BS EN 62810:2015

BSI Standards Publication

Cylindrical cavity method to measure the complex permittivity of low-loss dielectric rods

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National foreword

This British Standard is the UK implementation of EN 62810:2015.
It is identical to IEC 62810:2015.

The UK participation in its preparation was entrusted to Technical Committee EPL/46, Cables, wires and waveguides, radio frequency connectors and accessories for communication and signalling.

A list of organizations represented on this committee can be obtained on request to its secretary.

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EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

EN 62810

May 2015

ICS 33.120.30

English Version

Cylindrical cavity method to measure the complex permittivity of low-loss dielectric rods (IEC 62810:2015)

Mesure de la permitivité complexe des barreaux diélectriques à faibles pertes par la méthode de la cavité cylindrique
(IEC 62810:2015)

Zylindrisches Hohlraumverfahren zur Messung der komplexen Permittivität von verlustarmen dielektrischen Stäben
(IEC 62810:2015)

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European Committee for Electrotechnical Standardization
Comité Européen de Normalisation Electrotechnique
Europäisches Komitee für Elektrotechnische Normung

CEN-CENELEC Management Centre: Avenue Marnix 17, B-1000 Brussels

Ref. No. EN 62810:2015 E

Foreword

The text of document 46F/242/CDV, future edition 1 of IEC 62810, prepared by SC 46F, "R.F. and microwave passive components", of IEC TC 46, "Cables, wires, waveguides, R.F. connectors, R.F. and microwave passive components and accessories" was submitted to the IEC-CENELEC parallel vote and approved by CENELEC as EN 62810:2015.

The following dates are fixed:

• latest date by which the document has to be implemented at national level by publication of an identical national standard or by endorsement	(dop)	2015-12-24
• latest date by which the national standards conflicting with the document have to be withdrawn	(dow)	2018-03-24

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In the official version, for Bibliography, the following note has to be added for the standard indicated:

IEC 60556	NOTE	Harmonised as EN 60556.

CONTENTS

INTERNATIONAL ELECTROTECHNICAL COMMISSION

CYLINDRICAL CAVITY METHOD TO MEASURE THE COMPLEX PERMITTIVITY OF LOW-LOSS DIELECTRIC RODS

FOREWORD

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International Standard IEC 62810 has been prepared by subcommittee 46F: R.F. and microwave passive components, of IEC technical committee 46: Cables, wires, waveguides, R.F. connectors, R.F. and microwave passive components and accessories.

The text of this standard is based on the following documents:

CDV	Report on voting
46F/242/CDV	46F/260/RVC

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

IMPORTANT – The 'colour inside' logo on the cover page of this publication indicates that it contains colours which are considered to be useful for the correct understanding of its contents. Users should therefore print this document using a colour printer.

CYLINDRICAL CAVITY METHOD TO MEASURE THE COMPLEX PERMITTIVITY OF LOW-LOSS DIELECTRIC RODS

1 Scope

This International Standard relates to a measurement method for complex permittivity of a dielectric rod at microwave frequency. This method has been developed to evaluate the dielectric properties of low-loss materials in coaxial cables and electronic devices used in microwave systems. It uses the TM_{010} mode in a circular cylindrical cavity and presents accurate measurement results of a dielectric rod sample, where the effect of sample insertion holes is taken into account accurately on the basis of the rigorous electromagnetic analysis.

In comparison with the conventional method described in IEC 60556 [2][1], this method has the following characteristics:

- the values of the relative permittivity ε' and loss tangent $\tan\delta$ of a dielectric rod sample can be measured accurately and non-destructively;
- the measurement accuracy is within 1,0 % for ε' and within 20 % for $\tan\delta$;
- the effect of sample insertion holes is corrected using correction charts presented;
- this method is applicable for the measurements on the following condition:
  - frequency: $1\text{ GHz} \leqq f \leqq 10\text{ GHz}$;
  - relative permittivity: $1 \leqq \varepsilon' \leqq 100$;
  - loss tangent: $10^{-4} \leqq \tan\delta \leqq 10^{-1}$.

2 Normative references

Void.

3 Measurement parameters

The measurement parameters are defined as follows:

$$\varepsilon_r = \varepsilon' - j\varepsilon'' \tag{1}$$

$$\tan\delta = \varepsilon''/\varepsilon' \tag{2}$$

where ε' and ε'' are the real and imaginary parts of the complex relative permittivity ε_r.

4 Theory and calculation equations

A resonator structure used in these measurements is shown in Figure 1. A cavity, made with copper, with diameter D and height H has sample insertion holes with diameter d_2 and depth g oriented coaxially. A dielectric rod sample of diameter d_1 having ε' and $\tan\delta$ is inserted into the holes.

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[1] Figures in square brackets refer to the Bibliography.

The TM_{010} mode, where the electric field component in the cavity is parallel to the sample rod, is used for the measurement. Taking account of the effect of sample insertion holes calculated on the basis of the rigorous electromagnetic field analysis, ε' and $\tan\delta$ are determined from the measured values of the resonant frequency f_0 and the unloaded Q-factor Q_u. To avoid the tedious numerical calculation and make the measurements easy, the following process is taken in this measurement:

