BS EN 61703:2016

BSI Standards Publication

Mathematical expressions for reliability, availability, maintainability and maintenance support terms

bsi.

National foreword

This British Standard is the UK implementation of EN 61703:2016. It is identical to IEC 61703:2016. It supersedes BS EN 61703:2002, which will be withdrawn on 16 September 2019.

The UK participation in its preparation was entrusted to Technical Committee DS/1, Dependability.

A list of organizations represented on this committee can be obtained on request to its secretary.

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English Version

Mathematical expressions for reliability, availability, maintainability and maintenance support terms (IEC 61703:2016)

Expressions mathématiques pour les termes de fiabilité, de disponibilité, de maintenabilité et de logistique de maintenance
(IEC 61703:2016)

Mathematische Ausdrücke für Begriffe der Zuverlässigkeit, Verfügbarkeit, Instandhaltbarkeit und Instandhaltungsbereitschaft
(IEC 61703:2016)

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European Committee for Electrotechnical Standardization
Comité Européen de Normalisation Electrotechnique
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European foreword

The text of document 56/1682/FDIS, future edition 2 of IEC 61703, prepared by IEC/TC 56 "Dependability" was submitted to the IEC-CENELEC parallel vote and approved by CENELEC as EN 61703:2016.

The following dates are fixed:

• latest date by which the document has to be implemented at national level by publication of an identical national standard or by endorsement	(dop)	2017-06-16
• latest date by which the national standards conflicting with the document have to be withdrawn	(dow)	2019-09-16

This document supersedes EN 61703:2002.

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The text of the International Standard IEC 61703:2016 was approved by CENELEC as a European Standard without any modification.

In the official version, for Bibliography, the following notes have to be added for the standards indicated:

IEC 61508 Series	NOTE	Harmonized as EN 61508 Series.
IEC 61511 Series	NOTE	Harmonized as EN 61511 Series.
IEC 61025	NOTE	Harmonized as EN 61025.
IEC 61078	NOTE	Harmonized as EN 61078.
IEC 61165	NOTE	Harmonized as EN 61165.

Annex ZA
(normative)

Normative references to international publications with their corresponding European publications

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE 1 When an International Publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

NOTE 2 Up-to-date information on the latest versions of the European Standards listed in this annex is available here: www.cenelec.eu

Publication	Year	Title	EN/HD	Year
IEC 60050-192	2015	International electrotechnical vocabulary - Part 192: Dependability	-	-
ISO 3534-1	2006	Statistics - Vocabulary and symbols - Part -1: General statistical terms and terms used in probability	-	-

CONTENTS

INTERNATIONAL ELECTROTECHNICAL COMMISSION

MATHEMATICAL EXPRESSIONS FOR RELIABILITY, AVAILABILITY, MAINTAINABILITY AND MAINTENANCE SUPPORT TERMS

FOREWORD

International Standard IEC 61703 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition published in 2001. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition:

a) standard made as self containing as possible;
b) item split between individual items and systems;
c) generalization of the dependability concepts for systems made of several components;
   – introduction of the conditional failure intensity (Vesely failure rate);
   – introduction of the state-transition and the Markovian models;

   – generalization of the availability to production availability;

d) introduction of curves to illustrate the various concepts.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/1682/FDIS	56/1693/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

This International Standard is to be used in conjunction with IEC 60050-192:2015.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

INTRODUCTION

IEC 60050-192 provides definitions for dependability and its influencing factors, reliability, availability, maintainability and maintenance support, together with definitions of other related terms commonly used in this field. Some of these terms are measures of specific dependability characteristics, which can be expressed mathematically.

It is important for the users to understand the mathematical meaning of those expressions and how they are established. This is the purpose of the present International Standard which, used in conjunction with IEC 60050-192, provides practical guidance essential for the quantification of those measures. For those requiring further information, for example on detailed statistical methods, reference should be made to the IEC 60605 series [23][1].

Annex A provides a diagrammatic explanation of the relationships between some basic dependability terms, related random variables, probabilistic descriptors and modifiers.

Annex B provides a summary of measures related to time to failure.

Annex C compares some dependability measures for continuously operating items.

The bibliography gives references for the mathematical basis of this standard, in particular, the mathematical material is based on references [2], [6], [8], [9], [13], [14] and [18]; the renewal theory (renewal and alternating renewal processes) can be found in [6], [8], [9], [10], [11], [13], [15], and [17]; and more advanced treatment of renewal theory can be found in references [1], [3], [12], [16], [19] and [20]. More information on the theory and applications of Markov processes can be found in references [3], [9], [10], [15], [16], [17] and [19].

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1 Numbers in brackets refer to the Bibliography.

MATHEMATICAL EXPRESSIONS FOR RELIABILITY, AVAILABILITY, MAINTAINABILITY AND MAINTENANCE SUPPORT TERMS

1 Scope

This International Standard provides mathematical expressions for selected reliability, availability, maintainability and maintenance support measures defined in IEC 60050-192:2015. In addition, it introduces some terms not covered in IEC 60050-192:2015. They are related to aspects of the system of item classes (see hereafter).

According to IEC 60050-192:2015, dependability [192-01-22] is the ability of an item to perform as and when required and an item [192-01-01] can be an individual part, component, device, functional unit, equipment, subsystem, or system.

To account for mathematical constraints, this standard splits the items between the individual items considered as a whole (e.g. individual components) and the systems made of several individual items. It provides general considerations for the mathematical expressions for systems as well as individual items but the individual items which are easier to model are analysed in more detail with regards to their repair aspects.

The following item classes are considered separately:

- Systems;
- Individual items:
  - non-repairable [192-01-12];
  - repairable [192-01-11]:
    - i) with zero (or negligible) time to restoration;
    - ii) with non-zero time to restoration.

In order to explain the dependability concepts which can be difficult to understand, keep the standard self-contained and the mathematical formulae as simple as possible, the following basic mathematical models are used in this standard to quantify dependability measures:

- Systems:
  - state-transition models;
  - Markovian models.
- Individual items:
  - random variable (time to failure) for non-repairable items;
  - simple (ordinary) renewal process for repairable items with zero time to restoration;
  - simple (ordinary) alternating renewal process for repairable items with non-zero time to restoration.

The application of each dependability measure is illustrated by means of simple examples.

This standard is mainly applicable to hardware dependability, but many terms and their definitions may be applied to items containing software.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-192:2015, *International electrotechnical vocabulary – Part 192: Dependability* (available at http://www.electropedia.org)

ISO 3534-1:2006, *Statistics – Vocabulary and symbols – Part 1: General statistical terms and terms used in probability*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 60050-192:2015, ISO 3534-1 and the following apply.

NOTE To facilitate the location of the full definition, the IEC 60050-192 reference for each term is shown [in square brackets] immediately following each term, for example:

mean time to restoration **[192-07-23]**

The terms and definitions given in Clause 3, which do not appear in IEC 60050-192, are used in order to facilitate the presentation of mathematical expressions of some IEC 60050-192 terms. Some terms have been taken from IEC 60050-192 and modified for the needs of this standard.

3.1
instantaneous restoration intensity
restoration intensity
restoration frequency
$\nu(t)$
limit, if it exists, of the quotient of the mean number of restorations [192-06-23] of a repairable item [192-01-11] within time interval [t, $t + \Delta t$], and Δt, when Δt tends to zero, given that the item is as good as new at $t = 0$

$$\nu(t) = \lim_{\Delta t \to 0+} \frac{E\left[N_{\mathrm{R}}(t+\Delta t) - N_{\mathrm{R}}(t) \mid \text{as good as new at } t=0\right]}{\Delta t}$$

where

$N_{\mathrm{R}}(t)$ is the number of restorations in the time interval [0, t];

E denotes the expectation.

Note 1 to entry: The difference between the restoration intensity and the repair rate comes from the conditions: the item is as good as new at time $t = 0$ for the restoration intensity and for the repair rate the repair starts at time $t = 0$. From a mathematical point of view, the restoration intensity is similar to the unconditional failure intensity (see 3.8).

Note 2 to entry: The unit of measurement of instantaneous restoration intensity is the unit of time to the power-1.

3.2
instantaneous repair rate
repair rate
$\mu(t)$
limit, if it exists, of the quotient of the conditional probability that the repair is completed within time interval [t, $t + \Delta t$] and Δt, when Δt tends to zero, given that the repair started at $t = 0$ and had not been completed before time t

Note 1 to entry: The difference between the restoration intensity and the repair rate comes from the conditions: the item is as good as new at time $t = 0$ for the restoration intensity and for the repair rate the repair starts at time $t = 0$. From a mathematical point of view, the repair rate is similar to the failure rate (see 3.6).

[SOURCE: IEC 60050-192:2015, 192-07-20, modified — Note 1 to entry has been replaced and Note 2 to entry deleted]

3.3
mean time between failures
METBF
expectation of the time which elapses between successive failures

Note 1 to entry: The concept of mean time between failures has been omitted from IEC 60050-192. It was defined in IEC 60050-191 as "the expectation of time between failures". The definition has been modified to explain the acronym METBF (mean elapsed time between failures) which is used in this standard to avoid any confusion between the mean time between failure (METBF) and the mean operating time between failures (MTBF or MOTBF).

[SOURCE: IEC 60050-191:1990, 191-12-08, modified — acronym and Note 1 to entry added]

3.4
up-time distribution function
function giving, for every value of t, the probability that an up-time will be less than, or equal to, t

Note 1 to entry: If the up-time is (strictly) positive and a continuous random variable, then $F_U(0) = 0$ and

$$F_U(t) = 1 - \exp\left(-\int_0^t \lambda_U(x)\,dx\right)$$

where

$\lambda_U(t)$ is the instantaneous up-time hazard rate function.

Note 2 to entry: The up-time distribution function is the general up-time distribution valid for both COI (continuously operating item) and IOI (intermittently operating item). For COIs, $F_U(t) = F(t)$

Note 3 to entry: If the up time is exponentially distributed, then

$$F_U(t) = 1 - \exp(-t/\text{MUT})$$

where MUT is the mean up-time.

In this case, the reciprocal of MUT is denoted by λ_U and $\lambda_U = 1/\text{MUT}$

3.5
instantaneous up-time hazard rate function
up-time hazard rate function
$\lambda_U(t)$
limit, if it exists, of the quotient of the conditional probability that the up-time will end within time interval $[t, t + \Delta t]$ and Δt, when Δt tends to zero, given that the up-time started at $t = 0$ and had not been finished before time t

Note 1 to entry: The instantaneous up-time hazard rate function is expressed by the formula:

$$\lambda_U(t) = \lim_{\Delta t \to 0+} \frac{1}{\Delta t} \frac{F_U(t + \Delta t) - F_U(t)}{1 - F_U(t)} = \frac{f_U(t)}{1 - F_U(t)}$$

where $F_U(t)$ is the up-time distribution function and $f_U(t)$ is the probability density function of the up-time.

Note 2 to entry: $\lambda_U(t)$ is the general hazard rate for the up times valid for both COI and IOI. For COIs, $\lambda_U(t) = \lambda(t)$ (see 3.6).

Note 3 to entry: If the up time is exponentially distributed, then the instantaneous up-time hazard rate function is constant in time and is denoted by λ_U.

Note 4 to entry: The unit of measurement of instantaneous up-time hazard rate function is the unit of time to the power −1.

3.6
instantaneous failure rate
failure rate
$\lambda(t)$
limit, if it exists, of the quotient of the conditional probability that the failure of an item occurs within time interval $[t, t+\Delta t]$, by Δt, when Δt tends to zero, given that failure has not occurred within time interval $[0, t]$

Note 1 to entry: The instantaneous failure rate is expressed by the formula:

$$\lambda(t) = \lim_{\Delta t \to 0+} \frac{1}{\Delta t} \frac{F(t+\Delta t) - F(t)}{R(t)} = \frac{f(t)}{R(t)}$$

where $F(t)$ and $f(t)$ are, respectively, the distribution function and the probability density at the failure instant, and where $R(t)$ is the reliability function, related to the reliability $R(t_1, t_2)$ by $R(t) = R(0, t)$.

Note 2 to entry: The restriction to non-repairable items in the definition provided by IEC 60050-192 can be removed to generalize this definition for any kind of items i.e. systems or individual items, repairable or not.

Note 3 to entry: The instantaneous failure rate is the up-time hazard rate for COIs. In this case, $\lambda(t) = \lambda_U(t)$ (see 3.5).

Note 4 to entry: When $\Delta t \to 0+$, the failure rate is the conditional probability per unit of time that the item fails between t and $t + \Delta t$, given it is in up state all over the time interval $[0, t]$. It is usually assumed that the item is as good as new at time 0.

Note 5 to entry: The instantaneous failure rate may be also expressed by the formula:

$$\lambda(t) = \lim_{\Delta t \to 0+} \frac{E[N(t+\Delta t) - N(t) \mid \text{up state over } [0,t]]}{\Delta t}$$

where $N(t)$ is the number of failures in the time interval $[0, t]$, where E denotes the expectation.

This form of the definitions allows comparisons of the failure rate to the conditional failure intensity and to the unconditional failure intensity.

[SOURCE: IEC 60050-192:2015, 192-05-06, modified — generalized to systems with repairable components and Notes to entry added]

3.7
conditional failure intensity
Vesely failure rate
$\lambda_V(t)$
limit, if it exists, of the quotient of the mean number of failures of a repairable item within time interval $[t, t+\Delta t]$, by Δt, when Δt tends to zero, given that the item is in up state at time t and as good as new at time 0

Note 1 to entry: The instantaneous failure intensity is expressed by the formula:

$$\lambda_V(t) = \lim_{\Delta t \to 0+} \frac{E[N(t+\Delta t) - N(t) \mid \text{up state at } t \text{ and as good as new at time } 0]}{\Delta t}$$

where $N(t)$ is the number of failures in the time interval $[0, t]$, where E denotes the expectation.

Note 2 to entry: When $\Delta t \to 0+$, the conditional failure intensity is the probability per unit of time that the item fails between t and $t + \mathrm{d}t$, given it is in up state at time t and as good as new at time 0. In particular cases (quick restoration of failures), it provides good approximations of the failure rate. This parameter introduced in 1970 by W. E. Vesely [26] is also called Vesely failure rate.

Note 3 to entry: According to the definitions, $\lambda_V(t)$ and $z(t)$ are linked by the formula: $\lambda_V(t) = z(t)/A(t)$. where $A(t)$ is the item instantaneous availability at time t.

3.8
unconditional failure intensity
instantaneous failure intensity
failure intensity
failure frequency
$z(t)$
limit, if it exists, of the quotient of the mean number of failures of a repairable item within time interval [t, $t + \Delta t$], by Δt, when Δt tends to zero, given that the item is as good as new at time t= 0

Note 1 to entry: The instantaneous failure intensity is expressed by the formula:

$$z(t) = \lim_{\Delta t \to 0+} \frac{E[N(t+\Delta t) - N(t) \mid \text{as good as new at } t = 0]}{\Delta t}$$

where $N(t)$ is the number of failures in the time interval [0, t], where E denotes the expectation and where the implicit condition that the item is as good as new at time t = 0 has been added.

Note 2 to entry: The unconditional failure intensity is the failure intensity as defined in IEC 60050-192:2015 [192-05-08]. It is also sometimes named ROCOF (rate of occurrence of failure).

Note 3 to entry: When $\Delta t \to 0+$, the unconditional failure intensity is the probability per unit of time that the item fails between t and $t + \mathrm{d}t$ given that the item is in up state at time t = 0. Here the item may be in any state at time t and this is why the adjective unconditional is used.

Note 4 to entry: According to the definitions, $\lambda_V(t)$ and $z(t)$ are linked by the formula: $z(t) = A(t) . \lambda_V(t)$ where $A(t)$ is the item instantaneous availability at time t.

[SOURCE: IEC 60050-192:2015, 192-05-08, modified — synonyms have been added and the entry has been revised]

3.9
continuously operating item
COI
item for which operating time [192-02-05] is equal to its enabled time [192-02-17]

3.10
intermittently operating item
IOI
item for which operating time [192-02-05] is less than its enabled time [192-02-17]

Note 1 to entry: In this case, the enabled time of the item is made of the sum of the times spent in the operating [192-02-04], idle [192-02-14] and standby [192-02-10] states.

4 Glossary of symbols and abbreviations

4.1 General

The symbols and abbreviations given in Clause 4 are widely used and recommended, however, they are not mandatory. For consistency of presentation, the notation in this document may differ from that used in a referenced document.

4.2 Acronyms used in this standard

Acronym/abbreviation	Meaning
COI	Continuously operating item
IOI	Intermittently operating item
MACMT	Mean active corrective maintenance time, i.e. the expectation of the active corrective maintenance time

Acronym/abbreviation	Meaning
$\widehat{\text{MACMT}}$	Point estimate of the mean active corrective maintenance time
MAD	Mean administrative delay
$\widehat{\text{MAD}}$	Point estimate of the mean administrative delay
MADT(t_1, t_2)	Mean accumulated down time over the time interval [t_1, t_2]
$\widehat{\text{MADT}}(t_1,t_2)$	Point estimate of the mean accumulated down time over the time interval [t_1, t_2]
MAUT(t_1, t_2)	Mean accumulated up time over the time interval [t_1, t_2]
$\widehat{\text{MAUT}}(t_1,t_2)$	Point estimate of the mean accumulated up time over the time interval [t_1, t_2]
MDT	Mean down time
$\widehat{\text{MDT}}$	Point estimate of the mean down time
METBF	Mean (elapsed) time between failures
MFDT	Mean fault detection time, i.e. the expectation of the fault detection time
MLD	Mean logistic delay
$\widehat{\text{MLD}}$	Point estimate of the mean logistic delay
MMAT	Mean maintenance action time, i.e. the expectation of a given maintenance action time
MRT	Mean repair time
$\widehat{\text{MRT}}$	Point estimate of the mean repair time
MOTBF	Mean operating time between failures
MTD	Mean technical delay, i.e. the expectation of the technical delay
MTTF	Mean time to failure
$\widehat{\text{MTTF}}$	Point estimate of the mean time to failure
MTTR	Mean time to restoration
$\widehat{\text{MTTR}}$	Point estimate of the mean time to restoration
MUT	Mean up time
$\widehat{\text{MUT}}$	Point estimate of the mean up time
RT_i	Observed repair time of item i
TTF_i	Time to failure of item i
VRT	The variance of repair time, VRT = $Var[\zeta] = E[\zeta^2] - \text{MRT}^2$, where ζ is a random variable representing the repair time, Var denotes the variance and E denotes the expectation

4.3 Symbols used in this standard

Symbol	Meaning
$[t_1, t_2]$	Interval where t_1 is the lower bound and t_2 the upper bound ($t_1 < t_2$)
$Ast_i(t_1, t_2)$	Mean accumulated sojourn time in state i over the interval $[t_1, t_2]$
A	Asymptotic availability
$A(t)$	Instantaneous availability (availability function), i.e. the probability of the item being in an up state at the instant of time t
$\overline{A}(t_1,t_2)$	Mean availability for the time interval $[t_1, t_2]$
$\overline{A}$	Asymptotic mean availability
$\hat{\overline{A}}(t_1,t_2)$	Point estimate of the mean availability for the time interval $[t_1, t_2]$
Δt	Small strictly positive increment of time. $\Delta t > 0$
$\mathrm{d}t$	Infinitesimal strictly positive increment of time (i.e. Δt which tends to zero)
$E[.]$	Expectation
$F_U(t)$	Up time distribution function
$f_U(t)$	Probability density function of the up times
	NOTE For COIs, $f_U(t) = f(t)$.
$f(t)$	Probability density function of the (operating) times to failure
$\hat{f}(t)$	Point estimate at time t of the probability density function of the (operating) time to failure
$f_{R+U}(t)$	Probability density function of the sum of the time to restoration and the following up time
$G(t)$	Distribution function of the repair times
$G_{ACM}(t)$	Distribution function of the active corrective maintenance time
$G_R(t)$	Distribution function of the times to restoration
$g(t)$	Probability density function of the repair times
$\hat{g}(t)$	Point estimate at time t of the probability density function of the repair time
$g_{ACM}(t)$	Probability density function of the active corrective maintenance times
$g_{AD}(t)$	Probability density function of the administrative delays
$g_D(t)$	Probability density function of the down times
$g_{LD}(t)$	Probability density function of the logistic delays
$g_{MA}(t)$	Probability density function of the given maintenance action time
$g_R(t)$	Probability density function of the times to restoration
$h_{CTTF}^{(n)}(t)$	Probability density function of calendar time to the nth failure, $n \geq 1$
K_A, K_S , K_i	Nominal (production) capacity related to an item A, a system S or a state i
$K(t)$	Instantaneous (production) capacity related to a system
k	Number of repair times during a given period of observation

Symbol	Meaning
k_{ACM}	Number of active corrective maintenance times during a given period of observation
k_{AD}	Number of administrative delays during a given period of observation
k_D	Number of down times during a given period of observation
k_F	Number of failures during a given period of observation
k_{LD}	Number of logistic delays during a given period of observation
k_O	Number of failures while operating during a given period of observation
k_R	Number of times to restoration during a given period of observation
k_U	Number of up times during a given period of observation
λ	Constant failure rate, i.e. the reciprocal of mean time to failure (MTTF) when time to failure is exponentially distributed
$\lambda(t)$	Instantaneous failure rate
$\lambda(\infty)$	Asymptotic failure rate
$\hat{\lambda}$	Point estimate of the constant failure rate
$\hat{\lambda}(t)$	Point estimate of the instantaneous failure rate at time t
$\bar{\lambda}(t_1, t_2)$	Mean failure rate for the time interval $[t_1, t_2]$
λ_U	Constant up-time hazard rate function, i.e. the reciprocal of the mean up time (MUT) when up times are exponentially distributed
	NOTE 1 For COIs, $\lambda_U = \lambda$
$\lambda_U(t)$	The up-time hazard rate function
	NOTE 2 For COIs, $\lambda_U(t) = \lambda(t)$
$\lambda_V(t)$	Conditional failure intensity (Vesely failure rate)
$\lambda_V(\infty)$	Asymptotic conditional failure intensity (Asymptotic Vesely failure rate)
$M(t)$	Maintainability function, i.e. the probability of completing a given maintenance action within time t: $M(t) = M(t_1, t_2)$ for $t_1 = 0$ and $t_2 = t$
$\hat{M}(t)$	Point estimate at time t of the maintainability function
$M(t_1, t_2)$	Maintainability for the time interval $[t_1, t_2]$
m	Number of observed maintenance action times
$m_{MAT}(t)$	Number of maintenance action times with duration greater that t ($m_{MAT}(0) = m$)
μ	Constant repair rate, i.e. the reciprocal of the mean repair time (MRT) when the repair times are exponentially distributed
$\mu(t)$	Instantaneous repair rate
$\hat{\mu}(t)$	Point estimate at time t of the instantaneous repair rate
μ_{ACM}	Reciprocal of the mean active corrective maintenance time (MACMT) when the active corrective maintenance times are exponentially distributed

Symbol	Meaning
μ_{AD}	Reciprocal of the mean administrative delay (MAD) when the administrative delays are exponentially distributed
μ_{D}	Reciprocal of the mean down time (MDT) when the down times are exponentially distributed
μ_{LD}	Reciprocal of the mean logistic delay (MLD) when the logistic delays are exponentially distributed
μ_{MA}	Constant rate of the completion of a given maintenance action, when the maintenance action time is exponentially distributed
μ_{R}	Constant restoration rate, i.e. the reciprocal of the mean time to restoration (MTTR) when the times to restoration are exponentially distributed
$N(t)$	Number of failures occurring in the time interval $[0, t]$
$N_R(t)$	Number of restorations occurring in the time interval $[0, t]$
n	Number of items in the population
$n_D\{t\}$	Number of items that are in a down state at the instant of time t
$n_F(t, t + \Delta t)$	Number of failures observed during the time interval $[t, t + \Delta t]$, where the time scale includes both up and down times
$n_F(t_1, t_2)$	Number of failures observed during the time interval $[t_1, t_2]$, where the time scale includes both up and down times
$n_R(t)$	Number of repairable items that are still under repair at the instant of time t ($n_R(0) = n$)
$n_R(t + \Delta t) - n_R(t)$	Number of items with repair completed in the time interval $[t, t + \Delta t]$
$n_S(t)$	Number of (non-repairable) items that are still operational at the instant of time t ($n_S(0) = n$)
$n_S(t_1, t_2)$	Number of items that were operational at the instant of time t_1 and operated without failure in the time interval $[t_1, t_2]$
$n_S(t + \Delta t) - n_S(t)$	Number of items that fail in the time interval $[t, t + \Delta t]$
$n_U\{t\}$	Number of items that are in an up state at the instant of time t
$v(t)$	Instantaneous restoration intensity
$P(.)$	Probability. The dot stands for any relevant event or random variable
$P_i(t),\ P_i^{(av)}(t),\ P_i^{(rel)}(t)$	Probability of state i in state-transition models, probability of state i for an "availability" Markov graph (see 6.1.2.1), probability of state i for a "reliability" Markov graph (see 6.1.3.1)
$Prod(t)$	Instantaneous production at time t
$\overline{Prod}(t_1,t_2)$	Mean production availability over the time interval $[t_1, t_2]$
$R(t)$	Reliability function, i.e. the probability of survival until time t $R(t) = R(t_1, t_2)$ for $t_1 = 0$ and $t_2 = t$
$\hat{R}(t)$	Point estimate at time t of the reliability function
$R(t_1, t_2)$	Reliability for the time interval $[t_1, t_2]$
$\hat{R}(t_1,t_2)$	Point estimate for the time interval $[t_1, t_2]$ of the reliability function
$R(t, t + x \mid t)$	Conditional reliability for the time interval $[t, t + x]$, assuming that the item is in up state at time t
ρ	Production per unit of time

Symbol	Meaning
t	Instant in time
T	Instant in time or duration depending on the context
$\overline{Tbf}(t)$	Mean elapsed time between the consecutive failures occurring over the time interval [0, t]
τ	Instant in time or tests interval depending on the context
U	Asymptotic unavailability
$U(t)$	Instantaneous unavailability (unavailability function)
$\overline{U}(t_1,t_2)$	Mean unavailability for the time interval [t_1, t_2]
$\overline{U}$	Asymptotic mean unavailability
$\hat{\overline{U}}(t_1,t_2)$	Point estimate of the mean unavailability for the time interval [t_1, t_2]
$V(t_1, t_2)$	Mean number of restorations over the time interval [t_1, t_2]
$Z(t)$	Expected number of failures in the time interval [0, t] $Z(t) = E[N(t)]$, where E denotes the expectation
$z(t)$	Instantaneous failure intensity (failure frequency)
$z(\infty)$	Asymptotic failure intensity
$\hat{z}(t)$	Point estimate of the instantaneous failure intensity at time t
$\overline{z}(t_1,t_2)$	Mean failure intensity for the time interval [t_1, t_2]
$\hat{\overline{z}}(t_1,t_2)$	Point estimate of the mean failure intensity for the time interval [t_1, t_2]

5 General models and assumptions

5.1 Constituents of up and down times

In order to understand the mathematical expressions included in this standard, it is important to understand what constitutes up time and down time. IEC 60050-192 splits the up time (where the item performs as required) and down time (where the item does not perform as required) into their constituents. This is summarized in Figure 1 and Figure 2 in IEC 60050-192:2015 which also provides definitions of mean values for some of those constituents.

Figure 1 and Figure 2 (that have been adapted from IEC 60050-192) respectively summarize the constituents of up time and down time which are relevant to this standard. (Neither the “preventive maintenance time” nor the “externally disabled” time have been considered). The acronyms for the mean values included in this standard have been added in Figures 1 and 2.

<table>
<tr><td colspan="3">Up time [192-02-02]</td></tr>
<tr><td colspan="3">Enabled time [192-02-17]</td></tr>
<tr><td rowspan="2">Operating time [192-02-05]</td><td colspan="2">Non operating time [192-02-07]</td></tr>
<tr><td>Idle time [192-02-15]</td><td>Standby time [192-02-13]</td></tr>
<tr><td colspan="3">MUT [192-08-09]</td></tr>
</table>

IEC

Figure 1 – Constituents of up time

<table>
<tr><td colspan="7">Down time [192-02-21]</td></tr>
<tr><td colspan="7">Non-operating time [192-02-07]</td></tr>
<tr><td colspan="7">Time to restoration [192-07-06]</td></tr>
<tr><td colspan="5">Corrective maintenance time [192-07-07]</td><td rowspan="4">Fault detection time [192-07-11]</td><td rowspan="4">Administrative delay [192-07-12]</td></tr>
<tr><td rowspan="3">Logistic delay [192-07-13]</td><td colspan="4">Active corrective maintenance time [192-07-10]</td></tr>
<tr><td rowspan="2">Technical delay [192-07-15]</td><td colspan="3">Repair time [192-07-19]</td></tr>
<tr><td>Fault localization time [192-07-18]</td><td>Fault correction time [192-07-14]</td><td>Function checkout time [192-07-16]</td></tr>
<tr><td rowspan="2">MLD [192-07-27]</td><td>MTD</td><td colspan="3">MRT [192-07-21]</td><td rowspan="2">MFDT</td><td rowspan="2">MAD [192-07-26]</td></tr>
<tr><td colspan="4">MACMT [192-07-22]</td></tr>
<tr><td colspan="7">MTTR [192-07-23]</td></tr>
<tr><td colspan="7">MDT [192-08-10]</td></tr>
</table>

IEC

Figure 2 – Constituents of down time

IEC 60050-192 defines several acronyms for mean time related to the item failures occurring during the operating state. This is summarized in Figure 3.

