable. (See 14.2.)

5.5 Caution

From the definition of the AQL in 5.1, it follows that the desired protection can only be ensured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming items of product.

6 Switching rules for normal, tightened, and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving the production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When there is sufficient evidence from the control charts (see 23.1) that the variability is in statistical control, consideration should be given to switching to the σ-method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see Clause 26).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in Clauses 24, 25, and 26.

7 Relation to ISO 2859-1 and ISO 3951-1

7.1 Relation to ISO 2859-1

7.1.1 Similarities to ISO 2859-1

The similarities are as follows.

a) This part of ISO 3951 is complementary to ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans and the preferred values used in this part of ISO 3951 are identical with those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).

c) In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. General tables give the sample size to be taken and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the s-method and σ-method and for normal, tightened, and reduced inspection.

d) The switching rules are essentially equivalent.

e) The classification of nonconformities by degree of seriousness into class A, class B, etc., remains unchanged.

7.1.2 Differences from ISO 2859-1

The differences are as follows.

a) **Determination of acceptability.** Acceptability for an ISO 2859-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated process mean from the specification limit(s) in terms of the estimated process standard deviation. In this part of ISO 3951, two methods are considered: the s-method, for use when the process standard deviation, σ, is unknown, and the σ-method, for use when σ is presumed to be known. In the case of a class containing a single quality characteristic with a single specification limit, acceptability is determined most easily by comparing a quality statistic with a "Form k" acceptability constant (see 16.2 and 17.2). For more complicated classes with multiple quality characteristics and/or combined or complex control of double specification limits, acceptability is determined by comparing an estimate of the process fraction nonconforming for that class with a "Form p*" acceptability constant.

b) **Normality.** In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements on each quality characteristic should be distributed according to a normal distribution or at least a close approximation to a normal distribution.

c) **Independence.** In ISO 2859-1, there is no requirement relating to the independence of multiple quality characteristics. However, in this part of ISO 3951, for the efficient operation of a plan, it is necessary that the measurements for all quality characteristics in a class shall be independent or at least approximately so.

d) **Operating characteristic curves** (OC curves). The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves for unknown process standard deviation have been matched by minimizing the area between the curves representing the *squares* of the OC values, a method that gives greater emphasis to the match at the top of the OC curves. In most cases, the resulting match between the OC curves is so close that for most practical purposes, the attributes and variables OC curves may be considered to be identical. The plans for known process standard deviation were derived by minimizing the area between the squared OC functions subject to keeping the same Form p^* acceptability constant as for the corresponding case for unknown process standard deviation, i.e. only the sample size was open to choice, so the match was, in general, less perfect.

e) **Producer's risk**. For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1. (The producer's risks of the plans are given in Annex N.)

f) **Sample sizes.** The variables sample sizes for combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes for the same letters. This is particularly true for the σ-method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.

g) **Double sampling plans.** Double sampling plans by variables are presented separately, in ISO 3951-3.

h) **Multiple sampling plans.** No multiple sampling plans by variables are given in this part of ISO 3951.

i) **Average Outgoing Quality Limit (AOQL).** The AOQL concept is mainly of value when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

7.2 Relation to ISO 3951-1

7.2.1 Similarities to ISO 3951-1

The similarities are as follows.

a) This part of ISO 3951 is complementary to ISO 3951-1 and the two documents both present single sampling procedures for inspection by variables.

b) The procedures of ISO 3951-1 are included in this part of ISO 3951 and referred to as "Form *k*".

7.2.2 Differences from ISO 3951-1

The differences are as follows.

a) This part of ISO 3951 is more general than ISO 3951-1 as it includes multivariate procedures for independent quality characteristics and also includes procedures for separate or complex control of double specification limits.

b) Because Form *k* procedures may only be used for a single quality characteristic with a single AQL, this part of ISO 3951 also includes the more general Form p^* procedures.

NOTE For users who are familiar with MIL-STD-414,[19] Form *k* corresponds to form 1 of the Military Standard, and Form *p** corresponds to form 2. The new terminology is considered to be more helpful.

8 Consumer protection

8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal, and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Sometimes, specific individual plans are selected from this part of ISO 3951 and used without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as "inspection in compliance with ISO 3951-2". When used in such a way, ISO 3951-2 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

8.2 Consumer's risk quality (CRQ) tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give consumer's risk quality not more than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer's risk quality and a consumer's risk to be associated with it. Annex M gives values of consumer's risk quality levels for the *s*-method and *σ*-method corresponding to a consumer's risk of 10 %.

However, application of this part of ISO 3951 to isolated or short series of lots is deprecated, as the theory of sampling by variables applies to a *process*. For isolated or short series of lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from ISO 2859-2. (See also Reference [14] in the Bibliography.)

8.3 Producer's risk tables

Annex N gives the probability of non-acceptance under the *s*-method and *σ*-method for lots produced when the process fraction nonconforming equals the AQL. This probability is called the producer's risk.

8.4 Operating characteristic (OC) curves

The tables for consumer's risk quality and producer's risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any process quality level may, however, be judged from its operating characteristic curve. OC curves for the normal inspection *s*-method sampling plans of this part of ISO 3951 are given in Charts B to R of ISO 3951-1, which should be consulted when choosing a sampling plan. Also given in ISO 3951-1 are Tables B to R of process quality levels at nine standard probabilities of acceptance for all the *s*-method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the *s*-method. Most of them also provide a good approximation to the *σ*-method and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the *σ*-method, refer to Annex O.

9 Accommodating measurement variability

The master tables of this part of ISO 3951 are based on the assumption that the quality characteristic X of the items in the lots is normally distributed with unknown process mean, μ, and either known or unknown process standard deviation, σ. The assumption is also made that the measurement X is corrected for bias (if any) and contains no measurement variability, i.e. that the measurement of an item with the true value x_i results in the value x_i. However, the master tables can also be used, with appropriate adjustments, in the presence of measurement error.

If the measurement standard deviation is no greater than 10 % of the process standard deviation, it can be ignored. For measurement standard deviation greater than 10 % of the process standard deviation, the sample size will need to be increased, although the acceptability constant remains the same. Moreover, if neither the measurement standard deviation nor the process standard deviation is known, more than one measurement will need to be made on each sampled item and the total variability of the measurements will need to be separated into the components due to the measurements and to the process.

Details are provided in Annex P.

10 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge both of statistics and the product to be inspected. Clauses 11 to 13 of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

11 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time consuming and costly per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely how good the product is. Earlier warning will, therefore, be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in 16.3.2.2 and 16.3.2.4.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859-1) and the standard plans in this part of ISO 3951 reveals that the smallest samples tend to be required by the σ-method (used when the process standard deviation is presumed to be known). The sample sizes for the s-method (used when the process standard deviation is unknown) are also, in general, substantially smaller than for sampling by attributes.

e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

g) A variable scheme becomes relatively more complicated to operate as the number of quality characteristics and the number of measurements to be taken on each item increases.

h) This part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of each quality characteristic is normal and that the quality characteristics are independent. In case of doubt, the responsible authority should be consulted.

NOTE 1 ISO 16269-4 gives detailed procedures for tests for departure from normality.

NOTE 2 Departure from normality is also dealt with in Clause 2 of ISO 5725-2, which provides examples of graphical methods that can be used to verify that the distribution of the data are sufficiently normal to justify the use of sampling by variables.

12 Choice between the *s*-method and *σ*-method

If it is desired to apply inspection by variables, the next question is whether to use the *s*-method or the *σ*-method. The *σ*-method is usually the most economical in terms of sample size, but, before this method may be employed, the value of *σ* has to be established.

Initially, it will be necessary to begin with the *s*-method but, subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the *σ*-method? The size of the sample will generally be smaller and the acceptability criterion simpler under the *σ*-method. On the other hand, it will still be necessary to calculate the sample standard deviation, *s,* for record purposes and to keep the control charts up to date. (See Clause 22.) The calculation of *s* can appear daunting, but the difficulty is more apparent than real; this is especially true if an electronic calculator is available. Methods of calculating *s* are given in Annex K.

13 Choice of inspection level and AQL

For a standard sampling plan, the inspection level, in conjunction with the size of the lots and the AQL, determines the size of the sample to be taken and governs the severity of the inspection. The appropriate OC curve from Charts B to R of ISO 3951-1 or the appropriate table from Tables B to R of ISO 3951-1 shows the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

14 Choice of sampling scheme

14.1 Standard plans

The standard procedure can be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the *s*-method, has been found in practice to produce workable sampling schemes; but it rests on the assumption that the order of priority is the AQL first, the sample size second, and the limiting quality last.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see Clauses 23, 24, and 25), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE It should also be remembered that the limiting quality is the quality which, if offered for inspection, would have a 10 % probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

If, in certain circumstances, the limiting quality has a higher priority than the sample size, a suitable plan from this part of ISO 3951 may be selected by using Chart A. Construct a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample size code letter of a standard normal inspection plan that meets the specified requirements. (This may be verified by inspecting the OC curve from among Charts B to R of ISO 3951-1 relating to this code letter and AQL.)

However, the use of this method is deprecated for isolated lots or short series of lots. (See 8.2.)

EXAMPLE Suppose that an acceptable value for the limiting quality is 6,0 % nonconforming and that the desired quality with a 95 % probability of acceptance is 2,0 % nonconforming. A vertical line on Chart A at 6,0 % nonconforming and a horizontal line at 2,0 % nonconforming intersect just below the sloping line indexed by the letter L. Examining Chart L, it is seen that a plan with a sample size code letter L and an AQL of 1,5 % meets the requirements.

If the horizontal and vertical lines intersect at a point above the line marked R in Chart A, this implies that the specification cannot be met by any of the plans in this part of ISO 3951.

14.2 Special plans

If none of the standard plans are acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special plan is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

15 Preliminary operations

Perform the following checks before starting inspection by variables.

a) check that production is considered to be continuing and that the distribution of the quality characteristics can be considered to be normal and independent;

   NOTE 1 For tests for departure from normality, see, for example, ISO 16269-4.

   NOTE 2 If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

b) check separately for each quality characteristic whether the *s*-method is to be used initially or whether the process standard deviation is stable and known, in which case the σ-method should be used;

c) check that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) check, for every quality characteristic with double specification limits, whether the limits are under combined, separate, or complex control and to which class of nonconformity each limit has been assigned. For combined control, check that nonconformity beyond each limit is of equal importance;

e) check that an AQL has been designated for each class of nonconformity and that it is one of the preferred AQLs for use with this part of ISO 3951. If it is not, then the tables are not applicable.

16 Standard procedures for the univariate *s*-method

16.1 Obtaining a plan, sampling, and preliminary calculations

The procedure for obtaining and implementing a plan is as follows.

a) With the inspection level given (normally, this will be level II) and with the lot size, obtain the sample size code letter using Table A.1.

b) For a single specification limit, enter Table B.1, B.2, or B.3 as appropriate with this code letter and the AQL and obtain the sample size, n, and the Form k acceptability constant, k. For separate control of double specification limits, do this for both limits. For combined control of double specification limits, enter Table D.1, D.2, or D.3 as appropriate and obtain the sample size n and the Form p^* acceptability constant. For complex control of double specification limits, enter Table D.1, D.2, or D.3 as appropriate twice, once with the combined control part of the specification and once with the smaller AQL applying to the specification limit of greater concern.

c) Take a random sample of size n, measure the characteristic x in each item, and then calculate the sample mean, $\bar{x}$, and the estimate s of the process standard deviation (see Annex K). If $\bar{x}$ lies outside the specification limit(s), the lot can be judged unacceptable without even calculating s. It is, however, necessary to calculate s for record purposes. (See Clause 22.)

16.2 Form *k* acceptability criterion for the *s*-method

If single specification limits are given, or separate control of double specification limits is required, the most straightforward procedure is as follows. Calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s} \quad (1)$$

and/or

$$Q_L = \frac{\bar{x} - L}{s} \quad (2)$$

as appropriate, then compare the quality statistic (Q_U or Q_L) with the Form k acceptability constant obtained from Table B.1, B.2, or B.3 for normal, tightened, or reduced inspection, respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if it is less, the lot is not acceptable.

Thus, if only the upper specification limit, U, is given, the lot is

acceptable if $Q_U \geq k$, and

not acceptable if $Q_U < k$,

or, if only the lower specification limit, L, is given, the lot is

acceptable if $Q_L \geq k$, and

not acceptable if $Q_L < k$.

Under separate control of double specification limits, the Form k acceptability constants at L and U may be different. Denote them by k_L and k_U respectively. In this case, the lot is

acceptable if $Q_U \geq k_U$ and $Q_L \geq k_L$, and

not acceptable if $Q_U < k_U$ and/or $Q_L < k_L$.

EXAMPLE 1 Single upper specification limit.

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From Table A.1, the sample size code letter is found to be F; from Table B.1, it is seen that a sample size of 13 is required and that the acceptability constant k is 1,426. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	13
Sample mean: $\bar{x} = \sum x / n$	54,615 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See K.1.2, Annex K.)	3,330 °C
Specification limit (upper): U	60 °C
Upper quality statistic: $Q_U = (U - \bar{x}) / s$	1,617
Form k acceptability constant: k (from Table B.1)	1,426
Acceptability criterion: Is $Q_U \geq k$?	Yes (1,617 > 1,426)

The lot meets the acceptability criterion and is therefore acceptable.

EXAMPLE 2 Single lower specification limit, requiring the following of an arrow in the master table.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. The process standard deviation is unknown. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. From Table A.1, it is seen that the sample size code letter is J. However, on entering Table B.1 with sample size code letter J and AQL 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, i.e. sample size 28 and acceptability constant k = 2,580. A random sample of size 28 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95	6,04	6,68	6,63	6,65	6,52	6,59	6,40	6,44	6,34	6,04	6,15	6,29	6,63
6,44	7,15	6,70	6,59	6,51	6,80	5,94	6,35	7,17	6,83	6,25	6,96	7,00	6,38

Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	28
Sample mean: $\bar{x} = \sum x / n$	6,551 s
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See Annex K, K.1.2.)	0,3251 s
Lower specification limit: L	4,0 s
Lower quality statistic: $Q_L = (\bar{x} - L) / s$	7,847
Form k acceptability constant: k (from Table B.1)	2,580
Acceptability criterion: Is $Q_L \geq k$?	Yes (7,847 > 2,580)

The lot meets the acceptability criterion, so it is acceptable.

16.3 Form *p** acceptability criterion for the *s*-method

16.3.1 Introduction

This part of ISO 3951 also provides a Form *p** method for determining lot acceptability. Whereas Form *k* applies only to a single quality characteristic with either a single specification limit or with double specification limits that are to be controlled separately, Form *p** applies much more generally to a class consisting of single or multiple quality characteristics with any combination of single or double specification limits with combined, separate, or complex control.

16.3.2 Combined control for the *s*-method

16.3.2.1 General

If, for the univariate *s*-method, combined or complex control of both the upper and lower specification limits is required, i.e. there is an overall AQL for the percentage of the process outside the two specification limits, the first step is to check that the sample standard deviation, *s*, is not so large that lot acceptability is impossible. If the value of *s* exceeds the value of the maximum sample standard deviation (MSSD) determined from Table F.1, F.2, or F.3, no further calculation or reference to graphs is required and the lot shall be immediately judged unacceptable.

If the value of *s* does not exceed the value of the MSSD, the estimate $\hat{p}$ of the process fraction nonconforming shall be calculated and compared with the Form *p** acceptability constant. The lot is determined to be

acceptable if $\hat{p} \le p^*$, and

not acceptable if $\hat{p} > p^*$,

where

$$\hat{p} = \hat{p}_\mathrm{L} + \hat{p}_\mathrm{U} \tag{3}$$

with

$$\hat{p}_\mathrm{L} = G_{(n-2)/2}\left[\frac{1}{2}\left(1 - \frac{\bar{x} - L}{s}\frac{\sqrt{n}}{n-1}\right)\right] \tag{4}$$

$$\hat{p}_\mathrm{U} = G_{(n-2)/2}\left[\frac{1}{2}\left(1 - \frac{U - \bar{x}}{s}\frac{\sqrt{n}}{n-1}\right)\right] \tag{5}$$

in which $G_m(.)$ represents the distribution function of the symmetic beta distribution with both parameters equal to *m.* (See Annex L for details.)

Form *p** may also be applied to a single specification limit, although in that case, Form *k* is equivalent and easier to apply. However, an estimate of the process fraction nonconforming will not be obtained when using Form *k*.

In the absence of tables of the beta distribution or corresponding computer software, one of the following three procedures shall be used, depending on the sample size.

16.3.2.2 Combined control for the *s*-method with *n* = 3

It may be seen from Tables B.1, B.2, and B.3 that the required sample size is 3 for the *s*-method for several combinations of sample size code letter and AQL.

If combined control of double specification limits is required then, after calculating the sample mean $\bar{x}$ and the sample standard deviation s, the applicable value of the coefficient f_s shall be found from the corresponding cell of Table F.1, F.2, or F.3. Determine the MSSD (i.e. the maximum allowable) from Formula (6).

$$\text{MSSD} = s_{\max} = (U - L) f_s \ . \tag{6}$$

Then compare s with $s_{\max}$. If s is greater than $s_{\max}$, then the lot may be rejected without further calculation. Otherwise, determine the values of $Q_U = (U - \bar{x})/s$ and/or $Q_L = (\bar{x} - L)/s$. Multiply Q_U and/or Q_L by $\sqrt{n}/(n-1) = \sqrt{3}/2$ (i.e. approximately 0,866) and use Table H.1 to determine the estimates $\hat{p}_U$ and/or $\hat{p}_L$ of the fraction of items in the process that are nonconforming beyond the upper and/or lower limits respectively.

NOTE 1 Negative values of Q correspond to estimates of the process fraction nonconforming in excess of 0,500 0 at that specification limit and will consequently always result in lot non-acceptance under the provisions of this part of ISO 3951, as the largest value of p^* in the tables is 43,83 %, i.e. 0,438 3. However, in order to obtain a numerical value for record-keeping purposes, the estimate of the process fraction nonconforming may be obtained by entering Table H.1 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1,0. For example, if $Q_U = -0{,}156$, then $\sqrt{3}Q_U/2 = -0{,}135$; entering Table H.1 with 0,135 gives an estimate of 0,456 9; subtracting this from 1,0 gives $\hat{p}_U = 0{,}543\ 1$.

NOTE 2 The basis of Table H.1 is given in L.4 of Annex L. Instead of using Table H.1, the estimate of the process fraction nonconforming beyond each specification limit when n = 3 may be calculated directly as

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2/\sqrt{3} \\ \dfrac{2}{\pi}\arcsin\left[\sqrt{(1 - Q\sqrt{3}/2)/2}\right] & \text{if } -2/\sqrt{3} \le Q \le 2/\sqrt{3} \\ 1 & \text{if } Q < -2/\sqrt{3} \end{cases} \tag{7}$$

These two estimates are added to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table D.1, D.2, or D.3, the lot is considered to be acceptable; otherwise, the lot is considered unacceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 3.

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so a combined AQL requirement for double specification limits is appropriate. The specification limits are set at 10 m from the point of aim at a distance of 1 km, with an AQL of 4 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From Table A.1, the sample size code letter is found to be B. From Table B.1, it is seen that a sample of size 3 is required. Three torpedoes are tested, yielding deviations from the point of aim of −5,0m, 6,7m, and 8,8m. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	**Values obtained**
Sample size: n	3
Sample mean: $\bar{x} = \sum x / n$	3,5 m
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See Annex K, K.1.2.)	7,436 m
Value of f_s for MSSD (Table F.1)	0,475
MSSD = $s_{\max} = (U - L) f_s = [10 - (-10)] \times 0,475$	9,50

Since s = 7,436 < $s_{\max}$ = 9,50, the lot *may* be acceptable, so continue with the calculations.

$Q_U = (U - \bar{x}) / s =$ (10 − 3,5) / 7,436	0,874 1
$Q_L = (\bar{x} - L) / s =$ (3,5 + 10) / 7,436	1,815
$\sqrt{3} Q_U / 2$	0,757
$\sqrt{3} Q_L / 2$	1,572
$\hat{p}_U$ (from Table H.1)	0,226 7
$\hat{p}_L$ (from Table H.1)	0,000 0
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,226 7
p^* (from Table D.1 as it is normal inspection)	0,192 5

Since $\hat{p} > p^*$, the lot is not acceptable.

NOTE This lot is not acceptable even though all inspected items **in the sample** are within the specification limits.

16.3.2.3 Combined control for the *s*-method with *n* = 4

For sample size 4 under the *s*-method, calculate the sample mean $\bar{x}$ and the sample standard deviation, *s*, then find the applicable value of the coefficient f_s from Table F.1, F.2, or F.3. Determine the MSSD (i.e. the maximum allowable) from Formula (8).

$$\text{MSSD} = s_{\max} = (U - L) f_s \ . \tag{8}$$

Then, compare *s* with the MSSD. If *s* is greater than the MSSD, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x}) / s$ and $Q_L = (\bar{x} - L) / s$. Calculate

$$\hat{p}_U = \begin{cases} 1 & \text{if } Q_U \leq -1,5 \\ 0,5 - Q_U / 3 & \text{if } -1,5 < Q_U < 1,5 \\ 0 & \text{if } Q_U \geq 1,5 \end{cases} \tag{9}$$

and

$$\hat{p}_L = \begin{cases} 1 & \text{if } Q_L \leq -1,5 \\ 0,5 - Q_L / 3 & \text{if } -1,5 < Q_L < 1,5 \\ 0 & \text{if } Q_L \geq 1,5 \end{cases} \tag{10}$$

Add these two estimates to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table D.1, the lot is considered to be acceptable; otherwise, the lot is considered unacceptable.

NOTE The basis of Formulae (9) and (10) is given in L.5 of Annex L.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 4.

Items are being manufactured in lots of size 25. The lower and upper specification limits on their diameters are 82 mm to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming beyond either limit using an AQL of 2,5 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From Table A.1, the sample size code letter is found to be C. From Table D.1, it is seen that a sample of size 4 is required. The diameters of four items from the first lot are measured, yielding diameters 82,4 mm, 82,2 mm, 83,1 mm, and 82,3 mm. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	**Values obtained**
Sample size: n	4
Sample mean: $\bar{x} = \sum x / n$	82,50 mm
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$	0,408 2 mm
(See Annex K, K.1.2.)	
Upper specification limit: U	84,0 mm
Lower specification limit: L	82,0 mm
Value of f_s for MSSD (Table F.1)	0,365
MSSD $= s_{max} = (U - L) f_s =$ (84 – 82) × 0,365	0,730 mm

Since s = 0,4082 < s_{max} = 0,730, the lot *may* be acceptable, so continue with the calculations.

