Reliability growth — Statistical test and estimation methods

The European Standard EN 61164:2004 has the status of a British Standard

ICS 03.120.01; 03.120.30

British Standards

National foreword

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English version

Reliability growth - Statistical test and estimation methods
(IEC 61164:2004)

Croissance de la fiabilité -
Tests et méthodes
d'estimation statistiques
(CEI 61164:2004)

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CENELEC

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Foreword

The text of document 56/920/FDIS, future edition 2 of IEC 61164, prepared by IEC TC 56, Dependability, was submitted to the IEC-CENELEC parallel vote and was approved by CENELEC as EN 61164 on 2004-04-01.

This European Standard should be used in conjunction with EN 61014:2003.

The following dates were fixed:

– latest date by which the EN has to be implemented at national level by publication of an identical national standard or by endorsement	(dop)	2005-01-01
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Annex ZA has been added by CENELEC.

Endorsement notice

The text of the International Standard IEC 61164:2004 was approved by CENELEC as a European Standard without any modification.

CONTENTS

INTRODUCTION

This International Standard describes the power law reliability growth model and related projection model and gives step-by-step directions for their use. There are several reliability growth models available, the power law model being one of the most widely used. This standard provides procedures to estimate some or all of the quantities listed in Clauses 4, 6 and 7 of IEC 61014.

Two types of input are required. The first one is for reliability growth planning through analysis and design improvements in the design phase in terms of the design phase duration, initial reliability, reliability goal, and planned design improvements, along with their expected magnitude. The second input, for reliability growth in the project validation phase, is for a data set of accumulated test times at which relevant failures occurred, or were observed, for a single system, and the time of termination of the test, if different from the time of the final failure. It is assumed that the collection of data as input for the model begins after the completion of any preliminary tests, such as environmental stress screening, intended to stabilize the product's initial failure intensity.

Model parameters estimated from previous test results may be used to plan and predict the course of future reliability growth programmes, provided the conditions are similar.

Some of the procedures may require computer programs, but these are not unduly complex. This standard presents algorithms for which computer programs should be easy to construct.

RELIABILITY GROWTH –
STATISTICAL TEST AND ESTIMATION METHODS

1 Scope

This International Standard gives models and numerical methods for reliability growth assessments based on failure data, which were generated in a reliability improvement programme. These procedures deal with growth, estimation, confidence intervals for product reliability and goodness-of-fit tests.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050(191):1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

IEC 60300-3-5:2001, *Dependability management – Part 3-5: Application guide – Reliability test conditions and statistical test principles*

IEC 60605-4, *Equipment reliability testing – Part 4: Statistical procedures for exponential distribution – Point estimates, confidence intervals, prediction intervals and tolerance intervals*

IEC 60605-6, *Equipment reliability testing – Part 6: Tests for the validity of the constant failure rate or constant failure intensity assumptions*

IEC 61014:2003, *Programmes for reliability growth*

3 Terms and definitions

For the purposes of this document, the terms and definitions of IEC 60050(191) and IEC 61014, together with the following terms and definitions, apply.

3.1
reliability goal
desired level of reliability that the product should have at the end of the reliability growth programme

3.2
initial reliability
reliability that is estimated for the product in earlier design stages before any potential failure modes or their causes have been mitigated by the design improvement

3.3
reliability growth model for the design phase
mathematical model that takes into consideration potential design improvements, and their magnitude to express mathematically reliability growth from start to finish during the design period

3.4
average product failure rate
average product failure rate calculated from its reliability as estimated for a predetermined time period

NOTE The change in this failure rate as a function of time is a result of the modifications of the product design.

3.5
delayed modification
corrective modification, which is incorporated into the product at the end of a test

NOTE A delayed modification is not incorporated during the test.

3.6
improvement effectiveness factor
fraction by which the intensity of a systematic failure is reduced by means of corrective modification

3.7
type I test
time-terminated test
reliability growth test which is terminated at a predetermined time, or test with data available through a time which does not correspond to a failure

3.8
type II test
failure-terminated test
reliability growth test which is terminated upon the accumulation of a specified number of failures, or test with data available through a time which corresponds to a failure

4 Symbols

For the purposes of this standard, the following symbols apply.

a) For 6.1, clauses A.1 and B.3:

T	product lifetime such as mission, warranty period or operational time
$R_0(T)$	initial product reliability
λ_{a0}	initial average failure rate of product in design period
$d(t)$	number of design modifications at any time during the design period
α_D	reliability growth rate resultant from fault mitigation
D	total number of implemented design improvements
t_D	total duration of the design period available for the design improvements
t	time variable during the design period from 0 to t_D
$\lambda_a(t)$	average failure rate of product as a function of time during the design period
$\lambda_{aG}(t_D)$	goal average failure rate at the end of the design period t_D
$R_G(T)$	reliability goal of the product to be attained during design period
$R(t,T)$	reliability of product as a function of time and design improvements

b) For 6.2, clauses A.2 and B.4:

$R_G(T)$	reliability goal of the product to be attained during design period
t_D	total duration of the design period
α_D	reliability growth rate during design period
λ_{NS}	rate of non-systematic (or residual) failures
D	total number of predicted or implemented design improvements within design period to address weaknesses
K	total number of distinct classes of fault
j,k,i	general purpose indicators
p_{kj}	probability of j-th design weakness in fault class k resulting in failure during the specified life of the product
η_k	expected number of design weaknesses in fault class k resulting in failure during the specified life of the product
D_k	total number of predicted or implemented design improvements within design period to address faults in fault class k
λ_k	failure rate of design weaknesses categorized in fault class k
$R_I(T)$	initial reliability at time T
$R(T)$	reliability of product as a function of T
t_G	expected time to reach reliability goal

c) For 7.1.1, 7.1.2, Clauses 9, A.4, B.1, and B.2:

D	total number of design modifications carried out during product design period to mitigate identified faults
t_D	total duration of the design period available for potential design modifications
t	time variable (during design period $0 \le t \le t_D$)
$d(t)$	number of design modifications at any given time t during design period from 0 to t_D
α_D	reliability growth rate during the design period
λ_{a0}	initial average failure rate of a product in design
$\lambda_a(t)$	product average failure rate variable as a function of time during the design period (0 to t_D)
$R_0(T)$	initial product reliability calculated for a time T (mission or other predetermined time)
$R_G(T)$	product reliability goal to be attained through design improvement, calculated for a predetermined time

$R(t)$	product reliability increase as a function of time and design improvements
λ_{aG}	goal average failure rate
T	predetermined time during a product life (mission, warranty, life)
λ	scale parameter for the power law model
β	shape parameter for the power law model
CV	critical value for hypothesis test
d	number of intervals for grouped data analysis
$\overline{E}, E_i, E_j$	mean and individual improvement effectiveness factors
I	number of distinct types of category B failures observed
i, j	general purpose indices
K_A	number of category A failures
K_B	number of category B failures
K_i	number of i-th type category B failures observed: $K_B = \sum_{i=1}^{\ell} K_i$
M	parameter of the Cramér-von Mises test (statistical)
N	number of relevant failures
N_i	number of relevant failures in i-th interval
$N(T)$	accumulated number of failures up to test time T
$E[N(T)]$	expected accumulated number of failures up to test time T
$t(i-1)$; $t(i)$	endpoints of i-th interval of test time for grouped data analysis
T	current accumulated relevant test time
T_i	accumulated relevant test time at the i-th failure
T_N	total accumulated relevant test times for type II test
T^*	total accumulated relevant test times for type I test
$\chi^2_\gamma(\nu)$	γ fractile of the χ^2 distribution with ν degrees of freedom
z	general symbol for failure intensity
u_γ	γ fractile of the standard normal distribution
z_p	projected failure intensity
$z(T)$	current failure intensity at time T (relevant test time)
$\theta(T)$	current instantaneous mean time between failures
θ_p	projected mean time between failures
p_j	probability of success at the stage i of product modification in design phase described in Barlow, Proschan and Scheuer discrete reliability growth model
$N(t)$	number of non-random type faults remaining at time t>0 in IBM/Rosner continuous reliability growth model
g	fraction that a product/equipment is debugged as given in the IBM/Rosner reliability growth model

E	exposure time of item
λ_{NS}	failure rate of non-systematic (residual) failures
λ_S	failure rate of systematic failures
μ_k	failure rate of the k-th failure class
D_k	number of potential design weakness in failure class k
$p_{k,j}$	probability that the j-th potential design weakness associated with failure class k will result in failure
t_E	expected design phase time to achieve goal reliability, $R_G(T)$
t_D	duration of design phase
$R_I(T)$	initial reliability at time T
α	parameter to represent the expected growth rate
λ_k	the expected number of design weaknesses associated with failure class k that will result in failure
K_2	proportion of systematic (non-random) faults in product design at start of test
K_1	number of faults in the product design at start of test
q	fraction of faults removed through debugging on reliability growth test
$q(T)$	fraction of original faults removed by time t
t_q	Expected time for removing fraction q of systematic faults in test
$\theta(T)$	Cumulative time between failures

d) Symbols used in the discrete reliability growth model, 7.2:

R_i	reliability, or success probability, of the i-th configuration
f_i = 1- R_i	unreliability, or failure probability, of the i-th configuration
k	number of stages and configurations
n_i	number of trials for stage i
m_i	number of failures for stage i
$t_i = \sum_{j=1}^{i} n_j$	the cumulative number of trials through stage i
$N(t_i)$	the accumulated number of failures up through trial t_i
$\mathrm{E}\left[N(t_i)\right]$	the expected accumulated number of failure up through trial t_i
λ, β	scale and shape parameters for power law and discrete models

5 Reliability growth models in design and test

The basic principles of reliability growth of a product are the same during design and test. This is because both involve identifying and removing weaknesses to improve the product and both measure that improvement by comparing the estimated reliability with the reliability goal.

The difference lies in the tools used to conduct design and test analysis and the models used to measure reliability growth. IEC 61064 provides guidance on the construction of reliability growth programmes and the analysis tools used in design and test. This standard provides details about the models that can be used to measure reliability growth in different stages of the product life cycle and for different types of items, such as repairable or one-shot items.

The mathematical models for reliability growth are constructed to estimate the growth achieved and the projected reliability. Reliability growth models aim to support the planning of reliability improvement programmes by estimating the number and the magnitude of the changes during the design and development process or the test time required to reach a specified reliability goal.

The reliability growth models can be formulated in terms of the failure rate (or intensity) or probability of survival to a specified time (the reliability) as shown in Figure 1.

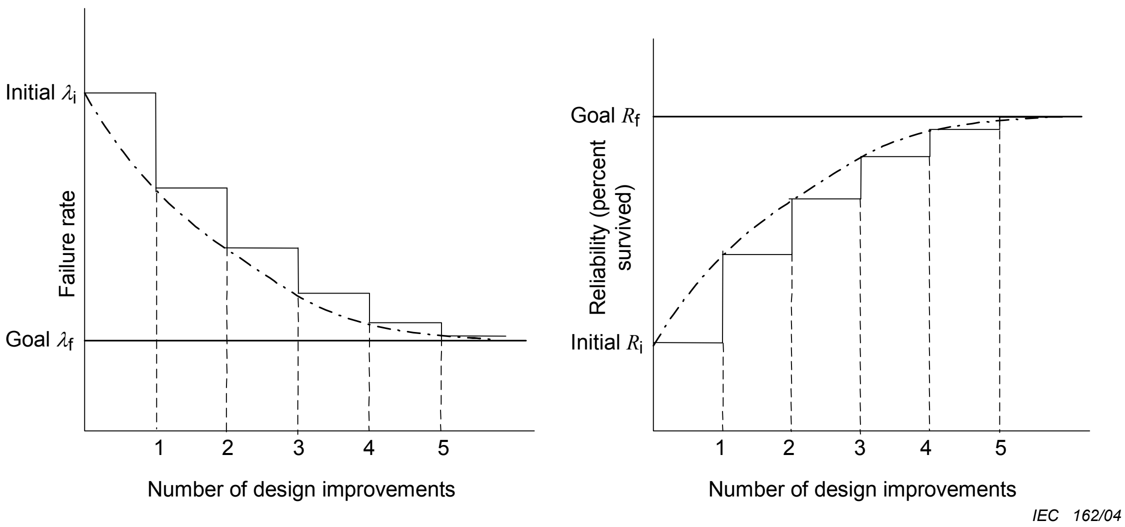

Figure 1 – Planned improvement of the average failure rate or reliability

Within this general framework many models for reliability growth exist. Table 1 provides a summary of the main categories. As well as the distinction between design and test, the type of data available will influence model selection. The continuous category refers to items that operate through time, for example, repaired items. The discrete category refers to data that are collected as if for a success/failure of a trial, for example, one-shot items. The procedures used to estimate reliability growth are labelled classical or Bayesian. The former uses the observed data only, while the latter uses both empirical data from design and test as well as engineering knowledge, for example, regarding the anticipated number of failure modes of concern.