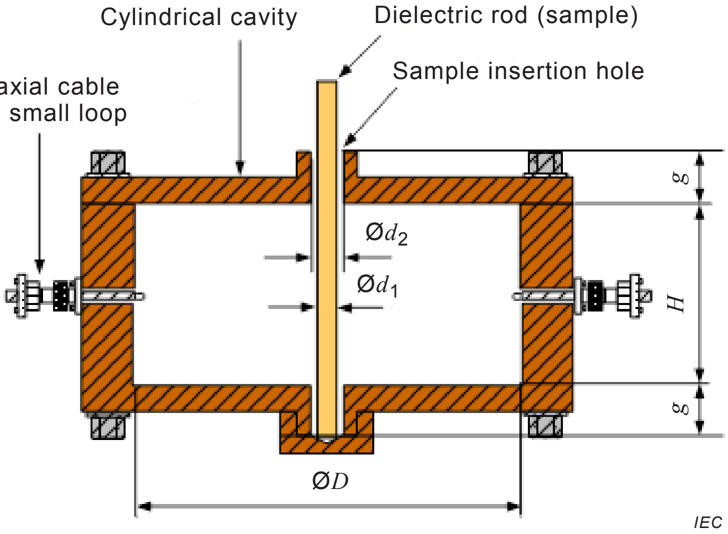


Figure 1 – Structure of a cylindrical cavity resonator

The following steps shall be taken:

1) At the first step, obtain approximate values ε_p and $\tan\delta_p$ from the f_0 and Q_u values by using the simple perturbation formulas, where the effect of sample insertion holes is neglected. The subscript p denotes the calculated values using the following perturbation formulas:

$$\varepsilon_p = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{D}{d_1}\right)^2 + 1 \qquad (3)$$

$$\tan\delta_p = \frac{1}{2\alpha\varepsilon_p}\left(\frac{D}{d_1}\right)^2\left(\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right) \qquad (4)$$

   where $\alpha = 1/J_1(x_{01})^2 = 1{,}855$.

   $J_n(x)$ is the Bessel function of order n of first kind and $x_{01} = 2{,}405$ is the first root of $J_0(x) = 0$. f_0 and Q_{u0} are the resonant frequency and unloaded Q-factor measured for the cavity without a sample, respectively. f_1 and Q_{u1} are ones measured for the cavity with a sample.

2) In the second step, obtain accurate values ε' and $\tan\delta$ from ε_p and $\tan\delta_p$ values by using the following equations with correction factors calculated based on the rigorous analysis:

$$\varepsilon' = C_1\varepsilon_p \qquad (5)$$

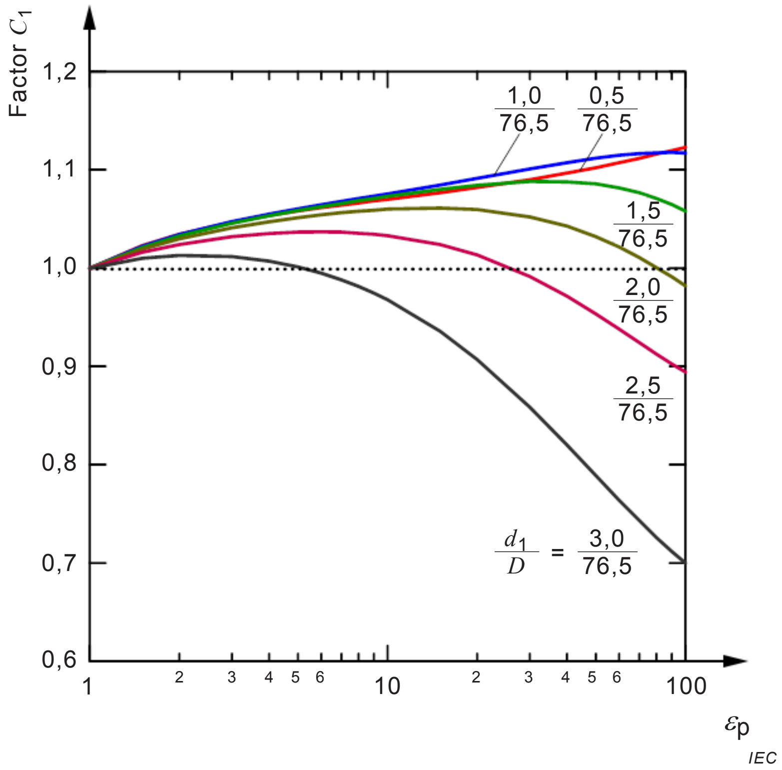
$$\tan\delta = C_2\tan\delta_p \qquad (6)$$

where correction factors C_1 and C_2, due to the sample insertion holes and errors included in the perturbation formulas, are calculated numerically by using the Ritz-Galerkin method [3][5], as shown in Figure 2 and Figure 3, and the corresponding data are listed in detail in Table 1, 2, and 3. The missing data of C_1 and C_2 can be obtained by interpolation or extrapolation from the tables. The correction factors shown in these figures are calculated for the cavity with D = 76,5 mm, H = 20,0 mm, d_2 = 3,0 mm, and g = 10,0 mm, where the resonant frequency is about 3 GHz. C_1 is also used for a cavity having the same aspect ratios as H/D, d_2/D and g/D.

It is found from the analysis for a cavity with insertion holes which constitute a cut-off TM_{01} mode cylindrical waveguide that f_0 converges to a constant value for $g>10$ mm and d_2 = 3 mm. Therefore, the correction factors shown in Figure 2 and Figure 3 are applicable to a dielectric sample rod with $d_1<3$ mm and ε' below the value calculated by the following equation for the measured value of the resonant frequency:

$$\varepsilon' \le \left(\frac{x_{01}c}{\pi d_2 f_0}\right)^2 \tag{7}$$

where c is the velocity of light in a vacuum (c = 2,9 979 × 108 m/s).



