BS EN 61649:2008

BSI British Standards

Weibull analysis

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National foreword

This British Standard is the UK implementation of EN 61649:2008. It is identical to IEC 61649:2008. It supersedes BS IEC 61649:1997 which is withdrawn.

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(IEC 61649:2008)

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CENELEC

European Committee for Electrotechnical Standardization
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Foreword

The text of document 56/1269/FDIS, future edition 2 of IEC 61649, prepared by IEC TC 56, Dependability, was submitted to the IEC-CENELEC parallel vote and was approved by CENELEC as EN 61649 on 2008-10-01.

The following dates were fixed:

– latest date by which the EN has to be implemented at national level by publication of an identical national standard or by endorsement	(dop)	2009-07-01
– latest date by which the national standards conflicting with the EN have to be withdrawn	(dow)	2011-10-01

Annex ZA has been added by CENELEC.

Endorsement notice

The text of the International Standard IEC 61649:2008 was approved by CENELEC as a European Standard without any modification.

In the official version, for Bibliography, the following notes have to be added for the standards indicated:

IEC 60300-1	NOTE Harmonized as EN 60300-1:2003 (not modified).
IEC 60300-2	NOTE Harmonized as EN 60300-2:2004 (not modified).
IEC 60300-3-1	NOTE Harmonized as EN 60300-3-1:2004 (not modified).
IEC 60300-3-2	NOTE Harmonized as EN 60300-3-2:2005 (not modified).
IEC 60300-3-4	NOTE Harmonized as EN 60300-3-4:2008 (not modified).
IEC 61703	NOTE Harmonized as EN 61703:2002 (not modified).

Annex ZA
(normative)

Normative references to international publications with their corresponding European publications

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE When an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

Publication	Year	Title	EN/HD	Year
IEC 60050-191	1990	International Electrotechnical Vocabulary (IEV) - Chapter 191: Dependability and quality of service	-	-
IEC 60300-3-5	2001	Dependability management - Part 3-5: Application guide - Reliability test conditions and statistical test principles	-	-
IEC 61810-2	- [1]	Electromechanical elementary relays - Part 2: Reliability	EN 61810-2	2005 [2]
ISO 2854	1976	Statistical interpretation of data - Techniques of estimation and tests relating to means and variances	-	-
ISO 3534-1	2006	Statistics - Vocabulary and symbols - Part 1: General statistical terms and terms used in probability	-	-

[1] Undated reference.

[2] Valid edition at date of issue.

CONTENTS

INTRODUCTION

The Weibull distribution is used to model data regardless of whether the failure rate is increasing, decreasing or constant. The Weibull distribution is flexible and adaptable to a wide range of data. The time to failure, cycles to failure, mileage to failure, mechanical stress or similar continuous parameters need to be recorded for all items. A life distribution can be modelled even if not all the items have failed.

Guidance is given on how to perform an analysis using a spreadsheet program. Guidance is also given on how to analyse different failure modes separately and identify a possible weak population. Using the three-parameter Weibull distribution can give information on time to first failure or minimum endurance in the sample.

WEIBULL ANALYSIS

1 Scope

This International Standard provides methods for analysing data from a Weibull distribution using continuous parameters such as time to failure, cycles to failure, mechanical stress, etc.

This standard is applicable whenever data on strength parameters, e.g. times to failure, cycles, stress, etc. are available for a random sample of items operating under test conditions or in-service, for the purpose of estimating measures of reliability performance of the population from which these items were drawn.

This standard is applicable when the data being analysed are independently, identically distributed. This should either be tested or assumed to be true (see IEC 60300-3-5).

In this standard, numerical methods and graphical methods are described to plot data, to make a goodness-of-fit test, to estimate the parameters of the two- or three-parameter Weibull distribution and to plot confidence limits. Guidance is given on how to interpret the plot in terms of risk as a function of time, failure modes and possible weak population and time to first failure or minimum endurance.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary – Part 191: Dependability and quality of service*

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 61810-2, *Electromechanical elementary relays – Part 2: Reliability*

ISO 2854:1976, *Statistical interpretation of data – Techniques of estimations and tests relating to means and variances*

ISO 3534-1:2006, *Statistics – Vocabulary and symbols – Part 1: General statistical terms and terms in probability*

3 Terms, definitions, abbreviations and symbols

For the purposes of this document, the definitions, abbreviations and symbols given in IEC 60050-191 and ISO 3534-1 apply, together with the following.

3.1 Terms and definitions

3.1.1
censoring
terminating a test after either a given duration or a given number of failures

NOTE A test terminated when there are still unfailed items may be called a “censored test", and test time data from such tests may be referred to as “censored data”.

3.1.2
suspended item
item upon which testing has been curtailed without relevant failure

NOTE 1 The item may not have failed, or it may have failed in a mode other than that under investigation.

NOTE 2 An “early suspension” is one that was suspended before the first failure. A “late suspension” is suspended after the last failure.

3.1.3
life test
test conducted to estimate or verify the durability of a product

NOTE The end of the useful life will often be defined as the time when a certain percentage of the items have failed for non-repairable items and as the time when the failure intensity has increased to a specified level for repairable items.

3.1.4
non-repairable item
item that cannot, under given conditions, after a failure, be returned to a state in which it can perform as required

NOTE The given conditions may be technical, economic, ecological and/or others.

3.1.5
operating time
time interval for which the item is in an operating state

NOTE ”Operating time” is generic, and should be expressed in units appropriate to the item concerned, e.g. calendar time, operating cycles, distance run, etc. and the units should always be clearly stated.

3.1.6
relevant failure
failure that should be included in interpreting test or operational results or in calculating the value of a reliability performance measure

NOTE The criteria for inclusion should be stated.

3.1.7
reliability test
experiment carried out in order to measure, quantify or classify a reliability measure or property of an item

NOTE 1 Reliability testing is different from environmental testing where the aim is to prove that the items under test can survive extreme conditions of storage, transportation and use.

NOTE 2 Reliability tests may include environmental testing.

3.1.8
repairable item
item that can, under given conditions, after a failure, be returned to a state in which it can perform as required

NOTE The given conditions may be technical, economic, ecological and/or others.

3.1.9
time to failure
operating time accumulated from the first use, or from restoration, until failure

NOTE In applications where the time in storage or on standby is significantly greater than “operating time”, the time to failure may be based on the time in the specified service.

3.1.10
time between failures
time duration between consecutive failures

NOTE 1 The time between failures includes the up time and the down time.

NOTE 2 In applications where the time in storage or on standby is significantly greater than operating time, the time to failure may be based on the time in the specified service.

3.1.11
B life
L percentiles
age at which a given percentage of items have failed

NOTE "B_{10}" life is the age at which 10 % of items (e.g. bearings) have failed. Sometimes it is denoted by the L (life) value. B lives may be read directly from the Weibull plot or determined more accurately from the Weibull equation. The age at which 50 % of the items fail, the B_{50} life, is the median time to failure.

3.2 Abbreviations

ASIC	application specific integrated circuit
BGA	ball grid array
CDF	cumulative distribution function
PDF	probability density function
MLE	maximum likelihood estimation
MRR	median rank regression
MTTF	mean time to failure

3.3 Symbols

t	time – variable
η	Weibull characteristic life or scale parameter
β	Weibull shape parameter
t_0	starting point or origin of the distribution, failure free time
r^2	coefficient of determination
f(t)	probability density function
F(t)	cumulative distribution function
h(t)	hazard function
λ(t)	instantaneous failure rate
H(t)	cumulative hazard function
F_1	number of failures with failure mode 1
F_2	number of failures with failure mode 2
F_3	number of failures with failure mode 3

4 Application of the techniques

Table 1 shows the circumstances in which particular aspects of this standard are applicable. It shows the three main methods for estimating parameters from the Weibull distribution, namely graphical, computational and WeiBayes, and indicates the type of data requirements for each of these three methods.

Table 1 – Guidance for using IEC 61649

Method/ Kinds of data	Graphical methods	Computational methods	WeiBayes
Interval censored	√	NC	√
Multiple censored	√	NC	√
Singly censored	√	√	√
Zero failures	NC	NC	√
Small sample (≤20)	√	NC	√
Large sample	√	√	NC
Curved data	√	NC	NC
Complete data	√	√	√
NOTE NC means not covered in this standard.			

5 The Weibull distribution

5.1 The two-parameter Weibull distribution

The two-parameter Weibull distribution is by far the most widely used distribution for life data analysis. The Weibull probability density function (PDF) is shown in Equation (1):

$$\mathrm{f}(t) = \beta \cdot \frac{t^{\beta - 1}}{\eta^{\beta}} \cdot e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{1}$$

where

t is the time, expressed as a variable;

η is the characteristic life or scale parameter;

β is the shape parameter.

The Weibull cumulative distribution function (CDF) has an explicit equation as shown in Equation (2):

$$\mathrm{F}(t) = 1 - \mathrm{e}^{-(t/\eta)^{\beta}} \tag{2}$$

The two parameters are η, the characteristic life, and β, the shape parameter. The shape parameter indicates the rate of change of the instantaneous failure rate with time. Examples include: infant mortality, random or wear-out. It determines which member of the Weibull family of distributions is most appropriate. Different members have widely different shaped PDFs (see Figure 1). The Weibull distribution fits a broad range of life data compared with other distributions. The variable t is generic and can have various measures such as time, distance, number of cycles or mechanical stress applications.

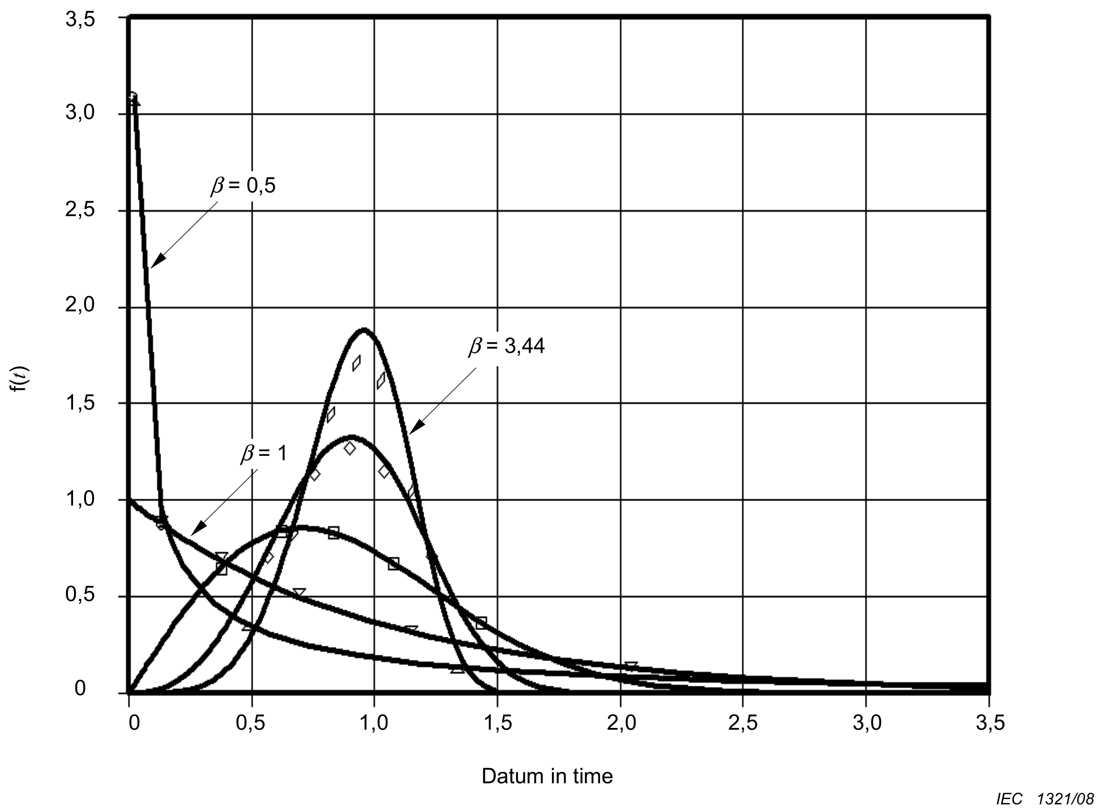


Figure 1 – The PDF shapes of the Weibull family for η = 1,0

From Figure 1, the PDF shape for β = 3,44 (indicated) looks like the normal distribution: it is a fair approximation, except for the tails of the distribution.

The instantaneous failure rate λ(t) (or h(t), the hazard function) of the two-parameter Weibull distribution is shown in Equation (3):

$$\lambda(t) = \mathrm{h}(t) = \beta \cdot \frac{t^{\beta-1}}{\eta^{\beta}} \tag{3}$$

Three ranges of values of the shape parameter, β, are salient:

- for β = 1,0 the Weibull distribution is identical to the exponential distribution and the instantaneous failure rate, λ(t), then becomes a constant equal to the reciprocal of the scale parameter, η;
- β > 1,0 is the case of increasing instantaneous failure rate; and
- β < 1,0, is the case of decreasing instantaneous failure rate.

Characteristic life, η, is the time at which 63,2 % of the items are expected to fail. This is true for all Weibull distributions, regardless of the shape parameter, β. If there is replacement of items, then 63,2 % of the times to failure are expected to be lower or equal to the characteristic life, η. Further discussion of the issues concerning repair and non-repairable items can be found in IEC 60300-3-5. The 63,2 % comes from setting $t = \eta$ in Equation (2) which results in Equation (4):

$$\mathrm{F}(\eta) = 1 - \mathrm{e}^{-(\eta/\eta)^{\beta}} = 1 - \mathrm{e}^{-(1)^{\beta}} = 1 - (1/\mathrm{e}) = 0{,}632 \tag{4}$$

5.2 The three-parameter Weibull distribution

Equation (5) shows the CDF of the three-parameter Weibull distribution:

$$F(t) = 1 - e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}} \tag{5}$$

The parameter t_0 is called the failure-free time, location parameter or minimum life.

The effect of location parameter is typically not understood well until a poor fit is observed with a 2-parameter Weibull plot. When a lack of fit is observed, engineers attempt to use other distributions that may provide them with a better fit. However, the lack of fit can be reconciled when the data is plotted with a 3-parameter Weibull distribution (see 8.5). Using the location parameter, it becomes evident that the product failures are offset by a fixed period of time, called the threshold. The effect of location parameter is normally observed when a product sees “shelf-life” after which the first failure occurs. A good indicator of the effect of a location parameter is the convex shape of a plot.

6 Data considerations

6.1 Data types

Life data are related to items that "age" to failure. Weibull failure data are usually life data but may also describe material data where the “aging” may be stress, force or temperature. "Age" may be operating time, starts and stops, landings, takeoffs, low-cycle fatigue cycles, mileage, shelf or storage time, cycles or time at high stress or high temperature, or many other continuous parameters. In this standard the “age” parameter will be called time. When required, “time” can be substituted by any of the “age” parameters listed above.

6.2 Time to first failure

The Weibull “time” variable is usually considered to be a measure of life consumption. The following interpretations can be used:

– time to first failure of a repairable item;
– time to failure of an non-repairable item;
– time from new to each failure of a repairable system if a non-repairable item in the system fails more than once during the period of observation. It has to be assumed that the repair (change of the item) does not introduce a new failure, so that the system after the repair can, with an approximation, be regarded as having the same reliability as immediately before the failure (commonly referred as the “bad as old” assumption);
– time to first failure of a non-repairable item, following scheduled maintenance, with the assumption that the failure is related to the previous maintenance.

6.3 Material characteristics and the Weibull distribution

Material characteristics such as creep, stress rupture or breakage and fatigue are often plotted on Weibull probability paper. Here the horizontal scale may be stress, cycles, load, number of load repetitions or temperature.

6.4 Sample size

Uncertainty with regard to the Weibull parameter estimation is related to the sample size and the number of relevant failures. Weibull parameters can be estimated using as few as two failures; however, the uncertainty of such an estimate would be excessive and could not confirm the applicability of the Weibull model. Whatever the sample size, confidence limits should be calculated and plotted in order to assess the uncertainty of the estimations.

As with all statistical analysis, the more data that is available, the better the estimation but if the data set is limited, then refer to the advice given in 11.3.

6.5 Censored and suspended data

When analysing life data it is necessary to include data on those items in the sample that have not failed, or have not failed by a failure mode analysis. This data is referred to as censored or suspended data (see IEC 60300-3-5). When the times to failure of all items are observed, the data are said to be complete.

An item on test that has not failed by the failure mode in question is a suspension or censored item. It may have failed by a different failure mode or not failed at all. An "early suspension" is one that was suspended before the first failure time. A "late suspension" is suspended after the last failure. Suspensions between failures are called random or progressive suspensions.

If items remain unfailed, then the corresponding data are said to be censored. If a test is terminated at a specified time, T, before all items have failed, then the data are said to be time censored. If a test is terminated after a specified number of failures have occurred, then the data are said to be failure censored.

Further discussion of censoring is covered in IEC 60300-3-5.

7 Graphical methods and goodness-of-fit

7.1 Overview

Graphical analysis consists of plotting the data on Weibull probability paper, fitting a line through the data, interpreting the plot and estimating the parameters using special probability paper derived by transforming the Weibull equation into a linear form. This is illustrated in Annex I.

Data is plotted after first organizing it from earliest to latest, a process called ranking. The time to failure data are plotted as the X coordinate on the Weibull probability paper.

The Y coordinate is the median rank as specified in 7.2.1. For sample sizes above 30 the median rank is, in practice, the same as the per cent of failures. If the plotted data follow a linear trend, a regression line may be drawn.

The parameters may then be read off the plot. The characteristic life, η, is the time to 63,2 % of the items failing, called the “B63,2 Life”. The shape parameter, β, is estimated as the slope on Weibull paper.

Median rank regression (MRR) is a method for estimating the parameters of the distribution using linear regression techniques with the variables being the median rank and lifetime or stress ,etc.

Another graphical method that is used for estimating parameters of a Weibull distribution is called hazard plotting. This is described in 7.3.

7.2 How to make the probability plot

In order to make a probability plot, a sequence of steps needs to be carried out. These steps are described in detail below.

7.2.1 Ranking

To make the Weibull plot, rank the data from the lowest to the highest times to failure. This ranking will set up the plotting positions for the time, *t*, axis and the ordinate, F(*t*), in percentage values. These will provide information for the construction of the Weibull line shown in Equation (6).

Median ranks are given in Annex C. Enter as an example the tables for 50 % median rank, for a sample size of five, and find the median ranks shown in Table 2 for five failure times shown in the middle column. The median rank plotting positions in Annex C are used with all types of probability paper, i.e. Weibull, log-normal, normal, and extreme value.

NOTE 1 If two data points have the same time, they are plotted at different median rank values.

Table 2 – Ranked flare failure rivet data

Order number I	Failure time *t* min (X)	Median rank % (Y)
1	30	12,94
2	49	31,38
3	82	50,00
4	90	68,62
5	96	87,06

The median estimate is preferred to the mean or average value for non-symmetrical distributions. Most life data distributions are skewed and, therefore, the median plays an important role.

