BRITISH STANDARD

BS ISO TR 7066-1 : 1997

Assessment of uncertainty in calibration and use of flow measurement devices

Part 1. Linear calibration relationships

ICS 17.120.01

British Standards

National foreword

This British Standard reproduces verbatim ISO/TR 7066-1 : 1997 and implements it as the UK national standard. It supersedes BS 7118 : Part 1 : 1990 which is withdrawn.

The UK participation in its preparation was entrusted by Technical Committee CPI/30, Measurement of fluid flow in closed conduits, to Subcommittee CPI/30/9, General topics, which has the responsibility to:

- aid enquirers to understand the text;
- present to the responsible international/European committee any enquiries on the interpretation, or proposals for change, and keep the UK interests informed;
- monitor related international and European developments and promulgate them in the UK.

A list of organizations represented on this subcommittee can be obtained on request to its secretary.

Cross-references

The British Standards which implement international or European publications referred to in this document may be found in the BSI Standards Catalogue under the section entitled 'International Standards Correspondence Index', or using the 'Find' facility of the BSI Standards Electronic Catalogue.

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Summary of pages

This document comprises a front cover, an inside front cover, the ISO/TR title page, pages ii to v, a blank page, pages 1 to 28, an inside back cover and a back cover.

Amendments issued since publication

Amd. No.	Date	Text affected

This British Standard, having been prepared under the direction of the Sector Board for Materials and Chemicals, was published under the authority of the Standards Board and comes into effect on 15 July 1997

ISBN 0 580 27554 X

TECHNICAL REPORT

ISO/TR 7066-1

First edition
1997-02-01

Assessment of uncertainty in calibration and use of flow measurement devices —

Part 1:
Linear calibration relationships

Évaluation de l'incertitude dans l'étalonnage et l'utilisation des appareils de mesure du débit —

Partie 1: Relations d'étalonnage linéaires

Reference number
ISO/TR 7066-1:1997(E)

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Descriptors: fluid flow, liquid flow, flow measurement, measuring instruments, calibration, error analysis, rules of calculation.

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards. In exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 7066-1, which is a Technical Report of type 1, was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, Subcommittee SC 9, *Uncertainties in flow measurement*.

This document is being issued as a type 1 Technical Report because no consensus could be reached between ISO TC 30/SC 9 and ISO TAG 4, *Metrology*, concerning the harmonization of this document with the *Guide to the expression of uncertainty in measurement*, which is a basic document in the ISO/IEC Directives. A future revision of this Technical Report will align it with the *Guide*.

This first edition as a Technical Report cancels and replaces the first edition as an International Standard (ISO 7066-1:1988), which has been technically revised.

ISO/TR 7066 consists of the following parts, under the general title *Assessment of uncertainty in calibration and use of flow measurement devices*:

— *Part 1: Linear calibration relationships*

— *Part 2: Non-linear calibration relationships*

Annex A forms an integral part of this part of ISO/TR 7066. Annexes B and C are for information only.

Introduction

One of the first International Standards to specifically address the subject of uncertainty in measurement was ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement,* published in 1978. The extensive use of ISO 5168 in practical applications identified many improvements to its methods; these were incorporated into a draft revision of this International Standard, which in 1990 received an overwhelming vote in favour of its publication. ISO 7066-1, *Assessment of uncertainty in the calibration and use of flow measurement devices — Part 1: Linear calibration relationships,* published in 1989, was drawn up according to the principles outlined in ISO 5168:1978. The draft revision of ISO 7066-1 is consistent with both the draft revision of ISO 5168 and with ISO 7066-2:1988.

However, the draft revisions of both ISO/TR 5168 and ISO/TR 7066-1 were withheld from publication for a number of years since, despite lengthy discussions, no consensus could be reached with the draft version of a document under development by a Working Group of ISO Technical Advisory Group 4, *Metrology* ISO TAG 4/WG 3). The TAG 4 document, *Guide to the expression of uncertainty in measurement* (GUM), was published in late 1993 as a basic document in the ISO/IEC Directives. At a meeting of the ISO Technical Management Board in May 1995 it was decided to publish the revisions of ISO 5168 and ISO 7066-1 as Technical Reports.

This document is published as a type 1 Technical Report instead of an International Standard because it is not consistent with the GUM. A future revision of this part of ISO/TR 7066 will align the two documents.

Assessment of uncertainty in calibration and use of flow measurement devices —

Part 1:
Linear calibration relationships

1 Scope

1.1 This part of ISO/TR 7066 describes the procedures to be used in deriving the calibration curve for any method of measuring flowrate in closed conduits or open channels, and of assessing the uncertainty associated with such calibrations. Procedures are also given for estimation of the uncertainty arising in measurements obtained with the use of the resultant graph, and for calculation of the uncertainty in the mean of a number of measurements of the same flowrate.

1.2 Only linear relationships are considered in this part of ISO/TR 7066; the uncertainty in non-linear relationships forms the subject of ISO/TR 7066-2. This part of ISO/TR 7066 is applicable, therefore, only if

a) the relationship between the two variables is itself linear,

   or

   one or both variables can be transformed in such a manner as to create a linear relationship between them, as, for instance by the use of logarithms,

   or

   the total range can be subdivided in such a way that within each subdivision the relationship between the two variables can be regarded as being linear; and if

b) systematic deviations from the fitted line are negligible compared with the uncertainty associated with the individual observations forming the graph.

NOTE — Examples of the application of the principles contained in this part of ISO/TR 7066 are given in annexes B and C.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO/TR 7066. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO/TR 7066 are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 772:1996, *Hydrometric determinations — Vocabulary and symbols.*

ISO 1100-2:—[1], *Liquid flow measurement in open channels — Part 2: Determination of the stage-discharge relationship.*

ISO 4006:1991, *Measurement of fluid flow in closed conduits — Vocabulary and symbols.*

ISO/TR 5168:—[2] *Measurement of fluid flow — Evaluation of uncertainties.*

ISO 7066-2:1988, *Assessment of uncertainty in the calibration and use of flow measurement devices — Part 2: Non-linear calibration relationships.*

3 Definitions and symbols

For the purposes of this part of ISO/TR 7066, the definitions and symbols given in ISO 772 and ISO 4006 and the following definitions and symbols apply.

3.1 Definitions

3.1.1 calibration graph: Curve drawn through the points obtained by plotting some index of the response of a flow meter against some function of the flowrate.

3.1.2 confidence limits: Upper and lower limits about an observed or calculated value within which the true value is expected to lie with a specified probability, assuming a negligible uncorrected systematic error.

3.1.3 correlation coefficient: Indicator of the degree of relationship between two variables.

NOTE — Such a relationship may be causal or may operate through the agency of a third variable, but a decision on this point cannot be made on statistical grounds alone.

3.1.4 covariance: First product moment measured about the variate means, i.e.

$$\mathrm{Cov}(x, y) = \left[\sum (x_i - \bar{x})(y_i - \bar{y})\right] / (n - 1)$$

3.1.5 error of measurement: Collective term meaning the difference between the measured value and the true value.

It includes both systematic and random components.

3.1.6 error, random: That component of the error of measurement which varies unpredictably from measurement to measurement.

NOTE — No correction is possible for this type of error, the cause of which may be known or unknown.

3.1.7 error, systematic: That component of the error of measurement which remains constant or varies predictably from measurement to measurement.

NOTE — The cause of this type of error may be known or unknown.

1) To be published. (Revision of ISO 1100-2:1982)

2) To be published.

3.1.8 error, spurious: Error which invalidates a measurement.

Such errors generally have a single cause, such as instrument malfunction or the misrecording of one or more digits of the measurement value.

3.1.9 function: Mathematical formula expressing the relationship between two or more variables.

3.1.10 line of best fit: Line drawn through a series of points in such a way as to minimize the variance of the points about the line.

3.1.11 residual: Difference between an observed value and the corresponding value calculated from the regression equation.

3.1.12 sample [experimental] standard deviation: Measure of the dispersion about the mean of a series of n values of a measurand, defined by the formula:

$$s(x) = \left[\sum (x_i - \bar{x})^2 / (n-1) \right]^{1/2}$$

NOTE — If the n measurements are regarded as a sample of the underlying population, then the formula below provides a sample estimate of the population standard deviation.

$$\sigma = \left[\sum (x_i - \mu)^2 / n \right]^{1/2}$$

3.1.13 systematic error limit: That component of the total uncertainty associated with the systematic error.

Its value cannot be reduced by taking many measurements.

3.1.14 uncertainty, random: Estimate characterizing the range of values within which it is asserted with a given degree of confidence that the true value of the measurand may be expected to lie.

Its magnitude in terms of mean values may be reduced by taking many measurements.

3.1.15 variance: Measure of dispersion based on the mean squared deviation from the arithmetic mean, defined as

$$\mathrm{Var}(x) = \sum (x_i - \bar{x})^2 / (n-1)$$

3.2 Symbols

NOTE — Symbols used in the open channel and closed conduit examples of annexes B and C where these differ from, or are in addition to, those listed below are included at the beginning of the respective annexes.

a	Intercept of the calibration curve on the ordinate
b	Gradient or slope of the calibration curve
c	Coefficient in a weighted least-squares equation
Cov( )	Covariance of variables in brackets
e_R( )	Random uncertainty of variable in brackets
e_S( )	Systematic error limits of variable in brackets
ln	Natural logarithm
n	Number of measurements used in deriving the calibration curve
Q	Flowrate
r	Correlation coefficient

$s(\ )$ Experimental standard deviation of variable in brackets

s_R Standard deviation (standard error) of points about best-fitting straight line

t "Student's" t (as obtained from ISO 5168 or from any set of statistical tables)

w_i ith weighting factor, in weighted least-squares

x Independent variable; variable subject to the smallest error

y Dependent variable; variable subject to the greatest error

U Total or overall uncertainty

U_{ADD} Uncertainty using the additive model; provides between approximately 95 % and 99 % coverage

$$U_{ADD} = e_S + e_R$$

U_{RSS} Uncertainty using the root-sum-square model; provides approximately 95 % coverage

$$U_{RSS} = (e_S^2 + e_R^2)^{1/2}$$

γ Ratio of the standard deviation of the independent, or x, variable to that of the dependent, or y, variable

Δ Difference between an observed and a calculated value

μ Population mean

σ Population standard deviation

θ Influence coefficient

NOTE — In a number of International Standards, the random uncertainty e_R and systematic error limits e_S are denoted by the symbols U_r and U_s or B respectively.