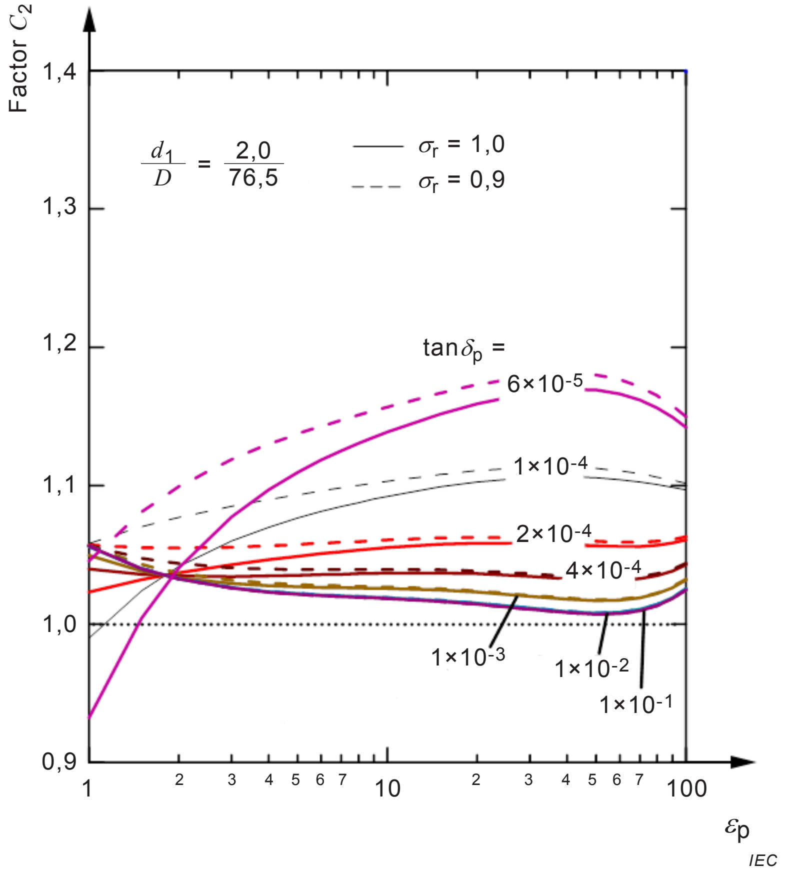
Assumptions

D	76,5 mm	d_2	3,0 mm
H	20,0 mm	g	10,0 mm

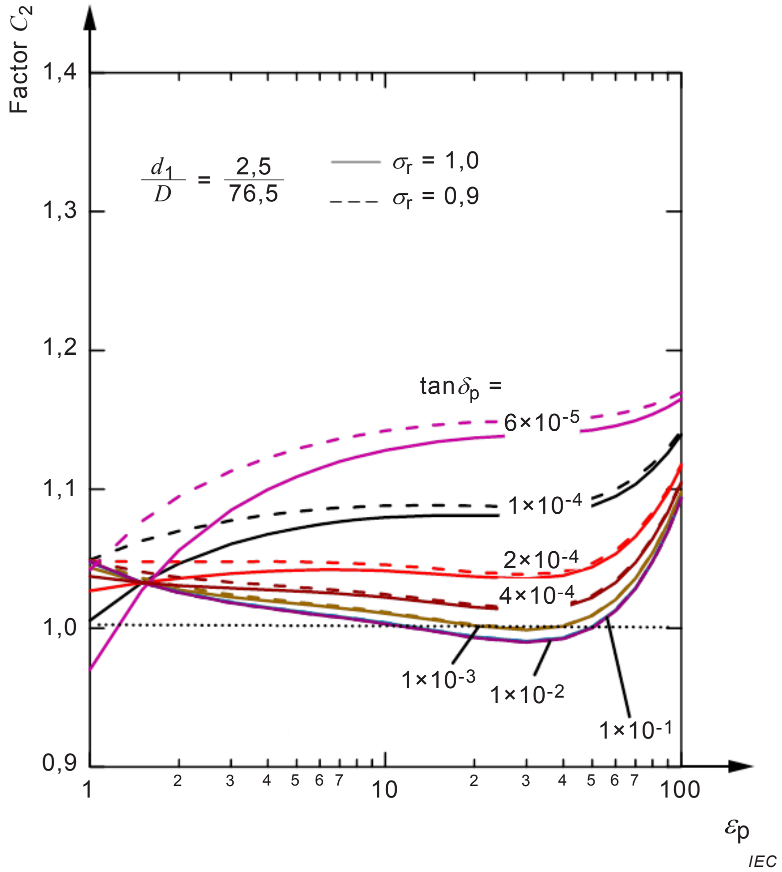
Figure 2 – Correction factor C_1 for ε'

Table 1 – Numerical values of correction factor C_1

ε_p	d_1(mm)					
	0, 5	1, 0	1, 5	2, 0	2, 5	3, 0
1	1, 000	1, 000	1, 000	1, 000	1, 000	1, 000
1, 5	1, 023	1, 022	1, 021	1, 019	1, 016	1, 010
2	1, 035	1, 034	1, 033	1, 030	1, 024	1, 013
3	1, 047	1, 047	1, 046	1, 041	1, 032	1, 012
4	1, 054	1, 055	1, 053	1, 047	1, 035	1, 007
5	1, 058	1, 060	1, 059	1, 051	1, 037	1, 001
6	1, 061	1, 064	1, 063	1, 054	1, 037	0, 995
7	1, 064	1, 068	1, 066	1, 056	1, 037	0, 988
8	1, 066	1, 071	1, 069	1, 058	1, 036	0, 981
9	1, 068	1, 073	1, 071	1, 059	1, 035	0, 975
10	1, 070	1, 076	1, 073	1, 060	1, 033	0, 968
15	1, 077	1, 085	1, 080	1, 061	1, 024	0, 936
20	1, 082	1, 091	1, 084	1, 060	1, 013	0, 907
30	1, 090	1, 101	1, 088	1, 052	0, 992	0, 859
40	1, 097	1, 107	1, 088	1, 043	0, 971	0, 820
50	1, 102	1, 112	1, 086	1, 032	0, 953	0, 789
60	1, 107	1, 115	1, 082	1, 021	0, 938	0, 764
70	1, 112	1, 117	1, 077	1, 011	0, 924	0, 743
80	1, 116	1, 118	1, 071	1, 001	0, 912	0, 726
90	1, 119	1, 118	1, 065	0, 991	0, 903	0, 712
100	1, 123	1, 117	1, 058	0, 982	0, 894	0, 700



