BS EN 13803-1:2010



BSI Standards Publication

Railway applications — Track — Track alignment design parameters — Track gauges 1435 mm and wider

Part 1: Plain line

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BS EN 13803-1:2010 BRITISH STANDARD

National foreword

This British Standard is the UK implementation of EN 13803-1:2010. It supersedes DD ENV 13803-1:2002 which is withdrawn.

The UK participation in its preparation was entrusted to Technical Committee RAE/2, Railway Applications - Track.

A list of organizations represented on this committee can be obtained on request to its secretary.

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CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration. Up-to-date lists and bibliographical references concerning such national standards may be obtained on application to the CEN Management Centre or to any CEN member.

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Cont	ents	age
Forewo	ord	4
1	Scope	5
2	Normative references	5
3	Terms and definitions	5
4	Symbols and abbreviations	8
5	Requirements	
5.1	Background	
5.1.1	General	
5.1.2	Track alignment design parameters	
5.1.3	Parameter quantification	10
5.2	Normal limits and exceptional limits for track alignment design parameters	10
5.2.1	Radius of horizontal curve R	10
5.2.2	Cant D	
5.2.3	Cant deficiency I	
5.2.4	Cant excess E	
5.2.5	Cant gradient dD/ds	
5.2.6	Rate of change of cant $\mathrm{d}D/\mathrm{d}t$	
5.2.7	Rate of change of cant deficiency dI/dt	
5.2.8	Length of transition curves in the horizontal plane L_K	
5.2.9	Length of circular curves and straights between two transition curves $L_{\rm i}$	
5.2.10		
5.2.11	Radius of vertical curve R_{v}	18
Annex	A (informative) Supplementary information for track alignment design related to shape and length of alignment elements	19
A.1	General	
A.2	Summary of the properties of different transition curves	
A.3	Further parameters that may be considered for track alignment curve design and a	
	progressive system of design rules	21
A.3.1	Symbols and abbreviations	
A.3.2	Objectives	
A.3.3	Progressive track alignment design	
A.3.4	Application	
Annex	B (informative) Length of alignment elements (circular curves and straights) between two transition curves L_i	
Annov	C (informative) Rules for converting parameter values for track gauges wider than 1435 mm .	44
C.1		
C.1	Scope	
C.2	Symbols and abbreviations	
C.4	Basic assumptions and equivalence rules	
C.4.1	Detailed conversion rules	
C.4.1 C.4.2	Radius of horizontal curve R_1 (5.2.1)	
C.4.2 C.4.3	Cant D_1 (5.2.2)	
C.4.3 C.4.4		
C.4.4 C.4.5	Cant excess E_1 (5.2.4)	
C.4.5 C.4.6	Cant gradient dD_1/ds (5.2.5)	
C.4.6 C.4.7	Rate of change of cant dD_1/dt (5.2.6 of the main body of the standard)	
C.4.7 C.4.8	Rate of change of cant deficiency dI_1/dt (5.2.7)	4ŏ ⊿o
C.4.8 C.4.9	Other parameters (5.2.9, 5.2.10 and 5.2.11)	
U.4.3	Other parameters (J.Z.S, J.Z. 10 and J.Z. 11)	43

	D (normative) Track alignment design parameter limits for track gauges wider than 1435 mm.	
D.1	Scope	
D.2	Requirements for a gauge of 1668 mm	
D.2.1	General	
D.2.2	Cant D_1	
D.2.3	Cant deficiency I_1	50
D.2.4	Cant excess E_1	51
D.2.5	Cant gradient dD ₁ /ds	
D.2.6	Rate of change of cant $\mathrm{d}D_1/\mathrm{d}t$	
D.2.7	Rate of change of cant deficiency $\mathrm{d}I_1/\mathrm{d}t$	
D.2.8	Length of circular curves and straights between two transition curves $L_{\rm il}$	
D.2.9	Vertical curves	
D.2.10	Radius of vertical curve	
D.3	Requirements for a gauge of 1524 mm	
D.3.1	General	
D.3.2	Cant <i>D</i> ₁	53
D.3.3	Cant deficiency I_1	54
D.3.4	Cant excess E_1	54
D.3.5	Cant gradient dD ₁ /ds	55
D.3.6	Rate of change of cant $\mathrm{d}D_1/\mathrm{d}t$	55
D.3.7	Rate of change of cant deficiency $\mathrm{d}I_1/\mathrm{d}t$	55
D.3.8	Length of circular curves and straights between two transition curves L_{i1}	56
D.3.9	Vertical curves	56
D.3.10	Radius of vertical curve	56
A	F (informative) The structure to be a letteral forces more untail by the nelling of sele	
	E (informative) Track resistance to lateral forces generated by the rolling stock	
E.1 E.2	Introduction The effect of alignment design parameters on lateral forces generated by the rolling stock	
E.2.1		
	Cant deficiency	
E.2.2	Cant excess	
E.2.3	The lateral strength limit of a track under loading (<i>Prud'homme</i> limit)	
E.2.4	Factors influencing the resistance to track lateral displacement	58
Annex	F (informative) Consequences on track resistance, stress and fatigue resulting from tilting	
	body train systems	59
F.1	General	
F.2	Basic principles applying to tilting body techniques	59
F.3	Safety requirements	
F.3.1	Lateral axle force and track lateral resistance (track lateral shift)	
F.3.2	Vehicle overturning	
	Comfort requirements	
	·	
Annex	G (informative) Constraints and risks associated with the use of exceptional limits	62
Annex	H (informative) Recapitulation of the work carried out by the ORE B 55 Committee -	
	Maximum permissible cant	63
H.1	Introduction	
H.2	Criteria for safety against derailment at low speed through wheel-climbing	
H.3	Limits for track twist.	
H.4	Rules applicable to the design of and checks performed on new vehicles with regard to	
	their capability of coping with track twist values	65
H.5	List of reports published by the ORE B 55 Committee	
Annex	I (informative) A-deviation Switzerland	66
Δημέν	ZA (informative) Relationship between this European Standard and the Essential	
, amic A	Requirements of EU Directive 2008/57/EC of the European Parliament and of the council of	
	17 June 2008 on the interoperability of the rail system within the Community	67
Bibliog	raphy	70

Foreword

This document (EN 13803-1:2010) has been prepared by Technical Committee CEN/TC 256 "Railway applications", the secretariat of which is held by DIN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by December 2010, and conflicting national standards shall be withdrawn at the latest by December 2010.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

This document supersedes ENV 13803-1:2002.

This document has been prepared under a mandate given to CEN by the European Commission and the European Free Trade Association, and supports essential requirements of EU Directive(s).

For relationship with EU Directive(s), see informative Annex ZA, which is an integral part of this document.

- Council Directive 96/48/EC of 23 July 1996 on the interoperability of the European high-speed network¹
- European Parliament and Council Directive 2004/17/EC of 31 March 2004 coordinating the procurement procedures of entities operating in the water, energy, transport and postal services sectors²
- Council Directive 91/440/EEC of 29 July 1991 on the development of the Community's railways³

EN 13803, Railway applications – Track – Track alignment design parameters – Track gauges 1435 mm and wider consists of the following parts:

- Part 1: Plain line
- Part 2: Switches and crossings and comparable alignment design situations with abrupt changes of the curvature

According to the CEN/CENELEC Internal Regulations, the national standards organizations of the following countries are bound to implement this European Standard: Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland and the United Kingdom.

 $^{^{\}rm 1}$ Official Journal of the European Communities N° L 235 of 1996-09-17

 $^{^2}$ Official Journal of the European Communities $\ensuremath{\text{N}^{\circ}}\ \ensuremath{\text{L}}\ \ 134$ of 2004-04-30

³ Official Journal of the European Communities N° L 237 of 1991-08-24

1 Scope

This European Standard specifies the rules and limits that determine permissible speed for a given track alignment. Alternatively, for a specified permissible speed, it defines limits for track alignment design parameters.

More restrictive requirements of the High Speed TSI Infrastructure and the Conventional Rail TSI Infrastructure, as well as other (national, company, etc.) rules will apply.

