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BSI Standards Publication

Guide for the statistical analysis of ageing test data

Part 1: Methods based on mean values of normally distributed test results

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National foreword

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Guide pour l'analyse statistique de données d'essais de vieillissement – Partie 1: Méthodes basées sur les valeurs moyennes de résultats d'essais normalement distribués

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CONTENTS

INTERNATIONAL ELECTROTECHNICAL COMMISSION

GUIDE FOR THE STATISTICAL ANALYSIS OF AGEING TEST DATA –

Part 1: Methods based on mean values of normally distributed test results

FOREWORD

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International Standard IEC 60493-1 has been prepared by IEC technical committee 112: Evaluation and qualification of electrical insulating materials and systems.

This second edition cancels and replaces the first edition, published in 1974, and constitutes a technical revision.

The main changes with respect to the first edition are that, besides a complete editorial revision, censored data sub-group are considered.

The text of this standard is based on the following documents:

CDV	Report on voting
112/172/CDV	112/192/RVC

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all the parts in the IEC 60493 series, published under the general title *Guide for the statistical analysis of ageing test data,* can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

INTRODUCTION

Procedures for estimating ageing properties are described in specific test procedures, or are covered by the general documents on test procedures for ageing tests with a specific environmental stress (e.g. temperature, radiation, partial discharges).

In many cases, a certain property is determined as a function of time at different ageing stresses, and a time to failure based on a chosen end-point criterion is found at each ageing stress. A plot of time to failure versus ageing stress may be used to obtain an estimate of the time to failure for similar specimens exposed to a specified stress, or to obtain an estimate of the value of stress which will cause failure in a specified time.

The physical and chemical laws governing the ageing phenomena may often lead to the assumption that a linear relationship exists between the property examined and the ageing time at fixed ageing stresses, or between certain mathematical functions of property and ageing time, e.g. square root or logarithm. Also, there may be a linear relationship between time to failure and ageing stress, or mathematical functions of these variables.

The methods described in this part of IEC 60493 apply to such cases of linear relationship. The methods are illustrated by the example of thermal ageing wherein the case of a simple chemical process it may be assumed that the degradation obeys the Arrhenius law, i.e. the logarithm of time to failure is a linear function of the reciprocal thermodynamic temperature. Numerical examples demonstrating the use of the methods in this case are given in IEC 60216-3 [1][1].

The calculation processes specified in this standard are based on the assumption that the data under examination are normally distributed. No test for normality of the data is specified, since the available tests are unreliable for small sample groups of data. However, the methods have been used for a considerable time without undesirable results and with no check on the normality of the data distributions.

1 Figures in square brackets refer to the bibliography.

GUIDE FOR THE STATISTICAL ANALYSIS OF AGEING TEST DATA –

Part 1: Methods based on mean values of normally distributed test results

1 Scope

This part of IEC 60493 gives statistical methods which may be applied to the analysis and evaluation of the results of ageing tests.

It covers numerical methods based on mean values of normally distributed test results.

These methods are only valid under specific assumptions regarding the mathematical and physical laws obeyed by the test data. Statistical tests for the validity of some of these assumptions are also given.

This standard deals with data from both complete test sets and censored test sets.

This standard provides data treatment based on the concept of "data sub-group" as defined in Clause 3. The validity of the coefficients used in the calculation processes to derive statistical parameters of the data groups are described in [1].

2 Normative references

None.

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this document, the following terms, definitions and symbols apply.

3.1.1
ordered data
set of data arranged in sequence so that in the appropriate direction through the sequence each member is greater than or equal to its predecessor

Note 1 to entry: "Ascending order" in this standard implies that the data is ordered in this way, the first being the smallest.

3.1.2
order-statistic
each individual value in a set of ordered data is referred to as an "order-statistic" identified by its numerical position in the sequence

3.1.3
incomplete data
ordered data, where the values above and/or below defined points are not known

3.1.4
censored data
incomplete data, where the number of unknown values is known

Note 1 to entry: If the censoring is begun above/below a specified numerical value, the censoring is Type I. If it is begun above/below a specified order-statistic, it is Type II. This standard is concerned only with Type II.

3.1.5
truncated data
incomplete data where the number of unknown values is not known

Note 1 to entry: This report is not concerned with truncated data.

3.1.6
Saw coefficient
one of the coefficients developed by J.G. Saw for calculating the primary statistical functions of a single sub-group

Note 1 to entry: There are four coefficients used in this standard. Saw originally defined five, the fifth being intended for estimating the variance of the variance estimate (see [7]).

3.1.7
degrees of freedom
number of data values minus the number of parameter values

3.1.8
variance of a data group
sum of the squares of the deviations of the data from a reference level

Note 1 to entry: The reference level may be defined by one or more parameters, for example a mean value (one parameter) or a line (two parameters, slope and intercept), divided by the number of degrees of freedom.

3.1.9
central second moment of a data group
sum of the squares of the differences between the data values and the value of the group mean, divided by the number of data in the group

3.1.10
covariance of data groups
for two groups of data with equal numbers of elements where each element in one group corresponds to one in the other, the sum of the products of the deviations of the corresponding members from their group means, divided by the number of degrees of freedom

3.1.11
regression analysis
process of deducing the best-fit line expressing the relation of corresponding members of two data groups by minimizing the sum of squares of deviations of members of one of the groups from the line

Note 1 to entry: The parameters are referred to as the regression coefficients.

3.1.12
correlation coefficient
number expressing the completeness of the relation between members of two data groups, equal to the covariance divided by the square root of the product of the variances of the groups

Note 1 to entry: The value of its square is between 0 (no correlation) and 1 (complete correlation.

3.1.13
data sub-group
single set of data which may be used with other sub-groups to form a compound group

3.2 Symbols

Symbol	Definition
a, b	Regression coefficient
e	Sample (point) estimate of e
e_1	Lower confidence limit of e
e_2	Upper confidence limit of e
f	Number of degrees of freedom
$f(x)$	Probability density
$f_1(t),\ f_3\ t)$	Arbitrary function of time
$f_2(\theta)$	Arbitrary function of stress
$f_4(p)$	Arbitrary function of property
F	Fisher-distributed stochastic variable
$F(x)$	Cumulative probability distribution
i	Order No. of partial sample
j	Order No. of specimen in partial sample No. i
k	Number of partial samples in total sample
m	Order No. of specimen
n	Number of observations in sample
n_i	Number of specimens in partial sample No. i
N	Total number of specimens
p	Arbitrary property of test specimens
$P(X \leq x)$	Probability that $X \leq x$
s^2	Sample variance
s_1^2	Variance within sets
s_2^2	Variance about regression line
s_{11}^2	Partial sample variance
t	Student-distributed stochastic variable
u	Standardized normal (Gaussian) distributed stochastic variable
x	Independent variable (for example $1/\theta$)
x_i	Partial sample value of x
$\bar{x}$	Sample mean
$\bar{x}$	Weighted mean of x
$\tilde{x}$	Sample median
X	Stochastic variable, specified value of x
y	Dependent stochastic variable (for example log v)
y_{ij}	Individual specimen value of y
$\bar{y}_i$	Partial sample mean of y
$\bar{y}$	Total sample mean of y
Y	Specified value of y
α	Significance level
ε	Arbitrary population parameter
Θ	Thermodynamic temperature/Kelvin
θ	Ageing stress (for example temperature)

Symbol	Definition
ξ	Mean value of X
$\bar{\xi}$	Median value of X
σ	Standard deviation of X
σ^2	Variance of X
χ^2	Chi-square-distributed test variable
$1 - \alpha$	Confidence level

4 Calculation procedures

4.1 General considerations

For these calculations:

– n is the number of values known in subgroup;
– m is the total number in subgroup (= n for complete data group);
– α, β, μ and ε are the "Saw" coefficients for these values of m and n.