If a table of median ranks and a means to calculate median ranks using the Beta distribution is not available, then Benard’s approximation, Equation (6), may be used:

$$F_i = \frac{(i-0,3)}{(N+0,4)}\% \qquad (6)$$

where N is the sample size and i is the ranked position of the data item of interest.

NOTE 2 This equation is mostly used for $N \leq 30$; for $N > 30$ the correction of the cumulative frequency can be neglected: $F_i = (i/N) \times 100$ %.

7.2.2 The Weibull probability plot

After transforming the data, the plot can be constructed using three different methods:

- Weibull probability paper – Annex F shows Weibull probability paper;
- a computer spreadsheet program – Annex E gives a spreadsheet example;
- commercial off-the-shelf software.

7.2.3 Dealing with suspensions or censored data

Non-failed items or items that fail by a different failure mode are "censored" or "suspended" items, respectively. These data cannot be ignored. The times on suspended items have to be included in the analysis.

The formula below gives the adjusted ranks without the need for calculating rank increments. It is used for every failure and requires an additional column for reverse ranks. The procedure is to rank the data with the suspensions and to use Equation (7) to determine the ranks, adjusted for the presence of the suspensions.

$$\text{Adjusted rank} = \frac{(\text{Reverse rank}) \text{ x } (\text{Previous adjusted rank}) + (N + 1)}{(\text{Reverse rank}) + 1} \quad (7)$$

The rank order numbers are adjusted for the effect of the three suspended items in Table 3.

Table 3 – Adjusted ranks for suspended or censored data

Rank	Time	Status	Reverse rank	Adjusted rank	Median rank %
1	10	Suspension	8	Suspended…	
2	30	Failure	7	[7 × 0 +(8+1)]/ (7+1) = 1,125	9,8
3	45	Suspension	6	Suspended…	
4	49	Failure	5	[5 × 1,125 +(8+1)]/ (5+1) = 2,438	25,5
5	82	Failure	4	[4 × 2,438 +(8+1)]/ (4+1) = 3,750	41,1
6	90	Failure	3	[3 × 3,750 +(8+1)]/ (3+1) = 5,063	56,7
7	96	Failure	2	[2 × 5,063 +(8+1)]/ (2+1) = 6,375	72,3
8	100	Suspension	1	Suspended…	

In this example, the adjusted ranks use Benard's approximation to calculate the median ranks as it is easier than interpolating in the table. The results in Table 3 are plotted in Figure 2.

NOTE If two items fail at the same age, they are assigned sequential rank order numbers. In case of later suspension, the procedure is to be repeated for the rest of the failures.

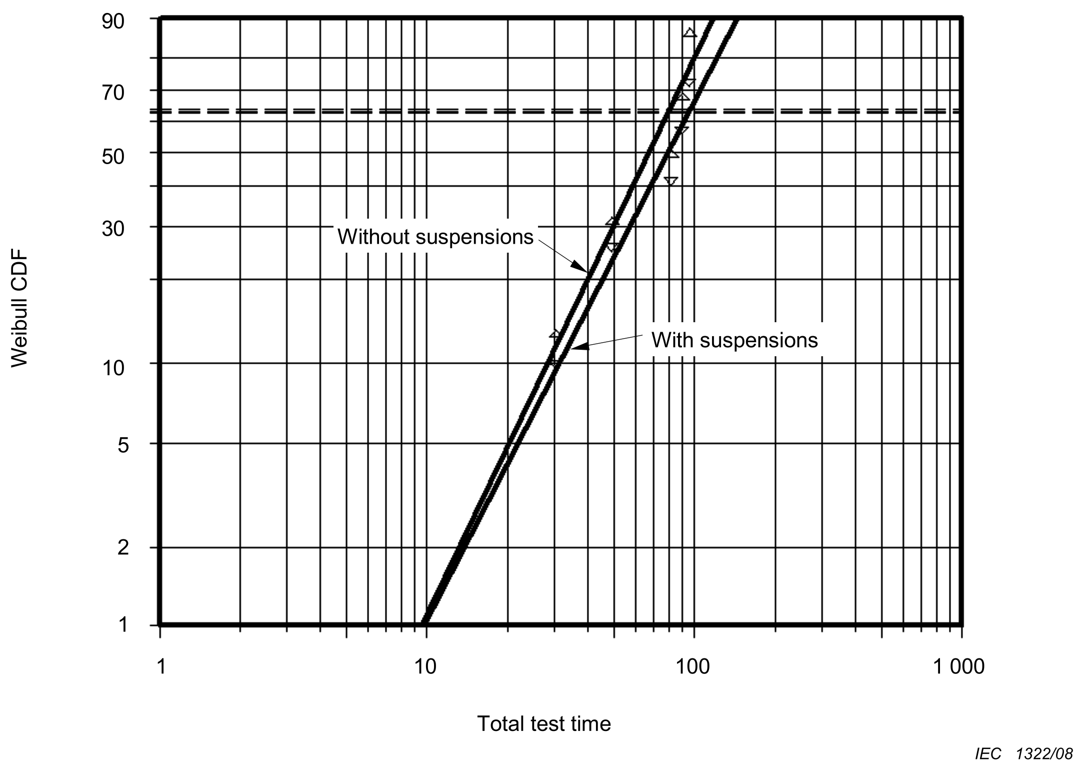


Figure 2 – Total test time (in minutes)

Benard's approximation for the median rank is sufficiently accurate when using suspension for plotting Weibull distributions and estimating the parameters. Here "i" is the adjusted rank and "N" is the sum of failures and suspensions. The median ranks are converted to percentages for plotting on Weibull paper. For example, for the first failure in Table 3 with an adjusted rank of 1,125:

$$Median\,Rank\,(\%) = \frac{(1{,}125 - 0{,}3)}{(8 + 0{,}4)} \times 100 = 9{,}82\% \quad (8)$$

Figure 2 shows the correct Weibull plot as it includes the suspensions.

The following are the steps to plot data sets with suspensions:

a) rank the times, both failures and suspensions, from earliest to latest;
b) calculate the adjusted ranks for the failures (suspensions are not plotted);
c) use Benard's approximation to calculate the median ranks;
d) plot the failure times (x) versus the median ranks (y) on Weibull paper;
e) estimate η by reading the B63,2 life from the plot;
f) estimate β with a ruler or use special beta scales usually given on the Weibull probability paper;
g) interpret the plot.

7.2.4 Probability plotting

Plotting the data on Weibull paper by hand or on a computer may be sufficient for checking the goodness-of-fit. Subjectively fitting a straight line by eye can give an indication of goodness-of-fit.

7.2.5 Checking the fit

If the data cluster around a straight line on a probability plot, it is evidence that the data is represented by the subject distribution. However, small samples make it difficult to gauge the goodness-of-fit. There are statistical measures of goodness-of-fit such as Chi-squared, Kolmogorov-Smirnoff and Nancy Mann's tests. This standard uses the correlation coefficient squared called the coefficient of determination.

This can be calculated using Equation (9):

$$r^2 = \frac{\left(\sum_{i=1}^{N} x_i y_i - \frac{\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N}\right)^2}{\left(\sum_{i=1}^{N} x_i^2 - N\left(\overline{x}\right)^2\right)\left(\sum_{i=1}^{N} y_i^2 - N\left(\overline{y}\right)^2\right)} \quad (9)$$

where x_i and y_i are the median rank and the failure time, respectively, $\overline{x}$ and $\overline{y}$ are averages of x_i and y_i and N is the sample size.

r^2 is the proportion of variation in the data that can be explained by the Weibull hypothesis. The closer this is to 1, the better the data are fitted to a Weibull distribution; the closer to 0 indicates a poor fit. The correlation coefficient, "r," is intended to measure the strength of a linear relationship between two variables. "r" is a number between -1 and +1, depending on the slope. Alternatively, if the Weibull plot is constructed using a spreadsheet, then when using linear regression techniques the correlation coefficient is often given as part of the output when data are fitted with a straight line (usually a selective option). Similarly, commercial software packages provide the coefficient of determination.

This should be used with care and only if visual inspection concurs with observation.

7.3 Hazard plotting

Weibull probability plotting techniques first estimate the cumulative proportion failed, F*(t)*, using median ranks, and then plot times to failure against respective estimated cumulative probabilities on the Weibull probability paper.

The hazard plotting technique begins with the estimation of the instantaneous failure rate or the hazard function,

$$\lambda(t) = h(t) = \frac{f(t)}{[1-F(t)]} \quad , \tag{10}$$

using an estimate of the cumulative hazard function.

The cumulative hazard function,

$$H(t) = \int_0^t h(t)dt = -\ln\left[1-F(t)\right] \quad , \tag{11}$$

is estimated by the cumulative sum of the estimated hazard functions.

For the Weibull distribution,

$$H(t) = \left(\frac{t}{\eta}\right)^{\beta} \quad , \tag{12}$$

taking natural logarithms of both sides, yields the following linear relationship for $\ln H(t)$ against ln$t$:

$$\ln(H(t)) = \beta\ln(t) - \beta\ln(\eta) \tag{13}$$

The Weibull hazard paper is therefore ln-ln paper. The nominal slope of the fitted line to the data is β, and $t = \eta$ when $H(t) = 1$.

NOTE Either natural logs or base 10 logs can be used for Weibull hazard paper.

Though the Weibull hazard paper is available for some countries, this technique can be used with the usual Weibull probability paper applying the transformation:

$$F(t) = 1 - \mathrm{e}^{-H(t)} \tag{14}$$

This can be very easily implemented using a spreadsheet program.

The hazard plotting procedure is as follows:

a) sort the times, both failures and suspensions simultaneously, from earliest to latest;

b) for each failure, calculate the instantaneous failure rate, given by 1/(number of items remaining after the previous failure or censoring);

c) for each failure, calculate the cumulative sum of instantaneous failure rates, as an estimate of the hazard function;

d) plot the estimated cumulative hazards against the time of failure on either log-log paper or ln-ln paper;

e) fit a straight line to the plots;

f) estimate the parameters.

Annex E gives worked examples. IEC 61810-2 also provides examples of using hazard plotting to estimate the parameters of a Weibull distribution.

8 Interpreting the Weibull probability plot

8.1 The bathtub curve

8.1.1 General

The often-used bathtub curve (see Figure 3) shows the relationship between the Weibull shape parameter, β, and the hazard function throughout the life of an item. Not all items, however, display all elements of the bathtub curve during their lifetimes.

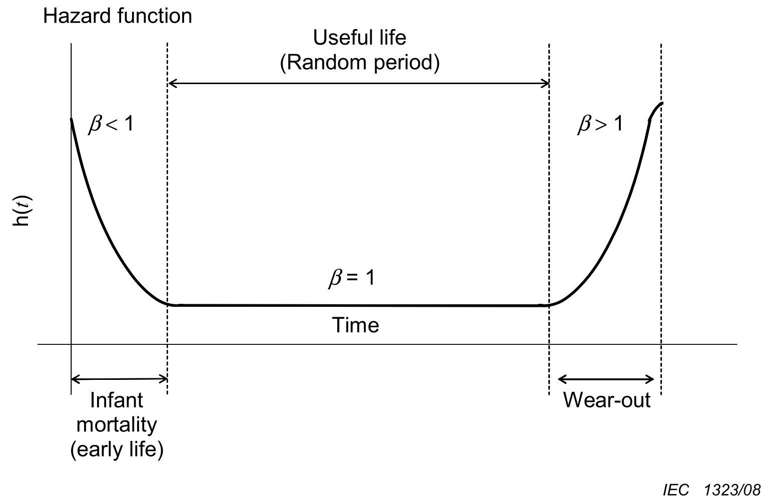


Figure 3 – Typical bathtub curve for an item

8.1.2 β <1 – Implies early failures

Both electronic and mechanical systems may initially have high failure rates. Manufacturers conduct production process control, production acceptance tests, "burn-in," or reliability stress screening (RSS), to prevent early failures before delivery to the customer. Therefore, shape parameters of less than one indicate the following:

– lack of adequate process control;

– inadequate burn-in or stress screening;

– production problems, mis-assembly, poor quality control;

– overhaul problems;

– mixture of populations;

– run-in or wear-in.

Many electronic components during their useful life show a decreasing instantaneous failure rate, thus featuring shape parameters less than 1. Preventive maintenance on such a component is not appropriate, as old parts are better than new.

8.1.3 β = 1 – Implies constant instantaneous failure rate

This is often called the random failure period as the failures occur randomly in time. These failure modes are considered independent of time. In this case, preventive maintenance would not improve the system.

Therefore, any of the following might be suspected:

– random maintenance errors, human errors;

– random overload;

– failures due to nature, foreign object damage, lightning strikes;

– mixtures of data from three or more failure modes (assuming they have different values of β) where no failure mechanism dominates the failure behaviour.

Here again, preventive maintenance is not appropriate. The Weibull distribution with β = 1 is identical to the exponential distribution. Of those that survive to time, t, a constant percentage fails in the next period of time. This is known as a constant instantaneous failure rate.

8.1.4 β > 1 – Implies wear-out

Some typical examples of these cases are as follows:

– wear;

– corrosion;

– crack propagation;

– fatigue;

– moisture absorption;

– diffusion;

– evaporation (weight loss);

– damage accumulation.

Design measures have to ensure that those phenomena do not significantly contribute to the probability of product failure during the expected operational life.

Three-parameter Weibull distribution estimates minimum time to first failure, which is highly advantageous in cases where the shape parameter is greater than 1 (see 5.2 or 8.5).

8.2 Unknown Weibull modes may be "masked"

The “masked" phenomenon might be encountered when there are two or more competing failure modes with high values of shape parameters and highly different scale parameters. This means that for a small sample size, most or all of the samples would fail in the failure mode with lower scale parameter. The other failure modes may not be identified until the first failure mode is eliminated. An example of two competing failure modes is shown in Figure 4.

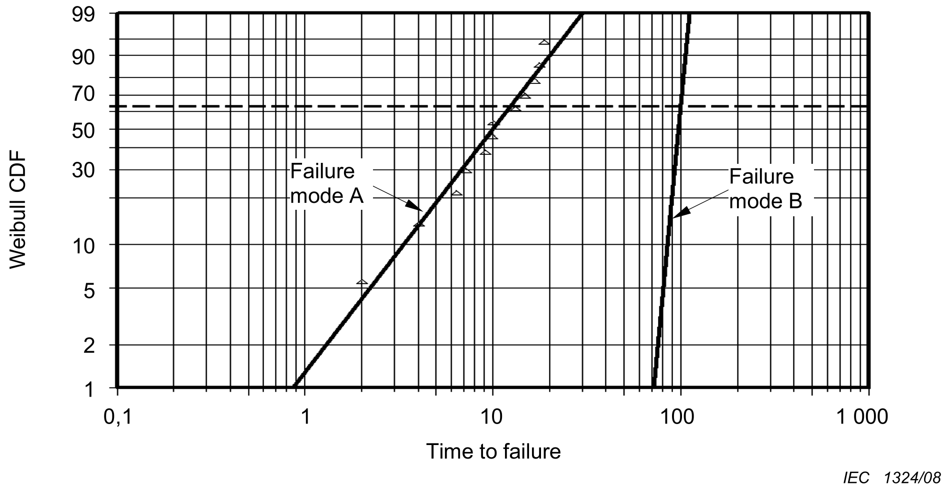


Figure 4 – Weibull failure modes may be "masked"

8.3 Small samples

Weibull analysis is possible from small samples. However, confidence limits are affected by sample size. Small samples will increase the uncertainty in estimating the life parameters.

Improvement in uncertainty with increasing sample size is illustrated in Figures 5 and 6. (The "90 % B life bounds" (confidence limits) contain the unknown "B life" with a frequency of 90 %. The interval is much smaller in Figure 6.)

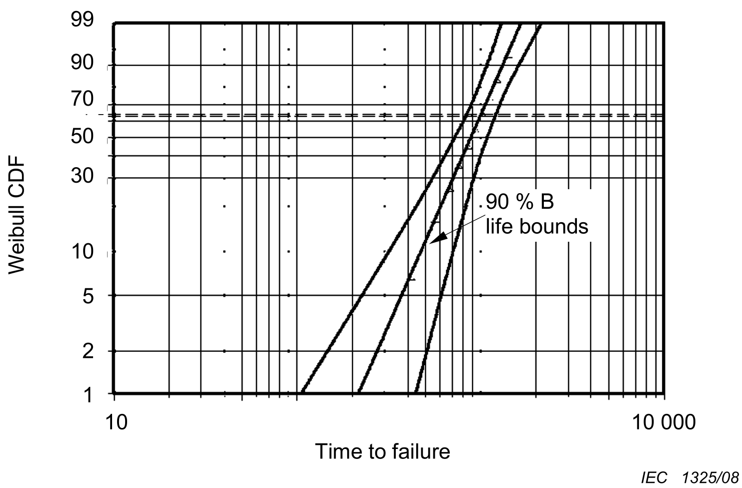


Figure 5 – Sample size: 10

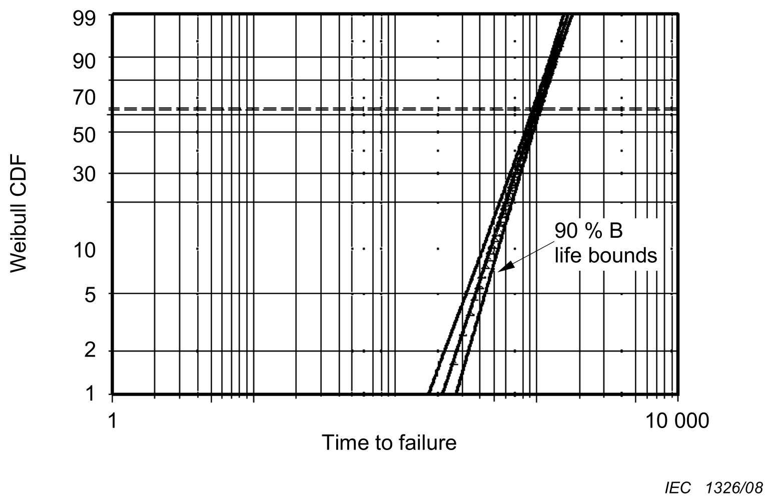


Figure 6 – Sample size: 100

The degree of uncertainty is largest in the tail of the Weibull distribution, as indicated by small values of F*(t)* when the fraction failing is small. Samples of greater than 20 failures and suspensions are required to differentiate the Weibull from other distributions.