Subscripts and superscripts

NOTE — In the following, the summation sign Σ is used to represent

$$\sum_{i=1}^{n}$$

unless otherwise noted; a bar above a symbol (¯) denotes the mean value of that quantity; a circumflex (^) denotes the value of the variable predicted by the equation of the fitted curve.

i ith value of a variable

ij ith value of the jth category

4 General

4.1 With the majority of calibrations considered in this part of ISO 7066, the relationship between the variables is of a functional nature and is defined by some form of mathematical expression. Any departure of the observed values from this relationship can then be attributed to errors of measurement of one kind or another, which may affect either or both variables and which may be random or systematic or a combination of the two.

4.2 The role of the calibration procedure is thus twofold: to assess the form of the underlying mathematical relationship and to provide an estimate of the uncertainty of the fitted line.

4.3 From a practical viewpoint there will exist pairs of values (x, y) for which the random uncertainties and systematic error limits in x and y will have been estimated by one of the methods given in clause 5. The choice of the procedure to be used in the calculation of the coefficients and uncertainty of the calibration equation will depend on the relative magnitudes of the random components $e_R(x)$ and $e_R(y)$.

4.4 Where the error in one or the other of the two variables can be assumed to be negligible, the methods set out in clauses 8, 9 and 11 shall be used, the underlying equation being taken to be of the form

$$y = a + bx \qquad \dots (1)$$

where

x is that variable with the smaller error;

a and b are coefficients of the fitted line to be determined.

Where both variables are subject to error and x is the variable with the smaller error, the methods described in clauses 8 and 9 can still be used if the x variable can be set to predetermined values during the calibration. This approach is known as the Berkson method.

4.5 A special case arises where y is effectively constant and independent of x, i.e. where the fitted line is parallel to the x-axis. In these cases, the methods specified in clause 10 shall be used in estimating the uncertainty.

4.6 To provide the information needed in selecting the fitting procedure to be used, a preliminary study of the data is essential. In particular, this should be directed towards establishing the uncertainties and systematic error limits in x and y and the adequacy of the linearity assumption. Where the relationship is known to be curvilinear, some attention should be given to the possibility of converting it to a linear form, thus simplifying the subsequent manipulation of the data.

5 Random uncertainties and systematic error limits in individual measurements

5.1 In determining the random uncertainties and systematic error limits in the two variables, there are no alternatives to the procedures given in ISO/TR 5168. As a first step in the estimation process, a table for each variable should be prepared indicating the various sources of error. These should include the errors in any basic measurements which have to be made and should list the random and systematic elements separately.

5.2 For variate values determined by direct measurement, the random uncertainty at a fixed value of the measurand x can be found by calculating the experimental standard deviation from a series of n measurements, using the formula

$$s = \left[\sum (x_i - \bar{x})^2 \Big/ (n-1)\right]^{1/2} \qquad \dots (2)$$

or, alternatively,

$$s = \left\{\left[n\sum x_i^2 - \left(\sum x_i\right)^2\right] \Big/ \left[n(n-1)\right]\right\}^{1/2} \qquad \dots (3)$$

and then substituting into

$$e_R(x) = ts(x) \qquad \dots (4)$$

5.3 In carrying out the above calculations, it should be remembered that the result obtained may vary depending on the magnitude of y at which x is measured. Similarly, the uncertainty in y, which can be found by substituting y for x in the above formulae, may also vary with the value of x at which it is measured. Since such variations will dictate the method to be used in the subsequent fitting of the calibration curve, it is essential that the estimation of uncertainty be carried out at a sufficient number of points to enable the extent of any problem to be accurately assessed.

5.4 Where the variate values are obtained as the sum or difference of a number of independent component measurements, the uncertainty shall be obtained by calculating the overall standard deviation from the formula:

$$s(x) = \left[\sum s(x_i)^2 \right]^{1/2} \qquad \ldots (5)$$

followed by substitution into equation (4).

In other cases, where the variables are derived from more complex functions of the constituent elements such as products or quotients, or where the elements are correlated, the overall standard deviation shall be determined by the methods given in annex A. The uncertainty may then again be obtained by substituting into equation (4).

5.5 The evaluation of the systematic error, which is somewhat more difficult, is described in ISO/TR 5168. Even when all known sources have been identified and allowed for, there will still remain a number of unidentified errors. In these cases any assessment will depend on a subjective judgement based on such evidence, e.g. past calibrations, previous history, etc., as is available.

5.6 Where the variate values are based on the sum of a number of elemental components, some difficulty may be experienced in determining the overall systematic error limits, due to the fact that, in a majority of cases, the sign of the components is unknown. In these instances the errors shall be combined using the root-sum-square procedure as defined by

$$e_S = \left(\sum_i e_{S,i}{}^2 \right)^{1/2} \qquad \ldots (6)$$

Where more complex funtions are involved, the systematic error limit, e_S, shall be found using the method given in annex A, replacing the variance terms by the corresponding $e_{S,i}{}^2$ terms.

5.7 The estimation process can be regarded as complete once all the sources of error have been identified and evaluated and the individual elements combined to give an overall assessment of the random uncertainty and systematic error limits for each variable.

6 Linearity of calibration graph

6.1 An initial investigation is also desirable to establish whether a linear calibration curve will provide an adequate and unbiased fit to the observed measurements. Of the methods available, the most effective are those based on a visual study of the deviations of the measurements from the fitted line. An approximation to this line can be obtained using Bartlett's method, as described in 6.2 to 6.5.

6.2 As a first step, the data should be ranked in ascending order in either the x or y direction, and the general means of the two variables found from the equations

$$\bar{x} = \sum x_i / n; \;\; \bar{y} = \sum y_i / n \qquad \ldots (7)$$

The data should now be divided into three equal and mutually exclusive groups and the means of the two end groups calculated as before. Denoting these by $\bar{x}_1, \bar{y}_1$ and $\bar{x}_3, \bar{y}_3$, respectively, the slope b of the approximate line can be found as

$$b = (\bar{y}_3 - \bar{y}_1) / (\bar{x}_3 - \bar{x}_1) \qquad \ldots (8)$$

Since the line must pass through the general means $\bar{x}, \bar{y}$ the equation of the curve can then be obtained from

$$(y_i - \bar{y}) = b(x_i - \bar{x}) \quad \text{. . . (9)}$$

or, substituting $\bar{y} - b\bar{x} = a$, as

$$\hat{y}_i = a + bx_i \quad \text{. . . (10)}$$

Finally the residuals can be determined from the formula

$$\Delta(y_i) = (y_i - \hat{y}_i) = (y_i - a - bx_i) \quad \text{. . . (11)}$$

As an alternative to the above procedure, a more accurate fit can be obtained by using the method of least squares as described in clause 8, with the residuals again being found from equation (11).

6.3 As a preliminary test, the residuals thus obtained should be ranked in ascending order and plotted as a cumulative frequency curve on normal probability paper. If the points lie in roughly a straight line with no evidence of any general curvature, the data can be regarded as being approximately normally distributed.

6.4 The opportunity should also be taken at this stage to examine any exceptionally large or small residuals, as the occurrence of these may seriously affect the position of the final fitted line and will inevitably increase the uncertainty. To assist in the identification of such "outliers", use can be made of Grubb's test as described in annex E of ISO/TR 5168. It must be emphasized, however, that even where the test result is positive, the decision to reject an observation should always be made on sound physical grounds following a careful study of all the relevant circumstances. In reaching a decision, it should be borne in mind that the point may be genuine and the size of the residual due to a lack of fit of the model to the observation. It should also be remembered that where an observation has been rejected, the whole of the fitting process and calculation of the residuals will need to be repeated.

6.5 Other tests which should be applied include the plotting of the residuals (Δy) against the observed values of the independent, (x), variable and against the predicted ($\hat{y}$) values. In either case if

a) the mathematical relationship is appropriate;

b) the fitting process has been correctly carried out;

c) the variance does not change significantly with x;

the points should lie in a horizontal band of uniform width [figure 1 a)]. Departures from this ideal form can include any one or more of the following:

a) the band forms a distinct upwards or downwards curve [figure 1 b)], implying that the relationship is curvilinear rather than linear in form;

b) a progressive widening or narrowing of the band, which remains horizontal [figure 1 c)]. This would indicate that the variance is not constant over the range of measurement and that some form of weighting procedure will be required in the final fitting process.

NOTES

1 As an alternative to weighting, it may be possible to transform the data to obtain uniform variance. As an example, if the variability increases with x, a plotting of $\log_{10} y$ against x or of $\log_{10} y$ against $\log_{10} x$ may give uniform variance in $\log_{10} y$. In making the transformation, care shall be taken that the calibration graph remains linear.

2 It should be noted that any transformation of the variables implies a weighting of the data and may be expected to give a curve fit somewhat different to that obtained from the original untransformed data.

c) the band shows a uniform straight-line upward or downward trend in its position [figure 1 d)], suggesting the presence of an error in either the fitting process itself or in the subsequent calculation of y.

7 Linearization of data

7.1 Where the initial tests have indicated that the calibration is best represented by some form of curvilinear relationship, serious consideration shall be given to the possibility of converting the data to linearity. The advantage of such an operation is that the fitting of the calibration curve and the determination of uncertainty then become relatively straightforward processes. Two procedures are possible.

7.1.1 The first of these involves an actual tranformation of the data and applies only where the relationship is of a mathematical nature. For this category, the form of transformation will usually be indicated by the form of function itself. Thus, for an open-channel calibration, where the relationship between water level and flow is expressed by the equation.

$$Q = c(h + h_0)^b \quad \text{. . . (12)}$$

where

h is the measured water level, expressed in metres;

h_0 is a datum correction denoting level at zero flow;

c is a coefficient;

b is an exponent;

the simple expedient of writing this in the logarithmic form

$$\ln Q = \ln c + b \ln (h + h_0) \quad \text{. . . (13)}$$

has the effect of linearizing the data.

7.1.2 In other cases, linearization may still be possible if the calibration curve can be divided into a number of sections, each of which can be treated as linear. Unlike the previously described method, the procedure is universally applicable and does not depend on the existence of a functional relationship between the two variables. To be successful, two conditions must be observed. The first of these is that each section of the curve should, wherever possible, be based on a similar number of observations, thus giving approximately the same degree of uncertainty to the whole of the fitted line. Secondly, to provide a smooth transition and avoid discontinuity, each section of the curve must have two or three points in common with any adjacent sections.