This European Standard applies to main lines with track gauges 1435 mm and wider with permissible speeds between 80 km/h and 300 km/h. Annex C (informative) describes the conversion rules which can be applied for tracks with gauges wider than 1435 mm. Normative Annex D is applied for track gauges wider than 1435 mm.

However, the values and conditions stated for this speed range can also be applied to lines where permissible speeds are less than 80 km/h, but in this case, more or less restrictive values may need to be used and should be defined in the contract.

This European Standard need not be applicable to certain urban and suburban lines.

This European Standard also takes account of vehicles that have been approved for high cant deficiencies.

For the operation of tilting trains, specific requirements are defined within this European Standard.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

EN 13803-2, Railway applications — Track — Track alignment design parameters — Track gauges 1435 mm and wider — Part 2: Switches and crossings and comparable alignment design situations with abrupt changes of curvature

EN 14363, Railway applications — Testing for the acceptance of running characteristics of railway vehicles — Testing of running behaviour and stationary tests

EN 15686, Railway applications — Testing for the acceptance of running characteristics of railway vehicles with cant deficiency compensation system and/or vehicles intended to operate with higher cant deficiency than stated in EN 14363:2005, Annex G

EN 15687, Railway applications — Testing for the acceptance of running characteristics of freight vehicles with static wheel axle higher than 225 kN and up to 250 kN

ISO 80000-3, Quantities and units — Part 3: Space and time

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1

alignment element

segment of the track with either vertical direction, horizontal direction or cant obeying a unique mathematical description as function of longitudinal distance

NOTE Unless otherwise stated, the appertaining track alignment design parameters are defined for the track centre line and the longitudinal distance for the track centre line is defined in a projection in a horizontal plane.

3.2

circular curve

alignment element of constant radius

3 3

transition curve

alignment element of variable radius

- NOTE 1 The clothoid (sometimes approximated as a 3rd degree polynomial, "cubic parabola") is normally used for transition curves, giving a linear variation of curvature. In some cases, curvature is smoothed at the ends of the transition.
- NOTE 2 It is possible to use other forms of transition curve, which show a non-linear variation of curvature. Informative Annex A gives a detailed account of certain alternative types of transitions that may be used in track alignment design.
- NOTE 3 Normally, a transition curve is not used for the vertical alignment.

3.4

compound curve

sequence of curved alignment elements, including two or more circular curves in the same direction

NOTE The compound curve may include transition curves between the circular curves and / or the circular curves and the straight tracks

3.5

reverse curve

sequence of curved alignment elements, containing alignment elements which curve in the opposite directions

NOTE A sequence of curved alignment elements, may be both a compound curve and a reverse curve.

3.6

cant

amount by which one running rail is raised above the other running rail

NOTE Cant is positive when the outer rail on curved track is raised above the inner rail and is negative when the inner rail on curved track is raised above the outer rail. Negative cant is unavoidable at switches and crossings on a canted main line where the turnout is curving in the opposite direction to the main line and, in certain cases, on the plain line immediately adjoining a turnout (see EN 13803-2).

3.7

equilibrium cant

cant at a particular speed at which the vehicle will have a resultant force perpendicular to the running plane

3.8

cant excess

difference between applied cant and a lower equilibrium cant

NOTE 1 When there is cant excess, there will be an unbalanced lateral force in the running plane. The resultant force will move towards the inner rail of the curve.

NOTE 2 Cant on a straight track results in cant excess, generating a lateral force towards the low rail.

3.9

cant deficiency

difference between applied cant and a higher equilibrium cant

NOTE When there is cant deficiency, there will be an unbalanced lateral force in the running plane. The resultant force will move towards the outer rail of the curve.

3.10

cant transition

alignment element where cant changes with respect to longitudinal distance

NOTE 1 Normally, a cant transition should coincide with a transition curve.

NOTE 2 Cant transitions giving a linear variation of cant are usually used. In some cases, cant is smoothed at the ends of the transition.

NOTE 3 It is possible to use other forms of cant transition, which show a non-linear variation of cant. Informative Annex A gives a detailed account of certain alternative types of transitions that may be used in track alignment design.

3.11

cant gradient

absolute value of the derivative (with respect to longitudinal distance) of cant

3.12

rate of change of cant

absolute value of the time derivative of cant

3.13

rate of change of cant deficiency

absolute value of the time derivative of cant deficiency (and/or cant excess)

3.14

maximum permissible speed

maximum speed resulting from the application of track alignment limits given in this standard

3.15

normal limit

limit not normally exceeded

NOTE The actual design values for new lines should normally have a margin to the normal limits. These values ensure maintenance costs of the track are kept at a reasonable level, except where particular conditions of poor track stability may occur, without compromising passenger comfort. To optimize the performance of existing lines it may be useful to go beyond the normal limits.

3.16

exceptional limit

extreme limit not to be exceeded

NOTE As exceptional limits are extreme, it is essential that their use is as infrequent as possible and subject to further consideration. Informative Annex H describes the constraints and risks associated with the use of exceptional limits.

4 Symbols and abbreviations

No.	Symbol	Designation	Unit
1	$a_{\rm i}$	quasi-static lateral acceleration, at track level, but parallel to the vehicle floor	m/s ²
2	a_{q}	non-compensated lateral acceleration in the running plane	m/s ²
3	C	factor for calculation of equilibrium cant	mm·m·h²/km²
4	da_i/dt	rate of change of quasi-static lateral acceleration, at track level, but parallel to the vehicle floor	m/s ³
5	da_q/dt	rate of change of non-compensated lateral acceleration	m/s ³
6	dD/ds	cant gradient	mm/m
7	dD/dt	rate of change of cant	mm/s
8	$\mathrm{d}I/\mathrm{d}t$	rate of change of cant deficiency (and/or cant excess)	mm/s
9	D	Cant	mm
10	D_{EQ}	equilibrium cant	mm
11	е	distance between the nominal centre points of the two contact patches of a wheelset (e.g. about 1500 mm for track gauge 1435 mm)	mm
12	E	cant excess	mm
13	g	acceleration due to gravity: 9,81 m/s ²	m/s ²
14	$h_{ m g}$	height of the centre of gravity	mm
15	I	cant deficiency	mm
16	$L_{ m D}$	length of cant transition	m
17	$L_{ m K}$	length of transition curve	m
18	$L_{\rm i}$	length of alignment elements (circular curves and straights) between two transition curves	m
19	lim	limit (index)	-
20	max	maximum value (index)	-
21	min	minimum value (index)	-
22	q_D	factor for calculation of length of cant transition	m·h/(km·mm)
23	q_I	factor for calculation of length of transition curve	m·h/(km·mm)
24	q_N	factor for calculation of length of cant transition or transition curve with non-constant gradient of cant and curvature, respectively	-
25	q_R	factor for calculation of vertical radius	m·h²/km²
26	q_V	factor for conversion of the units for vehicle speed: 3,6 km·s/(h·m)	km·s/(h·m)
27	R	radius of horizontal curve	m
28	$R_{ m v}$	radius of vertical curve	m
29	S	longitudinal distance	m
30	S_r	roll flexibility coefficient, equivalent to flexibility coefficient s in EN 15273-1	-
31	S_t	tilt compensation factor of a tilt system	-
32	t	time	s
33	V	speed	km/h
34	V_{max}	maximum speed of fast trains	km/h
35	V_{min}	minimum speed of slow trains	km/h
36	$\Delta a_{ m q}$	overall variation of non-compensated lateral acceleration	m/s ²
37	ΔD	overall variation of cant along a cant transition	mm
38	ΔI	overall variation of cant deficiency (and/or cant excess)	mm

5 Requirements

5.1 Background

5.1.1 General

The following technical normative rules assume that standards for acceptance of vehicle, track construction and maintenance are fulfilled.

A good compromise has to be found between train dynamic performance, maintenance of the vehicle and track, and construction costs. More restrictive limits than those in this European Standard may be specified in the contract.

Unnecessary use of the exceptional limits specified in this European Standard should be avoided. A substantial margin to them should be provided, either by complying with the normal limits or by applying a margin with respect to permissible speed.

For further details, see informative Annex G.