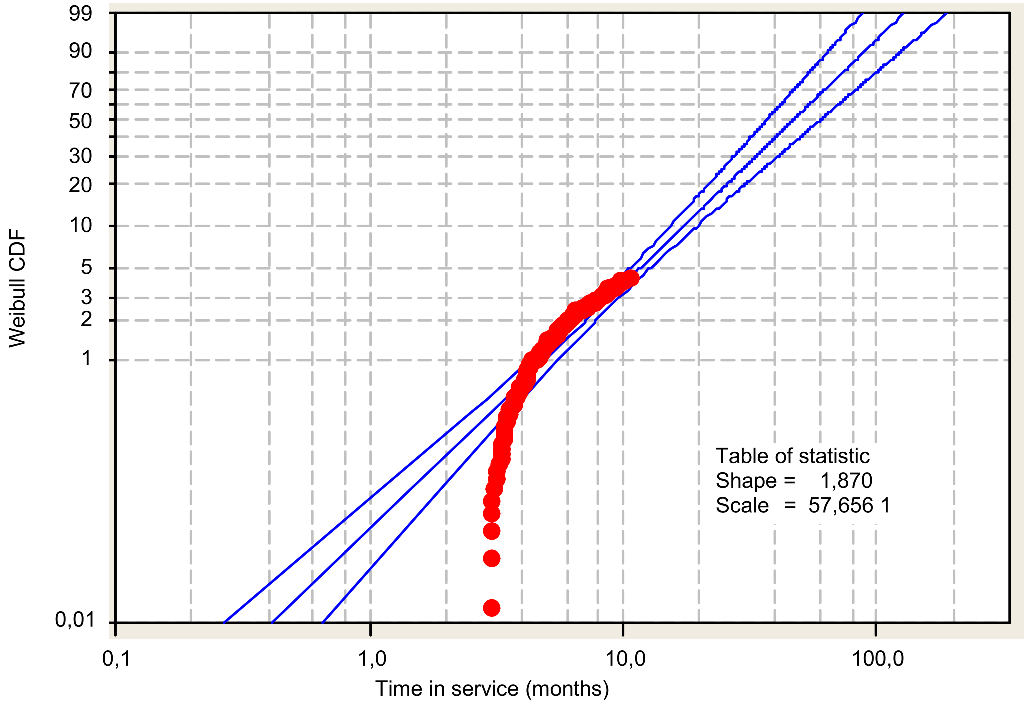
Decisions based upon results from a small sample should consider the uncertainty and, where possible, should be refined by collection and analysis of additional data.

8.4 Outliers

Sometimes, the first or last point in a data set is a wild point and not a member of the data set for some reason; such points are termed outliers. These points may be important to the life data analysis, and therefore require investigation of the engineering aspects of data recording, test records, instrumentation calibrations, etc. in order to identify the cause of extreme scatter of the point. Sometimes, the outliers may indicate a weak population or process flaws, and are therefore highly significant from the reliability assurance point of view.

8.5 Interpretation of non-linear plots

If the data on a Weibull plot appears curved, as illustrated in Figure 7, this indicates that the t_0 parameter may be non-zero. Before modifying the plot, it is necessary to check if the plot contains more than one failure mode. If so, refer to Annex G and investigate further whether those modes are competing with each other or are simple mixtures.



IEC 1327/08

Figure 7 – An example showing lack of fit with a two-parameter Weibull distribution

Minimum life does not mean at “zero time”, but rather that there exists a minimum life or a minimum endurance. “Zero age” is where none of the wear-out failure mechanisms of an item has started to operate whereas “zero time” is where an item has seen no operating time. For example, it may be physically impossible for the failure mode to produce failures instantaneously, or early in life. Figure 8 shows the same data as used in Figure 7 but with the origin shifted 2,99 months. This plot shows a linear fit to the data and is interpreted as a failure-free period (3 months), within which the probability of failure is zero.

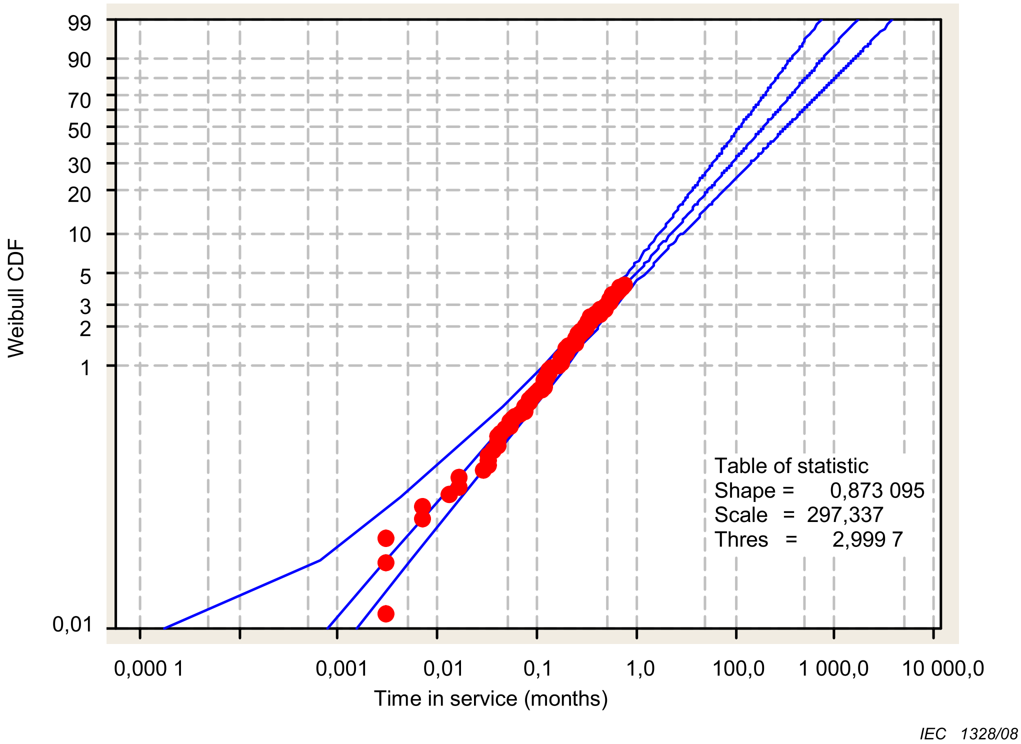


Figure 8 – The same data plotted with a three-parameter Weibull distribution shows a good fit with 3 months offset (location – 2,99 months)

The method consists of subtracting t_0 = 3 months from each data point in Figure 7 to obtain Figure 8. Note that the Weibull ordinate scale and the characteristic life are now in the t_0 domain. To convert back to real time, add t_0 back. This is an example of a three-parameter Weibull distribution with the third parameter, t_0, the failure-free period. A failure-free period should not be assumed without a technical justification. Annex H gives another example of the three-parameter Weibull distribution.

The three-parameter Weibull distribution will show a better fit than the two-parameter Weibull distribution fit simply because it is a more complex model. The following three criteria should always be met before using the three-parameter Weibull:

a) the Weibull plot shall show concave curvature;

b) there shall be a physical explanation of why failures cannot occur before t_0;

c) a larger sample size, at least 21 failures, shall be available. If there is prior knowledge from earlier Weibull distributions that the third parameter is appropriate, a smaller sample size, say eight to ten, may be acceptable.

Concave downward plots occur much more often than concave upward. Concave upward suggests a negative t_0, which may occur when items with wear-out failures were stressed before putting on test. There are several ways to estimate t_0. A curve may be plotted through the data and extrapolated down to the horizontal time scale. The intersection will be an approximate t_0. If the earliest portion of the data is missing, t_0 may compensate for the missing data, although this may not always be successful. For example, Figure 9 shows cable data grouped by vintage year and the aging scale in months.

NOTE t_0 will always be less than the first failure time.

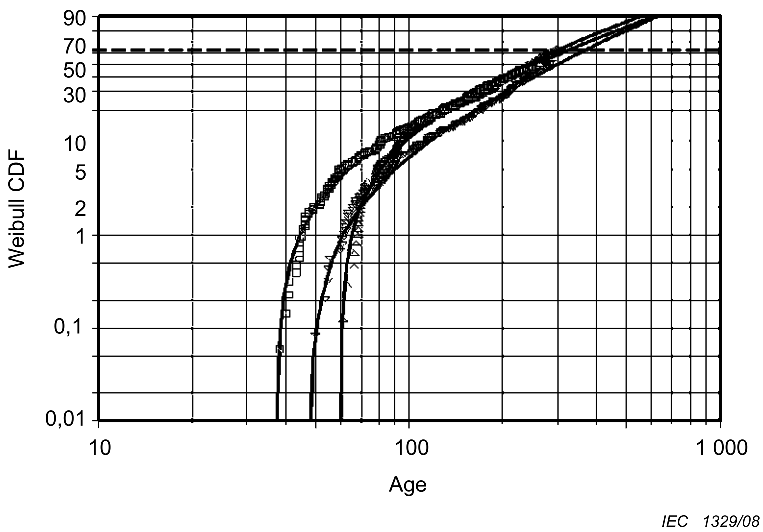


Figure 9 – Example of estimating t_0 by eye

In summary, concave downward plots indicate the origin needs to be shifted to the right, subtracting t_0 from each time to failure to get a straight line fit. Concave upward plots indicate the origin has to be shifted to the left and t_0 has to be added to each time to failure to get a straight line fit. The plot in "as recorded" time scale may be easier to understand.

8.5.1 Distributions other than the Weibull

There are other reasons for poor fit of a straight line, i.e. the data form a curve on Weibull paper. Another distribution may better describe the data. If this is true, the distribution that best describes the data should be used. For example, the log-normal distribution is not a member of the Weibull family of distributions but has application to life data analysis. Log-normal data plotted on Weibull paper are concave downward. The same data plotted on log-normal probability plot follow a straight line.

Data fitted to the three-parameter Weibull plot and the log-normal plot appear curved downward on a two-parameter Weibull plot. Both these distributions can model data with time to first failure.

8.5.2 Data inconsistencies and multimode failures

Based on the plotted Weibull data, an engineering hypothesis may be deduced. This hypothesis should then be confirmed by failure analysis of further investigation. Examples include:

a) failures are mostly low-time parts with high-time parts unaffected, suggesting a batch problem;
b) serial numbers of failed parts are close together, also suggesting a batch problem;
c) the data have a "dogleg" bend or cusps when plotted on Weibull paper, probably caused by a mixture of failure modes; and
d) the first or last point appears suspect as an outlier, indicating data problems or perhaps evidence of a different failure mode.

9 Computational methods and goodness-of-fit

9.1 Introduction

The maximum likelihood estimation (MLE) method is a computational method for large sample cases.

Among many computational methods to estimate the parameters of the Weibull distribution (see [9][1]), MLE has the advantage that it allows estimation of the parameters from the data sets with complicated censoring mechanisms and suspensions, when the number of items under a test is large. This clause describes MLE for the data sets without censoring. First the goodness-of-fit test is introduced to check the Weibull assumption. If the hypothesis is not rejected, then proceed to the MLE.

The methods in Clauses 7 and 8 deal with multiple and singly censored data; however, the methods in this clause deal with singly censored data only (not multiply censored data).

9.2 Assumptions and conditions

A sample of n non-repairable items, coming from the same population, is put on test at a given instant of time $t = 0$. The testing environment shall be the same for all items being subjected to the test, and failed items are not replaced once they fail. When the test is stopped at time T, there are r items that have failed (T can be equal to or greater than t_r). The time to failure of each failed item has to be known. There are r times to failure: t_1, t_2, ..., t_r so that $0 < t_i \le T, i = 1, 2, ..., r$.

NOTE 1 The statistical procedures of this standard assume access to some computing facility. Although most, if not all, formulas can be implemented in a small programmable calculator, it will be useful, from the user's standpoint, to have access to a programmable computer with a printer and some mass storage medium.

NOTE 2 Test time for each item has to be made from time zero.

9.3 Limitations and accuracy

These procedures are valid only if there are at least 10 relevant failures. The confidence intervals are approximate. Multiple censoring is not considered in this clause.

MLE is valid for larger sample sizes. The confidence intervals are approximately valid for large sample data sets. Though MLE can be applied to a wide variety of censoring mechanisms and suspensions, only the cases with single censoring are considered here.

9.4 Input and output data

The data to be analysed consist of times to failure of non-repairable items, which are put on test. These times to failure have to be known exactly, as opposed to knowledge of intervals of time. It is not necessary to have the times to failure of all the items tested, since the test can be stopped before all items have failed. All items shall be in operational condition at the start of the test, and the test shall be stopped for all operational items at the same time.

Input:

– number of items on test n;

– times to failure of each failed item, listed in ascending order: t_1, t_2, ..., t_r;

– significance level, γ, or confidence level $(1 - \gamma)$; to be specified.

Output:

– accept/reject goodness-of-fit;

– point estimates and confidence intervals of scale and shape parameters, η and β;

– point estimate of the mean time to failure;

– lower confidence limit for the expected time at which 10 % of the population will fail, B10;

– lower confidence limit for the reliability function R(t).

[1] Figures in square brackets refer to the Bibliography.

9.5 Goodness-of-fit test

Step 1 – Sort the r times to failure in ascending order and compute the natural logarithms of these times $\ln(t_1) = x_1$, $\ln(t_2) = x_2$, ..., $\ln(t_r) = x_r$.

NOTE 1 $x_1 \leq x_2 \leq, \ldots, \leq x_r$.

Step 2 – Compute the following ℓ_i quantities in Equation (15), for i = 1 to (r – 1):

$$\ell_i = \frac{x_i + 1 - x_i}{\ln\left[\ln\left(\frac{4(n-i-1)+3}{4n+1}\right)\Big/\ln\left(\frac{4(n-i)+3}{4n+1}\right)\right]} \tag{15}$$

Step 3 – Compute the quantity H using Equation (16) and the quantities obtained in step 2:

$$H = \frac{\displaystyle\sum_{i=\lfloor r/2\rfloor+1}^{r-1} \frac{\ell_i}{\lfloor (r-1)/2\rfloor}}{\displaystyle\sum_{i=1}^{\lfloor r/2\rfloor} \frac{\ell_i}{\lfloor r/2\rfloor}} \tag{16}$$

where the symbol $\lfloor x\rfloor$ is used to denote the largest integer less than or equal to x.

Step 4 – Reject the hypothesis that the data come from a Weibull distribution at the γ100 % significance level if $H \geq F_\gamma(2\lfloor (r-1)/2\rfloor, 2\lfloor r/2\rfloor)$ and do not proceed with the analysis.

Otherwise, no evidence has been detected to reject the Weibull nature of the times to failure and the analysis can proceed.

The values of the fractiles of the F distribution function can be found, for example, in Table IV of ISO 2854.

NOTE 2 It is recommended that, in case of rejection, the plotted data be examined for a possible mixture of populations, anomalous failure times or other artefacts. These situations are analysed by techniques beyond the scope of this standard.

9.6 MLE – point estimates of the distribution parameters β and η

The MLE of the two parameters of the Weibull distribution is obtained by numerically solving the equations below. The value of β that satisfies the first equation is the MLE of β. This value is used in the second equation to derive the MLE of η.

NOTE Any computer routine to solve equations can be used to obtain β from Equation (17), as the convergence to a single value is usually very fast.

Step 1 – Find the estimate of $\hat{\beta}$ that satisfies Equation (17):

$$\left[\frac{\displaystyle\sum_{i=1}^{r} t_i^{\beta} \ln(t_i) + (n-r)T^{\beta}\ln(T)}{\displaystyle\sum_{i=1}^{r} t_i^{\beta} + (n-r)T^{\beta}} - \frac{1}{\beta}\right] - \frac{1}{r}\sum_{i=1}^{r}\ln(t_i) = 0 \tag{17}$$

Step 2 – Compute $\hat{\eta}$ using Equation (18) and the value of $\hat{\beta}$, obtained in step 1, from:

$$\hat{\eta}=\left\{\frac{1}{r}\left[\sum_{i=1}^{r} t_i^{\hat{\beta}}+(n-r)T^{\hat{\beta}}\right]\right\}^{\frac{1}{\hat{\beta}}} \tag{18}$$

9.7 Point estimate of the mean time to failure

The point estimate of the mean time to failure, $\hat{m}$, is calculated using Equation (19) as:

$$\hat{m}=\hat{\eta}\,\Gamma(1+\frac{1}{\hat{\beta}}) \tag{19}$$

where $\hat{\beta}$ and $\hat{\eta}$ are obtained from steps 1 and 2 of 9.6 and where $\Gamma(z)$ is the gamma function of z as defined in NOTE 2 to definition 2.56 of ISO 3534-1.

Table D.1 gives the value of $\Gamma(1+1/\beta)$ as a function of β. For β values not listed in this table, a linear interpolation is acceptable.

NOTE 1 For cases where the lower confidence limit of β is greater than or equal to 1 (wear-out case), the confidence interval for η can be used as a rough measure of the confidence interval for the mean time to failure, since in these cases the gamma function always lies between 0,88 and 1.

NOTE 2 Mean time to failure (MTTF) is the mean value of a number of times to failure. The reliability tools used for exponentially distributed data (constant instantaneous failure rate) can only be used for Weibull distributed data if β = 1. It should be noted that the time to failure distribution is normally not symmetrical.

9.8 Point estimate of the fractile (10 %) of the time to failure

Compute $\hat{B}_{10}$ using Equation (20), the point estimate of B_{10}, the time by which 10 % of the population will have failed:

$$\hat{B}_{10}=\hat{\eta}\left[\text{In}\left(\frac{1}{0,9}\right)\right]^{1/\hat{\beta}} \tag{20}$$

9.9 Point estimate of the reliability at time t ($t \le T$)

The point estimate of the reliability at time t is given by Equation (21):

$$\hat{R}(t)=\mathrm{e}^{-(t/\hat{\eta})^{\hat{\beta}}} \tag{21}$$

9.10 Software programs

There are many statistical and reliability software packages that give estimates of the parameters of Weibull distributions using both graphical methods and/or MLE, not only for single and multiple censored data, but also for more general incomplete data due to complicated censoring mechanisms and suspensions.

10 Confidence intervals

10.1 Interval estimation of β

Step 1 – Compute the constants C, β_1 and β_2 using the ratio $q = r/n$ and Equations (22), (23) and (24):

$$C = 2{,}14628 - 1{,}361119\,q \tag{22}$$

$$\beta_1 = \chi^2_{\gamma/2}\left[(r-1)\,C\right] \quad (23)$$

$$\beta_2 = \chi^2_{1-\gamma/2}\left[(r-1)\,C\right] \quad (24)$$

where $\chi^2_p(\nu)$ is the p fractile of the χ^2 distribution with ν degrees of freedom.

Since the number of degrees of freedom, $(r - 1)C$, will not be an integer, the χ^2 fractiles should be calculated either using a computer program, or through interpolation in Table III of ISO 2854 or Table D.1 of IEC 60605-4:2001.