7.2 On completing the linearization process, it is essential that the tests for linearity described in clause 6 be repeated.

8 Fitting the best straight line

8.1 General

8.1.1 The preliminary tests will already have provided estimates of the standard deviations of the two variables, and in considering the fittting procedure it only remains to calculate the ratio

$$\gamma = s(y)/s(x) \quad \text{. . . (14)}$$

Where the value is large, say ⩾ 20, the classical least-squares method given in 8.2 shall be followed.

Where the value obtained lies below this limit, the procedure of 8.2 can still be used provided the x variable can be set to predetermined values as required by the Berkson method. In other cases, the methods needed are beyond the scope of this part of ISO/TR 7066.

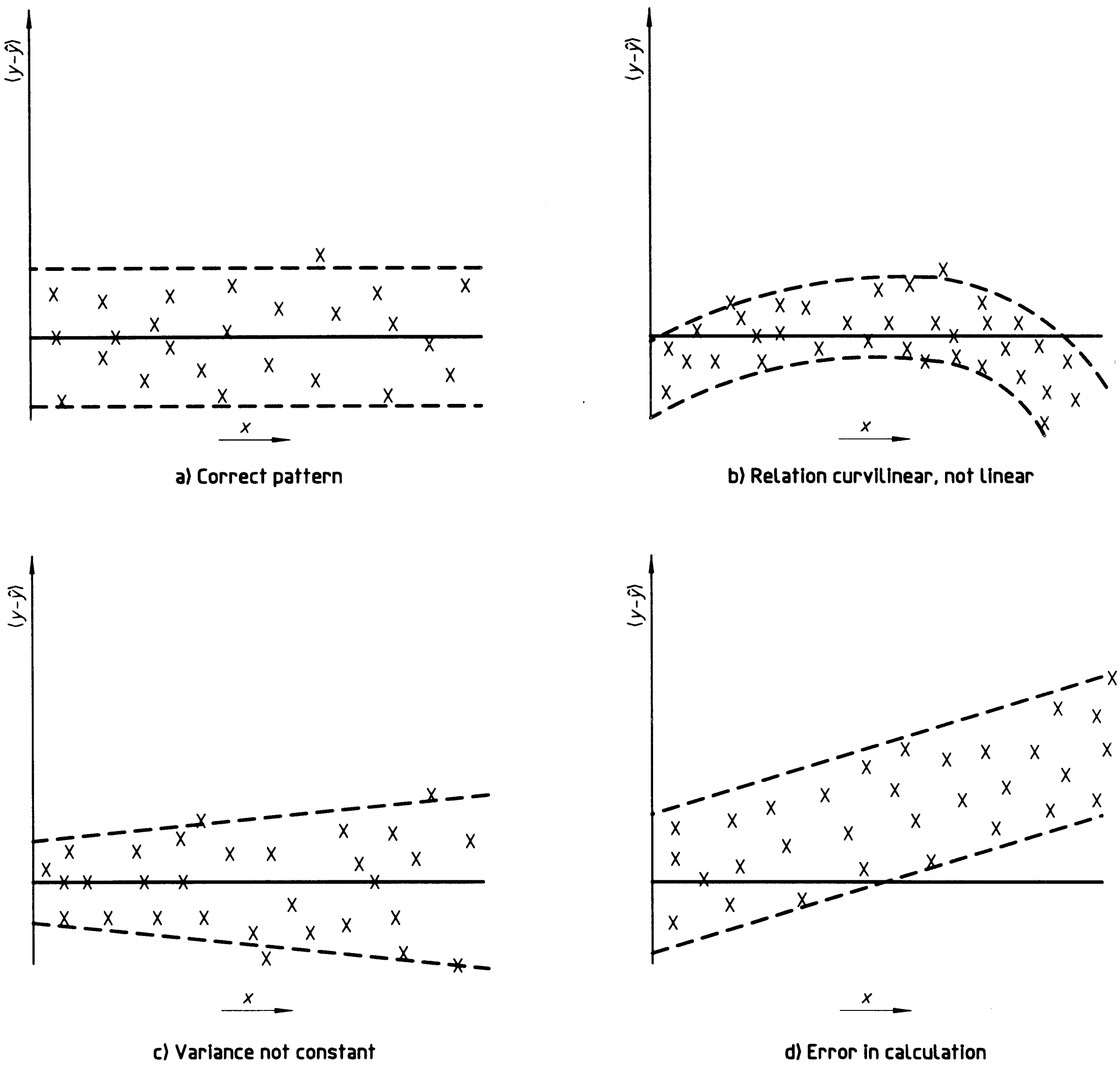


Figure 1 — Plot of residuals $(y - \hat{y})$ against x values

8.1.2 In describing the calibration procedure, it should be emphasized that the usual convention relating to dependent and independent variables has been abandoned. In the following sections, the x variable is always to be taken as that with the smallest error.

8.2 One variable only subject to error or Berkson method applies

8.2.1 Where the error in one variable can be regarded as negligible in comparison with that in the second variable, the fitting of the calibration curve shall be accomplished using a classical regression approach.

8.2.2 With this type of procedure, the slope b of the fitted line

$$\hat{y}_i = a + bx_i \qquad \text{. . . (15)}$$

can be found from the equation

$$b = \sum\left[(x_i - \bar{x})(y_i - \bar{y})\right] \Big/ \sum(x_i - \bar{x})^2 \qquad \ldots (16)$$

and the intercept from

$$a = \bar{y} - b\bar{x} \qquad \ldots (17)$$

Similarly the correlation coefficient, r, which expresses the strength of the relationship between x and y, can be determined from

$$r = \sum\left[(x_i - \bar{x})(y_i - \bar{y})\right] \Big/ \left[\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2\right]^{1/2} \qquad \ldots (18)$$

8.2.3 To complete the fitting process, the standard deviation of the observations about the fitted line should be calculated either from

$$s_R = \left[\sum(y_i - \hat{y}_i)^2 \Big/ (n-2)\right]^{1/2} \qquad \ldots (19)$$

$$= \left[\sum(y_i - a - bx_i)^2 \Big/ (n-2)\right]^{1/2} \qquad \ldots (20)$$

or from

$$s_R = s(y)\left(1 - r^2\right)^{1/2} \qquad \ldots (21)$$

where

$$s(y) = \left[\sum(y_i - \bar{y})^2 \Big/ (n-1)\right]^{1/2} \qquad \ldots (22)$$

Where equation (21) is used, a sufficient number of significant figures shall be retained to ensure the absence of any major rounding error.

8.2.4 Where modern computing facilities are not available b, r and $s(y)$ can be more conveniently obtained from the equations

$$b = \left[n\sum x_i y_i - \sum x_i \sum y_i\right] \Big/ \left[n\sum x_i^2 - \left(\sum x_i\right)^2\right] \qquad \ldots (23)$$

$$r = \left[n\sum x_i y_i - \sum x_i \sum y_i\right] \Big/ \left\{\left[n\sum x_i^2 - \left(\sum x_i\right)^2\right]\left[n\sum y_i^2 - \left(\sum y_i\right)^2\right]\right\}^{1/2} \qquad \ldots (24)$$

$$s(y) = \left\{\left[n\sum y_i^2 - \left(\sum y_i\right)^2\right] \Big/ n(n-1)\right\}^{1/2} \qquad \ldots (25)$$

with a again being found from equation (17).

Here also care shall be taken to retain a sufficient set of significant figures to avoid serious rounding errors.

8.2.5 Where the calibration curve consists of two or more sections, the point(s) of intersection of these shall be determined at this stage. Denote the equations of two adjacent sections by

$$y_1 = a_1 + b_1 x \text{ and } y_2 = a_2 + b_2 x \qquad \ldots (26)$$

Then, at the point of intersection, $y_1 = y_2$ and the common value of x will be given by

$$x = (a_1 - a_2)/(b_2 - b_1) \qquad \ldots (27)$$

The corresponding value of y can then be obtained by substituting into the appropriate equation (26).

9 Fitting the best weighted straight line

9.1 Where the preliminary linearity tests given in clause 6 indicate that the variance of y is not constant but varies with the value of x, the least-squares method given above is invalid and must be replaced by a weighted form of regression analysis if bias is to be avoided.

9.2 In such cases, a suitable procedure consists of replacing equations (16), (17) and (23) by

$$b = \left\{\sum c_i x_i y_i - \left[\left(\sum c_i x_i\right)\left(\sum c_i y_i\right)\right]/n\right\} \Big/ \left\{\sum c_i x_i^2 - \left[\left(\sum c_i x_i\right)^2/n\right]\right\} \quad \text{. . . (28)}$$

and

$$a = \left(\sum c_i y_i - b\sum c_i x_i\right)/n \quad \text{. . . (29)}$$

The standard deviation of the y_i values can then be obtained from equation (22) and the standard error of the fitted line from equation (19) or (20).

9.3 Where the variances of the individual observations are known, the weighting coefficient, c_i, should be found from the relationship

$$w_i = 1/\text{var } y_i; \quad c_i = w_i/\overline{w} \quad \text{. . . (30)}$$

In other cases, suitable values for the coefficients c_i can be obtained by

a) calculating the differences of the y_i values from the estimated calibration curve obtained as described in clause 6;

b) plotting the squares of the differences against the respective x_i values;

c) fitting a curve to the data by the methods given in ISO/TR 7066-2;

d) using a curve to obtain the mean squared differences $\Delta^2(y_i)$;

e) substituting the $\Delta^2(y_i)$ values for var y_i in equation (30) to obtain the c_i values.

10 Procedure when y is independent of x

10.1 A special case arises when the slope of the calibration curve is zero and y is constant over the range of x. In these circumstances y is independent of x, the calibration curve becomes a horizontal straight line and the calibration coefficient reduces to the arithmetic average of the y_i values, i.e.

$$\overline{y} = \sum y_i/n \quad \text{. . . (31)}$$

10.2 Where the evidence available suggests that a calibration of this form is appropriate, tests shall be carried out to determine whether or not the slope of the fitted line can be regarded as zero. For this purpose the value of

$$b \pm ts(b) \quad \text{. . . (32)}$$

should be calculated where

$$s(b) = s_R \Big/ \left[(n-1)^{1/2} s(x)\right] \quad \text{. . . (33)}$$

Where zero is included within the limits given by equation (32), it may be concluded that the line is effectively horizontal and that the coefficient will be given by equation (31).