Table 1 – Categories of reliability growth models with clause references

			Time	
			Continuous (time)	**Discrete (number of trials)**
Model type	Design	Classical	6.1	–
		Bayesian	6.2	–
	Test	Classical	7.1	7.2
		Bayesian	–	–

Many reliability models have been developed for analysing test data. This standard presents one of the most popular growth models, the power law (also known as the AMSAA or the Crow model) in both its continuous and discrete forms. This model is a generalization of the Duane reliability growth model due to Crow [1][1]. Although Bayesian variants of these models exist, they are not presented here. A review of the variety of reliability growth models available for analysing test data can be found in Jewell [2, 3] and Xie [4].

There are fewer documented reports of reliability growth models being used in design. Therefore a reliability growth planning model that is a modification of the power law for use in design and a Bayesian variant of the IBM-Rosner model adapted for design have been introduced. However, these are only given for products operating through continuous time.

In general, the choice of a reliability growth model involves a compromise between simplicity and realism. Selection should be made according to the aforementioned criteria such as stage of lifecycle and type of data, as well as by evaluating the validity of the assumptions underpinning a specific model for the context to which it is to be applied. Further details about the assumptions for the models described in this standard are given in Clauses 6 and 7. Note that reliability growth models should not be regarded as infallible nor should they be applied without discretion but used as statistical tools to aid engineering judgement.

6 Reliability growth models used for systems/products in design phase

6.1 Modified power law model for planning of reliability growth in product design phase

6.1.1 General

The statistical procedure for the modified power law model for the planned reliability growth in the product design phase concerns the necessary implementation of the design reliability improvements by mitigation of a failure mode, or by reduction of its probability of occurrence, and the time from the beginning of design to that improvement.

This model is used for planning purposes (and not for data analysis), to estimate the number or the magnitude of improvements in the original design to increase its reliability from that initially assessed to its goal value. The assumption of a power law for this model is justified by the fact that the early improvements will be those that will contribute the most to the reliability improvement, that is, the failure modes with the highest probability of occurrence will be addressed first, followed by improvements of lesser and lesser reliability contribution. The actual reliability values achieved in the course of the design are then plotted corresponding to the design time when they were realized and compared to the model. This model is thus used to plan the strategies necessary for reliability improvement of a design during the available time period from the initial design revision until the design is completed and released for production.

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1 Figures in square brackets refer to the bibliography.

6.1.2 Planning model for the reliability growth during the product design period

The Krasich reliability growth planning model [5] is derived in the following manner. An example of this model, as well as the spread sheet for its easy determination and plotting, given initial and reliability goal of a product, are given in Annex A.

If the initial product reliability for the predetermined product operational life time T was estimated by analysis or test to be $R_0(T)$, then, assuming that its average failure rate is constant, the initial average failure rate of that product corresponding to the time T is:

$$\lambda_{a0} = -\frac{\ln[R_0(T)]}{T} \quad (1)$$

Assumption of applicability of the power law is justified by the fact that the faults which are found to be the highest unreliability contributors are addressed first, and with design modification (fault mitigation) the product failure rate is continuously improved with a function $d(t)$. The failure rate of the product design at any time during the design period is:

$$\lambda_a(t) = \lambda_{a0} \cdot [1 + d(t)]^{-\alpha_D} \quad (2)$$

where

$d(t)$ is the number of design modifications at any time during the design period;

α_D is the reliability growth rate resultant from fault mitigation;

D is the total number of implemented design improvements;

t_D is the total duration of the design period available for the design improvements.

With a linear approximation, the number of design modifications as a function of time can be linearly distributed over the design period:

$$d(t) = D \cdot \frac{t}{t_D} \quad (3)$$

The average failure rate as a function of time then becomes:

$$\lambda_a(t) = \lambda_{a0} \cdot \left(1 + D \cdot \frac{t}{t_D}\right)^{-\alpha_D} \quad (4)$$

If the goal product average failure rate given the product reliability goal is expressed as $R_G(T)$, then the goal average failure rate at the end of design period t_D, is approximated by:

$$\lambda_{aG}(t_D) = \frac{-\ln[R_G(T)]}{T} \quad (5)$$

At the same time:

$$\lambda_{aG}(t_D) = \lambda_{a0} \cdot \left(1 + D \cdot \frac{t_D}{t_D}\right)^{-\alpha_D} = \lambda_{a0} \cdot (1 + D)^{-\alpha_D}$$
$$\lambda_{aG}(t_D) = -\frac{\ln[R_0(T)]}{T} \cdot (1 + D)^{-\alpha_D} \quad (6)$$

Substituting $\lambda_{aG}(t_D)$ with the expression containing reliability goal and solving for D

$$D = e^{\frac{-\ln\left(\frac{[\ln R_G(T)]}{\ln[R_0(T)]}\right)}{\alpha_D}} - 1 \tag{7}$$

Solving the same equation for the growth rate, expressed as a function of design modifications and initial and reliability goal gives:

$$\alpha_D = \frac{\ln\left(\frac{\ln[R_0(T)]}{\ln[R_G(T)]}\right)}{\ln(1+D)} \tag{8}$$

During the design period, continuous improvement of product reliability that it has to have at the time T is a function of time t (the reliability growth model in the time period from 0 to t_D) can be written as:

$$R(t,T) = \exp(-\lambda_a(t)\cdot T) \tag{9}$$

Substituting the expression for the average failure rate, the Krasich reliability growth model for the design phase $0 < t < t_D$, is derived as follows:

$$R(t,T) = \exp\left(-\lambda_{a0}\cdot\left(1 + D\cdot\frac{t}{t_D}\right)^{-\alpha_D}\cdot T\right) \tag{10}$$

$$R(t,T) = \exp\left(\frac{\ln[R_0(T)]}{T}\cdot\left(1 + D\cdot\frac{t}{t_D}\right)^{-\alpha_D}\cdot T\right) \tag{11}$$

In the above equation, expressing D in terms of initial and reliability goal, the reliability growth as a function of time in design period, available for design improvements becomes:

$$R(t,T) = R_0(T)^{\left\{\left[\frac{\left[t_D + t\cdot\left(\frac{\ln[R_G(T)]}{\ln[R_0(T)]}\right)^{-\frac{1}{\alpha_D}} - t\right]}{t_D}\right]^{-\alpha_D}\right\}} \tag{12}$$

6.1.3 Tracking the achieved reliability growth

Tracking of the achieved reliability growth means a simple recalculation of the assessed product reliability at the time the design was improved to account for the modifications. The reliability value calculated for the same predetermined life or mission period is simply plotted on the same plot with the reliability growth model for the corresponding design time.

The resultant entries to the graph can then be fitted with a best-fit line (power), or the values on the graph may be simply connected with the straight lines and the resultant achieved reliability is compared to the reliability growth model.

Use of fault tree analysis with commercially available software makes assessment of the reliability improvement easy and quick to accomplish and track as the product reliability is automatically calculated based on the changes.

After completion of the product design and with the introduction of the product validation phase, the planned reliability growth test may further improve product reliability or uncover failure modes that were not accounted for during analytical evaluations. The final reliability assessment of the completed design can then serve as the reliability goal for the reliability growth testing.

An example of practical derivation and application of the planning growth model for reliability improvement in design phase is shown in A.1.1. This real life example shows step by step how the model is constructed and how it is used.

6.2 Modified Bayesian IBM-Rosner model for planning reliability growth in design phase

6.2.1 General

A model is presented to describe the growth of reliability during the design phase of a repairable item prepared by Quigley and Walls [6] to [8] and is based on a Bayesian adaptation of the IBM-Rosner model [9] which was developed for analysing test data and is described in 7.1.2.

It is assumed a design has been developed to a sufficient level of detail to provide an initial estimate of reliability. It is further assumed that the reliability goal is specified. Modifications to the design will be made with a view to improving reliability until the goal is achieved. The model aims to capture the possible timings of the design modifications.

The model assumes that design review and re-assessments result in modifications with the aim of improving reliability and achieving the goal. The rate of growth as measured by advancing the initial reliability to the reliability goal is a function of the removal of aspects of design that contribute to systematic failures. It is assumed through using the model under consideration that there are greater improvements to reliability in the earlier re-design compared with later stages of re-design.

The model can be used in two ways:

a) to predict the length of time required to achieve reliability goal by forecasting the reliability of the design – this assumes that the expected growth rate can be quantified; or

b) to estimate the growth rate required to achieve the reliability goal from the initial estimate during a specified design period duration – this assumes that the duration of the design phase is fixed.

Details concerning the mathematical formulation of the model are given in Annex B.

6.2.2 Data requirements

Data are required concerning the reliability goal ($R_G(T)$) and <u>either</u> the duration of the design phase (t_D) <u>or</u> the expected growth rate during design (a_D) according to the purpose of the model application.

The failure rate of non-systematic failures (λ_{NS}) should be specified. This may be estimated from historical data for similar product designs operating in nominally identical environments.

All potential design improvements to mitigate the D potential weaknesses should be identified and may be allocated to one of K fault classes as appropriate.

The probability of each design weakness within each fault class resulting in failure during the specified life of the product should be estimated. This may be based on engineering judgement. The probability for the j-th design weakness in fault class k is given by p_{kj}.

The expected number of design weaknesses in fault class k (η_k) resulting in failure if no modifications are implemented to the product design can be calculated using

$$\eta_k = \sum_{j=1}^{D_k} p_{kj} \tag{13}$$

where D_k is the total number of design weaknesses expected in fault class k.

The failure rate for each fault class is required. These may be estimated from historical data for similar product designs operating in nominally identical environments. The failure rate for fault class k is given by λ_k.

6.2.3 Estimates of reliability growth and related parameters

In this subclause, equations are given to compute the key parameters of the reliability growth model.

The initial reliability of the design at time T is calculated by

$$R_\mathrm{I}(T) = \exp\left\{-\left[\lambda_\mathrm{NS} T + \sum_{k=1}^{K} \eta_k \left(1 - e^{-\lambda_k T}\right)\right]\right\} \tag{14}$$

The reliability growth of the design at time T is given by

$$R(T) = \exp\left\{-\left[\lambda_\mathrm{NS} T + \sum_{k=1}^{K} \eta_k e^{-\alpha_\mathrm{D} T} \left(1 - e^{-\lambda_k T}\right)\right]\right\} \tag{15}$$

If a growth rate is to be estimated, then the expected growth rate required to reach the reliability goal, given the reliability goal and the specified duration of the design period, is given by

$$\alpha_\mathrm{D} = \frac{\ln\left[\dfrac{\sum_{k=1}^{K} \eta_k \left(1 - e^{-\lambda_k T}\right)}{-\ln\left[R_\mathrm{G}(T)\right] - \lambda_\mathrm{NS} T}\right]}{t_\mathrm{D}} \tag{16}$$

If a growth rate has been specified, an estimate of the expected time to reach the reliability goal is given by

$$t_\mathrm{G} = \frac{\ln\left[\dfrac{\sum_{k=1}^{K} \eta_k \left(1 - e^{-\lambda_k T}\right)}{-\ln\left[R_\mathrm{G}(T)\right] - \lambda_\mathrm{NS} T}\right]}{\alpha_\mathrm{D}} \tag{17}$$

6.2.4 Tracking reliability growth during design phase

The tracking of the reliability growth with this modified Bayesian IBM-Rosner model is the same as that described in 6.1.3.

7 Reliability growth planning a tracking in the product reliability growth testing

7.1 Continuous reliability growth models

7.1.1 The power law model

The statistical procedures for the power law reliability growth model use the original relevant failure and time data from the test. Except in the projection technique (see 9.6), the model is applied to the complete set of relevant failures (as in IEC 61014, Figure 2 and Figure 4, characteristic 3) without subdivision into categories.

The basic equations for the power law model are given in this subclause. Background information on the model is given in Annex B.

The expected accumulated number of failures up to test time T is given by:

$$E[N(T)]=\lambda T^{\beta}, \text{with } \lambda > 0, \beta > 0, T > 0 \tag{18}$$

where

λ is the scale parameter;

β is the shape parameter (a function of the general effectiveness of the improvements; $0 < \beta < 1$, corresponds to reliability growth; $\beta = 1$ corresponds to no reliability growth; $\beta > 1$ corresponds to negative reliability growth).

The current failure intensity after T h of testing is given by:

$$z(T) = \frac{\mathrm{d}}{\mathrm{d}T}\mathrm{E}[N(T)]=\lambda\beta T^{\beta-1}, \text{with } T > 0 \tag{19}$$

Thus, parameters λ and β both affect the failure intensity achieved in a given time. The equation represents in effect the slope of a tangent to the $N(T)$ against T characteristic at time T as shown in IEC 61014, Figure 9.

The current mean time between failures after T h of testing is given by:

$$\theta(T) = \frac{1}{z(T)} \tag{20}$$

Methods are given in 9.1 and 9.2 for maximum likelihood estimation of the parameters λ and β. Subclause 9.3 gives goodness-of-fit tests for the model, and 9.4 and 9.5 discuss confidence interval procedures. An extension of the model for reliability growth projections is given in 9.6.