Step 2 – Compute the multiplying factors w_1 and w_2 using Equation (25) and (26):

$$w_1 = \left[\frac{\beta_1}{r\,C}\right]^{\frac{1}{1+q^2}} \quad (25)$$

$$w_2 = \left[\frac{\beta_2}{r\,C}\right]^{\frac{1}{1+q^2}} \quad (26)$$

Step 3 – Compute the $(1 - \gamma)100$ % confidence interval for β using Equation (27):

$$(w_1\hat{\beta}, w_2\hat{\beta}) \quad (27)$$

NOTE The confidence intervals for β can be used for comparisons. Since a value of β >1 constitutes evidence of wear and a value of β <1 indicates infant mortality, the confidence interval for β can be used to test these assumptions. Conversely, if the confidence interval for β contains the value β = 1, the items being tested may belong to a constant failure rate population. A formal test on constant failure rate is provided in [10].

10.2 Interval estimation of η

Step 1 – Compute the constants A_4, A_5 and A_6, using the ratio $q = r/n$, and Equations (28), (29) and (30):

$$A_4 = 0{,}49q - 0{,}134 + 0{,}622\, q^{-1} \quad (28)$$

$$A_5 = 0{,}2445\,(1{,}78 - q)\,(2{,}25 + q) \quad (29)$$

$$A_6 = 0{,}029 - 1{,}083\,\ln(1{,}325\, q) \quad (30)$$

Step 2 – Carry out step 2a if the test was stopped before all the items have failed, that is if $r < n$, or carry out step 2b if all the failure times are known, that is if $r = n$.

Step 2a ($r < n$) – Compute the constants A_3, d_1, d_2, A_1 and A_2: using Equations (31), (32), (33) and (34):

$$A_3 = -A_6\, x^2 \quad (31)$$

where $x = u_{(1-\gamma/2)}$ and u_p is the p fractile of the normal distribution given in Table D.2.

$$d_1 = \frac{A_3 + x\sqrt{x^2\left(A_6^2 - A_4 A_5\right) + rA_4}}{r - A_5 x^2} \quad (32)$$

$$d_2 = \frac{A_3 - x\sqrt{x^2\left(A_6^2 - A_4 A_5\right) + rA_4}}{r - A_5 x^2} \quad (33)$$

$$A_1 = \mathrm{e}^{\left(-d_1/\hat{\beta}\right)} \;;\; A_2 = \mathrm{e}^{\left(-d_2/\hat{\beta}\right)} \tag{34}$$

Step 2b ($r = n$) – Compute the quantities d_3, A_1 and A_2: using Equations (35), (36) and (37):

$$d_3 = t_{(1-\gamma/2)}(n-1) \tag{35}$$

where $t_p(r-1)$ is the p fractile of the Student t distribution with (r – 1) degrees of freedom and can be found in Table IIa of ISO 2854 (single-sided case).

$$A_1 = \mathrm{e}^{\left(\frac{-1{,}053 d_3}{\hat{\beta}\sqrt{n-1}}\right)} \tag{36}$$

$$A_2 = \mathrm{e}^{\left(\frac{1{,}053 d_3}{\hat{\beta}\sqrt{n-1}}\right)} \tag{37}$$

where $\hat{\beta}$ is obtained from step 1 of 9.6.

Step 3 – Compute the (1 – γ)100 % confidence interval for η using Equation (38):

$$(A_1\hat{\eta}, A_2\hat{\eta}) \tag{38}$$

where $\hat{\eta}$ is obtained from step 2 of 9.6.

10.3 MRR Beta-binomial bounds

The derivation of these bounds is directly related to the determination of median ranks. The bounds are calculated from the Beta-binomial distribution, a modified binomial distribution that is used to evaluate the Beta distribution as described by Johnson [14]. Suspensions require interpolation in the 5 % and 95 % ranks. These Beta-binomial intervals are slightly conservative (the width of the interval is too large), when compared to the Fisher's Matrix and likelihood ratio methods.

The method for converting the 5 % and 95 % ranks into intervals provides intervals for the time to failure. The following Equations (39) and (40) relate the 5 % and 95 % ranks in Annex C to the Weibull line:

$$t_{i,0,95} = \eta \left[\ln\left(\frac{1}{(1-F_{i(0,95)})} \right) \right]^{\left(\frac{1}{\beta}\right)} \tag{39}$$

$$t_{i,0,05} = \eta \left[\ln\left(\frac{1}{(1-F_{i(0,05)})} \right) \right]^{\left(\frac{1}{\beta}\right)} \tag{40}$$

10.4 Fisher's matrix bounds

There are significant advantages of using Fisher's matrix bounds over the Beta-binomial approach. Furthermore, for moderate size samples, the apparent confidence level is closer to the requested level, though more optimistic, than Beta-binomial. For 10 or fewer failures these bounds are too optimistic (see reference [20]).

10.5 Lower confidence limit for B_{10}

Compute the lower (1 – γ) 100 % confidence limit of B_{10} using Equations (41), (42), (43) and (44):

$$h_1 = \ln\left[-\ln(0,9)\right] \tag{41}$$

$$\delta_1 = \frac{-A_6 x^2 - r h_1 + x\sqrt{\left(A_6^2 - A_4 A_5\right)x^2 + r A_4 + 2 r h_1 A_6 + r A_5 h_1^2}}{r - x^2 A_5} \tag{42}$$

where $x = u_\gamma$ is the γ fractile of the normal distribution given in Table D.2, and A_4, A_5 and A_6 are computed according to step 1 of 10.2.

$$Q_1 = \mathrm{e}^{\left(-\frac{\delta_1 + h_1}{\hat{\beta}}\right)} \tag{43}$$

$$B_{10}\Big|_{\text{lower limit}} = Q_1 \hat{B}_{10} \tag{44}$$

10.6 Lower confidence limit for R

Compute the lower (1 – γ) 100 % confidence limit for the reliability at time t, $R_{1-\gamma}\Big|_{\text{lower limit}}$ using Equations (45) (46) and (47):

$$C_t = \hat{\beta}\ln\left(\frac{\hat{\eta}}{t}\right) \tag{45}$$

$$A_0 = A_4 + C_t^2 A_5 - 2C_t A_6 \tag{46}$$

where A_4, A_5 and A_6 are computed according to step 1 of 10.2.

$$R_{1-\gamma}\Big|_{\text{lower limit}} = \exp\left(-\exp\left[-C_t + x\sqrt{\frac{A_0}{r}}\right]\right) \tag{47}$$

where $x = u_\gamma$ is the γ fractile of the normal distribution given in Table D.2.

11 Comparison of median rank regression (MRR) and maximum likelihood estimation (MLE) estimation methods

11.1 Graphical display

Rank regression, MRR, provides a graphical display of the data. This helps to identify instances of poor fitting Weibull distribution plots perhaps suggesting another distribution, more than one failure mode affecting the items, mixtures of failure modes, batch problems or outliers. MLE does not provide a graphical display of the data.

11.2 B life estimates sometimes known as B or L percentiles

Rank regression provides more accurate estimates of "low" percentiles, like the B percentile life, from small sample sizes. These low B percentile lives and corresponding high reliabilities may be extremely important for safety problems, warranties, guaranties and contract obligations. Maximum likelihood B percentile lives tend to be optimistically biased for small numbers of failures.

11.3 Small samples

Rank regression failure forecasts are usually more accurate for small samples but much depends on the shape and age distribution of the suspensions. However, if the data set is introduced into a computer, it is recommended that both MRR and MLE should be used for small samples. In most cases the two sets of results will be in reasonably good agreement, providing some assurance of a good Weibull fit.

General advice for identifying the most appropriate method with respect to the sample size is as follows:

– For 20 or fewer data points, with or without censoring times, X on Y MRR is preferred.

– With data sets containing fewer than 10 data points, WeiBayes analysis is preferred when prior knowledge of the slope parameter, β, is available.

– For other data sets, MRR and MLE results should be compared. Closely similar results for MRR and MLE estimates, together with good regression and likelihood measures, will provide assurance that the data are correctly modelled by a Weibull distribution. (Comparison with other models should also be considered.) However, a large discrepancy between MRR and MLE estimates will indicate that the data are not correctly modelled and may contain multiple populations. Any such discrepancies should be investigated further.

11.4 Shape parameter β

MLE tends to overestimate the shape parameter, β, with small samples. The slope of the Weibull plot is often too steep. The slope of the log-normal and normal plots is similarly biased, too steep, as MLE standard deviation is underestimated.

11.5 Confidence intervals

Likelihood ratio interval estimates for MLE are rigorous; they are adjusted for small sample sizes. It is recommended to use pivotal interval estimates for MRR.

11.6 Single failure

MLE may provide a solution with one failure and some right or late suspensions. The capability has large uncertainties, but there are situations where it cannot be avoided as it provides the only solution if β is unknown. WeiBayes is preferred if there is prior knowledge of β.

11.7 Mathematical rigour

There is a mathematical objection to the use of the regression least-squares method for rank regression. The residual scatter about the line is not uniform. The results are such that the lower end of the line tends to be overweighed compared to the upper end. However, as all engineering interest is in the lower end of the curve, this is acceptable to engineers while it is unacceptable to statisticians. MLE does have attractive mathematical qualities.

11.8 Presentation of results

For presentations of results it is often best to keep it simple and concise in order to improve communications. Rank regression plots are preferred for this purpose. MLE plots with data points located with median ranks are not recommended as they can inspire comments about the poor fit of the Weibull line.

12 WeiBayes approach

12.1 Description

In WeiBayes analysis, the shape parameter, β, is assumed from historical failure data, prior experience, or from engineering knowledge of the physics of the failure. WeiBayes is defined as Weibull analysis with a given β parameter. It is a single parameter (η) Weibull distribution. WeiBayes can be used to analyse data sets with and without failures, where both types of data may have suspensions.

12.2 Method

Given β, Equation (48) may be derived using the method of maximum likelihood to determine the characteristic life, η:

$$\eta = \left[\sum_{i=1}^{N} \frac{t_i^{\beta}}{r} \right]^{1/\beta} \quad (48)$$

where

t is the time or cycles;

r is the number of failed items;

N is the total number of failures plus suspensions;

η is the maximum likelihood estimate of the characteristic life.

With β assumed and η calculated from Equation (48), a Weibull distribution is defined. A WeiBayes line is plotted on Weibull probability paper. The WeiBayes plot is used exactly like any other Weibull plot. Estimates of B lives, failure forecasts, and reliability are available from WeiBayes analysis.

12.3 WeiBayes without failures

In many WeiBayes problems, no failure has occurred. For example, a redesigned component may have been tested without any observed failures. In this case, a second assumption is required. The first failure is assumed to be imminent, i.e. in the equation, set r = 1,0. As no failures have occurred, this is a conservative engineering assumption. The resulting WeiBayes line is similarly conservative. Statistically, the WeiBayes line, based on assuming one failure, is a lower one-sided confidence estimate. That is, it may be stated with 63,2 % confidence that the true Weibull distribution lies to the right of the WeiBayes line, if the assumption of β is correct.

WeiBayes lines may be obtained at any level of confidence by employing larger or smaller denominators (assume imminent failures):

Confidence	50 %	63,2 %	90 %	95 %	99 %
Denominator	0,693	1,0	2,3	3,0	4,6

12.4 WeiBayes with failures

When the denominator is based on actual failures, the scale parameter, η, is an MLE estimate. A valuable characteristic of MLE estimates is that they are invariant under transformation. This means that the resulting WeiBayes line, B lives, and reliability estimates are all MLE estimates. The WeiBayes line is an MLE estimate of the true unknown Weibull distribution, a nominal Weibull.

Weibull distributions based on samples of 2 or 3 failures have large uncertainties. If there is good knowledge of β from prior data, significant improvements in accuracy may be obtained with WeiBayes. WeiBayes may offer cost reductions through reduced testing without loss of accuracy. A Weibull distribution library or data bank to provide Weibull distribution slope histories is strongly recommended in order to obtain the advantage of WeiBayes analysis.

The distinction between zero failure and one failure WeiBayes is worth reviewing. For example, assume five redesigned units have been tested without failure. A WeiBayes line is calculated based on the β value estimated from the original design. This is a lower one-sided confidence interval for the true unknown Weibull for the redesign. Now assume the same data set includes one failure and four suspensions.

The resulting WeiBayes is identical to the first zero failure WeiBayes, but the interpretation is different. With one failure, the WeiBayes is a nominal, MLE estimate of the true unknown Weibull distribution, not a confidence interval. However, a lower confidence bound for the MLE WeiBayes line may be calculated using Chi-squared [20].

If *r* is the number failures (≥1), the lower C % confidence limit for η is given by Equation (49):

$$\eta_{\mathrm{c}} = \eta_{MLE} \left(2\,\mathrm{r} / \chi_C{}^2 \left(2\mathrm{r} + 2\right)\right)^{(1/\beta)} \tag{49}$$

Using η_{c} and β, the lower confidence bound for the true WeiBayes line is defined.

12.5 WeiBayes case study

Fifteen compressor failures have been experienced in a large fleet of aircraft engines. Weibull analysis provides a β of approximately 5,0. Three redesigned compressor cases have been tested in engines to 1 600 h, 2 900 h and 3 100 h without failure. Is this enough testing to substantiate that the redesign is significantly better than the old design? Assuming β = 5,0 and the times on the three redesigned units, the characteristic life may be estimated for a WeiBayes solution.

$$n = \left[\frac{(1\,600)^5 + (2\,900)^5 + (3\,100)^5}{1} \right]^{1/5} = 3\,468\ \mathrm{h} \tag{50}$$

The WeiBayes line is plotted in Figure 10. It may be stated with 63 % confidence that the Weibull distribution for the redesigned units is to the right of this line and, therefore, significantly better than the parts in the bill-of-materials. It is possible that the redesign has eliminated this failure mode but that cannot be proven with this sample of data. As more time is put on these units without failure, the WeiBayes line will move further to the right and more assurance will be gained that the failure mode has been eliminated. The assumption of slope, in this case, is based on an established Weibull failure model.

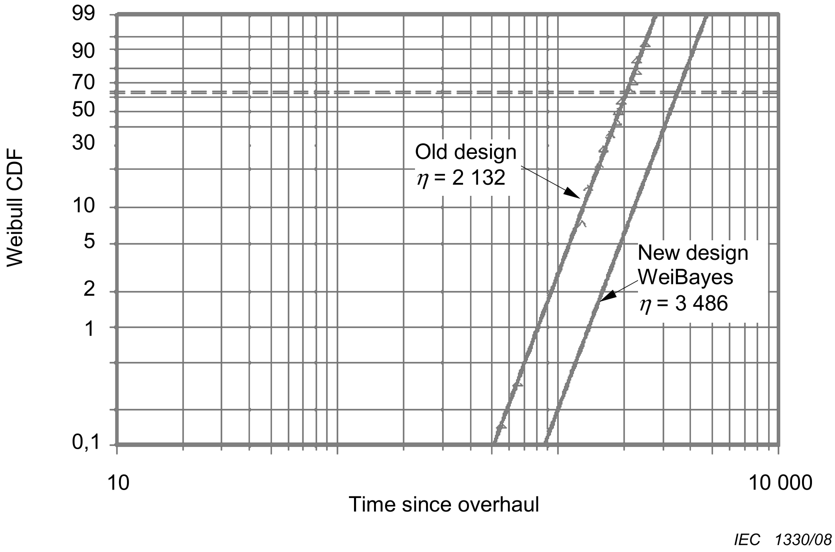


Figure 10 – New compressor design WeiBayes versus old design

When testing highly reliable items, a very small number of failures is often observed, i.e. zero failures or just one failure. This does not permit estimation of the parameters of a two- or three-parameter Weibull distribution.

In cases where the β value for the relevant failure mode is known from previous tests, a rough estimate can still be made with zero or one failure. Furthermore, the estimation of the best straight line through a small number of points can be improved if the β value is known. The available information can then be used to estimate the η value.

13 Sudden death method

Sudden death testing requires small subgroups of items for test, say three to eight items, the test consisting of running all the items simultaneously until the first failure. For a subgroup of four, this provides data on one failure and three suspensions at the same time. Perhaps four to ten items in each subgroup may be tested in typical sudden death plans. Ten sets of four provide ten failures and 30 suspensions. Sudden death provides a trade-off compared to testing all items to failure, i.e. an increase in uncertainty for a gross reduction in test time. In the bearing industry, for example, sudden death with subgroups of four is employed worldwide, and the analysis provides an estimate of L16 life. In other industries, estimates of L1 life are more common from sudden death testing.

The sudden death method is used to determine the time to a specified percentage of failures. This point of the Weibull curve will be determined with a higher precision while the rest of the Weibull curve, especially the slope of the plot, is determined with less precision than with a conventional Weibull test. The advantage of the sudden death method is that the test time is shorter than testing all samples to failure.

Often the required information is time to wear-out, i.e. time to a low, but significant percentage of failures, for example 10 %. This number is widely used for stating life of bearings, the so called L10 value, sometimes also called the B10 value. The estimated time to failure for other percentages, obtained from the Weibull plot, are shown in Table 4.

In a sudden death method, the L8,3 value (8,3 % failed) can be estimated, for example. This value can be stated as a conservative estimate of the L10 value, or be stated as the best estimate for the L8,3.

The following procedure is used to estimate the LX time (the time to X per cent failures):

a) divide the available number of samples randomly into A subgroups each consisting of B components according to Table 4;
b) set all subgroups to test;
c) record the time to first failure in each subgroup;
d) stop the test of a subgroup as soon as the first failure has occurred in that subgroup;
e) plot the time to first failure from each subgroup on a Weibull diagram, treating the rest of the items in each subgroup as suspended at the time of the first failure in that subgroup;
f) read the time to LX value on the plotted Weibull line as usual.

NOTE 1 In this case the LX point is estimated with higher precision, while the Weibull curve can be used to estimate the time to other percentages of failures, as well as estimating the slope of the Weibull curve. Uncertainty is larger, however, due to the large number of suspensions. This is especially important when time to other percentages of failures are read from the curve.

NOTE 2 Subgroups of four are often used for sudden death testing due to the saving in the number of test rigs required.

Table 4 – Subgroup size to estimate time to X % failures using the sudden death method

Subgroup size B	Precise median rank for 1 failure	LX estimated in test
2	0,292 9	L30
3	0,206 3	L20
4	0,159 1	L16
5	0,129 4	L13
6	0,109 1	L10
7	0,094 3	L9
8	0,083 0	L8
9	0,074 1	L7
10	0,067 0	L6
50	0,013 8	L1
70	0,009 94	L1

The LX value estimated in a sudden death test is almost as reliable as if all components in each subgroup had been tested to failure, but the confidence interval is approximately 50 % larger. A sudden death test can be performed much faster than testing all components to failure, however. For example, for β = 1 the test time is only 25 % of the time to test all components to failure provided the subgroups are tested sequentially. If all subgroups are tested simultaneously, the time required is only approximately 7 % of the time required to test all components to failure.

The ratio of test times may be estimated using the average times to failure. Manufacturers that employ sudden death reduce the test time by constructing sudden death rigs that test each subgroup together, subjected to the same load.