$Q_U = (U - \bar{x})/s =$ (84 – 82,5) / 0,408 2	3,674 7
$Q_L = (\bar{x} - L)/s =$ (82,5 – 82) / 0,408 2	1,224 9
$\hat{p}_U$ [from Formula (9)]	0,000 0
$\hat{p}_L$ [from Formula (10)]	0,091 7
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,091 7
p^* (from Table D.1, as it is normal inspection)	0,086 0

Since $\hat{p} > p^*$, the lot is not acceptable.

16.3.2.4 Combined control for the s-method with *n*≥5 — Exact method

After calculating the sample mean $\bar{x}$ and the sample standard deviation, *s*, find the applicable value of the coefficient f_s from Table F.1, F.2, or F.3. Determine the MSSD (i.e. the maximum allowable) from Formula (11).

$$\text{MSSD} = s_{max} = (U - L) f_s \qquad (11)$$

Then, compare *s* with s_{max}. If *s* is greater than s_{max}, then the lot may be rejected without further calculation.

Otherwise, compute the upper and lower quality statistics $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. If tables of the beta distribution function or corresponding software are available, determine estimates $\hat{p}_U$ and $\hat{p}_L$ of the process fractions nonconforming in accordance with L.2.1. Otherwise, use the method given in L.3.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 5 or more.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 80 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From Table A.1, the sample size code letter is found to be E; from Table D.1, it is seen that a sample of 13 is required, and from Table F.1, that the value of f_s for the MSSD under normal inspection is 0,274. Suppose the measurements obtained are as follows: 63,5 °C; 61,9 °C; 65,2 °C; 61,7 °C; 68,4 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size: n	13
Sample mean: $\bar{x} = \sum x / n$	64,223 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$ (See Annex K, K.1.2.)	2,789 9 °C
Upper specification limit: U	70,0 °C
Lower specification limit: L	60,0 °C
Value of f_s for MSSD (Table F.1 for normal inspection)	0,274
MSSD $= s_{max} = (U - L) f_s =$ (70 − 60) × 0,274	2,74 °C

Since the value of s exceeds s_{max}, the lot may immediately be adjudged unacceptable.

NOTE This lot is not acceptable even though all inspected items **in the sample** are within the specification limits.

Suppose that the AQL had been 2,5 % instead of 1,5 %. In that case, the value of f_s would be 0,285, so s_{max} is equal to (70 − 60) × 0,285 = 2,85 °C As s is now less than s_{max}, it is not possible to determine at this stage whether or not the lot is acceptable and further calculations are required.

Two methods of completing the necessary calculations are described. The first applies when tables or software are available for the beta distribution function (see L.2.1). Note that five significant figures are retained throughout the intermediate calculations.

Information needed	Values obtained
$Q_U = (U - \bar{x}) / s$	2,070 7
$x_U = \frac{1}{2}\left[1 - Q_U \sqrt{n} / (n-1)\right]$	0,188 92
$\hat{p}_U = G_{(n-2)/2}(x_U)$	0,0115 85
$Q_L = (\bar{x} - L) / s$	1,513 7
$x_L = \frac{1}{2}\left[1 - Q_L \sqrt{n} / (n-1)\right]$	0,272 59
$\hat{p}_L = G_{(n-2)/2}(x_L)$	0,0591 98
p^* (from Table D.1, with AQL 2,5 %)	0,064 66

The overall process fraction nonconforming is estimated as $\hat{p} = \hat{p}_L + \hat{p}_U = 0,0591\ 98$ + 0,011 585 = 0,070 78, which is greater than the acceptability constant p^*. The lot is therefore not accepted.

16.3.2.5 Combined control for the s-method with *n*≥5 — Approximative method

When beta distribution tables or software are not available, the highly accurate approximative method described in L.3 is recommended. It is demonstrated below by applying it to the foregoing example.

Information needed	Values obtained
$Q_U = (U - \bar{x})/s$	2,070 7
$x_U = \frac{1}{2}\left[1 - Q_U\sqrt{n}/(n-1)\right]$	0,188 92
a_n (from Table L.1)	1,583 745
$y_U = a_n \ln\left[x_U/(1-x_U)\right]$	−2,307 6
$w_U = y_U^2 - 3$	2,325 0
As $w_U \geq 0,\ t_U = \dfrac{12(n-1)y_U}{12(n-1)+w_U}$	−2,270 9
$\hat{p}_U = \Phi(t_U)$	0,011 577
$Q_L = (\bar{x} - L)/s$	1,513 7
$x_L = \frac{1}{2}\left[1 - Q_L\sqrt{n}/(n-1)\right]$	0,272 59
$y_L = a_n \ln\left[x_L/(1-x_L)\right]$	−1,554 5
$w_L = y_L^2 - 3$	−0,583 53
$w_L < 0,\ t_L = \dfrac{12(n-2)y_L}{12(n-2)+w_L}$	−1,561 4
$\hat{p}_L = \Phi(t_L)$	0,059 215
p^* (from Table G.1 as it is normal inspection)	0,115 4

The overall process fraction nonconforming is estimated as $\hat{p} = \hat{p}_L + \hat{p}_U$ = 0,059 215 + 0,011 577 = 0,070 79, which is less than the acceptability constant p^*. The lot is therefore accepted.

NOTE The approximative method is typically very accurate. In this example, the error in using it can be seen to be only one unit in the fourth significant figure, i.e. 0,070 79 instead of 0,070 78.

16.3.3 Separate control for the *s*-method

When separate AQLs apply to both specification limits, Table D.1, D.2, or D.3 is entered with the sample size code letter and the AQLs at the upper and lower limits to obtain p_U^* and p_L^*. The acceptance criterion is then $\hat{p}_U \leq p_U^*$ and $\hat{p}_L \leq p_L^*$.

16.3.4 Complex control for the *s*-method

Complex control consists of combined control of both specification limits and simultaneous control of one of the limits using a separate and smaller AQL. The lot is therefore accepted if $\hat{p} \le p^*$ and either $\hat{p}_U \le p_U^*$ or $\hat{p}_L \le p_L^*$, whichever of the latter is relevant.

17 Standard multivariate *s*-method procedures for independent quality characteristics

17.1 General methodology

The general methodology for dealing with a class containing *m* independent quality characteristics is as follows. Denoting the estimated process fraction nonconforming for the i^{th} quality characteristic in the class by $\hat{p}_i$, the estimated process fraction nonconforming for the class is given by

$$\hat{p} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_2)\ldots(1 - \hat{p}_m) \qquad (12)$$

i.e. 1 minus the product of the estimated process fractions *conforming*.

NOTE If $\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m$ are all small, say no greater than 0,01, then $\hat{p}$ is approximately equal to the sum of the individual estimates, i.e. $\hat{p} \approx \hat{p}_1 + \hat{p}_2 + \ldots + \hat{p}_m$.

If there is only one class, say class A, then the estimated process fraction nonconforming for the class may be denoted by $\hat{p}_A$. The lot is accepted if

$$\hat{p}_A \le p^*$$

and not accepted otherwise, where p^* is the Form p^* acceptability constant given in Table D.1, D.2, or D.3 for the inspection severity, applicable sample size code letter, and AQL applying to the class.

If there are two or more classes, say class A, class B, ... with acceptability constants p_A^*, p_B^*, ..., the lot is accepted if $\hat{p}_A \le p_A^*$ and $\hat{p}_B \le p_B^*$ and so on but not accepted if one or more of the inequalities is violated.

If there is more than one class of nonconformity, class A will contain nonconformities of the greatest level of seriousness and generally have the lowest AQL and, therefore, the lowest Form p^* acceptability constant; class B will contain nonconformities of the next lower level of seriousness and have a larger AQL and value of p^*; and so on. It is possible that different classes of nonconformity will be under inspection at different levels of severity at any one time.

17.2 Example

Consider a product that has five independent quality characteristics x_1, x_2, x_3, x_4, and x_5, none of whose process standard deviations are known. Two classes of nonconformity are specified, A and B, with an AQL of 0,25 % for class A and an AQL of 1,0 % for class B. Details of the classification are shown in the first four columns of Table 2. Lots are of size 400 and are to be inspected under general inspection level II, beginning with normal inspection. From Table A.1, it is found that the sample size code letter is H.

From Table D.1, the sample size is found to be 18 for class A and 24 for class B. This presents a slight problem for characteristics x_4 and x_5, which appear in both classes. The different sample sizes can be accommodated in one of two ways, either

a) by selecting two random samples from the lot, one of size 18 and one of size 24, or

b) by randomly selecting a subsample of 18 items from the random sample of 24 items.

Method b) minimizes the amount of measurement required, but care shall be taken to avoid bias in the subsampling.

The results are summarized in Table 2.

Table 2 — Example of requirements and results for five quality characteristics with unknown process standard deviations

Variable	Limits	Type of control	Class	Sample size	Sample mean	Sample standard deviation	Quality statistic Q	$\frac{1-Q\sqrt{n}/(n-1)}{2}$	$\hat{p}$
x_1	$U_1 =$ 70,0	Single	A	18	$\bar{x}_1 = 68{,}5$	$s_1 = 0{,}50$	3,000 0	0,187 5	0,000 418
x_2	$L_2 =$ 10,0	Single	B	24	$\bar{x}_2 = 10{,}4$	$s_2 = 0{,}20$	2,000 0	0,291 7	0,019 134
x_3	$U_3 =$ 4,05 $L_3 =$ 3,95	Combined	A	18	$\bar{x}_3 = 4{,}005$	$s_3 = 0{,}015$	3,000 0 3,666 7	0,187 5 0,118 1	0,000 418 0,000 004 0,000 422
x_4	$U_4 =$ 1,95 $L_4 =$ 1,75	Separate	B A	24 18	$\bar{x}_{4,U} = 1{,}862$ $\bar{x}_{4,L} = 1{,}830$	$s_{4,U} = 0{,}032$ $s_{4,L} = 0{,}030$	2,750 0 2,666 7	0,207 1 0,167 2	0,001 316 0,001 285
x_5	$U_5 =$ 214 $L_5 =$ 206	Complex, i.e. Separate and Combined	A B	18 24	$\bar{x}_{5,U} = 210{,}3$ $\bar{x}_{5,L} = 210{,}1$	$s_{5,U} = 1{,}25$ $s_{5,L} = 1{,}27$	2,960 0 3,070 9 3,228 3	0,130 6 0,173 0 0,156 2	0,000 231 0,000 264 0,000 103 0,000 367

From Table D.1, it is found that the Form p^* acceptability constants are $p_A^* = 0{,}007\ 546$ for class A and $p_B^* = 0{,}027\ 51$ for class B.

The fraction nonconforming for class A is estimated as

$$\begin{aligned}\hat{p}_A &= 1-(1-\hat{p}_1)(1-\hat{p}_3)(1-\hat{p}_{4,L})(1-\hat{p}_{5,U})\\ &= 1-(1-0{,}000\ 418)(1-0{,}000\ 422)(1-0{,}001\ 285)(1-0{,}000\ 231)\\ &= 1-0{,}999\ 582\times 0{,}999\ 578\times 0{,}998\ 715\times 0{,}999\ 769\\ &= 1-0{,}997\ 646\\ &= 0{,}002\ 354\end{aligned}$$

The fraction nonconforming for class B is estimated as

$$\begin{aligned}\hat{p}_B &= 1-(1-\hat{p}_2)(1-\hat{p}_{4,U})(1-\hat{p}_5)\\ &= 1-(1-0{,}019\ 134)(1-0{,}001\ 316)(1-0{,}000\ 367)\\ &= 1-0{,}980\ 866\times 0{,}998\ 684\times 0{,}999\ 633\\ &= 1-0{,}979\ 216\\ &= 0{,}020\ 784\end{aligned}$$

Since $\hat{p}_A < p_A^*$ and $\hat{p}_B < p_B^*$, the lot is accepted.

NOTE The corresponding approximate estimates of the process fractions nonconforming in each class obtained by simply adding the component estimates are

$$\begin{aligned}\hat{p}_A &\cong \hat{p}_1+\hat{p}_3+\hat{p}_{4,L}+\hat{p}_{5,U}\\ &= 0{,}000\ 418+0{,}000\ 422+0{,}001\ 285+0{,}000\ 231\\ &= 0{,}002\ 356\end{aligned}$$

and

$$\begin{aligned}\hat{p}_B &\cong \hat{p}_2+\hat{p}_{4,U}+\hat{p}_5\\ &= 0{,}019\ 134+0{,}001\ 316+0{,}000\ 367\\ &= 0{,}020\ 817\end{aligned}$$

18 Standard univariate *σ*-method procedures

18.1 Obtaining a plan, sampling, and preliminary calculations

The *σ*-method is to be used only when there is valid evidence that the standard deviation of the process can be considered constant and taken to be *σ*.

The procedure for obtaining and implementing a plan is as follows.

a) With the inspection level given (normally this will be level II) and with the lot size, obtain the sample size code letter using Table A.1.

b) For a single specification limit, enter Table C.1 or C.2 as appropriate with this code letter and the AQL and obtain the sample size *n*, and the Form *k* acceptability constant *k*. For separate control of double specification limits, do this for both limits. For combined control of double specification limits, enter Table E.1, E.2, or E.3 as appropriate and obtain the sample size *n*, and the Form *p** acceptability constant. For complex control of double specification limits, enter Table E.1, E.2, or E.3 as appropriate twice, once with the combined control part of the specification and once with the smaller AQL applying to the specification limit of greater concern.

c) Take a random sample of size *n*, measure the characteristic under inspection, *x*, for all items in the sample, and calculate the sample mean, $\bar{x}$. The estimate *s* of the process standard deviation (see Annex K) should also be calculated but only for the purpose of checking the continued stability of the process standard deviation. (See Clause 22). If $\bar{x}$ is outside the specification limit(s), the lot can be judged unacceptable without even calculating *s*.

18.2 Acceptability criterion for a single specification limit or for double specification limits with separate control

The acceptability criterion can be found by following the procedure given for the *s*-method. First, replace the *s* derived from the individual samples by *σ*, the presumed known value of the standard deviation of the process, and then compare the calculated value of *Q* with the value of the acceptability constant *k* obtained from one of Tables C.1 and C.2.

Note, for example, that the acceptability criterion $Q_U\ [=(U-\bar{x})/\sigma] \geq k$ for an upper specification may be written as $\bar{x} \leq U - k\sigma$. As *U*, *k*, and *σ* are all known in advance, the acceptance value $\bar{x}_U\ [=U-ks]$ should therefore be determined before inspection begins. For an upper specification limit, a lot will be

acceptable if $\bar{x} \leq \bar{x}_U\ [=U-k\sigma]$, and

not acceptable if $\bar{x} > \bar{x}_U\ [=U-k\sigma]$.

For a lower specification limit, a lot will be

acceptable if $\bar{x} \geq \bar{x}_L\ [=L+k\sigma]$, and

not acceptable if $\bar{x} < \bar{x}_L\ [=L+k\sigma]$.

EXAMPLE Determination of acceptability for a single specification limit using the *σ*-method.

The specified minimum yield point for certain steel castings is 400 N/mm^2. A lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 0,65 %, is to be used. The value of *σ* is considered to be 21 N/mm^2. From Table A.1, it is seen that the sample size code letter is H. Then, from Table C.1, it is seen that for an AQL of 1,0 %, the sample size, *n*, is 11 and the acceptability constant *k* is 2,046. Suppose the yield points of the sample specimens are as follows: 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400. Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Acceptability constant: *k*	2,046
Product: $k\sigma$	38,4 N/mm^2
Specification limit: *L*	400 N/mm^2
Acceptance value: $\bar{x}_L = L + k\sigma$	442,97 N/mm^2
Sum of measurement results: $\sum x$	4 713 N/mm^2
Sample size: *n*	11
Sample mean: $\bar{x}$	428,5 N/mm^2
Acceptability criterion: Is $\bar{x} \geq \bar{x}_L$?	No

The sample mean of the lot does not meet the acceptability criterion so the lot is not acceptable.

For double specification limits with separate control, the lot may at once be declared unacceptable if *σ* is greater than the MPSD derived from Table G.2. If *σ*≤MPSD, determine the acceptability constants for the upper and lower limits, say k_U and k_L. The lot will be

acceptable if $\bar{x} \leq \bar{x}_U\ [=U-k_U\sigma]$ and $\bar{x} \geq \bar{x}_L\ [=L+k_L\sigma]$, and

not acceptable if $\bar{x} > \bar{x}_U\ [=U-k_U\sigma]$ or $\bar{x} < \bar{x}_L\ [=L+k_L\sigma]$.

18.3 Acceptability criterion for double specification limits with combined or complex control

If there is a combined AQL requirement for the upper and the lower specification limits, i.e. an overall AQL for the percentage of the process outside both specification limits, the following procedure is recommended.

a) Before sampling, determine the value of the factor f_σ by entering Table G.1 (for combined control) with the single AQL or by entering Table G.3 (for complex control) with both AQLs.

b) Calculate the maximum allowable value of the process standard deviation using the formula $\sigma_{max} = (U - L) f_\sigma$ for the MPSD.

c) Compare the value of the process standard deviation σ with σ_{max}. If σ exceeds σ_{max}, the process is unacceptable and sampling inspection is discontinued until it is demonstrated that the process variability has been adequately reduced.

d) If $\sigma \le \sigma_{max}$, then use the lot size and given inspection level to determine the sample size code letter from Table A.1.

e) From the sample size code letter, AQL, and inspection severity (i.e. whether inspection is normal, tightened, or reduced), determine the sample size, n, and acceptability constant, p^*, from Table E.1, E.2, or E.3.

f) Select a random sample of size n from the lot and calculate the sample mean, $\bar{x}$.

g) Using the method given in L.2.2, calculate $\hat{p}_U$, $\hat{p}_L$, and $\hat{p} = \hat{p}_U + \hat{p}_L$.

h) If $\hat{p} > p^*$, the lot is not acceptable for either combined or complex control and no other calculations or comparisons are required.

i) For combined control, the lot is acceptable if $\hat{p} \le p^*$.

j) For complex control, determine from Table E.1, E.2, or E.3 the Form p^* acceptability constant for the single specification limit, i.e. p_U^* for an upper specification limit or p_L^* for a lower specification limit. For complex control that includes a separate AQL for the upper specification limit, the lot is acceptable if $\hat{p} \le p^*$ and $\hat{p}_U \le p_U^*$. For complex control that includes a separate AQL for the lower specification limit, the lot is acceptable if $\hat{p} \le p^*$ and $\hat{p}_L \le p_L^*$.

EXAMPLE Determination of acceptability for combined control under the σ-method.

The specification for electrical resistance of a certain electrical component is (520 ± 50) Ω. Production is at a rate of 1 000 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 1,5 %, is to be used for the two specification limits (470 Ω and 570 Ω). σ is known to be 18,5 Ω.

Information needed	**Values obtained**
Factor from Table G.1: f_σ	0,194
Upper specification limit: U	570 Ω
Lower specification limit: L	470 Ω
Maximum process standard deviation, $\sigma_{max} = (U - L) f_\sigma$	19,4 Ω
Known σ	18,5 Ω

Since σ is less than σ_{max}, the sample is analysed further with respect to lot acceptability.

Entering Table A.1 with the lot size and inspection level, it is found that the sample size code letter is J; from Table E.1, it is seen that a sample size of 20 is required under normal inspection, with a Form p^* acceptance constant of 4,241 %. Suppose that the 20 sample values of the resistance in Ω are as follows: 515; 491; 479; 507; 513; 521; 536; 483; 509; 514; 507; 484; 526; 532; 499; 530; 512; 492; 522; 488. Lot acceptability is to be determined.

The exact method of determining lot acceptability is as follows.

Further information needed	**Values obtained**
Sample size: n (from Table E.1)	20
Form p^* acceptability constant (from Table E.1): p^*	0,042 41
Sum of measurement results: Σx	101 60 Ω
Sample mean: $\bar{x}$	508,0 Ω
Lower quality statistic, $Q_L = (\bar{x} - L)/\sigma$	2,054 1
Estimate of process fraction nonconforming below L, $\hat{p}_L = \Phi\left(-Q_L\sqrt{\frac{n}{n-1}}\right)$	0,017 54
Upper quality statistic, $Q_U = (U - \bar{x})/\sigma$	3,351 4
Estimate of process fraction nonconforming above U, $\hat{p}_U = \Phi\left(-Q_U\sqrt{\frac{n}{n-1}}\right)$	0,000 29
Combined estimate, $\hat{p} = \hat{p}_L + \hat{p}_U$	0,017 83

Since the combined estimate is less than the Form p^* acceptability constant, the lot is accepted.

For sample sizes greater than 3, a simpler approximative method exists that avoids the necessity of calculating values of the standard normal distribution function, as shown below.

NOTE The disadvantage of this alternative method is that, besides only being approximate when σ is close to σ_{max}, no estimate of the process fraction nonconforming is produced for monitoring purposes.

Alternative further information needed	**Values obtained**
Sample size: n (from Table C.1)	20
Form k acceptability constant (from Table C.1): k	1,680
Sum of measurement results: Σx	101 60 Ω
Sample mean: $\bar{x}$	508,0 Ω
Upper bound for $\bar{x}$: $\bar{x}_U = U - k\sigma$	538,9 Ω
Lower bound for $\bar{x}$: $\bar{x}_L = L + k\sigma$	501,1 Ω

Since $\bar{x}$ at 511,0 Ω lies between the acceptance limits for $\bar{x}$ of 501,1 Ω and 538,9 Ω, the lot is acceptable.

NOTE If, for example, σ had been known to be 25, then σ exceeds the MPSD and a decision not to accept the lot could be made without any sampling inspection.