a) Dielectric sample rod with d_1 = 2,0 mm



b) **Dielectric sample rod with d_1 = 2,5 mm**

Assumptions

D	76,5 mm	d_2	3,0 mm
H	20,0 mm	g	10,0 mm

Figure 3 – Correction factor C_2 for tanδ with the different values of d_1

Table 2 – Numerical values of correction factor C_2

(Dielectric sample rod with d_1 = 2,0 mm)

σ_r=0,9

ε_p	$\tan\delta_p$						
	6×10^{-5}	1×10^{-4}	2×10^{-4}	4×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}
1	1,045	1,058	1,057	1,057	1,057	1,056	1,056
1,5	1,081	1,070	1,055	1,048	1,043	1,040	1,040
2	1,099	1,077	1,055	1,044	1,037	1,033	1,033
3	1,119	1,085	1,055	1,041	1,032	1,026	1,026
4	1,130	1,090	1,056	1,040	1,030	1,024	1,023
5	1,137	1,093	1,057	1,039	1,029	1,022	1,021
6	1,143	1,096	1,058	1,039	1,028	1,021	1,020
7	1,147	1,098	1,059	1,039	1,028	1,020	1,020
8	1,151	1,100	1,060	1,039	1,027	1,020	1,019
9	1,154	1,102	1,060	1,039	1,027	1,019	1,019
10	1,157	1,103	1,061	1,039	1,027	1,019	1,018
15	1,167	1,108	1,062	1,039	1,025	1,017	1,016
20	1,173	1,111	1,063	1,038	1,024	1,015	1,014
30	1,179	1,113	1,062	1,036	1,021	1,012	1,011
40	1,181	1,114	1,061	1,034	1,019	1,009	1,008
50	1,180	1,113	1,060	1,033	1,018	1,008	1,007
60	1,177	1,111	1,059	1,033	1,018	1,009	1,008
70	1,172	1,109	1,059	1,034	1,019	1,011	1,010
80	1,165	1,106	1,060	1,036	1,022	1,014	1,013
90	1,158	1,104	1,061	1,040	1,027	1,019	1,018
100	1,150	1,102	1,063	1,044	1,032	1,025	1,025

σ_r=1,0

ε_p	$\tan\delta_p$						
	6×10^{-5}	1×10^{-4}	2×10^{-4}	4×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}
1	0,932	0,990	1,023	1,040	1,050	1,056	1,056
1,5	1,004	1,024	1,032	1,036	1,038	1,040	1,040
2	1,040	1,042	1,037	1,035	1,033	1,033	1,032
3	1,077	1,060	1,043	1,034	1,029	1,026	1,026
4	1,097	1,070	1,046	1,035	1,028	1,023	1,023
5	1,110	1,077	1,049	1,035	1,027	1,022	1,021
6	1,118	1,081	1,051	1,036	1,026	1,021	1,020
7	1,125	1,085	1,052	1,036	1,026	1,020	1,020
8	1,131	1,088	1,053	1,036	1,026	1,020	1,019
9	1,135	1,090	1,054	1,037	1,026	1,019	1,019
10	1,139	1,092	1,055	1,037	1,026	1,019	1,018
15	1,152	1,099	1,058	1,037	1,024	1,017	1,016
20	1,159	1,103	1,058	1,036	1,023	1,015	1,014
30	1,167	1,106	1,058	1,034	1,020	1,012	1,011
40	1,170	1,107	1,057	1,033	1,018	1,009	1,008
50	1,169	1,106	1,056	1,032	1,017	1,008	1,007
60	1,166	1,104	1,056	1,032	1,017	1,008	1,008
70	1,162	1,103	1,056	1,033	1,019	1,010	1,010
80	1,156	1,101	1,057	1,035	1,022	1,014	1,013
90	1,150	1,099	1,059	1,038	1,026	1,019	1,018
100	1,142	1,097	1,061	1,043	1,032	1,025	1,025

Table 3 – Numerical values of correction factor C_2

(Dielectric sample rod with d_1 = 2,5 mm)

σ_r=0,9

ε_p	$\tan\delta_p$						
	6×10^{-5}	1×10^{-4}	2×10^{-4}	4×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}
1	1,042	1,049	1,049	1,048	1,048	1,048	1,048
1,5	1,077	1,063	1,048	1,040	1,036	1,033	1,033
2	1,095	1,070	1,048	1,037	1,030	1,026	1,026
3	1,113	1,078	1,048	1,033	1,024	1,019	1,018
4	1,123	1,081	1,048	1,031	1,021	1,015	1,014
5	1,129	1,084	1,048	1,030	1,019	1,012	1,012
6	1,133	1,086	1,047	1,028	1,017	1,010	1,009
7	1,136	1,087	1,047	1,027	1,015	1,008	1,008
8	1,139	1,087	1,047	1,026	1,014	1,007	1,006
9	1,141	1,088	1,046	1,025	1,013	1,005	1,004
10	1,142	1,088	1,046	1,024	1,011	1,004	1,003
15	1,146	1,088	1,043	1,020	1,006	0,998	0,997
20	1,148	1,088	1,040	1,017	1,002	0,994	0,993
30	1,150	1,088	1,039	1,014	0,999	0,991	0,990
40	1,150	1,089	1,041	1,016	1,002	0,993	0,992
50	1,152	1,094	1,047	1,023	1,009	1,001	1,000
60	1,154	1,100	1,056	1,034	1,021	1,013	1,012
70	1,157	1,108	1,068	1,048	1,036	1,029	1,028
80	1,161	1,118	1,083	1,065	1,055	1,048	1,048
90	1,165	1,130	1,100	1,084	1,075	1,070	1,069
100	1,170	1,142	1,118	1,106	1,098	1,094	1,094