Example: below are data from 12 failures in cycles.

Table 5 – Chain data – Cycles to failure

Subgroup 1	Subgroup 2	Subgroup 3	Subgroup 4
Suspended at 3 698	Suspended at 4 650	Suspended at 2 398	**Failed at 2 945**
Failed at 3 698	Suspended at 4 650	Suspended at 2 398	Suspended at 2 945
Suspended at 3 698	**Failed at 4 650**	**Failed at 2 398**	Suspended at 2 945

For the sudden death test, the mean rank, (1/(N + 1)), for the first of three is 1/4. The corresponding L life is B25. For a Weibull with β = 2,13, the B25 life is 0,557η and the ratio of MTTF to η is 0,8858. Therefore, the ratio of test times for sudden death is (0,557 x 4)/(0,885 8 x 12) or approximately 0,2. Comparing Fisher's matrix confidence bounds for the 10 sets of three with the Weibull for all 12 failures provides about a 27 % increase in the statistical uncertainty of B1 life compared to testing all 12 to failure. This increase in uncertainty is traded for an 80 % reduction in test time.

NOTE 3 Data can also be analysed by treating the subgroups as single samples.

14 Other distributions

If x is log-normally distributed, the distribution of x will be skewed to the right and log x will have the familiar bell-shaped normal distribution.

The log-normal distribution has many applications. The distribution of flaw sizes, radio frequency (RF) parameters and repair times are typical examples. Perhaps the most important is progressive deterioration such as performance loss, crack growth to rupture and increases in vibration amplitude if the rate of change increases with the deterioration. If the rate of change is linear these distributions will tend to be Weibull.

Physically, the log-normal distribution models a process where the time to failure results from the multiplication of effects. For example, deterioration may be progressive; a crack grows rapidly with high stress because the stress increases progressively as the crack grows. In this case, the growth rate will be log-normal. On the other hand, if the growth rate is linear with time, as it may be in a low stress area, the Weibull distribution will be more appropriate. The log-normal has many applications such as materials properties, personal incomes, inheritances, bank deposits, growth rate of cracks and the distribution of flaw sizes.

While there are many statistical distributions other than the Weibull, the log-normal distribution is the second choice for life data analysis. The log-normal distribution should be the first choice if there is good prior information and more than twenty failures. For example, many material characteristics employ the log-normal distribution. Times to repair and crack growth to rupture are often log-normal. Knowledge that the physics of failure indicates progressive deterioration is also a clue that the data may be log-normal. Some semiconductor chip failures are log-normal. The distribution of the Weibull parameter, β, is approximately log-normal, while η is more normally distributed.

Annex A
(informative)

Examples and case studies

A.1 Low time failures

Figure A.1 is an example of low-time part failures on main oil pumps. Gas turbine engines are tested before being shipped to the customer, and since there were over 1 000 of these engines in the field with no problems, what was going wrong? Upon examining the failed oil pumps, it was found that they contained oversized parts. Something had changed in the manufacturing process that created a batch problem. The oversized parts caused an interference with the gears in the pump that resulted in failure. This was traced to a machining operation and corrected. Low-time failures may suggest wear-out by having a slope greater than one, but more often, they will show infant mortality, with slopes less than one. Low-time failures provide a clue to a production or assembly process change, especially when there are many successful high-time items in the field. Overhaul and scheduled maintenance also may produce these "batch" effects. Times since overhaul or maintenance may provide a clue. The presence of many late suspensions may also be a clue that a batch problem exists.

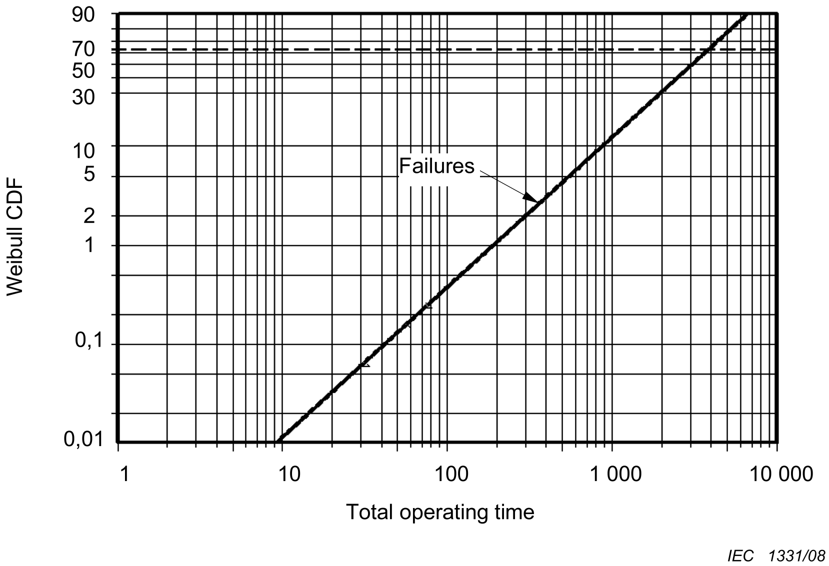


Figure A.1 – Main oil pump low times

A.2 Close serial numbers

The same reasoning can be extended to other particular failure groupings. For example, if low-time items have no failures, mid-time items have failures, and high-time items have no failures, then a batch problem is suspected. Something may have changed in the manufacturing process for a short period and then changed back. Closeness of serial numbers of the failed parts suggests a batch problem. Figure A.2 is a prime example of a process change that happened midstream in production. Bearings were failing in new augmenter pumps. The failures occurred in the 200 h to 400 h period. At least 650 items had more time than the highest time of failure. These failures were traced to a process change that was incorporated as a cost reduction for manufacturing of bearing cages. This example shows a poor fit to one Weibull distribution, as there are at least two dominant failure modes present in the data as shown in Figure A.2.

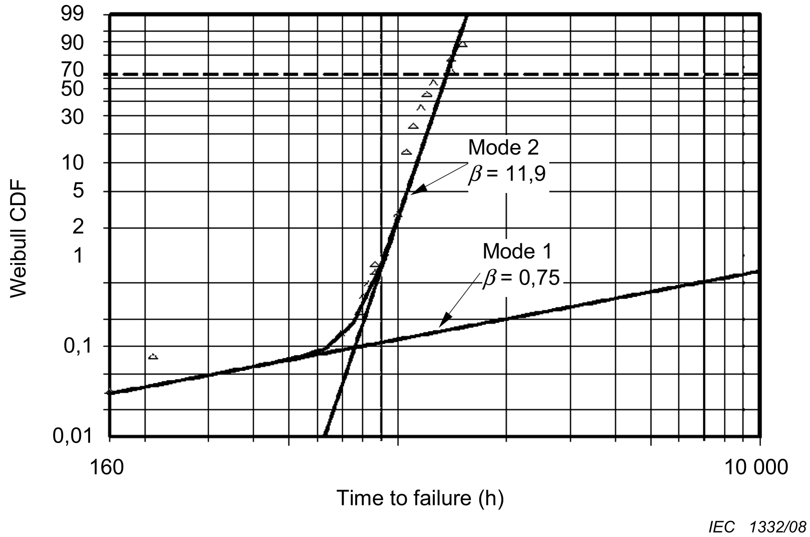


Figure A.2 – Augmenter pump bearing failure

A.3 Steep slopes

Caution should be exercised with values of β in excess of 4. A steep plot may be concealing curvature, outliers or doglegs, and the messages that they give about the data fit. The steep plot often hides incorrect Weibull data. All the messages from the data such as curves, outliers, doglegs tend to disappear. Apparently, good Weibulls may have poor fits. An example is given in Figure A.3. Here, at first glance, the plots appear to be good fits, but there is curvature and perhaps an outlier.

NOTE The steep slope of the Weibull curves is preferable from an engineering point of view, provided the value of η is large enough.

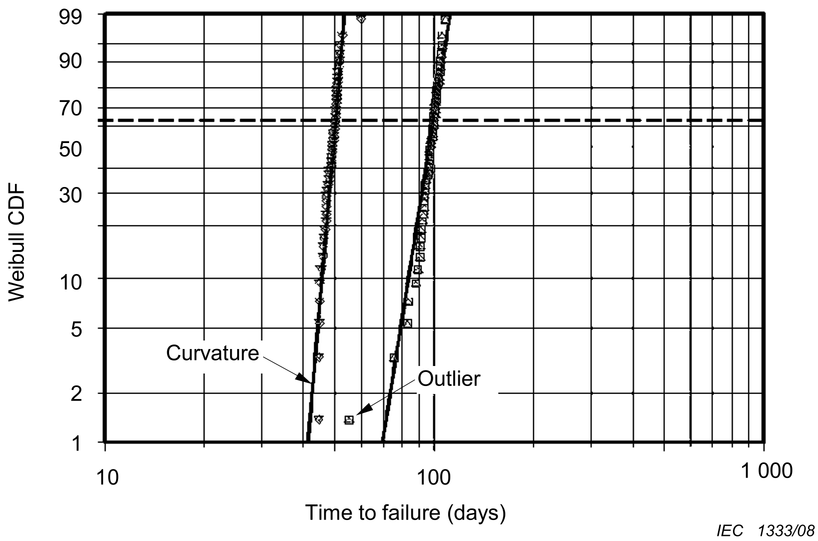


Figure A.3 – Steep β values hide problems

Annex B
(informative)

Example of computations

This example is provided as a numerical test case to verify the accuracy of computer programs implementing the MRR and MLE procedures of this standard.

Forty items are put under test. The test is stopped at the time of the 20th failure. The following are the times corresponding to the first 20 failures:

Table B.1 – Times to failure

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}	t_{15}	t_{16}	t_{17}	t_{18}	t_{19}	t_{20}
5	10	17	32	32	33	34	36	54	55	55	58	58	61	64	65	65	66	67	68

The first step is to plot the data as shown below in Figure B.1. Note that although both the MRR and MLE goodness-of-fit imply an acceptable fit, the plot shows the classic Bi-Weibull mixture of two failure modes, low slope followed by steep slope. Distribution analysis using the likelihood ratio test favours the three-parameter Weibull. The mixture analysis based on likelihood confirms at least two failure modes are present. This illustrates the merit of always plotting the data, not relying entirely on analytical methods.

Applying the numerical procedures of this standard yields the following results:

A goodness-of-fit likelihood test on this data set cannot reject, at the significance level of 10 %, the hypothesis that these times to failure come from a Weibull distribution since H = 0,36 and $F_{0,1}(18 ; 20)$ = 1,81. The MRR coefficient of determination is 93,9 %, above the critical 90 % value of 90,3 %. Both tests imply an acceptable, but not outstanding, fit.

The MLE/MRR values for β and η are $\hat{\beta}$ = 2,091/1,423 and $\hat{\eta}$ = 84/113.

The 90 % MLE/MRR confidence intervals are: [1,34/0,998 ; 2,742/2,029] for β and [70/79,05 ; 108/162,4] for η.

The MLE/MRR of B_{10} is 28,63/28,56 and the lower 90 % confidence limit for B_{10} is 20,43/23,29 (see Figure B.1). Note that as expected the MLE β is steeper than the MRR and the MLE B lives are optimistic compared to the MRR, even at the B10 level. At levels like B1 they are much more optimistic as shown in Table B.2. Note that t = 5,0 is approximately B1.

The MLE/MRR values for the reliability and their lower 90 % confidence limits for three arbitrary values of t are:

Table B.2 – Summary of results

t	$\hat{R}(t)$	$R_{0,9}\|_{\text{lower limit}}$
5,0	99,7/98,8	98,5/96,8
50,00	0,71	0,62
100,00	0,23	0,12

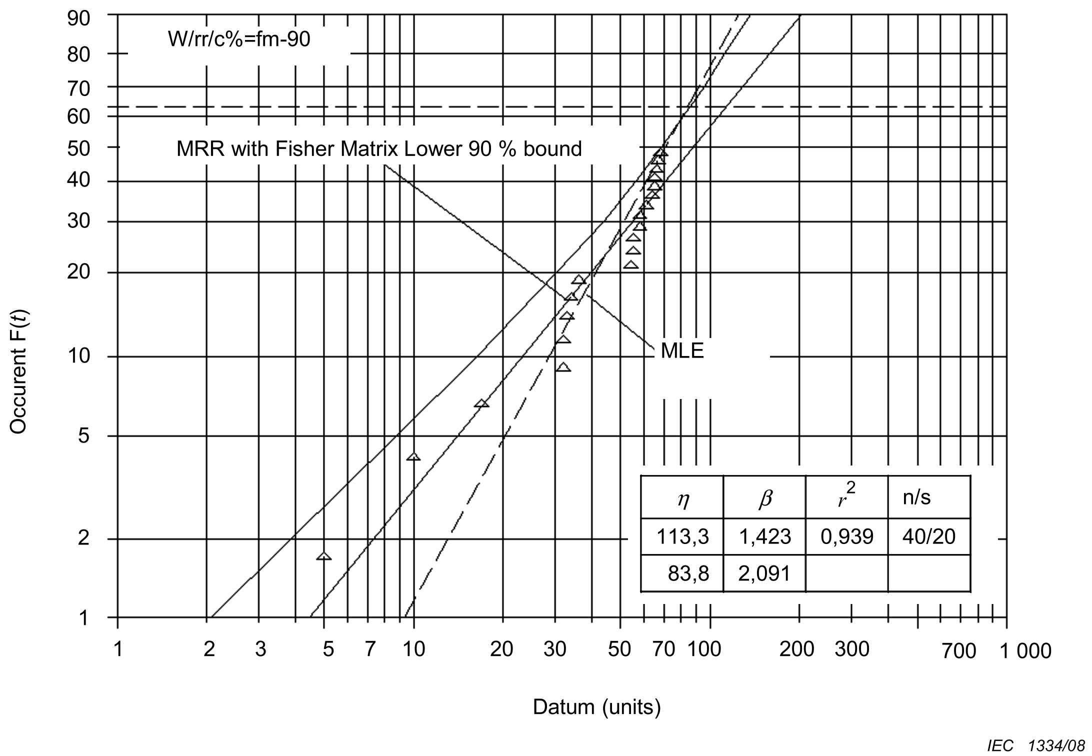


Figure B.1 – Plot of computations

Annex C
(informative)

Median rank tables

C.1 Median rank tables 5 % rank

Rank order	5 % ranks Sample size									
	1	2	3	4	5	6	7	8	9	10
1	5,0	2,53	1,70	1,27	1,02	0,85	0,73	0,64	0,57	0,51
2		22,36	13,54	9,76	7,64	6,28	5,34	4,64	4,10	3,68
3			36,84	24,86	18,93	15,32	12,88	11,11	9,77	8,73
4				47,29	34,26	27,13	22,53	19,29	16,88	15,00
5					54,93	41,82	34,13	28,92	25,14	22,24
6						60,70	47,93	40,03	34,49	30,35
7							65,18	52,93	45,04	39,34
8								68,77	57,09	49,31
9									71,69	60,58
10										74,11

Rank order	5 % ranks Sample size									
	11	12	13	14	15	16	17	18	19	20
1	0,47	0,43	0,39	0,37	0,34	0,32	0,30	0,28	0,27	0,26
2	3,33	3,05	2,81	2,60	2,42	2,27	2,13	2,01	1,90	1,81
3	7,88	7,19	6,60	6,11	5,68	5,31	4,99	4,70	4,45	4,22
4	13,51	12,29	11,27	10,40	9,67	9,03	8,46	7,97	7,53	7,14
5	19,96	18,10	16,57	15,27	14,17	13,21	12,38	11,64	10,99	10,41
6	27,12	24,53	22,40	20,61	19,09	17,78	16,64	15,63	14,75	13,96
7	34,98	31,52	28,70	26,36	24,37	22,67	21,19	19,90	18,75	17,73
8	43,56	39,09	35,48	32,50	30,00	27,86	26,01	24,40	22,97	21,71
9	52,99	47,27	42,74	39,04	35,96	33,34	31,08	29,12	27,39	25,87
10	63,56	56,19	50,54	46,00	42,26	39,10	36,40	34,06	32,01	30,20
11	76,16	66,13	58,99	53,43	48,92	45,17	41,97	39,22	36,81	34,69
12		77,91	68,37	61,46	56,02	51,56	47,81	44,60	41,81	39,36
13			79,42	70,33	63,66	58,34	53,95	50,22	47,00	44,20
14				80,74	72,06	65,62	60,44	56,11	52,42	49,22
15					81,90	73,60	67,38	62,33	58,09	54,44
16						82,93	74,99	68,97	64,06	59,90
17							83,84	76,23	70,42	65,63
18								84,67	77,36	71,74
19									85,41	78,39
20										86,09

Rank order	5 % ranks Sample size									
	21	22	23	24	25	26	27	28	29	30
1	0,24	0,23	0,22	0,21	0,20	0,20	0,19	0,18	0,18	0,17
2	1,72	1,64	1,57	1,50	1,44	1,38	1,33	1,28	1,24	1,20
3	4,01	3,82	3,65	3,50	3,35	3,22	3,10	2,98	2,88	2,78
4	6,78	6,46	6,17	5,90	5,66	5,43	5,22	5,03	4,85	4,69
5	9,88	9,41	8,98	8,59	8,23	7,90	7,59	7,31	7,05	6,81
6	13,24	12,60	12,02	11,49	11,01	10,56	10,15	9,77	9,42	9,09
7	16,82	15,99	15,25	14,57	13,95	13,38	12,85	12,37	11,92	11,50
8	20,57	19,56	18,63	17,80	17,03	16,33	15,68	15,09	14,53	14,02
9	24,50	23,27	22,16	21,16	20,24	19,40	18,62	17,91	17,25	16,63
10	28,58	27,13	25,82	24,64	23,56	22,57	21,66	20,82	20,05	19,33
11	32,81	31,13	29,61	28,24	26,99	25,84	24,79	23,83	22,93	22,11
12	37,19	35,25	33,51	31,94	30,51	29,21	28,01	26,91	25,89	24,95
13	41,72	39,52	37,54	35,76	34,14	32,66	31,31	30,07	28,93	27,87
14	46,41	43,91	41,68	39,68	37,86	36,21	34,70	33,31	32,03	30,85
15	51,26	48,45	45,95	43,71	41,68	39,84	38,16	36,62	35,20	33,89
16	56,30	53,15	50,36	47,86	45,61	43,57	41,71	40,00	38,44	36,99
17	61,56	58,02	54,90	52,13	49,64	47,38	45,34	43,46	41,75	40,16
18	67,08	63,09	59,61	56,53	53,78	51,30	49,05	47,00	45,12	43,39
19	72,94	68,41	64,51	61,09	58,05	55,32	52,86	50,62	48,57	46,69
20	79,33	74,05	69,64	65,82	62,46	59,46	56,77	54,33	52,10	50,06
21	86,71	80,19	75,08	70,77	67,04	63,74	60,79	58,13	55,71	53,49
22		87,27	80,98	76,02	71,83	68,18	64,94	62,03	59,40	57,01
23			87,79	81,71	76,90	72,81	69,24	66,06	63,20	60,61
24				88,27	82,39	77,71	73,73	70,23	67,11	64,30
25					88,71	83,02	78,47	74,58	71,16	68,10
26						89,12	83,60	79,18	75,39	72,04
27							89,50	84,15	79,84	76,14
28								89,85	84,66	80,47
29									90,19	85,14
30										90,50