<table>
<tr><td colspan="2">Non-repairable item [192-01-12]</td><td colspan="2"></td></tr>
<tr><td>Operating time to 1st failure [192-05-02]</td><td>Operating time to failure [192-05-01]</td><td>Operating time between failures [192-05-04]</td><td>(Elapsed) time between failures [192-05-03]</td></tr>
<tr><td>MTTFF [192-05-12]</td><td>MTTF [192-05-11]</td><td>MOTBF / MTBF [192-05-13]</td><td>METBF</td></tr>
<tr><td colspan="4">Repairable item [192-01-11]</td></tr>
</table>

IEC

Figure 3 – Acronyms related to failure times

NOTE In text books on the subject of reliability, the acronym MTBF is often used for the mean time between failures. According to IEC 60050-192, this acronym is used for the mean "operating" time between failures. Therefore to avoid any confusion, the acronym METBF (see 3.3) is used in this standard for the mean time between failures.

5.2 Introduction to models and assumptions

In order to derive correct mathematical expressions for selected dependability measures defined in IEC 60050-192, distinction is made between items considered as a whole (individual items) and items made of several individual items (systems). The following classes of items are considered separately in this standard:

- Systems;
- Individual items:
  - – non-repairable items [192-01-12];
  - – repairable items [192-01-11]:
    - i) with zero time to restoration;
    - ii) with non-zero time to restoration.

NOTE 1 The term "item" is used in this standard when it applies to systems and individual items.

NOTE 2 The term "non-repairable" item encompasses items which are either not repairable or are repairable but are not repaired when failed. The term "repairable" item encompasses only those items which are repairable and actually repaired when failed.

In order to keep the standard self-contained and the mathematical formulae as simple as possible, the following basic mathematical models are used to quantify dependability measures:

– state-transition models for systems;
– renewal processes for individual items.

The general way to model an item is to identify its various states and to analyse how it goes from state to state when the time is elapsing: this can be done by using state-transition models. Such models are useful in illustrating the development of the mathematical expressions for various dependability measures. When no particular assumptions are made regarding the probabilistic distributions, these models can be handled only in particular and simple cases using analytical calculations. Otherwise Monte Carlo simulation needs to be used. This is why, for systems, as well as individual items, the hypothesis of constant transition rates, is often made in order to use Markovian models which are well known and for which powerful analytical algorithms are available.

For individual items, under the assumption that they have only two states, the general formulae with non-constant transition rates can be derived to some extent. This is shown in 6.2, 6.3 and 6.4:

– non-repairable individual items: this is the simplest mathematical model as only one random variable applies: the time to failure of the item [192-05-01]. It can be characterized by its reliability $R(t)$, its instantaneous failure rate, $\lambda(t)$ [192-05-06], also referred to as the hazard rate associated with the time to failure distribution (i.e. the unreliability function) or its mean time to failure MTTF [192-05-11] which is also its mean time to first failure MTTFF [192-05-12].
– repairable individual items: the basic model is a simple (ordinary) renewal process, when the time to restoration of the item may be neglected, or a simple (ordinary) alternating renewal process in which the time to restoration of the item is non-zero. In the latter case, the item alternates between an up state and a down state, and, in addition to the common dependability measures, a widely used measure of reliability of the item is the failure intensity [192-05-08], which, in this case, is equal to the renewal density.

To avoid improper use of these mathematical expressions, which could yield erroneous results, the specific assumptions detailed in 5.4 and 5.5 should be observed.

In order to allow for easy comprehension, some definitions have been repeated in different parts of this standard, as needed.

5.3 State-transition approach

A simple way to introduce the dependability concepts is to analyse the behaviour of an item (i.e. an individual item or a system) during its evolution between up and down states.

This can be done by using a state-transition graph such as the one presented in the right hand side of Figure 4. It is related to the system represented by the reliability block diagram (see IEC 61078 [25]) on the left hand side of the figure. This system is made of three similar blocks (A_i) organized in a two out of three architecture and of one block B in standby position which immediately takes over the function of the first of the blocks A_i going to its down state. The switching between A_i and B is considered to be perfect (i.e. instantaneous and without failure).

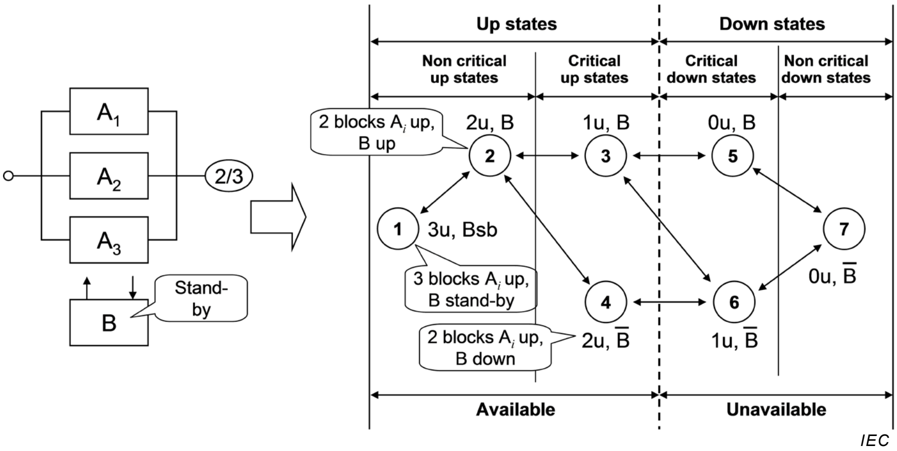


Figure 4 – Simple state-transition diagram

As the blocks (A_i) are similar, the similar states can be aggregated to obtain the state-transition diagram presented in the right hand side of Figure 4. It comprises seven states represented by circles and sixteen transitions represented by arrows. For example, the state 3 aggregates three similar states (A_1 and B in up states and other blocks in down states, A_2 and B in up states and other blocks in down states, A_3 and B in up states and other blocks in down states) and the line with arrows on each end between states 2 and 3 means that the system can have transitions from 2 to 3 and from 3 to 2.

This state-transition diagram is sufficient to identify the classes of states which are essential to define and understand the key dependability measures (e.g. availability, reliability, failure rate, failure intensities, failure density).

The states can be grouped into two main classes which can furthermore be subdivided in two sub classes:

- system up state class: states 1, 2, 3 and 4 (where at least 2 of the blocks are in up states). It is subdivided into:
  - non critical up state class: states 1 and 2 which are separated from the down state class by more than one transition;
  - critical up states class: states 3 and 4 which are separated from the down state class by only one transition;
- system down state class: 5, 6 and 7 (where less than 2 of the blocks are in up states). It is subdivided into:
  - non critical down state class: state 7 which is separated from the up state class by more than one transition;
  - critical down state class: states 5 and 6 which are separated from the up state class by only one transition.

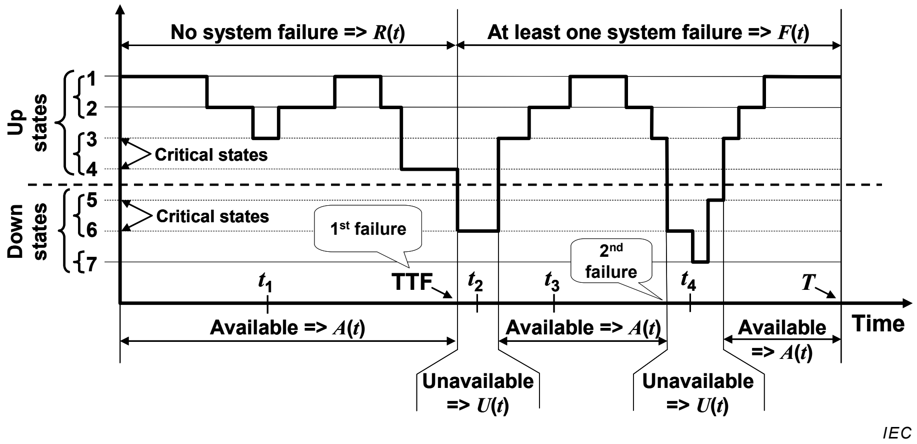


Figure 5 – Sample realization (chronogram) related to the system in Figure 4

When the transitions between the states occur randomly (e.g. according to failures and restorations of the various blocks) the behaviour of the system constitutes a stochastic process. One example of realization of this stochastic process (i.e. a trajectory of the process) over a time interval [0, T] is illustrated in Figure 5:

– the system is in up state (i.e. available) at time t_1, t_3 or T;

– the system is in down state (i.e. unavailable) at time t_2 or t_4;

– the system has been continuously in up state (i.e. reliable) until the first failure at TTF (time to failure).

This example encompasses all the cases which can be encountered.

5.4 Model and assumptions for non-repairable individual items

This applies both to individual items alone and individual items that are a part of a system. This is the simplest particular example which can be derived from Figure 5 because only two states have to be considered (see Figure 6 and Figure 7).

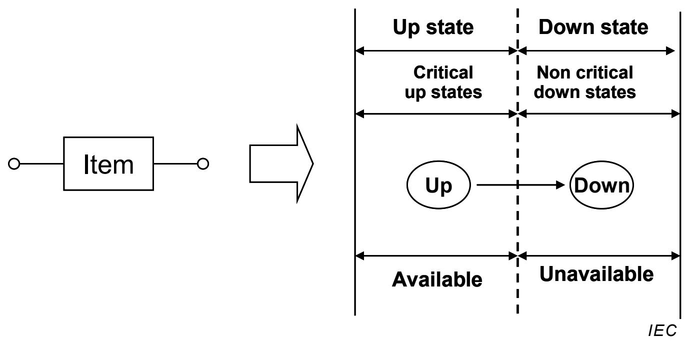


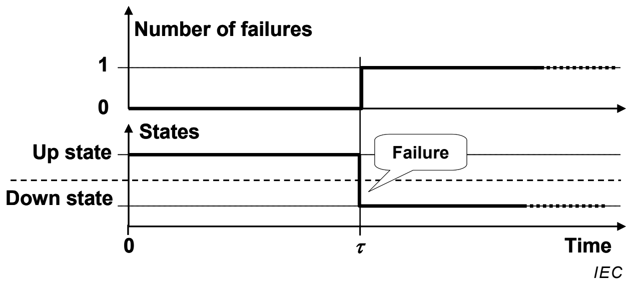
Figure 6 – State-transition diagram of a non-repairable individual item

At any instant of time, the non-repairable item will be either in:

– the up state from which the item can fail (i.e. goes to the down state) at time τ (time to failure);

– the down state where the item is failed and remains here for eternity.

Therefore in this simple case, the up state is also a critical up state and the down state is a non critical down state as no repair can occur. The behaviour of such an item is illustrated in Figure 7.



Key

τ Time to failure

Figure 7 – Sample realization of a non-repairable individual item

The number of observed failures, for this case, may be 0 or 1 and the number of repairs is equal to 0.

Unless otherwise stated, the assumptions used to derive the mathematical formulae are the following:

– When the item is in an up state, it is considered to be operating continuously.

NOTE The mathematical expressions given in 5.4 may not always be valid for IOIs.

– At time t = 0, the item is in an operating state, and is as good as new. Latent faults are not considered, which, if present, may invalidate some mathematical expressions.

– Preventive maintenance, or other planned actions that render the item incapable of performing a required function are not considered.

– The time to failure is a positive and continuous random variable with a probability density function and finite expectation.

5.5 Assumptions and model for repairable individual items

5.5.1 Assumption for repairable individual items

Unless otherwise stated, the assumptions used to derive the mathematical formulae are the following:

a) At time t = 0, the item is in an up state and as good as new. Therefore, $R(0) = A(0) = 1$. Latent faults are not considered.

b) When the item is in the up state, it is considered to be operating continuously.

c) Consecutive up times of the item are statistically independent, identically distributed, positive, continuous random variables with a common probability density function and finite expectation.

d) In the case of non-zero down-time duration, the consecutive down times of the item are statistically independent, identically distributed, positive, continuous random variables with a common probability density function and finite expectation.

e) The up times are statistically independent of the down times.

f) Preventive maintenance or other planned actions that render the item incapable of performing a required function are not considered.

g) Unless otherwise stated, other random variables (e.g. time to failure, repair time, logistic delay) considered in the standard, are positive and continuous random variables with probability density functions and finite expectations.

In summary:

– any transition from an up state to a down state is a failure;

– any transition from a down state to an up state is a restoration;

– any down state is a fault and, in consequence, the down time is equal to the time to restoration;

– after each restoration an individual item is as good as new.

NOTE 1 With the last assumption, all mathematical expressions for the reliability measures relating to the time to failure of a non-repairable individual item can also be applied to each time to failure of a continuously operating repairable individual item.

NOTE 2 The components of a system are as good as new after restoration but the whole system becomes as good as new only when all the failed components have been restored.

5.5.2 Instantaneous repair

This applies both to individual items alone and individual items that are a part of a system and are independent of each other. This is another simple example which can be derived from Figure 5. Here also only two states have to be considered (see Figure 8 and Figure 9) but the down state is a zero-duration state because repairs are instantaneous.

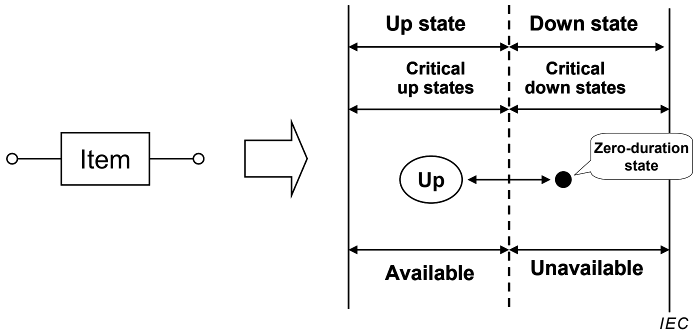


Figure 8 – State-transition diagram of an instantaneously repairable individual item

At any instant of time, the repairable item will be either in:

– the up state from which the item can fail (i.e. goes to the down state). In Figure 9, $S_{F,1}$, $S_{F,2}$, $S_{F,3}$ ... are instants of failure;

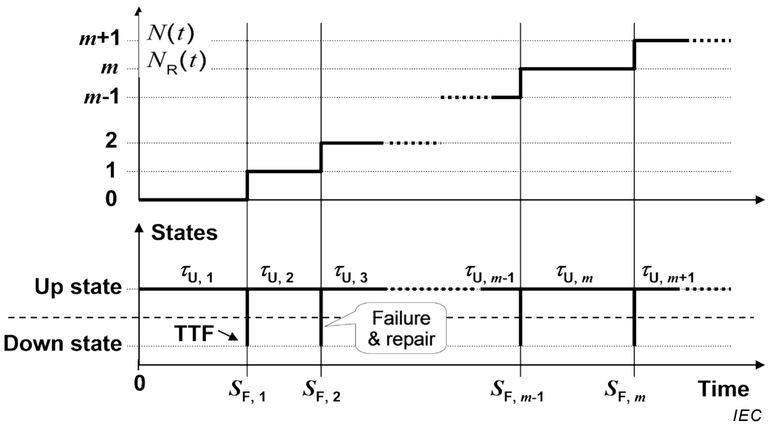
– the down state where the item is instantaneously repaired (i.e. goes to the up state). In Figure 9, $S_{F,1}$, $S_{F,2}$, $S_{F,3}$ ... are also instants of repairs.

The behaviour of such an item is illustrated in Figure 9. When the item is as good as new after repair, it can be modelled by a simple (ordinary) renewal process.

The repairable item being in up state at any instant of time, is available at any time. Therefore this model is mainly useful to count the number of failures occurring over a given time interval (see Figure 9).

When this approach is used to model an item without considering the times to restoration, the time considered in Figure 9 comprises only the operational times and this model allows to calculate the number of failures over a given accumulated duration of operational time.

When this approach is used to model an item where the times to restoration are small compared to the times to failures, the time considered in Figure 9 is the calendar time which encompasses both the operational and the restoration times. This calendar time overestimates the operational time and the calculated number of failures is conservative.



Key

$N(t)$	Number of failures during the time interval [0, t]
$N_R(t)$	Number of restorations during the time interval [0, t]
$S_{F,1}$, $S_{F,2}$, $S_{F,3}$	Consecutive instants of failure
$\tau_{U,1}$, $\tau_{U,2}$, $\tau_{U,3}$	Consecutive up times

Figure 9 – Sample realization of a repairable individual item with zero time to restoration

5.5.3 Non-instantaneous repair

This is the same case as in 5.5.2 except that the down state lasts for some time as the repairs are not instantaneous (see Figure 10 and Figure 11).

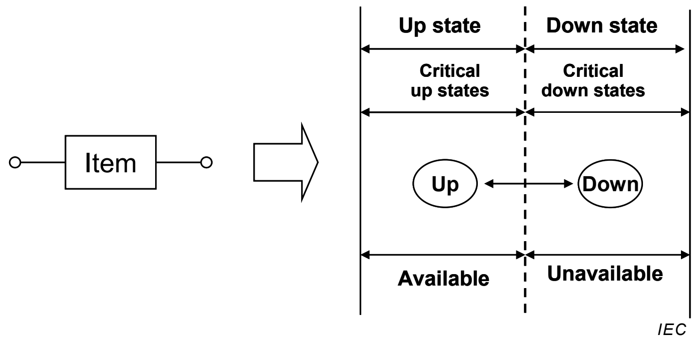


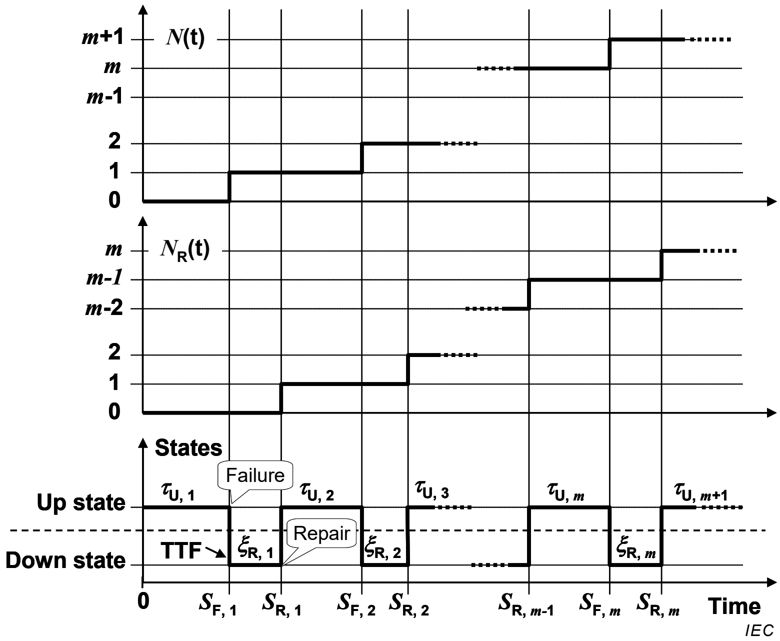
Figure 10 – State-transition diagram of a repairable individual item

At any instant of time, the repairable individual item will be either in:

– the up state from which the item can fail (i.e. can go to the down state). In Figure 11, $S_{F,1}$, $S_{F,2}$, $S_{F,3}$ ... are instants of failure;

– the down state where the item can be repaired (i.e. can go to the up state). In Figure 11, $S_{R,1}$, $S_{R,2}$, $S_{R,3}$ ... are instants of repairs;

The behaviour of such an item is illustrated in Figure 11. When the item is as good as new after repair it can be modelled by a simple (ordinary) alternating renewal process.



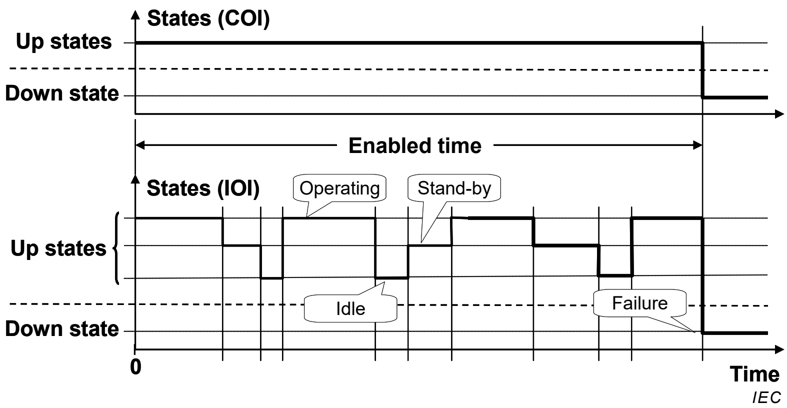
Key

$N(t)$	Number of failures during the time interval [0, t]
$N_R(t)$	Number of restorations during the time interval [0, t]
$S_{F,1}$, $S_{F,2}$, $S_{F,3}$	Consecutive instants of failure
$S_{R,1}$, $S_{R,2}$, $S_{R,3}$	Consecutive instants of restoration
$\tau_{U,1}$ $\tau_{U,2}$, $\tau_{U,3}$	Consecutive up times
$\xi_{R,1}$, $\xi_{R,2}$, $\xi_{R,3}$	Consecutive times to restoration

Figure 11 – Sample realization of a repairable individual item with non-zero time to restoration

5.6 Continuously operating items (COI) versus intermittently operating individual items (IOI)

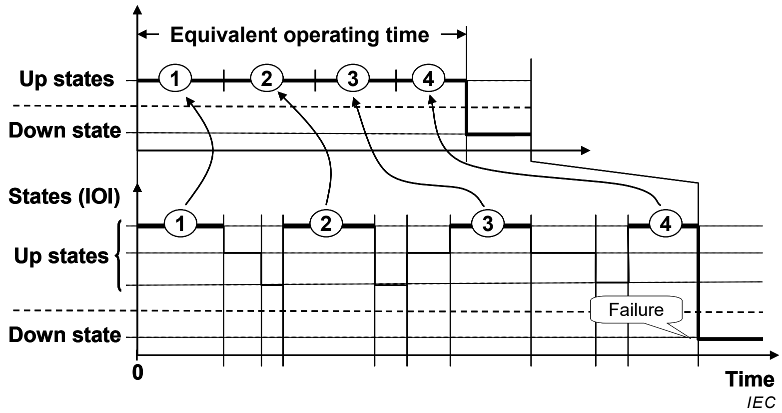
For continuously operating items (COI), the up state is reduced to the operating state, and then the up time is equal to the operating time. For intermittently operating items (IOI), the up state class is split between several types of states: for example operating state [192-02-04], idle state [192-02-14], and stand-by state [192-02-10] (see Figure 12).



NOTE For COI, Enabled time = Operating time; for IOI, Enabled time = Operating time + Standby time + Idle time.

Figure 12 – Comparison of an enabled time for a COI and an IOI

The expressions for reliability measures of continuously operating, repairable items may not be true for IOIs. Nevertheless, when it is assumed that the item cannot fail (i.e. go to the down state) when it is not in operating state, the expressions remain valid provided that the equivalent operating time is used as illustrated in Figure 13. If the item fails during non-operating states (e.g. idle or stand-by states), general stochastic processes like those presented in Figure 5 should be considered.



NOTE The equivalence illustrated is valid only if the item cannot fail during stand-by or idle positions.

Figure 13 – Equivalent operating time for IOI items

6 Mathematical models and expressions

6.1 Systems

6.1.1 General

The state graph drawn in Figure 14 will be used to explain the various terms. It models a system made of two redundant repairable components A and B. It comprises only 4 states: 3 up states of which 2 are critical and a single critical down state. This simple system is sufficient to illustrate the concepts of availability/unavailability, reliability/unreliability, failure rate, failure density and conditional and unconditional failure intensities.

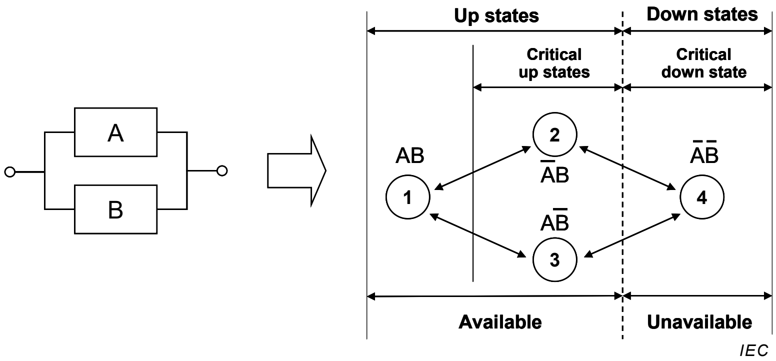


Figure 14 – State-transition graph for a simple redundant system

In this state-transition graph no assumptions are made about the rules of transition allowing the system to go at time t from a state, i, to another one, j. In the general case, this depends both on the states i and j but also on the time spent in the state i before the transition occurs and on the way the state i has been previously reached. Therefore, except in particular cases, there are generally no simple analytical expressions available and Monte Carlo simulation has to be implemented.

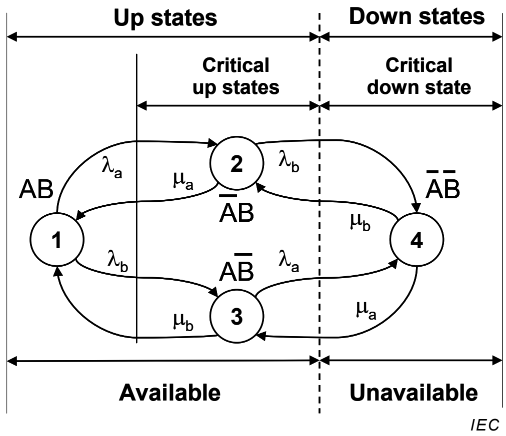


Figure 15 – Markov graph for a simple redundant system

Fortunately, the failure and repair rates of components can often be considered as reasonably constant and, the state-transition graph becomes a Markov graph [27] allowing analytical calculations. In this case the rules of transition from a state i to another one j are described by constant transition rates which depend only on the states i and j. Then, when the components A and B of the above example have constant failures (λ_a, λ_b) and repair rates (μ_a, μ_b), Figure 14 can be drawn as the Markov graph shown in Figure 15.

This Markov graph is used to derive mathematical expressions valid when the Markovian assumption is verified. Algorithms are available to calculate the probabilities of the various states. Figure 16 shows the typical evolution of the state probabilities when the time elapses. These curves have been drawn with the following parameters: λ_a = 2 year^{-1}, λ_b = 3 year^{-1} and μ_a = μ_b = 10 year^{-1}.

The failure rates have been chosen high and the repair rates relatively low: this leads to a rather low availability but allows to clearly visualize the transient period before the asymptotic values are reached.

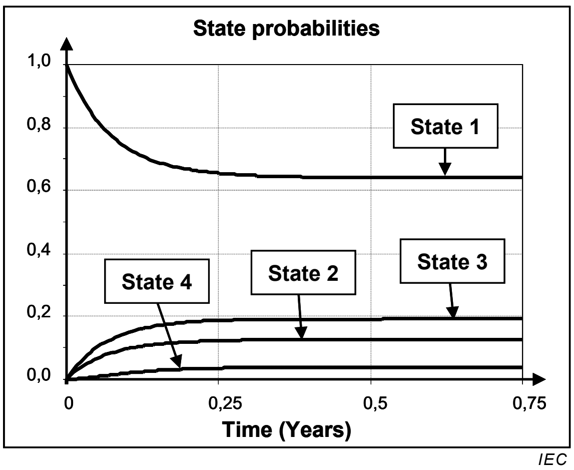


Figure 16 – Evolution of the state probabilities related to the Markov model in Figure 15

6.1.2 Availability related expressions

6.1.2.1 Instantaneous availability [192-08-01] and instantaneous unavailability [192-08-04]

According to its definition, the instantaneous availability, $A(t)$, is the probability that an item is in a state to perform as required at a given instant t.

According to its definition [192-02-01], an up state is a state of being able to perform as required.