The model has the following characteristic features:

– It is simple to evaluate.

- When the parameters have been estimated from past programmes, it is a convenient tool for planning future programmes employing similar conditions of testing and equal improvement effectiveness (see Clause 7, and IEC 61014, 6.4.1 to 6.4.7).
- It gives the unrealistic indications that $z(T) = \infty$ at T = 0 and that growth can be unending, that is $z(T)$ tends to zero as T tends to infinity. However, these limitations do not generally affect its practical use.
- It is relatively slow and insensitive in indicating growth immediately after a corrective modification, and so may give a low (that is, pessimistic) estimate of the final $\theta(T)$, unless projection is used (see 9.6).
- The normal evaluation method assumes the observed times to be exact times of failure, but an alternative approach is possible for groups of failures within a known time period (see 9.2.2).

7.1.2 The fixed number of faults model

This model, also known as the IBM/Rosner model [9], assumes the following:

- there are random (constant intensity function) failures occurring at a rate z; and
- there is a fixed, but unknown, number of non-random design, manufacturing and workmanship faults present in the product at the beginning of testing.

The model limitation is the assumption that the effectiveness factor for fault mitigation is equal to unity.

Rate of change of $N(T)$, with respect to time, is proportional to the number of non-random faults remaining at time T:

$$\frac{\mathrm{d}\,N(T)}{\mathrm{d}\,T} = -K_2 \cdot N(T)$$
$$\text{thus:} \tag{21}$$
$$N(T) = e^{-K_2 \cdot T + c}$$

If the number of faults at T = 0 is K_1, then:

$$N(T) = K_1 \cdot e^{-K_2 \cdot T} \tag{22}$$

with an assumption that

$$\mathrm{T} > 0, K_1, K_2 > 0$$

If $E[N(T)]$ is to be the expected cumulative number of faults up to time T, then:

$$E[N(T)] = z \cdot T + K_1 \cdot \left(1 - \mathrm{e}^{-K_2 \cdot T}\right) \tag{23}$$

The equation above means that by the time T, the total number of faults is equal to the sum of random and non-random faults. Here $E[N(0)]$ = 0.

With time approaching infinity:

$$E[N(T)] \to \infty \tag{24}$$

Since the model is non-linear, the estimate of λ, K_1, and K_2 has to be accomplished by iterative methods.

The model allows prediction of the time when the product is "q" fraction debugged (fraction of the original non-random faults removed, while $0<q<1$).

The number of non-random faults removed by the time t is:

$$N(0) - N(t) = K_1 - K_1 \cdot \mathrm{e}^{-K_2 \cdot T} \tag{25}$$

Hence the fraction of the original faults removed by time t is:

$$q(T) = \frac{K_1 - K_1 \cdot \mathrm{e}^{-K_2 \cdot T}}{K_1} = 1 - \mathrm{e}^{-K_2 \cdot T} \tag{26}$$

Thus, having estimated K_2 as ($\hat{K}_2$), the time necessary for a desired q, of non-random faults to have been removed is found as:

$$t_q = -\frac{\ln(1-q)}{\hat{K}_2} \tag{27}$$

To determine the number of non-random faults remaining at the time T is: $K_1 \cdot \mathrm{e}^{-\hat{K}_2 \cdot T}$

As in other continuous models, the dependent variable is the cumulative mean time between failures, $Y(t)$, where:

$$\theta(T) = \frac{t}{\text{Total number of failures in } (0,t)} \tag{28}$$

7.2 Discrete reliability growth model

7.2.1 Model description

This model, developed by L. Crow [1], is the discrete version of the power law model for reliability growth.

For this discrete model the data consist of sequences of dichotomous occurrences representing success or failure outcomes from successive testing of a product. The product testing is conducted in trials over successive stages with corrective action taking place after each stage. Each trial results in either a success or failure. The system configuration is held fixed during each stage of testing so that each trial within a stage has the same probability of a success or failure. Based on information obtained from the observed failures during each stage of testing, corrective actions are made to improve the product reliability. At the end of each stage these corrective actions are introduced into the next configuration. This updated configuration is tested over the next stage, which consists of a fixed number of trials. This discrete model is applicable to one-shot systems such as missiles.

This situation for discrete data is similar to the grouped data case for the power law model with continuous data and test time. Data are grouped in both cases. In the discrete case, data consists of the number of trials in each group or stage, in addition to the number of trials in each stage resulting in failure. In the continuous case, the data consist of the test time in each group and the number of failures observed.

An important feature of the discrete model is that it has the same growth pattern and learning curve characteristic as the power law model for continuous data.

It is assumed that the system configuration is being modified in k stages and that the reliability is held constant during each stage of testing. At stage i the reliability of the i-th configuration is R_i, i =1, ..., k, and $R_1 > 0$. Testing is conducted in trials, and each trial during stage i has the same probability R_i of success. This model provides maximum likelihood estimates $\hat{R}_i$ of the reliability for each configuration, i =1, ..., k. The reliability estimate of the final configuration tested is $\hat{R}_k$. If no further changes are made to the product then the estimate $\hat{R}_k$ represents the final reliability resulting from the testing and corrective actions.

There are k ordered stages with stage 1 being the first and stage k being last. For stage i data consist of m_i failures in n_i trials, i, where i =1, ..., k.

Let $t_i = \sum_{j=1}^{i} n_j$ be the cumulative number of trials through stage i. That is, at the end of the i-th stage, data consist of a sequence of t_i dichotomous trials representing success or failure outcomes from successive testing of the product.

During the i-th stage, the reliability R_i is held constant. That is, for each trial during the i-th stage of testing, the probability of success is R_i and the probability of failure is

$$f_i = 1 - R_i \tag{29}$$

Under this model the failure probability for configuration i is given by

$$f_i = \frac{\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta}}{n_i} \tag{30}$$

where

$t_0 = 0$, $n_i = t_i - t_{i-1}$ and $\lambda > 0, \beta > 0$ are parameters;

λ is a scale parameter;

β is the shape parameter.

Let $N(t_i)$ be the accumulated number of failures up through trial t_i, also, let $E[N(t_i)]$ be the expected accumulated number of failure up through trial t_i .

Then, under this discrete model,

$$E[N(t_i)] = \sum_{j=1}^{i} n_i f_i = \lambda t_i^{\beta} \tag{31}$$

This is the same reliability growth characteristic exhibited by the power law model for continuous data.

7.2.2 Estimation

These methods are suitable when testing consists of discrete trials over k successive stages. Corrective actions to improve reliability are incorporated at the end of the stages, resulting in an improvement in the reliability of the next configuration. At stage i the reliability of the i-th configuration is R_i, i = 1, …, k.

Data consist of the number of trials n_i for each configuration and the corresponding number of failures m_i, i = 1, …, k. It is recommended that m_1 > 0.

The reliability R_i is given as

$$R_i = 1 - f_i \tag{32}$$

where $f_i = \dfrac{\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta}}{n_i}$, $\lambda > 0, \beta > 0$

The maximum likelihood estimates of the parameters λ and β are values satisfying

$$\sum_{i=1}^{K} [H_i] \times [S_i] = 0 \tag{33}$$

$$\sum_{i=1}^{K} [U_i] \times [S_i] = 0 \tag{34}$$

where

$$H_i = \left[t_i^{\hat{\beta}} \ln(t_i) - t_{i-1}^{\hat{\beta}} \ln(t_{i-1}) \right] \tag{35}$$

$$U_i = \left[t_i^{\hat{\beta}} - t_{i-1}^{\hat{\beta}} \right] \tag{36}$$

$$S_i = \left[\frac{m_i}{\hat{\lambda} t_i^{\hat{\beta}} - \hat{\lambda} t_{i-1}^{\hat{\beta}}} - \frac{n_i - m_i}{n_i - \hat{\lambda} t_i^{\hat{\beta}} + \hat{\lambda} t_{i-1}^{\hat{\beta}}} \right] \tag{37}$$

For the maximum likelihood estimates $\hat{\lambda}$ and $\hat{\beta}$, the estimates of the failure probability and reliability for the i-th configuration are given by

$$\hat{f}_i = \frac{\hat{\lambda} t_i^{\hat{\beta}} - \hat{\lambda} t_{i-1}^{\hat{\beta}}}{n_i} \tag{38}$$

and $\hat{R}_i = 1 - \hat{f}_i$, respectively, i = 1, …, k.

The reliability of the system configuration during the last stage, k, is estimated by $\hat{R}_k$.

8 Use of the power law model in planning reliability improvement test programmes

As inputs to the procedure described in 6.4.2.3 and 6.4.4 of IEC 61014, two quantities have to be predicted by means of reliability growth models:

- the accumulated relevant test time in hours expected to be necessary to meet the aims of the programme;
- the number of relevant failures expected to occur during this time period.

The accumulated relevant test time is then converted to calendar time from the planned test time per week or month, making allowance for the predicted total downtime (see below) and other contingencies, and the number of relevant failures is increased by judgement to include non-relevant failures and used to predict total downtime.

The inputs to the model for these calculations will be the assumed parameters for the model, as already estimated from one or more previous programmes, and judged to be valid for the future application by similarity of the test items, test environment, management procedures and other significant influences.

9 Statistical test and estimation procedures for continuous power law model

9.1 Overview

The procedures in 9.2 utilize product failure data during a test programme to estimate the progress of reliability growth, and to estimate, in particular, the final product reliability at the end of the test. The reliability growth, which is assessed, is the result of corrective modifications incorporated into the product during test. The procedures discussed in 9.2.1 assume that the accumulated test time to each relevant failure is known. Subclause 9.2.2 addresses the situation where actual failure times are not known and failures are grouped in intervals of test time.

Type I tests, which are concluded at T^*, which is not a failure time, and type II tests, which are concluded at failure time T_N, use slightly different formulae, as indicated in 9.2.1.

An appropriate goodness-of-fit test, as described in 9.3, shall be performed after the growth test procedures of 9.2.1 and 9.2.2.

Subclause 9.6 addresses the situation where the corrective modifications are incorporated into the product at the end of the test as delayed modifications. The projection technique estimates the product reliability resulting from these corrective modifications.

9.2 Growth tests and parameter estimation

9.2.1 Case 1 – Time data for every relevant failure

This method applies only where the time of failure has been logged for every failure.

Step 1: Exclude non-relevant failures by reference to 6.4.5 of IEC 61014 and/or other appropriate documentation.

Step 2: Assemble into a data set the accumulated relevant test times (as defined in 7.4 of IEC 60300-3-5) at which each relevant failure occurred. For type I tests, note also the time of termination of the test.

Step 3: Calculate the test statistic

$$U=\frac{\sum_{i=1}^{N} T_\text{i} - N\frac{T^*}{2}}{T^*\sqrt{\frac{N}{12}}} \qquad \text{[type I]} \qquad (39)$$

or

$$U=\frac{\sum_{i=1}^{N-1} T_\text{i} - (N-1)\frac{T_N}{2}}{T_N\sqrt{\frac{N-1}{12}}} \qquad \text{[type II]} \qquad (40)$$

where

N is the total number of relevant failures;

T^* is the total accumulated relevant test time for type I test;

T_N is the total accumulated relevant test times for type II test;

T_i is the accumulated relevant test time at the i-th failure.

Under the hypothesis of zero growth (i.e. the failure times follow a homogeneous Poisson process), the statistic U is approximately distributed as a standard normal random variable with mean 0 and standard deviation 1. The statistic U can be used to test if there is evidence of reliability growth, positive or negative, independent of the reliability growth model.

A two-sided test for positive or negative growth at the α significance level has critical values $u_{1-\alpha/2}$ and $-u_{1-\alpha/2}$, where $u_{1-\alpha/2}$ is the $(1-\alpha/2)\cdot 100$ % fractile of the standard normal distribution. If:

$$U < -u_{1-\alpha/2} \text{ or } U > u_{1-\alpha/2} \qquad (41)$$

then there is evidence of positive or negative reliability growth, respectively, and the analysis is continued with step 4.

If, however,

$$-u_{1-\alpha/2} < U < u_{1-\alpha/2} \qquad (42)$$

then there is not evidence of positive or negative reliability growth at the α significance level and the growth analysis is terminated. In this case, the hypothesis of exponential times between successive failures (or a homogeneous Poisson process) is accepted at the α significance level. The critical values $u_{1-\alpha/2}$ and $-u_{1-\alpha/2}$ correspond to a one-sided test for positive or negative growth, respectively, at the $\alpha/2$ significance level.

At the 0,20 significance level, the critical values for a two-sided test are 1,28 and −1,28. The critical value 1,28 corresponds to a one-sided test for positive growth at the 10 % significance level. For other levels of significance, choose the appropriate critical values from a table of fractiles for the standard normal distribution.