19 Standard multivariate σ-method procedures for independent quality characteristics

19.1 General methodology

The general methodology for dealing with a class containing *m* independent quality characteristics $x_1, x_2, ..., x_m$ under the σ-method is similar to that for the multivariate *s*-method, i.e. denoting the estimated process fraction nonconforming for the i^{th} quality characteristic of the class by $\hat{p}_i$, the estimated process fraction nonconforming for the class is given by

$$\hat{p} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_2)...(1 - \hat{p}_m) , \quad (13)$$

i.e. 1 minus the product of the estimated process fractions *conforming*.

If there is only one class, say class A, then the estimated process fraction nonconforming for the class may be denoted by $\hat{p}_A$. The lot is accepted if $\hat{p}_A \le p^*$ and not accepted otherwise, where p^* is the Form p^* acceptability constant given in Table E.1, E.2, or E.3 for the inspection severity, applicable sample size code letter, and AQL applying to the class.

If there are two or more classes, say class A, class B, … with acceptability constants p_A^*, p_B^*,… the lot is accepted if $\hat{p}_A \le p_A^*$ and $\hat{p}_B \le p_B^*$ and so on but not accepted if one or more of the inequalities is violated.

If there is more than one class of nonconformity, class A will contain nonconformities of the greatest level of seriousness and generally have the lowest AQL and, therefore, the lowest Form p^* acceptability constant; class B will contain nonconformities of the next lower level of seriousness and have a higher AQL and value of p^*; and so on. It is possible that different classes of nonconformity will be under inspection at different levels of severity at any one time.

The only difference from the multivariate *s*-method is that the process fraction nonconforming for each characteristic is estimated in accordance with L.2.2 instead of L.2.1.

19.2 Example

The example given in 17.2 is repeated with the sample standard deviations redesignated as process standard deviations.

Consider a product that has five independent quality characteristics x_1, x_2, x_3, x_4, and x_5, for all of which the process standard deviations are known. The sample size code letter is H and the sample size is 12 under normal inspection for all five characteristics. Suppose that the requirements and results are as summarized in Table 3.

Suppose also that the AQL for class A nonconformity is 0,25 % and the AQL for class B is 1,0 %. From Table E.1, it is found that the corresponding sample sizes are 6 and 10, and the corresponding Form p^* acceptability constants are p_A^* = 0,007 546 and $p_B^* = 0{,}027\ 51$.

Table 3 — Example of requirements and results for five quality characteristics with known process standard deviations

Variable	Limits	Type of control	Class	Sample size	Sample mean	Process standard deviation	Quality statistic Q	$Q\sqrt{\frac{n}{n-1}}$	$\hat{p}$
x_1	$U_1 =$ 70,0	Single	A	6	$\bar{x}_1 = 68{,}5$	$\sigma_1 = 0{,}50$	3,000 0	3,286 3	0,000 508
x_2	$L_2 =$ 10,0	Single	B	10	$\bar{x}_2 = 10{,}4$	$\sigma_2 = 0{,}20$	2,000 0	2,097 6	0,017 970
x_3	$U_3 =$ 4,05 $L_3 =$ 3,95	Combined	A	6	$\bar{x}_3 = 4{,}005$	$\sigma_3 = 0{,}015$	3,000 0 3,666 7	3,286 3 4,016 6	0,000 508 0,000 030 0,000 538
x_4	$U_4 =$ 1,95 $L_4 =$ 1,75	Separate	B A	10 6	$\bar{x}_{4,\mathrm{U}} = 1{,}862$ $\bar{x}_{4,\mathrm{L}} = 1{,}830$	$\sigma_4 = 0{,}032$	2,750 0 2,500 0	2,884 2 2,738 6	0,001 962 0,003 085
x_5	$U_5 =$ 214 $L_5 =$ 206	Complex, i.e. Separate and Combined	A B	6 10	$\bar{x}_{5,\mathrm{U}} = 210{,}3$ $\bar{x}_{5,\mathrm{L}} = 210{,}1$	$\sigma_5 = 1{,}25$	2,960 0 3,280 0	3,242 5 3,985 5	0,000 592 0,000 034 0,000 626

The fraction nonconforming for class A is estimated as

$$\begin{aligned}\hat{p}_\mathrm{A} &= 1-(1-\hat{p}_1)(1-\hat{p}_3)(1-\hat{p}_{4,\mathrm{L}})(1-\hat{p}_{5,\mathrm{U}})\\ &= 1-(1-0{,}000\ 508)(1-0{,}000\ 538)(1-0{,}003\ 085)(1-0{,}000\ 592)\\ &= 1-0{,}999\ 492\times 0{,}999\ 462\times 0{,}996\ 915\times 0{,}999\ 408\\ &= 1-0{,}995\ 283\\ &= 0{,}004\ 7\end{aligned}$$

The fraction nonconforming for class B is estimated as

$$\begin{aligned}\hat{p}_\mathrm{B} &= 1-(1-\hat{p}_2)(1-\hat{p}_{4,\mathrm{U}})(1-\hat{p}_5)\\ &= 1-(1-0{,}017\ 970)(1-0{,}001\ 962)(1-0{,}000\ 626)\\ &= 1-0{,}982\ 030\times 0{,}998\ 038\times 0{,}999\ 374\\ &= 1-0{,}979\ 490\\ &= 0{,}020\ 51\end{aligned}$$

Since $\hat{p}_\mathrm{A} < p_\mathrm{A}^*$ and $\hat{p}_\mathrm{B} < p_\mathrm{B}^*$, the lot is accepted.

20 Standard multivariate combined *s*-method and *σ*-method procedures for independent quality characteristics

20.1 General methodology

Cases may arise in which the process standard deviations of some of the quality characteristics in a class are known and some are unknown. The general methodology for dealing with such a class containing *m independent* quality characteristics is, as before, to estimate the process fraction nonconforming for the class by

$$\hat{p} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_2)\ldots(1 - \hat{p}_m) \quad (14)$$

If there is only one class, say class A, then the estimated process fraction nonconforming for the class may be denoted by $\hat{p}_A$. The lot is accepted if

$$\hat{p}_A \leq p^*$$

and not accepted otherwise, where p^* is the Form p^* acceptability constant given in Table D.1, D.2, or D.3 (or, equivalently, Table E.1, E.2, or E.3) for the applicable inspection severity, sample size code letter, and AQL applying to the class.

If there are two or more classes, say class A, class B, etc., with acceptability constants p_A^*, p_B^*, etc., the lot is accepted if $\hat{p}_A \leq p_A^*$, $\hat{p}_B \leq p_B^*$, etc., but not accepted if one or more of the inequalities is violated.

The estimate of the process fraction nonconforming for each characteristic whose process standard deviation is unknown is obtained in accordance with L.2.1; for known process standard deviation, the estimate is obtained in accordance with L.2.2.

20.2 Example

Consider, as before, a product that has five independent quality characteristics x_1, x_2, x_3, x_4, and x_5, classified into class A with an AQL of 0,25 % or class B with an AQL of 1 %. However, in this case, only characteristics x_1 and x_4 have process standard deviations whose values are known. The sample size code letter is H and the sample sizes for class A and class B are 18 or 24, respectively, under normal inspection for the characteristics with unknown process standard deviation and 6 and 10 where the process standard deviation is known. Suppose that the requirements and results are as summarized in Table 4.

Table 4 — Example of requirements and results for five quality characteristics, some with known and some with unknown process standard deviations

Variable	Limits	Type of control	Class	Sample size, n	Sample mean	Standard deviation	Quality statistic Q	$\frac{1-Q\frac{\sqrt{n}}{n-1}}{2}$	$Q\sqrt{\frac{n}{n-1}}$	$\hat{p}$
x_1	$U_1 = 70,0$	Single	A	6	$\bar{x}_1 =$ 68,5	$\sigma_1 = 0,50$	3,000 0		3,286 3	0,000 508
x_2	$L_2 = 10,0$	Single	B	24	$\bar{x}_2 =$ 10,4	$s_2 = 0,20$	2,000 0	0,291 7		0,019 134
x_3	$U_3 = 4,05$ $L_3 = 3,95$	Combined	A	18	$\bar{x}_3 =$ 4,005	$s_3 =$ 0,015	3,000 0 3,666 7	0,187 5 0,118 1		0,000 418 0,000 004 0,000 422
x_4	$U_4 = 1,95$ $L_4 = 1,75$	Separate	B A	10 6	$\bar{x}_{4,U} =$ 1,862 $\bar{x}_{4,L} =$ 1,830	$\sigma_4 =$ 0,032	2,750 0 2,500 0		2,884 2 2,738 6	0,001 962 0,003 085
x_5	$U_5 = 214$ $L_5 = 206$	Complex, i.e. separate and combined	A B	18 24	$\bar{x}_{5,U} =$ 210,3 $\bar{x}_{5,L} =$ 210,1	$s_{5,U} =$ 1,25 $s_{5,L} =$ 1,27	2,960 0 3,070 9 3,228 3	0,130 6 0,173 0 0,156 2		0,000 231 0,000 264 0,000 103 0,000 367

Again, suppose that the AQL for class A nonconformities is 0,25 % and the AQL for class B is 1,0 %, so from Table G.1, it is found that the corresponding Form p^* acceptability constants are $p_A^* = 0{,}007\,546$ and $p_B^* = 0{,}027\,51$.

The fraction nonconforming for class A is estimated as

$$\begin{aligned}\hat{p}_A &= 1-(1-\hat{p}_1)(1-\hat{p}_3)(1-\hat{p}_{4,L})(1-\hat{p}_{5,U})\\ &= 1-(1-0{,}000\,508)(1-0{,}000\,422)(1-0{,}003\,085)(1-0{,}000\,231)\\ &= 1-0{,}999\,492\times 0{,}999\,578\times 0{,}996\,915\times 0{,}999\,769\\ &= 1-0{,}995\,758\\ &= 0{,}004\,242\end{aligned}$$

The fraction nonconforming for class B is estimated as

$$\begin{aligned}\hat{p}_B &= 1-(1-\hat{p}_2)(1-\hat{p}_{4,U})(1-\hat{p}_5)\\ &= 1-(1-0{,}019\,134)(1-0{,}001\,962)(1-0{,}000\,367)\\ &= 1-0{,}980\,866\times 0{,}998\,038\times 0{,}999\,633\\ &= 1-0{,}978\,582\\ &= 0{,}021\,42\end{aligned}$$

Since $\hat{p}_A < p_A^*$ and $\hat{p}_B < p_B^*$, the lot is accepted.

21 Procedure during continuing inspection

Since a variable sampling inspection plan can only operate efficiently if

a) the characteristic being inspected is normally distributed,

b) records are kept, and

c) the switching rules are obeyed,

it is necessary to ensure that these requirements are being met.

22 Normality and outliers

22.1 Normality

The responsible authority should have checked for normality before sampling began. In case of doubt, a statistician should advise whether the distribution appears suitable for sampling by variables or whether use should be made of the tests for departure from normality given in ISO 5479. Normality should be reconfirmed periodically, particularly if there is a significant change of any kind in production, e.g. in personnel, design, materials, or production method.

22.2 Outliers

An outlier (or an outlying observation) is one that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, will produce an increase in variability and change the mean and may consequently lead to non-acceptance of the lot (see, for example, ISO 16269-4). When outliers are detected, the disposition of the lot should be a matter for negotiation between the vendor and vendee.

23 Records

23.1 Control charts

One of the advantages of inspection by variables is that trends in the quality level of the product can be detected and a warning given before an unacceptable standard is reached, but this is only possible if adequate records are kept.

Whatever the method used, s-method or σ-method, records should be kept of the values of $\bar{x}$ and s, preferably in the form of control charts. (See, for example, ISO 7870.)

This procedure should be applied especially with the σ-method in order to verify that the values of s obtained from the samples fall within the limits of the prescribed value of σ.

For double specification limits with a combined AQL requirement, the value of the MSSD, derived from Table F.1, F.2, or F.3, should be plotted on the s control chart, as an indication of an unacceptable value.

NOTE Control charts are used to detect trends. The ultimate decision as to the acceptability of an individual lot is governed by the procedures given in Clauses 16 to 20.

23.2 Lots that are not accepted

Particular care shall be taken to record all lots that are not accepted and to see that switching rules are implemented. Any lot not accepted by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

24 Operation of switching rules

The standard switching rules are as follows.

24.1 Normal inspection is used at the start of inspection (unless otherwise designated) and shall continue to be used during the course of inspection until tightened inspection becomes necessary or reduced inspection is allowed.

24.2 Tightened inspection shall be instituted when two lots on original normal inspection are not accepted within any five or fewer successive lots.

Tightened inspection is generally achieved by increasing the value of the Form k acceptability constant and correspondingly decreasing the value of the Form p^* acceptability constant. The values are tabulated in Tables B.2 and D.2 for the s-method and Tables C.2 and E.2 for the σ-method. For neither method is there a change in the size of the sample in switching from normal to tightened inspection, unless the AQL is so small that the tables indicate, with a downward arrow, that an increase in sample size is necessary.

24.3 Tightened inspection shall be relaxed when five successive lots on original inspection have been accepted on tightened inspection; then, normal inspection shall be reinstated.

24.4 Reduced inspection may be instituted after 10 successive lots have been accepted under normal inspection, provided that

a) these lots would have been acceptable if the AQL had been one step tighter,

NOTE If a value of k for this tighter AQL is not given in Table B.1 (s-method) or Table C.1 (σ-method) or a value of p^* is not given in Table D.1 or Table E.1, refer to Table J.1.

b) production is in statistical control, and

c) reduced inspection is considered desirable by the responsible authority.

Reduced inspection is generally conducted on a much smaller sample than normal inspection and the value of the acceptability constant is also decreased. The values of n and k for reduced inspection are given in Table B.3 for the s-method and Table C.3 for the σ-method. The values of n and p^* for reduced inspection are given in Table D.3 for the s-method and Table E.3 for the σ-method.

24.5 Reduced inspection shall cease and normal inspection be reinstated if any of the following occur on original inspection:

a) a lot is not accepted;

b) production becomes irregular or delayed;

c) reduced inspection is no longer considered desirable by the responsible authority.

25 Discontinuation and resumption of inspection

If the cumulative number of lots that is not accepted in a sequence of consecutive lots on original tightened inspection reaches 5, the acceptance procedures of this part of ISO 3951 shall be discontinued.

Inspection under the provisions of this part of ISO 3951 shall not be resumed until action has been taken by the supplier to improve the quality of the submitted product or service. Tightened inspection shall then be used as if 23.2 had been invoked.

26 Switching between the *s*-method and *σ*-method

26.1 Estimating the process standard deviation

While this part of ISO 3951 is being used, the weighted root mean square of the values of s shall be calculated periodically as estimates of the process standard deviation, σ, under both the s-method and the σ-method. (See Annex K, K.2.) The value of σ shall be estimated at five-lot intervals, unless the responsible authority specifies another interval. The estimate shall be based on the preceding 10 lots, unless the responsible authority specifies another number of lots.

26.2 State of statistical control

Calculate the upper control limit for each of the 10 lots (or other number of lots specified by the responsible authority) from the expression $c_{\mathrm{U}}\sigma$, where c_{U} is a factor that depends on the sample size, n, and is given in Table I.1. If none of the sample standard deviations, s_i, exceed the corresponding control limit, then the process may be considered to be in a state of statistical control; otherwise, the process shall be considered to be out of statistical control.

NOTE 1 If the sample sizes from the lots are all equal, then the value of $c_{\mathrm{U}}\sigma$ is common to all the lots.

NOTE 2 If the sample sizes from each lot vary, it is not necessary to calculate $c_{\mathrm{U}}\sigma$ for those lots for which the sample standard deviation, s_i, is less than or equal to σ.

26.3 Switching from the *s*-method to the *σ*-method

If the process is considered to be in a state of statistical control under the s-method, then the σ-method may be instituted using the latest value of σ.

NOTE This switch is made at the discretion of the responsible authority.

26.4 Switching from the *σ*-method to the *s*-method

It is recommended that a control chart for s be kept even under the σ-method. As soon as there is any doubt that the process remains in statistical control, inspection shall be switched to the s-method.

Annex A
(normative)

Table for determining the sample size code letter

Table A.1 — Sample size code letters and inspection levels

Lot or batch size	Special inspection levels				General inspection levels		
	S-1	S-2	S-3	S-4	I	II	III
2 to 8	B	B	B	B	B	B	B
9 to 15	B	B	B	B	B	B	C
16 to 25	B	B	B	B	B	C	D
26 to 50	B	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1 200	C	C	E	F	G	J	K
1 201 to 3 200	C	D	E	G	H	K	L
3 201 to 10 000	C	D	F	G	J	L	M
10 001 to 35 000	C	D	F	H	K	M	N
35 001 to 150 000	D	E	G	J	L	N	P
150 001 to 500 000	D	E	G	J	M	P	Q
500 000 and over	D	E	H	K	N	Q	R

The sample size code letters and inspection levels in this part of ISO 3951 correspond to those given in ISO 2859-1.

Annex B
(normative)

Form *k* single sampling plans: *s*-method

Table B.1 — Single sampling plans of Form *k* for normal inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *K*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,950	4 0,735	4 0,586
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,242	6 1,061	6 0,939	5 0,550
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 1,476	9 1,323	9 1,218	6 0,887	7 0,507
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	9 1,696	13 1,569	13 1,475	9 1,190	9 0,869	9 0,618
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	11 1,889	17 1,769	18 1,682	13 1,426	14 1,147	14 0,935	14 0,601
G	↓	↓	↓	↓	↓	↓	↓	↓	15 2,079	22 1,972	23 1,893	18 1,659	20 1,411	21 1,227	21 0,945	21 0,724
H	↓	↓	↓	↓	↓	↓	↓	18 2,254	28 2,153	30 2,079	24 1,862	27 1,636	30 1,471	32 1,225	33 1,036	33 0,806
J	↓	↓	↓	↓	↓	↓	23 2,425	36 2,331	38 2,263	31 2,061	37 1,853	41 1,702	46 1,482	49 1,316	52 1,120	53 0,911
K	↓	↓	↓	↓	↓	28 2,580	44 2,493	47 2,428	40 2,237	48 2,043	54 1,904	63 1,702	69 1,552	75 1,377	79 1,195	82 0,946
L	↓	↓	↓	↓	34 2,737	54 2,653	58 2,592	50 2,412	61 2,230	71 2,101	84 1,914	94 1,777	105 1,619	115 1,456	124 1,239	↑
M	↓	↓	↓	40 2,882	64 2,802	69 2,744	60 2,573	76 2,400	89 2,279	108 2,104	124 1,977	143 1,832	159 1,683	178 1,488	↑	↑
N	↓	↓	47 3,023	75 2,948	82 2,892	73 2,728	93 2,564	110 2,449	137 2,285	159 2,166	186 2,031	213 1,894	247 1,715	↑	↑	↑
P	↓	55 3,161	88 3,089	96 3,036	86 2,879	112 2,723	134 2,614	171 2,459	202 2,347	239 2,220	277 2,092	332 1,928	↑	↑	↑	↑
Q	63 3,288	101 3,219	110 3,167	102 3,016	132 2,867	159 2,762	207 2,615	244 2,508	293 2,388	348 2,268	424 2,114	↑	↑	↑	↑	↑
R	116 3,351	127 3,301	120 3,156	155 3,012	189 2,912	247 2,771	298 2,670	362 2,556	438 2,443	541 2,298	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.2 — Single sampling plans of Form *k* for tightened inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,950	4 0,735
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,242	6 1,061	6 0,939
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 1,476	9 1,323	9 1,218	6 0,887
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	9 1,696	13 1,569	13 1,475	9 1,190	9 0,869
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	11 1,889	17 1,769	18 1,682	13 1,426	14 1,147	14 0,935
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	15 2,079	22 1,972	23 1,893	18 1,659	20 1,411	21 1,227	21 0,945
H	↓	↓	↓	↓	↓	↓	↓	↓	18 2,254	28 2,153	30 2,079	24 1,862	27 1,636	30 1,471	32 1,225	33 0,954
J	↓	↓	↓	↓	↓	↓	↓	23 2,425	36 2,331	38 2,263	31 2,061	37 1,853	41 1,702	46 1,482	50 1,245	53 1,010
K	↓	↓	↓	↓	↓	↓	28 2,580	44 2,493	47 2,428	40 2,237	48 2,043	54 1,904	63 1,702	71 1,489	78 1,281	82 1,045
L	↓	↓	↓	↓	↓	34 2,737	54 2,653	58 2,592	50 2,412	61 2,230	71 2,101	84 1,914	99 1,720	111 1,533	122 1,329	↑
M	↓	↓	↓	↓	40 2,882	64 2,802	69 2,744	60 2,573	76 2,400	89 2,279	108 2,104	131 1,924	150 1,752	170 1,564	↑	↑
N	↓	↓	↓	47 3,023	75 2,948	82 2,892	73 2,728	93 2,564	110 2,449	137 2,285	169 2,117	201 1,958	233 1,785	↑	↑	↑
P	↓	↓	55 3,161	88 3,089	96 3,036	86 2,879	112 2,723	134 2,614	171 2,459	214 2,300	260 2,152	312 1,992	↑	↑	↑	↑
Q	↓	63 3,288	101 3,219	110 3,167	102 3,016	132 2,867	159 2,762	207 2,615	262 2,464	323 2,324	395 2,174	↑	↑	↑	↑	↑
R	90 3,408	116 3,351	127 3,301	120 3,156	155 3,012	189 2,912	247 2,771	320 2,628	398 2,495	498 2,354	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.3 — Single sampling plans of Form *k* for reduced inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B – D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,950	4 0,850	4 0,735	4 0,586	7 0,218
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,242	6 1,155	6 1,061	6 0,939	5 0,550	9 0,162
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 1,476	8 1,406	9 1,323	9 1,218	6 0,887	7 0,507	8 0,231
G	↓	↓	↓	↓	↓	↓	↓	↓	9 1,696	11 1,642	13 1,569	13 1,475	9 1,190	9 0,869	9 0,618	12 0,237
H	↓	↓	↓	↓	↓	↓	↓	11 1,889	15 1,835	17 1,769	18 1,682	13 1,426	14 1,147	14 0,935	14 0,601	13 0,454
J	↓	↓	↓	↓	↓	↓	15 2,079	19 2,033	22 1,972	23 1,893	18 1,659	20 1,411	21 1,227	21 0,945	21 0,830	21 0,626
K	↓	↓	↓	↓	↓	18 2,254	24 2,209	28 2,153	30 2,079	24 1,862	27 1,636	30 1,471	32 1,225	33 1,126	33 0,954	33 0,806
L	↓	↓	↓	↓	23 2,425	30 2,385	36 2,331	38 2,263	31 2,061	37 1,853	41 1,702	46 1,482	48 1,394	50 1,245	52 1,120	↑
M	↓	↓	↓	28 2,580	37 2,543	44 2,493	47 2,428	40 2,237	48 2,043	54 1,904	63 1,702	66 1,622	71 1,489	75 1,377	↑	↑
N	↓	↓	34 2,737	44 2,701	54 2,653	58 2,592	50 2,412	61 2,230	71 2,101	84 1,914	90 1,842	99 1,720	105 1,619	↑	↑	↑
P	↓	40 2,882	52 2,848	64 2,802	69 2,744	60 2,573	76 2,400	89 2,279	108 2,104	117 2,037	131 1,924	143 1,832	↑	↑	↑	↑
Q	47 3,023	61 2,991	75 2,948	82 2,892	73 2,728	93 2,564	110 2,449	137 2,285	149 2,222	169 2,117	186 2,031	↑	↑	↑	↑	↑
R	71 3,131	88 3,089	96 3,036	86 2,879	112 2,723	134 2,614	171 2,459	187 2,399	214 2,300	239 2,220	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex C
(normative)