Therefore the instantaneous availability $A(t)$, is the probability to be in up state at a given time t:

$$A(t) = P(\text{up state at time } t)$$

In the same way, the instantaneous unavailability $U(t)$ is the probability to be in down state at a given time t:

$$U(t) = P(\text{down state at time } t)$$

and

$$A(t) = 1 - U(t)$$

With regard to the state-transition graph (Figure 14), or the Markov graph (see Figure 15) the instantaneous availability and the instantaneous unavailability of the modelled item are the following:

$$A(t) = P_1(t) + P_2(t) + P_3(t)$$

$$U(t) = P_4(t)$$

The evolution of $A(t)$ and $U(t)$ is illustrated in Figure 17. The curves in this figure have been drawn by using the values of the state probabilities presented in Figure 16.

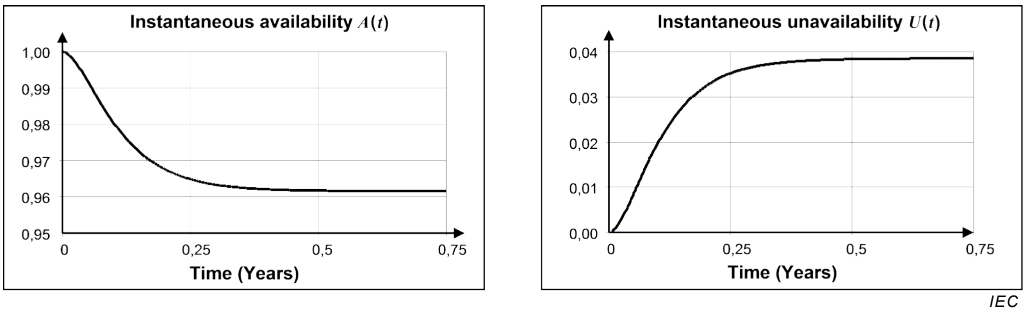


Figure 17 – Evolution of $A(t)$ and $U(t)$ related to the Markov model in Figure 15

NOTE The Markov graph in Figure 15 allows to calculate the system availability. To highlight this property, such a Markov graph is called "availability Markov graph".

6.1.2.2 Steady state availability [192-08-07] and steady state unavailability [192-08-08]

The steady state availability A is the limit, if it exists, of the instantaneous availability:

$$A = \lim_{t \to \infty} A(t)$$

In the same way the steady state unavailability is given by:

$$U = \lim_{t \to \infty} U(t)$$

The steady state values exist in the Markovian case because the probabilities $P_i(t)$ of the states reach asymptotical values P_i (see e.g. Figure 16). When a steady state is reached (see NOTE), the asymptotic probabilities represent also the proportion of the time spent in the corresponding states. This allows also to calculate the mean availability or the mean unavailability (see Figure 17).

NOTE The steady state does not characterize a state of the item itself but a state of the underlying stochastic process which becomes stationary. Such a steady state exists if, when the time increases, a statistical equilibrium is reached where the probability for the item to go into a given state becomes equal to the probability for the item to go out of this state. In this case, the probability to be in a given state reaches a steady state value (i.e. an asymptotic value) which characterizes the fact that the steady state is reached. By extension, the term "steady state of an item" is used to denote the steady state of its underlying stochastic process modelling how it goes from a state to another. See reference [27] for more details about the steady state of Markov processes.

6.1.2.3 Mean availability [192-08-05] and mean unavailability [192-08-06]

6.1.2.3.1 General formulae for mean availability and unavailability

The mean availability $\overline{A}(t_1,t_2)$ over an interval [t_1, t_2] is simply calculated through the integration of the instantaneous availability $A(t)$ divided by the time interval:

$$\overline{A}(t_1,t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} A(t)dt$$

It can also be calculated through the sum of the mean accumulated sojourn times $Ast_i(t_1,t_2)$ spent in the up states i over the time interval [t_1, t_2]:

$$Ast_i(t_1,t_2) = \int_{t_1}^{t_2} P_i(t)dt$$

Therefore, the mean availability is obtained by the following formula:

$$\overline{A}(t_1,t_2) = \frac{\sum_{i \in \text{up state}} Ast_i(t_1,t_2)}{t_2 - t_1}$$

In the same way, the mean unavailability $\overline{U}(t_1,t_2)$ is defined as:

$$\overline{U}(t_1,t_2) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} U(t)dt = \frac{\sum_{j \in \text{down states}} Ast_j(t_1,t_2)}{t_2 - t_1}$$

NOTE The mean unavailability is called PFDavg (average of the probability of failure on demand) when it is used in the context of the functional safety of safety instrumented systems (e.g. IEC 61508 [21] or IEC 61511 [22]).

For the state-transition graph (see Figure 14) or the Markov graph (see Figure 15), the mean availability and the mean unavailability of the modelled item are as follows:

$$\overline{A}(t_1,t_2) = \frac{Ast_1(t_1,t_2) + Ast_2(t_1,t_2) + Ast_3(t_1,t_2)}{t_2 - t_1}$$

$$\overline{U}(t_1,t_2) = \frac{Ast_4(t_1,t_2)}{t_2 - t_1}$$

Algorithms are available to calculate the mean accumulated sojourn times in the Markovian model. This is illustrated in Figure 18.

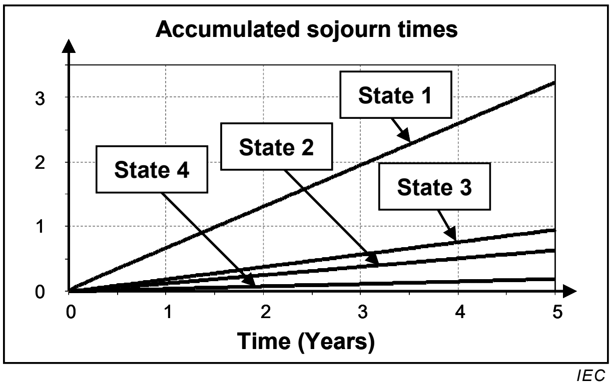


Figure 18 – Evolution of the $Ast_i(0, t)$ related to the Markov model in Figure 15

The mean accumulated time spent in the various states can be obtained by collecting the field feedback of the item. Therefore the mean availability or unavailability can be statistically estimated. This makes the link between mathematical calculations and actual observed data (field feedback).

6.1.2.3.2 Asymptotic mean availability and asymptotic mean unavailability

When a steady state exists, $A(t)$ has an asymptotic value A and the unavailability $U(t)$ an asymptotic value U. It follows from elementary calculus (see NOTE hereafter and reference [7] for more details) that in this case, these asymptotic values are also the average values over the interval of time $[t_1, t_2]$ when $t_2 \to \infty$. Therefore, the asymptotic average availability is given by:

$$\overline{A} \overset{\text{def.}}{=} \lim_{t_2 \to \infty} \overline{A}(t_1,t_2) = \lim_{t_2 \to \infty} \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} A(t)\mathrm{d}t = A$$

As $t_2 > t_1$, the above formula is also valid for an interval of time $[t_1, t_2]$ when $t_1 \to \infty$ because in this case t_2 also $\to \infty$. Therefore it is valid for a time interval $[t_1, t_2 \to \infty]$ where the steady state is established at t_2 (e.g. a large time interval $[0, t_2 \to \infty]$) and also for a time interval $[t_1 \to \infty, t_2]$ where the steady state is established at t_1 (e.g. a small time interval $[t_1 \to \infty, t_1 + x]$).

The average availability for such intervals is given by:

$$\lim_{t1\to\infty} \overline{A}(t_1,t_2) = \lim_{t_2\to\infty} \overline{A}(t_1,t_2) = \overline{A} = A$$

And, in the same way: $\lim_{t_1\to\infty} \overline{U}(t_1,t_2) = \lim_{t_2\to\infty} \overline{U}(t_1,t_2) = \overline{U} = U$

NOTE Mathematically speaking, the asymptotic value A and the steady state are reached when $t\to\infty$ but numerically speaking in many practical industrial cases $A(t)$ tends to A – with a sufficient accuracy – rather quickly. For example, for Markovian models where all the components have MTTR much smaller than MTTF the asymptotic value and the steady state are reached after duration equal to two or three times the highest MTTR of the components of the system.

With regard to the state-transition graph (see Figure 14) or the Markov graph (see Figure 15) and if a steady state exists, the steady state availability and the instantaneous unavailability of the modelled item are the following:

$$A = P_1 + P_2 + P_3$$

$$U = P_4$$

Those asymptotic values are illustrated in Figure 17.

At this stage it is possible to make the link between the asymptotic values and the mean up time (MUT) and the mean down time (MDT). When a steady state exists, the asymptotic availability and the asymptotic mean availability is given by (see references [9] and [10]):

$$\overline{A} = A = \frac{\text{MUT}}{\text{MUT} + \text{MDT}}$$

In the same way, the asymptotic unavailability and the asymptotic mean unavailability are given by:

$$\overline{U} = U = \frac{\text{MDT}}{\text{MUT} + \text{MDT}}$$

The above formulae have been established under the assumption that $A(t)$ or $U(t)$ reach asymptotic values. They can be used to estimate A and U from the data collection of field feedback about the up an down times. In certain cases they are still valid for $\overline{A}$ and $\overline{U}$ even if A and U do not exist. This is the case, for example, for systems implementing periodically tested components where $A(t)$ and $U(t)$ have no asymptotic values but where $\overline{A} = \lim_{t_2\to\infty} \overline{A}(t_1,t_2)$ and $\overline{U} = \lim_{t_2\to\infty} \overline{U}(t_1,t_2)$ can exist.

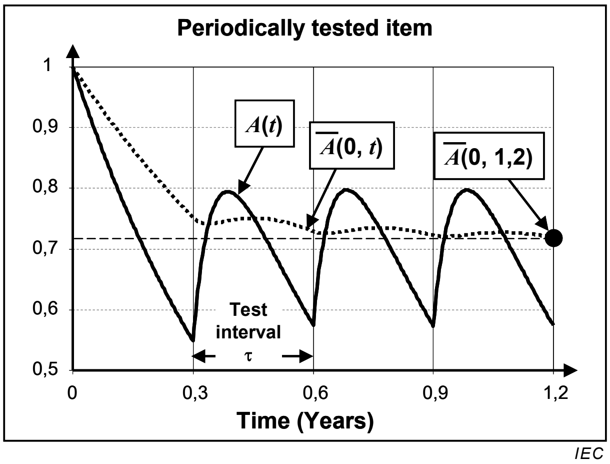


Figure 19 – Instantaneous availability and mean availability of a periodically tested item

Figure 19 illustrates the evolution of a normally dormant periodically tested item with a failure rate of 2 year^{-1}, a repair rate of 20 years^{-1} (MTTR = 438 h) and a test interval τ of 0,3 years (4 months). The instantaneous availability $A(t)$ decreases during the first test interval. If a failure occurs during the first test interval, its repair is started at the beginning of the second test interval. From this point, there is a competition between the cases where the item was in up state at the beginning of the second test interval (and which may go to the down state) and the cases where the system is under repair at the beginning of the second test interval (and which may go to the up state). Therefore $A(t)$ increases until a time roughly equal to the MTTR and then decreases until the next test. This gives the typical saw tooth curve shape which is the characteristic of the instantaneous availability of such periodically tested items. The instantaneous availability $A(t)$ has no asymptotic value but after some test interval the curve of $A(t)$ reaches a limit shape (i.e. the curve becomes identical from an interval to the next one) and keeps it for the further test intervals. Although $A(t)$ has no asymptotic value, the average availability $\overline{A}(0,t)$ does converge towards an asymptotic value (see the curve in dotted lines in Figure 19) which is equal to the average availability within a test interval located at infinity:

$$\overline{A} = \overline{A}(\infty) = \lim_{t \to \infty} \overline{A}(0,t) = \operatorname{Lim}_{i \to \infty} \overline{A}(i.\tau, [i+1] \cdot \tau) = \frac{\text{MUT}}{\text{MUT} + \text{MDT}}$$

As preventive maintenance is not taken under consideration in this document, the MDT can be replaced by the MTTR (see Figure 2) in the above formulae.

6.1.2.4 Extension of the availability concept to multi-states items

As defined earlier, availability concept is related to the up states of the item under consideration. No distinction is made between the states that are considered to be up states. The item in all of its up states is considered to provide the same service to the user. While this hypothesis may be relevant for individual items, this can be questioned when systems are involved. This is particularly the case for production systems used in industry to produce products like oil, gas, electricity, water, etc.

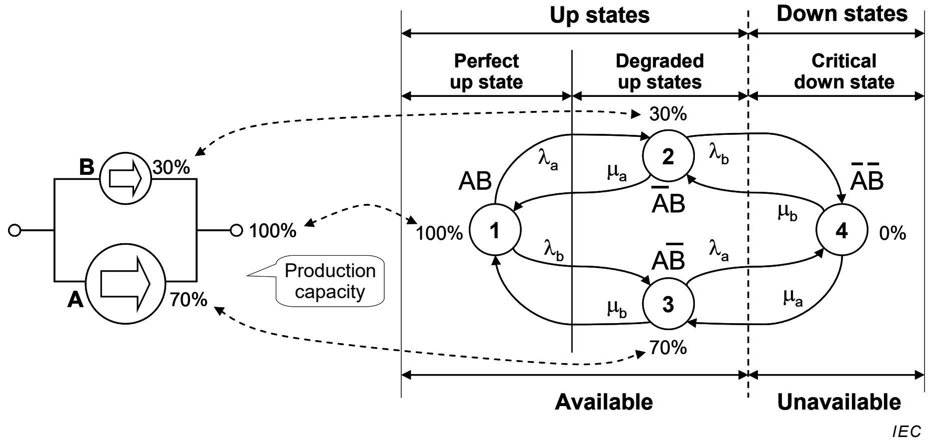


Figure 20 – Example of a simple production system

The left hand side of Figure 20 illustrates a simple production system made of 2 production units A and B. The nominal production per unit of time of the system is ρ (e.g. 5 m^3 of water per hour, 1 500 loafs of bread per hour, ...). The unit A has a production capacity K_A equal to 70 % when the unit B has a capacity K_B equal to 30 %. Therefore the whole production system has a production capacity K_S equal to (70 % + 30 %) = 100 %.

NOTE A production item A with a production capacity of K_A provides a production of $K_A \cdot \rho$ per unit of time.

The Markov graph modelling this system is the same as the one presented in Figure 15. The difference is that the level of service provided is different for each up state:

– in state 1 the production per unit of time of the system is $1 \cdot \rho = \rho$
– in state 2 the production per unit of time of the system is $0{,}7 \cdot \rho$
– in state 3 the production per unit of time of the system is $0{,}3 \cdot \rho$

The down state 4 is characterized by a production per unit of time of $0 \cdot \rho = 0$.

From a production point of view, it is not possible to simply split the states into up and down states and it is necessary to refine the classification. Such a system is called multi-states (see [28]) because its states have to be split in more than two classes. The measure of interest of such a system is not its availability nor its reliability but the mean expected value of its production over a given time interval.

According to the above assumptions, the instantaneous production capacity $K(t)$ of the system at time t is equal to:

$$K(t) = 100\% \cdot P_1(t) + 30\% \cdot P_2(t) + 70\% \cdot P_3(t) + 0\% \cdot P_4(t)$$

This formula can easily be generalized to systems with n states:

$$K(t) = \sum_{i=1}^{n} K_i \cdot P_i(t)$$

From the production capacity, the instantaneous expected production $Prod(t)$ is given by:

$$Prod(t) = K(t) \cdot \rho$$

As a particular case of $K(t)$, it is found the conventional instantaneous availability [192-08-01] for which K_i = 100 % for all the up states and K_i = 0 % for all the down states. It is the same for $Prod(t)$ when, in addition, ρ is equal to 1. In these cases, $A(t) \equiv K(t) \equiv Prod(t)$.

Figure 21 illustrates the evolution of the instantaneous availability $A(t)$ and of the production capacity $K(t)$ at time t according to the above hypothesis.

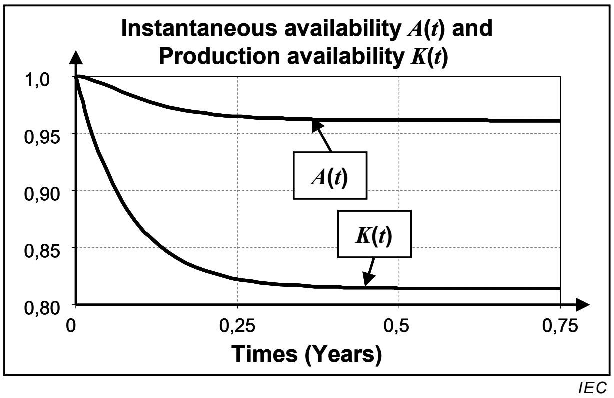


Figure 21 – Evolution of $A(t)$ and $K(t)$

The expected production $Prod(t_1,t_2)$ of the system over an interval [t_1, t_2] can be calculated through the mean accumulated sojourn times spent in the various states:

$$Prod(t_1,t_2) = [100\% \cdot Ast_1(t_1,t_2) + 30\% \cdot Ast_2(t_1,t_2) + 70\% \cdot Ast_3(t_1,t_2) + 0\% \cdot Ast_4(t_1,t_2)] \cdot \rho$$

And therefore the mean expected production is found by:

$$\overline{Prod}(t_1,t_2) = \frac{Prod(t_1,t_2)}{(t_2 - t_1) \cdot \rho}$$

where $(t_2 - t_1) \cdot \rho$ is the maximum possible production over [t_1, t_2].

Finally, the mean expected production over [t_1, t_2] is given by:

$$\overline{Prod}(t_1,t_2) = \frac{100\% \cdot Ast_1(t_1,t_2) + 30\% \cdot Ast_2(t_1,t_2) + 70\% \cdot Ast_3(t_1,t_2) + 0\% \cdot Ast_4(t_1,t_2)}{t_2 - t_1}$$

This formula can be generalized to systems with n states:

$$\overline{Prod}(t_1,t_2) = \frac{\sum_{i=1}^{n} K_i \cdot Ast_i(t_1,t_2)}{t_2 - t_1}$$

where K_i is the production capacity of the state i.

Of course, when a steady state exists the mean expected production tends to an asymptotic value which is equal to the asymptotic value of the instantaneous production capacity $K(\infty)$. This is illustrated in Figure 21.

Such a measure which is an extension of the mean availability and the mean expected production is often called "production availability" of the system. More generally it is also called "effectiveness" of the item. It is useful each time the service provided by a system state is proportional to the time spent in this state.

6.1.3 Reliability related expressions

6.1.3.1 Reliability [192-05-05] and unreliability

According to its definition [192-05-05], Note 3, the reliability $R(t)$ is the probability of performing as required for the time interval [0, t], under given conditions. Figure 22 illustrates a reliable behaviour over [0, t] of the system modelled by the state-transition graph proposed in Figure 14. It was in the perfect up state 1 (i.e. as good as new) at time t equal to 0 and has stayed in the up states 1, 2 and 3 all over [0, t].

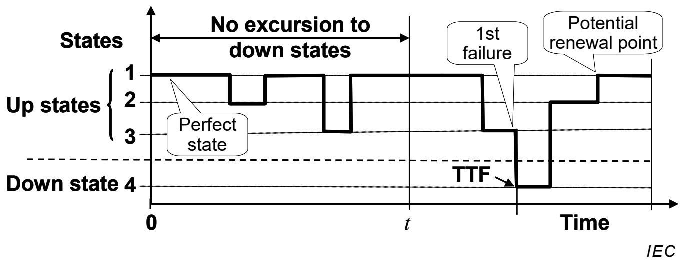


Figure 22 – Illustration of a system reliable behaviour over [0, t]

This is a typical example where the system failure rate $\lambda(t)$ [192-05-06] is non-constant because it may at least change according to the considered up state.

The reliability function is directly linked to the failure rate $\lambda(t)$ and to the failure density function $f(t)$ by the following formulae (see 6.1.5.1 NOTE 1):

$$R(t) = \exp[-\int_0^t \lambda(\tau)d\tau] = \int_t^\infty f(\tau)d\tau$$

If observed failure data are available for n items, from a homogenous population, the estimated value of $R(t)$ can be estimated by

$$\hat{R}(t) = \frac{n_S(t)}{n}$$

where $n_S(t)$ is the number of items that have had no failures over [0, t] and $n = n_S(0)$.

NOTE If the system is as good as new after restoration to the perfect state then any time of restoration to the perfect state can be used as initial time 0 (renewal point, see Figure 22).

According to its definition [192-05-05] the reliability $R(t_1,t_2)$ is the probability of performing as required for the time interval [t_1, t_2], under given conditions. Therefore the reliability of a system is the probability that it will be in up state all over [t_1, t_2]. It means that:

– the system is in up state at time t_1, i.e. it is available;

– the system never goes to the down state from t_1 to t_2.

This is illustrated in Figure 23 from the state-transition graph shown in Figure 14. What has happened before t_1 does not matter, except that the system has to be available at time t_1. What happens after t_2 does not matter either.

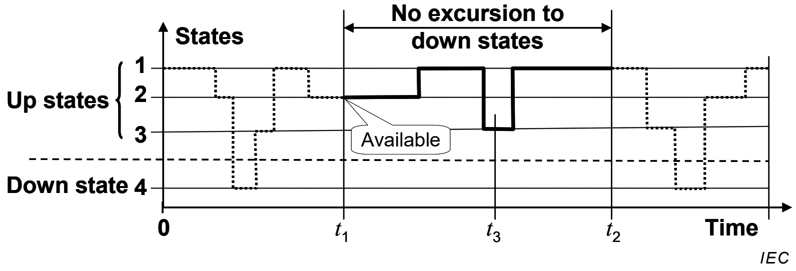


Figure 23 – Illustration of a system reliable behaviour over time interval [t_1, t_2]

From the instant t_1 (see Figure 23), the system has reached the state 3 at time t_3 through the sequence 2→1→3. But the graphs presented in Figure 14 or Figure 15 encodes all the sequences of events from state 2 to state 3. Therefore they encode two types of sequences:

a) sequences going from 2 to 3 without going through the down state (e.g. 2→1→3);

b) sequences going from 2 to 3 through the down state (e.g. 2→4→3).

Only the sequences of type a) ensure that the system remains in up state all over a time interval. Therefore the sequences of type b) do not participate in the system reliability and have to be excluded from the reliability calculation. This can be achieved by preventing the system from coming back to the up states when it has reached the down state as has been done in Figure 24.

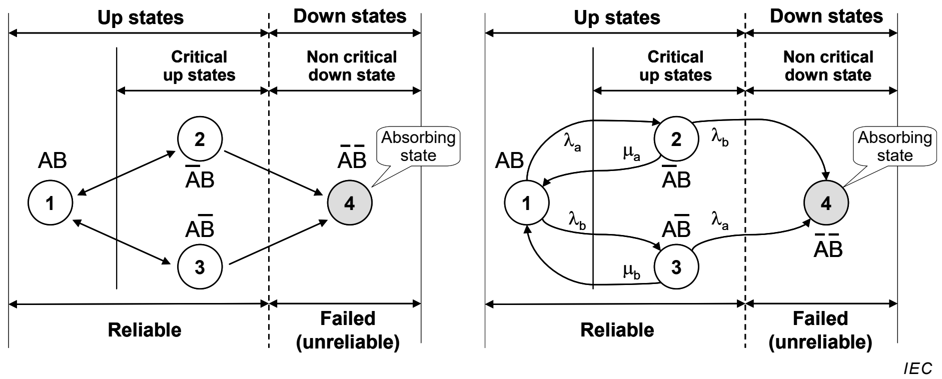


Figure 24 – State-transition and Markov graphs for reliability calculations

NOTE The Markov graph in Figure 24 allows to calculate the system reliability. To highlight this property, such a Markov graph is called “reliability Markov graph”.

In Figure 24 any up state can only be reached from other up states. When the system reaches the down state it remains stuck there for eternity. This is why it is called "absorbing" state. At the end the probability to be in this state is 1 and $\lim_{t_2 \to \infty} R(t_1, t_2) = 0$.

When the system is modelled by a Markov graph, the reliability $R(t_1, t_2)$ of the example can be calculated in two steps:

1) calculations of the probabilities $P_1^{(av)}(t_1)$, $P_2^{(av)}(t_1)$, $P_3^{(av)}(t_1)$ and $P_4^{(av)}(t_1)$ by using the Markov graph without absorbing state (see Figure 15);

2) calculations of the probabilities $P_1^{(rel)}(\theta)$, $P_2^{(rel)}(\theta)$, $P_3^{(rel)}(\theta)$ and $P_4^{(rel)}(\theta)$ for $\theta = t_2 - t_1$, by using the Markov graph with the absorbing state (left hand side of Figure 24) and the probabilities calculated at step 1) as initial conditions.

This gives both the reliability $R(t_1,t_2)$ and the unreliability $F(t_1,t_2)$:

$$R(t_1,t_2) = P_1^{(rel)}(\theta) + P_2^{(rel)}(\theta) + P_3^{(rel)}(\theta)$$

$$F(t_1,t_2) = P_4^{(rel)}(\theta)$$

where $\theta = t_2 - t_1$

And therefore: $F(t_1,t_2) = 1 - R(t_1,t_2)$.

When t_1 is equal to 0 and the calculation made for reliability/unreliability over the interval [0, t], the step 1) above is not required and the probabilities have to be calculated with the graphs with the absorbing states (see Figure 24).

Figure 25 illustrates the state probabilities of the system modelled by the reliability Markov graph (i.e. a Markov graph with an absorbing state) in Figure 24. As in the availability case those probabilities reach asymptotic values but these are 0 for the 3 up states and 1 for the down state.

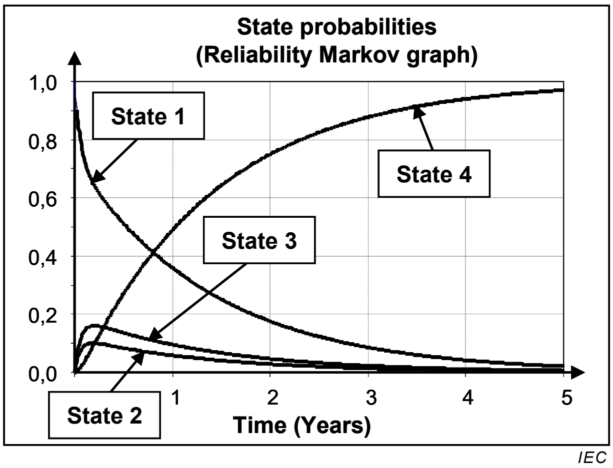


Figure 25 – Evolution of the state probabilities related to the Markov model in Figure 24

The state probabilities are presented in Figure 25 and therefore:

$$R(t) = P_1(t) + P_2(t) + P_3(t)$$

$$F(t) = P_4(t)$$

Due to the absorbing state, the evolution of the reliability and the unreliability are as illustrated in Figure 26.

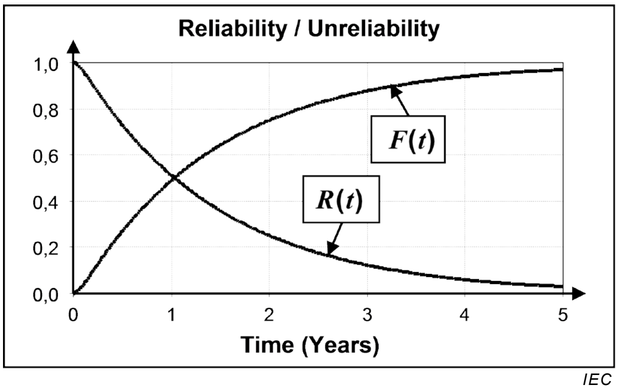


Figure 26 – Evolution of $R(t)$ and $F(t)$ related to the Markov model in Figure 24

The reliability/unreliability formulae are similar to those established for the availability/unavailability calculations. The difference of the results is only due to the presence or not of the absorbing state:

- graph without absorbing state: availability/unavailability calculations;
- graphs with absorbing state: reliability/unreliability calculation.

Looking at the graphs in Figure 24 it can be seen that the components of the system are repairable only if the whole item has not gone to the down state. Therefore the reliability calculations introduce systemic dependencies between the components and this makes the calculations more difficult than availability calculations.

Due to the absorbing state the reliability and the unreliability reach the following asymptotic values when the time increases.

$$\lim_{t\to\infty} R(t) = 0.$$

$$\lim_{t\to\infty} F(t) = 1.$$

This is illustrated in Figure 26.

6.1.3.2 Mean time to first failure – MTTFF [192-05-12]

According to the definition given in IEC 60050-192, the MTTFF is the expectation of the duration of the operating time to first failure. It is linked to the reliability and failure density functions by the following formula:

$$\text{MTTFF} = \int_0^\infty t f(t)\,\mathrm{d}t = \int_0^\infty R(t)\,\mathrm{d}t$$

In the case of COI items the up states are also operating states and therefore the MTTFF can be calculated by the sum of the mean accumulated sojourn time in up states of a reliability graph:

$$\text{MTTFF} = \lim_{t\to\infty} \sum_{i \in \text{up states}} Ast_i(0,t)$$

Figure 27 illustrates the evolution of the mean accumulated sojourn times $Ast_i(t)$ over [0, t] in the case of the reliability Markov graph presented in Figure 24.