σ_r=1,0

ε_p	$\tan\delta_p$						
	6×10^{-5}	1×10^{-4}	2×10^{-4}	4×10^{-4}	1×10^{-3}	1×10^{-2}	1×10^{-1}
1	0,970	1,006	1,027	1,037	1,044	1,048	1,048
1,5	1,027	1,033	1,033	1,033	1,033	1,033	1,033
2	1,056	1,046	1,036	1,031	1,028	1,026	1,026
3	1,085	1,060	1,039	1,029	1,022	1,019	1,018
4	1,100	1,068	1,041	1,028	1,020	1,015	1,014
5	1,109	1,072	1,042	1,027	1,018	1,012	1,012
6	1,115	1,075	1,042	1,026	1,016	1,010	1,009
7	1,120	1,077	1,042	1,025	1,014	1,008	1,008
8	1,123	1,078	1,042	1,024	1,013	1,007	1,006
9	1,126	1,079	1,042	1,023	1,012	1,005	1,004
10	1,128	1,080	1,041	1,022	1,011	1,004	1,003
15	1,134	1,081	1,039	1,018	1,006	0,998	0,997
20	1,137	1,081	1,037	1,015	1,002	0,994	0,993
30	1,139	1,081	1,035	1,012	0,999	0,990	0,990
40	1,141	1,083	1,038	1,015	1,001	0,993	0,992
50	1,143	1,088	1,044	1,022	1,009	1,001	1,000
60	1,146	1,095	1,054	1,033	1,021	1,013	1,012
70	1,150	1,104	1,066	1,047	1,036	1,029	1,028
80	1,154	1,114	1,081	1,064	1,054	1,048	1,048
90	1,159	1,126	1,098	1,084	1,075	1,070	1,069
100	1,165	1,139	1,116	1,105	1,098	1,094	1,094

The value of relative conductivity σ_r is determined from the measured unloaded Q-factor Q_{u0} at f_0 for the TM_{010} mode by the following equation:

$$\sigma_r = \left\{ Q_{u0} \frac{\delta_{s0}}{\lambda_0} \frac{2\pi\left(1+\dfrac{D}{2H}\right)}{x_{01}} \right\}^2 \tag{8}$$

where $\lambda_0 = c/f_0$ is the wave length, and the skin depth δ_{s0} at f_0 is defined as follows:

$$\delta_{s0} = \sqrt{\frac{1}{\pi f_0 \mu_0 \sigma_0}} \tag{9}$$

where μ_0 is the permeability of vacuum and $\sigma_0 = 5{,}8 \times 10^7$ S/m is the conductivity of standard copper.

Measurement uncertainties of ε' and $\tan\delta$, $u(\varepsilon')$ and $u(\tan\delta)$, are estimated as the mean square uncertainty and given respectively by

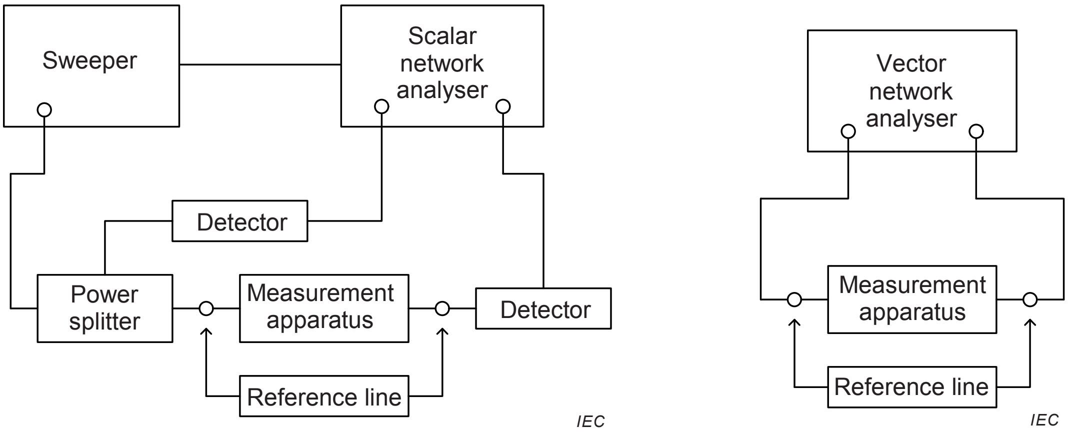
$$u(\varepsilon')^2 = \left(\frac{\partial \varepsilon'}{\partial f_0}\right)^2 u(f_0)^2 + \left(\frac{\partial \varepsilon'}{\partial f_1}\right)^2 u(f_1)^2 + \left(\frac{\partial \varepsilon'}{\partial d_1}\right)^2 u(d_1)^2 + \left(\frac{\partial \varepsilon'}{\partial D}\right)^2 u(D)^2 + \left(\frac{\partial \varepsilon'}{\partial C_1}\right)^2 u(C_1)^2 \tag{10}$$