For an uncensored subgroup, the values of the "Saw" coefficients are as follows:

$$\alpha = 1/(n-1) \tag{1}$$

$$\beta = -1/(n(n-1)) \tag{2}$$

$$\mu = 1 - 1/n \tag{3}$$

$$\varepsilon = 1 \tag{4}$$

If convenient, these coefficients may be used to calculate the primary statistical functions (mean and standard deviation) of complete data groups, using the formulae of 4.2.3 (in place of the usual formulae as in 4.2.2). "Saw" coefficients are given in Table B.1.

4.2 Single sub-group – Difference of mean and specified value

4.2.1 General

The purpose of the procedure is to test the significance of the difference between the sub-group mean and a specified numerical value.

4.2.2 Complete data sub-group

Calculate sub-group mean
$$\bar{y} = \sum_{i=1}^{n} y_i / n \tag{5}$$

Calculate sub-group variance
$$\sigma^2 = \frac{\left(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right)}{(n-1)} \tag{6}$$

Calculate t $$t = \bar{y} / \sqrt{\sigma^2 / n} \tag{7}$$

Compare the value of t with the tabulated t values.

4.2.3 Censored data sub-group

Calculate sub-group mean $$\bar{y} = (1-\mu) y_n + \mu \sum_{j=1}^{n-1} \frac{y_j}{(n-1)} \tag{8}$$

Calculate sub-group variance $$\sigma^2 = \alpha \sum_{j=1}^{n-1} \left(y_n - y_j\right)^2 + \beta \left[\sum_{j=1}^{n-1} \left(y_n - y_j\right) \right]^2 \tag{9}$$

Calculate adjustment for t $$a = \frac{(1 - n/m)}{(6{,}2 + n/6{,}4 - (m-n)/10{,}7)} \tag{10}$$

Calculate t $$t = \bar{y} / \sqrt{\varepsilon \sigma^2 / n} \tag{11}$$

Calculate t_a $$1/t_a = 1/t + a \tag{12}$$

Compare the value of t_a with the tabulated t values.

4.3 Two subgroups – Difference of means

4.3.1 General

The purpose of this procedure is to test the significance of the difference between the sub-group means.

For these calculations:

- n_i is the number of values known in subgroup i;
- m_i is the total number of values in subgroup i;
- α_i β_i μ_i and ε_i are the “Saw” coefficients for these values of m and n.

For a complete sub-group, ε_i =1.

4.3.2 Both sub-groups complete

Calculate sub-group means $$\bar{y}_i = \sum_{j=1}^{n_i} y_{ij} / n_i \tag{13}$$

Calculate sub-group variances $$\sigma_i^2 = \frac{\left(\sum_{j=1}^{n_i} y_{ij}^2 - n \bar{y}_i^2 \right)}{(n_i - 1)} \tag{14}$$

Calculate the group value of ε
$$e=\left(\frac{\varepsilon_1}{n_1}+\frac{\varepsilon_2}{n_2}\right) \quad (15)$$

Calculate the merged variance σ^2
$$\sigma^2=\frac{\left((n_1-1)\sigma_1^2+(n_2-1)\sigma_2^2\right)}{(n_1+n_2-2)} \quad (16)$$

Calculate t
$$t=\frac{(\bar{y}_1-\bar{y}_2)}{\sqrt{e\sigma^2}} \quad (17)$$

Determine probability by reference to tabulated values of t.

4.3.3 One or both subgroups censored

Calculate sub-group variances
$$\sigma_i^2=\alpha_i\sum_{j=1}^{n_i-1}\left(y_{in_i}-y_{ij}\right)^2+\beta_i\left[\sum_{j=1}^{n_i-1}\left(y_{in_i}-y_{ij}\right)\right]^2 \quad (18)$$

Calculate sub-group means
$$\bar{y}_i=(1-\mu_i)y_{in_i}+\mu_i\sum_{j=1}^{n_i-1}\frac{y_{ij}}{(n_i-1)} \quad (19)$$

Calculate the group value of ε
$$e=\left(\frac{\varepsilon_1}{n_1}+\frac{\varepsilon_2}{n_2}\right) \quad (20)$$

Calculate the merged variance σ^2
$$\sigma^2=\frac{\left((n_1-1)\sigma_1^2+(n_2-1)\sigma_2^2\right)}{(n_1+n_2-2)} \quad (21)$$

Calculate t
$$t=\frac{(\bar{y}_1-\bar{y}_2)}{\sqrt{e\sigma^2}} \quad (22)$$

Calculate adjustment, a
$$a=\frac{p}{(n_1+n_2)^2}\left[\frac{n_1}{m_1}-\frac{n_2}{m_2}\right]\left[\left(\frac{(n_1+n_2)}{20}\right)^2+2\right] \quad (23)$$

where p is the smaller of n_1 and n_2.

Apply adjustment
$$t_a=\frac{1}{\left(\frac{1}{t}+a\right)} \quad (24)$$

Determine probability by reference to tabulated values of t.

4.4 Two or more subgroups – Analysis of variance

Individual sub-groups may be complete or censored.

For these calculations:

n_i	is the number of values known in subgroup i;
m_i	is the total number in subgroup i;
α_i, β_i, μ_i and ε_i	are the “Saw” coefficients for these values of m and n;
k	is the number of subgroups;
c	is the intermediate value for χ^2 calculation;
A	is the adjustment factor for χ^2 calculation.

Calculate the total number of values $$M = \sum_{i=1}^{k} m_i \tag{25}$$

Calculate the total number of values known $$N = \sum_{i=1}^{k} n_i \tag{26}$$

Calculate subgroup means:

$$\bar{y}_i = (1-\mu_i)\, y_{in_i} + \mu_i \sum_{j=1}^{n_i-1} \frac{y_{ij}}{(n_i - 1)} \quad \text{(Censored data)}$$

$$\bar{y}_i = \frac{\sum_{j=1}^{n_i} y_{ij}}{n_i} \quad \text{(Complete data subgroup)} \tag{27}$$

Calculate group general mean $$\bar{\bar{y}} = \frac{\sum_{i=1}^{k} n_i \bar{y}_i}{N} \tag{28}$$

Calculate sub-group variances:

$$s_i^2 = \alpha_i \sum_{j=1}^{n_i-1} \left(y_{in_i} - y_{ij}\right)^2 + \beta_i \left[\sum_{j=1}^{n_i-1} \left(y_{in_i} - y_{ij}\right)\right]^2$$

$$s_i^2 = \frac{\sum_{j=1}^{n_i} \left(y_{ij}^2 - n_i \bar{y}_i^2\right)}{(n_i - 1)} \quad \text{(Complete data subgroup)} \tag{29}$$

Calculate mean variance factor $$\varepsilon = \frac{\sum_{i=1}^{k} \varepsilon_i}{k} \tag{30}$$

Calculate variance of means
$$s_N^2 = \frac{\left[\sum_{i=1}^{k} n_i \bar{y}_i^2 - N\, \bar{\bar{y}}^2\right]}{(k-1)} \quad (31)$$

Calculate residual variance
$$s_D^2 = \frac{\varepsilon\left[\sum_{i=1}^{k} s_i^2 (n_i - 1)\right]}{(N-k)} \quad (32)$$

Test equality of subgroup variances:

Calculate c
$$c = 1 + \frac{\left[\sum_{i=1}^{k} \frac{1}{(n_i - 1)} - \frac{1}{(N-k)}\right]}{3(k-1)} \quad (33)$$

Calculate adjustment factor
$$A = 1 + \frac{\left(1 - \frac{N}{M}\right) \times \left(1 - \frac{12}{M}\right)}{2} \quad (34)$$

Calculate χ^2
$$\chi^2 = \frac{A}{c}\left[(N-k)\ln\left(\frac{s_D^2}{\varepsilon}\right) - \sum_{i=1}^{k} (n_i - 1)\ln\left(s_i^2\right)\right] \quad (35)$$

Test equality of residual variance and variance of subgroup means.