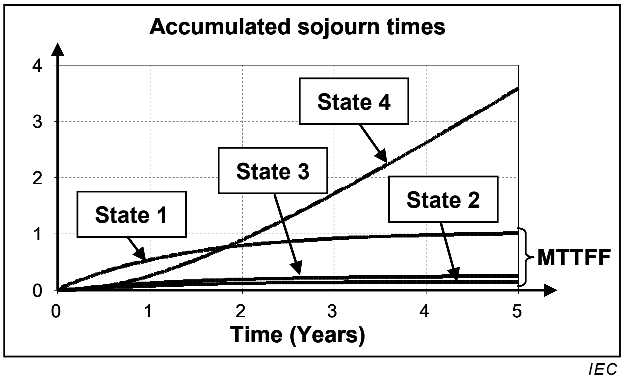


Figure 27 – Evolution of $Ast_i(0, t)$ related to the Markov model in Figure 24

When the time increases, the mean accumulated sojourn times spent in the up states (1, 2 and 3) tend to asymptotic values when the mean accumulated sojourn time spent in the down state (4) goes to infinity (this is the characteristic of an absorbing state). Therefore the MTTFF can be calculated as:

$$\text{MTTFF} = \lim_{t \to \infty} [Ast_1(0,t) + Ast_2(0,t) + Ast_3(0,t)]$$

The mean accumulated sojourn times in up states increase until they reach asymptotic values when the probability of the item to be in the down state is close to 1. Therefore the above formula converges more quickly for unreliable items than for reliable items.

6.1.4 Mean operating time between failures [192-05-13] and mean time between failures

IEC 60050-192 uses the acronyms MTBF or MOTBF for the mean operating time between failures [192-05-13] and therefore it is incorrect to use MTBF for the mean time between failures. This is why, in order to avoid any confusion, the acronym METBF is used in this standard for the mean time between failures (see definition 3.3).

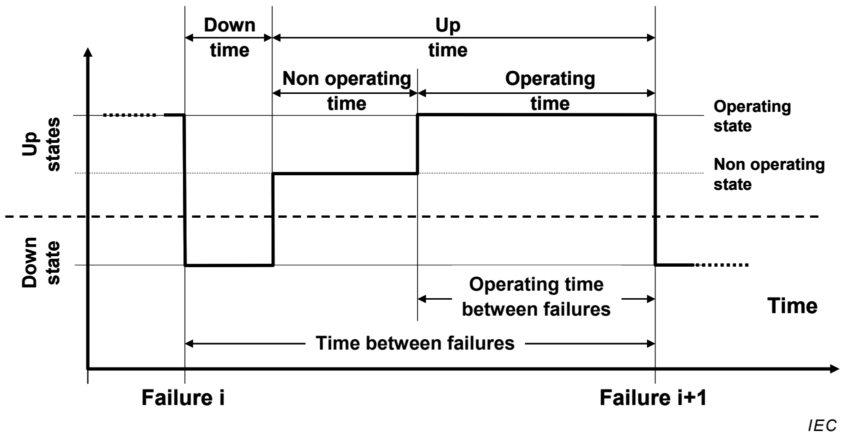


Figure 28 – Time between failures versus operating time between failures

Figure 28 illustrates the difference between the (elapsed) time between failures and the operating time between two consecutive failures: the time between failures is the sum of the up and down times when the operating time between failures is only the part of the up time when the item is in the operating state.

This leads to the general formula for the mean time between failures which is:

$$\text{METBF} = \text{MUT} + \text{MDT}$$

If the preventive maintenance is not considered, as is the case in this document, the MUT is equal to the MTTR (mean time to restoration) and therefore:

$$\text{METBF} = \text{MUT} + \text{MTTR}$$

For COIs, the mean up time is equal to the mean time to failure. Thus:

$$\text{METBF} = \text{MTTF} + \text{MTTR}$$

6.1.5 Instantaneous failure rate [192-05-06] and conditional failure intensity (Vesely failure rate)

6.1.5.1 Instantaneous values

According to the definition 3.6, the instantaneous failure rate is the limit, if it exists, of the quotient of the conditional probability that the failure of an item occurs within time interval $[t, t + \Delta t]$, and Δt, when Δt tends to zero, given that failure has not occurred within time interval $[0, t]$:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{F(t + \Delta t) - F(t)}{R(t)} = \frac{f(t)}{R(t)}$$

IEC 60050-192 provides a definition which is restricted to non-repairable items. But, in fact, it can be generalized as a failure density function $f(t)$ and a reliability function $R(t)$ can be defined for any item (individual items or systems, repairable or non-repairable).

According to the above definition, the failure rate $\lambda(t)$ is a conditional probability per unit of time:

$$\lambda(t) = \lim_{\Delta t \to 0^+} \frac{P(\text{failure between } t \text{ and } t + \Delta t \mid \text{as good as new at } t = 0 \text{ and up state over } [0, t])}{\Delta t}$$

NOTE 1 $\lim_{\Delta t \to 0^+} \lambda(t) \cdot \Delta t$ is the conditional probability that the system fails at time t given no failure has occurred since $t = 0$. Using the differential notation this can be written $\lambda(t)dt$ and then the increment $dF(t)$ of the probability of failure within $[t, t+dt]$ is equal to $R(t).\lambda(t)dt$. As $dF(t) = d[1- R(t)] = -dR(t)$ this gives $\lambda(t) = -dR(t)/R(t)$ and the failure rate is also directly linked to the logarithmic derivative of the reliability $R(t)$. The integration of this derivative leads to the equation $R(t) = \exp[-\int_0^t \lambda(\tau) d\tau]$ introduced in 6.1.3.1.

The condition "up state over [0, t]" (i.e. "no failure over [0, t]") is very strong and introduces the same systemic dependency as the one occurring with reliability calculations.

Looking at the example above in Figure 24, it can be seen that:

– the condition "up state over [0, t]" implies that the system

- cannot go back from down to up states; this is ensured by the absorbing state;
- is in up state at t; the probability of this event is the reliability given by $R(t) = P_1^{(rel)}(t) + P_2^{(rel)}(t) + P_3^{(rel)}(t)$;

– the item can go to the down state only from the critical states 2 and 3;

– the probability of failure between t and $t + \mathrm{d}t$ from state 2 is $\lambda_\mathrm{b}\mathrm{d}t$ by failure of component B;

– the probability of failure between t and $t + \mathrm{d}t$ from state 3 is $\lambda_\mathrm{a}\mathrm{d}t$ by failure of component A.

Therefore the following result is obtained:

$$\lambda(t) = \frac{\lambda_b P_2^{(rel)}(t) + \lambda_a P_3^{(rel)}(t)}{R(t)} = \frac{\lambda_b P_2^{(rel)}(t) + \lambda_a P_3^{(rel)}(t)}{P_1^{(rel)}(t) + P_2^{(rel)}(t) + P_3^{(rel)}(t)}$$

In the general case and when the item has Markovian behaviour, its failure rate can be calculated from its critical up states and the constant transition rates towards the down states.

The failure rate is a very important parameter as it is directly linked to the reliability function $R(t)$:

$$R(t) = e^{-\int_0^t \lambda(\tau)\mathrm{d}\tau}$$

If the same calculation is made from a graph without absorbing state (availability graph) such as Figure 14 or Figure 15 another important parameter is obtained which has not been defined in IEC 60050-192 yet: the conditional failure intensity also called Vesely failure rate $\lambda_\mathrm{V}(t)$. Formally it can be defined as:

$$\lambda_\mathrm{V}(t) = \lim_{\Delta t \to 0^+} \frac{P(\text{failure between } t \text{ and } t + \Delta t \mid \text{as good as new at 0 and up state at } t\,)}{\Delta t}$$

This is also a conditional probability per unit of time but the condition is weaker than for the actual failure rate, $\lambda(t)$: what happened during [0, t] does not matter (i.e. the item may have been in down state for several times before t) and the graph has no absorbing state. The only condition is that the item has to be available at t.

NOTE 2 $\lim_{\Delta t \to 0^+} \lambda_\mathrm{V}(t) \cdot \Delta t$ is the conditional probability that the system fails at time t given it is in up state at time t and was in up state at t = 0.

In the case of Figure 15 this gives:

$$\lambda_\mathrm{V}(t) = \frac{\lambda_b P_2^{(av)}(t) + \lambda_a P_3^{(av)}(t)}{A(t)} = \frac{\lambda_b P_2^{(av)}(t) + \lambda_a P_3^{(av)}(t)}{P_1^{(av)}(t) + P_2^{(av)}(t) + P_3^{(av)}(t)}$$

Figure 29 illustrates the difference between the failure rate $\lambda(t)$ obtained from Figure 15 and the Vesely failure rate $\lambda_\mathrm{V}(t)$ obtained from Figure 24. The two parameters behave in the same way when the time increases. They are equal for short times and converge to different asymptotic values, $\lambda(\infty)$ and $\lambda_\mathrm{V}(\infty)$, when the time becomes larger.

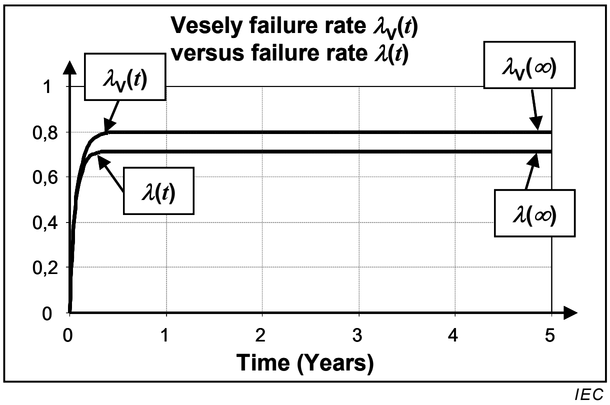


Figure 29 – Comparison between $\lambda(t)$ and $\lambda_V(t)$ related to the model in Figure 24

6.1.5.2 Mean and steady state (asymptotic) values

The mean failure rate is defined in IEC 60050-192. It has to be used cautiously because this is not really a failure rate and the fundamental relationship $R(t) = \exp[\int_0^t \lambda(\tau)d\tau]$ does not hold if $\lambda(\tau)$ is replaced by a mean failure rate. Therefore it cannot be used to calculate the item reliability $R(t)$ without care.

As shown in Figure 29, the following inequality $\lambda_V(\infty) > \lambda(\infty) > \lambda(t) \forall t$ is verified for the system modelled by the Markov process. More generally the inequality $\lambda_V(\infty) \geq \lambda(\infty)$ holds for systems made of independent repairable components with constant failure and repair rates.

In fact, the faster the restorations of the failed components are done and the more reliable/available the system is, the quicker the steady state and therefore the asymptotic values are reached and the closer $\lambda(\infty)$ and $\lambda_V(\infty)$ are. Using $\lambda_V(\infty)$ allows to obtain a very good approximation of $R(t)$ after a duration equal to two or three times the greater MTTR of the components of the system. This covers a large part of the studies made by reliability engineers.

NOTE The above results do not hold for systems with non repaired components or with no restoration from a failed state.

The conditions for calculating the Vesely failure rate are weaker than for the failure rate (see 6.1.5.1). Therefore it is easier to calculate than the failure rate. For actual studies on industrial systems with repairable components, the failure rate often cannot be calculated when the Vesely failure rate can be obtained rather easily. Thus, the Vesely failure rate is often used instead of the failure rate. This property is, for example, the basis of reliability calculations based on fault trees (see IEC 61025 [24]) and reliability block diagrams (see IEC 61078 [25]) modelling systems with repairable components.

When the failures are quickly detected and repaired (i.e. failure rates << repair rates) $\lambda(\infty) \approx \lambda_V(\infty)$ can be obtained directly from the Markov graph by using the following principle applied to the small example modelled in Figure 15 (or in Figure 24):

– identify a sequence from the perfect state 1 to the down state 4, e.g. 1→2→4;
– the item can go from 1 to 2 with the transition rate λ_a;
– the time spent in state 2 is negligible compared to the time spent in state 1. Therefore when the item reaches state 2 and according to the properties of constant failure rates, it

leaves it almost immediately for state 1 (probability $\frac{\mu_a}{\lambda_b + \mu_a}$) or state 4 (probability $\frac{\lambda_b}{\lambda_b + \mu_a}$);

- then the transition rate from 1 to 4 by the sequence 1→2→4 is $\lambda_a \frac{\lambda_b}{\lambda_b + \mu_a}$;
- continue with the other sequences (e.g. 1→3→4) until all sequences have been processed;
- gather all the results to obtain a good approximation of the asymptotic value of $\lambda(\infty) \approx \lambda_V(\infty)$.

In the example, only two sequences (1→2→4 and 1→3→4) have to be considered and the two Markov graphs lead to the same result:

$$\lambda(\infty) \approx \lambda_{\text{V}}(\infty) = \lambda_a \frac{\lambda_b}{\lambda_b + \mu_a} + \lambda_b \frac{\lambda_a}{\lambda_a + \mu_b}$$

The above asymptotic values also provide the mean failure rate over a large time interval.

6.1.6 Failure density and unconditional failure intensity [192-05-08]

The unreliability function $F(t)$ is the probability that the time to failure TTF is lower or equal to t. Therefore this is also the time to failure distribution or, in short, the failure distribution and the derivative of this function is the failure density $f(t)$. This means that $f(t)$.dt is the probability that the item fails between t and t+dt given that it was in up state (and as good as new) at $t = 0$:

$$f(t) = \lim_{\Delta t \to 0^+} \frac{P(t < \text{time to first failure} \leq\ t + \Delta t \mid \text{as good as new at } t = 0\ )}{\Delta t}$$

This formula is similar to the formula of the failure rate $\lambda(t)$ except for the condition to be in up state over [0, t] which is given by $R(t)$. Therefore $\lambda(t) = f(t) / R(t)$ and:

$$f(t) = \lambda(t) \cdot R(t)$$

In the case of the example this leads to:

$$f(t) = \lambda_b P_2^{(rel)}(t) + \lambda_a P_3^{(rel)}(t)$$

Another way to establish this formula is to realize that, due to the absorbing state of the Markov graph in Figure 24, the probability to have the first failure between t + dt is also the probability to be in a critical state at time t, e.g. $P_2(t)$, and to go to the down state between t and t + dt, e.g λ_{b}.dt. In the case of the example, it is found again:

$$f(t) = \frac{P_2^{(rel)}(t)\lambda_b \cdot \text{d}t + P_3^{(rel)}(t)\lambda_a \cdot \text{d}t}{\text{d}t} = \lambda_b P_2^{(rel)}(t) + \lambda_a P_3^{(rel)}(t)$$

If the same calculation is made from a graph without absorbing state (availability graph) such as Figure 14 or Figure 15 another important parameter is obtained: the unconditional failure intensity $z(t)$ which is formally defined as:

$$z(t) = \lim_{\Delta t \to 0^+} \frac{P(\text{failure between } t \text{ and } t + \Delta t \mid \text{as good as new at } t = 0)}{\Delta t}$$

Similarly as above:

$$z(t) = \lambda_V(t) \cdot A(t)$$

And, in the case of the example, the following formula is obtained:

$$z(t) = \lambda_b P_2^{(av)}(t) + \lambda_a P_3^{(av)}(t)$$

The unconditional failure intensity is the same parameter as the failure intensity [192-05-08] defined in another way in IEC 60050-192: limit, if it exists, of the quotient of the mean number of failures of a repairable item within time interval $[t,\ t + \Delta t]$, and Δt, when Δt tends to zero:

$$z(t) = \lim_{\Delta t \to 0^+} \frac{E[N(t + \Delta t) - N(t)]}{\Delta t}$$

In this formula $N(t)$ represents the number of failures occurring over $[0,\ t]$. When $\Delta t \to 0$, a physical item cannot fail several times within $[t,\ t + \Delta t]$ and $[N(t + \Delta t)\text{-}\ N(t)]$ is equal to 1 if a new failure occurs during the interval $[t,\ t + \Delta t]$ and equal to 0 otherwise. Finally, when $\Delta t \to 0$, $E[N(t + \Delta t)\text{-}\ N(t)]$ is equal to the probability to have one failure within the interval $[t,\ t + \Delta t]$ and the two formulae are equivalent.

According to its definition, $z(t)$ is also the derivative of the expected number of failures $Z(t)=E[N(t)]$ over $[0,\ t]$:

$$z(t) = \lim_{\Delta t \to 0^+} \frac{Z(t + \Delta t) - Z(t)}{\Delta t} = \frac{dZ(t)}{\mathrm{d}t} \text{ and } Z(t) = \int_0^t z(\tau)\mathrm{d}\tau .$$

When $\Delta t \to 0$, $z(t) \cdot \Delta t$ is the expected number of failures over $[t,\ \Delta t]$ then $z(t) \cdot \Delta t / \Delta t = z(t)$ is also the instantaneous failure frequency of the item at time t. This is why this measure is often called "failure frequency".

The average failure frequency over $[0,\ t]$ can be calculated as $\frac{Z(t)}{t}$. Therefore if a steady state exists, $z(t)$ reaches an asymptotic value and this leads to (see [7]):

$$\lim_{t \to \infty} \frac{Z(t)}{t} = \lim_{t \to \infty} \frac{\int_0^t z(\tau)\mathrm{d}\tau}{t} = z(\infty)$$

If $\overline{Tbf}(t)$ is the mean elapsed time between the consecutive failures occurring over an interval $[0,\ t]$ then, $t / \overline{Tbf}(t)$ is equal to the expected number of failures over $[0,\ t]$, $Z(t)$. When t tends to the infinity $\overline{Tbf}(\infty)$ is the METBF of the modelled system and when a steady state exists $z(t)$ reaches an asymptotic value $z(\infty)$. Then:

$$\lim_{t\to\infty}\frac{Z(t)}{t}=\lim_{t\to\infty}\frac{1}{\overline{Tbf}(t)}=\frac{1}{\text{METBF}}=z(\infty)$$

Therefore: $\text{METBF}=\frac{1}{z(\infty)}$.

More details can be found in reference [10].

Among the parameters analysed above (failure rate, Vesely failure rate and failure density), only the mean value of the unconditional failure intensity [192-05-09] is really useful:

$$\bar{z}(t_1,t_2)=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}z(t)\mathrm{d}t$$

This allows to calculate the number of failures occurring over a given time interval:

$$E[N(t_1,t_2)]=\bar{z}(t_1,t_2)\cdot(t_2-t_1)$$

NOTE The mean failure frequency is called PFH (probability of failure per hour) when it is used in the context of the functional safety of safety instrumented systems (e.g. IEC 61508 [21] or IEC 61511 [22]).

When the events are exponential (Markovian models) and when the failures are detected and repaired quickly, $A(t)$ and $\lambda_V(t)$ reach asymptotic (steady state) values A and $\lambda_V(\infty)$ after a duration equal to two or three times the greater MTTR of the components of the system. Then the failure frequency $z(t)$ also reaches an asymptotic value given by the following formula:

$$z(\infty) = A \cdot \lambda_V(\infty)$$

Once the steady state is achieved, the asymptotic values are also the mean values of these parameters.

Figure 30 shows the evolution of the failure density $f(t)$ obtained from Figure 24 and of the unconditional failure intensity $z(t)$ obtained from Figure 15. The behaviours are very different: $z(t)$ increases until it reaches an asymptotic value when $f(t)$ increases first and, after reaching a maximum value decreases to zero.

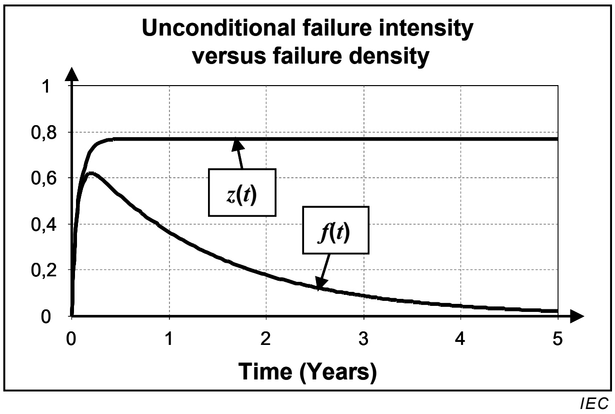


Figure 30 – Comparison between $z(t)$ and $f(t)$

Both $z(t)$ and $f(t)$ represent the system failure frequency but for $f(t)$ the system can fail only once and after it has failed it cannot fail any more. Therefore when $F(t)$ is close to 1 the probability to observe the failure is very low and $f(t)$ tends to 0.

6.1.7 Comparison of $\lambda(t)$, $\lambda_V(t)$, z(t) and f(t) for high and small MTTRs

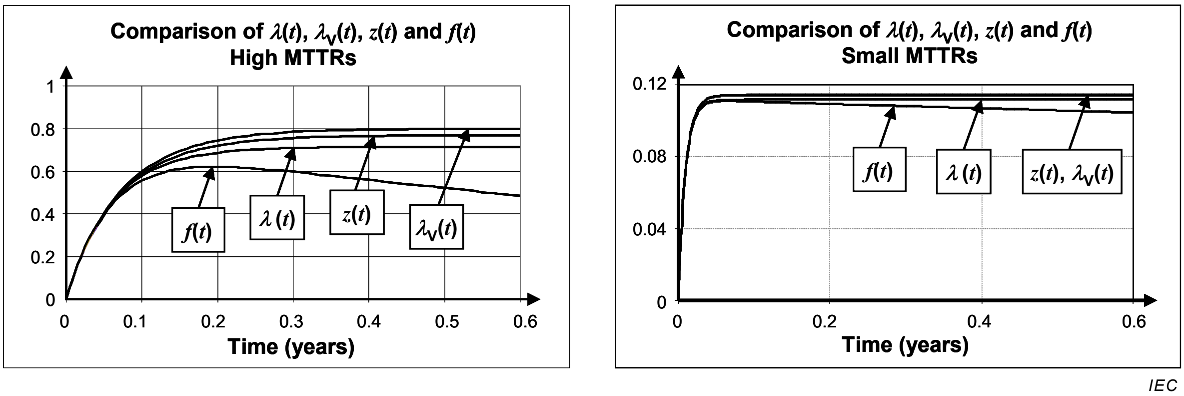


Figure 31 – Comparison of $\lambda(t)$, $\lambda_V(t)$, z(t) and f(t) for high and small values of MTTRs

On its left hand side, Figure 31 gathers the results presented in Figure 29 and in Figure 30. On the right hand side, it shows the same results when the MTTRS of the components A and B have been divided by 10. In both cases and on the short term, $\lambda(t)$, $\lambda_V(t)$, z(t) and f(t) have close numerical values, then, when the time increases, $\lambda(t)$, $\lambda_V(t)$, z(t) converge to asymptotic values and f(t) decreases to 0. These results are typical of Markovian models.

Figure 31 shows that, for Markovian models:

– The faster the failed items are restored, the faster the asymptotic values $\lambda(\infty)$, $\lambda_V(\infty)$ and $z(\infty)$ are reached and the closer they are. On the right hand side $z(t)$ and $\lambda_V(t)$ are even merged as the availability is very high. The asymptotic values are reached after about 3 times the highest MTTR, i.e. 0,3 y (3 x 876 h) on the left hand side and 0,03 y (3 x 87,6 h) on the right hand side. In actual cases, the restoration is often faster and the asymptotic values are reached almost immediately.
– $\lambda_V(\infty)$ is a conservative value of $\lambda(\infty)$. As $\lambda_V(\infty)$ is easier to calculate than $\lambda(\infty)$, it is commonly used instead of $\lambda(t)$ for reliability calculations.
– $z(\infty)$ is also a conservative value of $\lambda(\infty)$.
– Due to the similarities of the numerical values $\lambda(\infty)$, $\lambda_V(\infty)$ and $z(\infty)$ can be easily mixed-up.

The above results hold only for dependability models with underlying Markovian properties (e.g. simplified formulae, reliability block diagrams, fault trees, event trees, Petri nets, when they implement only constant failure and repair rates).

6.1.8 Restoration related expressions

6.1.8.1 Repair rate [192-07-20] and mean repair time [192-07-21]

The repair rate $\mu(t)$ is the limit, if it exists, of the quotient of the conditional probability that the repair is completed within time interval $[t, t + \Delta t]$ and Δt, when Δt tends to zero, given that the repair started at $t = 0$ and had not been completed before time t.

The difference between the restoration intensity and the repair rate is that, for the repair rate, the repair is considered to have started at $t = 0$ whereas for the restoration intensity the condition is that the item is as good as new at time $t = 0$.

The mean repair time (MRT) is the expectation of the repair time.

These definitions are similar (for the repairs) to the definitions given (for failures) for the failure rate and the MTTFF.

Looking at Figure 14 or Figure 15 and considering that the system is in state 4 at time equal 0:

– the repair rate is the sum of the transition rates from 4 to 2 and from 4 to 3;
– the MRT is the mean time spent in state 4 each time the item goes to the down state.

This leads to:

$$\mu = \mu_a + \mu_b$$

$$\text{MRT} = \frac{1}{\mu} = \frac{1}{\mu_a + \mu_b}$$

If now Figure 4 is considered, it can be seen that the system has 3 down states. Therefore, the condition "given that the repair started at $t = 0$" implies to define 3 different repair rates according to the probability to be in states 5, 6 and 7 at time $t = 0$. Therefore a system with several down states has not a single repair rate and this concept is relevant mainly for individual items considered as a whole.

6.1.8.2 Restoration intensity and mean time to restoration [192-07-23]

Restoration intensity is not defined in IEC 60050-192 and has been introduced in Clause 3 of the present standard: limit, if it exists, of the quotient of the mean number of restorations [192-06-23] of a repairable item [192-01-11] within time interval $[t, t + \Delta t]$, and Δt, when Δt tends to zero, given that the item is as good as new at time $t = 0$.

$$v(t) = \lim_{\Delta t \to 0^+} \frac{E[N_\text{R}(t + \Delta t) - N_\text{R}(t) \,|\, \text{as good as new at } t = 0]}{\Delta t}.$$

According to IEC 60050-192, the down time is made of the time to restoration plus a part of the preventive maintenance time. The hypothesis here is that the preventive maintenance is not considered and that the time to restoration is equal to the down time. See Figure 2.

In this formula $N_\text{R}(t)$ represents the number of restorations occurring over $[0, t]$. When $\Delta t \to 0$ several restorations of a physical item cannot finish within $[t, t + \Delta t]$, then $N_\text{R}(t + \Delta t) - N_\text{R}(t)]$ is equal to 1 if a new restoration occurs during the interval $[t, t + \Delta t]$ and equal to 0 otherwise. Therefore:

– $v(t)$ is the expected number of restorations per unit of time and is also the instantaneous restoration frequency of the item.
– When $\Delta t \to 0$, $E[N_\text{R}(t + \Delta t) - N_\text{R}(t)]$ is equal to the probability to have one restoration finishing within the interval $[t, t + \Delta t]$ and the restoration intensity may be defined as:

$$v(t) = \lim_{\Delta t \to 0^+} \frac{P(\text{restoration finishing between } t \text{ and } t + \Delta t \,\big|\, \text{as good as new at } t = 0)}{\Delta t}$$

This definition implies that the item is in a critical down state at t, and goes to one of the up states during $[t, t + \Delta t]$.

In the case of the example in Figure 15 it is found:

$$v(t) = (\mu_a + \mu_b)P_4(t)$$

This can be generalized to items with several critical down states like in Figure 4:

$$v(t) = \mu_{5,3}(t)P_5(t) + [\mu_{6,3}(t) + \mu_{6,4}(t)]P_6(t)$$

where $\mu_{i,j}(t)$ is the transition rate from state i to j at time t.

The mean restoration frequency over a given time interval [t_1, t_2] is obtained by:

$$\bar{v}(t_1,t_2) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)dt$$

Then the mean number of restorations over a given time interval [t_1, t_2] is given by:

$$V(t_1,t_2) = (t_2 - t_1)\bar{v}(t_1,t_2) = \int_{t_1}^{t_2} v(t)dt$$

The overall time of restoration spent over a given time interval [t_1, t_2] is equal to the sum of the accumulated times spent in the down states. Therefore the mean time to restoration is given by:

$$\text{MTTR} = \frac{\sum_{i \in \text{down states}} Ast_i(t_1,t_2)}{\overline{N}_\text{R}(t_1,t_2)}$$

When the preventive maintenance is not considered the MTTR is equal to the mean down time, MDT.

In the case of the example in Figure 15 it is found:

$$V(t_1,t_2) = (\mu_a + \mu_b)\int_{t_1}^{t_2} P_4(t)\text{d}t = (\mu_a + \mu_b).Ast_4(t_1,t_2)$$

and then

$$\text{MTTR} = \frac{1}{\mu_a + \mu_b}$$

6.2 Non-repairable individual items

6.2.1 General

This particular case is illustrated in Figure 6 and Figure 7.

All expressions in 6.2 are applicable to COIs only.

