

BS EN 61810-2:2011



BSI Standards Publication

# Electromechanical elementary relays

Part 2: Reliability

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**National foreword**

This British Standard is the UK implementation of EN 61810-2:2011. It is identical to IEC 61810-2:2011. It supersedes BS EN 61810-2:2005 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee EPL/94, General purpose relays and reed contact units.

A list of organizations represented on this committee can be obtained on request to its secretary.

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English version

**Electromechanical elementary relays -  
Part 2: Reliability  
(IEC 61810-2:2011)**Relais électromécaniques élémentaires -  
Partie 2: Fiabilité  
(CEI 61810-2:2011)Elektromechanische Elementarrelais -  
Teil 2: Funktionsfähigkeit (Zuverlässigkeit)  
(IEC 61810-2:2011)

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**CENELEC**European Committee for Electrotechnical Standardization  
Comité Européen de Normalisation Electrotechnique  
Europäisches Komitee für Elektrotechnische Normung**Management Centre: Avenue Marnix 17, B - 1000 Brussels**

## Foreword

The text of document 94/316/FDIS, future edition 2 of IEC 61810-2, prepared by IEC TC 94, All-or-nothing electrical relays, was submitted to the IEC-CENELEC parallel vote and was approved by CENELEC as EN 61810-2 on 2011-04-01.

This European Standard supersedes EN 61810-2:2005.

The main changes with respect to EN 61810-2:2005 are listed below:

- inclusion of both numerical and graphical methods for Weibull evaluation;
- establishment of full coherence with the second edition of the basic reliability standard EN 61649;
- deletion of previous Annex A and Annex D since both annexes are contained in EN 61810-1.

This standard is to be used in conjunction with EN 61649:2008.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN and CENELEC shall not be held responsible for identifying any or all such patent rights.

The following dates were fixed:

- |  |       |            |
|--|-------|------------|
| – latest date by which the EN has to be implemented at national level by publication of an identical national standard or by endorsement | (dop) | 2012-01-01 |
| – latest date by which the national standards conflicting with the EN have to be withdrawn   | (dow) | 2014-04-01 |

Annex ZA has been added by CENELEC.

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## Endorsement notice

The text of the International Standard IEC 61810-2:2011 was approved by CENELEC as a European Standard without any modification.

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## Annex ZA (normative)

### Normative references to international publications with their corresponding European publications

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE When an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

| <u>Publication</u> | <u>Year</u> | <u>Title</u>   | <u>EN/HD</u> | <u>Year</u> |
|--------------------|-------------|--|--------------|-------------|
| IEC 60050-191      | 1990        | International Electrotechnical Vocabulary (IEV) - Chapter 191: Dependability and quality of service                  | -            | -           |
| IEC 60050-444      | 2002        | International Electrotechnical Vocabulary - Part 444: Elementary relays  | -            | -           |
| IEC 60300-3-5      | 2001        | Dependability management - Part 3-5: Application guide - Reliability test conditions and statistical test principles | -            | -           |
| IEC 61649          | 2008        | Weibull analysis   | EN 61649     | 2008        |
| IEC 61810-1        | 2008        | Electromechanical elementary relays - Part 1: General requirements   | EN 61810-1   | 2008        |

## CONTENTS

|   |    |
|---|----|
| INTRODUCTION .....  | 5  |
| 1 Scope .....   | 6  |
| 2 Normative references .....  | 6  |
| 3 Terms and definitions .....   | 7  |
| 4 General considerations .....  | 9  |
| 5 Test conditions .....   | 10 |
| 5.1 Test items .....  | 10 |
| 5.2 Environmental conditions .....  | 10 |
| 5.3 Operating conditions .....  | 10 |
| 5.4 Test equipment .....  | 11 |
| 6 Failure criteria .....  | 11 |
| 7 Output data .....   | 11 |
| 8 Analysis of output data .....   | 11 |
| 9 Presentation of reliability measures .....                                    | 12 |
| Annex A (normative) Data analysis .....   | 13 |
| Annex B (informative) Example of numerical and graphical Weibull analysis ..... | 22 |
| Annex C (informative) Example of cumulative hazard plot .....                   | 26 |
| Annex D (informative) Gamma function .....                                      | 32 |
| Bibliography .....  | 33 |
| <br>  |    |
| Figure A.1 – An example of Weibull probability paper .....                      | 16 |
| Figure A.2 – An example of cumulative hazard plotting paper .....               | 18 |
| Figure A.3 – Plotting of data points and drawing of a straight line .....       | 18 |
| Figure A.4 – Estimation of distribution parameters .....                        | 19 |
| Figure B.1 – Weibull probability chart for the example .....                    | 24 |
| Figure C.1 – Estimation of distribution parameters .....                        | 28 |
| Figure C.2 – Cumulative hazard plots .....                                      | 30 |
| <br>  |    |
| Table B.1 – Ranked failure data .....   | 23 |
| Table C.1 – Work sheet for cumulative hazard analysis .....                     | 26 |
| Table C.2 – Example work sheet .....  | 29 |
| Table D.1 – Values of the gamma function .....                                  | 32 |

## INTRODUCTION

Within the IEC 61810 series of basic standards covering elementary electromechanical relays, IEC 61810-2 is intended to give requirements and tests permitting the assessment of relay reliability. All information concerning endurance tests for type testing have been included in IEC 61810-1.

NOTE According to IEC 61810-1, a specified value for the electrical endurance under specific conditions (e.g. contact load) is verified by testing 3 relays. None is allowed to fail. Within this IEC 61810-2, a prediction of the reliability of a relay is performed using statistical evaluation of the measured cycles to failure of a larger number of relays (generally 10 or more relays).

Recently the technical committee responsible for dependability (TC 56) has developed a new edition of IEC 61649 dealing with Weibull distributed test data. This second edition contains both numerical and graphical methods for the evaluation of Weibull-distributed data.

On the basis of this basic reliability standard, IEC 61810-2 was developed. It comprises test conditions and an evaluation method to obtain relevant reliability measures for electromechanical elementary relays. The life of relays as non-repairable items is primarily determined by the number of operations. For this reason, the reliability is expressed in terms of mean cycles to failure (MCTF).

Commonly, equipment reliability is calculated from mean time to failure (MTTF) figures. With the knowledge of the frequency of operation (cycling rate) of the relay within an equipment, it is possible to calculate an effective MTTF value for the relay in that application.

Such calculated MTTF values for relays can be used to calculate respective reliability, probability of failure, and availability (e.g. MTBF (mean time between failures)) values for equipment into which these relays are incorporated.

Generally it is not appropriate to state that a specific MCTF value is “high” or “low”. The MCTF figures are used to make comparative evaluations between relays with different styles of design or construction, and as an indication of product reliability under specific conditions.

# ELECTROMECHANICAL ELEMENTARY RELAYS –

## Part 2: Reliability

### 1 Scope

This part of IEC 61810 covers test conditions and provisions for the evaluation of endurance tests using appropriate statistical methods to obtain reliability characteristics for relays. It should be used in conjunction with IEC 61649.

This International Standard applies to electromechanical elementary relays considered as non-repaired items (i.e. items which are not repaired after failure), whenever a random sample of items is subjected to a test of cycles to failure (CTF).

The lifetime of a relay is usually expressed in number of cycles. Therefore, whenever the terms “time” or “duration” are used in IEC 61649, this term should be understood to mean “cycles”. However, with a given frequency of operation, the number of cycles can be transformed into respective times (e.g. times to failure (TTF)).

The failure criteria and the resulting characteristics of elementary relays describing their reliability in normal use are specified in this standard. A relay failure occurs when the specified failure criteria are met.

As the failure rate for elementary relays cannot be considered as constant, particularly due to wear-out mechanisms, the times to failure of tested items typically show a Weibull distribution. This standard provides both numerical and graphical methods to calculate approximate values for the two-parameter Weibull distribution, as well as lower confidence limits.

### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 60050-444:2002, *International Electrotechnical Vocabulary (IEV) – Part 444: Elementary relays*

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 61649:2008, *Weibull analysis*

IEC 61810-1:2008, *Electromechanical elementary relays – Part 1: General requirements*



### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-191 and IEC 60050-444, some of which are reproduced below, as well as the following, apply.

#### 3.1

##### **item**

any component that can be individually considered

[IEC 60050-191:1990, 191-01-01, modified]

NOTE For the purpose of this standard, items are elementary relays.