Step 4: Calculate the summation:

$$S_1 = \sum_{i=1}^{N} \ln\left(T^*/T_i\right) \qquad \text{[type I]} \tag{43}$$

or

$$S_1 = \sum_{i=1}^{N} \ln\left(T_N/T_i\right) \qquad \text{[type II]} \tag{44}$$

Step 5: Calculate the (unbiased) estimate of the parameter β from the formula:

$$\hat{\beta} = \frac{N-1}{S_1} \qquad \text{[type I]} \tag{45}$$

or

$$\hat{\beta} = \frac{N-2}{S_1} \qquad \text{[type II]} \tag{46}$$

Step 6: Calculate the estimate of the parameter λ from the formula:

$$\hat{\lambda} = N/\left(T^*\right)^{\hat{\beta}} \qquad \text{[type I]} \tag{47}$$

or

$$\hat{\lambda} = N/\left(T_N\right)^{\hat{\beta}} \qquad \text{[type II]} \tag{48}$$

Step 7: Calculate the estimated failure intensity $\hat{z}(T)$ and mean time between failures $\hat{\theta}(T)$, for any test time $T > 0$, from the formulae:

$$\hat{z}(T)=\hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \tag{49}$$

$$\hat{\theta}(T)=1/\hat{z}(T) \tag{50}$$

NOTE 1 $\hat{z}(T)$ and $\hat{\theta}(T)$ are estimates of the "current" failure intensity and MTBF at time $T > 0$ for T over the range represented by the data. "Extrapolated" estimates for a future time T during the test programme, or at its expected termination time, may be obtained similarly, but used with the usual caution associated with extrapolation. Extrapolated estimates should not extend past the expected termination time.

NOTE 2 If the test programme is completed, then $\hat{\theta}(T)$, for $T = T^*$ or $T = T_N$ (as appropriate), estimates the MTBF of the system configuration on test at the end of the test programme.

9.2.2 Case 2 – Time data for groups of relevant failures

This alternative method is employed when the data set consists of known time intervals, each containing a known number of failures. It is important to note that the interval lengths and the number of failures per interval need not be constant.

The test period is over the interval (0; T) and is partitioned into d intervals at times, $0 < t(1) < t(2) < \ldots < t(d)$. The i-th interval is the time period between $t(i-1)$ and $t(i)$, $i = 1, 2, \ldots, d$, $t(0) = 0$, $t(d) = T$. The partition times $t(i)$ may assume any values between 0 and T.

Step 1: Exclude non-relevant failures by reference to 6.4.5 of IEC 61014 and/or other appropriate documentation.

Step 2: Assemble into a data set the number of relevant failures N_i recorded in the i-th interval $\left(t(i-1); t(i)\right), i = 1, \ldots, d$.

The total number of relevant failures is

$$N = \sum_{i=1}^{d} N_{\mathrm{i}} \; . \qquad (51)$$

For each interval, $\rho_i N$ shall be not less than 5 (if necessary, adjacent intervals should be combined before this test) where:

$$\rho_{\mathrm{i}} = \frac{t(i) - t(i-1)}{t(d)} \qquad (52)$$

Step 3: For the d intervals (after combination if necessary) and corresponding failures N_{i}, calculate the statistic

$$X^2 = \sum_{i=1}^{d} \frac{\left(N_{\mathrm{i}} - \rho_{\mathrm{i}} N\right)^2}{\rho_{\mathrm{i}} N} \qquad (53)$$

Under the hypothesis of zero growth (i.e. the failure times follow a homogeneous Poisson process), the statistic X^2 is approximately distributed as a χ^2 random variable with d -1 degrees of freedom. The statistic X^2 can be used to test if there is evidence of reliability growth, positive or negative, independent of the reliability growth model.

A two-sided test for positive or negative growth at the α significance level has critical value

$$CV = \chi^2_{1-\alpha}(d-1) \qquad (54)$$

If $X^2 \geq CV$, then there is evidence of positive or negative reliability growth and the analysis is continued with step 4.

If $X^2 < CV$, then there is no evidence of positive or negative reliability growth at the α significance level and the growth analysis is terminated. In this case, the hypothesis of exponential times between successive failures (or a homogeneous Poisson process) is accepted at the α significance level.

Critical values $\chi^2_{1-\alpha}(d-1)$ for various significance levels α and several degrees of freedom ($d-1$) can be found in tables of the χ^2 distribution, for example in IEC 60605-4 and IEC 60605-6.

Step 4: For the original data set assembled in step 2, calculate the maximum likelihood estimate of the shape parameter β. The maximum likelihood estimate of β is the value $\hat{\beta}$, which satisfies the following equation:

$$\sum_{i=1}^{d} N_i \left[\frac{t(i)^{\hat{\beta}} \ln t(i) - t(i-1)^{\hat{\beta}} \ln t(i-1)}{t(i)^{\hat{\beta}} - t(i-1)^{\hat{\beta}}} - \ln t(d) \right] = 0 \tag{55}$$

Note that $t(0) = 0$ and also $t(0) \cdot \ln t(0) = 0$ All $t(\cdot)$ terms may be normalized with respect to $t(d)$ and then the final term $\ln t(d)$ disappears. An iterative method has to be used to solve this equation for $\hat{\beta}$.

Step 5: Calculate the estimate of the parameter λ from the formula

$$\hat{\lambda} = N/t(d)^{\hat{\beta}} \tag{56}$$

Step 6: Calculate the estimated failure intensity $\hat{z}(T)$ and mean time between failures $\hat{\theta}(T)$ for any test time $T > 0$ from the formulae:

$$\hat{z}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} \tag{57}$$

$$\hat{\theta}(T) = 1/\hat{z}(T) \tag{58}$$

NOTE 1 $\hat{z}(T)$ and $\hat{\theta}(T)$ are estimates of the "current" failure intensity and MTBF at time $T > 0$, for T over the range represented by the data. "Extrapolated" estimates for a future time T during the test phase, or at its expected termination time, may be obtained similarly, but used with the usual caution associated with extrapolation. Extrapolated estimates should not extend past the expected termination time.

NOTE 2 If the test programme is completed, then $\hat{\theta}(T)$ for $T = t(d)$, estimates the MTBF of the system configuration on test at the end of the test phase.

9.3 Goodness-of-fit tests

9.3.1 General

If individual failure times are available, use case 1; otherwise, use case 2.

9.3.2 Case 1 – Time data for every relevant failure

The estimation method included in 9.2.1 shall first be used to estimate the shape parameter β. The Cramér-von Mises statistic is then given by the following expression:

$$C^2(M) = \frac{1}{12M} + \sum_{i=1}^{M} \left[\left(\frac{T_i}{T} \right)^{\hat{\beta}} - \frac{2i-1}{2M} \right]^2 \tag{59}$$

where

$M = N$ and $T = T^*$ for type I tests;

$M = N - 1$ and $T = T_N$ for type II tests;

$T_1 < T_2 < ... < T_M$.

Table 2 gives critical values of this statistic for 10 % significance level. If the statistic $C^2(M)$ exceeds the critical value corresponding to M in the table, then the hypothesis that the power law model adequately fits the data shall be rejected. Otherwise, the model shall be accepted.

When the failure times are known, the graphical procedure described below may be used to obtain additional information about the correspondence between the model and the data.

For the graphical procedure, an estimate of the expected time to the j-th failure, $E[T_j]$, is plotted against the observed time to the j-th failure, T_j. From Annex B, $E[T_j]$ may be estimated by:

$$\hat{E}[T_j] = \left(\frac{j}{\hat{\lambda}}\right)^{1/\hat{\beta}}, \text{ with } j = 1,...,N \tag{60}$$

The expected failure times, $\left(\frac{j}{\hat{\lambda}}\right)^{1/\hat{\beta}}$, are then plotted against the observed failure times, T_j, on identical linear scales, as in the example of Figure A.3. The visual agreement of these points with a line at 45° through the origin is a subjective measure of the applicability of the model.

9.3.3 Case 2 – Time data for groups of relevant failures

This test is suitable only when $\hat{\beta}$ has been estimated using grouped data, as in 9.2.2. The expected number of failures in the time interval $(t(i-1); t(i))$ is approximated by:

$$e_i = \hat{\lambda}\left[t(i)^{\hat{\beta}} - t(i-1)^{\hat{\beta}}\right] \tag{61}$$

For each interval, e_i shall not be less than 5, and if necessary, adjacent intervals should be combined before the test. For d intervals (after combination if necessary) and with N_i the same as in 9.2.2, calculate the statistic:

$$X^2 = \sum_{i=1}^{d} \frac{(N_i - e_i)^2}{e_i} \tag{62}$$

The critical values of this statistic for d–2 degrees of freedom can be found in tables of the χ^2 distribution, for example in IEC 60605-4 and IEC 60605-6. If the critical value at a 10 % level of significance is exceeded, then the hypothesis that the power law model adequately fits the grouped data shall be rejected.

When the data set consists of known time intervals, each containing a known number of failures, the graphical procedure described below may be used to obtain additional information about the correspondence between the model and the data.

For each interval endpoint $t(i)$, the number of observed failures from 0 to $t(i)$ is

$$N(t(i)) = \sum_{j=1}^{i} N_j \qquad (63)$$

The expected number of failures $E[N(t(i))]$ is estimated by

$$\hat{E}[N(t(i))] = \hat{\lambda} t(i)^{\hat{\beta}} \qquad (64)$$

This gives

$$\frac{\hat{E}[N(t(i))]}{t(i)} = \hat{\lambda} t(i)^{\hat{\beta}-1} \qquad (65)$$

The graphical procedure consists of plotting

$$\ln\left\{\frac{N(t(i))}{t(i)}\right\}, i = 1,\ldots, d \qquad (66)$$

and also plotting the line:

$$\ln\hat{\lambda} + (\hat{\beta}-1)\ln T, T > 0 \qquad (67)$$

as in the example of Figure A.4. See Annex B for the relationship between $\ln\hat{\lambda}$ and δ and $(\hat{\beta}-1)$ and $-\alpha$.

For $\hat{\beta} < 1$, this line is decreasing. The visual agreement of these points with this line is a subjective measure of the applicability of the model.

9.4 Confidence intervals on the shape parameter

9.4.1 General

The shape parameter β in the power law reliability growth model determines if the model reflects growth and to what degree. If $0 < \beta < 1$, there is positive reliability growth, if $\beta = 1$, there is no reliability growth, and if $\beta > 1$, there is negative reliability growth.

For a two-sided confidence interval on β when individual failure times are available, use case 1. For grouped failure times, use case 2.

9.4.2 Case 1 – Time data for every relevant failure

Step 1: Calculate $\hat{\beta}$ from step 5 in 9.2.1.

Step 2:

a) Type I test

For a two-sided 90 % confidence interval on β, calculate

$$D_{\mathrm{L}} = \frac{\chi^2_{0,05}(2N)}{2(N-1)} \tag{68}$$

$$D_{\mathrm{U}} = \frac{\chi^2_{0,95}(2\mathrm{N})}{2(N-1)} \tag{69}$$

The fractiles can be found in tables of the χ^2 distribution, for example in IEC 60605-4 and IEC 60605-6.

The lower confidence limit on β is

$$\beta_{\mathrm{LB}} = D_{\mathrm{L}} \cdot \hat{\beta} \tag{70}$$

The upper confidence limit on β is

$$\beta_{\mathrm{UB}} = D_{\mathrm{U}} \cdot \hat{\beta} \tag{71}$$

One-sided 95 % lower and upper limits on β are β_{LB} and β_{UB}, respectively.

b) Type II test

For a two-sided 90 % confidence interval on β, calculate

$$D_{\mathrm{L}} = \frac{\chi^2_{0,05}(2(N-1))}{2(N-2)} \tag{72}$$

$$D_{\mathrm{U}} = \frac{\chi^2_{0,95}(2(N-1))}{2(N-2)} \tag{73}$$

The lower confidence limit on β is

$$\beta_{\mathrm{LB}} = D_{\mathrm{L}} \cdot \hat{\beta} \tag{74}$$

The upper confidence limit on β is

$$\beta_{\mathrm{UB}} = D_{\mathrm{U}} \cdot \hat{\beta} \tag{75}$$

One-sided 95 % lower and upper limits on β are β_{LB} and β_{UB}, respectively.

9.4.3 Case 2 – Time data for groups of relevant failures

These confidence interval procedures are suitable when $\hat{\beta}$ has been estimated from grouped data as in 9.2.2.

Step 1: Calculate $\hat{\beta}$ as in 9.2.2, step 4.

Step 2: Calculate

$$P(i) = \frac{t(i)}{t(d)}, \text{with } i = 1, 2, ..., d \tag{76}$$

Step 3: Calculate the expression

$$A=\sum_{i=1}^{d}\frac{\left[P(i)^{\hat{\beta}}\cdot\ln P(i)^{\hat{\beta}}-P(i-1)^{\hat{\beta}}\cdot\ln P(i-1)^{\hat{\beta}}\right]^2}{\left[P(i)^{\hat{\beta}}-P(i-1)^{\hat{\beta}}\right]} \tag{77}$$

Step 4: Calculate

$$C=\frac{1}{\sqrt{A}} \tag{78}$$

Step 5: For an approximate two-sided 90 % confidence interval on β, calculate

$$S=\frac{(1{,}64)\cdot C}{\sqrt{N}} \tag{79}$$

where N is the total number of failures.