For each measure, the following are presented:

a) the generic expression;
b) the most common expression (for exponentially distributed time to failure of the item);
c) a simple example of application where necessary.

6.2.2 Instantaneous availability [192-08-01]

Symbol $A(t)$

As the item is not repaired, the probability $A(t)$ for the item to be in up state at a time t is also the probability $R(t)$ to have been in the up state all over [0, t].

Then $A(t) \equiv R(t)$ and in this case, the two concepts of reliability and availability are merged. In particular, the asymptotic value A of the availability is equal to 0.

6.2.3 Reliability [192-05-05]

Symbols $R(t_1, t_2)$ for $0 \le t_1 < t_2$ and $R(t) = R(0,t)$ for $t_1 = 0$ and $t_2 = t$

In this case the more commonly used expressions are:

– the reliability function $R(t) = R(0, t)$, with $R(0) = 1$, and
– the conditional reliability $R(t, t + x \mid t)$, when no failure has occurred in time [0, t].

With regard to reliability and availability calculations the main properties of non-repairable items are the following:

– $R(t) \equiv A(t)$ (see 6.2.2);
– the probability $R(t_1, t_2)$ to be in up state all over a given time interval [t_1, t_2], $0 \le t_1 < t_2$, is also the probability $R(t_2)$ to have been in the up state all over [0, t_2].Then $R(t_1,t_2) \equiv R(t_2)$.

a) From a mathematical point of view the reliability function is given by the following formula:

$$R(t) = \exp\left(-\int_0^t \lambda(x)\,\mathrm{d}x\right) = \int_t^\infty f(x)\,\mathrm{d}x$$

where

$\lambda(x)$ is the instantaneous failure rate of the item;

$f(x)$ is the probability density function of the time to failure of the item, i.e. for small values Δx, $f(x)\cdot\Delta x$ is approximately equal to the probability that the failure of the item will occur during $(x, x + \Delta x)$.

b) If observed failure data (field feedback) are available for n non-repairable items, from a homogenous population, the estimated value of $R(t)$ is given by:

$$\hat{R}(t) = \frac{n_\mathrm{S}(t)}{n}$$

where

$n_\mathrm{S}(t)$ is the number of items that are still operational at the instant of time t.

n $n_\mathrm{S}(0)$.

c) The probability that the item will fail during the time interval [t_1, t_2], $0 \le t_1 < t_2$, is given by

$$R(t_1) - R(t_2) = \int_{t_1}^{t_2} f(t)\mathrm{d}t$$

d) The conditional reliability, $R(t,\ t + x \mid t)$, is defined as the conditional probability that an item can perform a required function for a given time interval $[t,\ t + x]$ provided that the item is in an operating state at the beginning of the time interval.

$$R(t, t + x \mid t) = \exp\left(-\int_t^{t+x} \lambda(t)\mathrm{d}t\right) = \frac{R(t + x)}{R(t)}$$

When $\lambda(t) = \lambda$ = constant, i.e. when the (operating) time to failure is exponentially distributed

$$R(t) = \exp(-\lambda t)$$

$$R(t,\ t + x \mid t) = \exp(-\lambda x)$$

NOTE This result comes from the fundamental Markovian property: if the item survives until t, the future of the modelled system does not depend on what has happened before t. This is known as the memorylessness property.

e) For an item with a constant failure rate of $\lambda = 1$ year^{-1} and a required time of operation of six months, the reliability is given by

$$R(6 \text{ months}) = \exp(-1 \times \frac{6}{12}) = 0{,}61$$

6.2.4 Instantaneous failure rate [192-05-06]

Symbol $\lambda(t)$

According to definition [192-05-06]:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{R(t) - R(t + \Delta t)}{R(t)} = \frac{f(t)}{R(t)}$$

For small values of Δt, $\lambda(t)\cdot\Delta t$ is approximately equal to the conditional probability that failure of the item will occur during $[t,\ t + \Delta t]$, given that the item has survived to time t.

Using the failure rate, the probability that the item will fail during the time interval $[t_1,\ t_2]$ is given by

$$F(t_1, t_2) = R(t_1) - R(t_2) = \exp\left(-\int_0^{t_1} \lambda(t)\mathrm{d}t\right) - \exp\left(-\int_0^{t_2} \lambda(t)\mathrm{d}t\right)$$

a) If observed failure data are available for n non-repairable items, from a homogenous population, the estimated value of $\lambda(t)$ at time t is given by:

$$\hat{\lambda}(t) = \frac{n_S(t) - n_S(t + \Delta t)}{n_S(t)\Delta t}$$

where

$n_S(t)$	is the number of items that are still operational at the instant of time t;
n	$n_S(0)$;
$n_S(t) - n_S(t + \Delta t)$	is the number of items that fail in the time interval $[t,\ t + \Delta t]$.

NOTE The estimated value of the failure density function $f(t)$, at time t, is given by

$$\hat{f}(t) = \frac{n_S(t) - n_S(t+\Delta t)}{n\Delta t}$$

b) When the time to failure is exponentially distributed, i.e. $\lambda(t) = \lambda$ for all values of t,

$$f(t) = \lambda \exp(-\lambda t)$$

and

$$R(t) = \exp(-\lambda t)$$

c) If observed failure data are available for n non-repairable items, from a homogenous population, with constant failure rate, then the estimated value of λ is given by the following maximum likelihood estimate:

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} \mathrm{TTF}_i}$$

where TTF_i is the time to failure of item i.

For 10 non-repairable items, from a homogenous population, with a constant failure rate, the observed total operating time to failures of all the items is $\sum_{i=1}^{10} \mathrm{TTF}_i$ = 2 years. Hence

$$\hat{\lambda} = \frac{10}{12} = 5 \text{ year}^{-1}$$

d) If the time to failure of a non-repairable item has a two-parameter Weibull distribution with scale parameter $\alpha > 0$ and shape parameter $\beta > 0$, then (see [9])

$$R(t) = \exp(-(\alpha t)^{\beta})$$

and

$$f(t) = \frac{-\mathrm{d}R(t)}{\mathrm{d}t} = \alpha\beta\,(\alpha t)^{\beta-1} \exp(-(\alpha t)^{\beta})$$

hence

$$\lambda(t) = \frac{f(t)}{R(t)} = \alpha\beta\,(\alpha t)^{\beta-1}$$

For $\beta = 2$ and $\alpha = 0{,}5$ year^{-1}

$$\lambda(6 \text{ months}) = 0{,}5 \times 2 \times (0{,}5 \times \frac{6}{12}) = 0{,}25 \text{ year}^{-1}$$

$$\lambda(1 \text{ year}) = 0{,}5 \times 2 \times (0{,}5 \times 1) = 0{,}5 \text{ year}^{-1}$$

6.2.5 Mean failure rate [192-05-07]

Symbol $\bar{\lambda}(t_1, t_2)$, $0 \le t_1 < t_2$

a) As $R(t) = e^{-\int_0^t \lambda(\tau) d\tau}$ the mean failure rate is given by:

$$\bar{\lambda}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \lambda(t) \mathrm{d}t = \frac{1}{t_2 - t_1} \ln \frac{R(t_1)}{R(t_2)}$$

Warning: The formula $R(t) = e^{-\int_0^t \lambda(\tau) d\tau} = e^{-t \cdot \frac{1}{t} \int_0^t \lambda(\tau) d\tau} = e^{-t \cdot \bar{\lambda}(0,t)}$ shows that the "mean failure rate" can be used to calculate $R(t)$. Nevertheless this should be done with care as $\bar{\lambda}(0,t)$ is not a failure rate (see 6.1.5.2).

b) When the time to failure is exponentially distributed

$$\bar{\lambda}(t_1, t_2) = \lambda$$

for all values of t_1 and t_2.

c) Let t_1 = 6 months, $R(t_1)$ = 0,8 and t_2 = 12 months, $R(t_2)$ = 0,5, then

$$\bar{\lambda}(6,\ 12) = \frac{1}{12 - 6} \ln \frac{0{,}8}{0{,}5} = \ln(1{,}6)/6 = \frac{0{,}47}{6} = 0{,}078 \text{ month}^{-1}$$

while ($R(0)$ = 1)

$$\bar{\lambda}(0,\ 6) = \frac{1}{6 - 0} \ln \frac{1}{0{,}8} = \ln(1{,}25)/6 = \frac{0{,}22}{6} = 0{,}037 \text{ month}^{-1}$$

6.2.6 Mean operating time to failure [192-05-11]

MTTF (abbreviation)

In the case of non-repairable items, the MTTF is also the MTTFF (mean time to first failure). It can be calculated by the following general formula:

$$\text{MTTF} = \text{MTTFF} = \int_0^\infty t f(t) \mathrm{d}t = \int_0^\infty R(t) \mathrm{d}t$$

a) If observed failure data (field feedback) are available for n non-repairable items, from a homogenous population, then an estimate of MTTF is given by

$$\text{M}\hat{\text{T}}\text{TF} = \text{M}\hat{\text{T}}\text{TFF} = \frac{\sum_{i=1}^{n} \text{TTF}_i}{n}$$

where TTF_i is the observed time to failure of item i.

NOTE The above formula is valid only if all the n items have failed during the observation period. If this is not the case, the duration T of the observation period can be used for TTFs of the non-failed items in order to obtain a conservative estimate of the MTTF.

b) When the time to failure is exponentially distributed, i.e. $\lambda(t) = \lambda$ for all values of t,

$$\text{MTTF} = \frac{1}{\lambda}$$

and the constant failure rate can be estimated by:

$$\hat{\lambda} = \frac{1}{\widehat{\text{MTTF}}}$$

c) For a non-repairable item with a constant failure rate of λ = 0,5 year^{-1},

$$\text{MTTF} = 2 \text{ years} = 17\,520 \text{ h}$$

d) If the time to failure of a non-repairable item has a two-parameter Weibull distribution with a scale parameter $\alpha > 0$ and shape parameter $\beta > 0$, then

$$R(t) = \exp(-(\alpha t)^{\beta})$$

and

$$\text{MTTF} = \frac{\Gamma(1+\frac{1}{\beta})}{\alpha}$$

where

$$\Gamma(x) = \int_0^{\infty} t^{x-1}\,\text{e}^{-t}\,\text{d}t$$

is the complete gamma function. (See [9])

For β = 2 and α = 0,5 year^{-1}:

$$\text{MTTF} = \frac{\Gamma(1+\frac{1}{2})}{0{,}5} = 2 \times \Gamma(1+\frac{1}{2})$$

but

$$\Gamma(1+\frac{1}{2}) = \Gamma(\frac{1}{2})/2 = \sqrt{\pi}/2$$

hence

$$\text{MTTF} = \sqrt{\pi} \approx 1{,}8 \text{ years} = 21 \text{ months}$$

6.3 Repairable individual items with zero time to restoration

6.3.1 General

This particular case is illustrated in Figure 8 and Figure 9.

All expressions in 6.3 are applicable to COIs. Where they are applicable to IOIs, this is stated.

For each measure, the following are presented:

a) the generic expression obtained through a simple renewal process [11];
b) the most common expression (for the cases when the times to failure of the item are exponentially distributed);
c) a simple example of application where necessary.

6.3.2 Reliability [192-05-05]

Symbol $R(t_1, t_2)$, $0 \leq t_1 < t_2$

The reliability $R(t_1, t_2)$ over the time interval $[t_1, t_2]$ is also known as the interval reliability.

a) The reliability of an item for the time interval $[t_1, t_2]$ is illustrated in Figure 32.

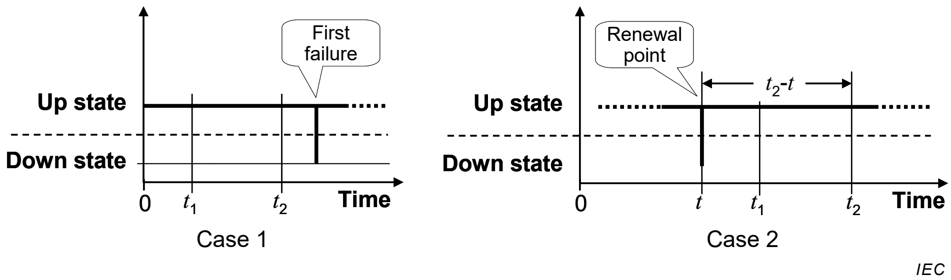


Figure 32 – Illustration of reliable behaviour over $[t_1, t_2]$ for a zero time to restoration individual item

Figure 32 shows that two cases have to be considered:

– No failure has occurred over $[0, t_2]$. This is the ordinary reliability $R(t_2)$.

– At least one failure has occurred and the faulty item has been repaired at an instant t before t_1, and no failure has occurred between t_1 and t_2:

- the probability that one failure has occurred (and has been repaired) at time t is the unconditional failure intensity $z(t)$,
- the probability that the item has no failure during the interval $[t, t_2]$, under the hypothesis that it is as good as new after repair, is the reliability over the duration t_2-t, $R(t_2-t)$.

As t may vary from 0 to t_1, $R(t_1, t_2)$ may be written as (see [9] and [13] for more information):

$$R(t_1, t_2) = R(t_2) + \int_0^{t_1} R(t_2 - t) z(t) \mathrm{d}t$$

$z(t)$ is the unconditional instantaneous failure intensity of the item. It is also the renewal density of the underlying renewal process i.e. for small values of Δt, $z(t)\cdot\Delta t$ is approximately equal to the (unconditional) probability that a failure of the item occurs (and therefore a repair for a zero time to restoration item) during $[t, t + \Delta t]$, and

$R(t) = R(0, t)$ is the reliability function of the item

$$R(t) = \int_t^{\infty} f(s) \mathrm{d}s$$

where $f(t)$ is the probability density function (also referred to as the failure density function) of the times to failure of the item, i.e. for small values of Δt, $f(t)\cdot\Delta t$ is approximately equal to the probability that the item fails during the time interval $[t, t + \Delta t]$. More precisely, it is approximately the probability that a given time to failure terminates in the time interval $[t, t + \Delta t]$, assuming that the time to failure started at time $t = 0$.

b) If observed failure data are available for n repairable items, from a homogenous population, then an estimate of $R(t_1, t_2)$ is given by

$$\hat{R}(t_1, t_2) = \frac{n_S(t_1, t_2)}{n}$$

where $n_S(t_1, t_2)$ is the number of items that were operational at the instant of time t_1 and did not fail during the time interval $[t_1, t_2]$.

c) By setting $t_1 = t$ and $t_2 = t + x$, one can obtain the asymptotic interval reliability (see [13]):

$$\lim_{t \to \infty} R(t, t+x) = \frac{1}{\text{MTTF}} \int_x^{\infty} R(s)\,\mathrm{d}s$$

which, for large values of t, can be used as an approximation of the $R(t,\ t + x)$, where MTTF is the mean time to failure.

This asymptotic expression follows from the key renewal theorem (see [9]).

This asymptotic interval reliability should not be confused with the asymptotic reliability $R(\infty)$ which is always equal to 0.

d) When $\lambda(t) = \lambda$ and is constant, i.e. when the times to failure are exponentially distributed,

$$R(t_1,\ t_2) = \exp\left[-\lambda \cdot (t_2 - t_1)\right]$$

In this case, the asymptotic interval reliability is given by

$$\lim_{t \to \infty} R(t, t+x) = \exp\,(-\lambda x)$$

e) For a repairable item with a constant failure rate $\lambda = 1\ \text{year}^{-1}$, its reliability over six months is given by

$$R(t,\ t + 6) = \exp\,(-1 \times \frac{6}{12}) = 0{,}61$$

where t is the starting point of the six month interval.

6.3.3 Instantaneous failure intensity [192-05-08]

Symbol $z(t)$

The expressions included in 6.3.3 also apply to IOIs.

a) As per the definition [192-05-08], $z(t)$ is the derivative of the expected number of failures, $Z(t) = E[N(t)]$, in the time interval [0, t] where $N(t)$ is the number of failures during the time interval [0, t] and E denotes the expectation.

$$z(t) = \lim_{\Delta t \to 0^+} \frac{Z(t+\Delta t) - Z(t)}{\Delta t} = \frac{\mathrm{d}Z(t)}{\mathrm{d}t}$$

For small values of Δt, $z(t) \cdot \Delta t$ is approximately equal to the (unconditional) probability that a failure of the item occurs during $[t,\ t + \Delta t]$.

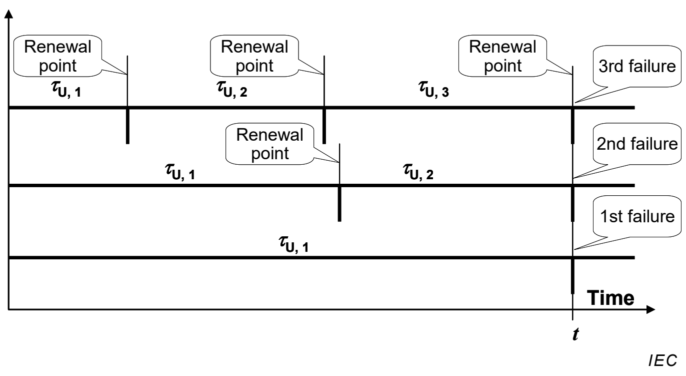


Figure 33 – Sample of possible number of failures at the renewal time *t*

Figure 33 shows that when one failure/repair occurs at time t, it may be the first, the second, the third ... the n^{th} failure/repair.

The random variable is defined as $\Theta_n(t) = \sum_{i=1}^{n} \tau_{\mathrm{U},i}(t)$. Then the probability to observe the n^{th} failure between t and $t + \mathrm{d}t$ is equal to the probability that $t < \Theta_n(t) \le t + \mathrm{d}t$. This is given by the probability density function of $\Theta_n(t)$. This is noted $h_{\mathrm{CTTF}}^{(n)}(t)$ in the renewal models (see [9]). It follows that $z(t)$ may be written as

$$z(t) = \sum_{n=1}^{\infty} h_{\mathrm{CTTF}}^{(n)}(t)$$

$h_{\mathrm{CTTF}}^{(n)}(t)$ is the probability density function of calendar time until the nth failure of the item. It is related to a sum of random variables. Therefore, according to the basic properties of random variables, it is given by the convolution products of the probability density functions of the random variables within the sum. Due to the "as good as new" hypothesis, all the probability density function of the $\tau_{\mathrm{U,i}}$ are equal to $f_{\mathrm{U}}(t)$ which is the probability density function of the up times of the item, i.e. for small values of Δt, $f_{\mathrm{U}}(t)\cdot\Delta t$ is approximately equal to the probability that a given up time of the item terminates during (t, $t + \Delta t$), assuming that the up time started at time t = 0.

NOTE 1 When the item operates continuously, $f_{\mathrm{U}}(t)$ is equal to $f(t)$.

This gives

$$h_{\mathrm{CTTF}}^{(n)}(t) = f_{\mathrm{U}}^{(n)}(t)$$

where $f_{\mathrm{U}}^{(n)}(t)$ is the convolution of $f_{\mathrm{U}}(t)$ n times with itself.

Therefore $h_{\mathrm{CTTF}}^{(n)}(t) = f_{\mathrm{U}}^{(n)}(t) = (f_{\mathrm{U}}^{(n-1)} * f_{\mathrm{U}})(t) = (h_{\mathrm{CTTF}}^{(n-1)} * f_{\mathrm{U}})(t)$

where "*" is used to note the convolution product,

NOTE 2 The convolution product of two functions $x(t)$ and $y(t)$ is given by $x(t) * y(t) = \int_0^t x(t-\tau) y(\tau) d\tau$

Finally $h_{\mathrm{CTTF}}^{(n)}(t)$ may be calculated by the following recursive relationship:

$$h_{\mathrm{CTTF}}^{(1)}(t) = f_{\mathrm{U}}(t)$$

$$h_{\mathrm{CTTF}}^{(n)}(t) = \int_0^t f_{\mathrm{U}}(x)\, h_{\mathrm{CTTF}}^{(n-1)}(t-x)\,\mathrm{d}x \text{, for } n > 1$$

Coming back to $z(t)$, the following formulae are obtained:

$$z(t) = h_{\mathrm{CTTF}}^{(1)}(t) + \sum_{n=2}^{\infty} h_{\mathrm{CTTF}}^{(n)}(t) = f_{\mathrm{U}}(t) + \sum_{n=2}^{\infty} \int_0^t f_{\mathrm{U}}(x)\, h_{\mathrm{CTTF}}^{(n-1)}(t-x)\,\mathrm{d}x$$

$$= f_{\mathrm{U}}(t) + \sum_{j=1}^{\infty} \int_0^t f_{\mathrm{U}}(x)\, h_{\mathrm{CTTF}}^{(j)}(t-x)\,\mathrm{d}x = f_{\mathrm{U}}(t) + \int_0^t f_{\mathrm{U}}(x) \sum_{j=1}^{\infty} h_{\mathrm{CTTF}}^{(j)}(t-x)\,\mathrm{d}x$$

The instantaneous failure intensity, $z(t)$, satisfies the following integral equation (see [9] and [11]):

$$z(t) = f_U(t) + \int_0^t f_U(x)\, z(t-x)\,dx$$

which may be solved by numerical methods.

b) If observed failure data are available for n repairable items, from a homogenous population, then an estimate of $z(t)$ is given by

$$\hat{z}(t) = \frac{n_F(t, t+\Delta t)}{n\Delta t}$$

where $n_F(t,\ t + \Delta t)$ is the number of failures observed during the time interval $[t,\ t + \Delta t]$.

c) When the up times are exponentially distributed the unconditional failure intensity is equal to $A(t).\lambda_U$. The item being repaired immediately after failure, it is available at any time so $A(t)$=1 and:

$$z(t) = \lambda_U$$

For a continuously operating item, λ_U equals λ.

6.3.4 Asymptotic failure intensity [192-05-10]

Symbol $z(\infty)$

The expressions included in 6.3.4 also apply to IOIs.

a) As per the definition, [192-05-10], $z(\infty)$ is the limit, if it exists, of the instantaneous failure intensity $z(t)$, when time t tends to infinity:

$$z(\infty) = \lim_{t\to\infty} z(t)$$

By definition the expected number of failures during the time interval [0, t], $Z(t)$ is equal to $Z(t) = \int_0^t z(\tau)\,d\tau$. If $z(t)$ reaches an asymptotic value when t tends to infinity, then (see 6.1.6):

$$\lim_{t\to\infty} \frac{Z(t)}{t} = z(\infty)$$

As per the definition of the mean time between failures METBF, the expected number of failures $Z(t)$ over [0, t] tends towards $\frac{t}{\text{METBF}}$ when t increases. In the case of zero time to restoration, MDT is equal to 0 and METBF = MUT. Then:

$$\lim_{t\to\infty} \frac{Z(t)}{t} = \frac{1}{\text{MUT}}$$

Finally, when it exists, the asymptotic failure intensity $z(\infty)$ is given by the equation

$$z(\infty) = \lim_{t\to\infty} z(t) = \frac{1}{\text{MUT}}$$

Under appropriate assumptions on $f_U(t)$, the above equation follows from the renewal density theorem (see [5], [9], [10] and [12]).

NOTE The easiest way to check conditions for the existence of $z(\infty)$ are the following:

– MUT < ∞;

– $f_U(t)$ is a bounded function over [0, +∞] and tends to 0 as $t \to +\infty$.

b) If observed failure data are available for n repairable items, from a homogenous population, and the time t is large enough, then an estimate of $z(\infty)$ is given by

$$\hat{z}(\infty) = \hat{z}(t) = \frac{n_F(t, t+\Delta t)}{n\Delta t}$$

where $n_F(t, t + \Delta t)$ is the number of failures observed during the time interval $[t, t + \Delta t]$.

For small values of Δt and large values of t, $z(\infty)\cdot\Delta t$ is approximately equal to the (unconditional) probability that a failure of the item occurs during $[t, t + \Delta t]$.

c) When the up times are exponentially distributed (see 6.3.3 c)),

$$z(\infty) = \lambda_U$$

For a continuously operating item, λ_U equals λ.(see 3.5, Note 2 and 3.6, Note 3)

6.3.5 Mean failure intensity [192-05-09]

Symbol $\bar{z}(t_1, t_2)$, $0 \le t_1 < t_2$

The expressions included in 6.3.5 also apply to IOIs.

a) $\bar{z}(t_1, t_2) = \dfrac{1}{t_2 - t_1}\displaystyle\int_{t_1}^{t_2} z(t)\mathrm{d}t$

The integral $\int_{t_1}^{t_2} z(t)\mathrm{d}t$ is equal to the expected number of failures of the item in the time interval $[t_1, t_2]$, hence $\bar{z}(t_1, t_2)$ may be interpreted as the expected number of failures per unit of time in $[t_1, t_2]$.

b) If observed failure data are available for n repairable items, from a homogenous population, then an estimate of $\bar{z}(t_1, t_2)$ is given by

$$\hat{\bar{z}}(t_1, t_2) = \frac{n_F(t_1, t_2)}{n.(t_1 - t_2)}$$

where $n_F(t_1, t_2)$ is the number of failures observed in the time interval $[t_1, t_2]$.

c) By setting $t_1 = t$ and $t_2 = t + x$, the asymptotic mean failure intensity can be obtained:

$$\lim_{t\to\infty} \bar{z}(t, t+x) = \frac{1}{\text{METBF}}$$

NOTE This equality comes from the equilibrium reached by the renewal process when t goes to infinity. When this equilibrium is reached, the number of failures observed within the interval $[t, t + x]$ tends towards x/METBF and therefore the mean number of failures tends towards 1/METBF. This equality can be demonstrated in a more mathematical way by using the Blackwell's theorem (see [9]).

In the zero repair time case, METBF is equal to MUT and:

$$\lim_{t\to\infty} \bar{z}(t, t+x) = \frac{1}{\text{MUT}}$$

which, for large values of t, can be used as an approximation of $\bar{z}(t, t + x)$.

d) When the up times are exponentially distributed, (see 6.3.3 c) then

$$\bar{z}(t_1, t_2) = \lambda_U$$

For a continuously operating item, λ_U equals λ (see 3.5, Note 2 and 3.6, Note 3). Then $\bar{z}(t_1, t_2) = \lambda$

6.3.6 Mean time between failures (see 3.3)

The expressions included in 6.3.6 also apply to IOIs.

a) In this case the MDT is equal to zero, the mean time between failures is reduced to the MUT (see 6.1.4):

$$\text{METBF} = \text{MUT} = \int_0^\infty t f_U(t)\,\mathrm{d}t$$

where $f_U(t)$ is the probability density function of the up times of the item (including its operating, idle, standby and external disabled times).

NOTE In cases where the assumption in 5.5.1 f) is not valid, i.e. when function-preventing preventive actions are performed, the time between failures includes the times for such actions. In this case,

METBF > MUT

If the item operates continuously,

mean time between failures

= mean time to failure (MTTF)

= mean operating time between failures (MOTBF)

= mean up time (MUT).

b) If the up times are exponentially distributed,

mean time between failures = $\dfrac{1}{\lambda_U}$

If the item operates continuously, λ_U equals λ.

6.3.7 Mean operating time to failure [192-05-11]

MTTF (abbreviation)

a) In the case of an individual repairable item and using the "as good as new" hypothesis, the MTTF has the same value as the MTTFF (mean time to first failure). It can be calculated by the following general formula:

$$\text{MTTF} = \text{MTTFF} = \int_0^\infty t f(t)\,\mathrm{d}t = \int_0^\infty R(t)\,\mathrm{d}t$$

b) When all observed operating times to failure of n items, from a homogenous population, are available, then an estimate of MTTF is given by

$$\widehat{\text{MTTF}} = \frac{\text{total operating time}}{k_F} = \frac{\sum_{i=1}^{n}(\text{operating time})_i}{k_F}$$

where

"total operating time" is the aggregate operating time of all n items during a given time period;

k_F is the total number of failures observed during the given time period;

(operating time)$_i$ is the aggregate operating time of the ith item during the given time period.

c) When the times to failure are exponentially distributed

$$\text{MTTF} = \frac{1}{\lambda}$$

d) For a repairable item with a constant failure rate of 0,5 year^{-1}

$$\text{MTTF} = 2 \text{ years} = 17\,520 \text{ h}$$

6.3.8 Mean operating time between failures [192-05-13]

MOTBF

a) As the time to restoration is equal to zero, the MOTBF is equal to the MTTF:

$$\text{MOTBF} = \text{MTTF} = \int_0^\infty t f(t)\,\mathrm{d}t = \int_0^\infty R(t)\,\mathrm{d}t$$

NOTE For continuously operating items, the MOTBF is equal to the whole MUT. Then, in this case MOTBF = MTTF = MUT.

b) If the times to failure are exponentially distributed,

$$\text{MOTBF} = \frac{1}{\lambda}$$

6.3.9 Instantaneous availability [192-08-01], mean availability [192-08-05] and asymptotic availability [192-08-07]

As the item is repaired immediately, its instantaneous availability is equal to 1 at any time:

$$A(t) \equiv 1, \forall t \in [0, \infty]$$

Therefore, the mean and the steady state availabilities are also identical to 1:

$$\overline{A}(t_1, t_2) = A = A(t) \equiv 1, \forall t \in [0, \infty]$$

Therefore this model is not very useful from an availability calculations point of view.