Calculate F
$$F = \frac{s_N^2}{s_D^2} \quad (36)$$

Degrees of freedom for F N - k (denominator), k -*1* (numerator)

Calculate significance of general mean:

Calculate t
$$t = \bar{\bar{y}}\sqrt{\frac{N}{s_T^2}} \quad (37)$$

Adjust t for censoring
$$1/t_a = 1/t + a \quad (38)$$

Determine probability by reference to tabulated values of t with N-1 degrees of freedom.

4.5 Three or more subgroups – Regression analysis

4.5.1 Regression analysis – General considerations

These data differ from those of (4.4) in that the y-values in each subgroup are associated with a value of another variable, referred to in this section as x_i. The objective of the analysis is to determine whether there is a linear relationship between x and y and, if so, its parameters and properties.

The parameters and properties in question are as follows:

– slope(b) and intercept (a) of regression line;
– equality of variance of subgroups (χ^2);
– linearity of regression (F);
– confidence intervals of regression estimates.

For these calculations:

n_i	is the number of values known in subgroup i;
m_i	is the total number in subgroup i;
α_i, β_i, μ_i and ε_i	are the “Saw” coefficients for these values of m and n;
k	is the number of subgroups;
c	is the intermediate value for χ^2 calculation;
A	is the adjustment factor for χ^2 calculation;
b and a	are the slope and intercept of the regression line;
$t_{p,n\text{-}1}$	is the tabulated value of t for probability p and n-1 degrees of freedom.

Sub-groups may be either complete or censored. Values of y_{ij} are the actual values of variable j in subgroup i.

4.5.2 Calculations

Calculate the total number of values
$$M = \sum_{i=1}^{k} m_i \tag{39}$$

Calculate the total number of values known
$$N = \sum_{i=1}^{k} n_i \tag{40}$$

Calculate subgroup means:

$$\bar{y}_i = (1 - \mu_i) y_{in_i} + \mu_i \sum_{j=1}^{n_i - 1} \frac{y_{ij}}{(n_i - 1)} \quad \text{(Censored data)}$$
$$\bar{y}_i = \frac{\sum_{j=1}^{n_i} y_{ij}}{n_i} \quad \text{(Complete data subgroup)} \tag{41}$$

Calculate sub-group variances:

$$s_i^2 = \alpha_i \sum_{j=1}^{n_i - 1} \left(y_{in_i} - y_{ij} \right)^2 + \beta_i \left[\sum_{j=1}^{n_i - 1} \left(y_{in_i} - y_{ij} \right) \right]^2 \quad \text{(Censored data)}$$
$$s_i^2 = \frac{\sum_{j=1}^{n_i} \left(y_{ij}^2 - n_i \bar{y}_i^2 \right)}{(n_i - 1)} \quad \text{(Complete data subgroup)} \tag{42}$$

Calculate x-mean $$\overline{x}=\frac{\sum_{i=1}^{k}n_i x_i}{N}$$ (43)

Calculate group general mean $$\overline{\overline{y}}=\frac{\sum_{i=1}^{k}n_i \overline{y}_i}{N}$$ (44)

Calculate mean variance factor $$\varepsilon=\frac{\sum_{i=1}^{k}\varepsilon_i}{k}$$ (45)

Calculate residual variance $$s_D^2=\frac{\varepsilon\left[\sum_{i=1}^{k}s_i^2(n_i-1)\right]}{(N-k)}$$ (46)

Calculate y-sum of squares $$SSy=\sum_{i=1}^{k}n_i\overline{y}_i^2-N\,\overline{\overline{y}}^2$$ (47)

Calculate x-sum of squares $$SSx=\sum_{i=1}^{k}n_i x_i^2-N\,\overline{x}^2$$ (48)

Calculate xy-sum of products $$SPxy=\sum_{i=1}^{k}n_i x_i\overline{y}_i-N\overline{x}\overline{y}$$ (49)

Calculate slope b $$b=\frac{SPxy}{SSx}$$ (50)

Calculate intercept a $$a=\overline{y}-b\overline{x}$$ (51)

Calculate r^2 $$r^2=\left(\frac{SPxy}{SSx}\right)\left(\frac{SPxy}{SSy}\right)$$ (52)

(r is the correlation coefficient)

Calculate non-linearity variance $$s_N^2=\frac{(1-r^2)SSy}{(k-2)}$$ (53)

4.5.3 Test equality of subgroup variances

Calculate c $$c=1+\frac{\left[\sum_{i=1}^{k}\frac{1}{(n_i-1)}-\frac{1}{(N-k)}\right]}{3(k-1)}$$ (54)

Calculate adjustment factor $$A = 1 + \frac{\left(1 - \frac{N}{M}\right) \times \left(1 - \frac{12}{M}\right)}{2}$$ (55)

Calculate χ^2 $$\chi^2 = \frac{A}{c}\left[(N-k)\ln\left(\frac{s_D^2}{\varepsilon}\right) - \sum_{i=1}^{k}(n_i - 1)\ln\left(s_i^2\right)\right]$$ (56)

4.5.4 Test significance of deviations from linearity

Calculate F $$F = \frac{s_N^2}{s_D^2}$$ (57)

Degrees of freedom for F N-k (denominator), k-2 (numerator)

4.5.5 Estimate and confidence limit of y

Central second moment of x $$\mu_{2(x)} = \frac{SSx}{N}$$ (58)

Non regression variance $$s_T^2 = \frac{\left((N-k)s_D^2 + (k-2)s_N^2\right)}{(N-2)}$$ (59)

Adjust for extrapolation $$s_C^2 = \frac{s_T^2}{N}\left(1 + \frac{(X - \overline{x})^2}{\mu_{2(x)}}\right)$$ (60)

Correction to t for censoring $$a = \frac{\left(1 - \frac{N}{M}\right)}{\left(6{,}2 + \frac{N}{6{,}4} - \frac{(M-N)}{10{,}7}\right)}$$ (61)

Correction applied $$t_c = \left(\frac{1}{t_{p,N-2}} - a\right)$$ (62)

Estimate of y $$\hat{y} = a + b\,X$$ (63)

Confidence limit of estimate of y $$\hat{y}_c = \hat{y} + t_c\sqrt{s_c^2}$$ (64)

4.5.6 Estimate and confidence limit of x

Estimate $$\hat{x} = \frac{(y-a)}{b}$$ (65)

For simplicity, calculate several temporary variables:

$$b_r = b - \frac{t_c^2\, s_T^2}{N\, b\, \mu_{2,(x)}} \tag{66}$$

$$s_r^2 = \frac{s_T^2}{N}\left(\frac{b_r}{b} + \frac{(\hat{x} - \overline{x})^2}{\mu_{2,(x)}}\right) \tag{67}$$

Confidence limit of estimate of x

$$\hat{x}_c = \overline{x} + \frac{(y - \overline{y})}{b_r} + \frac{t_c\sqrt{s_r^2}}{b_r} \tag{68}$$

Annex A
(informative)

Statistical background

A.1 Statistical distributions and parameters

The distribution of a stochastic variable X is described by the distribution function:

$$F(x) = P\left(X \leq x\right) \tag{A.1}$$

where $P(X \leq x)$ is the probability that the value of X is $\leq x$. Here $F(x)$ goes from 0 to 1 and is a never-decreasing function of x. If $F(x)$ is a continuous function of x, then the probability density is determined as:

$$f(x) = \frac{dF(x)}{dx} \tag{A.2}$$

The distribution may be characterized by parameters, of which the most important are:

– the mean value:

$$\xi = \int_{-\infty}^{\infty} x\, f(x) dx \tag{A.3}$$

– the variance:

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\xi)^2 f(x) dx \tag{A.4}$$

The square root of the variance is termed the standard deviation σ.