11 Calculation of uncertainty

11.1 The random uncertainty in the fitted line at the point $x = x_k$ shall be obtained from the equation

$$e_R(\hat{y}) = ts_R\left\{(1/n) + \left[(x_k - \bar{x})^2 / \sum (x_i - \bar{x})^2\right]\right\}^{1/2} \quad \text{. . . (34)}$$

whilst the uncertainty in an individual observation at the point $x = x_k$ can be found from

$$e_R(y_k) = ts_R\left\{1 + (1/n) + \left[(x_k - \bar{x})^2 / \sum (x_i - \bar{x})^2\right]\right\}^{1/2} \quad \text{. . . (35)}$$

The two values are substantially different; $e_R(\hat{y})$ represents the uncertainty in a value calculated from the equation of the line, whereas $e_R(y_k)$ is the uncertainty in the prediction of an individual measurement.

11.2 In both the above equations, the uncertainty interval is parabolic in form, with its narrowest width at the mean value of x. The width will also depend on the level of confidence required, being wider at the 99 % level than at the 95 % level.

12 Systematic error limits and reporting procedure

12.1 To complete the analysis, the systematic error limits in the calibration shall be estimated in accordance with the principles set out in ISO/TR 5168 and clause 5 of this part of ISO/TR 7066. In view of the difficulties in determining the signs and sizes of such errors, the individual components should be combined by the root-sum-square method. Where the variate values are obtained as the sum or difference of the elemental values, the overall systematic error limit can be otained from the equation

$$e_S = \left(\sum e_{S,i}^{\,2}\right)^{1/2} \quad \text{. . . (36)}$$

Where, however, the variate values are based on more complex functions such as products or quotients, the method of annex A shall be used, with the $e_{S,i}$ values replacing the respective variances.

12.2 As indicated in ISO/TR 5168, the random uncertainty and systematic error limit should always be reported separately. Where a single figure representing the combined uncertainty is also required, this shall be obtained from either the additive or root-sum-square models as defined by

$$U_{ADD} = e_S + ts \quad \text{. . . (37)}$$

or

$$U_{RSS} = \left[e_S^{\,2} + (ts)^2\right]^{1/2} \quad \text{. . . (38)}$$

the latter always providing the smaller estimate.

12.3 The value of U_{ADD} or U_{RSS} thus obtained should not be regarded as a confidence interval in the strict statistical sense since the systematic error limit is, by definition, based on subjective judgement.

13 Extrapolated values

13.1 Calibration coefficients and their associated uncertainties obtained by extrapolation of the calibration curve do not strictly fall within the scope of this part of ISO/TR 7066, since unpredictable effects may invalidate the results. Nevertheless, circumstances can arise which require the evaluation of flowrates outside the range of calibration and, in these instances, the procedures described below shall be used.

13.2 In figure 2 let the solid line represent the calibration graph of a flow measurement device as determined between the limits x_1 and x_2, and let the dotted line be the extension of this to the required point x_3. Also, let the dashed lines be the uncertainty limits extrapolated to the point x_3.

13.3 Where the calibrated device is covered by some International Standard which predicts the flowrates and uncertainties then, provided the extrapolated $\hat{y}$ value and its uncertainty limits lie wholly within the limits predicted by the International Standard as shown in figure 2 a), the extrapolated confidence limits shall be accepted. Where, however, the extrapolated limits lie outside those predicted by the International Standard, as in figure 2 b), then the confidence limits given by the International Standard shall be accepted. Finally, where the value of $\hat{y}$ lies outside the interval predicted by the International Standard or where the device is not covered by an International Standard, the extrapolated limits shall again be used.

14 Uncertainty in the use of the calibration graph for a single flowrate measurement

14.1 Where a calibrated meter is subsequently used to measure flowrate, any uncertainty in the position of the calibration curve will be transferred to the calculated value as a systematic error. Except in the case where the slope of the line is effectively zero and equation (15) thus reduces to a constant over the range of calibration, the uncertainty in the flowrate will always be greater than that due to the calibration graph alone. This will be so even when the measurement conditions are nominally identical with those existing during the calibration process, the difference arising due to the uncertainty in locating the position to be used on the calibration graph.

14.2 This additional uncertainty, together with any added contributions arising from data acquisition and reduction, shall be evaluated using the procedures given in ISO/TR 5168, with the random uncertainty and systematic error limit being determined separately and later combined using the U_{RSS} model. The total uncertainty in the measurement, denoted by $U_{\mathrm{RSS}}(\hat{y})$, is obtained using the formula

$$U_{\mathrm{RSS}}(\hat{y}) = \left[U_{\mathrm{RSS}}^2(\hat{y}_{\mathrm{o}}) + U_{\mathrm{RSS}}^2(\hat{y}_{\mathrm{c}})\right]^{1/2} \qquad \text{. . . (39)}$$

where

$U_{\mathrm{RSS}}(\hat{y}_{\mathrm{o}})$ is the total additional uncertainty;

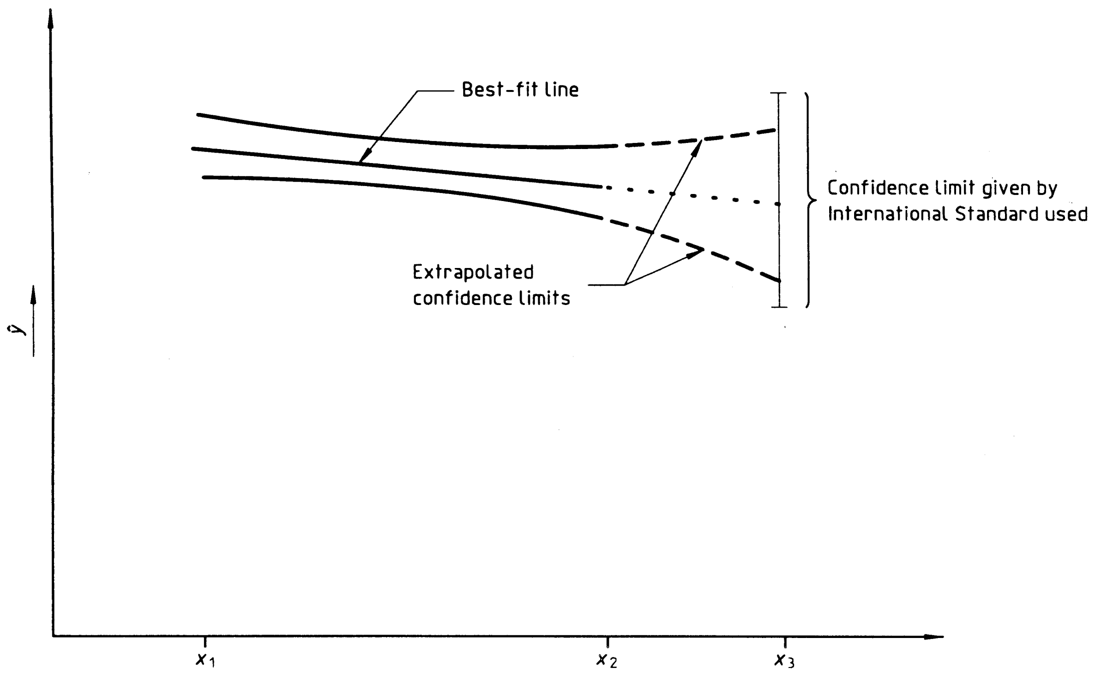
$U_{\mathrm{RSS}}(\hat{y}_{\mathrm{c}})$ is the total uncertainty in the calibration graph.

The manner in which the additional uncertainty contributes to the overall uncertainty in the flowrate will depend on the nature of the calibration graph. There are two main conditions, which are described in 14.3 and 14.4.

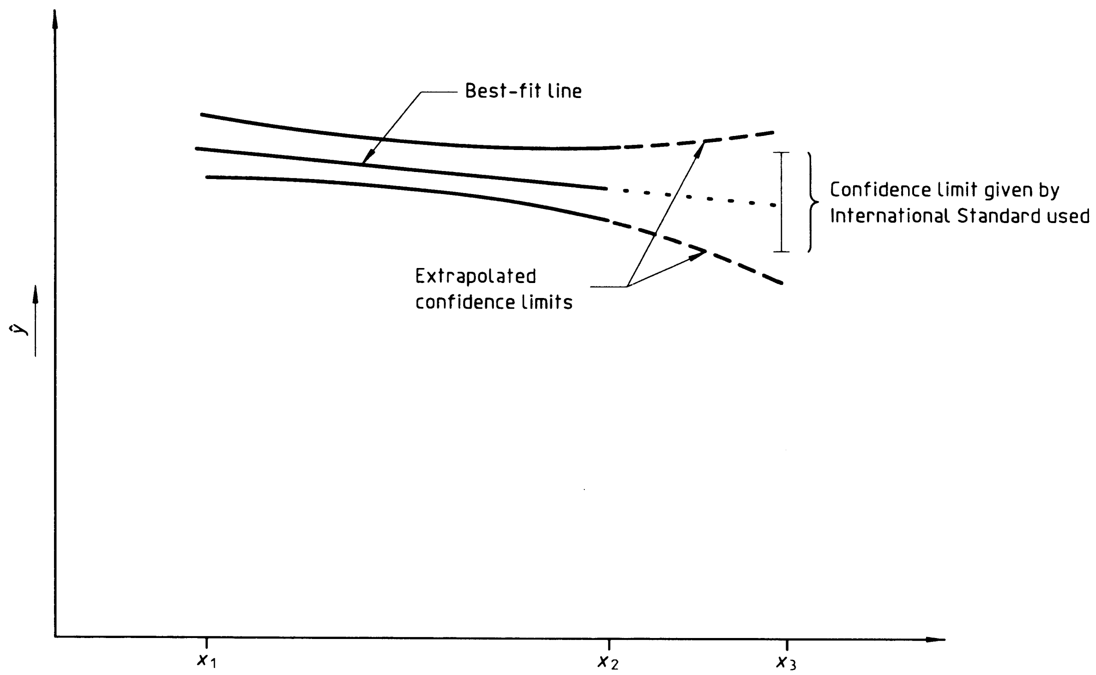
14.3 Where the slope of the calibration curve is zero, the flowrate is obtained simply by multiplying some function of the output of the meter by a coefficient which is independent of the flowrate. There is, therefore, no additional uncertainty and equation (39) reduces to

$$U_{\mathrm{RSS}}(\hat{y}) = U_{\mathrm{RSS}}(\hat{y}_{\mathrm{c}}) \qquad \text{. . . (40)}$$

and the uncertainty in the flowrate measurement becomes equal to the uncertainty in the calibration graph.



a) Extrapolated limits within those given by the International Standard used



b) Extrapolated limits outside those given by the International Standard used

Figure 2 — Criteria for confidence limits

14.4 Where the slope of the line is not zero, the calibration coefficient will itself be a function of the flowrate, and in estimating the latter some form of iterative process will be necessary. For this purpose an initial estimate of the calibration coefficient is used to obtain a first approximation to the flowrate, and this is used to obtain a more precise value for the calibration coefficient. The process is then repeated until successive estimates of the flowrate are identical. In this case any error in measuring the meter output will introduce an error in the coefficient used, and the overall uncertainty will be given by equation (39).