Step 6: The lower confidence limit on β is

$$\beta_{\mathrm{LB}}=\hat{\beta}(1-S) \tag{80}$$

The upper confidence limit on β is

$$\beta_{\mathrm{UB}}=\hat{\beta}(1+S) \tag{81}$$

One-sided 95 % lower and upper limits on β are β_{LB} and β_{UB}, respectively.

9.5 Confidence intervals on current MTBF

9.5.1 General

From 9.2.1, step 7, and 9.2.2, step 6, $\hat{\theta}(T)$ estimates the current MTBF, $\theta(T)$. For confidence intervals on $\hat{\theta}(T)$ when individual failure times are available, use case 1. For grouped failure times, use case 2.

9.5.2 Case 1 – Time data for every relevant failure

Step 1: Calculate $\hat{\theta}(T)$ from 9.2.1, step 7.

Step 2: For a two-sided 90 % confidence interval, refer to Table 3, type I, or Table 4, type II, and locate the values L and U for the appropriate sample size N.

Step 3: The lower confidence limit on $\theta(T)$ is

$$\theta_{\mathrm{LB}}=L\cdot\hat{\theta}(T) \tag{82}$$

The upper confidence limit on $\theta(T)$ is

$$\theta_{\mathrm{UB}} = U \cdot \hat{\theta}(T) \tag{83}$$

One-sided 95 % lower and upper limits on $\theta(T)$ are θ_{LB} and θ_{UB}, respectively.

9.5.3 Case 2 – Time data for groups of relevant failures

These confidence interval procedures are suitable when $\hat{\beta}$ has been estimated from grouped data as in 9.2.2.

Step 1: Calculate $\hat{\beta}$ as in 9.2.2, and calculate $\hat{\theta}(T)$ as in 9.2.1, step 7

Step 2: Calculate

$$P(i) = \frac{T(i)}{T(d)}, \text{ with } i = 1, 2, ..., d \tag{84}$$

Step 3: Calculate the expression

$$A = \sum_{i=1}^{d} \frac{\left(P(i)^{\hat{\beta}} \cdot \ln P(i)^{\hat{\beta}} - P(i-1)^{\hat{\beta}} \cdot \ln P(i-1)^{\hat{\beta}}\right)^2}{\left(P(i)^{\hat{\beta}} - P(i-1)^{\hat{\beta}}\right)} \tag{85}$$

Step 4: Calculate

$$D = \sqrt{\frac{1}{A} + 1} \tag{86}$$

Step 5: For an approximate two-sided 90 % confidence interval on $\theta(T)$, calculate

$$S = \frac{(1{,}64) \cdot D}{\sqrt{N}} \tag{87}$$

where N is the total number of failures.

Step 6: The lower confidence limit on $\theta(T)$ is

$$\theta_{\mathrm{LB}} = \hat{\theta}(T)(1 - S) \tag{88}$$

The upper confidence limit on $\theta(T)$ is

$$\theta_{\mathrm{UB}} = \hat{\theta}(T)(1 + S) \tag{89}$$

One-sided 95 % lower and upper limits on $\theta(T)$ are θ_{LB} and θ_{UB}, respectively.

9.6 Projection technique

The following technique is appropriate when the corrective modifications have been incorporated into the product at the end of the test as delayed modifications. The objective is to estimate the product reliability resulting from these corrective modifications.

Step 1: Separate the Category A and Category B failures (see IEC 61014:2003, definitions 3.14 and 3.15).

Step 2: Identify the time of first occurrence of each distinct type of failure in Category B, as a separate data set. Let I be the number of these distinct types.

Step 3: Perform steps 1 to 5 of 9.2.1 upon this data set, in order to estimate β, using N = I and T^* or T_N as applicable to the complete set of data.

Step 4: Assign to each of the I distinct types of category B failures in the data set of step 2 an improvement effectiveness factor, E_i, $i = 1,..., \mathrm{I}$. For each of the I distinct types of Category B failures, E_i, $0 \leq E_i \leq 1$, is an engineering assessment of the expected decrease in failure intensity resulting from an identified corrective modification (see definition 3.5).

From these assigned values, calculate the average $\overline{E}$, or if preferred, postulate an average improvement effectiveness factor (e.g. 0,7) instead of individually assigning the E_i, $i = 1,...\mathrm{I}$, as described above.

Step 5: Estimate the projected failure intensity and MTBF:

$$z_\mathrm{p} = \frac{1}{T}\left[K_\mathrm{A} + \sum_{i=1}^{\mathrm{I}} K_\mathrm{i}\left(1 - E_\mathrm{i}\right) + \mathrm{I} \cdot \hat{\beta} \cdot \overline{E}\right] \tag{90}$$

where

K_A is the number of Category A failures;

K_i is the number of observed failures for the i-th type of Category B failures;

$T = T^*$ or T_N, as used in step 3 above.

If the individual E_i values are not assigned and only the mean $\overline{E}$ is available, then the middle term in the square brackets becomes:

$$K_\mathrm{B}\left(1 - \overline{E}\right) \tag{91}$$

where K_B is the number of Category B failures.

In this case, the projected failure intensity is

$$z_\mathrm{p} = \frac{1}{T}\left[K_\mathrm{A} + K_\mathrm{B}\left(1 - \overline{E}\right) + \mathrm{I} \cdot \hat{\beta} \cdot \overline{E}\right] \tag{92}$$

The projected MTBF is

$$\theta_\mathrm{p} = 1/z_\mathrm{p} \tag{93}$$

Table 2 – Critical values for Cramér-von Mises goodness-of-fit test at 10 % level of significance

M	**Critical value of statistic**
3	0,154
4	0,155
5	0,160
6	0,162
7	0,165
8	0,165
9	0,167
10	0,167
11	0,169
12	0,169
13	0,169
14	0,169
15	0,169
16	0,171
17	0,171
18	0,171
19	0,171
20	0,172
30	0,172
≥ 60	0,173
NOTE For Type I tests, $M = N$; for Type II tests, $M = N - 1$.	

Table 3 – Two-sided 90 % confidence intervals for MTBF from Type I testing

N	L	U	N	L	U
3	0,175	6,490	21	0,570	1,738
4	0,234	4,460	22	0,578	1,714
5	0,281	3,613	23	0,586	1,692
6	0,320	3,136	24	0,593	1,641
7	0,353	2,826	25	0,600	1,653
8	0,381	2,608	26	0,606	1,635
9	0,406	2,444	27	0,612	1,619
10	0,428	2,317	28	0,618	1,604
11	0,447	2,214	29	0,623	1,590
12	0,464	2,130	30	0,629	1,576
13	0,480	2,060	35	0,652	1,520
14	0,494	1,999	40	0,672	1,477
15	0,508	1,947	45	0,689	1,443
16	0,521	1,902	50	0,703	1,414
17	0,531	1,861	60	0,726	1,369
18	0,543	1,825	70	0,745	1,336
19	0,552	1,793	80	0,759	1,311
20	0,561	1,765	100	0,783	1,273

NOTE For $N > 100$

$$L \doteq \frac{N-1}{N}\left(1+u_{0,5+\gamma/2}\sqrt{\frac{1}{2N}}\right)^{-2}$$

$$U \doteq \frac{N-1}{N}\left(1-u_{0,5+\gamma/2}\sqrt{\frac{1}{2N}}\right)^{-2}$$

where $u_{0,5+\gamma/2}$ is the $100\cdot(0,5+\gamma/2)$-th fractile of the standard normal distribution.

Table 4 – Two-sided 90 % confidence intervals for MTBF from Type II testing

N	*L*	*U*	*N*	*L*	*U*
3	0,1712	4,746	21	0,6018	1,701
4	0,2587	3,825	22	0,6091	1,680
5	0,3174	3,254	23	0,6160	1,659
6	0,3614	2,892	24	0,6225	1,641
7	0,3962	2,644	25	0,6286	1,623
8	0,4251	2,463	26	0,6344	1,608
9	0,4495	2,324	27	0,6400	1,592
10	0,4706	2,216	28	0,6452	1,578
11	0,4891	2,127	29	0,6503	1,566
12	0,5055	2,053	30	0,6551	1,553
13	0,5203	1,991	35	0,6763	1,501
14	0,5337	1,937	40	0,6937	1,461
15	0,5459	1,891	45	0,7085	1,428
16	0,5571	1,876	50	0,7212	1,401
17	0,5674	1,814	60	0,7422	1,360
18	0,5769	1,781	70	0,7587	1,327
19	0,5857	1,752	80	0,7723	1,303
20	0,5940	1,726	100	0,7938	1,267

NOTE For $N > 100$

$$L \doteq \frac{N-2}{N}\left(1 + u_{0,5+\gamma/2}\sqrt{\frac{2}{N}}\right)^{-1}$$

$$U \doteq \frac{N-2}{N}\left(1 - u_{0,5+\gamma/2}\sqrt{\frac{2}{N}}\right)^{-1}$$

where $u_{0,5+\gamma/2}$ is the $100 \cdot (0,5 + \gamma/2)$-th fractile of the standard normal distribution.

Annex A
(informative)

Examples for planning and analytical models used in design and test phase of product development

A.1 Reliability growth planning in product design phase

A.1.1 Power law planning model example

The real industry example below explains how to construct a power law model and how to compare the actual reliability growth to the one planned.

The information needed for planning is a rough estimate of the beginning product reliability. This estimate may be achieved based on the product similarity to another product, taking into account differences in complexity. Other factors needed for the model are as follows:

– reliability goal set for the product;
– estimate of the magnitude and number of possible design modifications;
– duration of design period during which it is possible to make and implement the desired modifications.

A.1.2 Construction of the model and monitoring of reliability growth

The planning model is constructed as follows.

The required life of the product is 15 years: $T = 365 \times 24 \times 15 = 1{,}314 \times 10^5$ h

Given duration of the design period of 140 days, $t_D = 140 \times 24 = 3\ 360$ h

A product's initial 15-year reliability is estimated to be $R_0(T = 15 \text{ years}) = 0{,}72$. The product reliability goal was to achieve a 15-year reliability of 0,95.

From the initial reliability and the reliability goal, the initial and goal average failure rates are calculated as follows:

$$R_0(T) = \exp\left[-\lambda_{a0} \cdot T\right] = 0{,}72$$

$$R_G(T) = 0{,}95$$

$$\lambda_{a0} = -\frac{\ln\left[R_0(T)\right]}{T} = 2{,}5 \cdot 10^{-6}$$

$$\lambda_{aG} = -\frac{\ln\left[R_G(T)\right]}{T} = 3{,}9 \cdot 10^{-7}$$

Given the assumed number of potential design modifications, the growth rate becomes:

$$\alpha_D = \frac{\ln\left(\dfrac{\ln\left[R_0(T)\right]}{\ln\left[R_G(T)\right]}\right)}{\ln(1+D)} = 0{,}774$$

and the reliability growth model is plotted from the equation

$$R(t,T) = R_0(T)^{\left\{ \frac{\left[t_D + t \cdot \left(\frac{\ln[R_G(T)]}{\ln[R_0(T)]} \right)^{-\frac{1}{\alpha_D}} - t \right]}{t_D} \right\}^{-\alpha_D}}$$

The actual product reliability growth is then plotted and compared to the model as shown in Figure A.1.

The model can be prepared using a spreadsheet, with equations embedded into it as shown in Table A.1.