A.2 Estimates of parameters

From a sample of n stochastic independent specimens from a population, estimates of the parameters of the population (see Clause A.1) may be derived.

An estimate of the mean value of the population (Formula (A.3)) is calculated as the average of the individual sample values:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{A.5}$$

where

x_i represents the individual sample values (i = 1, 2, ... n).

An estimate of the variance of the population (Formula (A.4)) is the sample variance:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{(n-1)} = \frac{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}}{n-1} = \frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n(n-1)} \qquad \text{(A.6)}$$

where $n - 1 = f$ is called the number of degrees of freedom of s^2.

A.3 Distribution types

A.3.1 General

The following distribution types are relevant to this application, the t, F, and χ^2 distributions being the distributions of secondary functions derived from the mean and variance parameter estimates of normally distributed data.

A.3.2 The normal distribution

The calculation processes specified in this standard are based on the assumption that the data under examination are normally distributed. No test for normality of the data is specified, since the available tests are unreliable for small sample groups of data. However, the methods have been used for a considerable time without undesirable results and with no check on the normality of the data distributions.

The normal (Gaussian) distribution is defined by:

$$f(x) = \frac{\exp\left\{-(x-\xi)^2/2\sigma^2\right\}}{\sqrt{2\pi\sigma^2}} \qquad \text{(A.7)}$$

and is completely characterized by its mean value ξ and variance σ^2.

The standardized normal distribution:

$$f(u) = \frac{\exp -\frac{u^2}{2}}{\sqrt{2\pi}} \qquad \text{(A.8)}$$

where

$$u = \frac{x-\xi}{\sigma} \qquad \text{(A.9)}$$

and the corresponding distribution function $F(u)$ have been tabulated and computer routines for their calculation are available (see [1]).

The above use of F should not be confused with the F distribution below.

The mean value $\bar{x}$ of a sample of n specimens from a normal distribution is itself a normally distributed stochastic variable with mean value $\xi_{\bar{x}} = \xi$ and variance $\sigma_{\bar{x}}^2 = \sigma^2/n$ and the corresponding standardized variable is:

$$u = \frac{x - \xi}{\frac{\sigma}{\sqrt{n}}} \quad \text{(A.10)}$$

A.3.3 The *t* distribution

If the true variance of the normal distribution σ^2 is not known, the sample estimate s^2 from Formula (A.6) may be substituted and the standardized sample mean value becomes:

$$u = \frac{x - \xi}{\frac{s}{\sqrt{n}}} \quad \text{(A.11)}$$

The distribution of this variable is called the t distribution (or Student's t) and depends on the parameter $f = n - 1$ (the number of degrees of freedom for s^2). The t distribution has been tabulated for different values of f. It is derived from the “Incomplete Beta function.

A.3.4 The *F* distribution

To test if two sample variances, determined from two different samples, may reasonably be considered to be estimates of the same theoretical variance (population parameter), the following test variable is calculated:

$$F = \frac{s_1^2}{s_2^2} \quad \text{(A.12)}$$

The distribution of this variable is called the F distribution (or Fisher) and depends on the parameters $f_1 = n_1 - 1$ and $f_2 = n_2 - 1$ (the number of degrees of freedom for s_1^2 and s_2^2). The F distribution has been tabulated for different values of f_1 and f_2. It is derived from the “Incomplete Beta function”

A.3.5 The χ^2 distribution

To test if several sample variances, each determined from a different sample, may reasonably be considered to be estimates of the same theoretical variance, the following test variable is calculated (Bartlett’s χ^2).

$$\chi^2 = \frac{2,3\left(f \lg s^2 - \sum_{i=1}^{k} f_i \lg s_i^2 \right)}{c} \quad \text{(A.13)}$$

where

$$c = 1 + \frac{\left[\sum_{i=1}^{k} \frac{1}{f_i} \right] - \frac{1}{f}}{3(k-1)} \quad \text{(A.14)}$$

k is the number of variances, s_i^2 the individual sample variance (i = 1, 2, ... k) with f_1 degrees of freedom, and $s^2 = \frac{\sum f_i s_i^2}{\sum f_i}$ is a pooled variance with $f = \sum f_i$ degrees of freedom. The test hypothesis is that all k variances s_i^2 are estimates of the same theoretical variance σ^2.

The calculated value χ^2 is compared with the tabulated value χ^2 (1 – α, k – 1) which is a function of k – 1, the number of degrees of freedom for χ^2 and of α, the significance level. If $\chi^2 > \chi^2$ (1 – α, k – 1), the hypothesis is rejected on significance level α.

The distribution of this variable is called the Bartlett's χ^2 distribution and depends on the parameter $f = k - 1$. The χ^2 distribution has been tabulated for different values of f . It is derived from the "Incomplete Gamma function".

Bartlett's test is an approximate test, but a good approximation if the number of degrees of freedom f_i of all the individual sample variances s_i^2 is greater than 2.

If the hypothesis is accepted, s^2 is taken as a pooled estimate of the common variance with f degrees of freedom.

Annex B
(informative)

Statistical tables

B.1 Use of the tables

Statistical tables of cumulative distribution functions $F\ (x)$ of a stochastic variable X are usually arranged in such a way that they give that value of x which, for a specified probability, P, satisfies the condition:

$$F\ (x,\ \delta) = P\ (X \leq x)$$

where δ represents possible parameters, which cannot be taken care of by standardization of the variable For instance, in the case of χ^2 distribution, Table B.5 gives for P = 0,95 and f = 6 a value of χ^2 = 12,6. This means that when f =6, the probability of getting a value of χ^2 equal to or less than $\chi^2(P, f)$ = χ^2(0,95, 6) = 12.6, is 95 %, or:

$$P\ (\chi^2 \leq 12{,}6) = 0{,}95 \qquad f = 6$$

Expressed in another way, P = 95 % of the χ^2 distribution lies below 12,6, and $\alpha = 1 - P$ = 5 % above this value when f = 6. α may be considered as a significance level, for example, if by hypothesis testing we use the interval $12{,}6 < \chi^2 < +\infty$ as reject interval, the risk of making a false decision by rejecting the hypothesis although true is 5 %. In some cases, α is used as entrance to the tables instead of P, for example where in Table B.5 for 6 degrees of freedom and a probability of 0,05, a value of χ^2 = 12,6 means that the probability of χ^2 being greater than12,6 is 5 %:

$$P(\chi^2 > 12{,}6) = 0{,}05 \qquad f = 6$$

Table B.1 – Coefficients for censored data calculations

m	n	α	β	μ	ε
5	3	614,4705061728	–100,3801985597	0,0000000000	860,4482888889
5	4	369,3153100012	–70,6712934899	472,4937150842	874,0745894447
6	4	395,4142139605	–58,2701183523	222,6915218468	835,7650306465
6	5	272,5287238052	–44,0988850936	573,5126123815	887,1066681426
7	4	415,5880351563	–46,5401552734	0,0000000000	841,7746734375
7	5	289,1914470089	–38,0060438107	364,2642153815	837,3681267819
7	6	215,5146796875	–30,1363662109	642,2345606152	898,7994404297
8	5	302,2559543304	–32,0455510095	173,7451925589	823,1325022970
8	6	227,1320334900	–26,7149242720	462,3946896558	845,5891673417
8	7	178,0192047851	–21,8909055649	692,0082911498	908,7175231765
9	5	312,9812000000	–26,3842700000	0,0000000000	830,5022000000
9	6	236,3858000000	–23,2986100000	296,0526300000	821,3172600000
9	7	186,6401000000	–19,7898900000	534,4601800000	855,2096700000
9	8	151,5120000000	–16,6140800000	729,7119900000	917,0583200000
10	6	244,1191560890	–20,0047740729	142,3739002847	815,8210886826
10	7	193,6205880047	–17,6663604814	386,9526017618	825,7590437753
10	8	158,2300608320	–15,2437931582	589,6341322307	864,6219294884
10	9	131,8030382363	–13,0347627976	759,2533663842	924,0989192531
11	6	250,6859320988	–16,8530354295	0,0000000000	822,9729127315
11	7	199,4695468487	–15,5836545374	249,2599953079	812,6308986254
11	8	163,6996121337	–13,8371182557	457,2090965743	832,5488161799
11	9	137,2299243827	–12,1001907793	633,2292924678	873,3355410880
11	10	116,5913210464	–10,4969569718	783,0177949444	930,0880372994
12	7	204,5349924229	–13,5767110244	120,5748554921	810,9803051840
12	8	168,3292196600	–12,4439880795	332,5519557674	814,7269021330
12	9	141,6425229674	–11,1219466676	513,1493415383	840,0625045817
12	10	121,0884792448	–9,8359507754	668,5392651269	881,2400322962
12	11	104,5060800375	–8,6333795848	802,5441292356	935,2282230049
13	7	208,9406118284	–11,6456142827	0,0000000000	817,5921863390
13	8	172,3464251400	–11,0865264201	215,2023355151	807,2699422973
13	9	145,4178687827	–10,1472348992	399,3236520338	819,3180095090
13	10	124,7371924225	–9,1300085328	558,7461589055	847,5908596926
13	11	108,3018058633	–8,1510819663	697,7158560873	888,3591181189
13	12	94,6796149706	–7,2252117874	818,8697028778	939,6794196639

NOTE α, β, μ and ε are all in units of 1×10^{-3}.

Table B.1 *(continued)*

m	*n*	α	β	μ	ε
14	8	175,9018422090	–9,7746826098	104,5543516980	807,5106793327
14	9	148,7066543210	–9,1891433745	291,5140765844	807,9273940741
14	10	127,8816896780	–8,4224506929	454,0609002065	825,0398828063
14	11	111,3817699729	–7,6266971302	596,6235832604	854,8238304463
14	12	97,9278246914	–6,8636059259	722,2249188477	894,7614153086
14	13	86,5363075231	–6,1355268822	832,7192524487	943,5668941976
15	8	179,0513405762	–8,5071530762	0,0000000000	813,5568182129
15	9	151,6274451540	–8,2566923172	189,3157319524	803,6572346196
15	10	130,6387362674	–7,7228786289	354,3906973785	810,9441335713
15	11	114,0457797966	–7,0973951863	499,7526628800	831,1920110198
15	12	100,5718881836	–6,4648224487	628,5859288205	861,6352648315
15	13	89,3466123861	–5,8578554309	743,0997382709	900,5262051665
15	14	79,6796956870	–5,2751393667	844,6143938637	946,9889014846
16	9	154,2518689085	–7,3527348129	92,2865976624	804,8901545650
16	10	133,0926552303	–7,0374903483	259,4703005026	803,4179489468
16	11	116,3971900144	–6,5718807983	407,1074446942	815,2259119510
16	12	102,8620227960	–6,0590262781	538,4703518878	837,4056164917
16	13	91,6475110414	–5,5485234808	655,9153003723	867,9864133589
16	14	82,1334839298	–5,0573990501	761,0897304685	905,7302132374
16	15	73,8281218530	–4,5839766095	854,9400915790	950,0229759376
17	9	156,6104758421	–6,4764602745	0,0000000000	810,4190113397
17	10	135,3069770991	–6,3698625234	168,9795641122	801,0660748802
17	11	118,4974933487	–6,0543187349	318,5208867246	805,3180627394
17	12	104,8944939376	–5,6546733211	451,9486020413	820,1513691949
17	13	93,6414079430	–5,2310447166	571,6961830632	843,4861778660
17	14	84,1578079201	–4,8133017972	679,5480456810	873,8803351313
17	15	75,9876912684	–4,4100612544	776,7517032846	910,4428918550
17	16	68,7761850391	–4,0203992390	863,9866274899	952,7308021373
18	10	137,3196901001	–5,7208401228	82,5925913725	802,8356541137
18	11	120,3965503416	–5,5477052124	233,7625216775	800,2584198483
18	12	106,7179571420	–5,2548692706	368,9237739923	808,4878348626
18	13	95,4179152353	–4,9135219393	490,5582072725	825,3579958906
18	14	85,9129822797	–4,5606570913	600,5193900565	849,3339891000
18	15	77,7846697341	–4,2145025451	700,1840825530	879,3395044075
18	16	70,6902823246	–3,8792292982	790,5080136386	914,7252389325
18	17	64,3706903919	–3,5548196830	871,9769987244	955,1618993620

NOTE α, β, μ and ε are all in units of 1×10^{-3}.

Table B.1 *(continued)*

m	*n*	α	β	μ	ε
19	10	139,1496250000	–5,0900181250	0,0000000000	807,9096187500
19	11	122,1302375000	–5,0534809375	152,5838471875	799,1198428125
19	12	108,3704562500	–4,8618459375	289,2318165625	801,3602625000
19	13	97,0188250000	–4,5986040625	412,4553559375	812,3967434375
19	14	87,4809000000	–4,3069634375	524,1508350000	830,6312000000
19	15	79,3443750000	–4,0105056250	625,7586734375	854,9021531250
19	16	72,2973312500	–3,7204237500	718,3571609375	884,3945350000
19	17	66,0780873071	–3,4385965290	802,6848402810	918,6300659873
19	18	60,4951234568	–3,1657522324	879,0853247478	957,3563882895
20	11	123,7207246907	–4,5719038494	74,7399526898	801,1790116264
20	12	109,8822471135	–4,4770355488	212,6836623662	797,9482811738
20	13	98,4738232381	–4,2879392332	337,2732389272	803,6777212196
20	14	88,8993849835	–4,0546864523	450,4338248217	816,7130862373
20	15	80,7401190433	–3,8047814139	553,6438253890	835,8437320329
20	16	73,6945982033	–3,5536092812	648,0414354618	860,1735387686
20	17	67,5243573136	–3,3080573368	734,4814490502	889,0784510048
20	18	62,0270511202	–3,0688692068	813,5380807649	922,2028072690
20	19	57,0593311634	–2,8372923418	885,4495276379	959,3470694381
21	11	125,1805042688	–4,1027870814	0,0000000000	805,8572211871
21	12	111,2748584476	–4,1010407267	139,0856144175	797,6054376202
21	13	99,8073278954	–3,9827324033	264,8685742314	798,4725915308
21	14	90,1927034195	–3,8051093799	379,2915229528	806,7637854827
21	15	82,0068958400	–3,5996961022	483,8588877922	821,2259618217
21	16	74,9465754505	–3,3846323534	579,7432762887	840,9212051713
21	17	68,7848146833	–3,1701150195	667,8566522885	865,1477737296
21	18	63,3357410645	–2,9603773312	748,8832650493	893,4238105389
21	19	58,4412075437	–2,7556394531	823,2713052490	925,4824714209
21	20	53,9924872844	–2,5574642897	891,1802616762	961,1609917803
22	12	112,5622493763	–3,7339426543	68,2498992309	799,8134564378
22	13	101,0383585659	–3,6836764565	195,0883064772	796,2007530338
22	14	91,3804604560	–3,5591868547	310,6161204268	800,1345064303
22	15	83,1655367136	–3,3963282816	416,3557572996	810,3242215503
22	16	76,0857406653	–3,2157585729	513,5029910799	825,8000270835
22	17	69,9154470902	–3,0297597429	603,0017042238	845,8218001615
22	18	64,4795955687	–2,8451649972	685,5911008461	869,8337436326
22	19	59,6311106506	–2,6645660073	761,8232057644	897,4613278831
22	20	55,2451821096	–2,4879744683	832,0484727783	928,5025656429
22	21	51,2381885483	–2,3171119644	896,3673255616	962,8206447076

NOTE α, β, μ and ε are all in units of 1×10^{-3}.