5.1.2 Track alignment design parameters

The following parameters are specified in	5.2:
---	------

- radius of horizontal curve R (m) (*S);
- cant D (mm) (*S);
- cant deficiency I (mm) (*S);
- cant excess E (mm);
- cant gradient dD/ds (mm/m) (*S);
- rate of change of cant dD/dt (mm/s);
- rate of change of cant deficiency (and/or cant excess) dI/dt (mm/s);
- length of cant transitions $L_{\rm D}$ (m) (*S);
- length of transition curves in the horizontal plane $L_{\rm K}$ (m);
- length of alignment elements (circular curves and straights) between two transition curves L_i (m);
- radius of vertical curve R_v (m);
- speed V (km/h) (*S).

Parameters followed by the (*S) note indicate safety-related parameters.

5.1.3 Parameter quantification

For most of the parameters, two different types of limits are specified:

- a normal limit;
- an exceptional limit which may have two different meanings:
 - a) For **safety-related parameters**, it shall be the absolute maximum limit of this parameter; this maximum limit may depend upon the actual track mechanical and geometrical state.

NOTE 1 The exceptional limits are safety-related and may (for most parameters) induce a reduced comfort level. These limits are extreme and should be used only under special circumstances or after specific safety-case analysis.

NOTE 2 The limits are defined for normal service operations. If and when running trials are conducted, for example to ascertain the vehicle dynamic behaviour (by continually monitoring of the vehicle responses), exceeding the limits (particularly in terms of cant deficiency) should be permitted and it is up to the infrastructure manager to decide any appropriate arrangement. In this context, safety margins are generally reinforced by taking additional steps such as ballast consolidation, monitoring of track geometric quality, etc.

b) For non-safety related parameters, the limits shall be considered as the limit above which passenger comfort may be affected and the need for track maintenance increased; however, to cope with special situations, values in excess of the limits may be used, but they shall not exceed any safety limit.

The use of exceptional limits should be avoided, especially use of exceptional limits for several parameters at the same location along the track.

For cant deficiency, not all vehicles are approved for the normal or exceptional limits. For such vehicles, the operational limit shall be consistent with the approved maximum cant deficiency.

5.2 Normal limits and exceptional limits for track alignment design parameters

5.2.1 Radius of horizontal curve R

The largest curve radii and transition permitted by track design constraints should be used where possible. Normal limit for radius is 190 m and exceptional limit is 150 m. Note that these small radii will result in a permissible speed less than 80 km/h. Hence, normal and exceptional limits for the radius shall also be derived from the requirements below.

The parameters that shall be considered in the determination of the minimum curve radius are:

- the maximum and minimum speeds;
- the applied cant;
- the limits for cant deficiency and cant excess.

For every combination of maximum speed $V_{\rm max}$ and maximum cant deficiency $I_{\rm lim}$, the minimum permissible curve radius shall be calculated using the following equation:

$$R_{\min} = \frac{C}{D + I_{\lim}} \cdot V_{\max}^2$$
 [m]

where $C = 11.8 \text{ mm} \cdot \text{m} \cdot \text{h}^2/\text{km}^2$

Where $D > E_{\text{lim}}$, the maximum permissible curve radius for the minimum speed V_{min} shall be calculated using the following equation:

$$R_{\rm max} = \frac{C}{D - E_{\rm lim}} \cdot V_{\rm min}^2 \qquad [m]$$

where $C = 11.8 \text{ mm} \cdot \text{m} \cdot \text{h}^2/\text{km}^2$, and $D > E_{\text{lim}}$

NOTE 1 It is recommended that the radius of tracks alongside platforms should not be less than 500 m. This is to restrict the gap between platform and vehicles to facilitate safe vehicle access and egress by passengers.

NOTE 2 Small radius curves may require gauge widening in order to improve vehicle curving.

5.2.2 Cant D

Cant shall be determined in relation to the following considerations:

- high cant on small-radius curves increases the risk of low-speed freight wagons derailing. Under these conditions, vertical wheel loading applied to the outer rail is much reduced, especially when track twist (defined in EN 13848-1) causes additional reductions;
- cant exceeding 160 mm may cause freight load displacement and the deterioration of passenger comfort when a train makes a stop or runs with low speed (high value of cant excess). Works vehicles and special loads with a high centre of gravity may become unstable;
- high cant increases cant excess values on curves where there are large differences between the speeds of fast trains and slow trains.

Normal limit for cant is 160 mm.

NOTE It is recommended that cant should be restricted to 110 mm for tracks adjacent to passenger platforms. Some other track features, such as level crossings, bridges and tunnels may also, in certain local circumstances, impose cant restrictions

Exceptional limit for cant is 180 mm.

To avoid the risk of derailment of torsionally-stiff freight wagons on small radius curve (R < 320 m), cant should be restricted to the following limit:

$$D_{\text{lim}} = \frac{R - 50\text{m}}{1,5\text{m/mm}}$$
 [mm]

The application of this limit assumes a high maintenance standard of the track, especially regarding twist. For further information, see informative Annex H.

5.2.3 Cant deficiency I

For given values of local radius R and cant D, the cant deficiency I shall determine the maximum permissible speed through a full curve such that:

$$I = C \cdot \frac{V^2}{R} - D = D_{EQ} - D \le I_{\lim}$$
 [mm]

where $C = 11.8 \text{ mm} \cdot \text{m} \cdot \text{h}^2/\text{km}^2$

NOTE 1
$$I_{\text{lim}}$$
 can be replaced with the value $(a_{\text{q}})_{\text{lim}}$: $a_{\text{q}} = \left(\frac{V}{q_V}\right)^2 \frac{1}{R} - \frac{g \cdot D}{e} = \frac{g \cdot I}{e} \le \left(a_{\text{q}}\right)_{\text{lim}} = \frac{g \cdot I_{\text{lim}}}{e} \text{ [m/s}^2]$

Normal and exceptional limits for cant deficiency are given in Table 1. These limits apply to all trains operating on a line. It is assumed that every vehicle has been tested and approved according to the procedures in EN 14363, EN 15686 and/or EN 15687 in conditions covering its own range of operating cant deficiency.

Table 1 — Cant deficiency $I_{\rm lim}$

	Normal limits	Exceptional limits		
Non-tilting trains				
80 km/h < V ≤ 200 km/h	130 mm	183 mm		
200 km/h < V ≤ 230 km/h	130 mm	168 mm ^{ab}		
230 km/h < V ≤ 250 km/h	130 mm	153 mm ^{ab}		
250 km/h < V ≤ 300 km/h	100 mm	130 mm ^{abc}		
Tilting trains				
80 km/h ≤ V ≤ 260 km/h ^d	275 mm ^b	306 mm ^b		
	·			

^a Trains complying with EN 14363, equipped with a cant deficiency compensation system other than tilt, may be permitted by the Infrastructure Manager to run with higher cant deficiency values.

NOTE 1 The European signalling system ERTMS includes vehicle limits of cant deficiency $I_{\rm lim}$ of 92 mm, 100 mm, 115 mm, 122 mm, 130 mm, 153 mm, 168 mm, 183 mm, 245 mm, 275 mm and 306 mm. These values reflect the current practice of operating different train categories in Europe.

NOTE 2 Freight vehicles are normally approved for a cant deficiency in the range 92 mm to 130 mm.

NOTE 3 Non-tilting passenger vehicles are normally approved for a cant deficiency of 130 mm to 168 mm.

NOTE 4 Depending on the characteristics of certain special features in track, such as certain switches and crossings in curves, bridges carrying direct-laid ballastless track, tracks with jointed rails, certain sections of line exposed to very strong cross winds, etc., it may be necessary to restrict the permissible cant deficiency. Rules in respect of these restrictions cannot be formulated beforehand since they will be dictated by the design of the special features; definition of such a frame of reference can only be left to the initiative of the Infrastructure Manager.