C.2 Median rank tables 95 % rank

Rank order	95 % ranks Sample size									
	1	2	3	4	5	6	7	8	9	10
1	95,00	77,64	63,16	52,71	45,07	39,30	34,82	31,23	28,31	25,89
2		97,47	86,46	75,14	65,74	58,18	52,07	47,07	42,91	39,42
3			98,30	90,24	81,07	72,87	65,87	59,97	54,96	50,69
4				98,73	92,36	84,68	77,47	71,08	65,51	60,66
5					98,98	93,72	87,12	80,71	74,86	69,65
6						99,15	94,66	88,89	83,12	77,76
7							99,27	95,36	90,23	85,00
8								99,36	95,90	91,27
9									99,43	96,32
10										99,49

Rank order	95 % ranks Sample size									
	11	12	13	14	15	16	17	18	19	20
1	23,84	22,09	20,58	19,26	18,10	17,07	16,16	15,33	14,59	13,91
2	36,44	33,87	31,63	29,67	27,94	26,40	25,01	23,77	22,64	21,61
3	47,01	43,81	41,01	38,54	36,34	34,38	32,62	31,03	29,58	28,26
4	56,44	52,73	49,46	46,57	43,98	41,66	39,56	37,67	35,94	34,37
5	65,02	60,91	57,26	54,00	51,08	48,44	46,05	43,89	41,91	40,10
6	72,88	68,48	64,52	60,96	57,74	54,83	52,19	49,78	47,58	45,56
7	80,04	75,47	71,30	67,50	64,04	60,90	58,03	55,40	53,00	50,78
8	86,49	81,90	77,60	73,64	70,00	66,66	63,60	60,78	58,19	55,80
9	92,12	87,71	83,43	79,39	75,63	72,14	68,92	65,94	63,19	60,64
10	96,67	92,81	88,73	84,73	80,91	77,33	73,99	70,88	67,99	65,31
11	99,53	96,95	93,40	89,60	85,83	82,22	78,81	75,60	72,61	69,80
12		99,57	97,19	93,89	90,33	86,79	83,36	80,10	77,03	74,13
13			99,61	97,40	94,32	90,97	87,62	84,37	81,25	78,29
14				99,63	97,58	94,69	91,54	88,36	85,25	82,27
15					99,66	97,73	95,01	92,03	89,01	86,04
16						99,68	97,87	95,30	92,47	89,59
17							99,70	97,99	95,55	92,86
18								99,72	98,10	95,78
19									99,73	98,19
20										99,74

Rank order	95 % ranks Sample size									
	21	22	23	24	25	26	27	28	29	30
1	13,29	12,73	12,21	11,73	11,29	10,88	10,50	10,15	9,81	9,50
2	20,67	19,81	19,02	18,29	17,61	16,98	16,40	15,85	15,34	14,86
3	27,06	25,95	24,92	23,98	23,10	22,29	21,53	20,82	20,16	19,53
4	32,92	31,59	30,36	29,23	28,17	27,19	26,27	25,42	24,61	23,86
5	38,44	36,91	35,49	34,18	32,96	31,82	30,76	29,77	28,84	27,96
6	43,70	41,98	40,39	38,91	37,54	36,26	35,06	33,94	32,89	31,90
7	48,74	46,85	45,10	43,47	41,95	40,54	39,21	37,97	36,80	35,70
8	53,59	51,55	49,64	47,87	46,22	44,68	43,23	41,87	40,60	39,39
9	58,28	56,09	54,05	52,14	50,36	48,70	47,14	45,67	44,29	42,99
10	62,81	60,48	58,32	56,29	54,39	52,62	50,95	49,38	47,90	46,51
11	67,19	64,75	62,46	60,32	58,32	56,43	54,66	53,00	51,43	49,94
12	71,42	68,87	66,49	64,24	62,14	60,16	58,29	56,54	54,88	53,31
13	75,50	72,87	70,39	68,06	65,86	63,79	61,84	60,00	58,25	56,61
14	79,43	76,73	74,18	71,76	69,49	67,34	65,30	63,38	61,56	59,84
15	83,18	80,44	77,84	75,36	73,01	70,79	68,69	66,69	64,80	63,01
16	86,76	84,01	81,37	78,84	76,44	74,16	71,99	69,93	67,97	66,11
17	90,12	87,40	84,75	82,20	79,76	77,43	75,21	73,09	71,07	69,15
18	93,22	90,59	87,98	85,43	82,97	80,60	78,34	76,17	74,11	72,13
19	95,99	93,54	91,02	88,51	86,05	83,67	81,38	79,18	77,07	75,05
20	98,28	96,18	93,83	91,41	88,99	86,62	84,32	82,09	79,95	77,89
21	99,76	98,36	96,35	94,10	91,77	89,44	87,15	84,91	82,75	80,67
22		99,77	98,43	96,50	94,34	92,10	89,85	87,63	85,47	83,37
23			99,78	98,50	96,65	94,57	92,41	90,23	88,08	85,98
24				99,79	98,56	96,78	94,78	92,69	90,58	88,50
25					99,80	98,62	96,90	94,97	92,95	90,91
26						99,80	98,67	97,02	95,15	93,19
27							99,81	98,72	97,12	95,31
28								99,82	98,76	97,22
29									99,82	98,80
30										99,83

C.3 Median rank tables 50 % rank

Rank order	Median ranks (50 %) Sample size									
	1	2	3	4	5	6	7	8	9	10
1	50	29,29	20,63	15,91	12,94	10,91	9,43	8,30	7,41	6,70
2		70,71	50,00	38,57	31,38	26,44	22,85	20,11	17,96	16,23
3			79,37	61,43	50,00	42,14	36,41	32,05	28,62	25,86
4				84,09	68,62	57,86	50,00	44,02	39,31	35,51
5					87,06	73,56	63,59	55,98	50,00	45,17
6						89,09	77,15	67,95	60,69	54,83
7							90,57	79,89	71,38	64,49
8								91,70	82,04	74,14
9									92,59	83,77
10										93,30

Rank order	Median ranks (50 %) Sample size									
	11	12	13	14	15	16	17	18	19	20
1	6,11	5,61	5,19	4,83	4,52	4,24	4,00	3,78	3,58	3,41
2	14,80	13,60	12,58	11,70	10,94	10,27	9,68	9,15	8,68	8,25
3	23,58	21,67	20,04	18,65	17,43	16,37	15,42	14,58	13,83	13,15
4	32,38	29,76	27,53	25,61	23,94	22,47	21,18	20,02	18,99	18,05
5	41,19	37,85	35,02	32,58	30,45	28,59	26,94	25,47	24,15	22,97
6	50,00	45,95	42,51	39,54	36,97	34,71	32,70	30,92	29,32	27,88
7	58,81	54,05	50,00	46,51	43,48	40,82	38,47	36,37	34,49	32,80
8	67,62	62,15	57,49	53,49	50,00	46,94	44,23	41,82	39,66	37,71
9	76,42	70,24	64,98	60,46	56,52	53,06	50,00	47,27	44,83	42,63
10	85,20	78,33	72,47	67,42	63,03	59,18	55,77	52,73	50,00	47,54
11	93,89	86,40	79,96	74,39	69,55	65,29	61,53	58,18	55,17	52,46
12		94,39	87,42	81,35	76,06	71,41	67,30	63,63	60,34	57,37
13			94,81	88,30	82,57	77,53	73,06	69,08	65,51	62,29
14				95,17	89,06	83,63	78,82	74,53	70,68	67,20
15					95,48	89,73	84,58	79,98	75,85	72,12
16						95,76	90,32	85,42	81,01	77,03
17							96,00	90,85	86,17	81,95
18								96,22	91,32	86,85
19									96,42	91,75
20										96,59

Rank order	Median ranks (50 %) Sample size									
	21	22	23	24	25	26	27	28	29	30
1	3,25	3,10	2,97	2,85	2,73	2,63	2,53	2,45	2,36	2,28
2	7,86	7,51	7,19	6,90	6,62	6,37	6,14	5,92	5,72	5,53
3	12,53	11,97	11,46	10,99	10,55	10,15	9,78	9,44	9,11	8,81
4	17,21	16,44	15,73	15,09	14,49	13,94	13,43	12,96	12,52	12,10
5	21,89	20,91	20,01	19,19	18,43	17,74	17,09	16,48	15,92	15,40
6	26,57	25,38	24,30	23,30	22,38	21,53	20,74	20,01	19,33	18,69
7	31,26	29,86	28,58	27,41	26,32	25,32	24,40	23,54	22,74	21,99
8	35,94	34,33	32,86	31,51	30,27	29,12	28,06	27,07	26,14	25,28
9	40,63	38,81	37,15	35,62	34,22	32,92	31,71	30,59	29,55	28,58
10	45,31	43,29	41,43	39,73	38,16	36,71	35,37	34,12	32,96	31,87
11	50,00	47,76	45,72	43,84	42,11	40,51	39,03	37,65	36,37	35,17
12	54,69	52,24	50,00	47,95	46,05	44,31	42,68	41,18	39,77	38,46
13	59,37	56,71	54,28	52,05	50,00	48,10	46,34	44,71	43,18	41,76
14	64,06	61,19	58,57	56,16	53,95	51,90	50,00	48,24	46,59	45,06
15	68,74	65,67	62,85	60,27	57,89	55,69	53,66	51,76	50,00	48,35
16	73,43	70,14	67,14	64,38	61,84	59,49	57,32	55,29	53,41	51,65
17	78,11	74,62	71,42	68,49	65,78	63,29	60,97	58,82	56,82	54,94
18	82,79	79,09	75,70	72,59	69,73	67,08	64,63	62,35	60,23	58,24
19	87,47	83,56	79,99	76,70	73,68	70,88	68,29	65,88	63,63	61,54
20	92,14	88,03	84,27	80,81	77,62	74,68	71,94	69,41	67,04	64,83
21	96,75	92,49	88,54	84,91	81,57	78,47	75,60	72,93	70,45	68,13
22		96,90	92,81	89,01	85,51	82,26	79,26	76,46	73,86	71,42
23			97,03	93,10	89,45	86,06	82,91	79,99	77,26	74,72
24				97,15	93,38	89,85	86,57	83,52	80,67	78,01
25					97,27	93,63	90,22	87,04	84,08	81,31
26						97,37	93,86	90,56	87,48	84,60
27							97,47	94,08	90,89	87,90
28								97,55	94,28	91,19
29									97,64	94,47
30										97,72

C.4 Generating ranks using a spreadsheet program

Ranks can be generated in a spreadsheet program using the following function:

BETAINV(C, J, N-J+1)

where

C is the confidence level;

J is the rank order;

N is the sample size.

Annex D
(normative)

Statistical tables

D.1 Table of the gamma function

Table D.1 is used in conjunction with 9.7.

Table D.1 – Values of the gamma function

β	Γ(1+1/*β*)	*β*	Γ(1+1/*β*)	*β*	Γ(1+1/*β*)
0,20	120	1,50	0,902 7	3,60	0,9011
0,25	24	1,55	0,899 4	3,70	0,9024
0,30	9,2603	1,60	0,896 6	3,80	0,9038
0,35	5,0295	1,65	0,894 2	3,90	0,9051
0,40	3,3233	1,70	0,892 2	4,00	0,9064
0,45	2,5055	1,75	0,890 6	4,10	0,9076
0,50	2,0000	1,80	0,889 2	4,20	0,9089
0,55	1,7024	1,85	0,888 2	4,30	0,9101
0,60	1,5045	1,90	0,887 4	4,40	0,9113
0,65	1,3603	1,95	0,886 7	4,50	0,9125
0,70	1,2657	2,00	0,886 2	4,60	0,9137
0,75	1,1906	2,10	0,885 7	4,70	0,9149
0,80	1,1330	2,20	0,885 6	4,80	0,9160
0,85	1,0878	2,30	0,885 9	4,90	0,9171
0,90	1,0522	2,40	0,886 5	5,00	0,9182
0,95	1,0238	2,50	0,887 2	5,20	0,9202
1,00	1,0000	2,60	0,888 2	5,40	0,9222
1,05	0,9808	2,70	0,889 3	5,60	0,9241
1,10	0,9649	2,80	0,890 3	5,80	0,9260
1,15	0,9517	2,90	0,891 7	6,00	0,9277
1,20	0,9406	3,00	0,893 0	6,20	0,9293
1,25	0,9314	3,10	0,894 3	6,40	0,9309
1,30	0,9236	3,20	0,895 6	6,60	0,9325
1,35	0,9169	3,30	0,897 0	6,80	0,9340
1,40	0,9114	3,40	0,898 4	7,00	0,9354
1,45	0,9067	3,50	0,899 7	8,00	0,9417

D.2 Fractiles of the normal distribution

Table D.2 gives values of the fractiles of the normal distribution u_p for commonly required values of the argument p.

Table D.2 – Fractiles of the normal distribution

p	0,010	0,025	0,050	0,100
u_p	2,326 3	1,960 0	1,644 9	1,281 6

Annex E
(informative)

Spreadsheet example

E.1 Example of Weibull analysis using a spreadsheet

Table E.1 – Practical analysis example

	A	B	C	D	E
1	Failure No. i	Failure time t_i	Median rank, $F_i(t)$ = $(i$ - 0,3)/(n + 0,4)	X =ln(t)	Y =ln$(1/\ln(1-F(t)))$
2	1	12	0,067 3	2,484 9	-2,663 8
3	2	20	0,163 5	2,995 7	-1,723 3
4	3	34	0,259 6	3,526 4	-1,202 0
5	4	65	0,355 8	4,174 4	-0,821 7
6	5	91	0,451 9	4,510 9	-0,508 6
7	6	134	0,548 1	4,897 8	-0,230 4
8	7	178	0,644 2	5,181 8	0,032 9
9	8	246	0,740 4	5,505 3	0,299 0
10	9	378	0,836 5	5,934 9	0,594 0
11	10	512	0,932 7	6,238 3	0,992 7
12	Regress X on Y, since there is more uncertainty in t			Alternative method, if more uncertainty in Y	
13	y = 1,111 5 x + 5,126 5			y = 0,8839x - 4,5403	
14	R^2 = 0,982 4			R^2 = 0,9824	
15	Intercept	5,1265		intercept	-4,540 3
16	β	0,8997		β	0,883 9
17	η	168,42		η	170,15
18					

Method	Equation	Spreadsheet model	β	η
X on Y (Standard)	$(1/\beta)\ln(-\ln(1-F)) + \ln(\eta) = \ln(t)$	y = 1,111 5 x - 5,126 5	= 1/1,115 = 0,899 7	=exp(5,1265) =168,42
Y on X (Special)	$\ln(-\ln(1-F)) = \beta\ln(t) - \beta\ln(\eta)$	y = 0,883 9 x - 4,540 3	= 0,883 9	=exp(4,5403/0,8839) = 170,15

Cells shaded in Table E.1 are selected for a plot. The plot type is scattered, without lines. Once the scattered graph is completed, the data are fitted with a straight regression line, and the fitting selection should specify the equation of the fitted line to be displayed on the graph along with the correlation coefficient, R^2, value. The plot obtained is shown in Figure E.1. It is convenient to copy the equation from the plot onto the data sheet for calculation of the scale parameter, as shown on the bottom of the Table E.1. The LINEST function in a spreadsheet model can also be used to calculate the fit.

There is usually more uncertainty with the time to failure, t, and less uncertainty with the failure number. Least squares regression should regress the variable with more uncertainty against the variable with less uncertainty. Commercial software will regress X on Y instead of the customary Y on X often used in spreadsheet linear plots. The rows on the bottom of Table E.1 show the comparison between the two methods.

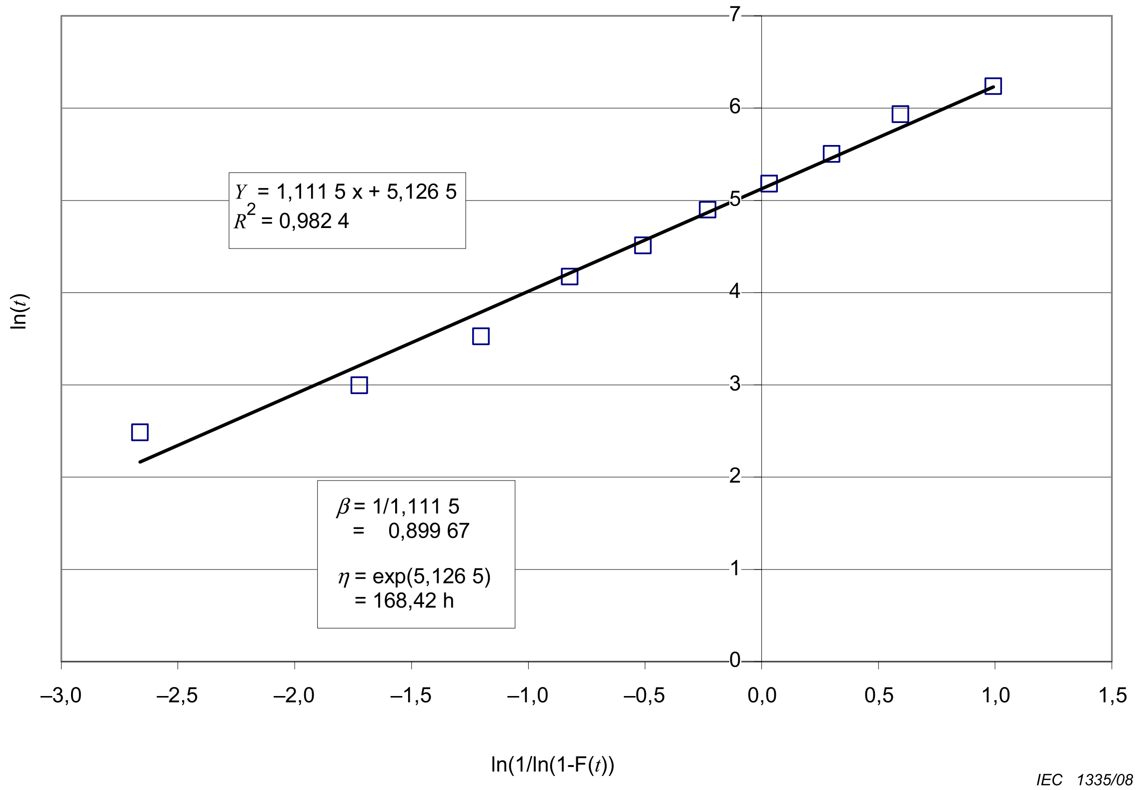


Figure E.1 – Weibull plot for graphical analysis

The slope of the line in the plot defines the shape parameter, β, and the scale parameter, η. These are calculated as shown in Table E.1.