14.5 The uncertainty will be still further increased if the conditions of use differ from those under which the calibration was carried out, e.g. different layout, fluid, instrumentation, etc., and in these cases it will usually be necessary to evaluate the confidence limits for each situation as it arises.

Annex A
(normative)

Calculation of the variance of a general function

A.1 If the overall variance is based on the product or quotient of two or more component variances, then the simple combinatorial equation (5) will not apply, and the more complex expression associated with the standard error of a general function must be used. The differential terms ($\partial X/\partial x_i$) included in the equation are identical in all repects to the influence coefficients ($\theta = \partial R/\partial Y_i$) used in combining elemental errors in ISO/TR 5168.

A.2 If $X = f(x_1, x_2 ..., x_n)$, where f is any function, then

$$\begin{aligned} \text{Var } X &= (\partial X/\partial x_1)^2 \text{ Var } x_1 + (\partial X/\partial x_2)^2 \text{ Var } x_2 + ... + (\partial X/\partial x_n)^2 \text{ Var } x_n + \\ &+ 2\{[(\partial X/\partial x_1)(\partial X/\partial x_2)] \text{ Cov}(x_1, x_2) + [(\partial X/\partial x_1)(\partial X/\partial x_3)] \text{ Cov}(x_1, x_3) + \\ &+ ... + [(\partial X/\partial x_{n-1})(\partial X/\partial x_n)] \text{ Cov}(x_{n-1}, x_n)\} \end{aligned} \quad ...(41)$$

If the terms involving higher differentials can be ignored and the covariances are zero, i.e. the variables are independent, then equation (41) reduces to the first line only.

A.3 As an example, consider the equation for flow through a segment of an open channel current-meter gauging section

$$Q_i = b_{c,i} d_i \bar{v}_i \quad ...(42)$$

where b_c, d and v are dependent variables.

Using the first line of equation (41)

$$\begin{aligned} \text{Var } Q_i &= (\partial Q_i/\partial b_{c,i})^2 \text{ Var } b_{c,i} + (\partial Q_i/\partial d_i)^2 \text{ Var } d_i + (\partial Q_i/\partial \bar{v}_i)^2 \text{ Var } \bar{v}_i \\ &= (d_i \bar{v}_i)^2 \text{ Var } b_{c,i} + (b_{c,i}\bar{v}_i)^2 \text{ Var } d_i + (b_{c,i} d_i)^2 \text{ Var } \bar{v}_i \end{aligned} \quad ...(43)$$

Annex B
(informative)

Example of an open channel calibration

B.1 Symbols used

h_0 datum correction denoting stage at zero flow, expressed in metres;

h measured stage, expressed in metres;

c coefficient;

b exponent;

Q flowrate, expressed in cubic metres per second.

B.2 The information is given in table B.1 for the determination of a stage-discharge relation. Calculate the rating equation and compute the standard deviation of the points about the best-fitting straight line (s_R) and the random uncertainty $e_R(Q)$ for the relationship.

NOTE — In a number of International Standards concerning flow measurement, random uncertainty $e_R(Q)$ is denoted by the symbol $2s_{mr}$ where s_{mr} is defined as the standard error of the mean relationship.

B.3 In the case of an open channel flow-measurement station where calibration is by the velocity-area method, the relationship between stage and discharge may be expressed by the equation

$$Q = c(h + h_0)^b \qquad \text{. . . (44)}$$

which, on writing in logarithmic form gives

$$\ln Q = \ln c + b \ln(h + h_0) \qquad \text{. . . (45)}$$

and substituting

$$\ln(h + h_0) = x;\ \ln Q = y;\ \ln c = a;$$

reduces to the linear equation

$$y = a + bx$$

as given in equation (1).

B.4 With this type of calibration, the error in the determination of stage is almost always much less than that incurred in the measurement of flowrate, giving a value for γ in equation (14) greater than 20. Fitting can, therefore, be carried out using the classical least-squares method given in 8.2 of this part of ISO/TR 7066.

Substituting from table B.1 into first equation (23) and then equation (17) gives the slope of the calibration curve as

$$b = \left\{\left[32(-2{,}933\,7)\right] - \left[93{,}785\,5\,(-15{,}579\,8)\right]\right\} \Big/ \left\{\left[32(35{,}509\,3)\right] - (-15{,}579\,8)^2\right\} = 1{,}530\,1 \qquad \text{. . . (46)}$$

and the intercept as

$$\ln c = 2{,}930\,8 - 1{,}530\,1(-0{,}486\,9) = 3{,}675\,7 \qquad \ldots (47)$$

Hence

$$\ln Q = 3{,}675\,7 + 1{,}530\,1\ \ln(h - 0{,}115) \qquad \ldots (48)$$

or, alternatively

$$Q = 39{,}479\,(h - 0{,}115)^{1{,}530\,1} \qquad \ldots (49)$$

The rating curve is drawn in figure B.1 with stage on the ordinate and flowrate on the abscissa, following normal hydrometric practice.

Table B.1 — Typical data used in the manual computation of a stage-discharge curve by the method of least squares

Obs. No.	Q m³/s	Stage (h) m	($h + h_0$) m	ln Q_i (y_i)	ln($h + h_0$) (x_i)	xy	x^2
1	2,463	0,272	0,157	0,901 4	– 1,851 5	– 1,668 9	3,428 0
2	2,325	0,273	0,158	0,843 7	– 1,845 2	– 1,556 8	3,404 8
3	2,923	0,303	0,188	1,072 6	– 1,671 3	– 1,792 6	2,793 2
4	3,242	0,307	0,192	1,176 2	– 1,650 2	– 1,941 0	2,723 2
5	3,841	0,334	0,219	1,345 7	– 1,518 7	– 2,043 7	2,306 4
6	4,995	0,374	0,259	1,608 4	– 1,350 9	– 2,172 8	1,824 9
7	5,410	0,393	0,278	1,688 2	– 1,280 1	– 2,161 1	1,638 6
8	5,422	0,394	0,279	1,690 5	– 1,276 5	– 2,157 9	1,629 4
9	5,883	0,402	0,287	1,772 1	– 1,248 3	– 2,212 1	1,558 2
10	6,154	0,410	0,295	1,817 1	– 1,220 8	– 2,218 3	1,490 4
11	7,376	0,463	0,348	1,998 2	– 1,055 6	– 2,109 3	1,114 3
12	9,832	0,520	0,405	2,285 6	– 0,903 9	– 2,066 0	0,817 0
13	11,321	0,548	0,433	2,426 6	– 0,837 0	– 2,031 1	0,700 6
14	12,372	0,576	0,461	2,515 4	– 0,774 4	– 1,947 9	0,599 7
15	11,825	0,580	0,465	2,470 2	– 0,765 7	– 1,891 4	0,586 3
16	13,826	0,616	0,501	2,626 6	– 0,691 1	– 1,815 2	0,477 6
17	14,102	0,626	0,511	2,646 3	– 0,671 4	– 1,776 7	0,450 8
18	19,020	0,721	0,606	2,945 5	– 0,500 9	– 1,475 4	0,250 9
19	19,790	0,739	0,624	2,985 2	– 0,471 6	– 1,407 8	0,222 4
20	20,280	0,747	0,632	3,009 6	– 0,458 9	– 1,381 1	0,210 6
21	21,204	0,796	0,681	3,054 2	– 0,384 2	– 1,173 4	0,147 6
22	23,996	0,846	0,731	3,177 9	– 0,313 3	– 0,995 6	0,098 2
23	36,242	1,041	0,926	3,590 2	– 0,076 9	– 0,276 1	0,005 9
24	54,591	1,340	1,225	3,999 9	0,202 9	0,811 6	0,041 2
25	67,327	1,526	1,411	4,409 6	0,344 3	1,449 4	0,118 5
26	79,050	1,761	1,646	4,370 1	0,498 3	2,177 6	0,248 3
27	110,783	2,010	1,895	4,707 6	0,639 2	3,009 1	0,408 6
28	162,814	2,632	2,517	5,092 6	0,923 1	4,701 0	0,852 1
29	227,600	3,265	3,150	5,427 6	1,147 4	6,227 6	1,316 5
30	228,800	3,280	3,165	5,432 8	1,152 2	6,259 7	1,327 6
31	228,500	3,306	3,191	5,431 5	1,160 3	6,302 2	1,346 3
32	236,600	3,340	3,225	5,466 4	1,170 9	6,400 6	1,371 0
Totals				93,785 5	– 15,579 8	– 2,933 7	35,509 3

NOTE — Datum correction $h_0 = -0{,}115$ m

B.5 As defined by equation (19), the standard deviation of the points about the best-fit line is given by the equation

$$s_R = \left[\sum\left(\ln Q_i - \overline{\ln Q}\right)^2 / (n-2)\right]^{1/2} \qquad \text{. . . (50)}$$

from which, on substituting from table B.2

$$s_R = (0{,}029\,18/30)^{1/2} = 0{,}031 \qquad \text{. . . (51)}$$

B.6 The random percentage uncertainty in $\widehat{\ln Q}$ calculated from the fitted line at the point $(h + h_0)_k$ may be found by using equation (34) in the form

$$e'_R(\widehat{\ln Q}) = ts_R \left\{ \frac{1}{n} + \frac{\left[\ln(h+h_0)_k - \overline{\ln(h+h_0)}\right]^2}{\sum\left[\ln(h+h_0) - \overline{\ln(h+h_0)}\right]^2} \right\}^{1/2} \times 100$$

$$= 6{,}3\left\{0{,}031\,25 + \left[\ln(h - 0{,}115)_k + 0{,}486\,9\right]^2 / 27{,}923\,8\right\}^{1/2} \qquad \text{. . . (52)}$$

Similarly, the random percentage uncertainty for individual values of $\widehat{\ln Q_i}$ may be calculated using equation (35) in the form

$$e'_R(\widehat{\ln Q_i}) = ts_R \left\{ 1 + \frac{1}{n} + \frac{\left[\ln(h+h_0)_k - \overline{\ln(h+h_0)}\right]^2}{\sum\left[\ln(h+h_0) - \overline{\ln(h+h_0)}\right]^2} \right\}^{1/2} \times 100$$

$$= 6{,}3\left\{1{,}031\,25 + \left[\ln(h - 0{,}115)_k + 0{,}486\,9\right]^2 / 27{,}923\,8\right\}^{1/2} \qquad \text{. . . (53)}$$

B.7 The value of $e_R(\widehat{\ln Q})$ for the calculated flowrates at each observed $(h + h_0)_k$ may be evaluated from equation (52) and the results plotted on either side of the stage-discharge curve to give the symmetrical confidence limits for the logarithm of flow, the minimum width being at $\overline{\ln(h + h_0)}$.