Table A.1 – Calculation of the planning model for reliability growth in design phase

	A	B	C	D	E	F	G	H	I
1	**Symbol or function**	α_D	D	$R_0(T)$	$R_G(T)$	T h	λ_{a0}	λ_{aG}	t_D h
2	**Value**	**0,774 42**	**10**	**0,72**	**0,95**	**131 400**	**2,50E-06**	**3,90E-07**	**3 360**
3	**Formula ***	**=LN(LN(D3)/LN(E3))/LN(1+C3)**					**=(-LN(D3)/F3)**	**=(-LN(E3)/F3)**	
4	**Formula $R(t,T)$_model**	**=$D$2^((($I$2+A4*24*(LN($E$2)/LN($D$2))^(-1/$B$2)-A4*24)/$I$2)^(-$B$2))**							
5	t(days)	$R(t,T)$_model	$R_G(T)$	$R(t,T)$_actual					
6	0	0,72	0,95						
7	5	0,771 577 8	0,95						
8	10	0,805 411 6	0,95						
9	15	0,829 525 2	0,95						
10	20	0,847 690 1	0,95						
11	25	0,861 927 9	0,95						
12	30	0,873 425 9	0,95	0,699					
13	35	0,882 930 2	0,95						
14	40	0,890 934 8	0,95						
15	45	0,897 780 3	0,95						
16	50	0,903 710 2	0,95	0,77					
17	55	0,908 902 9	0,95						
18	60	0,913 492 8	0,95						
19	65	0,917 582 8	0,95						
20	70	0,921 253 3	0,95						
21	75	0,924 568	0,95						
22	80	0,927 578 2	0,95	0,82					
23	85	0,930 325 6	0,95						
24	90	0,932 844 4	0,95						
25	95	0,935 163 1	0,95						
26	100	0,937 305 5	0,95						
27	105	0,939 291 8	0,95						
28	110	0,941 139 1	0,95						
29	115	0,942 862 1	0,95						
30	120	0,944 473 3	0,95						
31	125	0,945 983 7	0,95	0,93					
32	130	0,947 402 9	0,95						
33	135	0,9487392	0,95						
34	140	0,95	0,95						

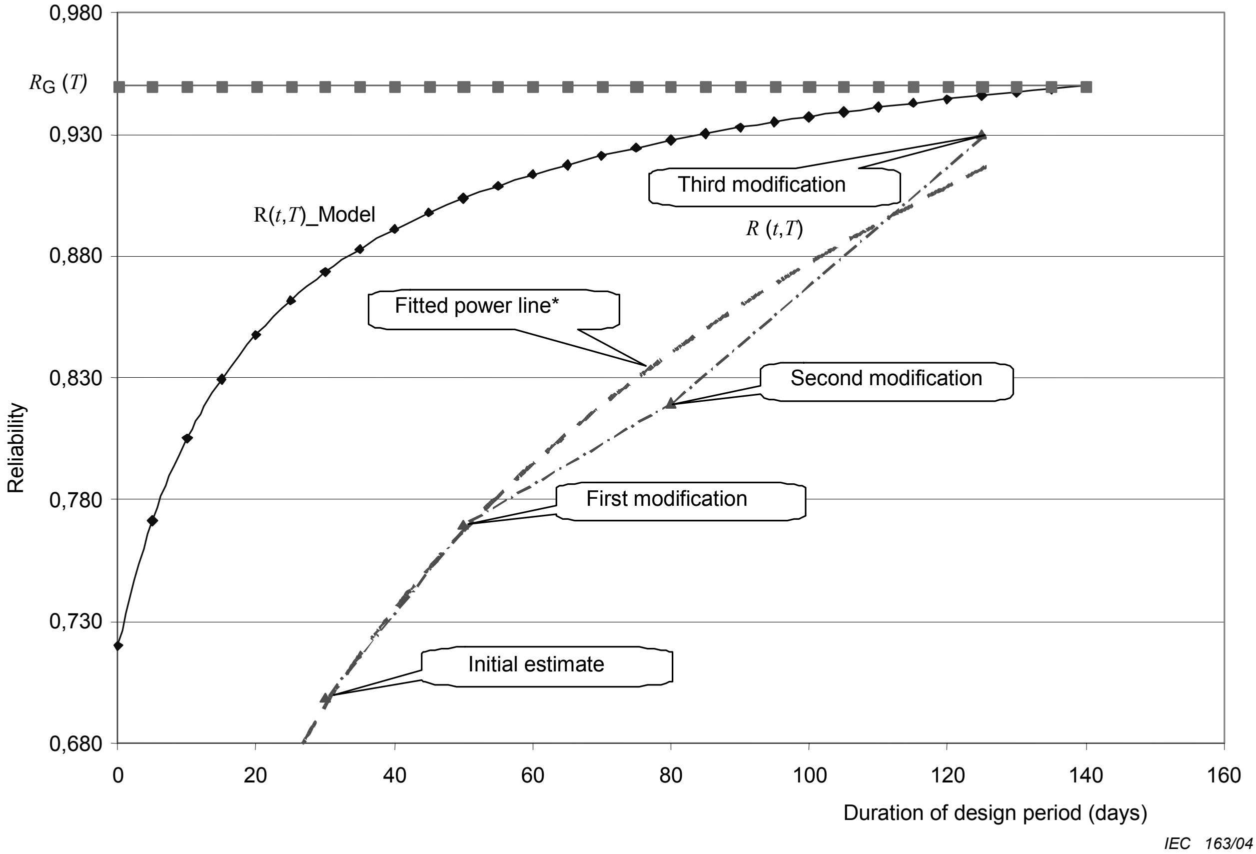
* The formulae shown in rows 3 and 4 come from an Excel spreadsheet.

In this example, improvements are made in three steps; however, many modifications of the same nature were lumped into one change. The first step was change of the capacitor types to those with much better dielectric properties and higher reliability (106 capacitors of various values and various contribution to unreliability were considered one design modification).

The second change was introduction of parts with higher ratings, as there were some parts (capacitors on semiconductors) that, because of improper electrical rating, demonstrated high unreliability.

The third change was also a compilation of several modifications: more reliable IC components, some switching field effect transistors (FET) to be obtained from a more reliable vendor, reduction in some discrete semiconductor components.

Further improvement was not considered cost effective, and the final reliability estimate was accepted as satisfactory.



*There is no need to fit the actual data with a power line. The fit in the example given is an illustration of how the actual improvements actually do follow the power law – fewer larger improvements – the steeper the curve.

Figure A.1 – Planned and achieved reliability growth – Example

As noted in Figure A.1, there is no need to fit the actual reliability growth with a power line, as the results can be represented as discrete values. The results of this actual example are fitted with the power line to illustrate how in reality, the faults that are highest contributors to unreliability are indeed addressed first, and how the power law is well applicable to the actual results.

A.2 Example of Bayesian reliability growth model for the product design phase

Reliability goal of an item, $R_G(T)$, about to commence the design phase is 0,95 h at 2 000 h of operation (T). There are 50 potential design weaknesses (D). Each design weakness is associated with a class of failure. There are two classes of failure. There are 20 potential design weaknesses associated with failure class 1 (D_1) and 30 associated with failure class 2 (D_2). For each potential design weakness there is an associated probability of resulting in failure (p_{kj}). The expected number of failures resulting from these weaknesses for failure class 1 is 7 (λ_1) and the expected number for class 2 is 10 (λ_2). Historically, there has been one non-systematic failure every 100 000 h of operation, failures associated with failure class 1 occur at a rate of 1 every 200 000 h and failures associated with failure class 2 occur at a rate of 1 every 75 000 h. The growth parameter is 0,00833.

The initial estimate of reliability of this design is:

$$R_I(2\,000) = \exp(-[0{,}000\,01 \times 2\,000 + 7 \times (1 - e^{-0{,}000\,005 \times 2\,000}) + 10 \times (1 - e^{-0{,}000\,013\,3 \times 2\,000})])$$
$$= \exp(-[0{,}02 + 7 \times (1 - e^{-0{,}01}) + 10(1 - e^{-0{,}026\,666})])$$
$$= 0{,}702\,722$$

The reliability growth curve is calculated as:

$$R(t) = \exp\{-[0{,}02 + e^{-0{,}008\,333 \times t} \times [7 \times (1 - e^{0{,}01}) + 10 \times (1 - e^{-0{,}026\,666})]\}$$
$$= \exp(-[0{,}02 + 0{,}332\,794 \times e^{-0{,}008\,33 \times t}])$$

Plotting $R(t)$ against design phase measured in days:

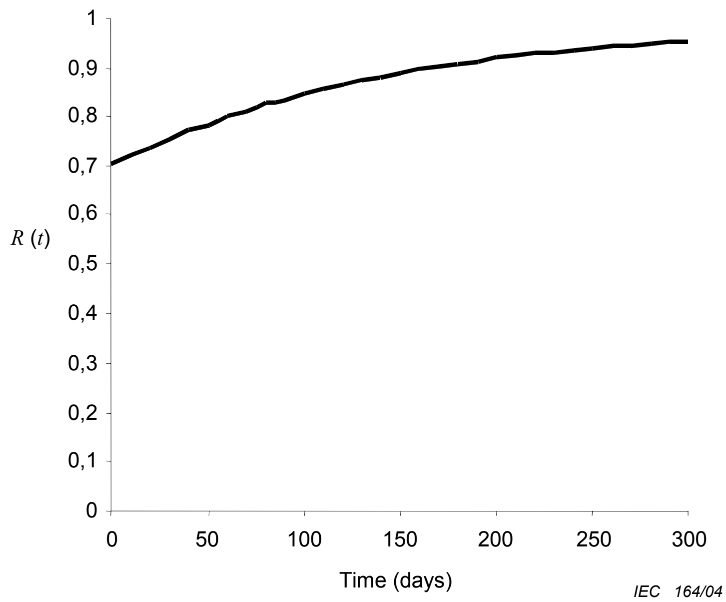


Figure A.2 – Planned reliability growth using Bayesian reliability growth model

This plot shows the projected growth in reliability throughout a year of the design phase. It is expected that such an intensive design phase would result in the reliability goal being met after 284 days.

$$t_G = \frac{\ln\left[\dfrac{0{,}332\,794}{-\ln(0{,}95) - 0{,}02}\right]}{0{,}008\,33}$$
$$= 283{,}807\,9$$

If a shorter period of design phase time were required such as 100 days then a more intensive program is required as reflected in the growth parameter.

$$\alpha = \frac{\ln\left[\dfrac{0{,}332\,794}{-\ln(0{,}95) - 0{,}02}\right]}{100}$$
$$= 0{,}023\,641$$

A.3 Failure data for discrete trials

A missile system undergoes reliability growth development testing for a total of 68 trials. Delayed corrected actions were incorporated after the 14th, 33rd and 48th trials. From trial 49 to trial 68 the configuration was not changed. For the k = 4 stages of testing we have

Stage 1: Trials 1-14

Stage 2: Trials 15-33

Stage 3: Trials 34-48

Stage 4: Trials 49-68.

This gives

$n_i = 14, n_2 = 19, n_3 = 15, n_4 = 20,$ and

$t_0 = 0$, $t_1 = 14$, $t_2 = 33$, $t_3 = 48$, $t_4 = 68$.

The failure data for estimating the model parameters are:

Configuration 1 experienced $m_1 = 5$ failures.

Configuration 2 experienced $m_2 = 3$ failures.

Configuration 3 experienced $m_3 = 4$ failures.

Configuration 4 experienced $m_4 = 4$ failures.

The maximum likelihood estimates for the parameters λ and β are

$\hat{\lambda}$ =0,595 and $\hat{\beta}$ =0,780

These estimates give

$\hat{f}_1 = 0{,}333$, $\hat{f}_2 = 0{,}234$, $\hat{f}_3 = 0{,}206$, $\hat{f}_4 = 0{,}190$.

The estimates of the reliability over stages 1, 2, 3 are $\hat{R}_1 = 0{,}667$, $\hat{R}_2 = 0{,}766$, $\hat{R}_3 = 0{,}794$, and the reliability estimate of the configuration during the final stage of testing is $\hat{R}_4 = 0{,}810$.

A.4 Examples of reliability growth through testing

A.4.1 Introduction

The following numerical examples show the use of the procedures discussed in Clause 9. Table A.1 is a complete data set used to illustrate the reliability growth methods when the relevant failure times are known, and Table A.2 shows these data combined within intervals suitable for the grouped data analysis. Tables A.3 and A.4 provide data for the projection technique when corrective modifications are delayed to the end of test. Goodness-of-fit tests, as described in 9.3, are applied when applicable. These examples may be used to validate computer programs designed to implement the methods given in Clause 9.

A.4.2 Current reliability assessments

A.4.2.1 General

The data set in Table A.2 corresponds to a test finishing at 1 000 h. These data are used in the examples given in A.4.2.2 and A.4.2.3 for Type I and Type II tests, respectively, and combined in Table A.3 for the example given in A.4.2.4 for grouped failures.

A.4.2.2 Example 1: Type I test – Case 1 – Time data for every relevant failure

This case is covered in 9.2.1. Data from Table A.2 are used with a test finishing at 1 000 h.

a) Test for growth

U = –3,713. At the 0,20 significance level, the critical values for a two-sided test are 1,28 and –1,28. Since U <–1,28, there is evidence of positive reliability growth and the analysis is continued.

b) Parameter estimation

The estimated parameters of the power law model are

$\hat{\lambda}$ = 1,069 4

$\hat{\beta}$ = 0,562 3.

c) Current MTBF

The estimated current MTBF at 1 000 h is 34,2 h.

d) Goodness-of-fit

$C^2(M)$ = 0,038 with M = 52. At the 0,10 significance level, the critical value from Table 2 is 0,173. Since $C^2(M)$ < 0,173, the power law model is accepted (see 9.3 and Figure A.3).

e) Confidence interval on β

A two-sided 90 % confidence interval on β is (0,449 1; 0,710 1).

f) Confidence interval on current MTBF

A two-sided 90 % confidence interval on the current MTBF at 1 000 h is (24,2 h; 48,1 h).