E.2 Example using suspended data

Censored or suspended data, especially when large, can be analysed using a computer spreadsheet, much in a similar way as was shown in the previous section for spreadsheet Weibull analysis. The difference is that the order number which is used for the calculation of median rank, previously designated as *i,* is modified to account for suspensions using the following expressions:

$$i_{t_i} = i_{t_{i-1}} + m_{t_i}$$

$$m_{t_i} = \frac{(n+1) - i_{t_{i-1}}}{1 + (n - \text{number of preceding items})}$$

$$F_i(t_i) = \frac{i_{t_i} - 0{,}3}{n + 0{,}4}$$

The relationship between the above two equations, $i_{t_i} = i_{t_{i-1}} + m_{t_i}$ and m_{t_i} are derived from Equation (7) of 7.2.3.

Tables E.2 and E.3 below show how a spreadsheet is to be set up for a given set of data.

Table E.2 – Spreadsheet set-up for analysis of censored data

	A	B	C	D	E	F	G
1	**Event number** *j*	**Adjusted failure No.** *i*	**Event time** t_i	**Event**	**Adjusted rank** $F_i(t)$ **(*i* - 0,3)/(*n* + 0,4)**	*x*	*y*
2	1	=A2	t_1	F	=(B2-0,3)/($A12+0,4)	ln(C2)	ln{ln[1/(1-E2)]}
3	2		t_2	S			
4	3		t_3	S			
5	4	=B2+((A12+1)-B2)/(1+(A12-A4))	t_4	F	=(B5-0,3)/($A12+0,4)	ln(C5)	ln{ln[1/(1-E5)]}
6	5	=B5+((A12+1)-B5)/(1+(A12-A5))	t_5	F	=(B6-0,3)/($A12+0,4)	ln(C6)	ln{ln[1/(1-E6)]}
7	6		t_6	S	,		
8	7	=B6+((A12+1)-B6)/(1+(A12-A7))	t_7	F	=(B8-0,3)/($A12+0,4)	ln(C8)	ln{ln[1/(1-E8)]}
9	8		t_8	S			
10	9	=B8+((A12+1)-B8)/(1+(A12-A9))	t_9	F	=(B10-0,3)/($A12+0,4)	ln(C10)	ln{ln[1/(1-E10)]}
11	10	=B10+((A12+1)-B10)/(1+(A12-A10))	t_{10}	F	=(B11-0,3)/($A12+0,4)	ln(C11)	ln{ln[1/(1-E11)]}
12	11		t_{11}	S			

Table E.3 – Example of Weibull analysis for suspended data

	A	B	C	D	E	F	G
1	**Event number** *j*	**Failure No.** *i*	**Failure time** t_i	**Event**	**Median rank, $F_i(t)$ (*i* - 0,3)/ (*n* + 0,4)**	*x*	*y*
2	1	1,000 0	12	F	0,0614	2,484 9	-2,758 8
3	2		20	S			
4	3		34	S			
5	4	2,222 2	65	F	0,168 6	4,174 4	-1,689 2
6	5	3,444 4	91	F	0,275 8	4,510 9	-1,130 9
7	6		134	S			
8	7	4,870 4	178	F	0,400 9	5,181 8	-0,668 8
9	8		246	S			
10	9	6,652 8	378	F	0,557 3	5,934 9	-0,204 8
11	10	8,435 2	450	F	0,713 6	6,109 2	0,223 5
12	11		512	S			
13							
14		y = 1,230 5x + 6,010 2					
15		R^2 = 0,9833					
16		Intercept	6,010 2				
17		β	0,812 7		= 1/1,230 5		
18		η	407,55		= exp(6,010 2)		
19							

The Weibull plot from the above example is shown in Figure E.2.

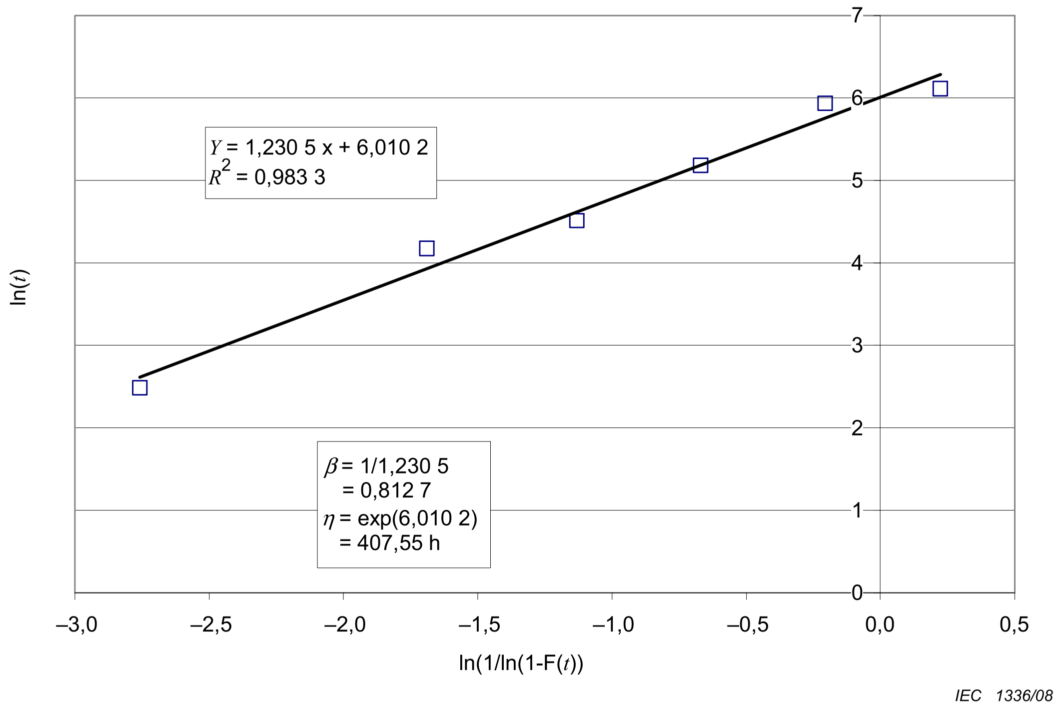


Figure E.2 – Weibull plot of censored data

E.3 Hazard plotting example 1

Another suitable way to analyse data with suspensions is to plot the cumulative hazard function. The plotted points correspond to the failure times, but the cumulative hazard calculation accounts for the suspensions. This method is considerably simpler than adjustment of the failure numbers, producing comparable results. IEC 61810-2 describes how to perform hazard plotting.

The spreadsheet with hazard plotting to account for suspensions is shown in Tables E.4 and E.5.

Table E.4 – Example of spreadsheet application for censored data

	A	B	C	D	E	F	G	H	I
1	**Event number** i	**Event time** t_i	**Event**	**Reverse rank**	**Hazard function** $h(t)$	**Cumulative hazard** $H(t)$	**ln*(t)***	**ln(*H(t)*)**	**F*(t)***
2	1	t_1	Failed	11	=1/D2	=E2	=LN(B2)	=LN(F2)	=1-EXP(-F2)
3	2	t_2	Censored	10					
4	3	t_3	Censored	9					
5	4	t_4	Failed	8	=1/D5	=F2+E5	=LN(B5)	=LN(F5)	=1-EXP(-F5)
6	5	t_5	Failed	7	=1/D6	=F5+E6	=LN(B6)	=LN(F6)	=1-EXP(-F6)
7	6	t_6	Censored	6					
8	7	t_7	Failed	5	=1/D8	=F6+E8	=LN(B8)	=LN(F8)	=1-EXP(-F8)
9	8	t_8	Censored	4					
10	9	t_9	Failed	3	=1/D10	=F8+E10	=LN(B10)	=LN(F10)	=1-EXP(-F10)
11	10	t_{10}	Failed	2	=1/D11	=F10+E11	=LN(B11)	=LN(F11)	=1-EXP(-F11)
12	11	t_{11}	Censored	1					

Table E.5 – Example spreadsheet

Time to failure *t*	Number	Event	Reverse rank	Hazard function h(*t*)	Cumulative hazard H(*t*)	ln(*t*)	ln(H(*t*))	F(*t*)
12	1	Failed	11	0,091	0,091	2,485	-2,398	0,087
20	2	Censored	10					
34	3	Censored	9					
65	4	Failed	8	0,125	0,216	4,174	-1,533	0,194
91	5	Failed	7	0,143	0,359	4,511	-1,025	0,301
134	6	Censored	6					
178	7	Failed	5	0,200	0,559	5,182	-0,582	0,428
246	8	Censored	4					
378	9	Failed	3	0,333	0,892	5,935	-0,114	0,590
450	10	Failed	2	0,500	1,392	6,109	0,331	0,751
512	11	Censored	1					

Plot (LN is the natural logarithm) LN cumulative hazard against LN time giving Figure E.3.

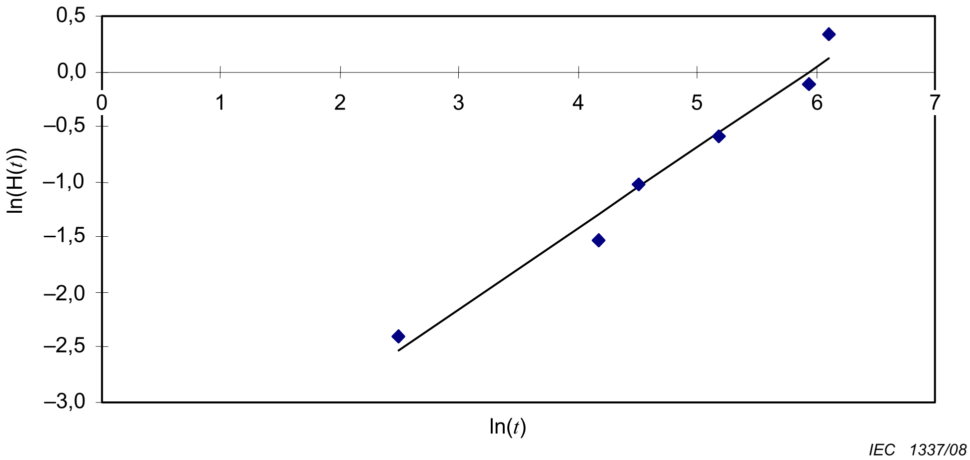


Figure E.3 – Cumulative hazard plot for data of Table E.4

From the regression analysis of ln[H(*t*)] on ln(*t*), a straight line fitted to the data is:

$$\ln[H(t)] = 0{,}729\ln(t) - 4{,}338$$

with $R^2 = 0{,}973$. The parameter estimates are:

$$\beta = 0{,}729$$

and

$$\eta = e^{\left(\frac{4{,}338}{0{,}729}\right)} = 384$$

E.4 Hazard plotting example 2

Both the hazard plotting technique and the technique described in 7.3 can analyse data sets with multiple modes of failure, by treating failure modes other than that to be analyzed as censorings. The following data set, provided by IEC/TC94, is the result of the reliability experiments on a relay. The lifetime scale is measured with the number of switching times. This relay suffered from two failure modes, weldings (mode 1) and erosion contacts (mode 2).

Table E.6 shows the data set along with an example to estimate the cumulative hazard function for failure mode 1. In analysing failure records with mode 1, those with mode 2 are treated as censored.

Table E.6 – A relay data provided by ISO/TC94 and Hazard analysis for failure mode 1

Times	Number	Event	Reverse rank	Hazard function h1(*t*)	Cumulative hazard H1(*t*)	ln(*t*)	ln(H1(*t*))	F1(*t*)
984 182	1	Failed - mode 1	30	0,033	0,033	13,800	-3,401	0,033
103 598 9	2	Failed - mode 1	29	0,034	0,068	13,851	-2,691	0,066
108 632 0	3	Failed - mode 1	28	0,036	0,104	13,898	-2,268	0,098
116 708 2	4	Failed - mode 2	27					
116 843 7	5	Failed - mode 2	26					
119 624 3	6	Failed - mode 2	25					
119 895 4	7	Failed - mode 1	24	0,042	0,145	13,997	-1,930	0,135
123 715 8	8	Failed - mode 2	23					
126 636 3	9	Failed - mode 1	22	0,045	0,191	14,052	-1,657	0,174
128 005 4	10	Failed - mode 2	21					
129 248 1	11	Failed - mode 2	20					
130 758 8	12	Failed - mode 1	19	0,053	0,243	14,084	-1,414	0,216
130 857 5	13	Failed - mode 2	18					
134 196 6	14	Failed - mode 2	17					
136 270 8	15	Failed - mode 1	16	0,063	0,306	14,125	-1,185	0,263
142 846 6	16	Failed - mode 1	15	0,067	0,372	14,172	-0,988	0,311
143 192 3	17	Failed - mode 2	14					
143 327 1	18	Failed - mode 1	13	0,077	0,449	14,175	-0,800	0,362
145 822 6	19	Failed - mode 2	12					
146 155 9	20	Failed - mode 2	11					
152 838 6	21	Failed - mode 2	10					
156 312 3	22	Failed - mode 2	9					
162 708 2	23	Failed - mode 2	8					
205 187 7	24	Failed - mode 2	7					
224 022 4	25	Failed - mode 1	6	0,167	0,616	14,622	-0,484	0,460
231 958 5	26	Failed - mode 1	5	0,200	0,816	14,657	-0,203	0,558
247 604 7	27	Failed - mode 1	4	0,250	1,066	14,722	0,064	0,656
248 000 0	28	Censored	3					
248 000 0	29	Censored	2					
248 000 0	30	Censored	1					

Similar analysis is performed on the failure mode 2 and the resulting plots and the fitted lines are shown in Figure E.4.

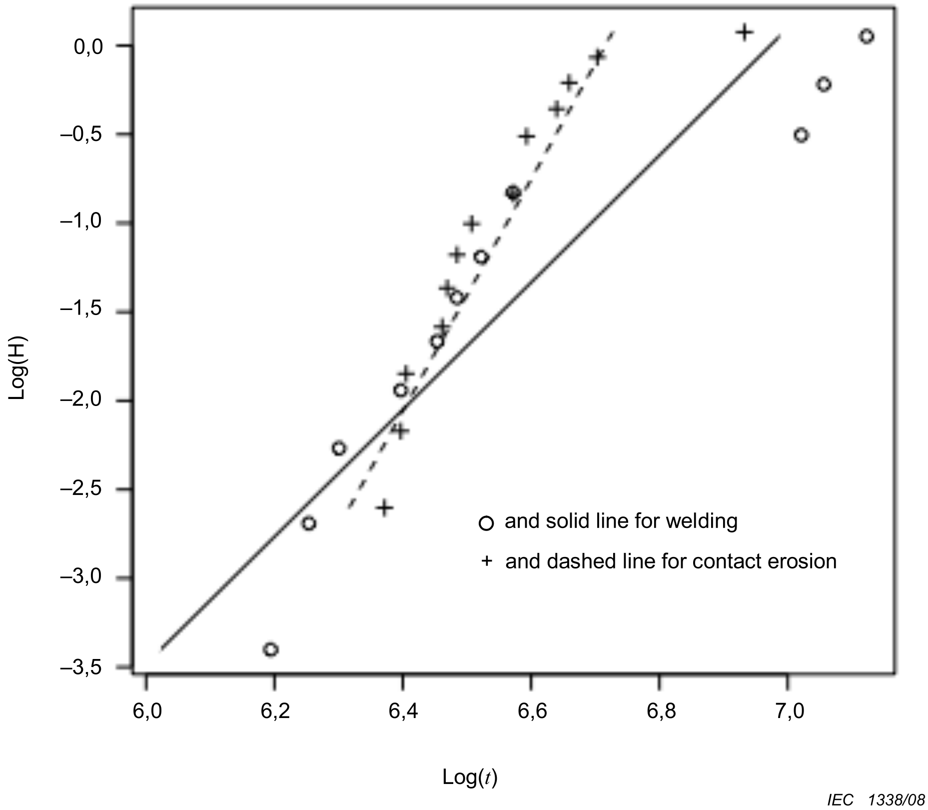


Figure E.4 – Cumulative hazard plots for Table E.6

The estimated parameters are $\beta_1 = 3{,}59$, $n_1 = 1066$, $\beta_2 = 6{,}53$ and $\eta = 825$, respectively.

Annex F
(informative)

Example of Weibull probability paper

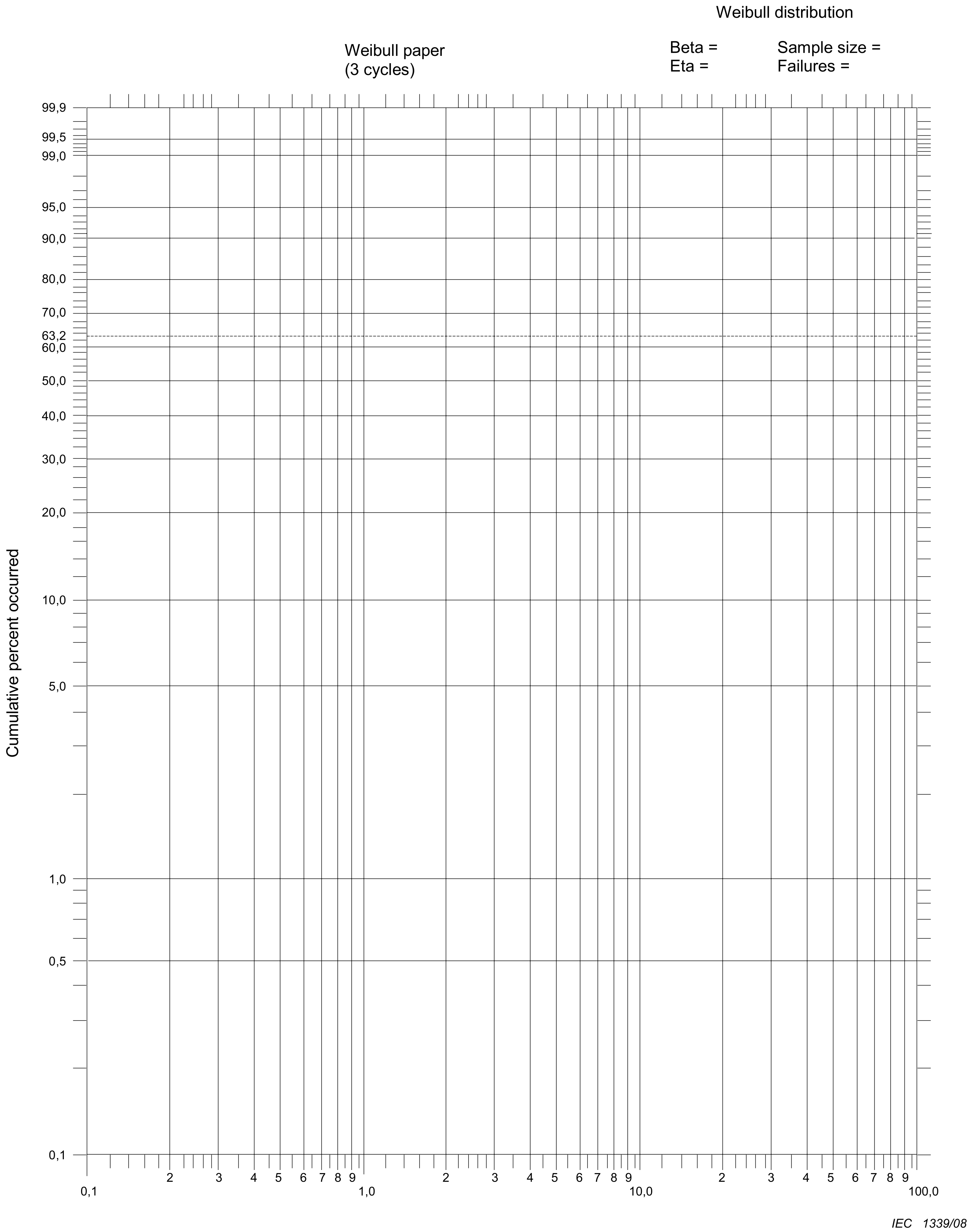


Figure F.1 – Weibull probability paper

Annex G
(informative)

Mixtures of several failure modes

G.1 Description

A Weibull plot containing a dogleg bend is a clue to the potential of multiple competitive failure modes. An example of this was a problem in a compressor start bleed system. Upon examination of the data, 10 out of 19 failures had occurred at one installation base. It was concluded that the location of this base was contributing to the problem. The base was located on the ocean and the salt air was the factor. The data were categorized into separate Weibull plots with this engineering knowledge. The first Weibull had a slope of 0,75. This could be considered an infant mortality problem, while the ocean base Weibull had a stress corrosion wear-out failure mechanism with β = 11,9. More attention to maintenance resolved the problem.