Table B.1 *(continued)*

m	*n*	α	β	μ	ε
23	12	113,7531148245	–3,3756614624	0,0000000000	804,1474989583
23	13	102,1805929155	–3,3910352539	127,7797799222	796,3938026565
23	14	92,4787143782	–3,3175684539	244,2868537307	796,3022860399
23	15	84,2320650394	–3,1954304107	351,0543166209	802,5583945106
23	16	77,1306716018	–3,0479567336	449,2947092156	814,1621969549
23	17	70,9462283196	–2,8891287395	539,9727159165	830,3483079467
23	18	65,5067730517	–2,7274270713	623,8577387905	850,5239901982
23	19	60,6745439037	–2,5674672567	701,5547596051	874,2452031077
23	20	56,3317476641	–2,4108249757	773,5119026250	901,2192903703
23	21	52,3789712444	–2,2574588071	840,0031107829	931,2919267259
23	22	48,7509673306	–2,1091382204	901,0843478372	964,3448710375
24	13	103,2433478819	–3,1048361929	62,7962963934	798,6676773352
24	14	93,4998991613	–3,0805979769	180,1796657031	794,8416535458
24	15	85,2198224000	–2,9975602432	287,8595445984	797,4563606400
24	16	78,0948480411	–2,8818367882	387,0601172274	805,4868099248
24	17	71,8941228860	–2,7491283441	478,7611920828	818,1445544127
24	18	66,4447043048	–2,6091448018	563,7511952496	834,8153636945
24	19	61,6126915633	–2,4678299408	642,6640805184	855,0189514195
24	20	57,2879088000	–2,3283191552	715,9989838080	878,3981891840
24	21	53,3750541943	–2,1915606657	784,1214493452	904,7233952180
24	22	49,7942298597	–2,0575307047	847,2450550466	933,8754408471
24	23	46,4937670005	–1,9279731656	905,3922645488	965,7495722974
25	13	104,2328856132	–2,8250501511	0,0000000000	802,7013015441
25	14	94,4531438920	–2,8483968323	118,1726878830	795,4024387937
25	15	86,1396570015	–2,8030714582	226,6706783537	794,6305407379
25	16	78,9893820242	–2,7178609400	326,7248166681	799,3466482243
25	17	72,7708231743	–2,6102752025	419,3261414998	808,7406844958
25	18	67,3090611675	–2,4911808278	505,2766660081	822,1806443986
25	19	62,4704476832	–2,3674613613	585,2280935412	839,1665981029
25	20	58,1487801571	–2,2433309433	659,7075915449	859,3057874761
25	21	54,2547721412	–2,1209279325	729,1297472481	882,3094990236
25	22	50,7106344695	–2,0008151849	793,7938286936	907,9968030989
25	23	47,4515824664	–1,8830136520	853,8654746859	936,2746548624
25	24	44,4360844355	–1,7691959649	909,3419372255	967,0482582508
26	14	95,3449516529	–2,6209763465	58,1493461856	797,6921057014
26	15	87,0000110601	–2,6121400166	167,3864953313	793,7600455859
26	16	79,8235563470	–2,5563562391	268,2070524346	795,3879797465
26	17	73,5857124933	–2,4729505699	361,6097876691	801,7490889242

NOTE α, β, μ and ε are all in units of 1×10^{-3}.

Table B.1 *(continued)*

m	*n*	α	β	μ	ε
26	18	68,1102550823	–2,3739926858	448,4070425448	812,1940558767
26	19	63,2623426172	–2,2671844225	529,2628483474	826,2086884902
26	20	58,9367927789	–2,1574866751	604,7212241224	843,3813295696
26	21	55,0480429364	–2,0479145245	675,2239918901	863,3884840036
26	22	51,5229352216	–1,9399299519	741,1174467776	885,9958055677
26	23	48,2974664808	–1,8338615040	802,6472197534	911,0602971929
26	24	45,3186434134	–1,7297802734	859,9406706514	938,5082900934
26	25	42,5525832097	–1,6292615541	912,9761491712	968,2524787108
27	14	96,1799524157	–2,3983307950	0,0000000000	801,4620973787
27	15	87,8072339510	–2,4248248792	109,9084470023	794,5762402919
27	16	80,6049085854	–2,3975186913	211,4227629241	793,3158799216
27	17	74,3466955590	–2,3374412078	305,5459346098	796,8463988729
27	18	68,8563635730	–2,2578996096	393,0980364707	804,5028770565
27	19	63,9978278861	–2,1674187844	474,7524530084	815,7576173705
27	20	59,6654080049	–2,0715671753	551,0645680133	830,1867939907
27	21	55,7749666285	–1,9739678436	622,4924148013	847,4482192120
27	22	52,2566505091	–1,8767936096	689,4087818496	867,2739476397
27	23	49,0499538933	–1,7810451416	752,1042681971	889,4731593849
27	24	46,1018252034	–1,6869108574	810,7807829691	913,9324867793
27	25	43,3685376227	–1,5945075052	865,5349833899	940,5926719763
27	26	40,8220442466	–1,5053002920	916,3311456462	969,3721656679
28	15	88,5660259125	–2,2411328182	54,1424788011	796,8511527567
28	16	81,3393701057	–2,2414413970	156,2886460959	792,8824325275
28	17	75,0603739817	–2,2039406070	251,0647971307	793,7610598373
28	18	69,5541501973	–2,1431471452	339,2962419039	798,8109866022
28	19	64,6839991218	–2,0684387560	421,6636413417	807,4913705645
28	20	60,3433137634	–1,9859661249	498,7308586043	819,3677667693
28	21	56,4479788544	–1,8998226295	570,9665830432	834,0869174986
28	22	52,9297209889	–1,8126830339	638,7593370987	851,3622108001
28	23	49,7308667081	–1,7261222115	702,4254764256	870,9707362940
28	24	46,8009654263	–1,6408249821	762,2097935366	892,7567254884
28	25	44,0957340937	–1,5568981499	818,2783352524	916,6300223837
28	26	41,5787804887	–1,4744958266	870,7030442470	942,5420886878
28	27	39,2265620349	–1,3949691273	919,4378349773	970,4159065169
29	15	89,2798839506	–2,0610683539	0,0000000000	800,3884370370
29	16	82,0315098349	–2,0881547819	102,7240365303	793,8770365482
29	17	75,7321296520	–2,0725597540	198,0959608027	792,2635822278

NOTE α, β, μ and ε are all in units of 1×10^{-3}.