#### 3.2

##### **non-repaired item**

item which is not repaired after a failure

[IEC 60050-191:1990, 191-01-03, modified]

#### 3.3

##### **cycle**

operation and subsequent release/reset

[IEC 60050-444:2002, 444-02-11]

#### 3.4

##### **frequency of operation**

number of cycles per unit of time

[IEC 60050-444:2002, 444-02-12]

#### 3.5

##### **reliability**

ability of an item to perform a required function under given conditions for a given number of cycles or time interval

[IEC 60050-191:1990, 191-02-06, modified]

NOTE 1 It is generally assumed that the item is in a state to perform this required function at the beginning of the time interval.

NOTE 2 The term “reliability” is also used as a measure of reliability performance (see IEC 60050-191:1990, 191-12-01).

#### 3.6

##### **reliability test**

experiment carried out in order to measure, quantify or classify a reliability measure or property of an item

[IEC 60300-3-5:2001, 3.1.27]

#### 3.7

##### **life test**

test with the purpose of estimating, verifying or comparing the lifetime of the class of items being tested

[IEC 60300-3-5:2001, 3.1.17, modified]

**3.8  
cycles to failure****CTF**

total number of cycles of an item, from the instant it is first put in an operating state until failure

**3.9  
mean cycles to failure****MCTF**

expectation of the number of cycles to failure

**3.10  
time to failure****TTF**

total time duration of operating time of an item, from the instant it is first put in an operating state until failure

[IEC 60050-191:1990, 191-10-02, modified]

**3.11  
mean time to failure****MTTF**

expectation of the time to failure

[IEC 60050-191:1990, 191-12-07]

**3.12  
useful life**

number of cycles or time duration until a certain percentage of items have failed

NOTE In this standard, this percentage is defined as 10 %.

**3.13  
failure**

termination of the ability of an item to perform a required function

[IEC 60050-191:1990, 191-04-01, modified]

**3.14  
malfunction**

single event when an item does not perform a required function

**3.15  
contact failure**

occurrence of break and/or make malfunctions of a contact under test, exceeding a specified number

**3.16  
failure criteria**

set of rules used to decide whether an observed event constitutes a failure

[IEC 60300-3-5:2001, 3.1.10]

**3.17  
contact load category**

classification of relay contacts dependent on wear-out mechanisms

NOTE Various contact load categories are defined in IEC 61810-1.

## 4 General considerations

The provisions of this part of IEC 61810 are based on the relevant publications on dependability. In particular, the following documents have been taken into account: IEC 60050-191, IEC 60300-3-5 and IEC 61649.

The aim of reliability testing as given in this standard is to obtain objective and reproducible data on reliability performance of elementary relays representative of standard production quality. The tests described and the related statistical tools to gain reliability measures based on the test results can be used for the estimation of such reliability measures, as well as for the verification of stated measures.

NOTE 1 Examples for the application of reliability measurements are:

- establishment of reliability measures for a new relay type;
- comparison of relays with similar characteristics, but produced by different manufacturers;
- evaluation of the influence, on a relay, of different materials or different manufacturing solutions;
- comparison of a new relay with a relay which has already worked for a specific period of time;
- calculation of the reliability of an equipment or system incorporating one or more relays.

According to Clauses 8 and 9 of IEC 60300-3-5, for non-repaired items showing a non-constant failure rate the Weibull model is the most appropriate statistical tool for evaluation of reliability measures. This analysis procedure is described in IEC 61649.

Elementary relays within the scope of this standard are considered as non-repaired items. They generally do not exhibit a constant failure rate but a failure rate increasing with time, being tested until wear-out mechanisms become predominant. The cycles to failure of a random sample of tested items typically show the Weibull distribution.

NOTE 2 In cases where no wear-out mechanisms prevail, random failures with constant failure rate can be assumed. Then the shape parameter  $\beta$  of the Weibull distribution equals 1 and the reliability function becomes the well-known exponential law. For relay tests where only very few failures (or even no failures at all) occur, the WeiBayes approach of IEC 61649 might be appropriate. Another option may be the application of the sudden death method described in Clause 13 of IEC 61649.

The statistical procedures of this standard are valid only when at least 10 relevant failures are recorded.

Upon special agreement between manufacturer and user, the test may be performed with even less than 10 relays, provided the uncertainty of the estimated Weibull parameters is acceptable to them. In such a case the minimum number of tested relays shall be specified; this number then replaces the minimum number of 10 relays wherever prescribed in this standard. However, it shall be noted that this reduction of relay specimens is only acceptable where the graphical methods of A.5.1 are applied. For the numerical method of A.5.2 at least 10 failures are required, since the maximum likelihood estimation (MLE) is a computational method for larger sample sizes, i.e. when at least 10 relevant failures are recorded (see 9.3 of IEC 61649).

The first step in the analysis of the recorded cycles to failure (CTF) of the tested relays is the determination of the two distribution parameters of the Weibull distribution. In a second step, the mean cycles to failure (MCTF) is calculated as a point estimate. In a third step, the useful life is determined as the lower confidence limit of the number of cycles by which 10 % of the relay population will have failed ( $B_{10}$ ).

With a given frequency of operation these reliability measures expressed in number of cycles (MCTF) can be transformed into respective times (MTTF), see Annex B for an example.

The statistical procedures require some appropriate computing facility. Software for evaluation of Weibull distributed data is commercially available on the market. Such software

may be used for the purpose of this standard provided it shows equivalent results when the data given in Annex B are used.

Since the number of cycles to failure highly depends on the specific set of test conditions (particularly the electrical loading of the relay contacts), values for MCTF and useful life derived from test data apply only to this set of test conditions, which have to be stated by the manufacturer together with the reliability measures.

## **5 Test conditions**

### **5.1 Test items**

As a minimum of 10 failures need to be recorded to perform the analysis described in this standard, 10 or more items (relays) should be submitted to the test. This allows the test to be truncated when at least 10 relays have failed. When the test is truncated at a specific number of cycles, all relays that have not yet failed may be considered to fail at that number of cycles (worst case assumption). However, at least 70 % of the tested relays shall fail physically. This allows the test to be carried out with 10 relays only, even when the test is truncated before all relays have physically failed (with a minimum of 7 physical failures recorded).

The items shall be selected at random from the same production lot and shall be of identical type and construction. No action is allowed on the test items from the time of sampling until the test starts.

Where any particular burn-in procedure or reliability stress screening is employed by the manufacturer prior to sampling, this shall apply to all production. The manufacturer shall describe and declare such procedures, together with the test results.

Unless otherwise specified by the manufacturer, all contacts of each relay under test shall be loaded as stated and monitored continuously during the test.

The test starts with all items and is stopped at some number of cycles. At that instant a certain number of items (minimum: 10 items) have failed. The number of cycles to failure of each of the failed items is recorded.

Items failed during the test are not replaced once they fail.

### **5.2 Environmental conditions**

The testing environment shall be the same for all items.

- The items shall be mounted in the manner intended for normal service; in particular, relays for mounting onto printed circuit-boards are tested in the horizontal position, unless otherwise specified.
- The ambient temperature shall be as specified by the manufacturer.
- All other influence quantities shall comply with the values and tolerance ranges given in Table 1 of IEC 61810-1, unless otherwise specified.

### **5.3 Operating conditions**

The set of operating conditions

- rated coil voltage(s);
- coil suppression (if any);
- frequency of operation;
- duty factor;

- contact load(s)

shall be as specified by the manufacturer.

Recommended values should be chosen from those given in Clause 5 of IEC 61810-1.

The test is performed on each contact load and each contact material as specified by the manufacturer.

All specified devices (for example, protective or suppression circuits), if any, which are part of the relay or stated by the manufacturer as necessary for particular contact loads, should be operated during the test.

The contacts shall be continuously monitored to detect malfunctions to open and malfunctions to close, as well as unintended bridging (simultaneous closure of make and break side of a changeover contact).

The contacts are connected to the load(s) in accordance with Table 12 of IEC 61810-1 as specified and indicated by the manufacturer.

#### **5.4 Test equipment**

The test circuit described in Annex C of IEC 61810-1 shall be used, unless otherwise specified by the manufacturer and explicitly indicated in the test report.

### **6 Failure criteria**

Whenever any contact of a relay under test fails to open or fails to close or exhibits unintended bridging, this shall be considered as a malfunction.