Dogleg Weibulls are caused by mixtures of more than one failure mode. These are usually competitive failure modes, competing to produce failure. However there are several types of mixtures described in this annex. For instance, fuel pump failures can be due to bearings, housing cracks, leaks, etc. If these different failure modes are plotted on one Weibull plot, one or more dogleg bends will result. When this occurs, a close examination of the failed parts is the best way to separate the data into different failure modes. If this is done correctly, separate good Weibulls will result. There can be mixtures of modes and populations, perhaps batches and competing failure modes. A steep slope followed by a shallow slope usually indicates a batch problem, as there are some "perpetual survivors" that are not subject to the failure mode. For example, there may be defects in some, but not all, parts; a batch problem.

It is always preferable to separate the failure modes based on analysis of the parts (and environment) and to analyse them separately, rather than rely on statistical methods.

Supposing a data set of 50 parts, and 20 of them have one failure mode and the other 30 have a different failure mode. The first set should be analysed as 20 failures (of F_1) and 30 suspensions (for F_2). The second set would be 30 failures (of F_2) and 20 suspensions (for F_1). These two sets of parameters can then be used to predict the cumulative failure distribution.

When parts are not available for physical analysis, the data may be split into groups based on plotting position. This can cause errors, because a small percentage of wear-out failures will occur at an "early" life, and a percentage of infant mortality failures will occur at later life.

A minimum of 20 failures is needed for credible results from a mixture of two failure modes, and 50 or more failures for the other mixtures.

The following are brief descriptions of the more common methods for handling mixtures:

– p indicates the portion or batch of the total population that has a particular failure distribution (F_1 in the simple mixture);
– F_1, F_2, and F_3 indicate failure distributions;
– R_1, R_2 a-nd R_3 are the corresponding reliability distributions;

The population cumulative failure distributions are F and R.

The descriptions are given without describing the particular distribution shape (e.g. Weibull, log-normal, normal, or exponential). An appropriate distribution shape needs to be substituted for each F_n.

G.2 Competing risk

$$F = 1 - (1 - F_1)(1 - F_2) \quad \text{(G.1)}$$

Competing risk occurs when a population has two or more failure modes and the entire population is at risk from either failure mode. Even though a Weibull plot of these data will appear curved, this is not a mixture of subpopulations; it is a homogeneous population. If one defines a mixture to be a mixture of failure modes, then this model would be a mixture as well, since there are two different failure modes.

NOTE This is simply a series reliability problem: $R = R_1 \cdot R_2$.

An example of competing risk is an ASIC component in a plastic encapsulation with micro BGA solder connections. The ASIC may fail through crack propagation in the solder balls or through moisture penetration through the plastic. The two failure modes are independent of each other, but will compete in causing the ASIC to fail.

G.3 Simple mixture

$$F = pF_1 + (1 - p)F_2 \quad \text{(G.2)}$$

This is a mixture of two independent subpopulations with no common failure modes. Each subpopulation has its own unique failure modes.

Although listed as a simple mixture, there are very few applications that truly fit this model. Most mixtures have at least one common failure mode. The simple mixture may be used as an approximation for more complex distributions, such as the competing risk mixture, described in Clause G.4. An example could be the solder balls of the micro BGA of the ASIC. Some of the solder balls have one or more voids. A crack will propagate to the void reducing the life time significantly, compared to the solder balls where the crack has to propagate through the whole length of the solder.

G.4 Competing risk mixture

$$F = p[1 - (1 - F_1)(1 - F_2)] + (1 - p)F_2 \quad \text{(G.3)}$$

Most mixtures of subpopulations are competing risk mixtures. There is at least one failure mode (F_1) that is unique to one subpopulation, and there is a failure mode (F_2) that is common to both subpopulations.

In this case, one subpopulation is subject to failure modes 1 and 2, as indicated by the portion of the equation in brackets [ ]. This subpopulation by itself has competing risk. As an example, a tyre of a car may wobble due to being out of round (F_1), but the tyre may also get a puncture. In both situations, the tyres are within specification of roundness and the tyre that wobbled may get a puncture. So it is possible to get a puncture (F_2) on the way to the dealer to have the wobbling tyre replaced.

Mixtures of more than three failure modes will have a better fit, the doglegs will disappear and β will tend toward one. Thus Weibulls for a system or component with many modes mixed together will tend toward a β of one. These Weibulls should not be employed if there is any way to categorize the data into separate, more accurate failure modes. Using a Weibull plot with mixtures of many failure modes is the equivalent of assuming the exponential distribution applies. Exponential results are often misleading and yet this is common practice.

Annex H
(informative)

Three-parameter Weibull example

H.1 Example

Figure H.1 is a typical example of a three-parameter Weibull distribution using fracture toughness of steel plate as the data of interest. The model indicates it is physically impossible to fail the plate at a low level of stress (see Figure H.2 for the effect of the t_0 shift). There are many possible reasons for an origin shift. The manufacturer may have put time or mileage on the system as part of production acceptance, but reported that the items are "zero time." The purpose of production acceptance is to eliminate the infant mortality failures. Electronic components may be subjected to burn-in or environmental stress screening for the same purpose. In these cases, the items have aged before being delivered as "zero time" systems. Spare parts such as rubber, chemicals and ball bearings may age in storage and use part of their life on the shelf, requiring a negative t_0. For material properties, where the Weibull abscissa is stress or strain, it may be impossible for fracture or creep or other properties to produce failure near the origin on the scale.

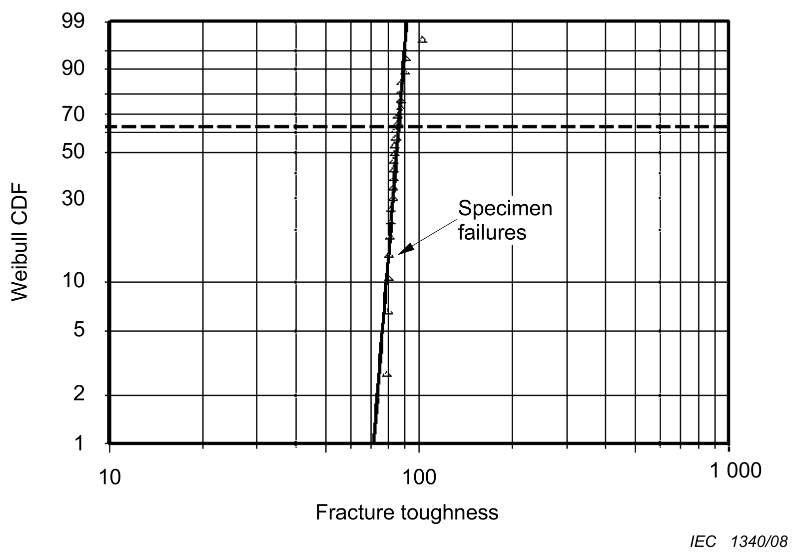


Figure H.1 – Steel-fracture toughness – Curved data

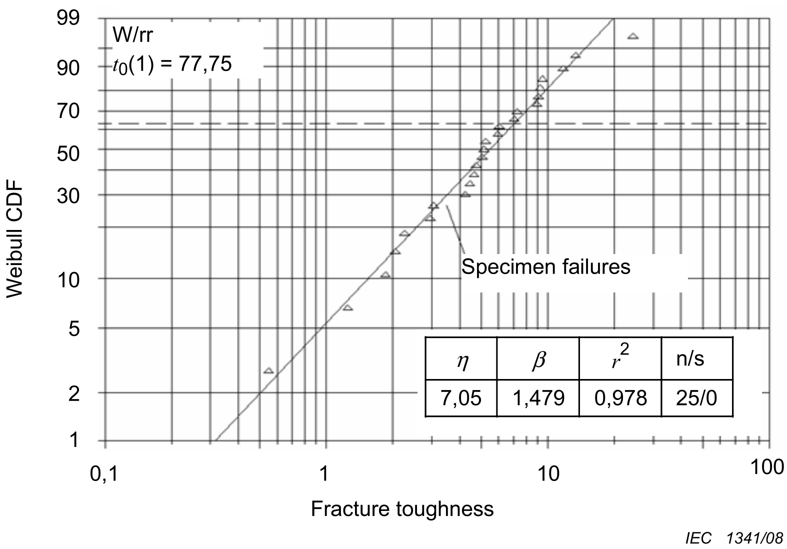


Figure H.2 – t_0 improves the fit of Figure H.1 data

For these reasons and others, the Weibull plot may be curved and needs an origin shift, from zero to t_0.

Three parameters, t_0, β and η, are included in the Weibull cumulative distribution function as shown in Equation (H.1):

$$F(t) = 1 - e^{-((t - t_0)/\eta)^{\beta}} \qquad \text{(H.1)}$$

where

t is the failure time;

t_0 is the starting point or origin of the distribution.

When the t_0 correction is applied to the data, the resulting plot will follow a straight line if the correction is appropriate. Figure H.2 shows the fracture data in Figure H.1 with the t_0 correction. Note that the Weibull ordinate scale and the characteristic life are now in the t_0 domain. To convert back to real time, add t_0 back.

Annex I
(informative)

Constructing Weibull paper

I.1 Weibull probability plotting paper

All probability papers have scales that transform the cumulative distribution function into a straight line. If data are plotted on the transformed scale and if they conform to a straight line, then this supports the contention that the distribution is appropriate.

Weibull graph paper can be constructed using the transformation as described in the following paragraphs.

The Weibull distribution may be defined mathematically as shown in Equation (I.1):

$$\mathrm{F}(t) = 1 - \mathrm{e}^{-(t/\eta)^{\beta}} \tag{I.1}$$

where F(t) defines the cumulative fraction of items that will fail by a time t. The fraction of items that has not failed up to time t is 1 – F(t).

This can be written as (I.2):

$$\frac{1}{1-\mathrm{F}(t)} = \mathrm{e}^{(t/\eta)^{\beta}} \tag{I.2}$$

Taking natural logarithms of both sides twice (decimal logarithms can also be used) gives an equation of a straight line, as shown in (I.3) below:

$$\ln\left[\ln\left[\frac{1}{1-\mathrm{F}(t)}\right]\right] = \beta\ln(t) - \beta\ln(n) \tag{I.3}$$

The equation above is a straight line of the form $y = mx + \mathrm{c}$. Weibull paper is constructed by plotting the cumulative probability of failure using a log-log reciprocal scale against t on a log scale. The slope of the straight line plotted in this manner will be the shape parameter, β, as shown in (I.4).

$$y = \ln\left[\ln\left[)\frac{1}{1-\mathrm{F}(t)}\right]\right]$$

$$m=\beta \tag{I.4}$$

$$x=\ln(t)$$

$$c=-\beta\cdot\ln(\eta)$$

The scale parameter is then calculated from the intercept (value of y for $x = \ln(t) = 0$, i.e. for $t = 1$) as shown in Equation (I.5):

$$\eta = \mathrm{e}^{-\frac{\text{intercept}}{\beta}} \tag{I.5}$$

Table I.1 – Construction of ordinate (Y)

F(t)	lnln(1/(1 - F(t)))	Col 2 Value + 6,91
0,001	–6,91	0
0,010	–4,60	2,31
0,1	–2,25	4,66
0,5	–0,37	6,54
0,9	0,83	7,74
0,99	1,53	8,44
0,999	1,93	8,84

Table I.2 – Construction of abscissa (t)

t h	ln(t)
1	0
2	0,69
3	1,10
4	1,39
5	1,61
10	2,30
15	2,71
20	3,00
100	4,61
1 000	6,91

The Weibull parameter β is estimated by measuring the slope of the line on the Weibull paper or plot.

MRR is the technique which combines the median rank as a plotting position and the least square regression on the Weibull paper as a fitting criterion.

I.2 Using a spreadsheet to construct Weibull plots

Weibull analysis can be carried out using any commercial spreadsheet computer software in a manner similar to the construction of the probability paper. Here, the paper is not prepared for manual plotting, but the spreadsheet graph presents the data in a manner appropriate for determination of Weibull parameters using linear regression.

Table I.3 – Content of data entered into a spreadsheet

Failure No. i	Failure time t_i	Median rank $F_i(t)$ (i - 0,3)/(n+0,4)	x_i	y_i
1	t_1	(1-0,3)/(n+0,4)	ln(t_1)	ln{ln[1/(1-F_1(t)]}
2	t_2	(2-0,3)/(n +0,4)	ln(t_2)	ln{ln[1/(1-F_2(t)]}
3	t_3	(3-0,3)/(n +0,4)	ln(t_3)	ln{ln[1/(1-F_3(t)]}
4	t_4	(4-0,3)/(n+0,4)	ln(t_4)	ln{ln[1/(1-F_4(t)]}
.	.	.	.	
.	.	.	.	
i	t_i	(i-0,3)/(n+0,4)	ln(t_i)	ln{ln[1/(1-F_i(t)]}
.	.	.	.	
.	.	.	.	
n	t_n	(n-0,3)/(n+0,4)	ln(t_n)	ln{ln[1/(1-F_n(t)]}

A practical example is shown in Annex E.

I.3 Commercial software

Commercial software is also available for analysis and Weibull plotting.

Annex J
(informative)

Technical background and references

Annex J gives information on the origin of the procedures of Clause 9 of this standard. The references quoted are all listed in Clause J.5.

J.1 Goodness-of-fit test

This is the Mann-Scheuer-Fertig (1973) test in the form presented by Lawless (1982). The expected values of the standard extreme value order statistics, necessary for the calculation of the ℓ_i, in 9.5, have been approximated as suggested by Blom (1958). This test has been shown to have power comparable to the Shapiro and Brain test (1987) and to Tiku's test as described by Lawless (1982). The latter was slightly better than any available empirical distribution function tests. In addition, the Mann-Scheuer-Fertig test can deal with censored samples.

J.2 Maximum likelihood estimates of *β* and *η*

The equations are those commonly used for singly censored samples. At present, they are the most widespread numerical techniques to obtain Weibull parameters. The form presented in this standard is that of Mann, Schafer and Singpurwalla (1974). Since the numerical procedure of this standard only applies to sample sizes greater than 10, the statistical bias is small.

J.3 Confidence intervals and lower confidence limits

The approach adopted is that of Bain and Engelhardt (1981) for complete samples and Bain and Engelhardt (1986) for censored samples. These references have coefficients generated by Monte Carlo methods, and use asymptotic approximations to adjust the results. Some simple linear and non-linear functions have been fitted to these tables eliminating the need for auxiliary tables. The differences are, in all cases, very minute (~1 %).

An alternative would have been to use Lawless's (1978) conditional methods, but this approach, although theoretically more appealing, would have led to a much more complicated procedure, requiring extensive numerical integration.

The purely asymptotic approach was rejected because the procedure needs to be robust for relatively small samples.

J.4 Accuracy of the standardized procedures

The procedures of this standard have been compared to results published using similar and different techniques. All the examples analysed obtain the same maximum likelihood estimates as the procedure of this standard. The only differences are in the confidence intervals and lower limits. The following is a summary of these comparisons.

J.4.1 Bain and Engelhardt (1986)

Since this is the origin of the standardized procedure, the need to compare the results could be questioned. The interest of the comparison lies in the accuracy of the approximating functions used in this standard. The comparison is as follows:

	Bain & Engelhardt	*Standardized procedure*
90 % confidence interval for β	[1,34 ; 2,73]	[1,34 ; 2,74]
90 % confidence interval for η	[70,7 ; 105,9]	[70 ; 108]
$R_{0,9}(t = 32,46)$	0,801	0,800

J.4.2 Lawless (1978)

The sample analysed has 28 failures for a sample size $n = 40$. Lawless only gives 90 % confidence intervals for k and 95 % lower confidence limits for B_{10} and for $R(t = e^{-1})$. The results are as follows:

	Lawless	*Standardized procedure*
90 % confidence interval for β	[0,783 ; 1,381]	[0,785 ; 1,370]
95 % lower confidence limit for B_{10}	0,066	0,074
$R_{0,95}(t = 0,368)$	0,647	0,644

J.4.3 Meeker and Nelson (1976)

This is an asymptotic technique. The example treated is a sample of 96 locomotives, 37 of them having failed. The censoring time T is slightly greater than the time of the last failure. Since the sample size is fairly large, the asymptotic approach should be accurate in this case. The authors only give a 95 % confidence interval for k and, since there is a 95 % confidence interval for B_{10}, we can derive the 97,5 % lower confidence limit for this quantity.

	Meeker & Nelson	*Standardized procedure*
90 % confidence interval for β	[1,72 ; 3,16]	[1,61 ; 3,04]
97,5 % lower confidence limit for B_{10}	55,4	54,2

J.4.4 Guida (1985)

This paper contains Monte Carlo generated tables to obtain exact lower limits for the maximum likelihood estimates of the reliability in small censored samples ($n \leq 20$). Some randomly generated Weibull distributed samples were used to compare the lower limits of the reliability calculated according to this standard and those obtained by Guida. In all cases, the differences were of the order of 1 % or less.

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