NOTE 5 For further considerations of rolling stock required to operate at high cant deficiencies, passenger comfort with respect to lateral acceleration may be analysed as follows:

- The quasi-static lateral acceleration a_i (at track level, but parallel to the vehicle floor) is a measure of the acceleration felt by passengers inside the vehicle;
- For a non-tilting train a_i is greater than the lateral non-compensated acceleration in the track plane a_q : $a_i = (1 + s_r) \cdot \frac{I}{e} \cdot g = (1 + s_r) \cdot a_q$ [m/s²];
- For a tilting train this can be expressed approximately by $a_i = (1 + s_r) \cdot (1 s_t) \cdot \frac{I}{e} \cdot g$ [m/s²];
- The roll flexibility coefficient s_r is positive for non-tilting vehicles, as the longitudinal rotation axis of the coach body is low (around the top of the suspension plane), hence the lateral acceleration felt by passengers due to cant deficiency is greater than that applied to the running plane. This coefficient can be reduced by choosing a dedicated suspension

^b The Infrastructure Manager may require qualification of a part of a line for the introduction of trains running at these or higher cant deficiencies, taking into account the required track quality and other conditions.

^c The limit may be raised to 153 mm for non ballasted track.

^d Currently, there are no lines used or planned where maximum speed for tilting trains exceeds 260 km/h.

system. With tilt techniques $s_t > 0$, the lateral acceleration felt by passengers for uncompensated acceleration in the running plane will be reduced;

The influence of lateral acceleration on passenger comfort is described in EN 12299.

NOTE 6 For further details regarding operations with tilting trains, see informative Annex F

5.2.4 Cant excess E

There is cant excess when the following has a positive value:

$$E = D - C \cdot \frac{V^2}{R} = D - D_{EQ}$$
 [mm]

where $C = 11.8 \text{ mm} \cdot \text{m} \cdot \text{h}^2/\text{km}^2$

Normal limit for cant excess $E_{\rm lim}$ is 110 mm.

The value of *E* affects inner-rail stresses induced by slow trains, since the quasi-static vertical wheel/rail force of an inner wheel is increased.

5.2.5 Cant gradient dD/ds

The following limits apply everywhere along the track where cant is varying:

$$\left(\frac{\mathrm{d}D}{\mathrm{d}s}\right)_{\mathrm{max}} \le \left(\frac{\mathrm{d}D}{\mathrm{d}s}\right)_{\mathrm{lim}}$$
 [mm/m]

Normal limit:
$$\left(\frac{dD}{ds}\right)_{v} = 2,25 \text{ mm/m}$$

Exceptional limit:
$$\left(\frac{dD}{ds}\right)_{lim} = 2,50 \text{ mm/m}$$

NOTE For permissible speed lower than 80 km/h, a higher cant gradient may be used after a safety-case analysis, see Annex H.

For cant transitions with constant cant gradient, $\left(\frac{\mathrm{d}D}{\mathrm{d}s}\right)_{\mathrm{max}}$ can be calculated from the overall cant variation ΔD and the length L_{D} :

$$\frac{\mathrm{d}D}{\mathrm{d}s} = \frac{\Delta D}{L_D} \le \left(\frac{\mathrm{d}D}{\mathrm{d}s}\right)_{\mathrm{lim}}$$
 [mm/m]

There are no further special limits for the tilting trains.

5.2.6 Rate of change of cant dD/dt

5.2.6.1 Rate of change of cant dD/dt for non-tilting trains

Cant transitions are normally found in transition curves. However, it may be necessary to provide cant transitions in circular curves and straights.

For cant transitions with constant cant gradient, the following relationship with ΔD being the cant variation shall apply:

$$\frac{\mathrm{d}D}{\mathrm{d}t} = \frac{\Delta D}{L_D} \cdot \frac{V}{q_V} \le \left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}}$$
 [mm/s]

where L_D is the length of the cant transition in metres, V is vehicle speed in km/h and q_V = 3,6 km·s/(h·m)

Normal and exceptional limits for rate of change of cant are given in Table 2.

Table 2 — Rate of change of cant $(dD/dt)_{lim}$ for constant cant gradients

	Normal limits	Exceptional limits	
Non-tilting trains <i>V</i> ≤200 km/h			
<i>I</i> ≤168 mm	50 mm/s	70 mm/s ^a	
168< <i>I<u><</u>183</i> mm	50 mm/s	50 mm/s	
Non-tilting trains 200 km/h< <i>V</i> <300 km/h			
	50 mm/s	60 mm/s	
^a Where I <153 mm and dI/dt <70 mm/s, the exceptional limit for dD/dt may be raised to 85 mm/s.			

For cant transitions with variable cant gradient, the value of dD/dt is not constant.

$$\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{max}} \le \left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{max}}$$
 [mm/s]

Normal limit:
$$\left(\frac{dD}{dt}\right)_{lim} = 55 \text{ mm/s}$$

Exceptional limit
$$\left(\frac{dD}{dt}\right)_{lim} = 76 \text{ mm/s}$$

NOTE 1 Due to limited experience with transitions with variable gradients, the limits for rate of change of cant are indicative. They may be replaced by limits for second derivative of cant with respect to time (d^2D/dt^2) , see A.3.

NOTE 2 Informative Annex A gives further information on linear cant transitions and alternative types of cant transitions.

5.2.6.2 Rate of change of cant dD/dt for tilting trains

Both active and the passive tilt systems need certain time to adapt the angle of tilt to the curve radius and it is for this reason that curves shall include transition sections of sufficient length.

The transition curves should coincide with the cant transitions. If they do not, then special running tests are recommended to determine to what extent the maximum cant deficiency may need to be reduced.

The clothoid is normally used for transition curves, giving a linear variation of curvature. Where using transition curves with non-constant gradients, the function of the tilt system shall be taken into account for the analysis of the complex interaction between the vehicle and the track.

Normal limit:
$$\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}} = 75 \, \mathrm{mm/s}$$

Exceptional limit:
$$\left(\frac{dD}{dt}\right)_{lim} = 95 \text{ mm/s}$$

5.2.7 Rate of change of cant deficiency dI/dt

For track elements with a variation of curvature and/or a variation of cant the following relationship has to be fulfilled.

$$\left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{\mathrm{max}} \le \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{\mathrm{lim}}$$
 [mm/s]

NOTE 1 The variation of non-compensated lateral acceleration in the running plane may be determined as $\left(\frac{da_q}{dt}\right)_{max} \le \left(\frac{da_q}{dt}\right)_{lim} [m/s^3]$

NOTE 2 The rate of change of the quasi-static lateral acceleration, at track level, but parallel to the vehicle floor (da_i/dt) , which is a measure of the rate of change of acceleration felt by the passenger inside the vehicle, is greater than the rate of change of non-compensated acceleration in the track plane (da_i/dt) : $\frac{da_i}{dt} = (1+s_r) \cdot \frac{da_q}{dt}$ [m/s³].

NOTE 3 The influence of rate of change of lateral acceleration on passenger comfort is described in EN 12299.

Normal and exceptional limits for rate of change of cant deficiency are given in Table 3.

Table 3 — Rate of change of cant deficiency (dI/dt)lim

	Normal limits	Exceptional limits		
Non-tilting trains	<i>V</i> ≤200 km/h			
<i>I</i> ≤168 mm	55 mm/s	100 mm/s		
168< <i>I</i> <u><</u> 183 mm	55 mm/s	90 mm/s		
Non-tilting trains 200 km/h< <i>V</i> <300 km/h				
	55 mm/s	75 mm/s		
		•		

In case of using tilting trains on given alignment, the values of $\mathrm{d}I/\mathrm{d}t$ are higher. The tilt control system creates transient states at the entry to curves, which may give rise to even more pronounced jerks. Both active and the passive tilt systems need certain time to adapt the angle of tilt to the curve radius and it is for this reason that curves shall include transition sections of sufficient length.

Normal limit:
$$\left(\frac{dI}{dt}\right)_{lim} = 100 \text{ mm/s}$$

For transitions with constant gradients of curvature and cant with ΔI the overall cant deficiency variation along the whole transition it follows:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\Delta I}{L_K} \cdot \frac{V}{q_V} \le \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{\mathrm{lim}}$$
 [mm/s]

where L_K is the length of the transition in metres, V is vehicle speed in km/h and q_V = 3,6 km·s/(h·m)

NOTE 4 For transitions with constant gradients of curvature and cant with $\Delta a_{\rm q}$ the overall variation of non-compensated lateral acceleration along the whole transition it follows: $\frac{da_q}{dt} = \frac{\Delta_q}{L_{\scriptscriptstyle K}} \quad \frac{V}{q_{\scriptscriptstyle V}} \leq \quad \frac{da_q}{dt} \quad \text{[m/s}^3\text{]}.$

The values of dI/dt and da_q/dt are not constant for transitions with non-linear curvature variation and non-linear cant application, see informative Annex A for further information.