Three severity levels are specified:

- severity A: the first detected malfunction is defined as a failure;
- severity B: the sixth detected malfunction or two consecutive malfunctions are defined as a failure;
- severity C: as specified by the manufacturer.

The severity level used for the test shall be as prescribed by the manufacturer and stated in the test report.

### **7 Output data**

The data to be analysed consists of cycles to failure (CTF) for each of the items put on test. These CTF values have to be known exactly. However, it is not necessary to gather the CTF values for all items under test, as the test may be stopped before all items have failed, provided at least 10 CTF values from different failed items are available.

### **8 Analysis of output data**

The evaluation of the CTF values obtained during the test shall be carried out in accordance with the procedures given in Annex A.

## 9 Presentation of reliability measures

The basic reliability measures applicable to elementary relays as described in this standard and obtained from the data analysis shall be provided.

However, since the values obtained for these reliability measures using the procedures of Annex A depend to a great extent on the basic design characteristics of the relay, the test conditions of Clause 5 and the failure criteria of Clause 6, the following information shall also be provided together with the test results:

- relay type for which the results are valid:
  - a) contact material;
  - b) deviations from standard types (if any);
  - c) type of termination;
- set of operating conditions (see 5.3):
  - a) rated coil voltage(s);
  - b) coil suppression (if any);
  - c) frequency of operation;
  - d) duty factor;
  - e) contact load(s);
  - f) ambient conditions;
- test schematic selected (see Clause C.3 of IEC 61810-1, or test circuit details, if different from the circuit described in Clause C.1 of IEC 61810-1);
- severity level (see Clause 6).

In addition basic data of the test and the related analysis (see Annex A) shall be given in the test report:

- number of items ( $n$ ) on test;
- number of failed items ( $r$ ) registered during the test (minimum 10);
- time (given in number of cycles) when the test was stopped ( $T$ );
- confidence level, if other than 90 %.

The test results are applicable to the samples specifically tested and variants, as stipulated by the manufacturer, provided that the relevant design characteristics remain the same.

NOTE Acceptable examples are coil variants with the same ampere-turns. Unacceptable examples are variants with AC in place of DC coils, or different contact dynamics.

When test results for various operating conditions (for example, contact loads) are available, they may be compiled as a family of curves or in suitable tables. However, it shall be ensured that a sufficient number of points are determined when plotting such curves.

## **Annex A** (normative)

### **Data analysis**

#### **A.1 General**

This annex has been derived from the reliability standard IEC 61649:2008 with certain modifications necessary to adopt the procedures to elementary relays. The distribution considered in the reliability standard is of the Weibull type, which has been empirically recognized to correspond to an appropriate data analysis for elementary relays.

The graphical method, as well as the numerical method are covered in IEC 61649. In addition, not only the Weibull probability analysis but also the Weibull hazard analysis is taken up in the graphical method. Here, Weibull hazard and Weibull probability analyses are applied to complete and incomplete data, respectively. The latter is especially useful for the reliability analysis of relays because many data sets obtained from life tests are incomplete (censored tests).

NOTE 1 Incomplete data are the data sets obtained from the test after either a certain number of failures or a certain number of cycles, when there are still items functioning, whereas complete data are the data sets without censoring.

This annex deals with the Weibull probability plot and the Weibull hazard plot for the graphical method based upon median rank regression (MRR) principles, and the maximum likelihood estimation (MLE) for the numerical method in accordance with the provisions of IEC 61649.

When more in-depth information is required, IEC 61649 is to be consulted.

The concept “time” is to be understood as “cycles” in the case of relays. However, with a given frequency of operation, the values indicated in numbers of cycles can be transformed into respective times.

NOTE 2 Whereas the variable “time” (symbol:  $t$ ) is used within IEC 61649, this standard therefore is based on the variable “cycles” (symbol:  $c$ ).

For the sake of consistency, the following symbols and equations are reproduced in accordance with IEC 61649.

#### **A.2 Abbreviations**

|      |                                  |
|------|----------------------------------|
| CDF  | Cumulative distribution function |
| MRR  | Median rank regression           |
| MLE  | Maximum likelihood estimation    |
| MCTF | Mean cycles to failure           |
| PDF  | Probability density function     |

#### **A.3 Symbols and definitions**

The following symbols are used in this Annex A, and in both Annex B and Annex C. Auxiliary constants and functions are defined in the text.

|        |  |
|--------|--|
| $f(c)$ | probability density function                           |
| $F(c)$ | cumulative distribution function (failure probability) |

|           |   |
|-----------|---|
| $h(c)$    | hazard function (or instantaneous failure rate)   |
| $H(c)$    | cumulative hazard function  |
| $R(c)$    | reliability function of the Weibull distribution (survival probability)                             |
| $B_{10}$  | expected time at which 10 % of the population have failed<br>(10 % fractile of the lifetime)        |
| $c$       | cycle – variable  |
| $\hat{m}$ | mean cycles to failure (MCTF)   |
| $\beta$   | Weibull shape parameter (indicating the rate of change of the instantaneous failure rate with time) |
| $\eta$    | Weibull scale parameter or characteristic life (at which 63,2 % of the items have failed)           |
| $\sigma$  | standard deviation  |

#### A.4 Weibull distribution

The fundamental Weibull formulae are defined as follows.

NOTE For more information, reference is made to IEC 61649.

The probability density function (PDF) of the Weibull distribution is:

$$f(c) = \beta \frac{c^{\beta-1}}{\eta^\beta} e^{-\left(\frac{c}{\eta}\right)^\beta} \quad (\text{A.1})$$

The cumulative distribution function (CDF), or the expected fraction failing at cycle  $c$ :

$$F(c) = 1 - e^{-(c/\eta)^\beta} \quad (\text{A.2})$$

The reliability function  $R(c)$ , or the expected fraction surviving at cycle  $c$ :

$$R(c) = 1 - F(c) = e^{-(c/\eta)^\beta} \quad (\text{A.3})$$

The hazard function (or instantaneous failure rate)  $h(c)$  is:

$$h(c) = \beta \frac{c^{\beta-1}}{\eta^\beta} \quad (\text{A.4})$$

The cumulative hazard function  $H(c)$  is:

$$H(c) = \left(\frac{c}{\eta}\right)^\beta \quad (\text{A.5})$$

#### A.5 Procedure

##### A.5.1 Graphical methods

##### A.5.1.1 Overview

Graphical analysis is performed by plotting the data on a suitably designed Weibull probability paper, fitting a straight line through the data, and estimating the distribution parameters (the



shape parameter, and the characteristic life or scale parameter). Then the reliability characteristics (i.e. MCTF,  $B_{10}$  value, and standard deviation) are calculated.

Graphical methods benefit from relatively straightforward processes and availability for data with a mixture of failure modes. The fundamentals of the analysis and an outline of the processes applied to Weibull probability and Weibull hazard plots are given in this clause.

### **A.5.1.2 Weibull probability plot**

#### **A.5.1.2.1 Ranking and plotting positions**

To make the Weibull plot, rank the data from the lowest to the highest number of cycles to failure ( $c_i$ ). This ranking will set up the plotting positions for the cycle ( $c$ ), axis and the ordinate, cumulative distribution function ( $F(c)$ ), in percentage values.

$F(c)$  is calculated by median rank regression (MRR).

An approximate value may be obtained using Benard's approximation (see 7.2.1 of IEC 61649:2008):

$$F(c_i) = (i - 0,3) / (n + 0,4) \% \quad (\text{A.6})$$

where

$n$  is the number of tested items;

$i$  is the ranked position of the data item.

Data points of  $(c_i, F(c_i))$  are plotted on the Weibull probability plotting paper.

For details, see 7.2.1 and 7.2.2 of IEC 61649:2008.

#### **A.5.1.2.2 Weibull probability plotting paper**

The design of Weibull probability paper is shown below.