$$\begin{aligned} u(\tan\delta)^2 = {} & \left(\frac{\partial \tan\delta}{\partial \varepsilon_P}\right)^2 u(\varepsilon_P)^2 + \left(\frac{\partial \tan\delta}{\partial d_1}\right)^2 u(d_1)^2 + \left(\frac{\partial \tan\delta}{\partial D}\right)^2 u(D)^2 + \\ & \left(\frac{\partial \tan\delta}{\partial Q_{u0}}\right)^2 u(Q_{u0})^2 + \left(\frac{\partial \tan\delta}{\partial Q_{u1}}\right)^2 u(Q_{u1})^2 + \left(\frac{\partial \tan\delta}{\partial C_2}\right)^2 u(C_2)^2 \end{aligned} \tag{11}$$

where $u(f_0)$, $u(f_1)$, $u(d_1)$, $u(D)$, and $u(C_1)$ are the standard uncertainties of f_0, f_1, d_1, D, and C_1, respectively. Also, $u(\tan\delta)$ is mainly attributed to measurement uncertainty of ε_p, d_1, D, Q_{u0}, Q_{u1}, and C_2. $u(\varepsilon_p)$, $u(d_1)$, $u(D)$, $u(Q_{u0})$, $u(Q_{u1})$, and $u(C_2)$ are the standard uncertainties of them, respectively.

5 Measurement system

Figure 4 shows a schematic diagram of two equipment systems required for microwave measurement. For the measurement of dielectric properties, only the information on the amplitude of transmitted power is needed, that is, the information on the phase of the transmitted power is not required. Therefore, a scalar network analyser can be used for the measurement shown in Figure 4a. However, a vector network analyser, as shown in Figure 4b, has an advantage in precision of the measurement.



a) Scalar network analyzer system **b) Vector network analyzer system**

Figure 4 – Schematic diagram of measurement systems

The structure of the TM_{010} mode cylindrical cavity resonator used in the complex permittivity measurement is shown in Figure 1. The cavity has D = 76,5 mm, H = 20,0 mm, d_2 = 3,0 mm, and g = 10,0 mm for the measurement around 3 GHz. A sample with diameter $d_1 < d_2$ is coaxially inserted into the holes and excited magnetically by a pair of semi-rigid coaxial cables with a small loop at the top. The transmission-type resonator is constituted and under-coupled equally to the input and output loops with setting $S_{11} = S_{22}$.

The resonant frequency f_0, half-power band width f_{BW}, and the insertion attenuation IA_0 (dB) at f_0 are measured using a network analyser by means of the swept-frequency method, as shown in Figure 5. The value of Q_{u} is given by

$$Q_{\mathrm{u}} = \frac{Q_{\mathrm{L}}}{1-10^{IA_0(\mathrm{dB})/20}}, \quad Q_{\mathrm{L}} = \frac{f_0}{f_{\mathrm{BW}}} \tag{12}$$

1) At the first step, obtain approximate values ε_{p} and $\tan\delta_{\mathrm{p}}$ from the f_0 and Q_{u} values by using the simple perturbation formulas, where the effect of sample insertion holes is neglected. The subscript p denotes the calculated values using the following perturbation formulas:

$$\varepsilon_{\mathrm{p}} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{D}{d_1}\right)^2 + 1 \tag{3}$$

$$\tan\delta_{\mathrm{p}} = \frac{1}{2\alpha\varepsilon_{\mathrm{p}}}\left(\frac{D}{d_1}\right)^2\left(\frac{1}{Q_{\mathrm{u1}}} - \frac{1}{Q_{\mathrm{u0}}}\right) \tag{4}$$

where $\alpha = 1/J_1(x_{01})^2 = 1{,}855$. $J_{\mathrm{n}}(x)$ is the Bessel function of order n of first kind and $x_{01} = 2{,}405$ is the first root of $J_0(x) = 0$. f_0 and Q_{u0} are the resonant frequency and unloaded Q-factor measured for the cavity without a sample, respectively. f_1 and Q_{u1} are ones measured for the cavity with a sample.

2) In the second step, obtain accurate values ε' and $\tan\delta$ from ε_{p} and $\tan\delta_{\mathrm{p}}$ values by using the following equations with correction factors calculated based on the rigorous analysis:

$$\varepsilon' = C_1\,\varepsilon_{\mathrm{p}} \tag{5}$$

$$\tan\delta = C_2 \tan\delta_{\mathrm{p}} \tag{6}$$

where correction factors C_1 and C_2 due to the sample insertion holes and errors included in the perturbation formulas are calculated numerically by using the Ritz-Galerkin method [3][5], as shown in Figure 2 and Figure 3, and the corresponding data are listed in detail in Table 1, 2, and 3. The missing data of C_1 and C_2 can be obtained by interpolation or extrapolation from the tables. The correction factors shown in these figures are calculated for the cavity with D = 76,5 mm, H = 20,0 mm, d_2 = 3,0 mm, and g = 10,0 mm, where the resonant frequency is about 3 GHz. C_1 is also used for a cavity having the same aspect ratios as H/D, d_2/D and g/D.