5.2.8 Length of transition curves in the horizontal plane L_K

For transition curves coinciding with cant transitions, $L_K = L_D$, with a constant gradient of curvature and cant, the minimum length shall be determined using the parameters from 5.2.5, 5.2.6 and 5.2.7 in the following manner:

- cant gradient $\frac{dD}{ds}$;
- rate of change of cant $\frac{dD}{dt}$;
- rate of change of cant deficiency $\frac{dI}{dt}$;

and by the following formulae:

$$L_D \ge \Delta D \cdot \left(\frac{dD}{ds}\right)_{lim}^{-1}$$
 [m]

$$L_D \ge \frac{V}{q_V} \cdot \Delta D \cdot \left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}}^{-1} [\mathrm{m}]$$

$$L_K \ge \frac{V}{q_V} \cdot \Delta I \cdot \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{\mathrm{lim}}^{-1} [\mathrm{m}]$$

NOTE 1 Certain local standards present the requirement for $\mathrm{d}D/\mathrm{d}t$ in the form $L_D \geq \frac{(q_D)_{\lim} \cdot V \cdot \Delta D}{1000}$ [m], where the limit for q_D is defined from $q_D = \frac{1000}{q_V} \cdot \left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)^{-1}$ [m·h/(km·mm)].

NOTE 2 Certain local standards present the requirement for dI/dt in the form $L_K \ge \frac{\left(q_I\right)_{\lim} \cdot V \cdot \Delta I}{1000}$ [m], where the limit for q_I is defined from $q_I = \frac{1000}{q_V} \cdot \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)^{-1}$ [m·h/(km·mm)].

For transition curves coinciding with cant transitions, $L_K = L_D$, with a non-constant gradient of curvature and cant, the minimum length shall be determined using the parameters from 5.2.5, 5.2.6 and 5.2.7 in the following manner:

$$L_D \ge q_N \cdot \Delta D \cdot \left(\frac{dD}{ds}\right)_{lim}^{-1}$$
 [m]

$$L_{K} \ge q_{N} \cdot \frac{V}{q_{V}} \cdot \Delta D \cdot \left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\lim}^{-1} [m]$$

$$L_K \geq q_N \cdot \frac{V}{q_V} \cdot \Delta I \cdot \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{\mathrm{lim}}^{-1} \text{ [m]}$$

NOTE 3 Certain local standards present the requirement for dD/dt in the form $L_D \ge \frac{(q_D)_{\lim} \cdot V \cdot \Delta D}{1000}$ [m], where the limit

$$\label{eq:qD} \text{for } q_D \text{ is defined from } q_D = q_N \cdot \frac{1000}{q_V} \cdot \left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)^{\!-1} \, [\text{m·h/(km·mm)}].$$

NOTE 4 Certain local standards present the requirement for dI/dt in the form $L_K \ge \frac{(q_I)_{\lim} \cdot V \cdot \Delta I}{1000}$ [m], where the limit for

$$q_I \text{ is defined from } q_I = q_N \cdot \frac{1000}{q_V} \cdot \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)^{\!-1} \, [\mathrm{m\cdot h/(km\cdot mm)}].$$

For certain types of transitions with non-constant gradient of curvature and cant, the value of the factor q_N is defined in Table 4.

Table 4 — Factor q_N for transitions with non-constant gradient of curvature and cant

Bloss	Cosine	Helmert (Schramm)	Sine (Klein)
1,5	π/2	2	2

The length of transition curve shall comply with all three criteria. It has to be at least the largest value derived from the above formulae for the selected values of $\left(\frac{\mathrm{d}D}{\mathrm{d}s}\right)_{\mathrm{lim}}$, $\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}}$ and $\left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{\mathrm{lim}}$.

NOTE 5 Due to limited experience with transitions with variable cant gradients, this method limiting the rate of change of cant is indicative. If it is replaced by limits for second derivative of cant with respect to time (d^2D/dt^2) , other formulas for the minimum transition lengths should be applied, see A.3.

Where there is no transition curve or it is of insufficient length with respect to the dI/dt criterion, the limits of the abrupt change of cant deficiency, defined in EN 13803-2, shall be complied with.

5.2.9 Length of circular curves and straights between two transition curves L_i

The normal limit for the length of a straight or a circular curve placed between two transition curves is 20 m, $L_i \ge$ 20 m.

NOTE As an alternative to a short length of a straight or a circular curve, this alignment element may be omitted and the two transition curves connected directly to each other.

For an alternative method to define the minimum lengths, see informative Annex B.

5.2.10 Vertical curves

Vertical curves should be at least 20 m long and may be designed without vertical transition curves.

NOTE Vertical curves are normally designed as parabolas (2nd degree polynomials) or as circular curves.

A vertical curve shall be provided where the difference in slope between adjacent gradients is more than:

- 2 mm/m for permissible speeds up to 230 km/h;
- 1 mm/m for permissible speeds over 230 km/h.

The limits for the vertical radii are defined in 5.2.11.

5.2.11 Radius of vertical curve R_v

The normal limit for radius of vertical curve is $R_{\nu, \rm lim} = q_{R, \rm lim} \cdot V^2$ [m], where $q_{\rm R, lim} = 0.35 \, \rm m \cdot h^2/km^2$, without going under 2000 m vertical radius.

NOTE 1 On lines where most of the passengers may be standing, it is recommended that q_R should be greater than 0.77 m·h²/km².

The exceptional limit for radius of vertical curve is $R_{v, \rm lim} = q_{R, \rm lim} \cdot V^2$ [m], where $q_{\rm R, lim} = 0.13~\rm m\cdot h^2/km^2$ for hollow, and $q_{\rm R, lim} = 0.16~\rm m\cdot h^2/km^2$ for crest.

For sections with switches and crossings laid in vertical curves, the limits defined in EN 13803-2 shall be complied with.

Annex A

(informative)

Supplementary information for track alignment design related to shape and length of alignment elements

A.1 General

Among other features of track alignment design, all changes imposed in the lateral plane to vehicle trajectory are important for providing good ride comfort. In such situations, the vehicle is subjected to abrupt variations of cant and curvature gradients (second order derivatives). The dynamic response to these types of excitations depends upon the suspension design. However, a common feature is that this response lasts a few seconds before the effect is eliminated over a following alignment element (circular curve or straight).

This annex provides detailed analysis methods with respect to design of transition curves and track alignment elements.

Subclause A.2 includes a Table summarising the properties of the following transition curves, compared with the conventional clothoid, sometimes approximated as a third degree polynomial "cubic parabola":

- Bloss curve;
- Cosine curve:
- Helmert curve, also known as the Schramm curve;
- Sine curve, also known as the *Klein* curve.

Subclause A.3 entitled "Further parameters that may be considered for transition curve design and a progressive system of design rules" provides a more comprehensive analysis method of the vehicle behaviour in complex curving situations with segments of varying curvature or / and cant with respect to the roll movement and of the consequences in terms of track alignment assessment.

A.2 Summary of the properties of different transition curves

In addition to the symbols used in the main part of the European Standard the following symbols are used in the Table summarising the properties of different transition curves shapes and the maximum values of corresponding parameters for track with gauge 1435 mm:

- K horizontal curvature (m)
- v line speed (m/s)
- 1:n cant gradient
- $r_{\rm v}$ vertical radius of smoothed outer rail at the beginning and the end of the uniform slope (m)
- a_v vertical acceleration in the track centre line within the transition curve (mm/s²)
- f_L shift (m)

Table A.1 summarises the properties of different transition curves shapes compared with the conventional clothoid, which is the bases of this European Standard and the maximum values of corresponding parameters for track with gauge 1435 mm.