6.3.10 Mean up time [192-08-09]

MUT (abbreviation)

The expressions in 6.3.10 also apply to IOIs.

a) $\text{MUT} = \int_0^\infty t f_U(t)\,\mathrm{d}t = \int_0^\infty (1 - F_U(t))\,\mathrm{d}t$

where $f_U(t)$ is the probability density function of the up times of the item (including its operating, idle, standby and external disabled times).

In the zero repair time case MDT = 0 and METBF = MUT

For a continuously operating item,

MUT = MTTF = MTTFF = MOTBF = METBF

b) If the up times are exponentially distributed,

$$\text{MUT} = \frac{1}{\lambda_U}$$

If the item operates continuously, λ_U equals λ (see 3.5, Note 2 and 3.6, Note 3).

6.4 Repairable individual items with non-zero time to restoration

6.4.1 General

All expressions in 6.4 are applicable to COIs. Where they are applicable to IOIs, this is stated.

For each measure, the following are presented:

a) the generic expression;
b) the most common expression (for the cases when times to failure, up times, down times, times to restoration and repair times of the item are exponentially distributed);
c) a simple example of application where necessary.

6.4.2 Reliability [192-05-05]

Symbol $R(t_1, t_2)$, $0 \le t_1 < t_2$

The reliability $R(t_1, t_2)$ over the time interval $[t_1, t_2]$ is also known as the interval reliability.

a) The reliability of an item for the time interval $[t_1, t_2]$ is illustrated in Figure 34.

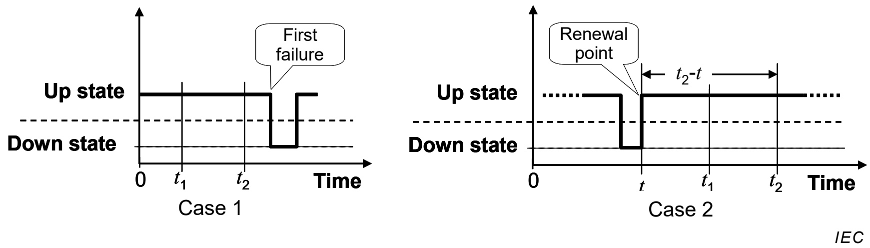


Figure 34 – Illustration of reliable behaviour over $[t_1, t_2]$ for a non-zero time to restoration individual item

Figure 34 is similar to Figure 32 but the renewal points occur after a repair has been performed. The difference is that the probability to have a repair finishing at time t is given by the repair intensity $v(t)$ instead of the failure intensity $z(t)$. Therefore, thanks to the same reasoning explained in 6.3.2, the reliability of a repairable item with non-zero time to restoration for the time interval $[t_1, t_2]$ may be written as (see [13] and [9]):

$$R(t_1, t_2) = R(t_2) + \int_0^{t_1} R(t_2 - t) v(t) \mathrm{d}t$$

where

the first term, $R(t_2)$, represents the probability of survival to time t_2, and the second term represents the probability of restoration (after a failure) at time t ($t < t_1$), and surviving to time t_2;

$v(t)$ is the instantaneous restoration intensity of the item, i.e. for small values of Δt, $v(t) \cdot \Delta t$ is approximately equal to the probability that a restoration of the item occurs during $[t, t + \Delta t]$ (see definition 3.1);

$R(t) = R(0, t)$ is the reliability function of the item

$$R(t) = \int_t^\infty f(s)\,\mathrm{d}s$$

where $f(t)$ is the probability density function of the times to failure of the item, i.e. for small values of Δt, $f(t)\cdot\Delta t$ is approximately equal to the probability that the item fails during the time interval $[t, t + \Delta t]$. More precisely, it is approximately the probability that a given time to failure terminates in the time interval $[t, t + \Delta t]$, assuming that the time to failure started at time $t = 0$.

NOTE 1 $R(t_1, t_2)$ is the (unconditional) probability of failure-free continuous operation of the item in the time interval $[t_1, t_2]$. The expression may not be true for IOIs.

b) If observed failure data are available for n repairable items, from a homogenous population, then an estimate of $R(t_1, t_2)$ is given by

$$\hat{R}(t_1,t_2) = \frac{n_S(t_1,t_2)}{n}$$

where $n_S(t_1, t_2)$ is the number of items which were operating at the instant of time t_1 and operated without failure in the time interval $[t_1, t_2]$.

c) By setting $t_1 = t$ and $t_2 = t + x$, one can obtain the asymptotic interval reliability (See [13] and [9]):

$$\lim_{t\to\infty} R(t, t+x) = \frac{1}{\mathrm{MTTF}+\mathrm{MTTR}} \int_x^\infty R(s)\,\mathrm{d}s$$

which, for large values of t, can be used as an approximation of the $R(t, t + x)$, where

MTTF is the mean time to failure, and

MTTR is the mean time to restoration.

This expression follows from the key renewal theorem (see [9]).

d) When the times to failure are exponentially distributed, then

$$R(t_1, t_2) = A(t_1)\exp(-\lambda\cdot(t_2 - t_1))$$

where $A(t_1)$ is the instantaneous availability at time t_1.

NOTE 2 The probability $R(t_1, t_2)$ for the item to be in up state over $[t_1, t_2]$ is equal to the probability to be in up state at time t_1 (i.e. available at time t_1, $A(t_1)$) multiplied by the conditional probability to have no failures over $[t_1, t_2]$ – i.e. exp $[-\lambda \times (t_2 - t_1)]$ as the exponential case is considered.

and

$$\lim_{t\to\infty} R(t, t+x) = \frac{\mathrm{MTTF}}{\mathrm{MTTF}+\mathrm{MTTR}} \exp(-\lambda x)$$

(See [9])

NOTE 3 The formula for $R(t_1, t_2)$ above can be related to IEC 60050-192:2015, 192-05-05, Note 2, by assuming that $t_1 = 0$, $t_2 = t$, $R(t_1, t_2) = R(0, t) = R(t)$ and $A(t_1) = A(0) = 1$.

e) When the times to failure and times to restoration are exponentially distributed, then, using either Markov techniques or the Laplace transform, the following is obtained:

$$R(t_1, t_2) = A(t_1)\, e^{-\lambda\times(t_2-t_1)} = \left(\frac{\mu_\mathrm{R}}{\lambda+\mu_\mathrm{R}} + \frac{\lambda}{\lambda+\mu_\mathrm{R}} e^{-(\lambda+\mu_\mathrm{R})t_1}\right) e^{-\lambda\times(t_2-t_1)}$$

and

$$\lim_{t\to\infty} R(t, t+x) = \frac{\mu_R}{\lambda+\mu_R}\exp(-\lambda x)$$

(See [9])

f) Figure 35 illustrates the application of the above formula for calculating $R(t, t + 1/4)$ for a COI with λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}:

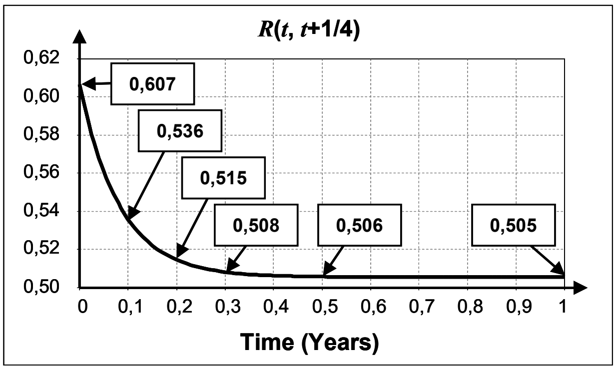


Figure 35 – Evolution of $R(t, t + 1/4)$

This curve shows that $R(t, t + 1/4)$ decreases from 0,607 when $t = 0$ to 0,505 when t goes to infinity.

6.4.3 Instantaneous failure intensity [192-05-08]

Symbol $z(t)$

The expressions in 6.4.3 also apply to IOIs.

a) As per the definition [192-05-08], $z(t)$ is the derivative of the expected number of failures, $Z(t) = E[N(t)]$, in the time interval [0, t], including up and down times, where $N(t)$ is the number of failures in the time interval [0, t], and E denotes the expectation, thus

$$z(t) = \lim_{\Delta t\to 0^+} \frac{Z(t+\Delta t) - Z(t)}{\Delta t} = \frac{\mathrm{d}Z(t)}{\mathrm{d}t}$$

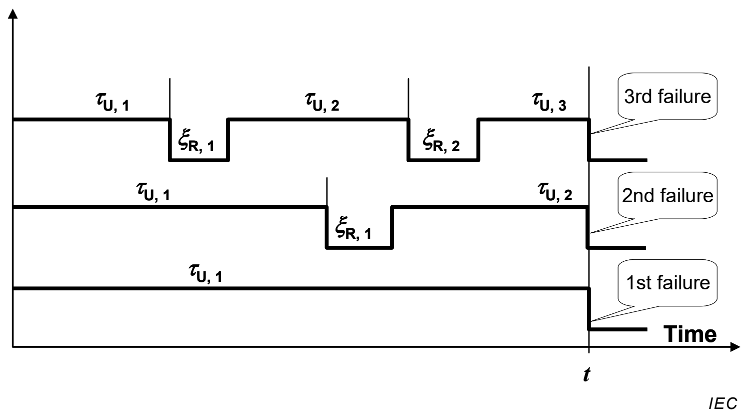


Figure 36 – Sample of possible number of failures at the renewal time t

Figure 36 shows that when one failure occurs at time t, it may be the first, the second, the third, ..., the n^{th} failure.

The random variable is defined as $\Theta_n(t)=\sum_{i=1}^{n}[\xi_{R,i}(t)+\tau_{U,i+1}(t)]$. Then the probability to observe the n^{th} failure between t and t + dt is equal to the probability that $t<\Theta_n(t)\le t+\mathrm{d}t$. This is given by the probability density function of $\Theta_n(t)$ and is noted $h_{CTTF}^{(n)}(t)$ in the alternating renewal process theory (see [9]). It follows that $z(t)$ may be written as

$$z(t)=\sum_{n=1}^{\infty}h_{CTTF}^{(n)}(t)$$

where $h_{CTTF}^{(n)}(t)$ is the probability density function of calendar time to the nth failure of the item. The formulae are similar to those developed in the zero-time to restoration case in 6.3.3 but the common probability density functions $f_U(t)$ of the $\tau_{U,i}$ has to be replaced by the common probability density function, $f_{R+U}(t)$ of the ($\xi_{R,i}+\tau_{U,i+1}$) and may be calculated by the following recursive relations:

$$h_{CTTF}^{(1)}(t)=f_U(t)$$

$$h_{CTTF}^{(n)}(t)=\int_0^t h_{CTTF}^{(n-1)}(x)f_{R+U}(t-x)\mathrm{d}x\text{, for } n>1$$

where

$f_U(t)$ is the probability density function of the up times of the item (including its operating, idle, standby and external disabled times). For small values of Δt, $f_U(t)\cdot\Delta t$ is approximately equal to the probability that a given up time of the item terminates during [t, $t+\Delta t$], assuming that the up time started at time t = 0;

$f_{R+U}(t)$ is the probability density function of the sum of the times to restoration ($\xi_{R,i}$) and the following up times ($\tau_{U,i+1}$). According to the properties of the probability density functions this is given by the following convolution product

$$f_{R+U}(t)=\int_0^t g_R(t-s)f_U(s)\mathrm{d}s$$

where $g_R(t)$ is the probability density function of the times to restoration of the item, i.e. for small values of Δt, $g_R(t)\cdot\Delta t$ is approximately equal to the probability that the item is restored from a fault to an up state in the time interval [t, t + Δt], assuming that a failure resulting in a fault occurred at time t = 0.

According to the commutative properties of the convolution product $f_{R+U}(t)=f_{U+R}(t)$.

For small values of Δt, $z(t)\cdot\Delta t$ is approximately equal to the (unconditional) probability that a failure of the item occurs during the time interval [t, $t+\Delta t$].

NOTE 1 Let $\tau_{U,1}$, $\xi_{R,1}$, $\tau_{U,2}$, $\xi_{R,2}$,…, $\tau_{U,n}$, $\xi_{R,n}$ … be consecutive up times (τ_U) and times to restoration (ξ_R) of the item. Then $h_{CTTF}^{(n)}(t)$ is the probability density function of the sum

$$\tau_{U,1}+(\xi_{R,1}+\tau_{U,2})+(\xi_{R,2}+\tau_{U,3})+\ldots+(\xi_{R,n-1}+\tau_{U,n})$$

while $f_{R+U}(t)$ is the probability density function of the sum $\xi_{R,m-1}+\tau_{U,m}$ for any $m>2$.

NOTE 2 The instantaneous failure intensity, $z(t)$, and the instantaneous restoration intensity, $v(t)$, fulfil the following simultaneous system of equations:

$$z(t) = f_U(t) + \int_0^t f_U(t-s)v(s)\mathrm{d}s$$

$$v(t) = \int_0^t g_R(t-s)z(s)\mathrm{d}s$$

Mathematically speaking, this constitutes a system of linear Volterra integral equations (see [14]) which can be solved by numerical methods.

These formulae can be established in the same way as this has been done in 6.3.3 for $z(t) = f_U(t) + \int_0^t f_U(t-s)z(s)\mathrm{d}s$ when the time to repair is equal to 0.

b) If observed failure data are available for n repairable items, from a homogenous population, then an estimate of $z(t)$ is given by

$$\hat{z}(t) = \frac{n_F(t, t+\Delta t)}{n\Delta t}$$

where $n_F(t,\ t+\Delta t)$ is the number of failures observed during the time interval $[t,\ t+\Delta t]$, where the time scale includes both up and down times.

c) When the up times are exponentially distributed, then (see [4])

$$z(t) = A(t)\lambda_U$$

where $A(t)$ is the instantaneous availability.

NOTE 3 For an individual item, the failure rate λ_U is equal to the conditional failure intensity of the item. Then the above formula comes directly from the fundamental relationship between the conditional and unconditional failure intensities (see 6.1.6).

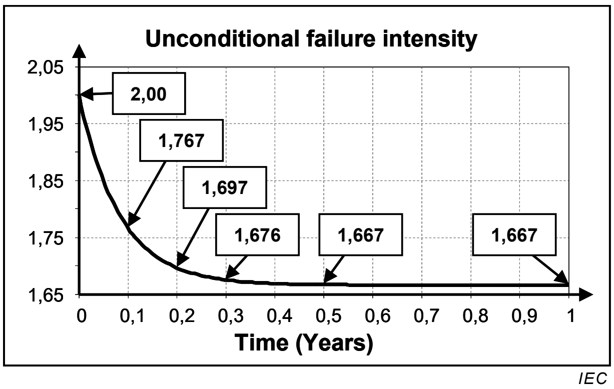
When the item operates continuously, $f_U(t) = f(t)$ and $\lambda_U = \lambda$.

d) When the up times and times to restoration are exponentially distributed, Markov techniques or the Laplace transform can be used to yield (see [14]):

$$z(t) = \frac{\lambda_U \mu_R}{\lambda_U + \mu_R} + \frac{\lambda_U^2}{\lambda_U + \mu_R}\exp[-(\lambda_U + \mu_R)t] = A(t)\lambda_U$$

For a COI, λ_U equals λ (see 3.5, Note 2 and 3.6, Note 3).

e) Figure 37 illustrates the application of the above formula for calculating $z(t)$ for a COI with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}.



NOTE Four digits have been kept to illustrate how $z(t)$ converges towards the limit value.

Figure 37 – Evolution of the failure intensity $z(t)$

6.4.4 Asymptotic failure intensity [192-05-10]

Symbol $z(\infty)$

The expressions included in 6.4.4 also apply to IOIs.

a) As per the definition [192-05-10], $z(\infty)$ is the limit, if it exists, of the instantaneous failure intensity $z(t)$, when time t tends to infinity:

$$z(\infty) = \lim_{t\to\infty} z(t)$$

By definition METBF = MUT + MDT is the mean time between two consecutive failures. Then the mean number of failures over an interval [0, t] is $Z(t) \approx t/\text{METBF}$. As shown in Note 1 hereafter and in 6.3.4, $z(\infty) = \lim_{t\to\infty} \frac{Z(t)}{t}$. Therefore, when it exists, the asymptotic failure intensity $z(\infty)$ is given by the equation

$$z(\infty) = \lim_{t\to\infty} z(t) = \frac{1}{\text{METBF}} = \frac{1}{\text{MUT}+\text{MDT}}$$

As preventive maintenance is not considered in this document, MDT = MTTR. Then

$$z(\infty) = \lim_{t\to\infty} z(t) = \frac{1}{\text{MUT}+\text{MTTR}}$$

which, under appropriate assumptions on $f_\text{U}(t)$ and $g_\text{R}(t)$, follows from the renewal density theorem (see [5], [9], [10] and [12] and Note 2).

NOTE 1 Using the elementary renewal theorem (see [9]):

$$\lim_{t\to\infty} \frac{Z(t)}{t} = \frac{1}{\text{MUT}+\text{MTTR}}$$

but

$$Z(t) = \int_0^t z(s)\,\text{d}s$$

hence, if $z(\infty)$ exists, then

$$z(\infty) = \lim_{t\to\infty} \frac{Z(t)}{t}$$

(see [7]).

NOTE 2 The easiest way to check conditions for the existence of $z(\infty)$ are the following: MUT $< \infty$, MTTR $< \infty$; at least one of $f_\text{U}(t)$ or $g_\text{R}(t)$ is a bounded function on [0, $+\infty$] tending to 0 as $t \to +\infty$.

b) If observed failure data are available for n repairable items, from a homogenous population, and the time t is large enough, then an estimate of $z(\infty)$ is given by

$$\hat{z}(\infty) = \hat{z}(t) = \frac{n_\text{F}(t, t+\Delta t)}{n\Delta t}$$

where $n_\text{F}(t, t+\Delta t)$ is the number of failures observed during the time interval [t, $t + \Delta t$].

For small values of Δt and large values of t, $z(\infty)\cdot\Delta t$ is approximately equal to the (unconditional) probability that a failure of the item occurs during [t, $t + \Delta t$].

c) When the up times are exponentially distributed, then (see [4])

$$z(\infty) = A \cdot \lambda_U$$

where A is the asymptotic availability.

When the item operates continuously, $f_U(t) = f(t)$ and $\lambda_U = \lambda$.

d) When the up times and times to restoration are exponentially distributed, then (see 6.4.3 d)

$$z(\infty) = \lim_{t \to \infty} z(t) = \frac{\lambda_U \mu_R}{\lambda_U + \mu_R} = \frac{1}{\dfrac{1}{\lambda_U} + \dfrac{1}{\mu_R}}$$

For a COI, λ_U equals λ (see 3.5, Note 2 and 3.6, Note 3).

e) For a COI with a failure rate of $\lambda = 2$ year^{-1} and a restoration rate of $\mu_R = 10$ year^{-1},

$$z(\infty) = \frac{20}{12} = 1{,}7 \text{ year}^{-1}$$

This is illustrated in Figure 37.

6.4.5 Mean failure intensity [192-05-09]

Symbol $\bar{z}(t_1, t_2)$, $0 \le t_1 < t_2$

The expressions in 6.4.5 also apply to IOIs.

a) By definition

$$\bar{z}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} z(t)\mathrm{d}t$$

The integral $\int_{t_1}^{t_2} z(t)\mathrm{d}t$ is equal to the expected number of failures of the item in the time interval $[t_1, t_2]$. Hence $\bar{z}(t_1, t_2)$ may be interpreted as the expected number of failures per unit of time in $[t_1, t_2]$.

b) If observed failure data are available for n repairable items, from a homogenous population, then an estimate of $\bar{z}(t_1, t_2)$ is given by

$$\hat{\bar{z}}(t_1, t_2) = \frac{n_F(t_1, t_2)}{n(t_1 - t_2)}$$

where $n_F(t_1, t_2)$ is the number of failures observed during the time interval $[t_1, t_2]$, where the time scale includes both up and down times.

c) By setting $t_1 = t$ and $t_2 = t + x$, the asymptotic mean failure intensity may be obtained:

$$\lim_{t \to \infty} \bar{z}(t, t + x) = \frac{1}{\text{METBF}} = \frac{1}{\text{MUT} + \text{MDT}}$$

NOTE This equality comes from the equilibrium reached by the renewal process when t goes to infinity. When this equilibrium is reached, the number of failures observed within the interval $[t, t + x]$ tends towards x/METBF and therefore the mean number of failures tends toward 1/METBF. This equality can be demonstrated in a more mathematical way by using the Blackwell's theorem (see [9]).

As preventive maintenance is not taken under consideration in this document, the MDT can be replaced by the MTTR in the formula:

$$\lim_{t\to\infty} \bar{z}(t, t+x) = \frac{1}{\text{MUT} + \text{MTTR}}$$

which, for large values of t, can be used as an approximation of $\bar{z}(t, t+x)$.

d) When the up times are exponentially distributed (see [4]),

$$\bar{z}(t_1, t_2) = \bar{A}(t_1, t_2)\lambda_U$$

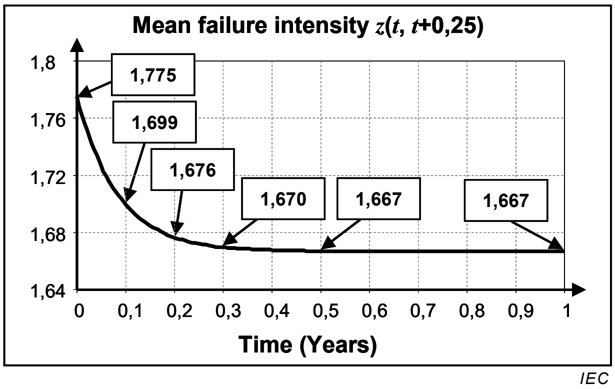
When the item operates continuously, $\lambda_U = \lambda$.

e) When the up times and times to restoration are exponentially distributed, then (see [14] and [18])

$$\bar{z}(t_1, t_2) = \frac{\lambda_U \mu_R}{\lambda_U + \mu_R} + \frac{\lambda_U^2}{(\lambda_U + \mu_R)^2} \frac{\exp[-(\lambda_U + \mu_R)t_1] - \exp[-(\lambda_U + \mu_R)t_2]}{t_2 - t_1} = \bar{A}(t_1, t_2)\lambda_U$$

For a COI, $\lambda_U = \lambda$.

f) Figure 38 illustrates the application of the above formula for calculating the mean failure intensity for a COI with a failure rate of $\lambda = 2$ year^{-1} and a restoration rate of $\mu_R = 10$ year^{-1}



NOTE Four digits have been kept to illustrate how the mean failure intensity converges towards the limit value.

Figure 38 – Evolution of the mean failure intensity $z(t, t + 1/4)$

6.4.6 Mean operating time to failure [192-05-11]

MTTF (abbreviation)

a) In the case of an individual repairable item and using the "as good as new" hypothesis, the MTTF has the same value as the MTTFF (mean time to first failure). It can be calculated by the following general formula:

$$\text{MTTF} = \text{MTTFF} = \int_0^\infty t f(t)\,\mathrm{d}t = \int_0^\infty R(t)\,\mathrm{d}t$$

where $R(t)$ is the reliability function of the item

$$R(t) = \int_t^{\infty} f(s)\,\mathrm{d}s$$

When observed (operating) times to failure of n items, from a homogenous population, are available, then an estimate of MTTF is given by

$$\hat{\text{MTTF}} = \frac{\text{total operating time}}{k_\text{O}} = \frac{\sum_{i=1}^{n} (\text{operating time})_i}{k_\text{O}}$$

where

"total operating time" is the aggregate operating time of all n items during a given time period;

k_O is the total number of failures of the items while operating during the given time period;

"(operating time)$_i$" is the total operating time of the ith item during the given time period.

b) When the times to failure are exponentially distributed,

$$\text{MTTF} = \frac{1}{\lambda}$$

c) For a COI with a failure rate of λ = 2 year^{-1}:

$$\text{MTTF} = \frac{1}{2} = 0{,}5 \text{ years} = 4\,380 \text{ h}$$

6.4.7 Mean time between failures (see 3.3)

METBF (abbreviation)

The expressions in 6.4.7 also apply to IOIs.

a) according to 6.1.4, METBF = MUT + MTTR:

$$\text{METBF} = \int_0^{\infty} t\, f_\text{U}(t)\,\mathrm{d}t + \int_0^{\infty} t\, g_\text{R}(t)\,\mathrm{d}t$$

If the item operates continuously:

$$\text{METBF} = \text{MTTF} + \text{MTTR}$$

NOTE In cases where the assumption 5.5.1 f) is not valid, the time between failures can include some down times due to preventive maintenance. Then, the formula established in 6.1.4 is still valid:

$$\text{METBF} = \text{MUT} + \text{MDT}$$

b) If observed failure data are available for n similar repairable items, then an estimate of the mean time between failures is given by

$$\hat{\text{METBF}} = \frac{\text{observation time}}{k_\text{F}} = \frac{\sum_{i=1}^{n} (\text{observation time})_i}{k_\text{F}} = \hat{\text{METBF}}$$

where

"observation time" is the aggregate calendar time of observation of all n items, including both up and down times;

"(observation time)$_i$" is the total calendar time of observation of the ith item, including both up and down times;

k_F is the total number of failures of the n items during a given period of observation.

c) If the up times and times to restoration are exponentially distributed

$$\text{METBF} = \frac{1}{\lambda_U} + \frac{1}{\mu_R} = \frac{\lambda_U + \mu_R}{\lambda_U \mu_R}$$

If the item operates continuously, λ_U equals λ.

d) For a continuously operating item with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}

$$\text{METBF} = \frac{12}{20} = 0{,}6 \text{ years} = 5\,256 \text{ h}$$

6.4.8 Mean operating time between failures [192-05-13]

MOTBF (abbreviation)

a) On the basis of the assumptions in 5.5.1,

MOTBF = MTTF (See 6.3.8.)

b) If the times to failure are exponentially distributed, then

$$\text{MOTBF} = \frac{1}{\lambda}$$

c) For a continuously operating item with a failure rate of λ = 2 year^{-1},

$$\text{MOTBF} = \frac{1}{2} = 0{,}5 \text{ years} = 4\,380 \text{ h}$$

6.4.9 Instantaneous availability [192-08-01]

Symbol $A(t)$

The expressions in 6.4.9 also apply to IOIs.

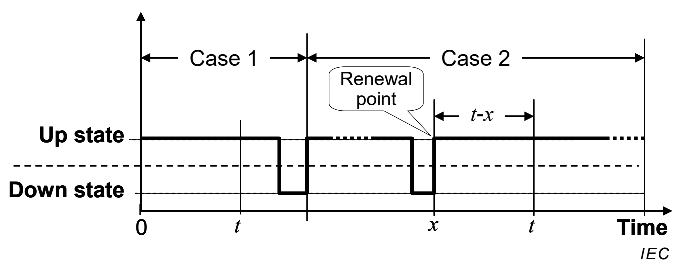


Figure 39 – Illustration of available behaviour at time t for a non-zero time to restoration individual item

a) As shown in Figure 39, two cases have to be analysed to establish the instantaneous availability $A(t)$ of the item at time t:

– No failure has occurred over [0, t]: $R(t)$

– At least one failure has occurred at time $x < t$ and no failure has occurred over [x, t]

Therefore the instantaneous availability can be established by using the restoration intensity $v(t)$ of the item which is the probability that a restoration finishes at the instant t:

$$A(t) = R(t) \; + \; \int_0^{t_1} R(t - x)\, v(x)\, \mathrm{d}t$$

Replacing the reliability $R(t)$ by $F_U(t)$ = 1-$R(t)$, the instantaneous availability of a repairable item with non-zero time to restoration at an instant of time t may be written as (see [13], [9] and [5]):

$$A(t) = 1 - F_U(t) \; + \; \int_0^t [1 - F_U(t - x)]\, v(x)\, \mathrm{d}x$$

where

$F_U(t)$ is the up time distribution function of the item:

$$F_U(t) = \int_0^t f_U(s)\, \mathrm{d}s$$

which is equal to the probability that a given up time will be less than or equal to t; then $F_U(t)$ is also the unreliability function and $R(t)$ =1- $F_U(t)$.

$v(t)$ is the instantaneous restoration intensity of the item.

NOTE The instantaneous availability, $A(t)$, is equal to the probability that the item is in an up state at the instant of time t . This arises if the item has had no failure before t or if the item was available at t-s and that no failure occurs over [t-s, t]. This leads to the following integral equation (see [9]):

$$A(t) = 1 - F_U(t) \; + \; \int_0^t f_{R+U}(s) A(t - s)\, \mathrm{d}s$$

which may be solved by numerical methods,

where

$$f_{R+U}(t) = \int_0^t g_R(t - s) f_U(s)\, \mathrm{d}s$$

is the probability density function of the sum of the up time and the corresponding time to restoration.

b) If observed up-state data are available for n repairable items, from a homogenous population, then an estimate of $A(t)$ is given by

$$\hat{A}(t) = \frac{n_U\{t\}}{n}$$

where $n_U\{t\}$ is the number of items which are in an up state at the instant of time t.