The equation (A.3) can be rewritten to the following equation:

$$\frac{1}{1 - F(c)} = e^{(c/\eta)^\beta} \quad (\text{A.7})$$

Taking normal logarithms of both sides of the equation (A.7) twice gives an equation of a straight line as shown below:

$$\ln \ln \frac{1}{1 - F(c)} = \beta \ln c - \beta \ln \eta \quad (\text{A.8})$$

The equation is a straight line of the form  $y = ax + b$ . Weibull paper is designed by plotting the cumulative probability of failure using a log log reciprocal scale against  $c$  on a log scale. When the equation is plotted as a function of  $\ln(c)$ , the slope of the straight line plotted in this manner will be  $\beta$ , the shape parameter, i.e.

$$y = \ln \ln \frac{1}{1 - F(c)} \tag{A.9}$$

where

$$a = \beta;$$

$$x = \ln (c);$$

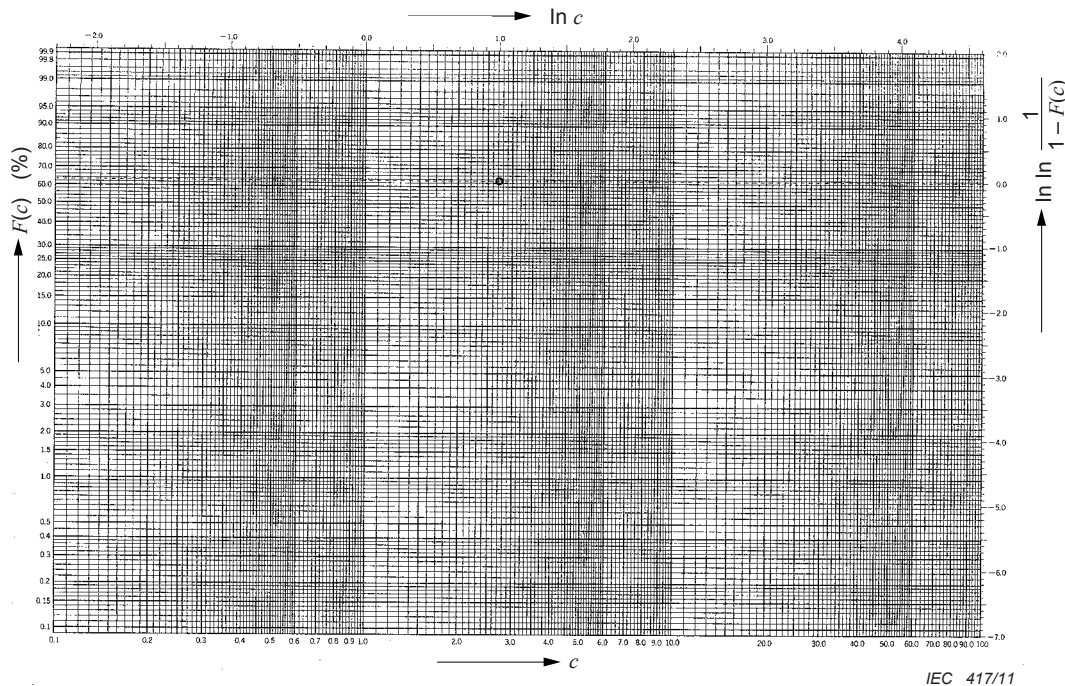
$$b = - \beta \ln (\eta).$$

The scale parameter is obtained from  $b = - \beta \ln (\eta)$  as follows:

$$\eta = \exp [- b_0/\beta] \tag{A.10}$$

where  $b_0$  is the value of  $y$  when  $c$  is equal to 1, that is  $\ln(c) = 0$ .

When data are following a Weibull distribution, those data plotted on a Weibull distribution paper become a straight line. Figure A.1 shows a blank Weibull distribution paper.



NOTE Partially rewritten on the basis of the paper published by JUSE PRESS

Figure A.1 – An example of Weibull probability paper

**A.5.1.3 Hazard plot**

**A.5.1.3.1 Ranking and plotting positions**

To perform the hazard plot, rank the data from the lowest to the highest number of cycles to failure. This ranking will set up the plotting positions for the cycle ( $c$ ), axis and the ordinate, cumulative hazard value  $H(c)$ , in percentage values.  $H(c)$  is calculated by hazard value  $h(c)$ . Data points of  $(c_i, H(c_i))$  are plotted on the cumulative hazard paper.

For details, see 7.3 of IEC 61649:2008.

#### **A.5.1.3.2 Cumulative hazard plotting paper**

The design of cumulative hazard paper is shown below.

Taking natural logarithms of both sides of equation (A.5) gives:

$$\ln H(c) = \beta \ln c - \beta \ln \eta \quad (\text{A.11})$$

The equation is a straight line of the form  $y = ax + b$ . Cumulative hazard paper is designed by plotting the cumulative probability of failure using a log reciprocal scale against  $c$  on a log scale. When the equation is plotted as a function of  $\ln(c)$ , the slope of the straight line plotted in this manner will be  $\beta$ , the shape parameter, i.e.

$$y = \ln H(c) \quad (\text{A.12})$$

where

$$a = \beta;$$

$$x = \ln (c);$$

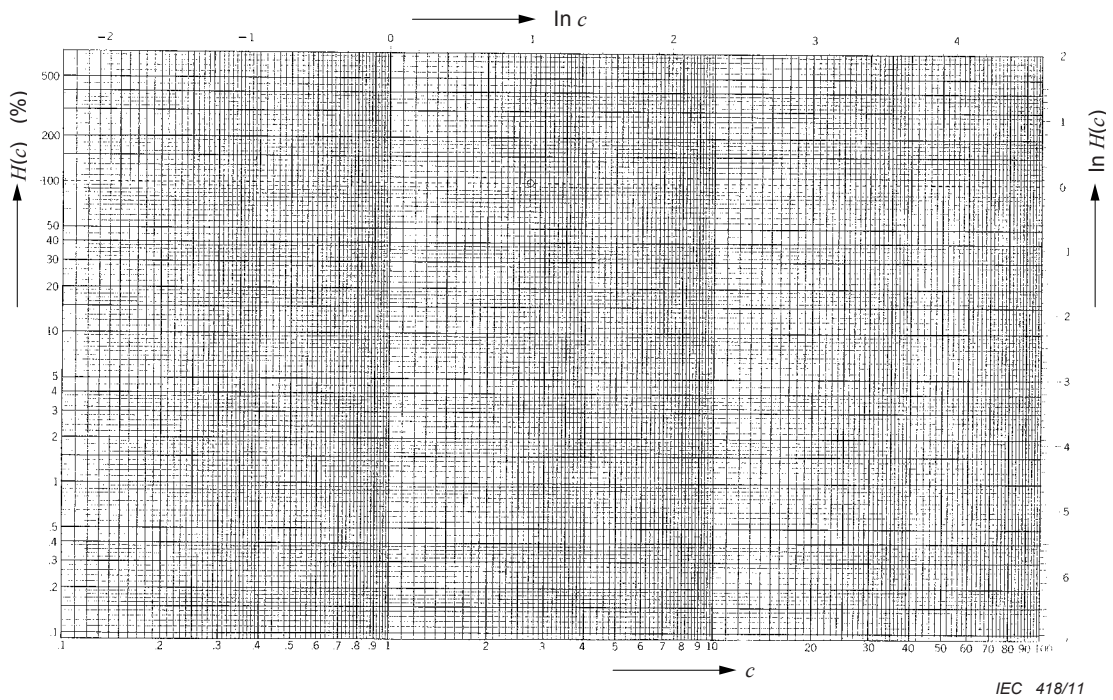
$$b = - \beta \ln (\eta).$$

The scale parameter is obtained from  $b = - \beta \ln (\eta)$  as follows:

$$\eta = \exp [- b_0/\beta] \quad (\text{A.13})$$

where  $b_0$  is the value of  $y$  when  $c$  is equal to 1, that is  $\ln(c) = 0$ .

When data points are following a cumulative hazard function, those data points plotted on a cumulative hazard paper become a straight line. Figure A.2 shows a blank cumulative hazard paper.



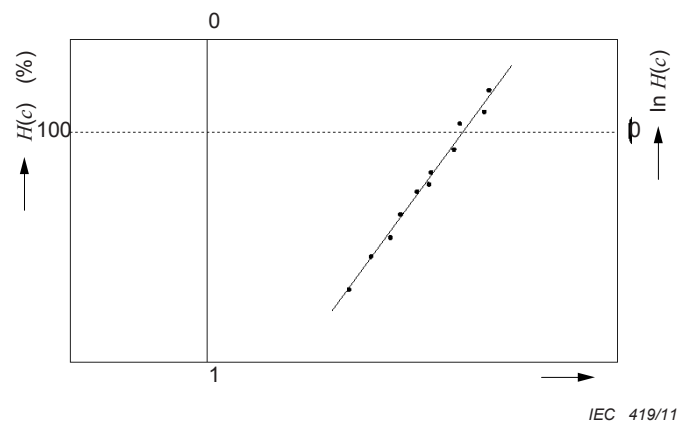
NOTE Partially rewritten on the basis of the paper published by JUSE PRESS

**Figure A.2 – An example of cumulative hazard plotting paper**

**A.5.1.4 Estimate values of distribution parameters and characteristics**

Distribution parameters and characteristics in the Weibull probability plot and the hazard plot are common.