Substituting for observation No. 1 in table B.2

$$e'_R(\widehat{\ln Q}) = 6{,}3\left\{0{,}031\,25 + \left[(-1{,}851\,5 + 0{,}486\,9)^2 / 27{,}923\,8\right]\right\}^{1/2}$$
$$= 1{,}97\ \%$$

Similarly, for observation No. 18

$$e'_R(\widehat{\ln Q_i}) = 6{,}3\left\{0{,}031\,25 + \left[(-0{,}500\,9 + 0{,}486\,9)^2 / 27{,}923\,8\right]\right\}^{1/2}$$
$$= 1{,}11\%$$

and for observation No. 32

$$e'_R(\widehat{\ln Q_i}) = 6{,}3\left\{0{,}031\,25 + \left[(-1{,}170\,9 + 0{,}486\,9)^2 / 27{,}923\,8\right]\right\}^{1/2}$$
$$= 2{,}27\ \%$$

A summary of these results, together with the values for the remaining observations, is given in the final column of table B.2.

Table B.2 — Values required for calculation of s_R and $e_R(\widehat{\ln Q})$

Obs. No.	$(h + h_0)$	$\ln(h + h_0)$ (x_i)	$(x_i - \bar{x})^2$	Q_i	$\ln Q_i$ (y_i)	$\widehat{Q}$	$\widehat{\ln Q}$ (y)	$(y_i - \bar{y})^2$	$e_R(\widehat{\ln Q})$
1	0,157	– 1,851 5	1,862 1	2,463	0,901 4	2,323	0,842 8	0,003 42	1,97
2	0,158	– 1,845 2	1,845 0	2,325	0,843 7	2,345	0,852 3	0,000 07	1,96
3	0,188	– 1,671 3	1,402 8	2,923	1,072 6	3,060	1,118 4	0,002 09	1,80
4	0,192	– 1,650 2	1,353 3	3,242	1,176 2	3,160	1,150 6	0,000 65	1,78
5	0,219	– 1,518 7	1,064 6	3,841	1,345 7	3,865	1,352 0	0,000 03	1,66
6	0,259	– 1,350 9	0,746 5	4,995	1,608 4	4,996	1,608 6	0,000 00	1,52
7	0,278	– 1,280 1	0,629 2	5,410	1,688 2	5,568	1,717 0	0,000 83	1,46
8	0,279	– 1,276 5	0,623 5	5,422	1,690 5	5,598	1,722 4	0,001 01	1,46
9	0,287	– 1,248 3	0,579 7	5,883	1,772 1	5,846	1,765 8	0,000 04	1,44
10	0,295	– 1,220 8	0,538 6	6,154	1,817 1	6,097	1,807 8	0,000 08	1,42
11	0,348	– 1,055 6	0,323 4	7,376	1,998 2	7,851	2,060 6	0,003 89	1,30
12	0,405	– 0,903 9	0,173 9	9,832	2,285 6	9,902	2,292 7	0,000 05	1,22
13	0,433	– 0,837 0	0,122 6	11,321	2,426 6	10,968	2,395 0	0,000 99	1,19
14	0,461	– 0,774 4	0,082 6	12,372	2,515 4	12,072	2,490 9	0,000 60	1,16
15	0,465	– 0,765 7	0,077 7	11,825	2,470 2	12,233	2,504 1	0,001 15	1,16
16	0,501	– 0,691 1	0,041 7	13,826	2,626 6	13,711	2,618 2	0,000 07	1,14
17	0,511	– 0,671 4	0,034 0	14,102	2,646 3	14,132	2,648 4	0,000 00	1,14
18	0,606	– 0,500 9	0,000 2	19,020	2,945 5	18,345	2,909 4	0,001 30	1,11
19	0,624	– 0,471 6	0,000 2	19,970	2,985 2	19,185	2,954 1	0,000 96	1,11
20	0,632	– 0,458 9	0,000 8	20,280	3,009 6	19,563	2,973 6	0,001 29	1,11
21	0,681	– 0,384 2	0,010 5	21,204	3,054 2	21,931	3,087 9	0,001 13	1,12
22	0,731	– 0,313 3	0,030 1	23,996	3,177 9	24,442	3,196 3	0,000 33	1,13
23	0,926	– 0,076 9	0,168 1	36,242	3,590 2	35,098	3,558 1	0,001 02	1,22
24	1,225	0,202 9	0,475 8	54,591	3,999 9	53,855	3,986 3	0,000 18	1,38
25	1,411	0,344 3	0,690 9	67,327	4,209 6	66,859	4,202 6	0,000 04	1,49
26	1,646	0,498 3	0,970 6	79,050	4,370 1	84,631	4,438 3	0,004 65	1,62
27	1,895	0,639 2	1,268 1	110,783	4,707 6	104,989	4,653 8	0,002 88	1,74
28	2,517	0,923 1	1,988 1	162,814	5,092 6	162,095	5,088 2	0,000 01	2,02
29	3,150	1,147 4	2,670 9	227,600	5,427 6	228,478	5,431 4	0,000 01	2,24
30	3,165	1,152 2	2,686 6	228,800	5,432 8	230,145	5,438 7	0,000 03	2,25
31	3,191	1,160 3	2,713 3	228,500	5,431 5	233,044	5,451 2	0,000 38	2,26
32	3,225	1,170 9	2,748 3	236,600	5,466 4	236,854	5,467 4	0,000 00	2,27

$\sum \ln(h + h_0) = -0{,}486\,9$; $\sum (x_i - \bar{x})^2 = 27{,}923\,8$; $\sum (y_i - \bar{y})^2 = 0{,}029\,18$

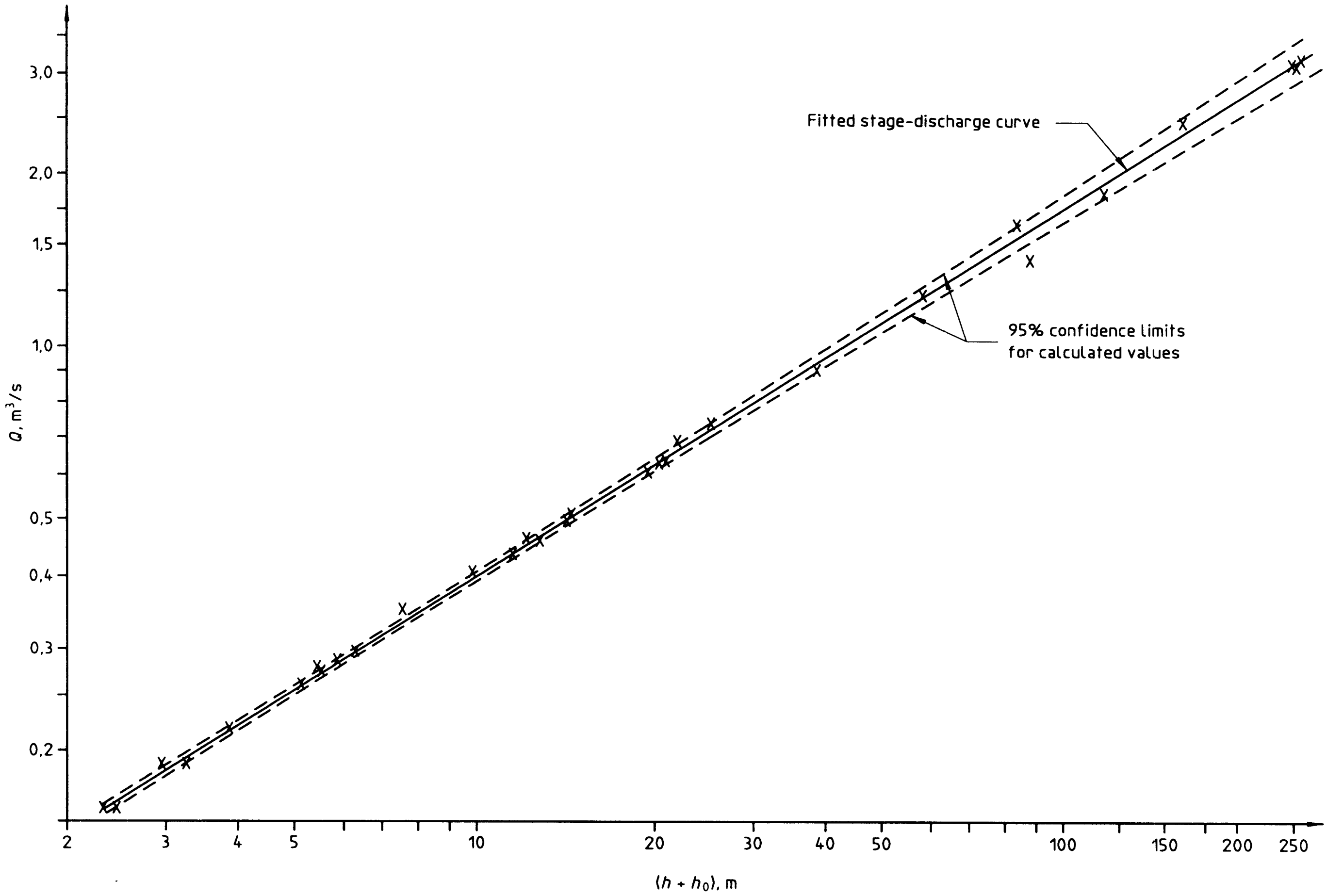


Figure B.1 — Stage-discharge curve based on data of table B.1

B.8 Asymmetrical limits for the untransformed flows may be obtained using the formulae

$100\left(e^{z}-1\right)$ for the upper 95 % confidence limit

and

$100\left(1-e^{-z}\right)$ for the lower 95 % confidence limit . . . (54)

where z is the right-hand side of equation (53) excluding the factor of 100.