Form *k* single sampling plans: *σ*-method

Table C.1 — Single sampling plans of Form *k* for normal inspection: *σ*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,709	4 0,571	3 0,417
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 1,115	5 0,945	5 0,821	4 0,436
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,406	6 1,240	6 1,128	5 0,770	5 0,431
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,595	7 1,506	8 1,419	7 1,115	7 0,792	7 0,555
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,845	8 1,720	9 1,635	8 1,366	10 1,094	9 0,877	11 0,564
G	↓	↓	↓	↓	↓	↓	↓	↓	5 2,006	9 1,934	10 1,856	9 1,610	12 1,370	13 1,186	13 0,906	15 0,694
H	↓	↓	↓	↓	↓	↓	↓	6 2,218	10 2,122	11 2,046	10 1,820	13 1,599	16 1,439	16 1,191	19 1,009	23 0,786
J	↓	↓	↓	↓	↓	↓	7 2,401	11 2,302	12 2,234	11 2,025	15 1,823	19 1,677	21 1,456	24 1,293	29 1,102	34 0,897
K	↓	↓	↓	↓	↓	7 2,541	12 2,468	13 2,401	13 2,210	17 2,018	21 1,882	27 1,683	29 1,533	35 1,361	42 1,182	53 0,937
L	↓	↓	↓	↓	8 2,710	13 2,629	15 2,573	14 2,387	19 2,209	24 2,083	32 1,900	34 1,761	42 1,606	52 1,446	66 1,231	↑
M	↓	↓	↓	8 2,844	14 2,780	16 2,726	15 2,550	21 2,382	27 2,264	36 2,092	39 1,963	50 1,821	61 1,674	79 1,481	↑	↑
N	↓	↓	9 2,996	15 2,929	17 2,874	17 2,709	24 2,550	30 2,437	40 2,274	45 2,155	57 2,022	72 1,887	94 1,710	↑	↑	↑
P	↓	10 3,141	17 3,069	19 3,023	19 2,865	26 2,711	33 2,603	45 2,450	51 2,337	65 2,212	82 2,086	110 1,923	↑	↑	↑	↑
Q	11 3,275	18 3,207	20 3,155	20 3,002	28 2,856	35 2,752	49 2,607	57 2,500	72 2,381	92 2,262	125 2,110	↑	↑	↑	↑	↑
R	19 3,339	21 3,289	22 3,145	30 3,002	38 2,903	54 2,764	64 2,663	81 2,550	105 2,438	142 2,294	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.2 — Single sampling plans of Form *k* for tightened inspection: *σ*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,709	4 0,571
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 1,115	5 0,945	5 0,821
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,406	6 1,240	6 1,128	5 0,770
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,595	7 1,506	8 1,419	7 1,115	7 0,792
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,845	8 1,720	9 1,635	8 1,366	10 1,094	9 0,877
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 2,006	9 1,934	10 1,856	9 1,610	12 1,370	13 1,186	13 0,906
H	↓	↓	↓	↓	↓	↓	↓	↓	6 2,218	10 2,122	11 2,046	10 1,820	13 1,599	16 1,439	16 1,191	20 0,929
J	↓	↓	↓	↓	↓	↓	↓	7 2,401	11 2,302	12 2,234	11 2,025	15 1,823	19 1,677	21 1,456	25 1,223	32 0,994
K	↓	↓	↓	↓	↓	↓	7 2,541	12 2,468	13 2,401	13 2,210	17 2,018	21 1,882	27 1,683	31 1,471	39 1,267	49 1,035
L	↓	↓	↓	↓	↓	8 2,710	13 2,629	15 2,573	14 2,387	19 2,209	24 2,083	32 1,900	37 1,705	47 1,521	61 1,316	↑
M	↓	↓	↓	↓	8 2,844	14 2,780	16 2,726	15 2,550	21 2,382	27 2,264	36 2,092	43 1,912	55 1,742	72 1,556	↑	↑
N	↓	↓	↓	9 2,996	15 2,929	17 2,874	17 2,709	24 2,550	30 2,437	40 2,274	49 2,106	65 1,950	85 1,779	↑	↑	↑
P	↓	↓	10 3,142	17 3,076	19 3,023	19 2,865	26 2,711	33 2,603	45 2,450	55 2,291	74 2,145	99 1,987	↑	↑	↑	↑
Q	↓	11 3,275	18 3,207	20 3,155	20 3,002	28 2,856	35 2,752	49 2,607	61 2,456	83 2,318	112 2,169	↑	↑	↑	↑	↑
R	14 3,391	19 3,339	21 3,289	22 3,145	30 3,002	38 2,903	54 2,764	68 2,621	92 2,490	126 2,350	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.3 — Single sampling plans of Form *k* for reduced inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*	*n* *k*
B – D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 0,709	4 0,679	4 0,571	3 0,417	6 0,187
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 1,115	5 1,047	5 0,945	5 0,821	4 0,436	8 0,145
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 1,406	5 1,314	6 1,240	6 1,128	5 0,770	5 0,431	7 0,204
G	↓	↓	↓	↓	↓	↓	↓	↓	4 1,595	6 1,581	7 1,506	8 1,419	7 1,115	7 0,792	7 0,555	11 0,220
H	↓	↓	↓	↓	↓	↓	↓	5 1,845	7 1,788	8 1,720	9 1,635	8 1,366	10 1,094	9 0,877	11 0,564	11 0,424
J	↓	↓	↓	↓	↓	↓	5 2,006	7 1,982	9 1,934	10 1,856	9 1,610	12 1,370	13 1,186	13 0,906	14 0,796	16 0,601
K	↓	↓	↓	↓	↓	6 2,218	8 2,171	10 2,122	11 2,046	10 1,820	13 1,599	16 1,439	16 1,191	18 1,096	20 0,929	23 0,786
L	↓	↓	↓	↓	7 2,401	9 2,355	11 2,302	12 2,234	11 2,025	15 1,823	19 1,677	21 1,456	22 1,369	25 1,223	29 1,102	↑
M	↓	↓	↓	7 2,541	10 2,518	12 2,468	13 2,401	13 2,210	17 2,018	21 1,882	27 1,683	26 1,601	31 1,471	35 1,361	↑	↑
N	↓	↓	8 2,710	10 2,669	13 2,629	15 2,573	14 2,387	19 2,209	24 2,083	32 1,900	31 1,825	37 1,705	42 1,606	↑	↑	↑
P	↓	8 2,844	11 2,822	14 2,780	16 2,726	15 2,550	21 2,382	27 2,264	36 2,092	38 2,024	43 1,912	50 1,821	↑	↑	↑	↑
Q	9 2,996	12 2,969	15 2,929	17 2,874	17 2,709	24 2,550	30 2,437	40 2,274	45 2,212	49 2,106	57 2,022	↑	↑	↑	↑	↑
R	13 3,113	17 3,076	19 3,023	19 2,865	26 2,711	33 2,603	45 2,450	50 2,390	55 2,291	65 2,212	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex D
(normative)

Form *p** single sampling plans: *s*-method

Table D.1 — Single sampling plans of Form *p for normal inspection: *s*-method**

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 19,25	4 25,50	4 30,47
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 8,600	6 14,53	6 17,93	5 30,74
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 5,220	9 8,717	9 10,82	6 19,46	7 31,49
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	9 3,279	13 5,195	13 6,466	9 11,43	9 19,61	9 27,43
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	11 1,958	17 3,295	18 4,144	13 7,204	14 12,45	14 17,61	14 27,71
G	↓	↓	↓	↓	↓	↓	↓	↓	15 1,245	22 2,011	23 2,518	18 4,381	20 7,627	21 10,85	21 17,29	21 23,62
H	↓	↓	↓	↓	↓	↓	↓	18 ,7546	28 1,266	30 1,592	24 2,751	27 4,799	30 6,857	32 10,94	33 15,00	33 21,09
J	↓	↓	↓	↓	↓	↓	23 ,4753	36 ,7878	38 ,9814	31 1,685	37 2,959	41 4,241	46 6,783	49 9,324	52 13,11	53 18,14
K	↓	↓	↓	↓	↓	28 ,3027	44 ,4976	47 ,6222	40 1,071	48 1,876	54 2,687	63 4,313	69 5,935	75 8,361	79 11,57	82 17,22
L	↓	↓	↓	↓	34 ,1880	54 ,3105	58 ,3872	50 ,6625	61 1,162	71 1,667	84 2,681	94 3,692	105 5,204	115 7,220	124 10,74	↑
M	↓	↓	↓	40 ,1180	64 ,1954	69 ,2436	60 ,4150	76 ,7337	89 1,052	108 1,694	124 2,335	143 3,290	159 4,571	178 6,804	↑	↑
N	↓	↓	47 ,07418	75 ,1217	82 ,1524	73 ,2605	93 ,4595	110 ,6602	137 1,063	159 1,467	186 2,069	213 2,873	247 4,286	↑	↑	↑
P	↓	55 ,04641	88 ,07599	96 ,09473	86 ,1614	112 ,2852	134 ,4100	171 ,6611	202 ,9127	239 1,290	277 1,793	332 2,668	↑	↑	↑	↑
Q	63 ,02960	101 ,04835	110 ,06042	102 ,1034	132 ,1817	159 ,2619	207 ,4220	244 ,5836	293 ,8248	348 1,146	424 1,707	↑	↑	↑	↑	↑
R	116 ,03011	127 ,03762	120 ,06433	155 0,1132	189 ,1631	247 ,2634	298 ,3637	362 ,5145	438 ,7143	541 1,065	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table D.2 — Single sampling plans of Form *p** for tightened inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 19,25	4 25,50
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 8,600	6 14,53	6 17,93
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 5,220	9 8,717	9 10,82	6 19,46
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	9 3,279	13 5,195	13 6,466	9 11,43	9 19,61
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	11 1,958	17 3,295	18 4,144	13 7,204	14 12,45	14 17,61
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	15 1,245	22 2,011	23 2,518	18 4,381	20 7,627	21 10,85	21 17,29
H	↓	↓	↓	↓	↓	↓	↓	↓	18 ,7546	28 1,266	30 1,592	24 2,751	27 4,799	30 6,857	32 10,94	33 17,03
J	↓	↓	↓	↓	↓	↓	↓	23 ,47 53	36 ,78 79	38 ,98 14	31 1,685	37 2,959	41 4,241	46 6,783	50 10,59	53 15,63
K	↓	↓	↓	↓	↓	↓	28 ,3027	44 ,4976	47 ,6222	40 1,071	48 1,876	54 2,687	63 4,313	71 6,738	78 9,963	82 14,80
L	↓	↓	↓	↓	↓	34 ,1880	54 ,3105	58 ,3872	50 ,6625	61 1,162	71 1,667	84 2,681	99 4,192	111 6,205	122 9,224	↑
M	↓	↓	↓	↓	40 ,1180	64 ,1954	69 ,2436	60 ,4150	76 ,7336	89 1,052	108 1,694	131 2,654	150 3,936	170 5,851	↑	↑
N	↓	↓	↓	47 ,07418	75 ,1218	82 ,1524	73 ,2605	93 ,4595	110 ,6602	137 1,063	169 1,666	201 2,470	233 3,679	↑	↑	↑
P	↓	↓	55 ,04641	88 ,07599	96 ,09473	86 ,1614	112 ,2852	134 ,4100	171 ,6611	214 1,039	260 1,540	312 2,292	↑	↑	↑	↑
Q	↓	63 ,02960	101 ,04835	110 ,06042	102 ,1034	132 ,1817	159 ,2619	207 ,4220	262 ,6640	323 ,9849	395 1,466	↑	↑	↑	↑	↑
R	90 ,02165	116 ,03011	127 ,03762	120 ,06433	155 ,1132	189 ,1631	247 ,2634	320 ,4141	398 ,6152	498 ,9152	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table D.3 — Single sampling plans of Form *p** for reduced inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**
B – D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 19,25	4 21,67	4 25,50	4 30,47	7 41,88
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 8,600	6 12,09	6 14,53	6 17,93	5 30,74	9 43,83
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	6 5,220	8 7,090	9 8,717	9 10,82	6 19,46	7 31,49	8 41,30
G	↓	↓	↓	↓	↓	↓	↓	↓	9 3,279	11 4,162	13 5,195	13 6,466	9 11,43	9 19,61	9 27,43	12 40,88
H	↓	↓	↓	↓	↓	↓	↓	11 1,958	15 2,670	17 3,295	18 4,144	13 7,204	14 12,45	14 17,61	14 27,71	13 32,84
J	↓	↓	↓	↓	↓	↓	15 1,245	19 1,613	22 2,011	23 2,518	18 4,381	20 7,627	21 10,85	21 17,29	21 20,45	21 26,75
K	↓	↓	↓	↓	↓	18 ,7546	24 1,016	28 1,266	30 1,592	24 2,751	27 4,799	30 6,857	32 10,94	33 12,96	33 17,03	33 21,09
L	↓	↓	↓	↓	23 ,4753	30 ,6246	36 ,7878	38 ,9814	31 1,685	37 2,959	41 4,241	46 6,783	48 8,059	50 10,59	52 13,11	↑
M	↓	↓	↓	28 ,3027	37 ,3976	44 ,4976	47 ,6222	40 1,071	48 1,876	54 2,687	63 4,313	66 5,129	71 6,738	75 8,361	↑	↑
N	↓	↓	34 ,1880	44 ,2451	54 ,3105	58 ,3872	50 ,6625	61 1,162	71 1,667	84 2,681	90 3,182	99 4,192	105 5,204	↑	↑	↑
P	↓	40 ,1180	52 ,1540	64 ,1954	69 ,2436	60 ,4150	76 ,7336	89 1,052	108 1,694	117 2,012	131 2,654	143 3,290	↑	↑	↑	↑
Q	47 ,07418	61 ,09633	75 ,1217	82 ,1524	73 ,2605	93 ,4595	110 ,6602	137 1,063	149 1,264	169 1,666	186 2,069	↑	↑	↑	↑	↑
R	71 ,05982	88 ,07599	96 ,09473	86 ,1614	112 ,2852	134 ,4100	171 ,6611	187 ,7874	214 1,039	239 1,290	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex E
(normative)

Form p^* single sampling plans: σ-method

Table E.1 — Single sampling plans of Form p^* for normal inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 19,25	4 25,50	3 30,47
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 8,600	5 14,53	5 17,93	3 30,74
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 5,220	6 8,717	6 10,82	5 19,46	5 31,49
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 3,279	7 5,195	8 6,466	7 11,43	7 19,61	7 27,43
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,958	8 3,295	9 4,144	8 7,204	10 12,45	9 17,61	11 27,71
G	↓	↓	↓	↓	↓	↓	↓	↓	5 1,245	9 2,011	10 2,518	9 4,381	12 7,627	13 10,85	13 17,29	15 23,62
H	↓	↓	↓	↓	↓	↓	↓	6 ,7546	10 1,266	11 1,592	10 2,751	13 4,799	16 6,857	16 10,94	19 15,00	23 21,09
J	↓	↓	↓	↓	↓	↓	7 ,4753	11 ,7878	12 ,9814	11 1,685	15 2,959	19 4,241	21 6,783	24 9,324	29 13,11	34 18,14
K	↓	↓	↓	↓	↓	7 ,3027	12 ,4976	13 ,6222	13 1,071	17 1,876	21 2,687	27 4,313	29 5,935	35 8,361	42 11,57	53 17,22
L	↓	↓	↓	↓	8 ,1880	13 ,3105	15 ,3872	14 ,6625	19 1,162	24 1,667	32 2,681	34 3,692	42 5,204	52 7,220	66 10,74	↑
M	↓	↓	↓	8 ,1180	14 ,1954	16 ,2436	15 ,4150	21 ,7336	27 1,052	36 1,694	39 2,335	50 3,290	61 4,571	79 6,804	↑	↑
N	↓	↓	9 ,07418	15 ,1217	17 ,1524	17 ,2605	24 ,4595	30 ,6602	40 1,063	45 1,467	57 2,069	72 2,873	94 4,277	↑	↑	↑
P	↓	10 ,04641	17 ,07599	19 ,09473	19 ,1622	26 ,2852	33 ,4100	45 ,6611	51 ,9127	65 1,290	82 1,793	110 2,668	↑	↑	↑	↑
Q	11 ,02960	18 ,04835	20 ,06042	20 ,1034	28 ,1817	35 ,2619	49 ,4220	57 ,5836	72 ,8248	92 1,146	125 1,707	↑	↑	↑	↑	↑
R	19 ,03011	21 ,03762	22 ,06433	30 ,1132	38 ,1631	54 ,2634	64 ,3637	81 ,5145	105 ,7143	142 1,065	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table E.2 — Single sampling plans of Form p^* for tightened inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$	n $100p^*$
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 19,25	4 25,50
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3 8,600	5 14,53	5 17,93
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 5,220	6 8,717	6 10,82	5 19,46
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	4 3,279	7 5,195	8 6,466	7 11,43	7 19,61
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,958	8 3,295	9 4,144	8 7,204	10 12,45	9 17,61
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	5 1,245	9 2,011	10 2,518	9 4,381	12 7,627	13 10,85	13 17,29
H	↓	↓	↓	↓	↓	↓	↓	↓	6 ,7546	10 1,266	11 1,592	10 2,751	13 4,799	16 6,857	16 10,94	20 17,03
J	↓	↓	↓	↓	↓	↓	↓	7 ,4753	11 ,7878	12 ,9814	11 1,685	15 2,959	19 4,241	21 6,783	25 10,59	32 15,63
K	↓	↓	↓	↓	↓	↓	7 ,3027	12 ,4976	13 ,6222	13 1,071	17 1,876	21 2,687	27 4,313	31 6,738	39 9,963	49 14,80
L	↓	↓	↓	↓	↓	8 ,1880	13 ,3105	15 ,3872	14 ,6625	19 1,162	24 1,667	32 2,681	37 4,192	47 6,205	61 9,224	↑
M	↓	↓	↓	↓	8 ,1180	14 ,1954	16 ,2436	15 ,4150	21 ,7336	27 1,052	36 1,694	43 2,654	55 3,936	72 5,851	↑	↑
N	↓	↓	↓	9 ,07419	15 ,1217	17 ,1524	17 ,2605	24 ,4595	30 ,6602	40 1,063	49 1,666	65 2,470	85 3,679	↑	↑	↑
P	↓	↓	10 ,04641	17 ,07599	19 ,09473	19 ,1622	26 ,2852	33 ,4100	45 ,6611	55 1,039	74 1,540	99 2,292	↑	↑	↑	↑
Q	↓	11 ,02960	18 ,04835	20 ,06042	20 ,1034	28 ,1817	35 ,2619	49 ,4220	61 ,6640	83 ,9849	112 1,466	↑	↑	↑	↑	↑
R	14 ,02165	19 ,03011	21 ,03762	22 ,06433	30 ,1132	38 ,1631	54 ,2634	68 ,4141	92 ,6152	126 ,9152	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table E.3 — Single sampling plans of Form *p** for reduced inspection: *σ*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**	*n* 100*p**
B – D											↓	3 19,25	4 21,67	4 25,50	3 30,47	6 41,88
E										↓	3 8,600	5 12,09	5 14,53	5 17,93	4 30,74	8 43,83
F									↓	4 5,220	5 7,090	6 8,717	6 10,82	5 19,46	5 31,14	7 41,30
G								↓	4 3,279	6 4,162	7 5,195	8 6,466	7 11,43	7 19,61	7 27,43	11 41,07
H							↓	5 1,958	7 2,670	8 3,295	9 4,144	8 7,204	10 12,45	9 17,61	11 27,71	11 32,84
J						↓	5 1,245	7 1,613	9 2,011	10 2,518	9 4,381	12 7,627	13 10,85	13 17,29	14 20,45	16 26,75
K					↓	6 ,7546	8 1,016	10 1,266	11 1,592	10 2,751	13 4,799	16 6,857	16 10,94	18 12,96	20 17,03	23 21,09
L				↓	7 ,4753	9 ,6246	11 ,7878	12 ,9814	11 1,685	15 2,959	19 4,241	21 6,783	22 8,059	25 10,59	29 13,11	↑
M			↓	7 ,3027	10 ,3976	12 ,4976	13 ,6222	13 1,071	17 1,876	21 2,687	27 4,313	26 5,129	31 6,738	35 8,361	↑	
N		↓	8 ,1880	10 ,2451	13 ,3105	15 ,3872	14 ,6625	19 1,162	24 1,667	32 2,681	31 3,182	37 4,192	42 5,204	↑		
P	↓	8 ,1180	11 ,1540	14 ,1954	16 ,2436	15 ,4150	21 ,7336	27 1,052	36 1,694	38 2,012	43 2,654	50 3,290	↑			
Q	9 ,07418	12 ,09633	15 ,1218	17 ,1524	17 ,2605	24 ,4595	30 ,6602	40 1,063	45 1,264	49 1,666	57 2,069	↑				
R	13 ,05982	17 ,07599	19 ,09473	19 ,1622	26 ,2852	33 ,4100	45 ,6611	50 ,7874	55 1,039	65 1,290	↑					

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex F
(normative)

Values of f_S for maximum sample standard deviation (MSSD)

Table F.1 — Values of f_S for maximum sample standard deviation (MSSD) for combined control of double specification limits: normal inspection, *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B													↓	0,475	0,447	0,479
C												↓	0,365	0,366	0,388	0,484
D											↓	0,303	0,312	0,328	0,399	0,494
E										↓	0,265	0,274	0,285	0,333	0,395	0,458
F									↓	0,241	0,248	0,257	0,292	0,334	0,375	0,461
G								↓	0,221	0,227	0,234	0,260	0,290	0,318	0,371	0,424
H							↓	0,206	0,211	0,216	0,237	0,260	0,280	0,316	0,350	0,401
J						↓	0,192	0,197	0,201	0,218	0,236	0,251	0,277	0,301	0,333	0,376
K					↓	0,182	0,185	0,189	0,203	0,218	0,230	0,250	0,268	0,291	0,319	0,367
L				↓	0,172	0,175	0,179	0,190	0,203	0,212	0,229	0,242	0,259	0,279	0,312	↑
M			↓	0,164	0,167	0,170	0,180	0,190	0,199	0,212	0,222	0,236	0,251	0,275	↑	
N		↓	0,157	0,160	0,162	0,171	0,180	0,187	0,198	0,206	0,217	0,230	0,248	↑		
P	↓	0,151	0,153	0,155	0,163	0,171	0,177	0,186	0,193	0,202	0,212	0,226	↑			
Q	0,145	0,147	0,149	0,156	0,163	0,168	0,176	0,183	0,190	0,199	0,210	↑				
R	0,142	0,144	0,150	0,156	0,161	0,168	0,173	0,180	0,187	0,196	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD f_S by the difference between the upper specification limit, *U*, and the lower specification limit, *L*, i.e. MSSD $= s_{\text{max}} = (U - L) f_S$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under normal inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, then there is a possibility, but not a certainty, that the lot will be accepted.