A.4.2.3 Example 2: Type II test – Case 1 – Time data for every relevant failure

This case is covered in 9.2.1. Data from Table A.2 are used with a test finishing at 975 h.

a) Test for growth

U = –3,764. At the 0,20 significance level, the critical values for a two-sided test are 1,28 and –1,28. Since U < –1,28, there is evidence of positive reliability growth and the analysis is continued.

b) Parameter estimation

The estimated parameters of the power law model are

$\hat{\lambda}$ = 1,106 7

$\hat{\beta}$ = 0,559 4.

c) Current MTBF

The estimated current MTBF at 975 h is 33,5 h.

d) Goodness-of-fit

$C^2(M)$ = 0,041 with M = 51. At the 0,10 significance level, the critical value from Table 1 is 0,173. Since $C^2(M)$ < 0,173, the power law model is accepted (see 9.3 and Figure A.3).

e) Confidence interval on β

A two-sided 90 % confidence interval on β is (0,445 8; 0,708 0).

f) Confidence interval on current MTBF

A two-sided 90 % confidence interval on the current MTBF at 975 h is (24,3 h; 46,7 h).

A.4.2.4 Example 3 – Case 2 – Time data for group relevant failures

This case is covered in 9.2.2. Data from Table A.2 are used. The failures have been grouped over intervals of 200 h to give the data set in Table A.3. The analysis of this data set gives the results described below.

a) Test for growth

X^2 = 14,730 8 with four degrees of freedom. At the 0,20 significance level, the critical value is 6,0. Since X^2 >6,0, there is evidence of positive or negative reliability growth and the analysis is continued.

b) Parameter estimation

The estimated parameters of the power law model are

$\hat{\lambda}$ = 0,961 5

$\hat{\beta}$ = 0,577 7.

c) Current MTBF

The estimated current MTBF at 1 000 h is 33,3 h.

d) Goodness-of-fit

X^2 = 2,175 with three degrees of freedom. At the 0,10 significance level, the critical value is 6,25. Since X^2 <6,25, the power law model is accepted (see 9.3 and Figure A.4).

e) Confidence interval on β

A two-sided 90 % confidence interval on β is (0,4083; 0,7471).

f) Confidence interval on current MTBF

A two-sided 90 % confidence interval on the current MTBF at 1 000 h is (20,94 h; 45,66 h).

A.4.3 Projected reliability estimates

A.4.3.1 General

This example illustrates the calculation of a projected reliability estimate (see 9.6) when the corrective modifications have been incorporated into the product at the end of test.

A.4.3.2 Example 4

The basic data used in this example are given in Table A.4. There are a total of $N = 45$ relevant failures with $K_A = 13$ Category A failures which received no corrective modification. At the end of the 4 000 h test, $I = 16$ distinct corrective modifications were incorporated into the product to address the $K_B = 32$ Category B failures. The category for each relevant failure is given in Table A.4. Each Category B failure type is distinguished by a number. Table A.5 provides additional information used for the projection.

The steps in the procedure are as follows.

Step 1: Identify Category A and B failures

Times of occurrence and the Category A and B failures are identified in Table A.4. The failure times for the 16 distinct Category B types are indicated in Table A.5, column 2.

Step 2: Identify first occurrence of distinct Category B types.

The times of first occurrence of the 16 distinct category B types are given in Table A.5, Column 3.

Step 3: Analyse first occurrence data.

The data set of Table A.5, Column 3, is analysed in accordance with steps 4 to 7 of 9.2.1. The results follow below.

Parameter estimation

The estimated parameters of the power law model are

$$\hat{\lambda} = 0{,}032\,6$$

$$\hat{\beta} = 0{,}747\,2\,.$$

First occurrence failure intensity estimation

The estimated current failure intensity for first occurrence of distinct Category B types at 4 000 h is 0,0030 h^{-1}.

Goodness-of-fit

$C^2(M) = 0{,}085$ with $M = 16$. At the 0,10 significance level, the critical value from Table 2 is 0,171. Since $C^2(M) < 0{,}171$, the power law model is accepted for the times of first occurrence of distinct Category B types.

Step 4: Assign effectiveness factors

An effectiveness factor is assigned, based on the opinion of the design engineer who is responsible for the specific change. If this information is not available, then the estimated numbers for individual effectiveness factors are to be estimated based on experience.

An example of assigned individual effectiveness factors for each corrective modification is given in Table A.5, column 5. The average of these 16 effectiveness factors is 0,72. An average in the range of 0,65 to 0,75 is typical, based on historical experience.

Step 5: Estimate projected failure intensity

To calculate the projected failure intensity, the following values are needed:

T = 4 000 h

K_A = 13

I = 16

$\hat{\beta}$ = 0,747 2

$\overline{E}$ = 0,72

K_i – Table A.5, Column 4

E_i – Table A.5, Column 5.

The estimated projected failure intensity at T = 4 000 h (the end of test) is 0,0074 h^{-1}.

Step 6: Estimate projected MTBF

The projected MTBF is 135,1 h.

NOTE With no reliability growth during the 4 000 h test, the MTBF over this period is estimated by (4 000/45) = 88,9 h. The projected MTBF is the estimated increase in MTBF due to the 16 corrective modifications and the corresponding effectiveness factors. The sensitivity of the projected MTBF to the assigned effectiveness factors is often of interest. If only an average effectiveness factor of 0,60 were assigned, the projected MTBF would equal 121,3 h. An average effectiveness factor of 0,80 would give a projected MTBF of 138,1 h.

Table A.2 – Complete data – All relevant failures and accumulated test times for Type I test

2	4	10	15	18	19	20	25	39
41	43	45	47	66	88	97	104	105
120	196	217	219	257	260	281	283	289
307	329	357	372	374	393	403	466	521
556	571	621	628	642	684	732	735	754
792	803	805	832	836	873	975		
T^* = 1 000 h, N = 52.								

Table A.3 contains failure times listed in increasing order.

Table A.3 – Grouped data for Example 3 derived from Table A.2

Group number	Number of failures	Accumulated relevant test time at end of group interval h
1	20	200
2	13	400
3	5	600
4	8	800
5	6	1 000

Table A.4 – Complete data for projected estimates in Example 4 – All relevant failures and accumulated test times

	Accumulated relevant test times, T_i Classification per Category A/B, including distinct Category B types h								
T_i	150	253	475	540	564	636	722	871	996
Category	B1	B2	B3	B4	B5	A	B5	A	B6
T_i	1 003	1 025	1 120	1 209	1 255	1 334	1 647	1 774	1 927
Category	B7	A	B8	B2	B9	B10	B9	B10	B11
T_i	2 130	2 214	2 293	2 448	2 490	2 508	2 601	2 635	2 731
Category	A	A	A	A	B12	A	B1	B8	A
T_i	2 747	2 850	3 040	3 154	3 171	3 206	3 245	3 249	3 420
Category	B6	B13	B9	B4	A	A	B12	B10	B5
T_i	3 502	3 646	3 649	3 663	3 730	3 794	3 890	3 949	3 952
Category	B3	B10	A	B2	B8	B14	B15	A	B16
T^* = 4 000 h, N = 45, K_A = 13, K_B = 32, I = 16.									

Table A.5 – Distinct types of Category B failures, from Table A.4, with failure times, time of first occurrence, number observed and effectiveness factors

Column no.				
1	**2**	**3**	**4**	**5**
Type	**Failure times** h	**Time at first occurrence** h	**Number observed**	**Assigned effectiveness factors**
1	150; 2 601	150	2	0,7
2	253; 1 209; 3 663	253	3	0,7
3	475; 3 502	475	2	0,8
4	540; 3 154	540	2	0,8
5	564; 722; 3 420	564	3	0,9
6	996; 2 747	996	2	0,9
7	1 003	1 003	1	0,5
8	1 120; 2 635; 3 730	1 120	3	0,8
9	1 255; 1 647; 3 040	1 255	3	0,9
10	1 334; 1 774; 3 249; 3 646	1 334	4	0,7
11	1 927	1 927	1	0,7
12	2 490; 3 245	2 490	2	0,6
13	2 850	2 850	1	0,6
14	3 794	3 794	1	0,7
15	3 890	3 890	1	0,7
16	3 952	3 952	1	0,5

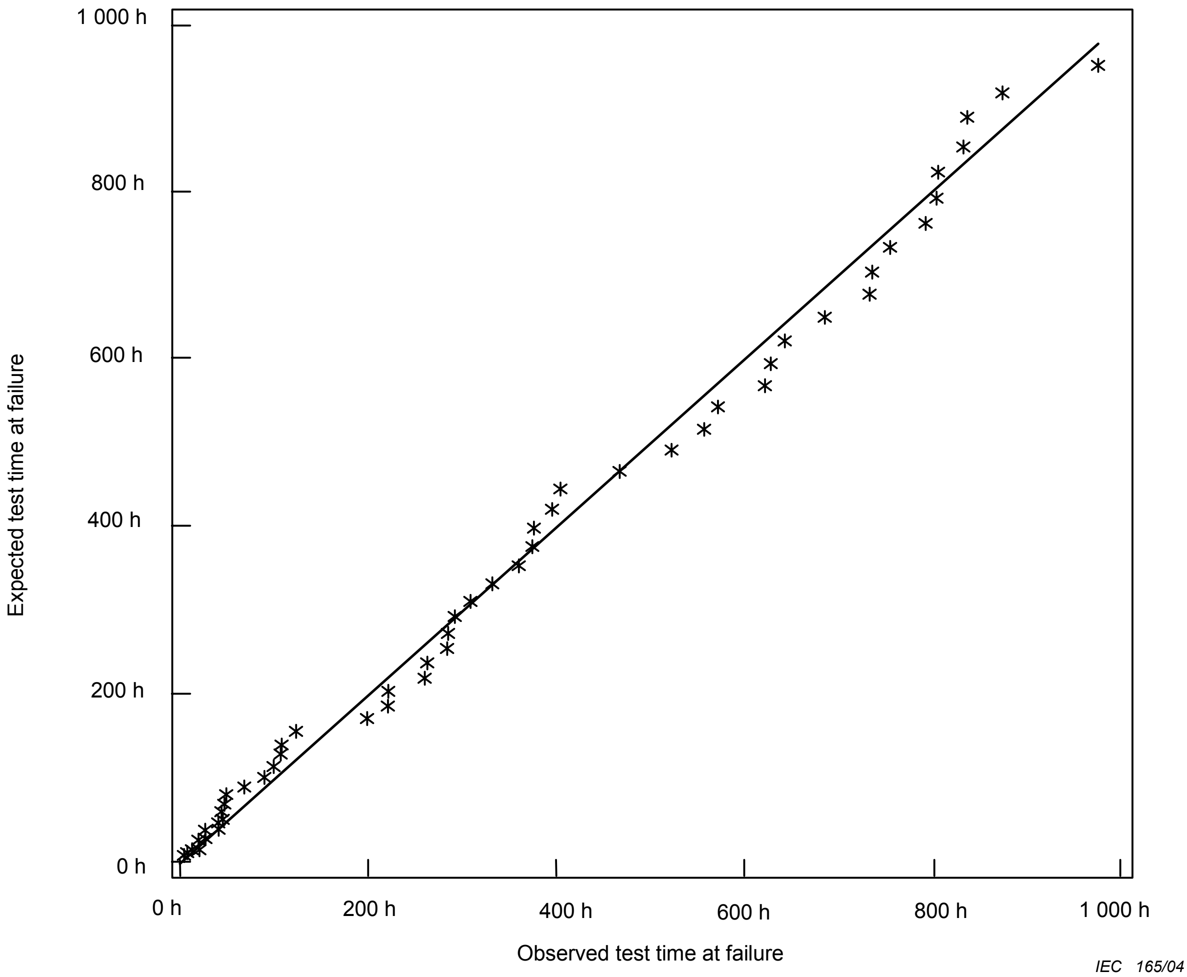


Figure A.3 – Scatter diagram of expected and observed test times at failure based on data of Table A.2 with power law model