EN 13803-1:2010 (E)

Table A.1 — Transition curves and the maximum values of corresponding parameters for track with gauge 1435 mm

NOTE (Unit length: m; unit time: s; speed v in m/s)					
	Clothoid	Bloss	Cosine	Helmert (Schramm)	Sine (Klein)
Horizontal curvature $_K$; Non-compensated lateral acceleration in the track plane $_{a_{\rm q}}$	$K = \frac{1}{R}; \ a_{q} = \frac{v^{2}}{R} - \frac{g \cdot D}{1,5}$	$K = \frac{1}{R}; \ a_{q} = \frac{v^{2}}{R} - \frac{g \cdot D}{1,5}$	$K = \frac{1}{R}; \ a_{q} = \frac{v^{2}}{R} - \frac{g \cdot D}{1,5}$	$K = \frac{1}{R}; a_{q} = \frac{v^{2}}{R} - \frac{g \cdot D}{1,5}$	$K = \frac{1}{R}; a_{q} = \frac{v^{2}}{R} - \frac{g \cdot D}{1,5}$
Cant <i>D</i> ; Maximum rate of change of cant as function of the length d <i>D</i> /d <i>s</i> through maximum values of 1: <i>n</i> _{max}	$1: n = \frac{D}{L}$	$1:n_{\text{max}}$ $1:n_{\text{max}} = \frac{3 \cdot D}{2 \cdot L}$	$1:n_{\text{max}}$ $1:n_{\text{max}} = \frac{\pi \cdot D}{2 \cdot L}$	$1:n_{\text{max}}$ $1:n_{\text{max}} = \frac{2 \cdot D}{L}$	$1:n_{\text{max}}$ $1:n_{\text{max}} = \frac{2 \cdot D}{L}$
Rate of change of non-compensated lateral acceleration $\mathrm{d}a_q/\mathrm{d}t$; rate of change of cant $\mathrm{d}D/\mathrm{d}t$; vertical velocity $v_{\mathrm{v}}^{\ a}$; maximum lateral jerk $\mathrm{d}a_q/\mathrm{d}t _{\mathrm{max}}$; maximum value of $\mathrm{d}D/\mathrm{d}t _{\mathrm{max}}$	$\frac{\mathrm{d}a_{\mathrm{q}}}{\mathrm{d}t} _{\mathrm{max}} = \frac{a_{\mathrm{q}} \cdot v}{L}; \frac{\mathrm{d}D}{\mathrm{d}t} _{\mathrm{max}} = \frac{D \cdot v}{L}$	$\frac{da_q}{dt}\Big _{mas} = \frac{3 a_q v}{2 L}; \frac{dD}{dt}\Big _{max} = \frac{3 \cdot D \cdot v}{2 \cdot L}$	$\frac{da_{q}}{dt} _{max} = \frac{\pi \cdot a_{q} \cdot \nu}{2 \cdot L}; \frac{dD}{dt} _{max} = \frac{\pi \cdot D \cdot \nu}{2 \cdot L}$	$\frac{da_{q}}{dt}\Big _{max} = \frac{2 \cdot a_{q} \cdot v}{L}; \frac{dD}{dt}\Big _{max} = \frac{2 \cdot D \cdot v}{L}$	$\frac{da_{q}}{dt} _{max} = \frac{2 \cdot a_{q} \cdot v}{L}; \frac{dD}{dt} _{max} = \frac{2 \cdot D \cdot v}{L}$
Rate of change of lateral jerk; vertical acceleration $a_{\rm v}^{\ a}$; maximum value of ${\rm d}^2 a_{\rm q} / {\rm d} t^2 _{\rm max}$; maximum value $a_{\rm v \ max}^{\ a}$	$\frac{d^{2}a_{v}}{dt^{2}}\Big _{max} = \infty; a_{v max} = \frac{v^{2}}{2 \cdot r_{v}}$	$\frac{d^{2}a_{q}}{dr^{2}} _{\max} = \frac{6 \cdot a_{q} \cdot v^{2}}{L^{2}}; a_{v\max} = \frac{3 \cdot D \cdot v^{2}}{L^{2}}$	$\frac{d^{2}a_{q}}{dr^{2}} _{\max} = \frac{\pi^{2} \cdot a_{q} \cdot v^{2}}{2 \cdot L^{2}}; a_{v\max} = \frac{\pi^{2} \cdot D \cdot v^{2}}{4 \cdot L^{2}}$	$\frac{\mathrm{d}^2 a_{\mathrm{q}}}{\mathrm{d}t^2} _{\mathrm{max}} = \frac{4 \cdot a_{\mathrm{q}} \cdot v^2}{L^2}; a_{\mathrm{vmax}} = \frac{2 \cdot D \cdot v^2}{L^2}$	$\frac{\mathrm{d}^2 a_{\mathbf{q}}}{\mathrm{d}t^2}\Big _{\mathrm{max}} = \frac{2 \cdot \pi \cdot a_{\mathbf{q}} \cdot \nu^2}{L^2}, a_{\mathrm{vmax}} = \frac{\pi \cdot D \cdot \nu^2}{L^2}$
Rate of change of vertical acceleration da_v/dt^a ; maximum vertical jerk $da_v/dt _{max}^a$			Φ Φ		$\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}t} = \frac{2 \cdot \pi^2 \cdot D \cdot v^3}{L^3}$
Shift $f_{\rm L}^{\ \ \rm b}$	$f_{\text{Cloth}} = \frac{L^2}{24 \cdot R}$	$f_{Bloss} = \frac{L^2}{40 \cdot R}$	$f_{\text{Cosine}} = \frac{L^2}{42,23 \cdot R}$	$f_{\text{Helmert}} = \frac{L^2}{48 \cdot R}$	$f_{\text{Sine}} = \frac{L^2}{61,2 \cdot R}$
Equivalent length of transition curve $(f_L = f_{Cloth})^b$	$L_{ m Cloth}$	L_{Bloss} = 1,291· L_{Cloth}	L_{Cosine} = 1,326· L_{Cloth}	L_{Helmert} = 1,414· L_{Cloth}	$L_{\rm Sine}$ = 1,597· $L_{\rm Cloth}$
Rate of value of horizontal jerk $(f_L = f_{Cloth})^b$	1,0	1,162	1,184	1,414	1,252
Rate of change of vertical acceleration $(f_L = f_{Cloth})^{a \ b}$	1,0	0,37	0,29	0,21	0,25
a The shape and the maximum valu	es only apply if the inner rail of t	he curve is kent on level and only th	e outer rail is elevated		

^a The shape and the maximum values only apply, if the inner rail of the curve is kept on level and only the outer rail is elevated.

^b The values for shift are approximations. For design, exact values should be used.

A.3 Further parameters that may be considered for track alignment curve design and a progressive system of design rules

A.3.1 Symbols and abbreviations

Table A.2 gives additional symbols used in this informative subclause.

Table A.2 — List of symbols and abbreviations

No.	Symbol	Designation	Dimension; unit
39	0	index for a limit usually referred to in this European Standard	-
40	1	index for a special value	-
41	С	index for a fixed value (in a circular curve)	-
42	a_0	non-compensated lateral acceleration at height h	length/time ²
43	a_V	vertical acceleration	length/time ²
44	\dot{a}_{Q0}	limit of non-compensated lateral jerk at height h	length/time ³
45	$\dot{a}_{ m V0}$	limit of vertical jerk	length/time ³
46	α	angular acceleration about roll axis	rad/time ²
47	$\dot{\alpha}_0$	limit of angular jerk about roll axis	rad/time ³
48	b; B	distance between wheel treads of an axle (on track with gauge 1435 mm: $B=1500$ mm; on track with gauge 1668 mm: $B=1733$ mm; on track with gauge 1524 mm: $B=1585$ mm)	length; mm
49	$oldsymbol{eta}_{ ext{Q}}$	angle of non-compensated lateral acceleration	rad
50	$oldsymbol{eta}_{ ext{ iny V}}$	ratio of vertical acceleration	1
51	$\dot{oldsymbol{eta}}_{ ext{Q}0}$	limit of angle of non-compensated lateral jerk	rad/time
52	$\dot{eta}_{ ext{V}0}$	limit of ratio of vertical jerk	1/time
53	c_{v1}	factor for the conversion of the units for vehicle speed $c_{v1} = \frac{5}{18}$	-; m·h/(km·s)
54	c_{v2}	factor for the conversion of the units for vehicle speed used for accelerations $c_{v2} = \frac{1}{12,96}$	-; (m·h/(km·s)) ²
55	c _{v3}	factor for the conversion of the units for vehicle speed used for jerks $c_{v3} = \frac{1}{46,656}$	-; (m·h/(km·s)) ³
56	c ₁₁	factor for the conversion of the units for the angular acceleration about roll axis	-; mm·(km·s/(m·h)) ²
57	c ₁₅	factor for the conversion of the units for the angular jerk about roll axis	-; mm·(km·s/(m·h)) ³
58	c ₂₁	factor for the conversion of the units for the cant deficiency due to curvature in the horizontal plane	-; mm·m·h²/km²