Table F.2 — Values of f_S for maximum sample standard deviation (MSSD) for combined control of double specification limits: tightened inspection, *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,475	0,447
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,365	0,366	0,388
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,303	0,312	0,328	0,399
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,265	0,274	0,285	0,333	0,395
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,241	0,248	0,257	0,292	0,334	0,375
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,221	0,227	0,234	0,260	0,290	0,318	0,371
H	↓	↓	↓	↓	↓	↓	↓	↓	0,206	0,211	0,216	0,237	0,260	0,280	0,316	0,367
J	↓	↓	↓	↓	↓	↓	↓	0,192	0,197	0,201	0,218	0,236	0,251	0,277	0,312	0,354
K	↓	↓	↓	↓	↓	↓	0,182	0,185	0,189	0,203	0,218	0,230	0,250	0,276	0,305	0,347
L	↓	↓	↓	↓	↓	0,172	0,175	0,179	0,190	0,203	0,212	0,229	0,248	0,269	0,298	↑
M	↓	↓	↓	↓	0,164	0,167	0,170	0,180	0,190	0,199	0,212	0,227	0,244	0,265	↑	↑
N	↓	↓	↓	0,157	0,160	0,162	0,171	0,180	0,187	0,198	0,210	0,224	0,240	↑	↑	↑
P	↓	↓	0,151	0,153	0,155	0,163	0,171	0,177	0,186	0,196	0,207	0,221	↑	↑	↑	↑
Q	↓	0,145	0,147	0,149	0,156	0,163	0,168	0,176	0,185	0,195	0,206	↑	↑	↑	↑	↑
R	0,140	0,142	0,144	0,150	0,156	0,161	0,168	0,175	0,183	0,192	↑	↑	↑	↑	↑	↑

NOTE The MSSD is obtained by multiplying the standardized MSSD f_S by the difference between the upper specification limit, *U*, and the lower specification limit, *L*, i.e. MSSD $= s_{\max} = (U - L) f_S$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under normal inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD then there is a possibility, but not a certainty, that the lot will be accepted.

Table F.3 — Values of f_S for maximum sample standard deviation (MSSD) for combined control of double specification limits: reduced inspection, *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B-D											↓	0,475	0,426	0,447	0,479	0,602
E										↓	0,365	0,350	0,366	0,388	0,484	0,632
F									↓	0,303	0,303	0,312	0,328	0,399	0,494	0,598
G								↓	0,265	0,267	0,274	0,285	0,333	0,395	0,458	0,599
H							↓	0,241	0,243	0,248	0,257	0,292	0,334	0,375	0,461	0,510
J						↓	0,221	0,223	0,227	0,234	0,260	0,290	0,318	0,371	0,397	0,452
K					↓	0,206	0,207	0,211	0,216	0,237	0,260	0,280	0,316	0,333	0,367	0,401
L				↓	0,192	0,194	0,197	0,202	0,218	0,233	0,251	0,277	0,289	0,312	0,333	↑
M			↓	0,182	0,183	0,185	0,189	0,203	0,218	0,230	0,250	0,259	0,276	0,291	↑	
N		↓	0,172	0,173	0,175	0,179	0,190	0,203	0,212	0,229	0,235	0,248	0,259	↑		
P	↓	0,164	0,165	0,167	0,170	0,180	0,190	0,199	0,212	0,217	0,227	0,236	↑			
Q	0,157	0,158	0,160	0,162	0,171	0,180	0,187	0,198	0,202	0,210	0,217	↑				
R	0,151	0,153	0,155	0,163	0,171	0,177	0,186	0,190	0,196	0,202	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD f_S by the difference between the upper specification limit, *U*, and the lower specification limit, *L*, i.e. $MSSD = s_{max} = (U - L) f_S$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under reduced inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, there is a possibility, but not a certainty, that the lot will be accepted.

Annex G
(normative)

Values of f_σ for maximum process standard deviation (MPSD)

Table G.1 — Values of f_σ for maximum process standard deviation for combined control of double specification limits: σ-method

Acceptance quality limit (in percent nonconforming)	f_σ
0,010	0,125
0,015	0,129
0,025	0,132
0,040	0,137
0,065	0,141
0,10	0,147
0,15	0,152
0,25	0,157
0,40	0,165
0,65	0,174
1,0	0,184
1,5	0,194
2,5	0,206
4,0	0,223
6,5	0,243
10	0,271

NOTE The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MPSD $= \sigma_{\max} = (U - L) f_\sigma$.

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for combined control of double specification limits when the process variability is known. If the process standard deviation is less than the MPSD, there is a possibility, but not a certainty, that the lot will be accepted.

Table G.2 — Values of f_σ for maximum process standard deviation (MPSD) for separate control of double specification limits: σ-

AQL% (lower limit)	Acceptance quality limit in percent nonconforming (upper limit)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ
0,010	0,131	0,133	0,134	0,137	0,139	0,142	0,145	0,147	0,151	0,154	0,158	0,163	0,167	0,173	0,179	0,187
0,015	0,133	0,134	0,136	0,139	0,141	0,144	0,147	0,150	0,153	0,157	0,161	0,165	0,170	0,176	0,183	0,191
0,025	0,134	0,136	0,138	0,141	0,144	0,146	0,149	0,152	0,156	0,160	0,164	0,168	0,173	0,179	0,186	0,195
0,040	0,137	0,139	0,141	0,144	0,146	0,149	0,152	0,155	0,159	0,163	0,168	0,172	0,177	0,184	0,191	0,200
0,065	0,139	0,141	0,144	0,146	0,149	0,152	0,155	0,158	0,162	0,167	0,171	0,176	0,181	0,188	0,196	0,205

Table G.2 *(continued)*

AQL% (lower limit)	Acceptance quality limit in percent nonconforming (upper limit)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ
0,10	0,142	0,144	0,146	0,149	0,152	0,155	0,159	0,162	0,166	0,170	0,175	0,180	0,186	0,193	0,201	0,211
0,15	0,145	0,147	0,149	0,152	0,155	0,159	0,162	0,165	0,170	0,174	0,179	0,185	0,190	0,198	0,207	0,217
0,25	0,147	0,150	0,152	0,155	0,158	0,162	0,165	0,168	0,173	0,178	0,183	0,189	0,195	0,203	0,212	0,223
0,40	0,151	0,153	0,156	0,159	0,162	0,166	0,170	0,173	0,178	0,183	0,189	0,195	0,201	0,210	0,219	0,231
0,65	0,154	0,157	0,160	0,163	0,167	0,170	0,174	0,178	0,183	0,189	0,195	0,201	0,207	0,217	0,227	0,240
1,0	0,158	0,161	0,164	0,168	0,171	0,175	0,179	0,183	0,189	0,195	0,201	0,208	0,215	0,225	0,236	0,250
1,5	0,163	0,165	0,168	0,172	0,176	0,180	0,185	0,189	0,195	0,201	0,208	0,215	0,222	0,233	0,245	0,260
2,5	0,167	0,170	0,173	0,177	0,181	0,186	0,190	0,195	0,201	0,207	0,215	0,222	0,230	0,242	0,255	0,271
4,0	0,173	0,176	0,179	0,184	0,188	0,193	0,198	0,203	0,210	0,217	0,225	0,233	0,242	0,255	0,269	0,288
6,5	0,179	0,183	0,186	0,191	0,196	0,201	0,207	0,212	0,219	0,227	0,236	0,245	0,255	0,269	0,286	0,306
10,0	0,187	0,191	0,195	0,200	0,205	0,211	0,217	0,223	0,231	0,240	0,250	0,260	0,271	0,288	0,306	0,330

NOTE The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MPSD $= \sigma_{\max} = (U - L) f_\sigma$.

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for separate control of double specification limits when the process standard deviation is known. If the process standard deviation is less than the MPSD, there is a possibility, but not a certainty, that the lot will be accepted.

Table G.3 — Values of f_σ for maximum process standard deviation (MPSD) for complex control of double specification limits: σ-

AQL % (single limit)	Acceptance quality limit in percent nonconforming (both limits combined)														
	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ
0,010	0,129	0,132	0,135	0,138	0,141	0,144	0,147	0,151	0,154	0,158	0,162	0,167	0,173	0,179	0,187
0,015		0,132	0,136	0,140	0,143	0,146	0,149	0,153	0,157	0,161	0,165	0,170	0,176	0,183	0,191
0,025			0,137	0,141	0,145	0,148	0,151	0,155	0,159	0,164	0,168	0,173	0,179	0,186	0,195
0,040				0,141	0,146	0,150	0,154	0,158	0,162	0,167	0,172	0,177	0,184	0,191	0,200
0,065					0,147	0,152	0,156	0,161	0,166	0,171	0,176	0,181	0,188	0,196	0,205
0,10						0,152	0,157	0,163	0,169	0,174	0,180	0,185	0,193	0,201	0,211
0,15							0,157	0,165	0,171	0,178	0,183	0,189	0,197	0,206	0,217
0,25								0,165	0,173	0,180	0,187	0,193	0,202	0,211	0,223
0,40									0,174	0,183	0,191	0,198	0,208	0,218	0,230
0,65										0,184	0,194	0,202	0,213	0,225	0,238
1,0											0,194	0,205	0,219	0,232	0,247
1,5												0,206	0,222	0,238	0,255
2,5													0,223	0,242	0,262
4,0														0,243	0,269
6,5															0,271

NOTE The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit, U, and the lower specification limit, L, i.e. MPSD $= \sigma_{max} = (U - L) f_\sigma$.

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for complex control of double specification limits when the process standard deviation is known. If the process standard deviation is less than the MPSD, there is a possibility, but not a certainty, that the lot will be accepted.

Annex H
(normative)

Estimating the process fraction nonconforming for sample size 3: *s*-method

Table H.1 — Estimated process fraction nonconforming, $\hat{p}$, as a function of the quality statistic Q

		Third decimal place of $Q\sqrt{3}/2$									
		0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
		$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
First two decimal places of $Q\sqrt{3}/2$	0,00	0,500 0	0,499 7	0,499 4	0,499 0	0,498 7	0,498 4	0,498 1	0,497 8	0,497 5	0,497 1
	0,01	0,496 8	0,496 5	0,496 2	0,495 9	0,495 5	0,495 2	0,494 9	0,494 6	0,494 3	0,494 0
	0,02	0,493 6	0,493 3	0,493 0	0,492 7	0,492 4	0,492 0	0,491 7	0,491 4	0,491 1	0,490 8
	0,03	0,490 4	0,490 1	0,489 8	0,489 5	0,489 2	0,488 9	0,488 5	0,488 2	0,487 9	0,487 6
	0,04	0,487 3	0,486 9	0,486 6	0,486 3	0,486 0	0,485 7	0,485 4	0,485 0	0,484 7	0,484 4
	0,05	0,484 1	0,483 8	0,483 4	0,483 1	0,482 8	0,482 5	0,482 2	0,481 8	0,481 5	0,481 2
	0,06	0,480 9	0,480 6	0,480 3	0,479 9	0,479 6	0,479 3	0,479 0	0,478 7	0,478 3	0,478 0
	0,07	0,477 7	0,477 4	0,477 1	0,476 7	0,476 4	0,476 1	0,475 8	0,475 5	0,475 1	0,474 8
	0,08	0,474 5	0,474 2	0,473 9	0,473 5	0,473 2	0,472 9	0,472 6	0,472 3	0,472 0	0,471 6
	0,09	0,471 3	0,471 0	0,470 7	0,470 4	0,470 0	0,469 7	0,469 4	0,469 1	0,468 8	0,468 4
	0,10	0,468 1	0,467 8	0,467 5	0,467 2	0,466 8	0,466 5	0,466 2	0,465 9	0,465 6	0,465 2
	0,11	0,464 9	0,464 6	0,464 3	0,464 0	0,463 6	0,463 3	0,463 0	0,462 7	0,462 4	0,462 0
	0,12	0,461 7	0,461 4	0,461 1	0,460 7	0,460 4	0,460 1	0,459 8	0,459 5	0,459 1	0,458 8
	0,13	0,458 5	0,458 2	0,457 9	0,457 5	0,457 2	0,456 9	0,456 6	0,456 3	0,455 9	0,455 6
	0,14	0,455 3	0,455 0	0,454 6	0,454 3	0,454 0	0,453 7	0,453 4	0,453 0	0,452 7	0,452 4
	0,15	0,452 1	0,451 8	0,451 4	0,451 1	0,450 8	0,450 5	0,450 1	0,449 8	0,449 5	0,449 2
	0,16	0,448 9	0,448 5	0,448 2	0,447 9	0,447 6	0,447 2	0,446 9	0,446 6	0,446 3	0,445 9
	0,17	0,445 6	0,445 3	0,445 0	0,444 7	0,444 3	0,444 0	0,443 7	0,443 4	0,443 0	0,442 7
	0,18	0,442 4	0,442 1	0,441 7	0,441 4	0,441 1	0,440 8	0,440 4	0,440 1	0,439 8	0,439 5
	0,19	0,439 2	0,438 8	0,438 5	0,438 2	0,437 9	0,437 5	0,437 2	0,436 9	0,436 6	0,436 2
	0,20	0,435 9	0,435 6	0,435 3	0,434 9	0,434 6	0,434 3	0,434 0	0,433 6	0,433 3	0,433 0
	0,21	0,432 7	0,432 3	0,432 0	0,431 7	0,431 4	0,431 0	0,430 7	0,430 4	0,430 0	0,429 7
	0,22	0,429 4	0,429 1	0,428 7	0,428 4	0,428 1	0,427 8	0,427 4	0,427 1	0,426 8	0,426 5
	0,23	0,426 1	0,425 8	0,425 5	0,425 1	0,424 8	0,424 5	0,424 2	0,423 8	0,423 5	0,423 2
	0,24	0,422 9	0,422 5	0,422 2	0,421 9	0,421 5	0,421 2	0,420 9	0,420 6	0,420 2	0,419 9
	0,25	0,419 6	0,419 2	0,418 9	0,418 6	0,418 3	0,417 9	0,417 6	0,417 3	0,416 9	0,416 6
	0,26	0,416 3	0,415 9	0,415 6	0,415 3	0,415 0	0,414 6	0,414 3	0,414 0	0,413 6	0,413 3
	0,27	0,413 0	0,412 6	0,412 3	0,412 0	0,411 7	0,411 3	0,411 0	0,410 7	0,410 3	0,410 0
	0,28	0,409 7	0,409 3	0,409 0	0,408 7	0,408 3	0,408 0	0,407 7	0,407 3	0,407 0	0,406 7

Table H.1 — *(continued)*

		Third decimal place of $Q\sqrt{3}/2$									
		0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
		$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
First two decimal places of $Q\sqrt{3}/2$	0,29	0,406 3	0,406 0	0,405 7	0,405 3	0,405 0	0,404 7	0,404 3	0,404 0	0,403 7	0,403 3
	0,30	0,403 0	0,402 7	0,402 3	0,402 0	0,401 7	0,401 3	0,401 0	0,400 7	0,400 3	0,400 0
	0,31	0,399 7	0,399 3	0,399 0	0,398 7	0,398 3	0,398 0	0,397 7	0,397 3	0,397 0	0,396 7
	0,32	0,396 3	0,396 0	0,395 6	0,395 3	0,395 0	0,394 6	0,394 3	0,394 0	0,393 6	0,393 3
	0,33	0,393 0	0,392 6	0,392 3	0,391 9	0,391 6	0,391 3	0,390 9	0,390 6	0,390 2	0,389 9
	0,34	0,389 6	0,389 2	0,388 9	0,388 6	0,388 2	0,387 9	0,387 5	0,387 2	0,386 9	0,386 5
	0,35	0,386 2	0,385 8	0,385 5	0,385 2	0,384 8	0,384 5	0,384 1	0,383 8	0,383 5	0,383 1
	0,36	0,382 8	0,382 4	0,382 1	0,381 8	0,381 4	0,381 1	0,380 7	0,380 4	0,380 0	0,379 7
	0,37	0,379 4	0,379 0	0,378 7	0,378 3	0,378 0	0,377 6	0,377 3	0,377 0	0,376 6	0,376 3
	0,38	0,375 9	0,375 6	0,375 2	0,374 9	0,374 5	0,374 2	0,373 9	0,373 5	0,373 2	0,372 8
	0,39	0,372 5	0,372 1	0,371 8	0,371 4	0,371 1	0,370 7	0,370 4	0,370 1	0,369 7	0,369 4
	0,40	0,369 0	0,368 7	0,368 3	0,368	0,367 6	0,367 3	0,366 9	0,366 6	0,366 2	0,365 9
	0,41	0,365 5	0,365 2	0,364 8	0,364 5	0,364 1	0,363 8	0,363 4	0,363 1	0,362 7	0,362 4
	0,42	0,362 0	0,361 7	0,361 3	0,361 0	0,360 6	0,360 3	0,359 9	0,359 6	0,359 2	0,358 9
	0,43	0,358 5	0,358 2	0,357 8	0,357 5	0,357 1	0,356 7	0,356 4	0,356 0	0,355 7	0,355 3
	0,44	0,355 0	0,354 6	0,354 3	0,353 9	0,353 6	0,353 2	0,352 8	0,352 5	0,352 1	0,351 8
	0,45	0,351 4	0,351 1	0,350 7	0,350 4	0,350 0	0,349 6	0,349 3	0,348 9	0,348 6	0,348 2
	0,46	0,347 8	0,347 5	0,347 1	0,346 8	0,346 4	0,346 1	0,345 7	0,345 3	0,345 0	0,344 6
	0,47	0,344 3	0,343 9	0,343 5	0,343 2	0,342 8	0,342 4	0,342 1	0,341 7	0,341 4	0,341 0
	0,48	0,340 6	0,340 3	0,339 9	0,339 5	0,339 2	0,338 8	0,338 5	0,338 1	0,337 7	0,337 4
	0,49	0,337 0	0,336 6	0,336 3	0,335 9	0,335 5	0,335 2	0,334 8	0,334 4	0,334 1	0,333 7
	0,50	0,333 3	0,333 0	0,332 6	0,332 2	0,331 9	0,331 5	0,331 1	0,330 8	0,330 4	0,330 0
	0,51	0,329 6	0,329 3	0,328 9	0,328 5	0,328 2	0,327 8	0,327 4	0,327 0	0,326 7	0,326 3
	0,52	0,325 9	0,325 6	0,325 2	0,324 8	0,324 4	0,324 1	0,323 7	0,323 3	0,322 9	0,322 6
	0,53	0,322 2	0,321 8	0,321 4	0,321 1	0,320 7	0,320 3	0,319 9	0,319 6	0,319 2	0,318 8
	0,54	0,318 4	0,318 0	0,317 7	0,317 3	0,316 9	0,316 5	0,316 1	0,315 8	0,315 4	0,315 0
	0,55	0,314 6	0,314 2	0,313 9	0,313 5	0,313 1	0,312 7	0,312 3	0,312 0	0,311 6	0,311 2
	0,56	0,310 8	0,310 4	0,310 0	0,309 6	0,309 3	0,308 9	0,308 5	0,308 1	0,307 7	0,307 3
	0,57	0,306 9	0,306 6	0,306 2	0,305 8	0,305 4	0,305 0	0,304 6	0,304 2	0,303 8	0,303 4
	0,58	0,303 1	0,302 7	0,302 3	0,301 9	0,301 5	0,301 1	0,300 7	0,300 3	0,299 9	0,299 5
	0,59	0,299 1	0,298 7	0,298 3	0,297 9	0,297 5	0,297 2	0,296 8	0,296 4	0,296 0	0,295 6
	0,60	0,295 2	0,294 8	0,294 4	0,294 0	0,293 6	0,293 2	0,292 8	0,292 4	0,292 0	0,291 6
	0,61	0,291 2	0,290 8	0,290 4	0,290 0	0,289 6	0,289 2	0,288 8	0,288 3	0,287 9	0,287 5
	0,62	0,287 1	0,286 7	0,286 3	0,285 9	0,285 5	0,285 1	0,284 7	0,284 3	0,283 9	0,283 5
	0,63	0,283 1	0,282 6	0,282 2	0,281 8	0,281 4	0,281 0	0,280 6	0,280 2	0,279 8	0,279 3
	0,64	0,278 9	0,278 5	0,278 1	0,277 7	0,277 3	0,276 9	0,276 4	0,276 0	0,275 6	0,275 2