Draw a straight line (that best fits the data) through the data points on the plotting paper (Figure A.3).



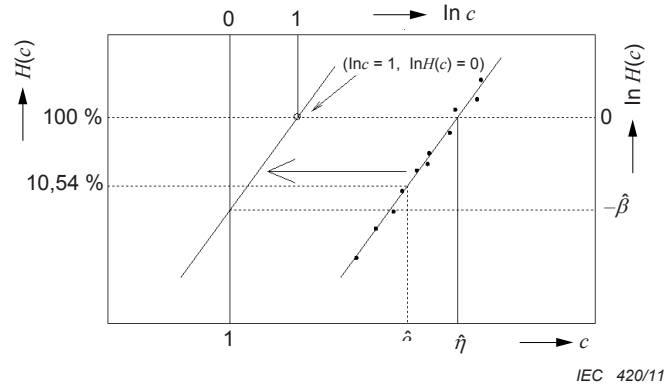
**Figure A.3 – Plotting of data points and drawing of a straight line**

- 1) The point estimate of the shape parameter,  $\hat{\beta}$

$\hat{\beta}$  is derived from the slope  $a$  of the plotted straight line.

A parallel line is drawn above the original plotted line, through the coordinate point  $(\ln c = 1, \ln(H(c) = 0))$ . The ordinate value of this point is equivalent to  $H(c) = 100\%$  (or  $F(c) = 63,2\%$ ).

$\hat{\beta}$  is read from the value of  $\ln H(c)$  corresponding to the cross point of this parallel line and a vertical line through  $\ln c = 0$ , as shown in Figure A.4.



**Figure A.4 – Estimation of distribution parameters**

2) The point estimate of the scale parameter,  $\hat{\eta}$

$\hat{\eta}$  is derived directly from the cross point of the original plotted line and a horizontal line through  $H(c) = 100\%$  (or  $F(c) = 63,2\%$ ) as shown in Figure A.4.

3) The point estimate of mean cycles to failure (MCTF),  $\hat{m}$

$\hat{m}$  is given by the following expression:

$$\text{MCTF} = \hat{m} = \hat{\eta} \times \Gamma(1+1/\hat{\beta}) \quad (\text{A.14})$$

with  $\hat{\eta}$  taken from step 2 above, and the gamma function value ( $\Gamma$  as defined e.g. in 2.56 of ISO 3534-1:2006) obtained with a handy scientific calculator or a convenient gamma functional table, respectively (see Annex D).

4) The point estimate of standard deviation,  $\hat{\sigma}$

$\hat{\sigma}$  is given by the following expression:

$$\hat{\sigma} = \hat{\eta} \times \sqrt{\Gamma(1+2/\hat{\beta}) - \Gamma^2(1+1/\hat{\beta})} \quad (\text{A.15})$$

Where  $\hat{\eta}$  and the gamma function value are obtained in the same way as mentioned under step 3 above.

5) The point estimate of the fractile (10 %) of the cycles to failure,  $B_{10}$

$B_{10}$  is derived directly from the cross point of the original plotted straight line and a horizontal line through  $F(c) = 10\%$  in the Weibull plot or  $H(c) = -\ln 0,9 = 10,54\%$  in the hazard plot as shown in Figure A.4.

#### A.5.1.5 Mixture of several failure modes

The Weibull probability plot or a hazard plot can result in a “dogleg curve”.

If the line is not straight, it is called “Dogleg Weibull”. This is caused by a mixture of more than one failure mode, i.e. usually competitive failure modes.

When this occurs, a close examination of the failed items is the best way to separate the data into different failure modes.

Suppose there is a data set of two kinds of failure modes (A and B). The first set should be analyzed as A mode data only, suspending the B mode data. Consequently, the second set would contain B mode data. These two sets of data can be used to predict the failure distribution.

If this is done correctly, plotting the two separate data sets will result in straight lines. A detailed description is shown in Annex G of IEC 61649:2008. In particular, it has to be noted that at least 10 failures are required for each failure mode.

### A.5.2 Numerical methods

#### A.5.2.1 Distribution parameters

Whereas the graphical method described in A.5.1 above applies to complete, single censored, or multiple censored data, the numerical method of this subclause does not deal with multiple censored data.

The estimate for the two parameters of the Weibull distribution is obtained by numerically solving the equations below. The value of  $\beta$  that satisfies the first equation is the maximum likelihood estimation (MLE) of  $\beta$ . This value is used in the second equation to derive the MLE of  $\eta$ .

NOTE 1 Any appropriate computer routine to solve equations can be used to obtain  $\beta$  from the first equation, as the convergence to a single value is usually very fast.

NOTE 2 Refer to IEC 61649 for interval estimation, lower limit, etc. of  $\beta$  and  $\eta$ . For the meaning of  $\beta <, =, >$  see Clause 8 of IEC 61649:2008.

Step 1 – Find the value of  $\beta$  that satisfies the equation below. The solution to this equation is the point estimate of the Weibull shape parameter  $\hat{\beta}$ .

$$\left[ \frac{\sum_{i=1}^r c_i^\beta \ln(c_i) + (n-r)C^\beta \ln(C)}{\sum_{i=1}^r c_i^\beta + (n-r)C^\beta} - \frac{1}{\beta} \right] - \frac{1}{r} \sum_{i=1}^r \ln(c_i) = 0 \quad (\text{A.16})$$

where

$n$  is the number of tested items;

$r$  is the number of failed items ( $i=1,2,\dots,r$  and  $r \leq n$ );

$C$  is the number of cycles when the test was stopped ( $0 < c_i \leq C$ ).

Step 2 – Compute  $\hat{\eta}$  using the value of  $\hat{\beta}$ , obtained in step 1, from:

$$\hat{\eta} = \left\{ \frac{1}{r} \left[ \sum_{i=1}^r c_i^{\hat{\beta}} + (n-r)C^{\hat{\beta}} \right] \right\}^{\frac{1}{\hat{\beta}}} \quad (\text{A.17})$$

#### A.5.2.2 Characteristics

##### A.5.2.2.1 Point estimate of mean cycles to failure MCTF, $\hat{m}$

$\hat{m}$  is calculated as:

$$\hat{m} = \hat{\eta} \Gamma(1 + 1/\hat{\beta}) \quad (\text{A.18})$$

where  $\hat{\beta}$  and  $\hat{\eta}$  are obtained from steps 1 and 2 in A.5.2.1 and the gamma function value  $\Gamma$  is defined in 2.56 of ISO 3534-1:2006. Alternatively, a suitable gamma function table may be used (see Annex D).

##### A.5.2.2.2 Standard deviation, $\hat{\sigma}$

$\hat{\sigma}$  is calculated as:

$$\hat{\sigma} = \hat{\eta} \times \sqrt{\Gamma(1 + 2/\hat{\beta}) - \Gamma^2(1 + 1/\hat{\beta})} \quad (\text{A.19})$$

##### A.5.2.2.3 Point estimate of the fractile (10 %) of cycles to failure, $\hat{B}_{10}$

$\hat{B}_{10}$  is calculated as:

$$\hat{B}_{10} = \hat{\eta} \left[ \ln\left(\frac{1}{0,9}\right) \right]^{1/\hat{\beta}} \quad (\text{A.20})$$

##### A.5.2.2.4 Point estimate of the reliability at cycle $c$

The calculation and indication of the relay reliability at cycle  $c$  is optional.

The point estimate of the reliability at cycle  $c$  is calculated as:

$$\hat{R}(c) = \exp\left[-(c/\hat{\eta})^{\hat{\beta}}\right] \quad (\text{A.21})$$

## Annex B (informative)

### Example of numerical and graphical Weibull analysis

#### B.1 General

This example is taken from Annex B of IEC 61649:2008 and adapted to the modifications necessary for elementary relays as indicated in Clause A.1 of this standard. It is provided as a numerical test case to verify the accuracy of computer programmes implementing the procedures of this standard. In order to demonstrate coherence with the graphical method for Weibull analysis, the given data are also plotted on Weibull probability paper.