Table A.2 (continued)

No.	Symbol	Designation	Dimension; unit
59	c ₂₂	factor for the conversion of the units for the cant deficiency due to the alignment induced roll motion	-; (m·h/km) ²
60	c ₂₅	factor for the conversion of the units for the rate of change of cant deficiency due to curvature in the horizontal plane	-; mm·h³/(km³·s)
61	c ₂₆	factor for the conversion of the units for the rate of change of cant deficiency due to the alignment induced roll motion	-; (m·h/km) ³ /s
62	c ₂₇	factor for the conversion of the units for the rate of change of cant deficiency and the lateral jerk	-; mm·s²/m
63	c ₃₁	factor for the conversion of the units for vertical acceleration due to curvature in the diagram of altitudes	-; m·(h/km)²
64	c ₃₅	factor for the conversion of the units for vertical jerk due to curvature in the diagram of altitudes	-; (s/m ²)·(km/h) ³
65	d	differential quantity	-
66	d; D	cant	length; mm
67	g	acceleration due to gravity according to ISO 80000-3	length/time ²
68	γ	angle of cant gradient (twist of the track)	rad/length
69	h	mean height of centre of gravity of vehicles (for all track gauges: $h = 1.8 \text{ m}$)	length
70	i;I	cant deficiency in the sense of unbalanced cant	length; mm
71	\dot{i}_0 ; \dot{I}_0	limit of rate of change of cant deficiency	length/time; mm/s
72	\mathcal{K}_{H}	curvature of the projection of the track centre line in the horizontal plane	1/length
73	K_{R}	curvature of a rail in the diagram of altitudes	1/length
74	$\kappa_{ m V}$	curvature of the track centre line in the diagram of altitudes	1/length
75	$\ell;L$	length of a transition curve and cant gradient	length; m
76	ω	angular velocity about roll axis	rad/time
77	arphi	direction angle in the horizontal plane	rad
78	Ψ	angle of cant	rad
79	r_H ; R_H	local radius of the track centre line in the horizontal plane	length; m
80	r_R	local radius of a rail in the diagram of altitudes	length
81	r_V ; R_V	local radius of the track centre line in the diagram of altitudes	length; m
82	s; S	length of path along the track centre line	length; m
83	t;T	time	time; s
84	θ	angle of inclination of the track centre line in the diagram of altitudes	rad
85	v; V	speed	length/time; km/h
NOT		iables with dimensions consisting of "length" and "time" any consistent syst	em of units can be

A.3.2 Objectives

In conventional track alignment, design conditions for the circular curve are defined. For the transition curve a more global approach is chosen with integral conditions for the whole transition. To be more realistic local restrictions all along the track are used.

A.3.3 Progressive track alignment design

A.3.3.1 General

In a differential geometric approach, the alignment of track is described by the three co-ordinates of a starting point and by the progress of two angles, the angle of direction $\varphi(s)$ in the horizontal plane and the angle of inclination $\theta(s)$ in the diagram of altitudes, all relative to the track centre line in the plane of the top edges of the rails. Finally, the third angle, the angle of cant $\psi(s)$, is the angle in which the horizontal axis perpendicular to the track centre line should be pivoted around the latter to bring it into the plane of the track. The gauge is measured in accordance with this pivoted axis perpendicular to the track centre line and they both define the track plane. These three angles can be used in a matrix of a spatial rotation.

In this subclause, the cant angle is defined as the ratio of the length of the arc of this matching circle of the third angle of rotation with its centre at the track centre line to the radius. The cant d(s) itself is the length of the arc of the circle with radius equal to the distance between wheel treads b of an axle. Applying cant gives different altitudes of the left and right rails. The cant can be indicated and measured approximately in a vertical plane perpendicular to the horizontal projection of the track centre line.

Along the track, all quantities vary with the curved abscissa s in the track centre line and, according to their definitions, allocated signs and therefore absolute values have to be taken for the comparison with limits.

The curvature in the horizontal plane at any point of the track equals the first local derivative of the direction angle, $|\kappa_{\rm H}(s)| = \left|\frac{{\rm d}\,\phi}{{\rm d}s}\right| \le \kappa_{\rm H0}$ and it has to be limited by a prescribed value $\kappa_{\rm H0}$. The radius in the horizontal plane $r_{\rm H}\left(s\right) = \frac{1}{\kappa_{\rm H}(s)}$ is the inverse of the curvature (see 5.2.1).

In the diagram of altitudes, the inclination angle is limited by the prescribed value $|\theta(s)| \le \theta_0$ and in the same way the vertical curvature $|\kappa_{\rm V}(s)| = \left|\frac{{\rm d}\,\theta}{{\rm d}s}\right| \le \kappa_{{
m V}0}$ is limited by $\kappa_{{
m V}0}$ taken from 5.2.11 regarding $r_{{
m V}}(s) = \frac{1}{\kappa_{{
m V}}(s)}$.

Also for static reasons, the cant angle has to be limited everywhere, $|\psi(s)| = \frac{|d(s)|}{b} \le \psi_0 = \frac{d_0}{b}$, especially in circular curves. For track with gauge 1435 mm, limits follow from 5.2.2, for D_0 , by $\psi_0 = \frac{D_0}{B}$.

The same has to be done for the angle of cant gradient $|\gamma(s)| = \left|\frac{d\psi}{ds}\right| = \frac{1}{b} \quad \left|\frac{dd}{ds}\right| \le \gamma_0 = \frac{dd}{ds}\Big|_0$. For track with gauge 1435 mm normal and exceptional limits result from 5.2.5: $\frac{dD}{dS}\Big|_0$, $\gamma_0 = \frac{1}{B} \cdot \frac{dD}{dS}\Big|_0$.

Thus, the geometry can be described and static criteria formulated by the three angles and their derivatives independently of and therefore valid for any gauge and also for systems without defined gauge as monorails and magnetic levitated trains.

To obtain the accelerations and jerks induced by the alignment of the track at every point – also in transition curves and other segments of varying curvature or / and cant – an exact physical model for a cross section of a rigid vehicle is used and the kinematic variables are expressed. A linearization gives the relation between track alignment geometry and accelerations and jerks which should be limited. This linearization also leads to a difference between a sloping length at a track gradient that can be approximated as the length projected in the horizontal plane.

In this subclause, also the effect on the non-compensated lateral acceleration and jerk due to the roll movement is to take into account, see Figure A.1. This is an improvement on the previous methods where a mass point travelling along the track centre line is taken as the basis. In reality, the centre of gravity of the vehicle, the passengers and the freight are always situated at a certain height above the track plane, which, for the purpose of this European Standard, is taken as mean height h (track alignment for centre of gravity).

In circular curves, all the direct geometric limitations remain unchanged relative to the text this European Standard and the two criteria containing the height convert for h=0 to the conventional rules.