Table H.1 — *(continued)*

		Third decimal place of $Q\sqrt{3}/2$									
		0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
		$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
First two decimal places of $Q\sqrt{3}/2$	0,65	0,274 8	0,274 3	0,273 9	0,273 5	0,273 1	0,272 7	0,272 2	0,271 8	0,271 4	0,271 0
	0,66	0,270 6	0,270 1	0,269 7	0,269 3	0,268 9	0,268 4	0,268 0	0,267 6	0,267 2	0,266 7
	0,67	0,266 3	0,265 9	0,265 4	0,265 0	0,264 6	0,264 1	0,263 7	0,263 3	0,262 8	0,262 4
	0,68	0,262 0	0,261 5	0,261 1	0,260 7	0,260 2	0,259 8	0,259 4	0,258 9	0,258 5	0,258 0
	0,69	0,257 6	0,257 2	0,256 7	0,256 3	0,255 8	0,255 4	0,255 0	0,254 5	0,254 1	0,253 6
	0,70	0,253 2	0,252 7	0,252 3	0,251 8	0,251 4	0,250 9	0,250 5	0,250 0	0,249 6	0,249 1
	0,71	0,248 7	0,248 2	0,247 8	0,247 3	0,246 9	0,246 4	0,246 0	0,245 5	0,245 1	0,244 6
	0,72	0,244 1	0,243 7	0,243 2	0,242 8	0,242 3	0,241 8	0,241 4	0,240 9	0,240 5	0,240 0
	0,73	0,239 5	0,239 1	0,238 6	0,238 1	0,237 7	0,237 2	0,236 7	0,236 2	0,235 8	0,235 3
	0,74	0,234 8	0,234 4	0,233 9	0,233 4	0,232 9	0,232 4	0,232 0	0,231 5	0,231 0	0,230 5
	0,75	0,230 1	0,229 6	0,229 1	0,228 6	0,228 1	0,227 6	0,227 2	0,226 7	0,226 2	0,225 7
	0,76	0,225 2	0,224 7	0,224 2	0,223 7	0,223 2	0,222 7	0,222 2	0,221 7	0,221 3	0,220 8
	0,77	0,220 3	0,219 8	0,219 3	0,218 8	0,218 3	0,217 7	0,217 2	0,216 7	0,216 2	0,215 7
	0,78	0,215 2	0,214 7	0,214 2	0,213 7	0,213 2	0,212 7	0,212 1	0,211 6	0,211 1	0,210 6
	0,79	0,210 1	0,209 6	0,209 0	0,208 5	0,208 0	0,207 5	0,206 9	0,206 4	0,205 9	0,205 4
	0,80	0,204 8	0,204 3	0,203 8	0,203 2	0,202 7	0,202 2	0,201 6	0,201 1	0,200 6	0,200 0
	0,81	0,199 5	0,198 9	0,198 4	0,197 8	0,197 3	0,196 7	0,196 2	0,195 6	0,195 1	0,194 5
	0,82	0,194 0	0,193 4	0,192 9	0,192 3	0,191 7	0,191 2	0,190 6	0,190 0	0,189 5	0,188 9
	0,83	0,188 3	0,187 8	0,187 2	0,186 6	0,186 0	0,185 5	0,184 9	0,184 3	0,183 7	0,183 1
	0,84	0,182 6	0,182 0	0,181 4	0,180 8	0,180 2	0,179 6	0,179 0	0,178 4	0,177 8	0,177 2
	0,85	0,176 6	0,176 0	0,175 4	0,174 8	0,174 2	0,173 6	0,172 9	0,172 3	0,171 7	0,171 1
	0,86	0,170 5	0,169 8	0,169 2	0,168 6	0,168 0	0,167 3	0,166 7	0,166 0	0,165 4	0,164 8
	0,87	0,164 1	0,163 5	0,162 8	0,162 2	0,161 5	0,160 9	0,160 2	0,159 5	0,158 9	0,158 2
	0,88	0,157 5	0,156 9	0,156 2	0,155 5	0,154 8	0,154 2	0,153 5	0,152 8	0,152 1	0,151 4
	0,89	0,150 7	0,150 0	0,149 3	0,148 6	0,147 9	0,147 2	0,146 5	0,145 7	0,145 0	0,144 3
	0,90	0,143 6	0,142 8	0,142 1	0,141 4	0,140 6	0,139 9	0,139 1	0,138 4	0,137 6	0,136 8
	0,91	0,136 1	0,135 3	0,134 5	0,133 8	0,133 0	0,132 2	0,131 4	0,130 6	0,129 8	0,129 0
	0,92	0,128 2	0,127 4	0,126 6	0,125 7	0,124 9	0,124 1	0,123 2	0,122 4	0,121 5	0,120 7
	0,93	0,119 8	0,118 9	0,118 1	0,117 2	0,116 3	0,115 4	0,114 5	0,113 6	0,112 7	0,111 8
	0,94	0,110 8	0,109 9	0,108 9	0,108 0	0,107 0	0,106 1	0,105 1	0,104 1	0,103 1	0,102 1
	0,95	0,101 1	0,100 1	0,099 0	0,098 0	0,096 9	0,095 9	0,094 8	0,093 7	0,092 6	0,091 5
	0,96	0,090 3	0,089 2	0,088 0	0,086 9	0,085 7	0,084 5	0,083 2	0,082 0	0,080 7	0,079 5
	0,97	0,078 2	0,076 8	0,075 5	0,074 1	0,072 7	0,071 3	0,069 9	0,068 4	0,066 9	0,065 3
	0,98	0,063 8	0,062 1	0,060 5	0,058 8	0,057 0	0,055 2	0,053 3	0,051 4	0,049 4	0,047 3
	0,99	0,045 1	0,042 7	0,040 3	0,037 7	0,034 9	0,031 8	0,028 5	0,024 7	0,020 1	0,014 2
	1,00	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0	0,000 0

NOTE For negative values of Q, enter the table with the absolute value of $Q\sqrt{3}/2$ and subtract the result from 1,0.

Annex I
(normative)

Values of c_U for upper control limit on the sample standard deviation

Table I.1 — Values of c_U for upper control limit on the sample standard deviation

Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sample size, n	Factor, c_U	Sam-ple size, n	Factor, c_U	Sample size, n	Factor, c_U
3	2,296 8	27	1,361 6	51	1,260 0	82	1,203 9	124	1,165 2	213	1,125 6
4	2,064 7	28	1,354 8	52	1,257 4	83	1,202 6	125	1,164 5	214	1,125 3
5	1,924 1	29	1,348 4	53	1,254 9	84	1,201 4	126	1,163 8	233	1,120 0
6	1,827 3	30	1,342 2	54	1,252 5	85	1,200 2	127	1,163 2	239	1,118 5
7	1,755 5	31	1,336 4	55	1,250 1	88	1,196 7	131	1,160 6	244	1,117 3
8	1,699 5	32	1,330 9	57	1,245 6	89	1,195 5	132	1,160 0	247	1,116 5
9	1,654 3	33	1,325 7	58	1,243 4	90	1,194 4	134	1,158 8	260	1,113 6
10	1,616 8	34	1,320 6	60	1,239 2	92	1,192 3	137	1,157 0	262	1,113 1
11	1,585 0	35	1,315 9	61	1,237 2	93	1,191 2	142	1,154 2	277	1,110 0
12	1,557 7	36	1,311 3	63	1,233 3	94	1,190 2	143	1,153 7	293	1,106 9
13	1,533 8	37	1,306 9	64	1,231 4	96	1,188 1	149	1,150 5	298	1,106 0
14	1,512 8	38	1,302 7	65	1,229 6	99	1,185 2	150	1,150 0	312	1,103 6
15	1,494 0	39	1,298 6	66	1,227 8	101	1,183 3	155	1,147 5	320	1,102 3
16	1,477 1	40	1,294 7	68	1,224 3	102	1,182 4	159	1,145 6	323	1,101 8
17	1,461 9	41	1,291 0	69	1,222 7	105	1,179 8	169	1,141 2	332	1,100 4
18	1,448 0	42	1,287 4	71	1,219 4	108	1,177 2	170	1,140 8	348	1,098 0
19	1,435 3	43	1,283 9	72	1,217 9	110	1,175 5	171	1,140 4	362	1,096 1
20	1,423 6	44	1,280 6	73	1.216 3	111	1,174 7	178	1,137 5	395	1,092 0
21	1,412 8	45	1,277 3	74	1,214 8	112	1,173 9	186	1,134 5	398	1,091 6
22	1,402 7	46	1,274 2	75	1,213 4	115	1,171 6	187	1,134 1	424	1,088 7
23	1,393 4	47	1,271 2	76	1,211 9	116	1,170 9	189	1,133 4	438	1,087 3
24	1,384 7	48	1,268 3	78	1,209 1	117	1.170 1	201	1,129 3	498	1,081 8
25	1,376 5	49	1,265 4	79	1,207 8	120	1,168 0	202	1,129 0	541	1,078 5
26	1,368 8	50	1,262 7	81	1,205 2	122	1,166 6	207	1,127 4		

NOTE Table entries are $\sqrt{\chi^2_{n-1,\gamma}/(n-1)}$ where $\chi^2_{n-1,\gamma}$ is the γ-fractile of the chi-squared distribution with $n-1$ degrees of freedom and $\gamma = 0{,}95^{0,1} = 0{,}994884$.

Annex J
(normative)

Supplementary acceptability constants for qualifying towards reduced inspection

Table J.1 — Supplementary acceptability constants for qualifying towards reduced inspection

Sample size code letter	AQL (%)	Form *k* acceptability constant for AQL that is one step tighter		Form *p** acceptability constant for AQL that is one step tighter	
		s-method		*σ*-method	
		k	*p** (%)	*k*	*p** (%)
B	4,0	1,114	8,502	0,918	13,04
C	2,5	1,409	3,041	1,325	5,230
D	1,5	1,601	3,241	1,562	3,562
E	1,0	1,825	2,103	1,752	2,151
F	0,65	2,029	1,164	2,013	1,219
G	0,40	2,209	0,775 1	2,161	0,784 5
H	0,25	2,390	0,448 2	2,379	0,458 4
J	0,15	2,530	0,318 8	2,523	0,320 8
K	0,10	2,689	0,197 9	2,667	0,198 6
L	0,065	2,857	0,116 4	2,847	0,117 0
M	0,040	2,995	0,074 39	2,972	0,074 36
N	0,025	3,143	0,044 98	3,131	0,044 94
P	0,015	3,254	0,031 32	3,246	0,031 16
Q	0,010	3,385	0,019 46	3,382	0,019 44
R	0,010	3,449	0,020 24	3,446	0,019 94

Annex K
(normative)

Procedures for obtaining *s* and *σ*

K.1 Procedure for obtaining *s*

K.1.1 The estimate from a sample of the standard deviation of a population is generally denoted by the symbol *s*. Its value may be obtained from Formula (K.1).

$$s=\sqrt{\frac{\sum_{j=1}^{n}(x_j-\bar{x})^2}{(n-1)}} \quad \text{(K.1)}$$

where x_j is the value of the quality characteristic of the j^{th} item in a sample of size *n*, and $\bar{x}$ is the mean value of the x_j, i.e.

$$\bar{x}=\sum_{j=1}^{n}x_j/n \quad \text{(K.2)}$$

K.1.2 Formula (K.1) for *s* is not recommended for the purpose of computation, as it tends to introduce an unnecessary amount of rounding error. An equivalent but computationally better formula is

$$s=\sqrt{\frac{n\sum_{j=1}^{n}x_j^2-(\sum_{j=1}^{n}x_j)^2}{n(n-1)}} \quad \text{(K.3)}$$

K.1.3 If the variability is very small relative to the mean, i.e. *s* is very small in comparison with $\bar{x}$, Formula (K.3) can be improved upon still further by subtracting a suitable arbitrary constant *a* from all the values of x_j before computing *s*, i.e.

$$s=\sqrt{\frac{n\sum_{j=1}^{n}(x_j-a)^2-\left[\sum_{j=1}^{n}\left(x_j-a\right)\right]^2}{n(n-1)}} \quad \text{(K.4)}$$

K.1.4 Many pocket calculators have a standard deviation function key. If it is planned to use a calculator function, or a computer program, it is important to check that the formula used by the machine is equivalent to Formula (K.1), as sometimes the sample size *n* is used by the machine in the denominator instead of $n-1$. A simple check is to find the standard deviation of the three numbers 0, 1, and 2. The sample size, *n*, is 3, the sample mean is 1, the deviations from the mean are −1, 0, and 1, the squares of the deviations are 1, 0, and 1, and the sum of squares of the deviations is 2. So from Formula (K.1), we have

$$s = \sqrt{\frac{2}{2}} = \sqrt{1} = 1$$

If the computer or calculator is erroneously using n instead of $n-1$ in the denominator, then the result of the calculation will be

$$s = \sqrt{\frac{2}{3}} = 0{,}816\ 5$$

Use of n in the denominator shall be avoided, for otherwise the acceptance criterion is weakened and the AQL protection to the consumer is lost.

NOTE It is instructive to work through the use of Formula (K.3) for this example. It is found that

$$s = \sqrt{\frac{3\times(0^2+1^2+2^2)-(0+1+2)^2}{3\times(3-1)}} = \sqrt{\frac{3\times(0+1+4)-3^2}{3\times2}} = \sqrt{\frac{3\times5-9}{6}} = \sqrt{\frac{6}{6}} = 1$$

as before.

K.2 Procedure for obtaining σ

K.2.1 If it appears from the control chart that the value of s is in control, σ may be presumed to be the weighted root mean square of s given by Formula (K.5):

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m}(n_i-1)s_i^2}{\sum_{i=1}^{m}(n_i-1)}} \qquad \text{(K.5)}$$

where

m is the number of lots;

n_i is the sample size from the i^{th} lot;

s_i is the sample standard deviation from the i^{th} lot.

K.2.2 If the sample sizes from each of the lots are equal, then Formula (K.5) simplifies to

$$\sigma = \sqrt{\sum_{i=1}^{m} s_i^2 / m} \qquad \text{(K.6)}$$

Annex L
(normative)

Estimating the process fraction nonconforming

L.1 General

For technical reasons, minimum variance unbiased estimators (MVUEs) are used to estimate the process fraction nonconforming from the sample results. The process fraction nonconforming is denoted by p and its estimator by $\hat{p}$. This annex presents the exact formula for $\hat{p}$ for the case of unknown process variability (the s-method) and also for the case of known process variability (the σ-method). Because the exact formula for $\hat{p}$ for the s-method generally requires reference to tables or software for the distribution function of the symmetric beta distribution, an approximative formula is presented that only requires reference to tables of the standard normal distribution. This formula is accurate enough for all practical purposes for sample sizes greater than 4. Accordingly, further details are presented to facilitate the application of the exact formula for the s-method for sample sizes 3 and 4.

L.2 Exact formulae

L.2.1 The exact MVUE estimator of *p* for the *s*-method

Denote the distribution function of the symmetric beta distribution by

$$G_m(y)=\begin{cases}0 & \text{if } y<0\\ \int_0^y \frac{t^{m-1}(1-t)^{m-1}}{B(m,m)}\,\mathrm{d}t & \text{if } 0\le y\le 1\\ 1 & \text{if } y>1\end{cases} \tag{L.1}$$

where $B(m,m)=\Gamma(m)\Gamma(m)/\Gamma(2m)$, with $\Gamma(m)$ representing the complete gamma integral, i.e.

$$\Gamma(m)=\int_0^\infty x^{m-1}e^{-x}\mathrm{d}x\ . \tag{L.2}$$

Then the general formula for the estimator of the process fraction nonconforming beyond either of the specification limits when the process standard deviation is unknown is

$$\hat{p}=G_{(n-2)/2}\left[\frac{1}{2}\left(1-Q\frac{\sqrt{n}}{n-1}\right)\right] \tag{L.3}$$

where *n* is the sample size and *Q* is the quality statistic for that specification limit. Thus, for the lower specification limit,

$$\hat{p}_L=G_{(n-2)/2}\left[\frac{1}{2}\left(1-Q_{\mathrm{L}}\frac{\sqrt{n}}{n-1}\right)\right]=G_{(n-2)/2}\left[\frac{1}{2}\left(1-\frac{\bar{x}-L}{s}\frac{\sqrt{n}}{n-1}\right)\right] \tag{L.4}$$

whereas for the upper specification limit,

$$\hat{p}_U=G_{(n-2)/2}\left[\frac{1}{2}\left(1-Q_{\mathrm{U}}\frac{\sqrt{n}}{n-1}\right)\right]=G_{(n-2)/2}\left[\frac{1}{2}\left(1-\frac{U-\bar{x}}{s}\frac{\sqrt{n}}{n-1}\right)\right]\ . \tag{L.5}$$

For combined control of double specification limits, the combined process fraction nonconforming is estimated by the sum of these two estimates, viz. $\hat{p}=\hat{p}_{\mathrm{L}}+\hat{p}_{\mathrm{U}}$.

L.2.2 The exact MVUE estimator of *p* for the *σ*-method

Denote the distribution function of the standard normal distribution by

$$\Phi(y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y}e^{-t^2/2}\mathrm{d}t\ . \tag{L.6}$$

Then the general formula for the estimator of the process fraction nonconforming below the lower specification limits when the process standard deviation is known is

$$\hat{p}_{\mathrm{L}}=\Phi\left(-Q_{\mathrm{L}}\sqrt{\frac{n}{n-1}}\right)=\Phi\left(\frac{L-\bar{x}}{\sigma}\sqrt{\frac{n}{n-1}}\right) \tag{L.7}$$

where *σ* is the process standard deviation whose value is presumed to be known.

The corresponding formula for the upper specification limit is

$$\hat{p}_U = \Phi\left(-Q_U\sqrt{\frac{n}{n-1}}\right) = \Phi\left(\frac{\bar{x}-U}{\sigma}\sqrt{\frac{n}{n-1}}\right). \quad \text{(L.8)}$$

Again, when required, the combined process fraction nonconforming is estimated by the sum of these two estimates.

L.3 Approximative procedure for the *s*-method with $n \geq 5$

If tables or software for the distribution function of the symmetric beta distribution are not available, the following procedure can be used to obtain an accurate approximation to $\hat{p}$ for the *s*-method when the sample size is 5 or more.

a) Calculate $Q = (U-\bar{x})/s$ and/or $(\bar{x}-L)/s$.

b) Calculate $x = \frac{1}{2}\left[1 - Q\sqrt{n}/(n-1)\right]$.

c) Calculate $y = a_n \ln\left[x/(1-x)\right]$ where a_n is given in Table L.1 below.

d) Calculate $w = y^2 - 3$.

e) If $w \geq 0$, set $t = \frac{12(n-1)y}{12(n-1)+w}$; otherwise, set $t = \frac{12(n-2)y}{12(n-2)+w}$.

Look up $\hat{p} = \Phi(t)$ in tables of the standard normal distribution function.

Table L.1 — Values of a_n for normal approximation to $\hat{p}$

Sample size, *n*	a_n	Sample size, *n*	a_n	Sample size, *n*	a_n	Sample size, *n*	a_n
3	0,318 310	39	3,000 385	82	4,444 216	155	6,164 458
4	0,551 329	40	3,041 751	83	4,472 252	159	6,245 041
5	0,731 350	41	3,082 562	84	4,500 114	169	6,442 088
6	0,880 496	42	3,122 841	85	4,527 805	170	6,461 463
7	1,009 784	43	3,162 607	88	4,609 879	171	6,480 779
8	1,125 182	44	3,201 879	89	4,636 914	178	6,614 414
9	1,230 248	45	3,240 676	90	4,663 792	186	6,763 908
10	1,327 276	46	3,279 015	92	4,717 090	187	6,782 363
11	1,417 833	47	3,316 910	93	4,743 514	189	6,819 124
12	1,503 044	48	3,354 378	94	4,769 792	201	7,035 654
13	1,583 745	49	3,391 432	96	4,821 918	202	7,053 398
14	1,660 575	50	3,428 086	99	4,899 068	207	7,141 457
15	1,734 040	51	3,464 352	101	4,949 833	213	7,245 716
16	1,804 542	52	3,500 243	102	4,975 022	214	7,262 947
17	1,872 410	53	3,535 769	105	5,049 833	233	7,582 899
18	1,937 919	54	3,570 943	108	5,123 553	239	7,681 169
19	2,001 296	55	3,605 773	110	5,172 115	244	7,762 110
20	2,062 737	57	3,674 445	111	5,196 227	247	7,810 272
21	2,122 408	58	3,708 303	112	5,220 226	260	8,015 630
22	2,180 453	60	3,775 111	115	5,291 573	262	8,046 758

Table L.1 *(continued)*

Sample size, n	a_n	Sample size, n	a_n	Sample size, n	a_n	Sample size, n	a_n
23	2,236 997	61	3,808 075	116	5,315 142	277	8,276 491
24	2,292 152	63	3,873 163	117	5,338 608	293	8,514 710
25	2,346 014	64	3,905 300	120	5,408 393	298	8,587 798
26	2,398 670	65	3,937 175	122	5,454 420	312	8,789 213
27	2,450 197	66	3,968 794	124	5,500 063	320	8,902 262
28	2,500 665	68	4,031 288	125	5,522 742	323	8,944 286
29	2,550 137	69	4,062 175	126	5,545 329	332	9,069 193
30	2,598 669	71	4,123 254	127	5,567 825	348	9,287 101
31	2,646 313	72	4,153 457	131	5,656 912	362	9,473 660
32	2,693 115	73	4,183 442	132	5,678 965	395	9,8995 06
33	2,739 119	74	4,213 214	134	5,722 817	398	9,9373 14
34	2,784 364	75	4,242 777	137	5,787 972	424	10,259 15
35	2,828 887	76	4,272 135	142	5,894 964	438	10,428 34
36	2,872 720	78	4,330 255	143	5,916 130	498	11,124 31
37	2,915 896	79	4,359 025	149	6,041 570	541	11,597 42
38	2,958 442	81	4,416 001	150	6,062 225		

L.4 Simplified exact formula for $\hat{p}$ for the *s*-method with *n* = 3

When *n* = 3, the *s*-method estimator is

$$\hat{p} = G_{\frac{1}{2}}\left[\left(1 - Q\sqrt{3}\,/\,2\right)/\,2\right] \tag{L.9}$$

Now

$$G_{\frac{1}{2}}(x) = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\int_0^x \frac{t^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}}{B(\frac{1}{2},\frac{1}{2})}\,\mathrm{d}t & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{L.10}$$

where

$$B(\tfrac{1}{2},\tfrac{1}{2}) = \Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{2})\,/\,\Gamma(1) = \sqrt{\pi}\sqrt{\pi}\,/\,1 = \pi$$

Writing $t = \sin^2\theta$, Formula (L.10) becomes

$$G_{\frac{1}{2}}(x) = \begin{cases} 0 \\ \displaystyle\frac{2}{\pi}\int_0^{\arcsin(\sqrt{x})} \mathrm{d}\theta \\ 1 \end{cases} = \begin{cases} 0 & \text{if } x < 0 \\ \displaystyle\frac{2}{\pi}\arcsin\left(\sqrt{x}\right) & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \tag{L.11}$$

Hence, substituting Formula (L.11) in Formula (L.10),

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2\,/\,\sqrt{3} \\ \displaystyle\frac{2}{\pi}\arcsin\left[\sqrt{\left(1 - Q\sqrt{3}\,/\,2\right)/\,2}\right] & \text{if } -2\,/\,\sqrt{3} \le Q \le 2\,/\,\sqrt{3} \\ 1 & \text{if } Q < -2\,/\,\sqrt{3} \end{cases} \tag{L.12}$$

This is the quantity tabulated in Annex H.