Forty items are put under test. The test is stopped at the time of the 20th failure. The following are the number of cycles ( $\times 10^3$ ) corresponding to the first 20 failures:

| $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ | $t_{11}$ | $t_{12}$ | $t_{13}$ | $t_{14}$ | $t_{15}$ | $t_{16}$ | $t_{17}$ | $t_{18}$ | $t_{19}$ | $t_{20}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 5     | 10    | 17    | 32    | 32    | 33    | 34    | 36    | 54    | 55       | 55       | 58       | 58       | 61       | 64       | 65       | 65       | 66       | 67       | 68       |

Applying the numerical procedures of this standard yields the following results:

#### B.2 Distribution parameters

The maximum likelihood estimate (MLE) values for  $\beta$  and  $\eta$  are:

$$\hat{\beta} = 2,091 \text{ and } \hat{\eta} = 84 \times 10^3$$

#### B.3 Mean cycles to failure (MCTF)

The point estimate of the mean cycles to failure  $m$  is:

$$\hat{m} = 74,39 \times 10^3$$

#### B.4 Value of $\hat{B}_{10}$

The point estimate of  $B_{10}$ , the time (in number of cycles) by which 10 % of the population will have failed is:

$$\hat{B}_{10} = 28,63 \times 10^3$$

#### B.5 Mean time to failure (MTTF)

Only where an estimate of the number of cycles per unit of time appropriate to a specific end use is known, then a mean time to failure (MTTF) for the relay can be determined.

Example: If the number of cycles per unit of time is equal to 100 cycles per day and the relay MCTF value is  $74,39 \times 10^3$ , the MTTF for the relay in this application can be calculated as follows:

$$\text{MTTF} = \text{MCTF} / \text{Number of cycles per unit of time} = 74,39 \times 10^3 / 100 = 743,9 \text{ days}$$



## B.6 Graphical method (Weibull probability plot)

For the ranking of data, the same failure times (in number of cycles) as given above for the first 20 failures are taken.

According to A.5.1.2.1 the values for  $F(c_i)$  are calculated using Benard's approximation, see Table B.1.

**Table B.1 – Ranked failure data**

| Order number<br><i>i</i> | Failure time<br>$c_i$ [ $\times 10^3$ cycles] | Median rank<br>$F(c_i)$ [%] |
|--------------------------|---|-----------------------------|
| 1                        | 5   | 1,75                        |
| 2                        | 10  | 4,2                         |
| 3                        | 17  | 6,7                         |
| 4                        | 32  | 9,2                         |
| 5                        | 32  | 11,6                        |
| 6                        | 33  | 14,1                        |
| 7                        | 34  | 16,6                        |
| 8                        | 36  | 19,1                        |
| 9                        | 54  | 21,5                        |
| 10                       | 55  | 24,0                        |
| 11                       | 55  | 26,5                        |
| 12                       | 58  | 29,0                        |
| 13                       | 58  | 31,4                        |
| 14                       | 61  | 33,9                        |
| 15                       | 64  | 36,4                        |
| 16                       | 65  | 38,9                        |
| 17                       | 65  | 41,3                        |
| 18                       | 66  | 43,8                        |
| 19                       | 67  | 46,3                        |
| 20                       | 68  | 48,8                        |

The coordinates  $(c_i, F(c_i))$  of each failure are plotted on the Weibull probability paper, see Figure B.1.

In order to show consistency between the numerical and graphical methods, the original straight line is drawn with the values of the distribution parameters obtained from the numerical method (see B.2 above):

$$\hat{\beta} = 2,091 \text{ and } \hat{\eta} = 84 \times 10^3$$

This can be verified using the procedures described in A.5.1.4, see also Figure A.4.

From the cross point of the original plotted line and a horizontal line at  $F(c) = 10\%$ , the value for  $B_{10}$  is estimated as  $\hat{B}_{10} = 28 \times 10^3$  cycles, in line with the numerical result of B.4.

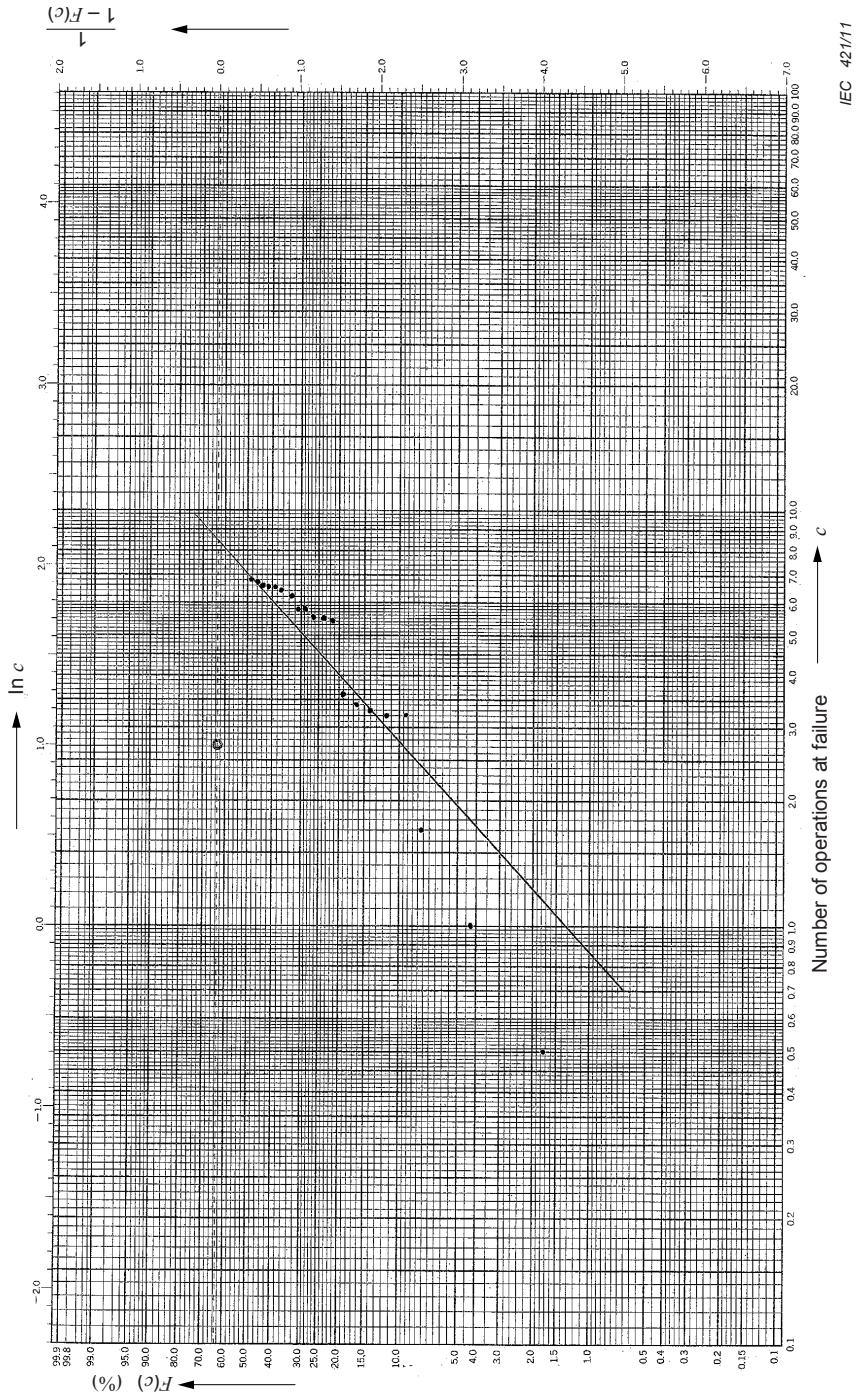


Figure B.1 – Weibull probability chart for the example

The plot shows a mixture of two failure modes, a low slope followed by a steep slope. Although the numerical method yields acceptable results, further analysis (see A.5.1.5) would be recommended. This illustrates the merit of plotting the data, not relying entirely on analytical methods.



#### Step 1

The ranking,  $i$  and the reverse ranking,  $K_i$  are entered in the respective columns. The value of  $K_i$  is calculated as follows:

$$K_i = n + 1 - i$$

where

$n$  is the number of tested items.