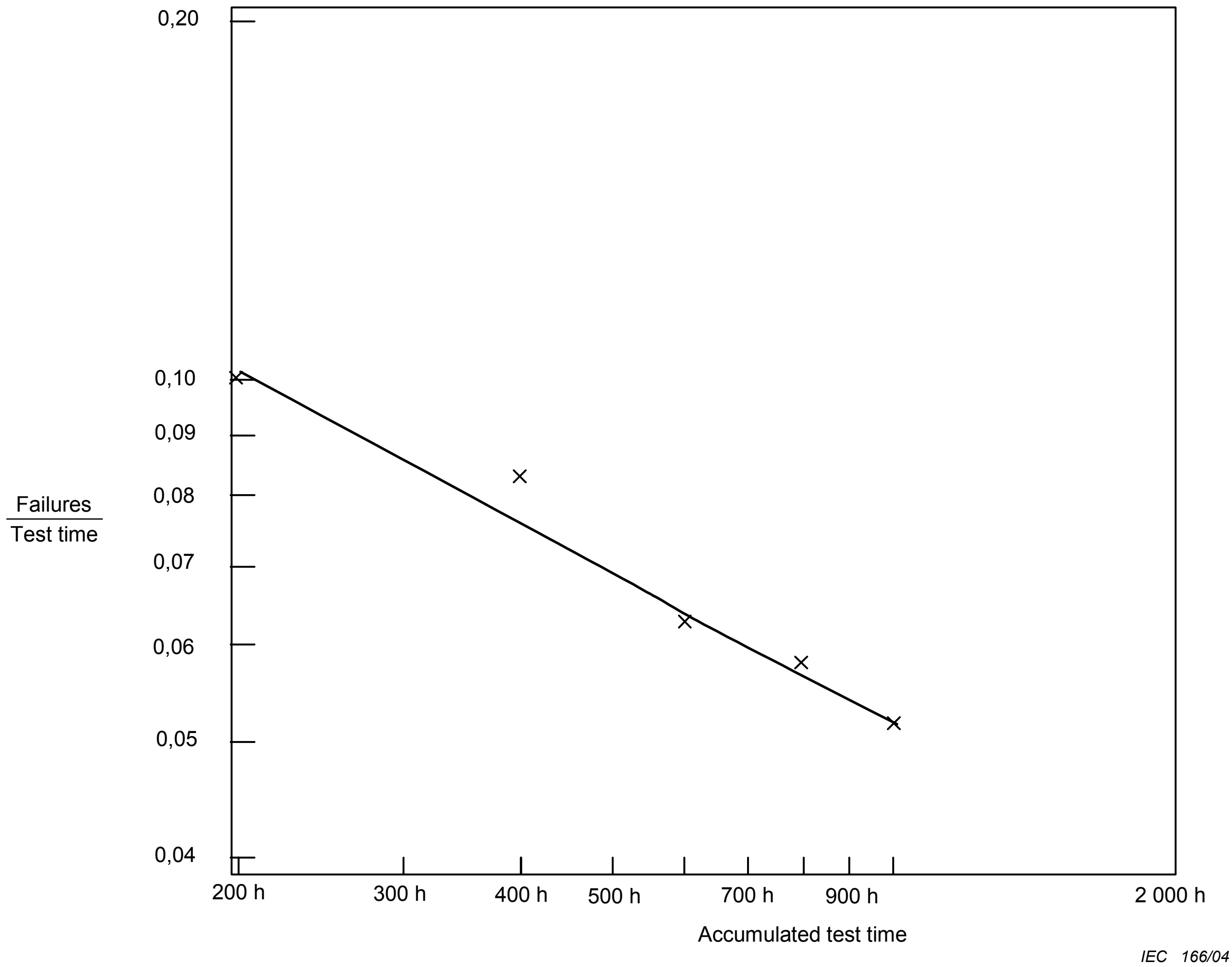


Figure A.4 – Observed and estimated accumulated failures/accumulated test time based on data of Table A.2 with power law model

Annex B
(informative)

The power law reliability growth model – Background information

B.1 The Duane postulate

The most commonly accepted pattern for reliability growth was reported in a paper by J.T. Duane in 1964 [11]. In this paper, Duane discussed his observations on failure data for a number of products during development testing. He observed that the accumulated number of failures $N(T)$, divided by the accumulated test time, T, was decreasing and fell close to a straight line when plotted against T on ln-ln scale. That is, approximately:

$$\ln\left(N(T)/T\right) = \delta - \alpha \ln T, \text{ with } \delta > 0,\ \alpha > 0$$

Duane interpreted these plots and concluded that the accumulated number of failures is approximated by the power law function:

$$N(T) \doteq \lambda T^{\beta}, \text{ with } \lambda > 0, \beta = 1 - \alpha$$

Based on this observation, Duane expressed the current instantaneous failure intensity at time T as:

$$\frac{\mathrm{d}}{\mathrm{d}T} N(T) = \lambda \beta T^{\beta - 1}, \text{ with } T > 0$$

which gives the instantaneous MTBF

$$\left(\lambda \beta T^{\beta - 1}\right)^{-1} = \frac{T^{1-\beta}}{\lambda \beta}, \text{ with } T > 0$$

The exponent $\alpha = 1 - \beta$ is sometimes called the "growth rate".

The Duane postulate is deterministic in the sense that it gives the expected pattern for reliability growth but does not address the associated variability of the data.

B.2 The power law model

L.H. Crow, in 1974 [1], considered the power law reliability growth pattern and formulated the underlying probabilistic model for failures as a non-homogeneous Poisson process (NHPP), $\{N(T), T > 0\}$, with mean value function

$$E[N(T)] = \lambda T^{\beta}$$

and intensity function

$$z(T) = \lambda \beta T^{\beta - 1}$$

The Crow NHPP power law model has exactly the same reliability growth pattern as the Duane postulate, for example, they both have the same expression λT^{β} for the expected number of failures by time T. However, the NHPP model gives the Poisson probability that $N(T)$ will assume a particular value, that is

$$\Pr[N(T)=n]=\frac{(\lambda T^{\beta})^{n}e^{-\lambda T^{\beta}}}{n!}, \text{ with } n=0,1,2,...$$

Also, under this model

$$E[\lambda T_j^{\beta}]=j, \text{ with } j=1,2,...$$

where T_j is the accumulated time to the j-th failure.

This gives the useful first order approximation

$$E[T_j]=\left(\frac{j}{\lambda}\right)^{1/\beta}, \text{ with } j=1,2,...$$

for the expected time to the j-th failure.

When β = 1, then $z(T)\equiv\lambda$, and the times between successive failures follow an exponential distribution with mean $1/\lambda$ (homogeneous Poisson process), indicating no reliability growth. The intensity function $z(T)$ is decreasing for $\beta<1$ (positive growth), and increasing for $\beta>1$ (negative growth).

The NHPP power law reliability growth model is a probabilistic interpretation of the Duane postulate and therefore allows for the development and use of rigorous statistical procedures for reliability growth assessments. These methods include maximum likelihood estimation of the model parameters and product reliability, confidence interval procedures and objective goodness-of-fit tests. The NHPP power law model was extended by Crow in 1983 [12] for reliability growth projections.

B.3 Modified power law model for planning of reliability growth in product design phase

M. Krasich, in 1998 developed and presented the power law model for application in modelling of the planned reliability improvement of a product in design phase, where test results were not available because the hardware design was not yet completed, and therefore the hardware was not available for testing. The model considers only the design improvements in the course of the design. The model has been also slightly modified to allow the initial average failure rate of the product, before any design modifications took place, to be a real number. In this model, the number of design modifications, taken by conscious decision, replace failure occurrences.

Design improvements are made in such a manner that those potential design flaws that are the highest contributors to the product unreliability are addressed first, ensuring the applicability of the power law model. Here, there is no risk of whether a test is appropriately designed to actually allow appearance of the failures with high probability of occurrence or the highest failure rate. In the model, the contributors are analytically evaluated, and the design is improved accordingly and at a relatively constant pace. Thus, the model continuity is assured, meaning that the discrete improvements of a product reliability are planned as a continuous model.

If the initial product reliability for the predetermined product operational time T is estimated to be $R_0(T)$, then the initial average failure rate corresponding to the time T is calculated to be:

$$\lambda_{a0}=-\frac{\ln[R_0(T)]}{T}$$

Assuming that the power law applies, and with modification such as to allow the product failure rate to be equal to the initial failure rate when no design improvements are achieved, failure rate of the design at any time during the design period is

$$\lambda_a(t) = \lambda_{a0} \times [1 + d(t)]^{-\alpha_D}, \text{ or}$$

$$\lambda_a(t) = \lambda_{a0} \times \left(1 + D \times \frac{t}{t_D}\right)^{-\alpha_D}$$

In the above equations, $d(t)$ is the number of design modifications at any time during the design period. With a linear approximation, the number of design modifications as a function of time is

$$d(t) = D \times \frac{t}{t_D}$$

The goal product average failure rate given the product reliability goal $R_G(T)$ is:

$$\lambda_{aG} = \frac{-\ln[R_G(T)]}{T}$$

During the design period, continuous improvement of product reliability as a function of time t (the reliability growth model)(from 0 to t_D) can be written as:

$$R(t) = \exp(-\lambda(t) \times T)$$

With the further derivations and substitutions, the reliability growth model for the design phase $0 < t < t_D$, becomes

$$R(t) = R_0(T)^{\left[\frac{\left[t_D + t \times \left(\frac{\ln[R_G(T)]}{\ln[R_0(T)]}\right)^{-\frac{1}{\alpha_D}} - t\right]}{t_D}\right]^{-\alpha_D}}$$

B.4 Modified Bayesian IBM-Rosner model for planning reliability growth in the design phase

This modification of the IBM-Rosner reliability growth was initially motivated by the analysis of test data [8] and [6]. The version presented in this annex is adapted for supporting planning decisions during the product design phase.

The model is based on a Bayesian approach that combines a prior distribution for the number of design weaknesses in the new product design with empirical data for the reliability of similar product designs to produce a posterior distribution for estimating the reliability of the new product design.

Like the IBM-Rosner model [9], this model assumes that a fixed number of weaknesses or potential faults are inherent in the product design and that, within the period between design modifications, the rate at which failures occur are constant. It is further assumed that modifications to the design to remove weaknesses are perfect.

In this model the inherent failure rate of the product design is, in addition, decomposed into the systematic and non-systematic (or residual or noise) failures. This allows the reliability growth profile to be modified as the systematic failure rate changes when design modifications are implemented, while always taking into account the impact of noise failures on the estimated reliability at a given time.

The non-systematic failures are assumed to occur at a constant rate (λ_{NS}) and can be estimated using data from similar product designs or using engineering judgement.

The systematic failures are assessed through a combination of expert judgement concerning design weaknesses and failure rates associated with fault classes from engineering experience.

To assess the effect of systematic failures on the reliability of the product design, all potential design weaknesses (D) should be identified and may be allocated to one of K fault classes, as appropriate. The probability of each design weakness within each fault class resulting in failure during the specified life of the product should be estimated using, for example, engineering judgement. Procedures for identifying design weaknesses and estimating their probability of resulting in failure using engineering judgement may be required. See, for example, [7].

The expected number of design weaknesses in fault class k (η_k) likely to result in failure if the design is not modified can be calculated using

$$\eta_k = \sum_{j=1}^{D_k} p_{kj}$$

where D_k is the total number of design weaknesses expected in fault class k and p_{kj} is the probability of the j-th design weakness in fault class k. This calculation is based on the assumption that the number of design weaknesses for each fault class is a Poisson random variable.

Systematic failure rates are also required for each fault class. These may be estimated using empirical or generic data on relevant existing product designs.

Given that the input data have been specified, the (posterior) estimator of the reliability of the initial product design can be found. This is the product of the reliability of the non-systematic failures and the reliability of the systematic failures. The rate of the former is the product of the (prior) distribution for the number of design weaknesses and the (empirical) data for the systematic failures. Thus, the reliability of the initial product design can be written as

$$R_I(T) = \exp\left\{-\left[\lambda_{NS}T + \sum_{k=1}^{K} \eta_k \left(1 - e^{-\lambda_k T}\right)\right]\right\}$$

Given that modifications will be implemented to remove design weaknesses, the reliability of the product design will grow. Therefore, to take into account the rate of reliability growth (α_D), the reliability of the modified product design at time T is given by

$$R(T) = \exp\left\{-\left[\lambda_{NS}T + \sum_{k=1}^{K} \eta_k e^{-\alpha_D T} \left(1 - e^{-\lambda_k T}\right)\right]\right\}$$

To estimate the rate of growth of the goal reliability, replace ($R_G(T)$) and the specified time of the design period (t_D) with $R(T)$ and the time index T on the growth rate (α_D), respectively, in the previous equation. Rearranging gives

$$\alpha_D = \frac{\ln\left[\dfrac{\sum_{k=1}^{K}\eta_k\left(1-e^{-\lambda_k T}\right)}{-\ln\left[R_G(T)\right]-\lambda_{NS}T}\right]}{t_D}$$

If a growth rate has been specified or estimated, then similarly an estimate of the expected time to reach the goal reliability is given by

$$t_G = \frac{\ln\left[\dfrac{\sum_{k=1}^{K}\eta_k\left(1-e^{-\lambda_k T}\right)}{-\ln\left[R_G(T)\right]-\lambda_{NS}T}\right]}{\alpha_D}$$

Annex ZA
(normative)

Normative references to international publications with their corresponding European publications

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE Where an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

Publication	Year	Title	EN/HD	Year
IEC 60050-191	1990	International Electrotechnical Vocabulary (IEV) Chapter 191: Dependability and quality of service	-	-
IEC 60300-3-5	2001	Dependability management Part 3-5: Application guide - Reliability test conditions and statistical test principles	-	-
IEC 60605-4	- [1]	Equipment reliability testing Part 4: Statistical procedures for exponential distribution - Point estimates, confidence intervals, prediction intervals and tolerance intervals	-	-
IEC 60605-6	- [1]	Part 6: Tests for the validity of the constant failure rate or constant failure intensity assumptions	-	-
IEC 61014	2003	Programmes for reliability growth	EN 61014	2003

[1] Undated reference.

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