Using the example for observation No. 1, the upper 95 % confidence limit thus becomes

$$100\left(e^{0,019\,7}-1\right)=100\left(1,019\,9-1\right)1,99\ \%$$

whilst the lower 95 % confidence limit becomes

$$100\left(1-e^{-0,019\,7}\right)=100\left[1-\left(1/e^{0,019\,7}\right)\right]=100\left(1-0,980\,4\right)=1,96\ \%.$$

Annex C
(informative)

Example of determination of uncertainty in calibration of a closed conduit

C.1 Introduction

C.1.1 This example describes the determination of the uncertainty in the calibration of an orifice plate with flange tappings and illustrates the calculation of the uncertainty in a measurement of flowrate obtained by using the orifice plate after calibration.

C.1.2 The calibration facility was one in which the water, after flowing through the orifice plate assembly in the test section of the circuit, normally passed into a sump, from which it was passed back to the inlet of the test section. When flow conditions were steady, the flow was diverted for a measured time interval into a weighing tank instead of into the sump.

C.1.3 During the time of diversion, the differential head across the orifice plate was measured using compressed air/water or mercury/water manometers, the procedure being repeated at 25 points covering the flowrate range over which the calibration was required. The temperature of the water in the test line and the ambient air temperature adjacent to the manometers were also noted at each point. Using a density bottle, the density of the water used relative to that of distilled water at the same temperature was obtained as 1,001 42 and this figure was used throughout the test.

C.2 Definition of symbols specific to present example

A_o	Area of orifice bore
C	Discharge coefficient
D	Pipe diameter
d	Diameter of orifice bore
g	Acceleration due to gravity
H, H'	Differential head across orifice plate
p_s	Absolute static pressure
Re_d	Reynolds number, given by $4Q/\pi \nu d$
t	Time of diversion
W	Mass of water collected during diversion procedure
β	Diameter ratio given by d/D
θ_w	Temperature of water in test section
ν	Kinematic viscosity
ρ_w	Density of pure water

C.3 Calculation of calibration coefficient

C.3.1 The mean diameter of the bore of the orifice plate was 164,34 mm and that of the upstream pipework 204,98 mm, giving a diameter ratio β of 0,801 7.

C.3.2 Test results for the 25 points are given in table C.1, the values of the flowrate being derived from the equation

$$Q = 1{,}001\,05 W / \rho_{\mathrm{w}} t \qquad \text{. . . (55)}$$

where the constant 1,001 05 is a correction factor for the effect of air buoyancy on the weighbridge reading.

$$\rho_{\mathrm{w}} = 1\,000{,}25 - 0{,}008\theta_{\mathrm{w}} - 0{,}004\,86\theta_{\mathrm{w}}^2 + \left(0{,}46 \times 10^{-6}\right) p_{\mathrm{s}}$$

and those of the calibration coefficient from

$$C = Q\left(1 - \beta^4\right)^{1/2} \Big/ A_{\mathrm{o}}\left(2gH\right)^{1/2} \qquad \text{. . . (56)}$$

C.4 Linearity of calibration graph

C.4.1 By plotting the calibration coefficients against the respective Reynolds numbers, it is immediately obvious that the relationship is curvilinear. From previous experience it is known, however, that a linear relationship may be obtained by plotting C against some function of the reciprocal of Re_{d}, and for this example $(1/Re_{\mathrm{d}})^{1/2}$ was chosen. The resulting graph is shown in figure C.1.

C.4.2 Examination of the data as described in clause 6 of this part of ISO/TR 7066 confirms the linearity of this latter plotting and suggests that the fitting of the line may be carried out using the classical least-squares procedure given in clause 8.

C.5 Uncertainty of individual calibration points

C.5.1 In accordance with the principles set out in annex A of this part of ISO/TR 7066, the percentage random uncertainty in C may be calculated from

$$e'_{\mathrm{R}}(C) = \left\{ e'^2_{\mathrm{R}}(Q) + e'^2_{\mathrm{R}}(g)/4 + e'^2_{\mathrm{R}}(H)\Big/4 + \left[2\Big/\left(1-\beta^4\right)\right]^2 e'^2_{\mathrm{R}}(d) + \left[2\beta^4\Big/\left(1-\beta^4\right)\right]^2 e'^2_{\mathrm{R}}(D) \right\}^{1/2} \qquad \text{. . . (57)}$$

C.5.2 The random uncertainty and systematic error limits in the six component quantities as determined using the principles given in ISO/TR 5168 are given in table C.2 and by substituting the former into equation (57) a percentage random uncertainty of 0,16 % was obtained. By using a similar equation with e'_{R} replaced by e'_{S} and substituting from the last column of table C.2, the systematic error limit was obtained as 0,75 %.

C.5.3 In the same way, defining

$$X = 1\Big/\left(Re_d\right)^{1/2} = \left(\pi v d/4Q\right)^{1/2} \qquad \text{. . . (58)}$$

the percentage random uncertainty in X may be found from the equation

$$e'_{\mathrm{R}}(X) = \left\{\left[e'^2_{\mathrm{R}}(v) + e'^2_{\mathrm{R}}(d) + e'^2_{\mathrm{R}}(Q)\right]\Big/4\right\}^{1/2} \qquad \text{. . . (59)}$$

Substituting the values for e'_{R} from table C.2 gives the random uncertainty in X as 0,08 %. Similarly, using the corresponding equation for $e'_{\mathrm{S}}(X)$, the systematic error limit is obtained as 0,28 %.

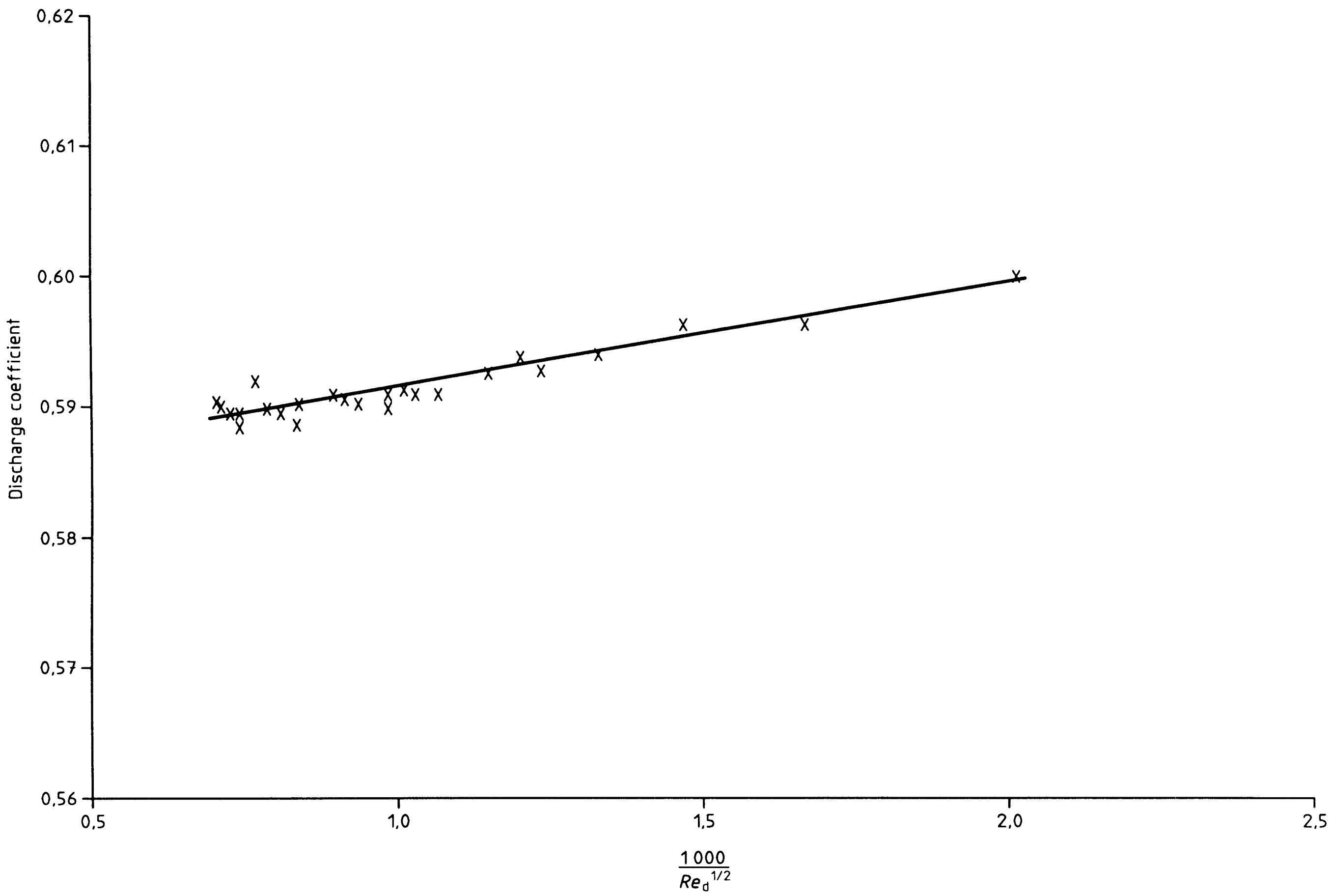


Figure C.1 — Discharge coefficient as a function of 1 000/$Re_d^{1/2}$

C.6 Fitting the best straight line

C.6.1 Using the above percentages and the figures in table C.3, the absolute random uncertainties in the mean values of the two variables become

$$e_R(1/Re_d)^{1/2} = 8{,}3 \times 10^{-7}; \quad e_R(C) = 9{,}5 \times 10^{-4}$$

from which, on substituting into equation (14)

$$\gamma = 1\ 144$$

C.6.2 The random uncertainty in $(1/Re_d)^{1/2}$ is thus negligible and the equation of the calibration curve may, therefore, be written in the form

$$C = a + b\left(10^3 / Re_d^{1/2}\right) \qquad \text{. . . (60)}$$

Table C.1 — Calibration results

Test result number	Flowrate, Q m^3/s	Discharge coefficient C (y)	Reynolds number[1)] $Re_d \times 10^{-6}$	$10^3/Re_d^{1/2}$ (x)
1	0,031 5	0,599 7	0,244 8	2,020 9
2	0,046 0	0,596 2	0,357 6	1,672 3
3	0,058 9	0,595 7	0,459 3	1,475 5
4	0,071 9	0,593 7	0,560 9	1,335 3
5	0,087 3	0,593 7	0,682 8	1,210 2
6	0,083 5	0,592 7	0,653 3	1,237 2
7	0,130 2	0,590 8	1,018 8	0,990 7
8	0,154,0	0,590 4	1,207 8	0,909 9
9	0,178 3	0,589 6	1,398 5	0,845 6
10	0,205 1	0,589 6	1,615 9	0,786 7
11	0,231 0	0,587 5	1,811 6	0,743 0
12	0,258 3	0,589 5	2,016 0	0,704 3
13	0,256 8	0,589 2	2,023 6	0,703 0
14	0,245 8	0,589 0	1,941 4	0,717 7
15	0,231 4	0,588 5	1,845 7	0,736 1
16	0,217 4	0,591 2	1,737 7	0,758 6
17	0,202 4	0,589 3	1,622 0	0,785 2
18	0,192 7	0,589 1	1,544 2	0,804 7
19	0,179 9	0,588 2	1,441 8	0,832 8
20	0,153 6	0,590 3	1,230 7	0,901 4
21	0,141 8	0,590 1	1,136 5	0,938 0
22	0,128 4	0,589 5	1,029 0	0,985 8
23	0,117 9	0,590 4	0,942 7	1,029 9
24	0,107 6	0,590 5	0,859 9	1,078 4
25	0,094 4	0,592 2	0,754 9	1,151 0

1) Based on throat diameter.