Table B.1 *(continued)*

m	*n*	α	β	μ	ε
29	18	70,2093506173	–2,0299142798	286,9437737860	794,8680140741
29	19	65,3267056512	–1,9704535803	369,9535738121	801,1382490326
29	20	60,9768578172	–1,9009261993	447,6958995522	810,6292670577
29	21	57,0751222222	–1,8258524691	520,6472033745	822,9791690123
29	22	53,5535949791	–1,7482834402	589,2061520394	837,8907417215
29	23	50,3561788346	–1,6702113813	653,7044516931	855,1224584510
29	24	47,4347950617	–1,5927829630	714,4118941152	874,4882360494
29	25	44,7470712190	–1,5164662295	771,5353211803	895,8606629483
29	26	42,2557943767	–1,4413224717	825,2112044901	919,1678051803
29	27	39,9304194120	–1,3675341081	875,4915371353	944,3690905327
29	28	37,7509219731	–1,2963396833	922,3227345445	971,3911639175
29	26	42,2557943767	–1,4413224717	825,2112044901	919,1678051803
29	27	39,9304194120	–1,3675341081	875,4915371353	944,3690905327
29	28	37,7509219731	–1,2963396833	922,3227345445	971,3911639175
30	16	82,6848208854	–1,9376644008	50,6520145730	796,1185722795
30	17	76,3662564658	–1,9433496245	146,5702475018	792,1584763197
30	18	70,8267629538	–1,9183170174	235,9811143595	792,4611315875
30	19	65,9309300986	–1,8736230328	319,5743135792	796,4671299289
30	20	61,5711678648	–1,8166270961	397,9256281462	803,7214545789
30	21	57,6621596530	–1,7522778385	471,5175920251	813,8538227737
30	22	54,1357421601	–1,6839506825	540,7560773794	826,5596155283
30	23	50,9363946094	–1,6139463118	605,9825649718	841,5868189642
30	24	48,0175200857	–1,5437595603	667,4818601282	858,7309102420
30	25	45,3387017070	–1,4742282493	725,4850166532	877,8361298288
30	26	42,8641163652	–1,4056715102	780,1672310812	898,7980905027
30	27	40,5622887708	–1,3381271218	831,6404696499	921,5591821698
30	28	38,4073685313	–1,2717973971	879,9405903814	946,0847402411
30	29	36,3821129993	–1,2078131518	925,0087226562	972,3044539924
31	16	83,3019925385	–1,7899787388	0,0000000000	799,4492403168
31	17	76,9661360877	–1,8163270487	96,4208836722	793,2775680743
31	18	71,4103278105	–1,8084196718	186,3489334956	791,4083191149
31	19	66,5009117575	–1,7780597559	270,4755669886	793,2803915678
31	20	62,1305948377	–1,7332089792	349,3804542382	798,4306924738
31	21	58,2136500316	–1,6792554773	423,5510178697	806,4805828440
31	22	54,6814518197	–1,6198892883	493,3986895403	817,1187339274
31	23	51,4784579361	–1,5576656387	559,2717351874	830,0864157799
31	24	48,5587515564	–1,4943363914	621,4644612582	845,1685682487
31	25	45,8832580326	–1,4310299785	680,2226141518	862,1913335039

NOTE α, β, μ and ε are all in units of 1×10^{-3}.

Table B.1 *(continued)*

m	*n*	α	β	μ	ε
31	26	43,4177502831	–1,3683601397	735,7447851016	881,0240582666
31	27	41,1317569496	–1,3065437907	788,1796327268	901,5811029053
31	28	38,9984874307	–1,2456083418	837,6187354827	923,8161235884
31	29	36,9958879027	–1,1857687888	884,0848862383	947,6988227000
31	30	35,1089424384	–1,1280548995	927,5156412107	973,1614917466
NOTE α, β, μ and ε are all in units of 1×10^{-3}.					

Table B.2 – Fractiles of the *F*-distribution, $F_{0,95}$

f	f_n								
f_d	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	4,965	4,103	3,708	3,478	3,326	3,217	3,135	3,072	3,02
11	4,844	3,982	3,587	3,357	3,204	3,095	3,012	2,948	2,896
12	4,747	3,885	3,49	3,259	3,106	2,996	2,913	2,849	2,796
13	4,667	3,806	3,411	3,179	3,025	2,915	2,832	2,767	2,714
14	4,6	3,739	3,344	3,112	2,958	2,848	2,764	2,699	2,646
15	4,543	3,682	3,287	3,056	2,901	2,79	2,707	2,641	2,588
16	4,494	3,634	3,239	3,007	2,852	2,741	2,657	2,591	2,538
17	4,451	3,592	3,197	2,965	2,81	2,699	2,614	2,548	2,494
18	4,414	3,555	3,16	2,928	2,773	2,661	2,577	2,51	2,456
19	4,381	3,522	3,127	2,895	2,74	2,628	2,544	2,477	2,423
20	4,351	3,493	3,098	2,866	2,711	2,599	2,514	2,447	2,393
25	4,242	3,385	2,991	2,759	2,603	2,49	2,405	2,337	2,282
30	4,171	3,316	2,922	2,69	2,534	2,421	2,334	2,266	2,211
40	4,085	3,232	2,839	2,606	2,449	2,336	2,249	2,18	2,124
50	4,034	3,183	2,79	2,557	2,4	2,286	2,199	2,13	2,073
100	3,936	3,087	2,696	2,463	2,305	2,191	2,103	2,032	1,975
500	3,86	3,014	2,623	2,39	2,232	2,117	2,028	1,957	1,899

f	f_n								
f_d	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
10	2,978	2,943	2,913	2,887	2,865	2,845	2,828	2,812	2,798
11	2,854	2,818	2,788	2,761	2,739	2,719	2,701	2,685	2,671
12	2,753	2,717	2,687	2,66	2,637	2,617	2,599	2,583	2,568
13	2,671	2,635	2,604	2,577	2,554	2,533	2,515	2,499	2,484
14	2,602	2,565	2,534	2,507	2,484	2,463	2,445	2,428	2,413
15	2,544	2,507	2,475	2,448	2,424	2,403	2,385	2,368	2,353
16	2,494	2,456	2,425	2,397	2,373	2,352	2,333	2,317	2,302
17	2,45	2,413	2,381	2,353	2,329	2,308	2,289	2,272	2,257
18	2,412	2,374	2,342	2,314	2,29	2,269	2,25	2,233	2,217
19	2,378	2,34	2,308	2,28	2,256	2,234	2,215	2,198	2,182
20	2,348	2,31	2,278	2,25	2,225	2,203	2,184	2,167	2,151
25	2,236	2,198	2,165	2,136	2,111	2,089	2,069	2,051	2,035
30	2,165	2,126	2,092	2,063	2,037	2,015	1,995	1,976	1,96
40	2,077	2,038	2,003	1,974	1,948	1,924	1,904	1,885	1,868
50	2,026	1,986	1,952	1,921	1,895	1,871	1,85	1,831	1,814
100	1,927	1,886	1,85	1,819	1,792	1,768	1,746	1,726	1,708
500	1,85	1,808	1,772	1,74	1,712	1,686	1,664	1,643	1,625

f	f_n							
f_d	**19**	**20**	**25**	**30**	**40**	**50**	**100**	**500**
10	2,785	2,774	2,73	2,7	2,661	2,637	2,588	2,548
11	2,658	2,646	2,601	2,57	2,531	2,507	2,457	2,415
12	2,555	2,544	2,498	2,466	2,426	2,401	2,35	2,307
13	2,471	2,459	2,412	2,38	2,339	2,314	2,261	2,218
14	2,4	2,388	2,341	2,308	2,266	2,241	2,187	2,142
15	2,34	2,328	2,28	2,247	2,204	2,178	2,123	2,078
16	2,288	2,276	2,227	2,194	2,151	2,124	2,068	2,022
17	2,243	2,23	2,181	2,148	2,104	2,077	2,02	1,973
18	2,203	2,191	2,141	2,107	2,063	2,035	1,978	1,929
19	2,168	2,155	2,106	2,071	2,026	1,999	1,94	1,891
20	2,137	2,124	2,074	2,039	1,994	1,966	1,907	1,856
25	2,021	2,007	1,955	1,919	1,872	1,842	1,779	1,725
30	1,945	1,932	1,878	1,841	1,792	1,761	1,695	1,637
40	1,853	1,839	1,783	1,744	1,693	1,66	1,589	1,526
50	1,798	1,784	1,727	1,687	1,634	1,599	1,525	1,457
100	1,691	1,676	1,616	1,573	1,515	1,477	1,392	1,308
500	1,607	1,592	1,528	1,482	1,419	1,376	1,275	1,159