It is found from the analysis for a cavity with insertion holes which constitute a cut-off TM_{01} mode cylindrical waveguide that f_0 converges to a constant value for g>10 mm and d_2 = 3 mm. Therefore, the correction factors shown in Figure 2 and Figure 3 are applicable to a dielectric sample rod with d_1<3 mm and ε' below the value calculated by the following equation for the measured value of the resonant frequency:

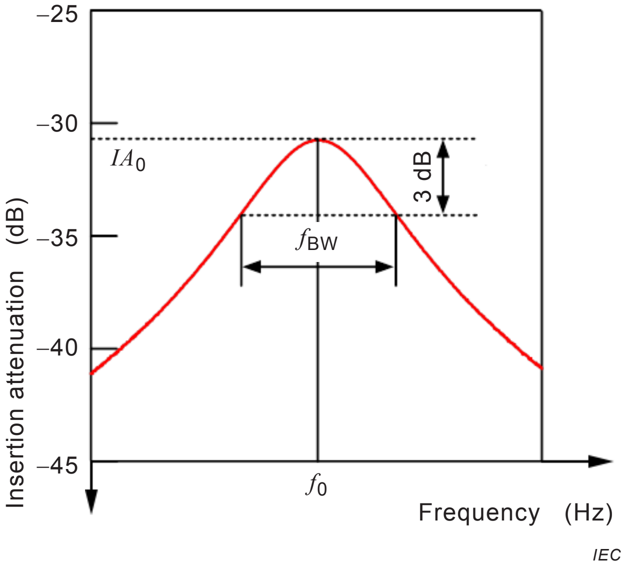


Figure 5 – Resonance frequency f_0, insertion attenuation IA_0 and half-power band width f_{BW}

6 Measurement procedure

6.1 Preparation of measurement apparatus

Set up the measurement equipment and apparatus as shown in Figure 4. The cavity resonator and dielectric samples shall be kept in a clean and dry state, as high humidity degrades unloaded Q. The relative humidity shall preferably be less than 60 %.

6.2 Measurement of reference level

The reference level, level of full transmission power, is measured first. Connect the reference line to the measurement equipment and measure the full transmission power level over the entire measurement frequency range.

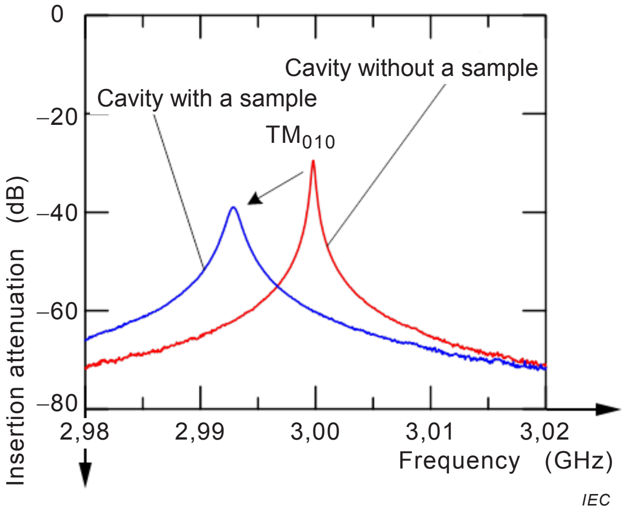
6.3 Measurement of cavity parameters: σ_r

Set the empty cavity and adjust the insertion attenuation IA_0 to be around 30 dB by changing the distance between two semi-rigid cables, as shown in Figure 5.

Measure f_0, f_{BW}, and IA_0 of the TM_{010} resonant mode. Calculate Q_{u0} by using Equation (12). Then, calculate σ_r by using Equation (8). Since the value of σ_r degrades due to oxidation of the metal surface, it shall be measured periodically. σ_r shall preferably be more than 0,9.

6.4 Measurement of complex permittivity of test sample: ε', tan δ

Insert the test sample into the holes. Figure 6 shows the frequency responses of the TM_{010} mode in the cavity with and without a sample. Measure the resonant frequency f_1, half-power band width f_{BW} and the insertion attenuation IA_0. Calculate the values of ε_p' and tan δ_p by using Equations (3) and (4), respectively. Then, calculate ε' and tanδ values by using Equations (5) and (6).



Assumptions

D	76,5 mm	d_2	3,0 mm
H	20,0 mm	g	10,0 mm

Figure 6 – Frequency responses of the TM_{010} mode of cylindrical cavity

Annex A
(informative)

Example of measurement results and accuracy

A.1 Measurement of ε' and $\tan\delta$ values

The measurement results of ε' and $\tan\delta$ for polyethylene rod sample are obtained as followed.

a) The parameters such as D, H and d_2 of the cavity and d_1 of the polyethylene sample used in the measurements are shown in Table A.1.

Table A.1 – The parameters of the cavity and the rod sample

D mm	H mm	d_2 mm	d_1 mm
76,50 ±0,02	20,00 ±0,01	3,00 ±0,01	2,52 ±0,01

b) The resonant frequency f_0 and unloaded Q-factor Q_{u0} of the TM_{010} mode in the cavity without a sample and f_1 and Q_{u1} in the cavity with a sample are measured and shown in Table A.2.

Table A.2 – The resonant frequencies and unloaded Q-factors

f_0 (GHz)	Q_{u0}	f_1 (GHz)	Q_{u1}
2,99992 ±0,00001	10264 ±5	2,99249 ±0,00001	10073 ±7

c) The approximate values ε_p and $\tan\delta_p$ and the value of relative conductivity σ_r are calculated numerically by Equations (3), (4), and (8), respectively, and the results are shown in Table A.3.

Table A.3 – The approximate values and the relative conductivity value

ε_p	$\tan\delta_p\ (\times 10^{-4})$	σ_r
2,233 ±0,010	2,055 ±0,095	0,889 ±0,001

d) The correction factors C_1 and C_2 are found from Figure 2 and Figure 3b, respectively, using the calculated values of ε_p, $\tan\delta_p$ and σ_r. The results are shown in Table A.4.

Table A.4 – Correction factors and the measurement results

C_1	C_2	ε'	$\tan\delta\ (\times 10^{-4})$
1,027 ±0,001	1,047 ±0,001	2,293 ±0,010	2,152 ±0,099

e) The accurate values ε' and $\tan\delta$ are obtained from Equations (5) and (6), and these results are also shown in Table A.4.