Table C.2 — Component uncertainties and error limits

Variable	Percentage random uncertainty	Percentage systematic error limit
d	0,00	0,20
D	0,00	0,20
g	negligible	negligible
H	0,10	0,05
Q	0,15	0,15
v	0,00	0,50

Table C.3 — Quantities required to calculate uncertainty in calibration graph

Quantity	Value
$\bar{x}$	1,014 17
$\bar{y}$	0,591 064
$s^2(x)$	1,087 864
$s^2(y)$	$8,101\ 213 \times 10^{-6}$
$\mathrm{Cov}(xy)$	$8,985\ 401 \times 10^{-7}$

By substituting the required sums of squares and products calculated from table C.3 into equations (16) and (17), this then gives

$$C = 0{,}582\,7 + \left(8{,}259\,7 / Re_d^{1/2}\right) \qquad \ldots (61)$$

the correlation coefficient as found from equation (18) being 0,957 0.

C.6.3 From equation (16) it will be noted that b is very small and that the coefficient of discharge changes only slowly with the Reynolds number. In view of this it would seem reasonable to enquire whether the slope of the line could be taken as zero. Using the values given in table C.3 to substitute into equations (32) and (33) gives the 95 % confidence limits for b as 0,008 259 7 ± (2,06 × 0,000 500 7), and since this interval does not include zero it must be concluded that b is non-zero.

C.6.4 On this basis, the random uncertainty in any value of C may be obtained from equation (34), the required value of s_R being first calculated from either equation (19), (20) or (21). Substituting into the latter,

$$s_R = 8{,}42 \times 10^{-4}$$

and, inserting this into equation (34)

$$e_R(\hat{C}) = 2{,}06 \times \left(8{,}42 \times 10^{-4}\right)\left\{0{,}04 + \left[\left(x_k - 1{,}014\,17\right)^2 / 2{,}610\,874\right]\right\}^{1/2} \qquad \ldots (62)$$

At $x_k = \bar{x} = 1{,}014\,17$: $\quad e_R(\hat{C}) = 3{,}469 \times 10^{-3}$;

at $x_k = 2{,}020\,9$: $\quad e_R(\hat{C}) = 1{,}134 \times 10^{-3}$;

at $x_k = 0{,}703\,0$: $\quad e_R(\hat{C}) = 4{,}8 \times 10^{-4}$.

C.6.5 Equation (62) gives the random uncertainty in the value of the calibration coefficient and this must now be combined with the systematic error limit. This latter is the same as that in any individual measured value of C, i.e.

$$e_S(C) = \left\{e_S^2(Q) + e_S^2(g)/4 + e_S^2(H)/4 + \left[2/\left(1-\beta^4\right)\right]^2 e_S^2(d) + \left[2\beta^4/\left(1-\beta^4\right)\right]^2 e_S^2(D)\right\}^{1/2} \qquad \ldots (63)$$

Substituting the values from table C.2 gives $e'_S(\hat{C}) = 0{,}75$ %.

The total uncertainty in any value of $\hat{C}$, corresponding to a 95 % coverage, is then obtained by combining, by the root-sum-square method, the random uncertainty given by equation (62) with the above systematic error limit, e.g.

At $x_k = \bar{x} = 1{,}014\,17$: $\quad U'_{RSS}(\hat{C}) = \left(0{,}035^2 + 0{,}75^2\right)^{1/2}\ \% = 0{,}75\ \%$;

at $x_k = 2{,}020\,9$: $\quad U'_{RSS}(\hat{C}) = \left(0{,}11^2 + 0{,}75^2\right)^{1/2}\ \% = 0{,}76\ \%$;

at $x_k = 0{,}703\,0$: $\quad U'_{RSS}(\hat{C}) = \left(0{,}05^2 + 0{,}75^2\right)^{1/2}\ \% = 0{,}75\ \%$.

This is the uncertainty associated with the actual value of $\hat{C}$ which shall be used, for instance, to estimate the uncertainty in standardized or tabulated values of the discharge coefficient. It may be noted that in such an example the systematic error is largely predominant.

C.6.6 Nevertheless, when the orifice plate previously calibrated as stated above is used for a flowrate measurement in conditions strictly identical to that prevailing during calibration (same fluid at the same pressure and temperature, same influence quantities, etc.), then the effect of the errors in the determination of the geometrical characteristics of the orifice plate disappears, for it may be considered that $C = kQH^{-1/2}$. The systematic error limit in $\hat{C}$ is then given by:

$$e'_{S}(\hat{C}) = \left[e_{S}^{2}(Q) + 0{,}25 e_{S}^{2}(H)\right]^{1/2} \qquad \text{. . . (64)}$$

and, on substituting the appropriate values from table C.2,

$$e'_{S}(\hat{C}) = \left[0{,}15^{2} + 0{,}25 \times 0{,}05^{2}\right]^{1/2} = 0{,}15\ \%$$

The total uncertainty in any value of $\hat{C}$ is then obtained by the same procedure as in C.6.5, e.g.

At $x = \bar{x} = 1{,}014\,17$: $U'_{RSS}(\hat{C}) = \left(0{,}035^{2} + 0{,}15^{2}\right)^{1/2}\ \% = 0{,}15\ \%$;

at $x = 2{,}020\,9$: $U'_{RSS}(\hat{C}) = \left(0{,}11^{2} + 0{,}15^{2}\right)^{1/2}\ \% = 0{,}19\ \%$;

at $x = 0{,}703\,0$: $U'_{RSS}(\hat{C}) = \left(0{,}05^{2} + 0{,}15^{2}\right)^{1/2}\ \% = 0{,}16\ \%$.

C.7 Uncertainty in a flowrate measurement using the calibrated orifice plate

C.7.1 Since the slope of the calibration graph is not zero, an additional uncertainty will be incurred in using the graph to estimate a flowrate. Bearing in mind the comments of the previous clause, only H and v will have any effect on the uncertainty in $1/(Re_{d})^{1/2}$, and equation (59) thus reduces to

$$e_{R}(X) = \left\{\left[e_{R}^{2}(v)\right]/4 + \left[e_{R}^{2}(H')\right]/8\right\}^{1/2} \qquad \text{. . . (65)}$$

with a similar expression in e_{S} for the systematic error limit.

C.7.2 The effect of the above on the value of C is dependent on the slope of the calibration graph and is given in absolute terms by

$$U_{RSS} = b\left[e_{R}^{2}(X) + e_{S}^{2}(X)\right]^{1/2} \qquad \text{. . . (66)}$$

The total uncertainty in C is then found by combining this value with the uncertainty for the calibration graph using the root-sum-square method

$$U_{RSS}(C) = \left[U_{RSS}^{2}(\hat{C}) + U_{RSS}^{2}(C_{0})\right]^{1/2} \qquad \text{. . . (67)}$$

C.7.3 In the same way, the random uncertainty in Q will be given by

$$e_{R}(Q) = 0{,}5 e_{R}(H') \qquad \text{. . . (68)}$$

and the systematic error limit by

$$e_S(Q) = \left[e_S{}^2(C) + 0{,}25 e_S{}^2(H')\right]^{1/2} \quad \text{. . . (69)}$$

C.7.4 For the present, let

$$e'_R(v) = 0{,}0\ \%; \quad e_S(v) = 1{,}0\ \%$$
$$e'_R(H') = 0{,}5\ \%; \quad e_S(H') = 1{,}0\ \%$$

Then, from equation (38) and the associated formula for the systematic error limit

$$e'_R(X) = 0{,}5/8^{1/2}\ \% = 0{,}18\ \%$$
$$e'_S(X) = (0{,}25 + 0{,}125)^{1/2}\ \% = 0{,}61\ \%$$

Thus, taking Re_d at the point where the iteration converges to be 8×10^5 (i.e. X_k = 0,001 12)

$$U_{RSS}(C_0) = 8{,}26\left[(0{,}001\ 12 \times 0{,}001\ 8)^2 + (0{,}001\ 12 \times 0{,}006\ 1)^2\right]^{1/2}$$
$$= 5{,}88 \times 10^{-5}$$

For the above value of Re_d, C = 0,591 9, giving the absolute uncertainty in the calibration curve as

$$U_{RSS}(\hat{C}) = (0{,}591\ 9 \times 0{,}001\ 6) = 9{,}470\ 4 \times 10^{-4}$$

Combining $U_{RSS}(\hat{C})$ with $U_{RSS}(C_0)$ using the root-sum-square method and dividing by the calibration coefficient gives

$$U'_{RSS}(C) = \left[(9{,}470\ 4 \times 10^{-4})^2 + (5{,}88 \times 10^{-5})^2\right]^{1/2} \Big/ 0{,}591\ 9 = 0{,}16\ \%$$

C.7.5 Finally, substituting into equations (68) and (69)

$$e'_R(Q) = (0{,}5 \times 0{,}5)\ \% = 0{,}25\ \%$$
$$e'_S(Q) = \left[0{,}16^2 + 0{,}25(1)^2\right]^{1/2}\ \% = 0{,}52\ \%$$

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