#### Step 2

Observed data are sorted from smallest to largest in order of cycles to failure, with the values for cycles to failure ( $c_i$ , corresponding to  $i$ ) filled in. The individual sample number is also entered in the column "No.", corresponding to  $c_i$ .

#### Step 3

The hazard values,  $h(c_i)$  are filled into the respective column corresponding to  $c_i$  and are calculated as follows:

$$h(c_i) = 1 / K_i \times 100(\%)$$

#### Step 4

If multiple failure modes appear, failure mode numbers are filled in the column of  $M_j$  corresponding to  $c_i$ . Here,  $j$  is the code number of a specific failure mode.

#### Step 5

Cumulative hazard values  $H_j(c_i)$  are filled in the respective column and each value is calculated according to the same failure mode ( $M_j$ ) as follows:

$$H_j(c_i) = \sum_{l \geq 1} h(c_l)$$

NOTE See Table C.2 for an example.

#### Step 6

Data points corresponding to ( $c_i, H_j(c_i)$ ) are plotted in a cumulative hazard chart. Then a straight line is drawn through the data points of each failure mode that best fits the data.

#### Step 7

If the distribution of data points is close to the straight line, proceed to C.2.3, as the result seems to be aligned with a Weibull distribution,  $\gamma = 0$ .

If it is difficult to draw the straight line, it might be better to review the failure modes and to carry out a detailed failure diagnosis of the relays used for the test, or to re-assess the test conditions, etc.

### C.2.3 Estimation of distribution parameters

Shape and scale parameters are derived from the plotting paper as follows:

- 1) The point estimate of the shape parameter,  $\hat{\beta}$

A parallel line is drawn above the original plotted line, through the coordinate point ( $\ln c = 1, \ln H(c) = 0$ ). The ordinate value of this point is equivalent to  $H(c) = 100\%$  (or  $F(c) = 63,2\%$ ).

$\hat{\beta}$  is read from the value of  $\ln H(c)$  corresponding to the cross point of this parallel line and a vertical line through  $\ln c = 0$ , as shown in Figure C.1.

- 2) The point estimate of the scale parameter,  $\hat{\eta}$

$\hat{\eta}$  is derived directly from the cross point of the original plotted line and a horizontal line through  $H(c) = 100\%$  (or  $F(c) = 63,2\%$ ) as shown in Figure C.1.

#### C.2.4 Estimation of distribution characteristics

The estimated values of the mean cycles to failure (MCTF)  $\hat{m}$ , the standard deviation  $\hat{\sigma}$  and the fractile (10 %) of cycles to failure  $\hat{B}_{10}$  are obtained as follows:

- 1) The point estimate of the mean cycles to failure (MCTF),  $\hat{m}$

$\hat{m}$  is obtained from equation (A.14) with the values of  $\hat{\eta}$  and  $\hat{\beta}$  from C.2.3 above and the gamma function value determined with a convenient scientific calculator or a suitable gamma function table.

- 2) The point estimate of the standard deviation,  $\hat{\sigma}$

$\hat{\sigma}$  is obtained in the same way from equation (A.15).

- 3) The point estimate of the fractile (10 %) of cycles to failure,  $\hat{B}_{10}$

$\hat{B}_{10}$  can be read from the value of  $c$  at the cross point of the original plotted line and a horizontal line through  $H(c) = 10,54\%$ , as shown in Figure C.1.

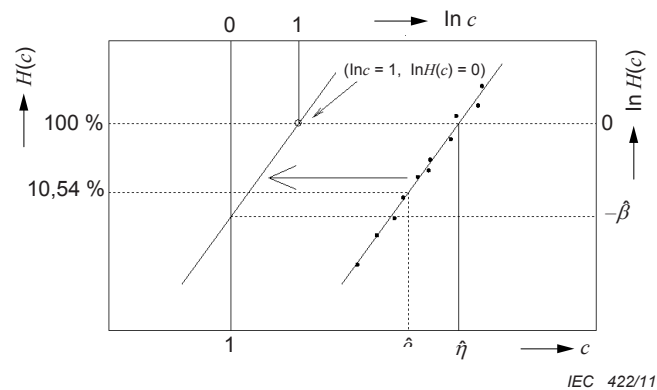


Figure C.1 – Estimation of distribution parameters

### C.3 Example applied to life test data

#### C.3.1 General

This example is provided to demonstrate the usefulness of reliability analysis by Weibull hazard plot based on life tests of elementary relays. Thirty items are put under test. The test is censored (truncated) at 1 240 000 cycles. The majority of items fail because of welding (failure mode 1) or erosion of contacts (failure mode 2).

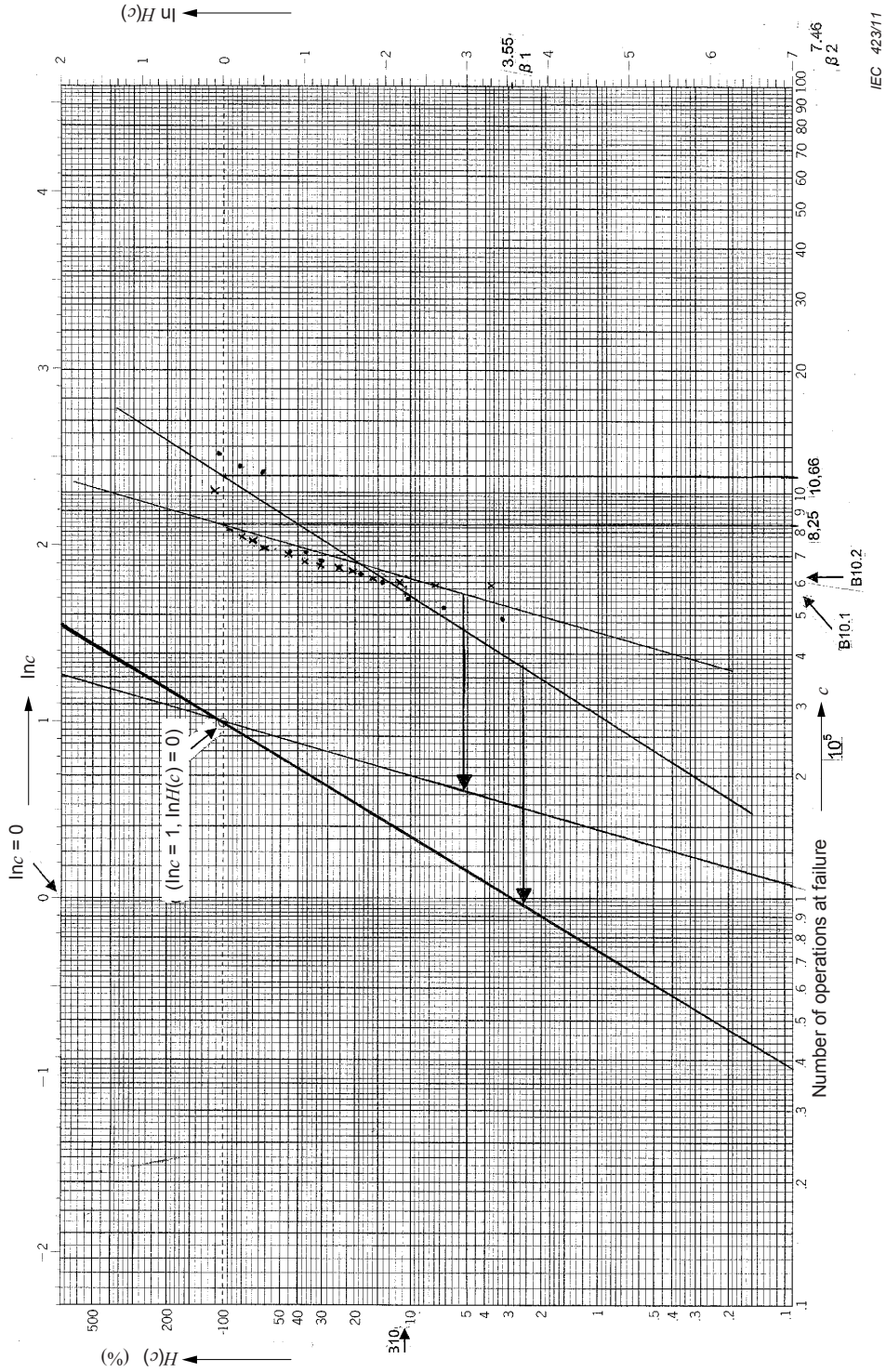
#### C.3.2 Ranking and plotting

The application of the procedure from step 1 to step 6 of C.2.2 for the work sheet and the hazard plot yields Table C.2 and Figure C.2.