A.2 Measurement uncertainty of ε' and $\tan\delta$

The measurement uncertainty (see ISO/IEC Guide 98-3) of ε' and $\tan\delta$ is calculated for the polyethylene sample mentioned above by Equation (10) and (11). Each sensitivity coefficients in Equations (10) and (11) are as follows:

$$\frac{\partial \varepsilon'}{\partial f_0} = \frac{1}{\alpha}\frac{1}{f_1}\left(\frac{D}{d_1}\right)^2 C_1$$

$$\frac{\partial \varepsilon'}{\partial f_1} = \frac{1}{\alpha}\left(-\frac{f_0}{f_1{}^2}\right)\left(\frac{D}{d_1}\right)^2 C_1$$

$$\frac{\partial \varepsilon'}{\partial d_1} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(-2\frac{D^2}{d_1{}^3}\right) C_1$$

$$\frac{\partial \varepsilon'}{\partial D} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{2D}{d_1{}^2}\right) C_1$$

$$\frac{\partial \varepsilon'}{\partial C_1} = \frac{1}{\alpha}\frac{f_0 - f_1}{f_1}\left(\frac{D}{d_1}\right)^2$$

$$\frac{\partial \tan\delta}{\partial \varepsilon_\mathrm{p}} = -\frac{1}{\varepsilon_\mathrm{p}{}^2}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\} C_2$$

$$\frac{\partial \tan\delta}{\partial d_1} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(-2\frac{D^2}{d_1{}^3}\right)\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\} C_2$$

$$\frac{\partial \tan\delta}{\partial D} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{2D}{d_1{}^2}\right)\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\} C_2$$

$$\frac{\partial \tan\delta}{\partial Q_{u0}} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{\frac{1}{Q_{u0}{}^2}\right\} C_2$$

$$\frac{\partial \tan\delta}{\partial Q_{u1}} = \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{-\frac{1}{Q_{u1}{}^2}\right\} C_2$$

$$\frac{\partial \tan\delta}{\partial C_2} == \frac{1}{\varepsilon_\mathrm{p}}\frac{1}{2\alpha}\left(\frac{D}{d_1}\right)^2\left\{\frac{1}{Q_{u1}} - \frac{1}{Q_{u0}}\right\}$$

The results are shown in Table A.5 and A.6.

Table A.5 – The measurement uncertainty of ε'

	$\frac{\partial \varepsilon'}{\partial f_0}u(f_0)$	$\frac{\partial \varepsilon'}{\partial f_1}u(f_1)$	$\frac{\partial \varepsilon'}{\partial d_1}u(d_1)$	$\frac{\partial \varepsilon'}{\partial D}u(D)$	$\frac{\partial \varepsilon'}{\partial C_1}u(C_1)$	$u(\varepsilon')$
Sensitivity	$1{,}7050\times10^{-7}$	$-1{,}7092\times10^{-7}$	$-1{,}0054\times10^{3}$	$3{,}3119\times10^{1}$	$1{,}2335\times10^{0}$	----
uncertainty	0,0017	0,0017	0,0101	0,0007	0,0012	0,0104

Table A.6 – The measurement uncertainty of tanδ

	$\frac{\partial \tan\delta}{\partial \varepsilon_P}u(\varepsilon_P)$	$\frac{\partial \tan\delta}{\partial d_1}u(d_1)$	$\frac{\partial \tan\delta}{\partial D}u(D)$	$\frac{\partial \tan\delta}{\partial Q_{u0}}u(Q_{u0})$	$\frac{\partial \tan\delta}{\partial Q_{u1}}u(Q_{u1})$	$\frac{\partial \tan\delta}{\partial C_2}u(C_2)$	$u(\tan\delta)$
Sensitivity	$-9{,}6313\times10^{-5}$	$-1{,}7073\times10^{-1}$	$5{,}6239\times10^{-3}$	$1{,}1053\times10^{-6}$	$-1{,}1476\times10^{-6}$	$2{,}0546\times10^{-4}$	----
uncertainty	$0{,}00972\times10^{-4}$	$0{,}01707\times10^{-4}$	$0{,}00112\times10^{-4}$	$0{,}05526\times10^{-4}$	$0{,}08033\times10^{-4}$	$0{,}00205\times10^{-4}$	$0{,}09949\times10^{-4}$

Bibliography

[1] ISO/IEC Guide 98-3, *Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

[2] IEC 60556, *Gyromagnetic materials intended for application at microwave frequencies – Measuring methods for properties*

[3] ESTIN, A.J. and H.E.BUSSEY, H.E., *Errors in dielectric measurements due to a sample insertion hol in a cavity*, IRE Trans. Microwave Theory & Tech., vol.MTT-8, no.6, pp.650-653, Nov. 1960

[4] KAWABATA, H., TANPO, H. and KOBAYASHI, Y., *An improvement of the perturbation method using a* TM_{010} *mode cylindrical cavity*, IEICE Trans. Electron., vol.E86-C, no.12, pp.2371-2379, Dec. 2003

[5] KANEKO, S., KAWABATA, H.and KOBAYASHI, Y., *Improved perturbation method of complex permittivity using correction charts for* TM_{010} *and* TM_{020} *modes of a circular cylindrical cavity*" Proc. 2010 Asia-Pacific Microwave Conf., TH3G-47, pp.1448-1451, Dec. 2010

[6] WEISSTEIN, Eric W., *Galerkin Method*, from MathWorld – A Wolfram Web Resource. http://mathworld.wolfram.com/GalerkinMethod.html (website checked 2014.12.22)

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