L.5 Simplified exact formula for $\hat{p}$ for the *s*-method with *n* = 4

When *n* = 4, the *s*-method estimator becomes

$$\hat{p} = B_1\left[\tfrac{1}{2}\left(1 - \tfrac{2}{3}Q\right)\right] = B_1\left(0.5 - Q / 3\right) \qquad \text{(L.13)}$$

Now

$$B_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ \int_0^x \frac{dt}{B(1,1)} & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \qquad \text{(L.14)}$$

where

$$B(1,1) = \Gamma(1)\Gamma(1) / \Gamma(2) = 1 \;.$$

Formula (L.14) can therefore be written as

$$B_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases} \qquad \text{(L.15)}$$

Hence, substituting Formula (L.15) in Formula (L.13),

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 1{,}5 \\ 0{,}5 - Q / 3 & \text{if } -1{,}5 \le Q \le 1{,}5 \\ 1 & \text{if } Q < -1{,}5 \end{cases} \qquad \text{(L.16)}$$

Annex M
(informative)

Consumer's risk qualities

M.1 For a given sampling plan, the consumer's risk quality is the process quality at which the probability of accepting a given lot is 10 %.

M.2 For the univariate *s*-method with a single specification limit, the consumer's risk quality is the solution in p to the equation $F_{n-1,\sqrt{n}K_p}(\sqrt{n}k)=0{,}90$, where n is the sample size, k is the Form k s-method acceptability constant, K_p is the upper p-fractile of the standard normal distribution, and $F_{n-1,\sqrt{n}K_p}(.)$ is the distribution function of the non-central *t*-distribution with $n-1$ degrees of freedom and non-centrality parameter $\sqrt{n}K_p$. In terms of the Form p^* *s*-method acceptability constant, the consumer's risk quality is the solution in p to the equation $F_{n-1,\sqrt{n}K_p}\left[(n-1)(1-2\beta_{(n-2)/2,\,p^*})\right]=0{,}90$, where $\beta_{(n-2)/2,\,p^*}$ is the p^*-fractile of the symmetric beta distribution with both parameters equal to $(n-2)/2$.

M.3 Consumer's risk qualities for the *s*-method plans of this part of ISO 3951 are given below in Tables M.1, M.3, and M.5 for normal, tightened, and reduced inspection, respectively.

M.4 For the univariate σ-method with a single specification limit, the consumer's risk quality is given by the formula $\Phi\{(1{,}2816/\sqrt{n})-k\}$ where n is the sample size, k is the Form k σ-method acceptability constant, and $\Phi(.)$ is the distribution function of the standard normal distribution. In terms of the Form p^* σ-method acceptability constant p^*, the consumer's risk quality is given by Formula (M.1).

$$\Phi\left[(1{,}2816-\sqrt{n-1}.K_{p^*})/\sqrt{n}\right] \tag{M.1}$$

M.5 Consumer's risk qualities for the σ-method plans of this part of ISO 3951 are given below in Tables M.2, M.4, and M.6 for normal, tightened, and reduced inspection respectively.

M.6 The tabulated risk qualities also apply approximately in the case of double specification limits and/or multiple quality characteristics.

Table M.1 — Consumer's risk quality (in percent) for normal inspection: *s*-method

Code	Acceptance quality limit (in percent nonconforming)															
letter	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	53,0	52,3	56,4
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	39,5	36,5	39,9	54,1
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	26,4	24,5	27,1	41,4	51,2
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	16,7	15,8	17,7	27,8	36,8	44,8
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	11,7	10,7	11,8	18,7	24,8	30,7	41,4
G	↓	↓	↓	↓	↓	↓	↓	↓	7,37	6,97	7,73	12,2	16,2	20,0	27,6	34,5
H	↓	↓	↓	↓	↓	↓	↓	4,96	4,54	5,01	7,96	10,7	13,1	18,0	22,6	29,4
J	↓	↓	↓	↓	↓	↓	3,11	2,86	3,18	5,09	6,78	8,41	11,5	14,5	18,7	24,3
K	↓	↓	↓	↓	↓	2,01	1,85	2,05	3,27	4,39	5,45	7,46	9,39	12,2	15,8	22,0
L	↓	↓	↓	↓	1,26	1,16	1,29	2,06	2,78	3,43	4,72	5,94	7,71	10,0	13,9	↑
M	↓	↓	↓	0,812	0,743	0,826	1,33	1,77	2,19	3,02	3,79	4,91	6,39	8,91	↑	↑
N	↓	↓	0,515	0,471	0,521	0,830	1,12	1,39	1,91	2,40	3,12	4,05	5,64	↑	↑	↑
P	↓	0,323	0,296	0,328	0,521	0,705	0,873	1,19	1,50	1,95	2,54	**3,53**	↑	↑	↑	↑
Q	0,207	0,190	0,211	0,336	0,453	0,562	0,766	0,968	1,26	1,63	2,27	↑	↑	↑	↑	↑
R	0,119	0,132	0,209	0,284	0,352	0,481	0,605	0,786	1,02	1,42	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table M.2 — Consumer's risk quality (in percent) for normal inspection: σ-method

Code	Acceptance quality limit (in percent nonconforming)															
letter	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	51,2	52,8	62,7
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	35,4	35,5	40,2	58,1
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	22,2	23,7	27,3	42,2	55,3
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	17,0	15,4	16,7	26,4	37,9	47,2
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,2	10,3	11,4	18,1	24,6	32,6	43,0
G	↓	↓	↓	↓	↓	↓	↓	↓	7,59	6,59	7,34	11,8	15,9	20,3	29,1	35,8
H	↓	↓	↓	↓	↓	↓	↓	4,50	4,30	4,85	7,85	10,7	13,2	19,2	23,7	30,2
J	↓	↓	↓	↓	↓	↓	2,76	2,77	3,12	5,07	6,79	8,33	12,0	15,1	19,4	24,9
K	↓	↓	↓	↓	↓	1,98	1,80	2,04	3,18	4,39	5,45	7,54	9,76	12,6	16,2	22,3
L	↓	↓	↓	↓	1,20	1,15	1,25	2,05	2,78	3,43	4,72	6,16	7,95	10,2	14,2	↑
M	↓	↓	↓	0,840	0,738	0,807	1,32	1,78	2,18	3,02	3,94	5,05	6,55	9,06	↑	↑
N	↓	↓	0,510	0,469	0,518	0,821	1,11	1,38	1,91	2,48	3,20	4,13	5,73	↑	↑	↑
P	↓	0,311	0,284	0,317	0,523	0,696	0,865	1,19	1,55	2,00	2,59	3,59	↑	↑	↑	↑
Q	0,193	0,184	0,206	0,330	0,448	0,562	0,768	0,990	1,29	1,66	2,30	↑	↑	↑	↑	↑
R	0,116	0,131	0,204	0,282	0,352	0,480	0,616	0,803	1,04	1,45	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table M.3 — Consumer's risk quality (in percent) for tightened inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B														↓	53,0	52,3
C													↓	39,5	36,5	39,9
D												↓	26,4	24,5	27,1	41,4
E											↓	16,7	15,8	17,7	27,8	36,8
F										↓	11,7	10,7	11,8	18,7	24,8	30,7
G									↓	7,37	6,97	7,73	12,2	16,2	20,0	27,6
H								↓	4,96	4,54	5,01	7,96	10,7	13,1	18,0	24,9
J							↓	3,11	2,86	3,18	5,09	6,78	8,41	11,5	15,9	21,5
K						↓	2,01	1,85	2,05	3,27	4,39	5,45	7,46	10,3	14,0	19,3
L					↓	1,26	1,16	1,29	2,06	2,78	3,43	4,72	6,52	8,85	12,2	↑
M				↓	0,812	0,743	0,826	1,33	1,77	2,19	3,02	4,17	5,68	7,85	↑	
N			↓	0,515	0,471	0,521	0,830	1,12	1,39	1,91	2,64	3,58	4,97	↑		
P		↓	0,323	0,296	0,328	0,521	0,705	0,873	1,19	1,66	2,25	3,11	↑			
Q	↓	0,207	0,190	0,211	0,336	0,453	0,562	0,766	1,06	1,44	2,00	↑				
R	0,116	0,119	0,132	0,209	0,284	0,352	0,481	0,666	0,906	1,25	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table M.4 — Consumer's risk quality (in percent) for tightened inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B														↓	51,2	52,8
C													↓	35,4	35,5	40,2
D												↓	22,2	23,7	27,3	42,2
E											↓	17,0	15,4	16,7	26,4	37,9
F										↓	10,2	10,3	11,4	18,1	24,6	32,6
G									↓	7,59	6,59	7,34	11,8	15,9	20,3	29,1
H								↓	4,50	4,30	4,85	7,85	10,7	13,2	19,2	26,0
J							↓	2,76	2,77	3,12	5,07	6,79	8,33	12,0	16,7	22,1
K						↓	1,98	1,80	2,04	3,18	4,39	5,45	7,54	10,7	14,4	19,7
L					↓	1,20	1,15	1,25	2,05	2,78	3,43	4,72	6,75	9,10	12,5	↑
M				↓	0,840	0,738	0,807	1,32	1,78	2,18	3,02	4,31	5,83	7,99	↑	
N			↓	0,510	0,469	0,518	0,821	1,11	1,38	1,91	2,72	3,67	5,05	↑		
P		↓	0,311	0,284	0,317	0,523	0,696	0,865	1,19	1,71	2,30	3,16	↑			
Q	↓	0,193	0,184	0,206	0,330	0,448	0,562	0,768	1,10	1,47	2,03	↑				
R	0,115	0,116	0,131	0,204	0,282	0,352	0,480	0,684	0,924	1,27	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table M.5 — Consumer's risk quality (in percent) for reduced inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	53,0	49,2	52,3	56,4	61,1
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	39,5	34,1	36,5	39,9	54,1	60,8
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	26,4	23,9	24,5	27,1	41,4	51,2	59,4
G	↓	↓	↓	↓	↓	↓	↓	↓	16,7	15,8	15,8	17,7	27,8	36,8	44,8	55,7
H	↓	↓	↓	↓	↓	↓	↓	11,7	10,5	10,7	11,8	18,7	24,8	30,7	41,4	47,2
J	↓	↓	↓	↓	↓	↓	7,37	6,85	6,97	7,73	12,2	16,2	20,0	27,6	31,0	37,8
K	↓	↓	↓	↓	↓	4,96	4,48	4,54	5,01	7,96	10,7	13,1	18,0	20,3	24,9	29,4
L	↓	↓	↓	↓	3,11	2,84	2,86	3,18	5,09	6,78	8,41	11,5	13,0	15,9	18,7	↑
M	↓	↓	↓	2,01	1,82	1,85	2,05	3,27	4,39	5,45	7,46	8,45	10,3	12,2	↑	↑
N	↓	↓	1,26	1,16	1,16	1,29	2,06	2,78	3,43	4,72	5,31	6,52	7,71	↑	↑	↑
P	↓	0,812	0,745	0,743	0,826	1,33	1,77	2,19	3,02	3,40	4,17	4,91	↑	↑	↑	↑
Q	0,515	0,473	0,471	0,521	0,830	1,12	1,39	1,91	2,15	2,64	3,12	↑	↑	↑	↑	↑
R	0,297	0,296	0,328	0,521	0,705	0,873	1,19	1,35	1,66	1,95	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table M.6 — Consumer's risk quality (in percent) for reduced inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	51,2	48,5	52,8	62,7	63,3
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	35,4	31,8	35,5	40,2	58,1	62,8
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	22,2	22,9	23,7	27,3	42,2	55,3	61,1
G	↓	↓	↓	↓	↓	↓	↓	↓	17,0	14,5	15,4	16,7	26,4	37,9	47,2	56,8
H	↓	↓	↓	↓	↓	↓	↓	10,2	9,61	10,3	11,4	18,1	24,6	32,6	43,0	48,5
J	↓	↓	↓	↓	↓	↓	7,59	6,71	6,59	7,34	11,8	15,9	20,3	29,1	32,5	39,0
K	↓	↓	↓	↓	↓	4,50	4,30	4,30	4,85	7,85	10,7	13,2	19,2	21,4	26,0	30,2
L	↓	↓	↓	↓	2,76	2,69	2,77	3,12	5,07	6,79	8,33	12,0	13,7	16,7	19,4	↑
M	↓	↓	↓	1,98	1,73	1,80	2,04	3,18	4,39	5,45	7,54	8,86	10,7	12,6	↑	↑
N	↓	↓	1,20	1,18	1,15	1,25	2,05	2,78	3,43	4,72	5,54	6,75	7,95	↑	↑	↑
P	↓	0,840	0,743	0,738	0,807	1,32	1,78	2,18	3,02	3,47	4,31	5,05	↑	↑	↑	↑
Q	0,510	0,467	0,469	0,518	0,821	1,11	1,38	1,91	2,16	2,72	3,20	↑	↑	↑	↑	↑
R	0,292	0,284	0,317	0,523	0,696	0,865	1,19	1,36	1,71	2,00	↑	↑	↑	↑	↑	↑

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted

Annex N
(informative)

Producer's risks

N.1 The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL, i.e. 1 minus the probability of accepting a given lot when the process fraction nonconforming is equal to the AQL.

N.2 For the univariate *s*-method with a single specification limit, the producer's risk is given by the formula $F_{n-1,\sqrt{n}K_p}(\sqrt{n}k)$, where n is the sample size, p is the AQL expressed as a fraction nonconforming, k is the Form k *s*-method acceptability constant, K_p is the upper p-fractile of the standard normal distribution, and $F_{n-1,\sqrt{n}K_p}(.)$ is the distribution function of the non-central *t*-distribution with $n-1$ degrees of freedom and non-centrality parameter $\sqrt{n}K_p$. In terms of the Form p^* *s*-method acceptability constant p^*, the producer's risk is given by the formula $F_{n-1,\sqrt{n}K_p}\left[(n-1)(1-2\beta_{(n-2)/2,\,p^*}\right]$, where $\beta_{(n-2)/2,\,p^*}$ is the p^*-fractile of the symmetric beta distribution with both parameters equal to $(n-2)/2$.

N.3 Producer's risks for the *s*-method plans of this part of ISO 3951 are given below in Tables N.1, N.3, and N.5 for normal, tightened, and reduced inspection, respectively.

N.4 For the univariate σ-method with a single specification limit, the producer's risk is given by the formula $\Phi\left[\sqrt{n}(k-K_p)\right]$, where n is the sample size, p is the AQL expressed as a fraction nonconforming, k is the Form k σ-method acceptability constant, K_p is the upper p-fractile of the standard normal distribution, and $\Phi(.)$ is the distribution function of the standard normal distribution. In terms of the Form p^* σ-method acceptability constant, the producer's risk is given by Formula (N.1).

$$\Phi(\sqrt{n-1}K_{p^*}-\sqrt{n}K_p) \qquad \text{(N.1)}$$

N.5 Producer's risks for the σ-method plans of this part of ISO 3951 are given below in Tables N.2, N.4, and N.6 for normal, tightened, and reduced inspection, respectively.

N.6 The tabulated producer's risks also apply approximately in the case of double specification limits and/or multiple quality characteristics.

Table N.1 — Producer's risk (in percent) for normal inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,8	7,46	8,93
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	12,2	8,00	10,8	7,37
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,8	7,52	10,3	8,74	2,50
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,3	6,88	10,4	9,07	4,62	3,18
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	10,8	7,12	8,54	8,14	3,77	3,34	0,908
G	↓	↓	↓	↓	↓	↓	↓	↓	9,81	7,62	9,99	7,49	3,94	3,35	1,45	1,10
H	↓	↓	↓	↓	↓	↓	↓	9,88	6,98	9,99	7,95	3,37	3,07	1,21	1,30	0,853
J	↓	↓	↓	↓	↓	↓	8,91	6,61	9,63	8,64	3,91	2,71	1,26	1,28	1,27	1,13
K	↓	↓	↓	↓	↓	9,16	5,79	9,08	7,65	3,99	3,14	0,891	1,12	1,01	1,48	0,568
L	↓	↓	↓	↓	9,45	6,29	8,16	7,54	3,78	3,51	1,24	0,891	1,08	1,37	1,05	↑
M	↓	↓	↓	9,01	6,54	8,99	6,77	3,51	3,12	1,39	1,19	0,685	1,23	0,787	↑	↑
N	↓	↓	8,76	6,26	9,48	7,30	2,97	2,98	1,20	1,43	1,07	0,803	0,741	↑	↑	↑
P	↓	8,09	6,12	9,15	7,88	3,60	2,55	1,18	1,27	1,42	1,44	0,462	↑	↑	↑	↑
Q	8,47	5,32	8,68	7,20	3,74	2,93	,806	1,10	1,07	1,66	0,759	↑	↑	↑	↑	↑
R	6,00	7,90	7,07	3,52	3,35	1,14	,821	1,05	1,42	1,18	↑	↑	↑	↑	↑	↑

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table N.2 — Producer's risk (in percent) for normal inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	3,57	2,96	6,72
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	7,17	3,59	6,06	4,54
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	6,33	3,89	6,37	4,81	2,86
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	7,17	3,94	6,29	4,62	2,81	2,74
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	7,65	4,32	5,42	4,66	1,89	2,80	0,865
G	↓	↓	↓	↓	↓	↓	↓	↓	7,44	4,96	6,87	4,66	2,04	2,09	1,41	1,15
H	↓	↓	↓	↓	↓	↓	↓	7,47	4,68	7,35	5,48	1,98	1,86	1,26	1,38	0,871
J	↓	↓	↓	↓	↓	↓	6,69	4,70	7,38	6,40	2,56	1,59	1,05	1,25	1,32	1,24
K	↓	↓	↓	↓	↓	7,32	4,16	7,17	5,56	2,74	2,10	0,572	1,08	1,06	1,58	0,602
L	↓	↓	↓	↓	7,64	4,82	6,30	5,80	2,67	2,48	0,788	0,854	1,09	1,39	1,07	↑
M	↓	↓	↓	7,52	5,16	7,26	5,29	2,56	2,19	0,933	1,17	0,682	1,28	0,829	↑	↑
N	↓	↓	7,30	5,02	7,95	5,82	2,04	2,12	0,844	1,36	1,07	0,808	0,774	↑	↑	↑
P	↓	6,70	4,77	7,55	6,30	2,64	1,82	0,832	1,23	1,42	1,46	0,481	↑	↑	↑	↑
Q	7,06	4,16	7,25	5,85	2,84	2,26	0,578	1,02	1,07	1,69	0,776	↑	↑	↑	↑	↑
R	4,89	6,71	5,76	2,73	2,68	0,830	0,738	1,04	1,43	1,20	↑	↑	↑	↑	↑	↑

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table N.3 — Producer's risk (in percent) for tightened inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	17,6	14,7
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	19,0	16,7	21,7
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	18,1	15,7	22,7	18,6
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	16,2	15,8	22,4	20,7	13,4
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	17,3	13,9	20,6	18,7	13,2	13,0
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	17,2	15,4	19,7	18,8	13,1	15,0	9,41
H	↓	↓	↓	↓	↓	↓	↓	↓	16,9	15,7	20,8	16,8	12,7	13,6	10,2	5,66
J	↓	↓	↓	↓	↓	↓	↓	16,4	14,9	22,1	18,8	11,4	13,8	9,92	7,33	5,18
K	↓	↓	↓	↓	↓	↓	15,1	14,3	20,7	19,0	12,5	11,6	9,13	6,21	6,13	4,11
L	↓	↓	↓	↓	↓	16,1	13,1	20,5	18,4	13,7	13,8	7,85	6,26	5,82	6,19	↑
M	↓	↓	↓	↓	16,5	14,2	18,7	18,1	12,6	14,8	9,52	4,65	5,34	5,00	↑	↑
N	↓	↓	↓	15,9	15,0	20,5	16,3	12,3	13,8	10,7	6,36	3,84	4,76	↑	↑	↑
P	↓	↓	15,7	14,5	21,8	18,2	11,2	13,9	10,1	7,75	6,00	3,38	↑	↑	↑	↑
Q	↓	14,5	13,9	20,5	18,7	12,4	11,5	9,18	6,35	6,64	4,86	↑	↑	↑	↑	↑
R	12,4	12,9	20,4	18,1	13,5	13,8	7,80	6,33	6,01	6,68	↑	↑	↑	↑	↑	↑