If the item operates continuously, then $R_U(t)$ = $R(t)$ and $f_U(t)$ = $f(t)$.

c) When the up times and times to restoration are exponentially distributed, then using either Markov techniques or the Laplace transform, the following is obtained (see [9], [14] and [18]):

$$A(t) = \frac{\mu_R}{\lambda_U + \mu_R} + \frac{\lambda_U}{\lambda_U + \mu_R} \exp[-(\lambda_U + \mu_R)t]$$

If the item operates continuously, λ_U = λ.

d) Figure 40 illustrates the application of the above formula for a COI with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}.

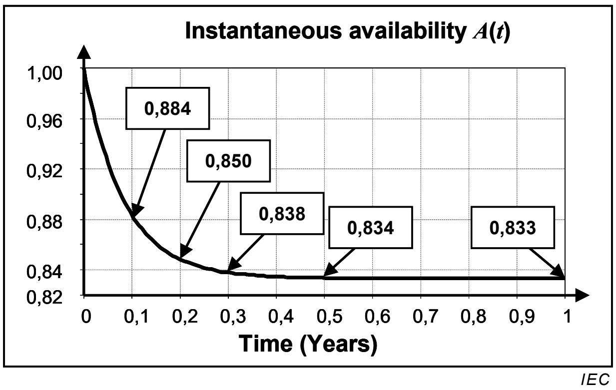


Figure 40 – Evolution of the instantaneous availability $A(t)$

6.4.10 Instantaneous unavailability [192-08-04]

Symbol $U(t)$

The expressions in 6.4.10 also apply to IOIs.

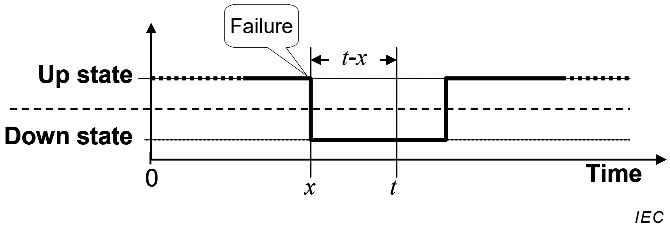


Figure 41 – Illustration of unavailable behaviour at time t for a non-zero time to restoration individual item

a) The instantaneous unavailability, $U(t)$, is equal to the probability that the item is in a down state at the instant of time t. As shown in Figure 41, the item is in the down state at the instant of time t if a failure has occurred at the instant of time x and the restoration has not been achieved over [x, t]. Therefore the instantaneous unavailability, $U(t)$, of a repairable item with non-zero time to restoration may be written as (see [5]):

$$U(t) = 1 - A(t) = \int_0^t [1 - G_R(t-x)]\, z(x)\, dx$$

where

$z(t)$ is the instantaneous failure intensity of the item;

$G_R(t)$ is the distribution function of the times to restoration of the item

$$G_R(t) = \int_0^t g_R(s)\, ds$$

which is equal to the probability that a restoration of the item is completed by time t. In this formula, $g_R(t)$ is the probability density function of the times to restoration of the item.

b) If observed down-state data are available for n repairable items, from a homogenous population, then an estimate of $U(t)$ is given by

$$\hat{U}(t) = \frac{n_D\{t\}}{n}$$

where $n_D\{t\}$ is the number of items which are in a down state at the instant of time t.

c) When the up times and times to restoration are exponentially distributed, then (see [14], and [18]),

$$U(t) = \frac{\lambda_U}{\lambda_U + \mu_R}(1 - \exp[-(\lambda_U + \mu_R)t])$$

If the item operates continuously, $\lambda_U = \lambda$.

d) Figure 42 illustrates the application of the above formula for a COI with a failure rate of $\lambda = 2$ year^{-1} and a restoration rate of $\mu_R = 10$ year^{-1}.

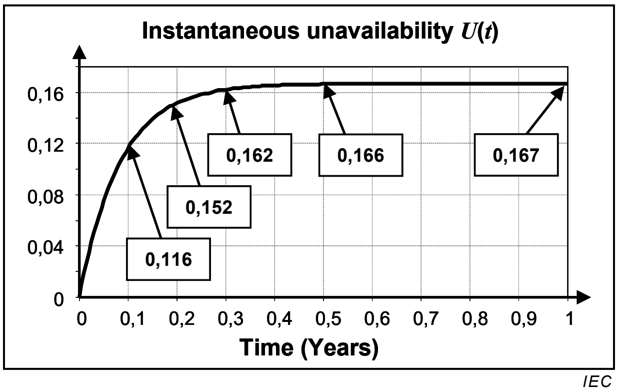


Figure 42 – Evolution of the instantaneous unavailability $U(t)$

6.4.11 Mean availability [192-08-05]

Symbol $\overline{A}(t_1, t_2)$, $0 \le t_1 < t_2$

The expressions in 6.4.11 also apply to IOIs.

a) According to its definition the mean availability is given by

$$\overline{A}(t_1, t_2) = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} A(t)\mathrm{d}t$$

The integral $\int_{t_1}^{t_2} A(t)\mathrm{d}t$ is equal to the expected up time accumulated in the time interval $[t_1, t_2]$, hence $\overline{A}(t_1, t_2)$ gives the expected fraction of the time interval $[t_1, t_2]$ that the item is in an up state.

It then follows that the mean availability, $\overline{A}(t_1, t_2)$, and the mean accumulated up time, MAUT(t_1, t_2), in the time interval $[t_1, t_2]$ are related as

$$\mathrm{MAUT}(t_1, t_2) = \int_{t_1}^{t_2} A(t)\mathrm{d}t = \overline{A}(t_1, t_2) \times (t_2 - t_1)$$

b) If observed up times in the interval $[t_1, t_2]$ are available for n repairable items, from a homogenous population, then an estimate of $\overline{A}(t_1, t_2)$ is given by

$$\hat{\bar{A}}(t_1, t_2) = \frac{\text{total up time}}{(t_2 - t_1)n} = \frac{\sum_{i=1}^{n} (\text{up time})_i}{(t_2 - t_1)n}$$

where

"total up time" is the aggregate up time of all n items during the time interval $[t_1, t_2]$;

"(up time)$_i$" is the total up time of the ith item during the time interval $[t_1, t_2]$.

c) An estimate of the mean accumulated up time, MAUT(t_1, t_2), in the time interval $[t_1, t_2]$ is given by

$$\text{M}\hat{\text{A}}\text{UT}(t_1, t_2) = \frac{\text{total up time}}{n} = \frac{\sum_{i=1}^{n} (\text{up time})_i}{n}$$

where

"total up time" is the aggregate up time of all n items during the time interval $[t_1, t_2]$;

"(up time)$_i$" is the total up time of the ith item during the time interval $[t_1, t_2]$.

d) Following from the assumptions given in 5.5.1, the asymptotic mean availability $\bar{A} = \lim_{t_2 \to \infty} \bar{A}(t_1, t_2)$ is equal to the asymptotic availability A (see 6.1.2.3 and [7]):

$$\bar{A} = A = \frac{\text{MUT}}{\text{MUT} + \text{MTTR}}$$

e) When the up times and times to restoration are exponentially distributed, then integrating $A(t)$ over the time interval $[t_1, t_2]$ gives:

$$\text{MAUT}(t_1, t_2) = \frac{(t_2 - t_1)\mu_\text{R}}{\lambda_\text{U} + \mu_\text{R}} + \frac{\lambda_\text{U}\{\exp[-(\lambda_\text{U} + \mu_\text{R})t_1] - \exp[-(\lambda_\text{U} + \mu_\text{R})t_2]\}}{(\lambda_\text{U} + \mu_\text{R})^2}$$

Dividing this expression by $t_2 - t_1$ yields:

$$\begin{aligned}\bar{A}(t_1, t_2) &= \frac{\text{MAUT}(t_1, t_2)}{t_2 - t_1} \\ &= \frac{\mu_\text{R}}{\lambda_\text{U} + \mu_\text{R}} + \frac{\lambda_\text{U}}{(\lambda_\text{U} + \mu_\text{R})^2} \frac{\exp[-(\lambda_\text{U} + \mu_\text{R})t_1] - \exp[-(\lambda_\text{U} + \mu_\text{R})t_2]}{t_2 - t_1} \\ &= \frac{\bar{z}(t_1, t_2)}{\lambda_\text{U}}\end{aligned}$$

If the item operates continuously, $\lambda_\text{U} = \lambda$.

f) Figure 43 illustrates the application of the above formula for a continuously operating item with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}.

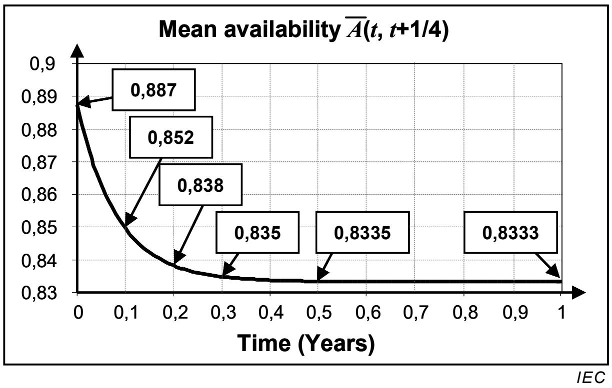


Figure 43 – Evolution of the mean availability $\overline{A}(t, t+1/4)$

The mean accumulated up time, MAUT(0,1), during the first year may be calculated as follows:

$$\text{MAUT}(0,1) = A(0,1) \times 1 = \left[\overline{A}\left(0, \frac{1}{4}\right) + \overline{A}\left(\frac{1}{4}, \frac{1}{2}\right) + \overline{A}\left(\frac{1}{2}, \frac{3}{4}\right) + \overline{A}\left(\frac{3}{4}, 1\right)\right] / 4$$

$$\text{MAUT}(0,1) = (0{,}887\,5 + 0{,}836\,0 + 0{,}833\,5 + 0{,}833\,3)/4 = 0{,}847 \text{ years} = 7\,424 \text{ h}$$

6.4.12 Mean unavailability [192-08-06]

Symbol $\overline{U}(t_1, t_2)$, $0 \le t_1 < t_2$

The expressions in 6.4.12 also apply to IOIs.

a) According to its definition the mean unavailability is given by

$$\overline{U}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} U(t)\,\mathrm{d}t \;=\; 1 - \overline{A}(t_1, t_2)$$

The integral $\int_{t_1}^{t_2} U(t)\,\mathrm{d}t$ is equal to the expected down time accumulated in the time interval [t_1, t_2]. Hence, $\overline{U}(t_1, t_2)$ gives the expected fraction of the time interval [t_1, t_2] spent in the down state.

It then follows that the mean unavailability, $\overline{U}(t_1, t_2)$, and the mean accumulated down time, MADT(t_1, t_2), in the time interval [t_1, t_2] are related as

$$\text{MADT}(t_1, t_2) = \int_{t_1}^{t_2} U(t)\,\mathrm{d}t = \overline{U}(t_1, t_2) \times (t_2 - t_1)$$

b) If observed down times in the interval [t_1, t_2] are available for n repairable items, from a homogenous population, then an estimate of $\overline{U}(t_1, t_2)$ is given by

$$\hat{\overline{U}}(t_1, t_2) = \frac{\text{total down time}}{(t_2 - t_1)n} = \frac{\sum_{i=1}^{n} (\text{down time})_i}{(t_2 - t_1)n}$$

c) An estimate of the mean accumulated down time, MADT(t_1, t_2), in the time interval [t_1, t_2] is given by

$$\hat{\text{MADT}}(t_1,t_2) = \frac{\text{total down time}}{n} = \frac{\sum_{i=1}^{n}(\text{down time})_i}{n}$$

where

"total down time" is the aggregate down time of all n items during the time interval [t_1, t_2];

"(down time)$_i$" is the total down time of the ith item during the time interval [t_1, t_2].

d) Following from the assumptions given in 5.5.1, the asymptotic mean unavailability $\overline{U} = \lim_{t_2 \to \infty} \overline{U}(t_1, t_2)$ is equal to the asymptotic unavailability U (see 6.1.2.3 and [7]):

$$\overline{U} = U = \frac{\text{MTTR}}{\text{MUT} + \text{MTTR}}$$

e) When the down times and times to restoration are exponentially distributed, then integrating $U(t)$ over the time interval [t_1, t_2] gives:

$$\text{MADT}(t_1,t_2) = \frac{(t_2 - t_1)\lambda_\text{U}}{\lambda_\text{U} + \mu_\text{R}} - \frac{\lambda_\text{U}\{\exp[-(\lambda_\text{U} + \mu_\text{R})t_1] - \exp[-(\lambda_\text{U} + \mu_\text{R})t_2]\}}{(\lambda_\text{U} + \mu_\text{R})^2}$$

Dividing this expression by $t_2 - t_1$ yields:

$$\begin{aligned}\overline{U}(t_1, t_2) &= \frac{\text{MADT}(t_1,t_2)}{t_2 - t_1} \\ &= \frac{\lambda_\text{U}}{\lambda_\text{U} + \mu_\text{R}} - \frac{\lambda_\text{U}}{(\lambda_\text{U} + \mu_\text{R})^2}\,\frac{\exp[-(\lambda_\text{U} + \mu_\text{R})t_1] - \exp[-(\lambda_\text{U} + \mu_\text{R})t_2]}{t_2 - t_1} \\ &= 1 - \frac{\overline{z}(t_1, t_2)}{\lambda_\text{U}}\end{aligned}$$

If the item operates continuously, $\lambda_\text{U} = \lambda$.

f) Figure 44 illustrates the application of the above formula for a COI with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}.

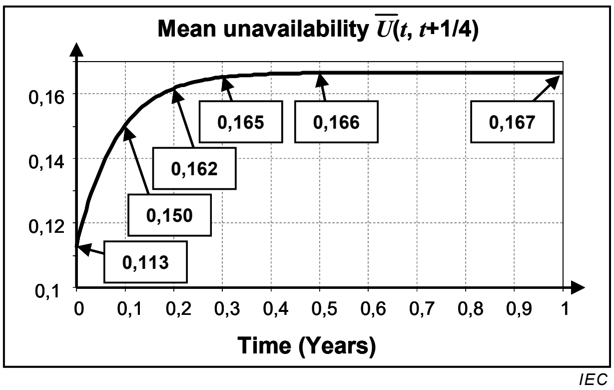


Figure 44 – Evolution of the mean unavailability $\overline{U}(t, t+1/4)$

The mean accumulated down time, MADT(0, 1), during the first year may be calculated as follows:

$$\text{MADT}(0,1) = \overline{U}(0,1) \times 1 = \left[\overline{U}\left(0, \frac{1}{4}\right) + \overline{U}\left(\frac{1}{4}, \frac{1}{2}\right) + \overline{U}\left(\frac{1}{2}, \frac{3}{4}\right) + \overline{U}\left(\frac{3}{4}, 1\right)\right] / 4$$

$$\text{MADT}(0,1) = (0{,}112\,5 + 0{,}164\,0 + 0{,}166\,5 + 0{,}166\,7)/4 = 0{,}152 \text{ years} = 1335 \text{ h}$$

6.4.13 Asymptotic availability [192-08-07]

Symbol A

The expressions in 6.4.13 also apply to IOIs.

a) When a steady state exists, the asymptotic availability is equal to the asymptotic mean availability and can be calculated by the general following formula (see 6.1.2.3):

$$\lim_{t \to \infty} A(t) = A(\infty) = A = \overline{A} = \frac{\text{MUT}}{\text{MUT} + \text{MDT}}$$

As preventive maintenance is not taken under consideration in this document, MDT = MTTR (see Figure 2) and

$$A = \frac{\text{MUT}}{\text{MUT} + \text{MTTR}}$$

If, in addition, the item operates continuously, then

$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

When the up times and times to restoration are exponentially distributed, then

$$A = \frac{\mu_R}{\lambda_U + \mu_R}$$

If the item operates continuously, $\lambda_U = \lambda$.

b) For a COI with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}

$$A = \frac{10}{12} = 0{,}83$$

This is illustrated in Figure 40.

6.4.14 Asymptotic unavailability [192-08-08]

Symbol U

The expressions in 6.4.14 also apply to IOIs.

a) When a steady state exists, the asymptotic unavailability is equal to the asymptotic mean unavailability and can be calculated by the general following formula (see 6.1.2.3):

$$\lim_{t \to \infty} U(t) = U(\infty) = \overline{U} = U = \frac{\text{MDT}}{\text{MUT} + \text{MDT}}$$

As the preventive maintenance is not taken under consideration in this document, MDT = MTTR (see Figure 2) and

$$U = \lim_{t \to \infty} U(t) = \frac{\text{MTTR}}{\text{MUT} + \text{MTTR}} = 1 - A$$

If, in addition, the item operates continuously, then

$$U = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}}$$

b) When the up times and times to restoration are exponentially distributed, then

$$U = \frac{\lambda_U}{\lambda_U + \mu_R}$$

If the item operates continuously, $\lambda_U = \lambda$.

c) For a COI with a failure rate of λ = 2 year^{-1} and a restoration rate of μ_R = 10 year^{-1}, then:

$$U = \frac{2}{12} = 0{,}17$$

This is illustrated in Figure 42.

6.4.15 Mean up time [192-08-09]

MUT (abbreviation)

The expressions in 6.4.15 also apply to IOIs.

a) $\text{MUT} = \int_0^\infty t f_U(t)\,\mathrm{d}t = \int_0^\infty (1 - F_U(t))\,\mathrm{d}t$

where

$f_U(t)$ is the probability density function of the up times of the item;

$F_U(t)$ is the up time distribution function of the item.

NOTE When the item operates continuously, then, according to the assumption in 5.5.1 f) (i.e no preventive maintenance):

$$\text{MUT} = \text{MTTF}$$

However, when function-preventing preventive maintenance is permitted, the relation between MUT and MTTF is more complex, and usually, MUT < MTTF.

b) If observed up times are available for n repairable items, from a homogenous population, then an estimate of MUT is given by

$$\widehat{\text{MUT}} = \frac{\text{total up time}}{k_U} = \frac{\sum_{i=1}^{n} (\text{up time})_i}{k_U}$$

where

"total up time" is the aggregate up time of all n items during the given period of observation;

k_U is the total number of up times of the items during the given period of observation;

"(up time)$_i$" is the total up time of the ith item during the given period of observation.

EXAMPLE Consider an item belonging to IOIs, which operates as described below. The item starts operating at time t = 0 and is required to operate (be in operating state) for a fixed time interval [0, X], $X > 0$ during which the item can fail with a constant failure rate of

$\lambda > 0$. If the item did not fail in this interval, it becomes not required for subsequent fixed time interval $[X, X+Y]$, $Y > 0$, during which the item cannot fail (i.e. the item is in a failure free idle state during this interval). At time $t = X + Y$ the next required time starts and the functioning process repeats as from the initial time $t = 0$, independently of previous history of the item functioning process. When the item fails during the required time $[0, X]$, it is repaired to the state as goods as new and then the functioning process repeats again as at the initial time $t = 0$, independently of the previous history of the item functioning process. Repair times are mutually independent and also independent of the previous history of the item functioning process. See Figure 45.

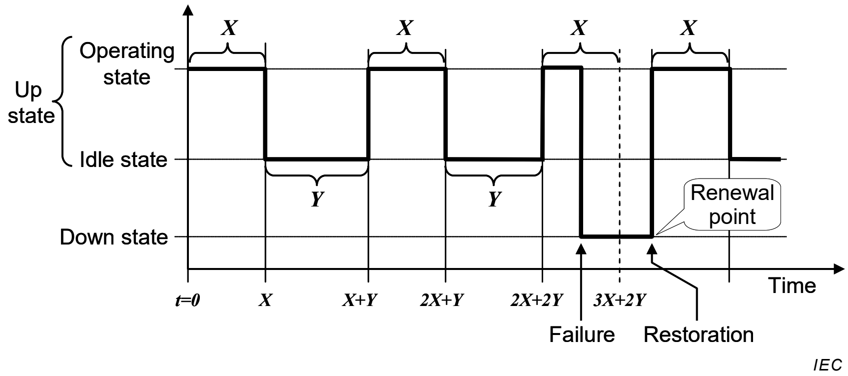


Figure 45 – Sample realization of the individual item state

It is clear from the above description that consecutive up times are statistically independent and are identically distributed positive, continuous random variables. Therefore, to calculate MUT, the up time to first failure of the item can be considered. The up-time hazard rate function $\lambda_U(t)$ is depicted in Figure 46.

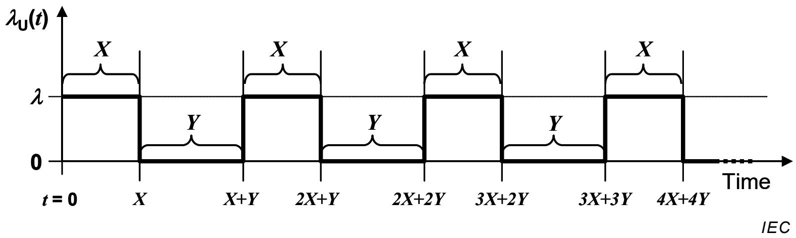


Figure 46 – Plot of the up-time hazard rate function $\lambda_U(t)$

The analytical form of $\lambda_U(t)$ is:

$$\lambda_U(t) = \begin{cases} \lambda & \text{for } n \cdot (X+Y) \le t < n \cdot (X+Y) + X \\ 0 & \text{for } n \cdot (X+Y) + X \le t < (n+1) \cdot (X+Y) \end{cases}, \quad n = 0, 1, 2, \ldots$$

Since

$$1 - F_U(t) = \exp\left(- \int_0^t \lambda_U(x)\,\mathrm{d}x \right),$$

then by integrating $\lambda_U(t)$, the above formula can be written as:

$$1 - F_U(t) = \begin{cases} \exp(-\lambda \cdot (t - n \cdot Y)) & \text{for } n \cdot (X+Y) \le t < n \cdot (X+Y) + X \\ \exp(-\lambda \cdot (n+1) \cdot X) & \text{for } n \cdot (X+Y) + X \le t < (n+1) \cdot (X+Y) \end{cases}, \quad n = 0, 1, 2, \ldots$$

By integrating $1-F_\mathrm{U}(t)$ over [0, ∞], MUT of the item is obtained:

$$\mathrm{MUT} = \frac{1}{\lambda} + Y \cdot \frac{\exp(-\lambda \cdot X)}{1-\exp(-\lambda \cdot X)} = \mathrm{MOTBF} + Y \cdot \frac{\exp(-\lambda \cdot X)}{1-\exp(-\lambda \cdot X)}$$

The second term of the above formula is equal to the expected accumulated idle time to failure of the item. For λ = 0,01 h^{-1} and X = 10 h, the following values of MUT for some values of Y are obtained:

MUT = 195 h for Y = 10 h, MUT = 290 h for Y = 20 h,

MUT = 575 h for Y = 50 h, MUT = 1 051 h for Y = 100 h,

whereas:

MOTBF = 100 h.

c) When the up times are exponentially distributed

$$\mathrm{MUT} = \frac{1}{\lambda_\mathrm{U}}$$

d) For a repairable item with λ_U = 2 year^{-1}

$$\mathrm{MUT} = \frac{1}{2} = 0{,}5 \text{ years} = 4\,380 \text{ h}$$

6.4.16 Mean down time [192-08-10]

MDT (abbreviation)

The expressions in 6.4.16 also apply to IOIs.

a) $\mathrm{MDT} = \int_0^\infty t g_\mathrm{D}(t)\mathrm{d}t$

where $g_\mathrm{D}(t)$ is the probability density function of the down time of the item (which is defined to include the time to restoration of the item, after failure and/or function-preventing preventive maintenance times), i.e. for small values of Δt, $g_\mathrm{D}(t)\cdot\Delta t$ is approximately equal to the probability that the item returns to its up state from its down state in the time interval [t, $t + \Delta t$], assuming that the down time started at time t = 0.

b) If observed down times are available for n repairable items, from a homogenous population, then an estimate of MDT is given by

$$\hat{\mathrm{MDT}} = \frac{\text{total down time}}{k_\mathrm{D}} = \frac{\sum_{i=1}^{n} (\text{down time})_i}{k_\mathrm{D}}$$

where

"total down" time is the aggregate down time of all n items during a given time period;

k_D is the total number of down times of the items in the given time period;

"(down times)$_i$" is the total down time of the ith item during the given time period.

c) If the down times are exponentially distributed with a parameter μ_D, i.e.

$$g_D(t) = \mu_D \exp(-\mu_D t)$$

then

$$\text{MDT} = \frac{1}{\mu_D}$$

NOTE According to the assumptions in 5.5.1 (any fault is the result of a failure, and no preventive maintenance), any down time is equal to the time to restoration, i.e.

$$\text{MDT} = \text{MTTR}$$

and, for exponentially distributed down times

$$\mu_D = \mu_R$$

d) For a repairable item with μ_D = 100 year^{-1}

$$\text{MDT} = \frac{1}{100} = 0{,}01 \text{ years} = 87{,}6 \text{ h}$$

6.4.17 Maintainability [192-07-01]

Symbol $M(t_1, t_2)$, $0 \le t_1 < t_2$

The expressions in 6.4.17 also apply to IOIs.

a) The probability that a given maintenance action on an item can be completed in the time interval $[t_1, t_2]$, assuming that the maintenance action started at time t = 0, is given by

$$M(t_1, t_2) = \int_{t_1}^{t_2} g_{MA}(t)\,dt$$

where $g_{MA}(t)$ is the probability density function of the time to complete a given maintenance action for the item, i.e. for small values of Δt, $g_{MA}(t) \cdot \Delta t$ is approximately equal to the probability of completing a given maintenance action during the time interval $[t, t + \Delta t]$, assuming that the maintenance action started at time t = 0.

NOTE 1 The probability density function $g_{MA}(t)$ is distinct from the restoration intensity which is the probability of completing a restoration action (e.g. a maintenance action) assuming that the item was in the perfect upstate at t = 0.

In practical applications, the maintainability function, $M(t)$, defined as

$$M(t) = M(0, t) = \int_0^t g_{MA}(x)\,dx$$

is used, with $M(0)$ = 0. This is equal to the probability that a given maintenance action will be completed before time t, assuming that the action started at time t = 0, i.e. $M(t)$ is the distribution function of that time. The maintainability, $M(t_1, t_2)$, and the maintainability function, $M(t)$, are related as follows:

$$M(t_1, t_2) = M(t_2) - M(t_1)$$

NOTE 2 $M(t)$ is the counterpart, for the maintenance actions of $F(t)$=1-$R(t)$, for the failures. Therefore the formulae can be established in the same way as was done when reliability was analysed. The maintainability function, $M(t)$, and the mean maintenance action time, MMAT, are related as follows:

$$\text{MMAT} = \int_0^\infty (1 - M(t))\,dt = \int_0^\infty t\, g_{MA}(t)\,dt$$

NOTE 3 The above formula is valid for any maintenance action. Then if the maintenance action time is equal to the entire active corrective maintenance time:

$$M(t) = G_{\mathrm{ACM}}(t),\ \mathrm{MMAT} = \mathrm{MACMT}$$

where $G_{\mathrm{ACM}}(t)$ is the distribution function of the active corrective maintenance time and MACMT is the mean active corrective maintenance time.

If the repair is considered as the maintenance action, then

$$M(t) = G(t),\ \mathrm{MMAT} = \mathrm{MRT}$$

where $G(t)$ is the distribution function of the repair time and MRT is the mean repair time.

Similarly, when the maintenance action consists of all actions attributed to the time to restoration, then

$$M(t) = G_{\mathrm{R}}(t),\ \mathrm{MMAT} = \mathrm{MTTR}$$

where $G_{\mathrm{R}}(t)$ is the distribution function of the time to restoration and MTTR is the mean time to restoration.

b) If observed data for m maintenance action times of a given type are available, from a homogenous population, the estimated value of $M(t)$ is given by:

$$\hat{M}(t) = \frac{m - m_{\mathrm{MAT}}(t)}{m}$$

where

$m_{\mathrm{MAT}}(t)$ is the number of maintenance action times with a duration greater that t, i.e. not finished up to time t, and

$m_{\mathrm{MAT}}(0) = m$.

c) If the given maintenance action times are exponentially distributed with a parameter μ_{MA}, i.e.

$$g_{\mathrm{MA}}(t) = \mu_{\mathrm{MA}} \exp(-\mu_{\mathrm{MA}} t)$$

then

$$M(t_1, t_2) = \exp(-\mu_{\mathrm{MA}} t_1) - \exp(-\mu_{\mathrm{MA}} t_2)$$

$$M(t) = 1 - \exp(-\mu_{\mathrm{MA}} t)$$

and

$$\mathrm{MMAT} = \frac{1}{\mu_{\mathrm{MA}}}$$

d) Figure 47 illustrates the application of the above formula for a repairable item with μ_{MA} = 1 000 year^{-1},(i.e. 0,114 2 h^{-1}) and $t_2 - t_1$ = 16 h (simplify by using hours, rather than years, as the unit).