Table B.3 – Fractiles of the F-distribution, $F_{0,995}$

f	f_n								
f_d	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	12,826	9,427	8,081	7,343	6,872	6,545	6,302	6,116	5,968
11	12,226	8,912	7,6	6,881	6,422	6,102	5,865	5,682	5,537
12	11,754	8,51	7,226	6,521	6,071	5,757	5,525	5,345	5,202
13	11,374	8,186	6,926	6,233	5,791	5,482	5,253	5,076	4,935
14	11,06	7,922	6,68	5,998	5,562	5,257	5,031	4,857	4,717
15	10,798	7,701	6,476	5,803	5,372	5,071	4,847	4,674	4,536
16	10,575	7,514	6,303	5,638	5,212	4,913	4,692	4,521	4,384
17	10,384	7,354	6,156	5,497	5,075	4,779	4,559	4,389	4,254
18	10,218	7,215	6,028	5,375	4,956	4,663	4,445	4,276	4,141
19	10,073	7,093	5,916	5,268	4,853	4,561	4,345	4,177	4,043
20	9,944	6,986	5,818	5,174	4,762	4,472	4,257	4,09	3,956
25	9,475	6,598	5,462	4,835	4,433	4,15	3,939	3,776	3,645
30	9,18	6,355	5,239	4,623	4,228	3,949	3,742	3,58	3,45
40	8,828	6,066	4,976	4,374	3,986	3,713	3,509	3,35	3,222
50	8,626	5,902	4,826	4,232	3,849	3,579	3,376	3,219	3,092
100	8,241	5,589	4,542	3,963	3,589	3,325	3,127	2,972	2,847
500	7,95	5,355	4,33	3,763	3,396	3,137	2,941	2,789	2,665

f	f_n								
f_d	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
10	5,847	5,746	5,661	5,589	5,526	5,471	5,422	5,379	5,34
11	5,418	5,32	5,236	5,165	5,103	5,049	5,001	4,959	4,921
12	5,085	4,988	4,906	4,836	4,775	4,721	4,674	4,632	4,595
13	4,82	4,724	4,643	4,573	4,513	4,46	4,413	4,372	4,334
14	4,603	4,508	4,428	4,359	4,299	4,247	4,2	4,159	4,122
15	4,424	4,329	4,25	4,181	4,122	4,07	4,024	3,983	3,946
16	4,272	4,179	4,099	4,031	3,972	3,92	3,875	3,834	3,797
17	4,142	4,05	3,971	3,903	3,844	3,793	3,747	3,707	3,67
18	4,03	3,938	3,86	3,793	3,734	3,683	3,637	3,597	3,56
19	3,933	3,841	3,763	3,696	3,638	3,587	3,541	3,501	3,465
20	3,847	3,756	3,678	3,611	3,553	3,502	3,457	3,416	3,38
25	3,537	3,447	3,37	3,304	3,247	3,196	3,151	3,111	3,075
30	3,344	3,255	3,179	3,113	3,056	3,006	2,961	2,921	2,885
40	3,117	3,028	2,953	2,888	2,831	2,781	2,737	2,697	2,661
50	2,988	2,9	2,825	2,76	2,703	2,653	2,609	2,569	2,533
100	2,744	2,657	2,583	2,518	2,461	2,411	2,367	2,326	2,29
500	2,562	2,476	2,402	2,337	2,281	2,23	2,185	2,145	2,108

f	f_n							
f_d	**19**	**20**	**25**	**30**	**40**	**50**	**100**	**500**
10	5,305	5,274	5,153	5,071	4,966	4,902	4,772	4,666
11	4,886	4,855	4,736	4,654	4,551	4,488	4,359	4,252
12	4,561	4,53	4,412	4,331	4,228	4,165	4,037	3,931
13	4,301	4,27	4,153	4,073	3,97	3,908	3,78	3,674
14	4,089	4,059	3,942	3,862	3,76	3,698	3,569	3,463
15	3,913	3,883	3,766	3,687	3,585	3,523	3,394	3,287
16	3,764	3,734	3,618	3,539	3,437	3,375	3,246	3,139
17	3,637	3,607	3,492	3,412	3,311	3,248	3,119	3,012
18	3,527	3,498	3,382	3,303	3,201	3,139	3,009	2,901
19	3,432	3,402	3,287	3,208	3,106	3,043	2,913	2,804
20	3,347	3,318	3,203	3,123	3,022	2,959	2,828	2,719
25	3,043	3,013	2,898	2,819	2,716	2,652	2,519	2,406
30	2,853	2,823	2,708	2,628	2,524	2,459	2,323	2,207
40	2,628	2,598	2,482	2,401	2,296	2,23	2,088	1,965
50	2,5	2,47	2,353	2,272	2,164	2,097	1,951	1,821
100	2,257	2,227	2,108	2,024	1,912	1,84	1,681	1,529
500	2,075	2,044	1,922	1,835	1,717	1,64	1,46	1,26

Table B.4 – Fractiles of the *t*-distribution, $t_{0,95}$

		p	
f	**0,95**	**0,99**	**0,995**
1	6,313751514	31,82051595	63,65674115
2	2,91998558	6,964556734	9,9248432
3	2,353363435	4,540702858	5,840909309
4	2,131846782	3,746947388	4,604094871
5	2,015048372	3,364929997	4,032142983
6	1,943180274	3,142668403	3,70742802
7	1,894578604	2,997951566	3,499483297
8	1,859548033	2,896459446	3,355387331
9	1,833112923	2,821437921	3,249835541
10	1,812461102	2,763769458	3,169272672
11	1,795884814	2,718079183	3,105806514
12	1,782287548	2,680997992	3,054539586
13	1,770933383	2,650308836	3,012275833
14	1,761310115	2,624494064	2,976842734
15	1,753050325	2,60248029	2,946712883
16	1,745883669	2,583487179	2,920781621
17	1,739606716	2,566933975	2,898230518
18	1,734063592	2,552379618	2,878440471
19	1,729132792	2,539483189	2,860934604
20	1,724718218	2,527977001	2,845339707
25	1,708140745	2,48510717	2,787435805
30	1,697260851	2,457261531	2,749995652
40	1,683851014	2,423256774	2,704459262
50	1,675905026	2,403271907	2,677793261
100	1,660234327	2,364217356	2,625890514
500	1,647906854	31,82051595	2,744041917

Table B.5 – Fractiles of the χ^2 -distribution

f	***p* = 0,95**	***p* = 0,99**	***p* = 0,995**
1	3,8	6,6	7,9
2	6,0	9,2	10,6
3	7,8	11,3	12,8
4	9,5	13,3	14,9
5	11,1	15,1	16,7
6	12,6	16,8	18,5
NOTE The significance level *P* is equal to (1-*p*), e.g. significance 0,05 corresponds to *p* = 0,95.			

Bibliography

[1] IEC 60216-1, *Electrical insulating materials – Properties of thermal endurance – Part 1: Ageing procedures and evaluation of test results*

[2] IEC 60216-3:2006, *Electrical insulating materials – Thermal endurance properties – Part 3: Instructions for calculating thermal endurance characteristics*

[3] IEC 60216-5, *Electrical insulating materials – Thermal endurance properties – Part 5: Determination of relative thermal endurance index (RTE) of an insulating material*

[4] IEC 60216-6, *Electrical insulating materials – Thermal endurance properties – Part 6: Determination of thermal endurance indices (TI and RTE) of an insulating material using the fixed time frame method*

[5] IEC/TR 60493-2, *Guide for the statistical analysis of ageing test data – Part 2: Validation of procedures for statistical analysis of censored normally distributed data*

[6] IEC 62539, *Guide for the statistical analysis of electrical insulation breakdown data*

[7] PRESS, W.H. et al., *Numerical Recipes*, Cambridge University Press, New York 1986, Ch 6

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