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table N.4 — Producer's risk (in percent) for tightened inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	8,17	7,75
C	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	13,6	10,2	15,2
D	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	13,4	10,5	17,2	12,6
E	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	12,5	11,5	17,4	14,5	9,78
F	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	14,1	10,2	16,5	13,9	9,18	11,3
G	↓	↓	↓	↓	↓	↓	↓	↓	↓	14,3	12,0	16,1	14,7	9,35	11,8	8,77
H	↓	↓	↓	↓	↓	↓	↓	↓	14,4	12,6	17,7	13,4	9,68	10,6	9,80	5,73
J	↓	↓	↓	↓	↓	↓	↓	14,1	12,3	19,3	15,9	8,94	10,9	8,86	7,29	5,19
K	↓	↓	↓	↓	↓	↓	13,0	12,0	18,3	16,2	10,2	9,37	7,54	5,99	6,15	4,19
L	↓	↓	↓	↓	↓	14,1	11,1	18,2	16,1	11,5	11,7	6,29	6,07	5,79	6,10	↑
M	↓	↓	↓	↓	14,7	12,3	16,7	16,0	10,8	12,7	7,96	4,50	5,31	4,97	↑	↑
N	↓	↓	↓	14,2	13,3	18,7	14,3	10,4	11,9	9,26	6,18	3,79	4,73	↑	↑	↑
P	↓	↓	14,2	12,7	20,1	16,3	9,49	12,1	8,77	7,61	5,94	3,40	↑	↑	↑	↑
Q	↓	13,0	12,3	18,8	17,0	10,7	10,1	8,07	6,26	6,54	4,84	↑	↑	↑	↑	↑
R	11,0	11,4	18,9	16,5	12,0	12,4	6,74	6,24	5,96	6,67	↑	↑	↑	↑	↑	↑

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table N.5 — Producer's risk (in percent) for reduced inspection: *s*-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D											↓	3,77	2,70	3,29	3,91	0,257
E										↓	4,92	2,65	3,69	4,52	2,05	0,041
F									↓	4,28	3,18	3,08	4,33	3,41	0,571	0,159
G								↓	3,30	3,37	3,30	3,98	3,67	1,18	0,655	0,017
H							↓	3,38	2,67	3,22	3,80	2,87	0,915	0,544	0,082	0,214
J						↓	2,67	2,70	3,10	4,34	3,23	0,838	0,573	0,112	0,338	0,290
K					↓	2,97	2,28	2,84	3,77	3,20	0,978	0,421	0,092	0,256	0,379	0,853
L				↓	2,81	2,45	2,38	3,69	3,08	1,03	0,557	0,072	0,273	0,370	1,27	↑
M			↓	2,49	2,35	2,53	3,09	2,68	0,849	0,581	0,087	0,184	0,325	1,01	↑	
N		↓	2,36	2,26	2,57	3,40	2,34	0,822	0,503	0,107	0,272	0,235	1,08	↑		
P	↓	2,09	2,14	2,36	3,56	2,66	0,631	0,457	0,084	0,309	0,333	0,685	↑			
Q	2,19	1,89	2,30	3,30	2,65	0,743	0,349	0,074	0,256	0,419	1,07	↑				
R	2,09	2,01	3,25	2,49	0,838	0,459	0,052	0,244	0,363	1,42	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table N.6 — Producer's risk (in percent) for reduced inspection: σ-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D											↓	0,570	0,519	0,913	2,87	0,367
E										↓	1,80	0,601	1,16	1,88	1,55	0,065
F									↓	1,56	1,18	1,14	2,08	1,42	0,772	0,217
G								↓	1,72	1,35	1,49	1,68	1,27	0,561	0,560	0,021
H							↓	1,57	1,12	1,54	1,90	1,15	0,307	0,439	0,081	0,222
J						↓	1,58	1,46	1,56	2,36	1,59	0,278	0,263	0,116	0,359	0,323
K					↓	1,64	1,21	1,51	2,23	1,80	0,438	0,173	0,105	0,275	0,443	0,871
L				↓	1,55	1,37	1,36	2,36	1,88	0,524	0,234	0,054	0,278	0,418	1,32	↑
M			↓	1,59	1,36	1,55	2,06	1,57	0,447	0,293	0,042	0,185	0,326	1,06	↑	
N		↓	1,47	1,53	1,72	2,25	1,49	0,456	0,266	0,047	0,261	0,235	1,09	↑		
P	↓	1,46	1,44	1,61	2,50	1,82	0,361	0,240	0,039	0,230	0,327	0,682	↑			
Q	1,51	1,26	1,62	2,43	1,84	0,407	0,181	0,038	0,158	0,412	1,07	↑				
R	1,44	1,31	2,31	1,67	0,498	0,258	0,026	0,161	0,368	1,42	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Annex O
(informative)

Operating characteristics for the σ-method

O.1 Formula for probability of acceptance

The exact probability of lot acceptance for a single specification limit at process fraction nonconforming p is given by Formula (O.1)

$$P_a = \Phi\left[\sqrt{n}(K_p - k)\right], \tag{O.1}$$

where $\Phi(.)$ denotes the standard normal distribution function, n is the sample size, K_p denotes the upper p-fractile of the standard normal distribution, and k is the σ-method Form k acceptability constant.

O.2 Example

Consider the calculation of the probability of acceptance at a process quality of 2,5 % nonconforming for a σ-method plan with AQL of 1,0 % and sample size code letter M under normal inspection. Entering Table C.1 with sample size code letter M and AQL of 1,0 %, it is found that the sample size n is 39 and the acceptability constant k is 1,962. The process fraction nonconforming under consideration is P = 0,025, and from tables of the standard normal distribution, it is found that $K_p = 1{,}960$. Hence, $P_a = \Phi\left[\sqrt{39}(1{,}960 - 1{,}962)\right] = \Phi(-0{,}01249)$, which, again from tables of the standard normal distribution, yields $P_a = 0{,}495$.

O.3 Comparison with tabulated value for the s-method

It is instructive to observe that this probability of acceptance for the σ-method is very roughly in agreement with the corresponding probability of acceptance for the s-method. From the column in the table below Chart M of ISO 3951-1:2013 for AQL 1,0 %, it is seen that a process quality level of 2,43 %, i.e. P = 0,0243, corresponds to a probability of acceptance of 50 %, i.e. to $P_a = 0{,}500$.

Annex P
(informative)

Accommodating measurement variability

P.1 General

The master tables of this part of ISO 3951 are based on the assumption that the quality characteristic X of the items in the lots is normally distributed with unknown process mean, μ, and either known or unknown process standard deviation σ. The assumption is also made that X can be measured without measurement error, i.e. that the measurement of an item with the true value x_i results in the value x_i. This annex explains how these master tables may also be used when measurement error is present.

In the presence of measurement error, the measured value of an item with true value x_i will differ from x_i. It is assumed that

— the measurement method is unbiased, i.e. the expectation of the measurement error is zero,

— measurement error inflates the perceived process variation and is independent of the actual process standard deviation, and

— measurement error is normally distributed with known or unknown measurement standard deviation σ_{m}.

It follows that the distribution of the measured values is a normal distribution with mean μ and standard deviation

$$\sigma_{\mathrm{total}} = \sqrt{\sigma^2 + \sigma_{\mathrm{m}}^2} \qquad \text{(P.1)}$$

It can be seen that σ_{total} is always larger than σ if measurement error exists.

If it is known that $\sigma_{\mathrm{m}} < 0{,}1\sigma$, i.e. the ratio $\gamma = \sigma_{\mathrm{m}} / \sigma$ of measurement standard deviation to process standard deviation is less than 10 %, the total standard deviation is

$$\sigma_{\mathrm{total}} < \sqrt{\sigma^2 + (0{,}1\sigma)^2} = \sigma\sqrt{1 + 0{,}01} = 1{,}005\sigma \qquad \text{(P.2)}$$

i.e. the standard deviation is increased by less than 0,5 %, which for most practical purposes is negligible and hence, the sampling plans do not need to be adjusted for measurement error.

In cases where $\sigma_{\mathrm{m}} \geq 0{,}1\sigma$, the sampling plans of this part of ISO 3951 shall be used with the following adjustments

1. Increase the sample size, n, in order to compensate for the perceived inflated variability, but do not alter the acceptability constant k or p^*.
2. When the process standard deviation, σ, is known, use σ in calculating the test statistic $\bar{x} \pm k\sigma$ or $\hat{p}$; otherwise, use an estimate s of σ in calculating the test statistic $\bar{x} \pm ks$ or $\hat{p}$.

Further details are given in the following sub-clauses for three distinct cases.

P.2 Process standard deviation, σ, and measurement standard deviation, σ_m, both known

1. Increase the sample size, *n*, of the sampling plan to

$$n^* = n(1+\gamma^2). \qquad \text{(P.3)}$$

2. Use the process standard deviation, σ, in calculating the test statistic $\bar{x} \pm k\sigma$ or $\hat{p}$.

P.3 Process standard deviation, σ, unknown but measurement standard deviation σ_m known

1. Increase the sample size, *n*, of the sampling plan to

$$n^* = n(1+\tilde{\gamma}^2) \qquad \text{(P.4)}$$

where $\tilde{\gamma}$ is an estimated upper bound of $\gamma = \sigma_m / \sigma$.

NOTE As $\tilde{\gamma}$ increases, the operating characteristic curve of the sampling plan turns clockwise around the indifference quality point ($p_{50\%}$, 0,5), i.e. the point where the probability of acceptance of the lot is 50 %, If γ is overestimated ($\tilde{\gamma}$ is larger than γ), the sampling plan is better than required, i.e. its probabilities of acceptance are larger than required for $p<p_{50\%}$ and smaller than required for $p>p_{50\%}$ Hence, overestimation of γ ensures a sampling plan that is better than required.

2. Use the estimate

$$s^* = \sqrt{s^2 - \sigma_m^2} \qquad \text{(P.5)}$$

of the process standard deviation instead of *s* in calculating the test statistic $\bar{x} \pm ks$ or $\hat{p}$.

If $s^2 - \sigma_m^2 < 0$, use $s^* = 0$.

P.4 Process standard deviation σ and measurement standard deviation, σ_m, both unknown

Increase the sample size, *n*, in accordance with Formula (P.4), perform duplicate (or multiple) measurements on each sampled item, and use the measurement results to estimate the process standard deviation separately from the measurement standard deviation, as shown below. Use this estimate instead of *s* in calculating the test statistic $\bar{x} \pm ks$ or $\hat{p}$.

Estimation of the process and measurement standard deviations

We denote the j^{th} measurement on the i^{th} item by x_{ij}, the mean for the ith item by $\bar{x}_i$, and the overall mean by $\bar{x}_{..}$. The number of measurements for the ith item will be denoted by n_i. The total sum of squares of the measurements about their overall mean can be partitioned as follows:

$$\begin{aligned}
\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(x_{ij}-\bar{x}_{..}\right)^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(x_{ij}-\bar{x}_{i.}+\bar{x}_{i.}-\bar{x}_{..}\right)^2 \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}\left[(x_{ij}-\bar{x}_{i.})^2+(\bar{x}_{i.}-\bar{x}_{..})^2+2(x_{ij}-\bar{x}_{i.})(\bar{x}_{i.}-\bar{x}_{..})\right] \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2+\sum_{i=1}^{n}n_i(\bar{x}_{i.}-\bar{x}_{..})^2+2\sum_{i=1}^{n}(\bar{x}_{i.}-\bar{x}_{..})\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.}) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2+\sum_{i=1}^{n}n_i(\bar{x}_{i.}-\bar{x}_{..})^2+0 \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_{i.})^2+\sum_{i=1}^{n}n_i(\bar{x}_{i.}-\bar{x}_{..})^2 \\
&= W+B
\end{aligned} \quad \text{(P.6)}$$

respectively, where W is the within-items sum of squares and B is the between-items sum of squares.

The expectations of these sums of squares are

$$E(W)=\sigma_{\mathrm{m}}^2\sum_{i=1}^{n}(n_i-1)=\sigma_{\mathrm{m}}^2(N-n) \quad \text{(P.7)}$$

where $N=\sum_{i=1}^{n} n_i$ is the total number of observations, and

$$E(B)=\sigma_{\mathrm{m}}^2(n-1)+(N-n)\sigma^2. \quad \text{(P.8)}$$

Hence, σ_{m}^2 can be estimated by

$$\hat{\sigma}_{\mathrm{m}}^2 = W/(N-n) \quad \text{(P.9)}$$

and σ^2 can be estimated by

$$s^2 = \hat{\sigma}^2 = \left[B-(n-1)\hat{\sigma}_m^2\right]/(N-n). \quad \text{(P.10)}$$

Example

A manufactured component has a dimension with an upper specification limit of 13,05 cm. The process standard deviation, σ, and measurement standard deviation, σ_{m}, are unknown, but from previous experience, it is known that the ratio $\sigma_{\mathrm{m}}/\sigma$ is greater than 0,20 but less than 0,25. Lots of size 1 000 of these components are to be inspected. Normal inspection is to be instituted with an AQL of 0,15 %.

From Table A.1, it is found that the sample size code letter is J. As only one specification limit is being controlled, Form k can be used; from Table B.1, the sampling plan in the absence of sampling error is $n=23$, $k=2{,}425$.

As σ_m / σ exceeds 0,1, it is necessary to adjust the sample size to allow for measurement variability.

In the presence of measurement error, the appropriate sample size (from P.3) is given by

$$n^* = n(1+\tilde{\gamma}^2) = 23\left[1+(0,25)^2\right] = 23 \times 1,062\ 5 = 24,44$$

The sample size must be an integer, so in order to be sure to provide *at least* the required AQL protection, n^* is rounded *up* to $n^* = 25$. A random sample of 25 of the components is taken from the next lot, and, in order to be able to assess the measurement variability, each component is measured twice. The results for the sample from the first lot are as follows.

Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}	Item, i	x_{i1}	x_{i2}
1	13,000 5	12,988 8	6	13,028 7	13,029 4	11	12,964 6	12,962 7	16	12,957 2	12,948 1	21	13,007 9	12,999 1
2	12,985 3	12,983 8	7	12,992 8	12,977 8	12	12,981 1	12,982 3	17	12,972 4	12,974 3	22	12,993 0	12,990 4
3	12,962 7	12,962 3	8	12,958 5	12,952 0	13	13,009 4	0,104 4	18	12,997 8	12,994 1	23	12,968 0	12,966 6
4	12,956 2	12,960 1	9	12,955 0	12,956 4	14	12,980 5	0,080 8	19	12,999 3	13,006 7	24	12,991 0	12,995 5
5	12,972 8	12,971 7	10	13,011 7	13,017 7	15	12,931 7	0,026 7	20	12,974 0	12,972 4	25	12,969 8	12,967 4

The accuracy of subsequent calculations can be improved by subtracting an arbitrary constant that reduces the number of significant figures. Denote the constant by c and set c = 12,9. The resulting values of $y_{ij} = x_{ij} - 12,9$ are as follows.

Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}	Item, i	y_{i1}	y_{i2}
1	0,100 5	0,088 8	6	0,128 7	0,129 4	11	0,064 6	0,062 7	16	0,057 2	0,048 1	21	0,107 9	0,099 1
2	0,085 3	0,083 8	7	0,092 8	0,077 8	12	0,081 1	0,082 3	17	0,072 4	0,074 3	22	0,093 0	0,090 4
3	0,062 7	0,062 3	8	0,058 5	0,052 0	13	0,109 4	0,104 4	18	0,097 8	0,094 1	23	0,068 0	0,066 6
4	0,056 2	0,060 1	9	0,055 0	0,056 4	14	0,080 5	0,080 8	19	0,099 3	0,106 7	24	0,091 0	0,095 5
5	0,072 8	0,071 7	10	0,111 7	0,117 7	15	0,031 7	0,026 7	20	0,074 0	0,072 4	25	0,069 8	0,067 4

The sum of y_{ij} is $\sum_{i=1}^{25}\sum_{j=1}^{2} y_{ij} = 3,993\ 4$.

The sample mean value of y is $\bar{y} = 3,9934/50 = 0,079\ 868$.

Hence, the sample mean value of x is $\bar{x} = c + \bar{y} = 12,9 + 0,079\ 868 = 12,979\ 868$.

The total sum of squares of y is $T = \sum_{i=1}^{25}\sum_{j=1}^{2} y_{ij}^2 = 0,343\ 882\ 92$.

The total sum of squares, *T*, about the overall sample mean is given by

$$\begin{aligned} T &= \sum_{i=1}^{25}\sum_{j=1}^{2} y_{ij}^2 - \sum_{i=1}^{25}\left[\left(\sum_{j=1}^{2} y_{ij}\right)^2 / 2\right] \\ &= 0{,}343\ 882\,92 - 0{,}318\,944\,87 \\ &= 0{,}024\,938\,05\ . \end{aligned} \qquad \text{(P.11)}$$

The within-items sum of squares, *W*, is given by

$$\begin{aligned} W &= \sum_{i=1}^{25}\sum_{j=1}^{2} (y_{ij} - \bar{y}_{i.})^2 \\ &= \sum_{i=1}^{25}\sum_{j=1}^{2} y_{ij}^2 - \sum_{i=1}^{25}\left(\sum_{j=1}^{2} y_{ij}\right)^2 / 2 \\ &= 0{,}343\,882\,92 - 0{,}343\,489\,84 \\ &= 0{,}000\,393\,08\ . \end{aligned} \qquad \text{(P.12)}$$

By subtraction, the between-item sum of squares *B* is given by

$$\begin{aligned} B &= T - W \\ &= 0{,}024\,938\,05 - 0{,}000\,393\,08 \\ &= 0{,}024\,544\,97\ . \end{aligned} \qquad \text{(P.13)}$$

The measurement error variance is estimated as

$$\hat{\sigma}_{\mathrm{m}}^2 = W / (N - n) = 0{,}000\ 393\ 08 / (50 - 25) = 0{,}000\ 015\ 723\ 2\ .$$

The process variance is estimated as

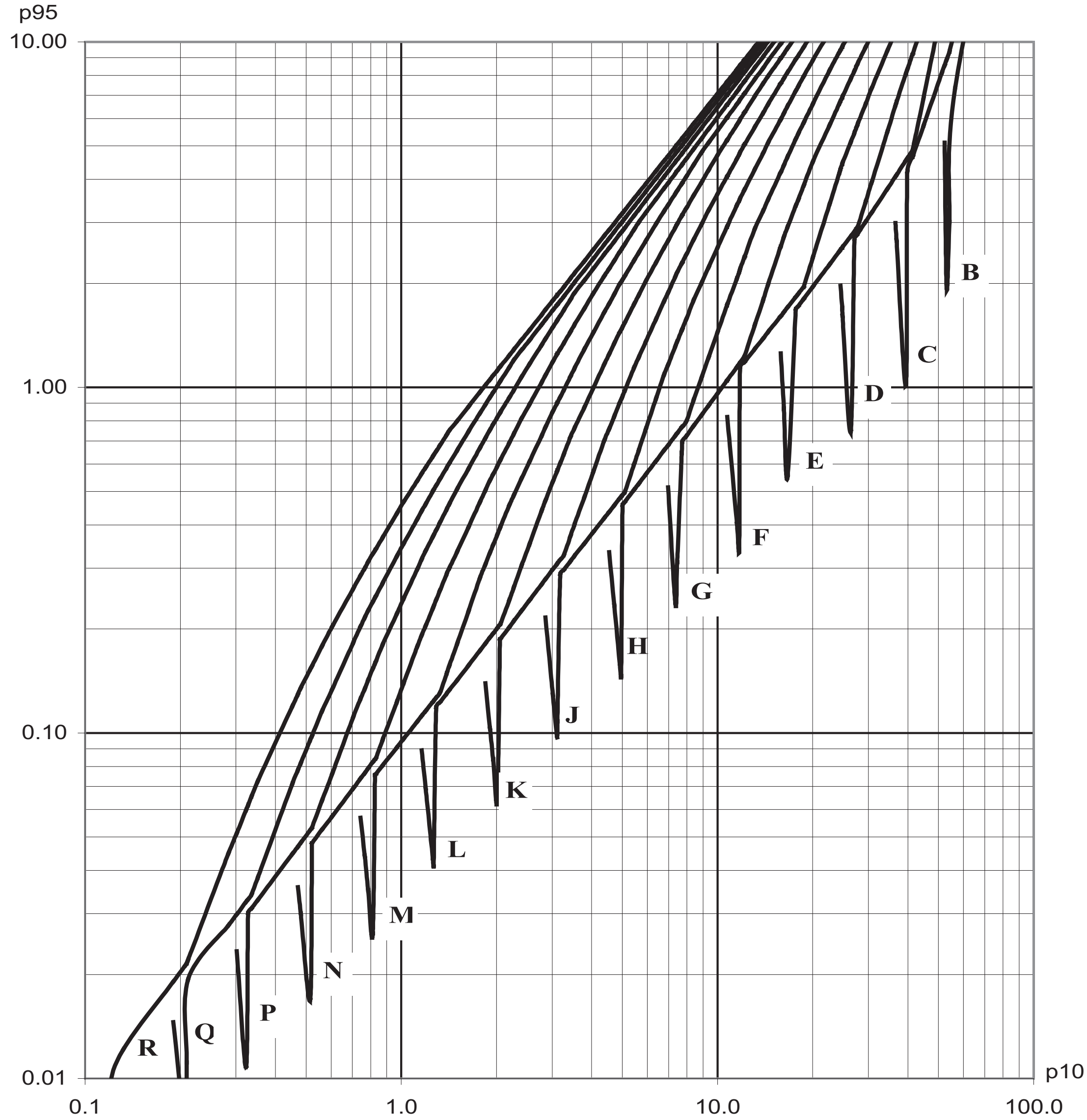
$$\begin{aligned} s^2 = \hat{\sigma}^2 &= \left[B - (n-1)\hat{\sigma}_m^2\right] / (N - n) \\ &= \left[0{,}024\ 544\ 97 - 24 \times 0{,}000\ 015\ 723\ 2\right] / (50 - 25) \\ &= 0{,}024\ 167\ 62 / 25 \\ &= 0{,}000\ 966\ 70\ , \end{aligned}$$

so the process standard deviation is estimated as

$$s = \hat{\sigma} = \sqrt{0{,}000\ 966\ 70} = 0{,}031\ 092\ .$$

$$U - 2{,}425s = 13{,}05 - 2{,}425 \times 0{,}031\ 092 = 12{,}975.$$

As $\bar{x} = 12{,}990 > 12{,}975$, the lot is not accepted.



Key

p10 quality level in percent nonconforming at probability of acceptance 10 %

p95 quality level in percent nonconforming at probability of acceptance 95 %

Figure P.1 — Chart A: Sample size code letters of standard single sampling plans for specified qualities at probabilities of acceptance 95 % and 10 %

Sample size code letters are shown on the chart in boldface type.

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