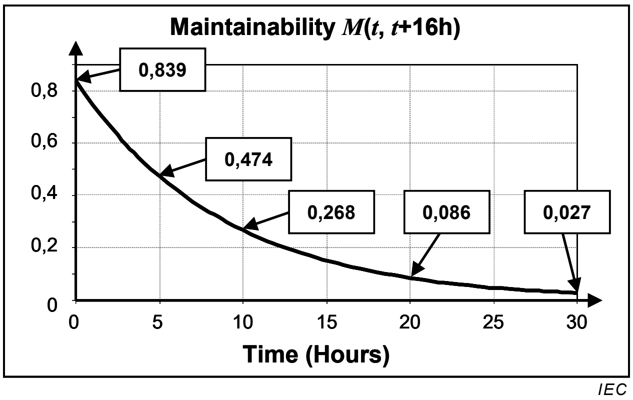


Figure 47 – Evolution of the maintainability $M(t, t$+16h)

6.4.18 Instantaneous repair rate [192-07-20]

Symbol $\mu(t)$

The expressions in 6.4.18 also apply to IOIs.

a) By definition,

$$\mu(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{G(t+\Delta t) - G(t)}{1 - G(t)} = \frac{g(t)}{1 - G(t)}$$

where

$g(t)$ is the probability density function of the repair time [192-07-19] of an item (excluding technical, logistic and administrative delays), i.e. for small values of Δt, $g(t)\cdot\Delta t$ is approximately equal to the probability that the repair started at time $t = 0$ will be completed during $[t, t + \Delta t]$;

$G(t)$ is the distribution function of the repair time of the item, i.e. $G(t)$ is the probability that the repair, started at time $t = 0$, with $G(0) = 0$, will be completed by time t:

$$G(t) = 1 - \exp\left(-\int_0^t \mu(x)\mathrm{d}x\right) = \int_0^t g(x)\mathrm{d}x$$

For small values of Δt, $\mu(t)\cdot\Delta t$ is approximately equal to the conditional probability that the repair will be completed in the time interval $[t, t + \Delta t]$, given that the repair started at time $t = 0$ and has not been completed by the instant of time t.

b) If observed repair data are available for n repairable items, from a homogenous population, the estimated value of $\mu(t)$ at time t is given by

$$\hat{\mu}(t) = \frac{n_\mathrm{R}(t) - n_\mathrm{R}(t + \Delta t)}{n_\mathrm{R}(t)\Delta t}$$

where

$n_\mathrm{R}(t)$ is the number of items that are still under repair at the instant of time t ($n_\mathrm{R}(0) = n$);

$n_\mathrm{R}(t) - n_\mathrm{R}(t + \Delta t)$ is the number of items with repair completed in the time interval $[t, t + \Delta t]$.

It should be noted that the estimated value of the repair density function $g(t)$, at time t, is given by

$$\hat{g}(t) = \frac{n_R(t) - n_R(t + \Delta t)}{n\Delta t}$$

c) When the time to repair is exponentially distributed,

$$g(t) = \mu \exp(-\mu t)$$

and

$$G(t) = 1 - \exp(-\mu t)$$

hence

$$\mu(t) = \mu$$

for all values of t.

In this case:

$$\text{MRT} = \frac{1}{\mu}$$

where MRT is the mean repair time.

d) If observed repair data are available for n repairable items, from a homogenous population, with constant repair rate, then the estimated value of μ is given by

$$\hat{\mu} = \frac{n}{\sum_{i=1}^{n} \text{RT}_i}$$

where RT_i is the repair time of item i.

For 10 repairable items, from a homogenous population, with a constant repair rate, the observed total repair time of all the items is $\sum_{i=1}^{10} \text{RT}_i$ = 5 h. Hence

$$\hat{\mu} = \frac{10}{5} = 2 \text{ h}^{-1}$$

e) If the repair time of a repairable item has a lognormal distribution with scale parameter m and shape parameter $\sigma > 0$, then (see Table B.2)

$$g(t) = \frac{1}{t\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln t - m)^2}{2\sigma^2}\right)$$

and

$$G(t) = \int_0^t g(x)\,\text{d}x$$

hence

$$\mu(t) = \frac{g(t)}{1 - G(t)}$$

f) Assuming that an item has a mean repair time (MRT) of 1,5 h and repair time variance (VRT) of 0,16 h^2, in order to compute the repair rate, the parameters m and σ of the lognormal distribution of repair times have first to be determined. According to the results given in Table B.2,

$$\mathrm{MRT} = \exp\left(m + \frac{\sigma^2}{2}\right)$$

$$\mathrm{VRT} = \exp(2m + 2\sigma^2) - \exp(2m + \sigma^2) = \mathrm{MRT}^2 \cdot [\exp(\sigma^2) - 1]$$

Solving the above equations yields:

$$m = \frac{1}{2}\ln\left(\frac{\mathrm{MRT}^4}{\mathrm{VRT} + \mathrm{MRT}^2}\right), \quad \sigma^2 = \ln\left(\frac{\mathrm{VRT} + \mathrm{MRT}^2}{\mathrm{MRT}^2}\right),$$

giving m = 0,37 and σ^2 = 0,069.

Figure 48 illustrates the evolution of the lognormal repair rate with the above parameters.

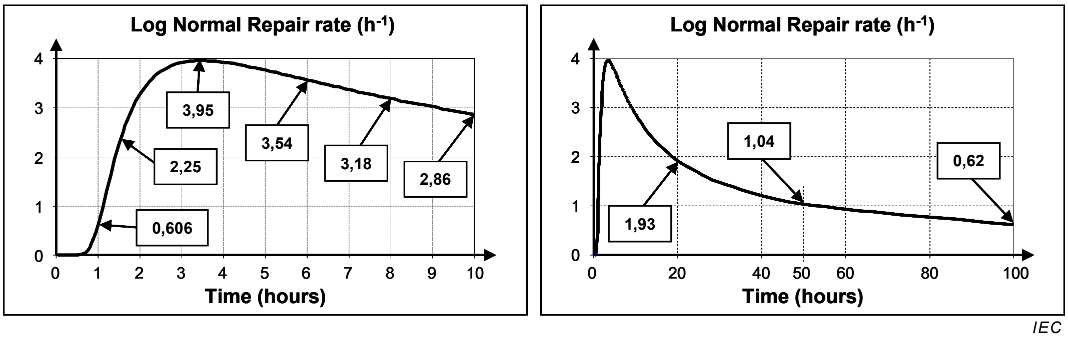


Figure 48 – Evolution of the lognormal repair rate $\mu(t)$

6.4.19 Mean repair time [192-07-21]

MRT (abbreviation)

The expressions in 6.4.19 also apply to IOIs.

a) MRT = $\int_0^\infty t g(t)\mathrm{d}t = \int_0^\infty (1 - G(t))\mathrm{d}t$

where $g(t)$ is the probability density function of the repair time of an item, and $G(t)$ is the distribution function of the repair time of the item.

NOTE From the definition of the repair time

MRT = MACMT − MTD

where

MTD is the mean technical delay;

MACMT is the mean active corrective maintenance time.

b) If observed repair times are available for n repairable items, from a homogenous population, then an estimate of MRT is given by

$$\widehat{\mathrm{MRT}} = \frac{\text{total repair time}}{k} = \frac{\sum_{i=1}^{n}(\text{repair time})_i}{k}$$

where

"total repair time" is the aggregate repair time of all n items during a given time period;

k is the total number of repair times of the items during the given time period;

"(repair time)$_i$" is the total repair time of the ith item during the given time period.

c) If the repair times are exponentially distributed with a parameter μ, i.e.

$$g(t) = \mu \exp(-\mu t)$$

then

$$\text{MRT} = \frac{1}{\mu}$$

d) For a repairable item with μ = 1 000 year^{-1}:

$$\text{MRT} = \frac{1}{1000} = 0{,}001 \text{ years} = 8{,}76 \text{ h}$$

6.4.20 Mean active corrective maintenance time [192-07-22]

MACMT (abbreviation)

The expressions in 6.4.20 also apply to IOIs.

a) $\text{MACMT} = \int_0^\infty (1 - G_{\text{ACM}}(t))\text{d}t = \int_0^\infty t\, g_{\text{ACM}}(t)\text{d}t$

where

$g_{\text{ACM}}(t)$ is the probability density function of the active corrective maintenance times of an item (including technical delay and repair time, but excluding logistic and administrative delays), i.e. for small values of Δt, $g_{\text{ACM}}(t) \cdot \Delta t$ is approximately equal to the probability that the active corrective maintenance of the item is completed in the time interval [t, $t + \Delta t$], assuming that the active corrective maintenance started at time t = 0;

$G_{\text{ACM}}(t)$ is the distribution function of the active corrective maintenance time of the item, i.e. $G_{\text{ACM}}(t)$ is the probability that the active corrective maintenance, started at time $t = 0$, will be completed by time t:

$$G_{\text{ACM}}(t) = \int_0^t g_{\text{ACM}}(x)\text{d}x$$

NOTE As per the definition of the active corrective maintenance time:

$$\text{MACMT} = \text{MRT} + \text{MTD}$$

where MTD is the mean technical delay.

b) If observed active corrective maintenance times are available for n repairable items, from a homogenous population, then an estimate of MACMT is given by

$$\widehat{\text{MACMT}} = \frac{\text{total active corrective maintenance time}}{k_{\text{ACM}}} = \frac{\sum_{i=1}^{n} (\text{active corrective maintenance time})_i}{k_{\text{ACM}}}$$

where

"total active corrective maintenance time" is the aggregate active corrective maintenance time of all n items during a given time period;

k_{ACM} is the total number of active corrective maintenance actions on the items during the given time period;

"(active corrective maintenance time)$_i$" is the total active corrective maintenance time of the ith item during the given time period.

c) If the active corrective maintenance times are exponentially distributed with parameter μ_{ACM}, i.e.

$$g_{ACM}(t) = \mu_{ACM} \exp(-\mu_{ACM} t)$$

then

$$\text{MACMT} = \frac{1}{\mu_{ACM}}$$

d) For a repairable item with the mean technical delay MTD = 5 h and the mean repair time MRT = 9 h:

$$\text{MACMT} = 5 + 9 = 14 \text{ h}$$

6.4.21 Mean time to restoration [192-07-23]

MTTR (abbreviation)

The expressions in 6.4.21 also apply to IOIs.

a) MTTR = $\int_0^\infty t g_R(t) \mathrm{d}t$

where $g_R(t)$ is the probability density function of the times to restoration of the item, i.e. for small values of Δt, $g_R(t) \cdot \Delta t$ is approximately equal to the probability that the item is restored from a fault to an up state in the time interval [t, $t + \Delta t$], assuming that a failure resulting in a fault occurred at time $t = 0$.

NOTE The mean time to restoration (of a faulty item), MTTR, can be written as the sum of the expected values of its constituent times (see Figure 2):

$$\text{MTTR} = \text{MFDT} + \text{MAD} + \text{MLD} + \text{MACMT} = \text{MFDT} + \text{MAD} + \text{MLD} + \text{MTD} + \text{MRT}$$

where

MFDT is the mean fault detection time;

MAD is the mean administrative delay;

MLD is the mean logistic delay;

MACMT is the mean active corrective maintenance time given by

$$\text{MACMT} = \text{MTD} + \text{MRT}$$

where

MTD is the mean technical delay;

MRT is the mean repair time.

b) If observed times to restoration are available for n repairable items, from a homogenous population, then an estimate of MTTR is given by

$$\widehat{\text{MTTR}} = \frac{\text{total time to restoration}}{k_R} = \frac{\sum_{i=1}^{n} (\text{time to restoration})_i}{k_R}$$

where

"total time to restoration" is the aggregate time to restoration of all n items during a given time period;

k_R is the total number of times to restoration of the items during the given time period;

"(time to restoration)$_i$" is the total time to restoration of the ith item during the given time period.

c) If the times to restoration are exponentially distributed, i.e.

$$g_R(t) = \mu_R \exp(-\mu_R t)$$

where μ_R is the constant restoration rate, then:

$$\text{MTTR} = \frac{1}{\mu_R}$$

d) For a repairable item with a restoration rate of μ_R = 100 year^{-1}

$$\text{MTTR} = \frac{1}{100} = 0{,}01 \text{ years} = 87{,}6 \text{ h}$$

6.4.22 Mean administrative delay [192-07-26]

MAD (abbreviation)

The expressions in 6.4.22 also apply to IOIs.

a) MAD = $\int_0^\infty t\, g_{AD}(t)\mathrm{d}t$

where $g_{AD}(t)$ is the probability density function of the administrative delay during a time to restoration of a faulty item, i.e. for small values of Δt, $g_{AD}(t)\cdot\Delta t$ is approximately equal to the probability that the delay ends in the time interval [t, $t + \Delta t$], assuming that the delay started at time t = 0.

b) If observed administrative delays are available for n repairable items, from a homogenous population, then an estimate of MAD is given by

$$\hat{\text{MAD}} = \frac{\text{total administrative delay}}{k_{AD}} = \frac{\sum_{i=1}^{n}(\text{administrative delay})_i}{k_{AD}}$$

where

"total administrative delay" is the aggregate administrative delay of all n items during the given time period;

k_{AD} is the total number of administrative delays during the given time period;

"(administrative delay)$_i$" is the total administrative delay of the ith item during the given time period.

c) If the administrative delays are exponentially distributed with parameter μ_{AD}, i.e.

$$g_{AD}(t) = \mu_{AD} \exp(-\mu_{AD} t)$$

then

$$\text{MAD} = \frac{1}{\mu_{\text{AD}}}$$

d) For a repairable item with μ_{AD} = 1 000 year^{-1}:

$$\text{MAD} = \frac{1}{1000} = 0{,}001 \text{ years} = 8{,}76 \text{ h}$$

6.4.23 Mean logistic delay [192-07-27]

MLD (abbreviation)

The expressions in 6.4.23 also apply to IOIs.

a) MLD = $\int_0^\infty t g_{\text{LD}}(t)\text{d}t$

where $g_{\text{LD}}(t)$ is the probability density function of the logistic delay during a maintenance time of a faulty item, i.e. for small values of Δt, $g_{\text{LD}}(t) \cdot \Delta t$ is approximately equal to the probability that the delay ends in the time interval [t, $t + \Delta t$], assuming that the delay started at time t = 0.

b) If observed logistic delays are available for n repairable items, from a homogenous population, then an estimate of MLD is given by

$$\hat{\text{MLD}} = \frac{\text{total logistic delay}}{k_{\text{LD}}} = \frac{\sum_{i=1}^{n} (\text{logistic delay})_i}{k_{\text{LD}}}$$

where

"total logistic delay" is the aggregate logistic delay of all n items during a given time period;

k_{LD} is the total number of logistic delays during the given time period;

"(logistic delay)$_i$" is the total logistic delay of the ith item during the given time period.

c) If the logistic delays are exponentially distributed with parameter μ_{LD}, i.e.

$$g_{\text{LD}}(t) = \mu_{\text{LD}} \exp(-\mu_{\text{LD}} t)$$

then

$$\text{MLD} = \frac{1}{\mu_{\text{LD}}}$$

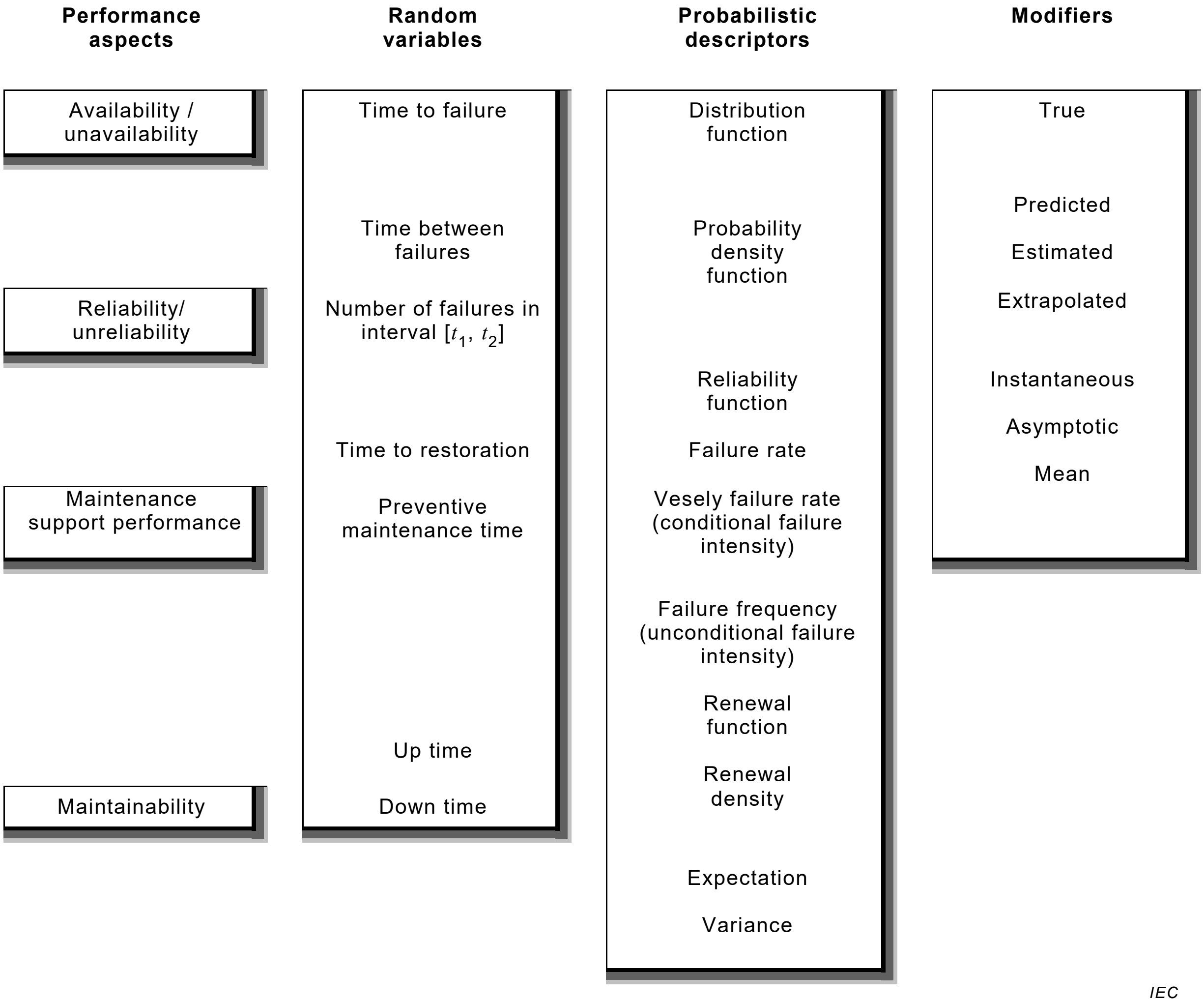
d) For a repairable item with μ_{LD} = 1 000 year^{-1}

$$\text{MLD} = \frac{1}{1000} = 0{,}001 \text{ years} = 8{,}76 \text{ h}$$

Annex A
(informative)

Performance aspects and descriptors

Performance aspects and descriptors are given in Figure A.1.



NOTE 1 There is no direct relationship between the various columns.

NOTE 2 A mathematical operation on a random variable results in a basic measure. The addition of a modifier to a basic measure results in a specific measure.

Figure A.1 – Performance aspects and descriptors

Annex B
(informative)

Summary of measures related to time to failure

A summary of measures related to time to failure is given in Tables B.1 and B.2. A summary of measures related to repair time is given in Table B.3.

Table B.1 – Relations among measures related to time to failure of continuously operating items

Measure	Relation to other measures			
	$F(t)$	$f(t)$	$R(t)$	$\lambda(t)$
$F(t)$		$\frac{\mathrm{d}F(t)}{\mathrm{d}t}$	$1 - F(t)$	$\frac{1}{1-F(t)}\frac{\mathrm{d}F(t)}{\mathrm{d}t}$
$f(t)$	$\int_0^t f(x)\mathrm{d}x$		$\int_t^\infty f(x)\mathrm{d}x$	$\frac{f(t)}{\int_t^\infty f(x)\mathrm{d}x}$
$R(t)$	$1 - R(t)$	$-\frac{\mathrm{d}R(t)}{\mathrm{d}t}$		$-\frac{1}{R(t)}\frac{\mathrm{d}R(t)}{\mathrm{d}t}$
$\lambda(t)$	$1-\exp\left(-\int_0^t \lambda(x)\,\mathrm{d}x\right)$	$\lambda(t)\exp\left(-\int_0^t \lambda(x)\,\mathrm{d}x\right)$	$\exp\left(-\int_0^t \lambda(x)\,\mathrm{d}x\right)$	
NOTE Similar relationships hold among functional measures of any random variable, for example time to first failure, up time, down time, time to restoration, corrective maintenance time, repair time.				

Table B.2 – Summary of characteristics for some continuous probability distributions of time to failure of continuously operating items

Distribution	Range	Probability density function $f(t)$	Reliability (survival) function $R(t)$	Failure rate $\lambda(t)$	Expected value MTTF	Variance
Exponential	$\lambda > 0$ $t \geq 0$	$\lambda \exp(-\lambda t)$	$\exp(-\lambda t)$	λ	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Weibull	$\alpha > 0, \beta > 0$ $t \geq 0$	$\beta\alpha(\alpha t)^{\beta-1}\exp\left(-(\alpha t)^{\beta}\right)$	$\exp\left(-(\alpha t)^{\beta}\right)$	$\beta\alpha(\alpha t)^{\beta-1}$	$\frac{\Gamma(1+\frac{1}{\beta})}{\alpha}$	$\frac{1}{\alpha^2}\left(\Gamma(1+\frac{2}{\beta})-\Gamma^2(1+\frac{1}{\beta})\right)$
Gamma	$\alpha > 0, \beta > 0$ $t \geq 0$	$\frac{\alpha(\alpha t)^{\beta-1}}{\Gamma(\beta)}\exp(-\alpha t)$	$\int_t^{\infty} f(u)\mathrm{d}u$	$\frac{f(t)}{R(t)}$	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha^2}$
Erlang	$\lambda > 0$ $k \in \{1, 2, \ldots\}$ $t \geq 0$	$\frac{\lambda(\lambda t)^{k-1}}{(k-1)!}\exp(-\lambda t)$	$\exp(-\lambda t)\sum_{i=0}^{k-1}\frac{(\lambda t)^i}{i!}$	$\frac{f(t)}{R(t)}$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$
Rayleigh	$k \in \{1, 2, \ldots\}$ $t \geq 0$	$kt\exp\left(-\frac{kt^2}{2}\right)$	$\exp\left(-\frac{kt^2}{2}\right)$	kt	$\sqrt{\frac{\pi}{2k}}$	$\frac{2}{k}\left(1-\frac{\pi}{4}\right)$
Lognormal	$-\infty < m < +\infty$ $\sigma > 0,\ t > 0$	$\frac{1}{t\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln t-m)^2}{2\sigma^2}\right)$	$\int_t^{\infty} f(u)\mathrm{d}u$	$\frac{f(t)}{R(t)}$	$\exp\left(m+\frac{\sigma^2}{2}\right)$	$\exp(2m+2\sigma^2)$ $-\exp(2m+\sigma^2)$
$\Gamma(x)$ is the complete gamma function defined as $\Gamma(x)=\int_0^{\infty} t^{x-1}e^{-t}\mathrm{d}t$, $x > 0$.						

Table B.3 – Summary of characteristics for some probability distributions of repair time

Distribution	Range	Probability density function $g(t)$	Maintainability function $M(t)$	Repair rate $\mu(t)$	Expected value MRT	Variance
Deterministic	$t \geq 0$	Dirac delta function $\delta(t\text{-}\theta)$ (Deterministic duration $\theta \geq 0$)	$M(t)=\begin{cases}0 & \text{for } t<\theta \\ 1 & \text{for } t \geq \theta\end{cases}$ (it is a Heaviside function)	NA	θ	0
Exponential	$\mu > 0$ $t \geq 0$	$\mu \exp(-\mu t)$	$1-\exp(-\mu t)$	μ	$\frac{1}{\mu}$	$\frac{1}{\mu^2}$
Lognormal	$-\infty < m < +\infty$ $\sigma > 0, \; t > 0$	$\frac{1}{t\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln t - m)^2}{2\sigma^2}\right)$	$\int_0^t g(u)\mathrm{d}u$	$\frac{g(t)}{1-M(t)}$	$\exp\left(m+\frac{\sigma^2}{2}\right)$	$\exp(2m+2\sigma^2)$ $-\exp(2m+\sigma^2)$

Annex C
(informative)

Comparison of some dependability measures for continuously operating items

A comparison of some dependability measures for continuously operating items is given in Table C.1.

Table C.1 – Comparison of some dependability measures of continuously operating items with constant failure rate λ and restoration rate μ_R

Measure	Non-repairable item ($\mu_R = 0$)	Repairable item with time to restoration equal to	
		zero ($\mu_R \to \infty$)	**non-zero** ($0 < \mu_R < \infty$)
Reliability function $R(t) = R(0, t)$	$\exp(-\lambda t)$	$\exp(-\lambda t)$	$\exp(-\lambda t)$
Reliability $R(t_1, t_2)$	$\exp(-\lambda t_2)$	$\exp(-\lambda \times (t_2 - t_1))$	$\left(\frac{\mu_R}{\lambda + \mu_R} + \frac{\lambda}{\lambda + \mu_R}\exp[-(\lambda + \mu_R)t_1]\right)\exp[-\lambda \times (t_2 - t_1)]$
Mean time to failure MTTF	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$
Mean operating time between failures MOTBF		$\frac{1}{\lambda}$	$\frac{1}{\lambda}$
Mean time between failures METBF	NA	$\frac{1}{\lambda}$	$\frac{1}{\lambda} + \frac{1}{\mu_R}$
Instantaneous failure intensity $z(t)$	$\lambda \exp(-\lambda t)$	λ	$\frac{\lambda \mu_R}{\lambda + \mu_R} + \frac{\lambda^2}{\lambda + \mu_R}\exp[-(\lambda + \mu_R)t]$
Asymptotic failure intensity $z(\infty)$	0	λ	$\frac{\lambda \mu_R}{\lambda + \mu_R}$
Mean failure intensity $\bar{z}(t_1, t_2)$	$\frac{\exp(-\lambda t_1) - \exp(-\lambda t_2)}{t_2 - t_1}$	λ	$\frac{\lambda \mu_R}{\lambda + \mu_R} + \frac{\lambda^2}{(\lambda + \mu_R)^2}\frac{\exp[-(\lambda + \mu_R)t_1] - \exp[-(\lambda + \mu_R)t_2]}{t_2 - t_1}$
Instantaneous availability $A(t)$	$\exp(-\lambda t)$	1	$\frac{\mu_R}{\lambda + \mu_R} + \frac{\lambda}{(\lambda + \mu_R)}\exp[-(\lambda + \mu_R)t]$
Mean availability $\bar{A}(t_1, t_2)$	$\frac{\exp(-\lambda t_1) - \exp(-\lambda t_2)}{\lambda \times (t_2 - t_1)}$	1	$\frac{\mu_R}{\lambda + \mu_R} + \frac{\lambda}{(\lambda + \mu_R)^2}\frac{\exp[-(\lambda + \mu_R)t_1] - \exp[-(\lambda + \mu_R)t_2]}{t_2 - t_1}$
Asymptotic availability A	0	1	$\frac{\mu_R}{\lambda + \mu_R}$

Measure	Non-repairable item ($\mu_R = 0$)	Repairable item with time to restoration equal to	
		zero ($\mu_R \to \infty$)	**non-zero** ($0 < \mu_R < \infty$)
Instantaneous unavailability $U(t)$	$1 - \exp(-\lambda t)$	0	$\frac{\lambda}{\lambda + \mu_R}(1 - \exp[-(\lambda + \mu_R)t])$
Mean unavailability $\overline{U}(t_1,t_2)$	$1 - \frac{\exp(-\lambda t_1) - \exp(-\lambda t_2)}{\lambda \times (t_2 - t_1)}$	0	$\frac{\lambda}{\lambda + \mu_R} - \frac{\lambda}{(\lambda + \mu_R)^2} \frac{\exp-(\lambda + \mu_R)t_1] - \exp[-(\lambda + \mu_R)t_2]}{t_2 - t_1}$
Asymptotic unavailability U	1	0	$\frac{\lambda}{\lambda + \mu_R}$

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