**Table C.2 – Example work sheet**

| Sample No. | Rank | Reverse rank | KCycles( $c_i$ ) | Failure mode | Hazard value | Cumulative hazard value ( $H_j\%$ ) |         |
|------------|------|--------------|------------------|--------------|--------------|-------------------------------------|---------|
|            | $i$  | $Ki=n-i+1$   |                  | $M_j$        | $h\%$        | $M1$                                | $M2$    |
| 12         | 1    | 30           | 490              | 1            | 3,333        | 3,333                               | -       |
| 27         | 2    | 29           | 520              | 1            | 3,448        | 6,782                               | -       |
| 3          | 3    | 28           | 545              | 1            | 3,571        | 10,353                              | -       |
| 10         | 4    | 27           | 585              | 2            | 3,704        | -                                   | 3,704   |
| 6          | 5    | 26           | 585              | 2            | 3,846        | -                                   | 7,550   |
| 22         | 6    | 25           | 600              | 2            | 4,000        | -                                   | 11,550  |
| 18         | 7    | 24           | 600              | 1            | 4,167        | 14,520                              | -       |
| 17         | 8    | 23           | 605              | 2            | 4,348        | -                                   | 15,898  |
| 30         | 9    | 22           | 635              | 1            | 4,545        | 19,065                              | -       |
| 9          | 10   | 21           | 640              | 2            | 4,762        | -                                   | 20,660  |
| 23         | 11   | 20           | 645              | 2            | 5,000        | -                                   | 25,660  |
| 28         | 12   | 19           | 655              | 1            | 5,263        | 24,328                              | -       |
| 21         | 13   | 18           | 655              | 2            | 5,556        | -                                   | 31,216  |
| 5          | 14   | 17           | 670              | 2            | 5,882        | -                                   | 37,098  |
| 15         | 15   | 16           | 680              | 1            | 6,250        | 30,578                              | -       |
| 1          | 16   | 15           | 715              | 1            | 6,667        | 37,245                              | -       |
| 8          | 17   | 14           | 715              | 2            | 7,143        | -                                   | 44,241  |
| 2          | 18   | 13           | 715              | 1            | 7,692        | 44,937                              | -       |
| 20         | 19   | 12           | 730              | 2            | 8,333        | -                                   | 52,574  |
| 4          | 20   | 11           | 730              | 2            | 9,091        | -                                   | 61,665  |
| 19         | 21   | 10           | 765              | 2            | 10,000       | -                                   | 71,665  |
| 29         | 22   | 9            | 780              | 2            | 11,111       | -                                   | 82,776  |
| 11         | 23   | 8            | 815              | 2            | 12,500       | -                                   | 95,276  |
| 26         | 24   | 7            | 1 025            | 2            | 14,286       | -                                   | 109,562 |
| 25         | 25   | 6            | 1 120            | 1            | 16,667       | 61,604                              | -       |
| 24         | 26   | 5            | 1 160            | 1            | 20,000       | 81,604                              | -       |
| 16         | 27   | 4            | 1 240            | 1            | 25,000       | 106,604                             | -       |
| 14         | 28   | 3            | 1 240            | C            | -            | -                                   | -       |
| 7          | 29   | 2            | 1 240            | C            | -            | -                                   | -       |
| 13         | 30   | 1            | 1 240            | C            | -            | -                                   | -       |

Mode 1 = Welding  
Mode 2 = Contact erosion  
C = Censored



NOTE Using the hazard paper partially rewritten on the basis of the paper published by JUSE PRESS

Figure C.2 – Cumulative hazard plots



Distribution of this sample is a “dogleg” Weibull type. Data points corresponding to  $(c_i, H_j(c_i))$  are plotted using filled circles (●) for welding failures (mode 1), and crosses (x) for contact erosion failures (mode 2).

### C.3.3 Estimation of distribution parameters

Applying the procedures of C.2.3 yields the following results:

$$\hat{\beta}_1 = 3,55 \quad \hat{\eta}_1 = 1,066 \times 10^6$$

$$\hat{\beta}_2 = 7,46 \quad \hat{\eta}_2 = 8,25 \times 10^5$$

### C.3.4 Estimation of distribution characteristics

Applying the procedures of C.2.4 yields the following results:

$$\hat{m}_1 = 9,60 \times 10^5 \quad \hat{\sigma}_1 = 2,83 \times 10^5 \quad \hat{B}_{10>1} = 5,60 \times 10^5$$

$$\hat{m}_2 = 7,74 \times 10^5 \quad \hat{\sigma}_2 = 1,22 \times 10^5 \quad \hat{B}_{10>2} = 6,10 \times 10^5$$

## C.4 Reference document

H. Shiomi, T. Mitsuhashi, M. Saito, A Masuda, *How to use probability paper in reliability*, 1983 (only available in Japanese)

**Annex D**  
(informative)

**Gamma function**

The gamma function is defined in 2.56 of ISO 3534-1:2006.

Table D.1 gives the value of  $\Gamma(1+1/k)$  as a function of  $k$ . For  $k$  values not listed in this table, a linear interpolation is acceptable.

**Table D.1 – Values of the gamma function**

|      |                 |      |                 |      |                 |
|------|-----------------|------|-----------------|------|-----------------|
| $k$  | $\Gamma(1+1/k)$ | $k$  | $\Gamma(1+1/k)$ | $k$  | $\Gamma(1+1/k)$ |
| 0,20 | 120             | 1,50 | 0,902 7         | 3,60 | 0,901 1         |
| 0,25 | 24              | 1,55 | 0,899 4         | 3,70 | 0,902 4         |
| 0,30 | 9,260 3         | 1,60 | 0,896 6         | 3,80 | 0,903 8         |
| 0,35 | 5,029 5         | 1,65 | 0,894 2         | 3,90 | 0,905 1         |
| 0,40 | 3,323 3         | 1,70 | 0,892 2         | 4,00 | 0,906 4         |
| 0,45 | 2,505 5         | 1,75 | 0,890 6         | 4,10 | 0,907 6         |
| 0,50 | 2,000 0         | 1,80 | 0,889 2         | 4,20 | 0,908 9         |
| 0,55 | 1,702 4         | 1,85 | 0,888 2         | 4,30 | 0,910 1         |
| 0,60 | 1,504 5         | 1,90 | 0,887 4         | 4,40 | 0,911 3         |
| 0,65 | 1,360 3         | 1,95 | 0,886 7         | 4,50 | 0,912 5         |
| 0,70 | 1,265 7         | 2,00 | 0,886 2         | 4,60 | 0,913 7         |
| 0,75 | 1,190 6         | 2,10 | 0,885 7         | 4,70 | 0,914 9         |
| 0,80 | 1,133 0         | 2,20 | 0,885 6         | 4,80 | 0,916 0         |
| 0,85 | 1,087 8         | 2,30 | 0,885 9         | 4,90 | 0,917 1         |
| 0,90 | 1,052 2         | 2,40 | 0,886 5         | 5,00 | 0,918 2         |
| 0,95 | 1,023 8         | 2,50 | 0,887 2         | 5,20 | 0,920 2         |
| 1,00 | 1,000 0         | 2,60 | 0,888 2         | 5,40 | 0,922 2         |
| 1,05 | 0,980 8         | 2,70 | 0,889 3         | 5,60 | 0,924 1         |
| 1,10 | 0,964 9         | 2,80 | 0,890 3         | 5,80 | 0,926 0         |
| 1,15 | 0,951 7         | 2,90 | 0,891 7         | 6,00 | 0,927 7         |
| 1,20 | 0,940 6         | 3,00 | 0,893 0         | 6,20 | 0,929 3         |
| 1,25 | 0,931 4         | 3,10 | 0,894 3         | 6,40 | 0,930 9         |
| 1,30 | 0,923 6         | 3,20 | 0,895 6         | 6,60 | 0,932 5         |
| 1,35 | 0,916 9         | 3,30 | 0,897 0         | 6,80 | 0,934 0         |
| 1,40 | 0,911 4         | 3,40 | 0,898 4         | 7,00 | 0,935 4         |
| 1,45 | 0,906 7         | 3,50 | 0,899 7         | 8,00 | 0,941 7         |

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