A.3.3.2 Non-compensated lateral acceleration

The following equation for the angle $\beta_{\rm Q}$ between the normal to the track plane and the resultant non-compensated acceleration in the car body, the dimensionless *Froude* number defined for all track guided systems, should be fulfilled everywhere along the track:

$$\left|\beta_{\mathbf{Q}}\right| = \frac{\left|a_{\mathbf{Q}}\right|}{g} = \frac{\left|i\right|}{b} = \left|\left(\kappa_{\mathbf{H}} + h \cdot \frac{\mathrm{d}^{2}\psi}{\mathrm{d}s^{2}}\right) \cdot \frac{v^{2}}{g} - \psi\right| \le \beta_{\mathbf{Q}0} = \frac{a_{\mathbf{Q}0}}{g} = \frac{i_{0}}{b} \tag{A.1}$$

The following number equation can be obtained:

$$|I| = \left| \left(\frac{c_{21}}{R_H} + c_{22} \cdot \frac{d^2 D}{dS^2} \right) \cdot V^2 - D \right| \le I_0$$
 (A.2)

where the constants for track with gauge 1435 mm and a mean height of centre of gravity h = 1,8 m are

$$c_{21} = \frac{625}{52,95591}$$
 and $c_{22} = \frac{1}{70,608}$.

Limits I_0 have to be taken from 5.2.3.

Proportionality between cant and horizontal curvature may be assumed as in conventional track alignment design:

$$\frac{\kappa_{\rm H}(s)}{\kappa_{\rm C}} = \frac{\psi(s)}{\psi_{\rm C}} = \frac{d(s)}{d_{\rm C}} \tag{A.3}$$

This assumption makes it possible to obtain the following equations from which either the curvature or the cant has been eliminated:

$$\left|\beta_{\mathbf{Q}}\right| = \frac{\left|a_{\mathbf{Q}}\right|}{g} = \left|\left(\frac{\kappa_{\mathbf{C}} \cdot v^{2}}{g \cdot \psi_{\mathbf{C}}} - 1\right) \cdot \psi + \frac{h \cdot v^{2}}{g} \cdot \frac{d^{2}\psi}{ds^{2}}\right| = \left|\left(\frac{v^{2}}{g} - \frac{\psi_{\mathbf{C}}}{\kappa_{\mathbf{C}}}\right) \cdot \kappa_{\mathbf{H}} + \frac{h \cdot v^{2} \cdot \psi_{\mathbf{C}}}{g \cdot \kappa_{\mathbf{C}}} \cdot \frac{d^{2}\kappa_{\mathbf{H}}}{ds^{2}}\right| \le \beta_{\mathbf{Q}0} = \frac{a_{\mathbf{Q}0}}{g} \tag{A.4}$$

For a circular curve, the conditions are reduced to the formulae in the main body of this European Standard as the derivatives of cant disappear and $\kappa_H = \kappa_C$, $\psi = \psi_C$ and $\beta_Q = \beta_{QC}$:

$$\left|\beta_{\mathrm{QC}}\right| = \frac{\left|a_{\mathrm{QC}}\right|}{g} = \frac{\left|i_{\mathrm{C}}\right|}{b} = \left|\frac{\kappa_{\mathrm{C}} \cdot v^{2}}{g} - \psi_{\mathrm{C}}\right| \le \beta_{\mathrm{Q0}} = \frac{a_{\mathrm{Q0}}}{g} = \frac{i_{\mathrm{0}}}{g} \tag{A.5}$$

If the fixed values are those of a circular curve adjacent to a transition curve, the angle of non-compensated lateral acceleration β_{OC} can be used to rewrite Equation (A.4):

$$\left|\beta_{Q}\right| = \frac{\left|a_{Q}\right|}{g} = \frac{\left|i_{C}\right|}{b} = \left|\frac{\beta_{QC} \cdot \psi}{\psi_{C}} + \frac{h \cdot v^{2}}{g} \cdot \frac{d^{2}\psi}{ds^{2}}\right| = \left|\frac{\beta_{QC} \cdot \kappa_{H}}{\kappa_{C}} + \frac{h \cdot v^{2} \cdot \psi_{C}}{g \cdot \kappa_{C}} \cdot \frac{d^{2}\kappa_{H}}{ds^{2}}\right| \leq \beta_{Q0} = \frac{a_{Q0}}{g} = \frac{i_{0}}{b} \quad (A.6)$$

A.3.3.3 Non-compensated lateral jerk

For the non-compensated lateral jerk, the jerk due to the roll movement has to be taken into account. With the same considerations as above, the following is obtained for all gauges, with consistent units:

$$\left| \frac{\mathrm{d}\beta_{\mathcal{Q}}}{\mathrm{d}t} \right| = \frac{1}{g} \cdot \left| \frac{\mathrm{d}a_{\mathcal{Q}}}{\mathrm{d}t} \right| = \frac{1}{b} \cdot \left| \frac{\mathrm{d}i}{\mathrm{d}t} \right| = \left| \left(\left(\frac{\mathrm{d}\kappa_H}{\mathrm{d}s} + h \cdot \frac{\mathrm{d}^3\psi}{\mathrm{d}s^3} \right) \cdot \frac{v^2}{g} - \frac{\mathrm{d}\psi}{\mathrm{d}s} \right) \cdot v \right| \le \dot{\beta}_{\mathcal{Q}0} = \frac{\dot{a}_{\mathcal{Q}0}}{g} = \frac{\dot{i}_0}{b}$$
(A.7)

The corresponding number equation can be obtained:

$$\left| \frac{\mathrm{d}I}{\mathrm{d}T} \right| = \left| \left(\left(-\frac{c_{25}}{R_H^2} \cdot \frac{\mathrm{d}R_H}{\mathrm{d}S} + c_{26} \cdot \frac{\mathrm{d}^3 D}{\mathrm{d}S^3} \right) \cdot V^2 - c_{v1} \cdot \frac{\mathrm{d}D}{\mathrm{d}S} \right) \cdot V \right| \le \dot{I}_0$$
(A.8)

where the constants for track with gauge 1435 mm and mean height of centre of mass are

$$c_{25} = \frac{625}{190,6413}$$
 and $c_{26} = \frac{1}{254,188}$

Limits are taken from 5.2.7,
$$\frac{\mathrm{d}I}{\mathrm{d}t}\Big|_{0} = \dot{I}_{0} = c_{27} \cdot \dot{a}_{Q0}$$
 with $c_{27} = 152,9574$ regarding $\dot{i}_{0} = \frac{b \cdot \dot{a}_{Q0}}{g} = b \cdot \dot{\beta}_{Q0}$.

The proportionality between cant and horizontal curvature, as already mentioned, makes it possible to eliminate either the cant or the curvature and the non-compensated lateral jerk equation can be reformulated with the values in the circular curve:

$$\left| \frac{\mathrm{d}\beta_{Q}}{\mathrm{d}t} \right| = \frac{1}{g} \cdot \left| \frac{\mathrm{d}a_{Q}}{\mathrm{d}t} \right| = \frac{|i_{C}|}{b} = \left| \left(\frac{\beta_{QC}}{\psi_{C}} \cdot \frac{\mathrm{d}\psi}{\mathrm{d}s} + \frac{h \cdot v^{2}}{g} \cdot \frac{\mathrm{d}^{3}\psi}{\mathrm{d}s^{3}} \right) \cdot v \right| =$$

$$= \left| \left(\frac{\beta_{QC}}{\kappa_{C}} \cdot \frac{\mathrm{d}\kappa_{H}}{\mathrm{d}s} + \frac{h \cdot v^{2} \cdot \psi_{C}}{g \cdot \kappa_{C}} \cdot \frac{\mathrm{d}^{3}\kappa_{H}}{\mathrm{d}s^{3}} \right) \cdot v \right| \leq \dot{\beta}_{Q0} = \frac{\dot{a}_{Q0}}{g} = \frac{\dot{a}_{0}}{b}$$
(A.9)

A.3.3.4 Angular acceleration about roll axis

As the angular velocity $\omega = \frac{\mathrm{d}\,\psi}{\mathrm{d}t} = \left(\frac{\mathrm{d}\,\psi}{\mathrm{d}s}\right) \cdot v = \gamma(s) \cdot v$ scarcely appears in the *Euler* equation, from a physical point of view, a criterion for non constant cant gradient should not be established based on it.

The angular roll acceleration α equal to the second time derivative of angle of cant is an appropriate parameter for track alignment design:

$$\left|\alpha\right| = \left|\frac{\mathrm{d}^2 \psi}{\mathrm{d}t^2}\right| = \left|\frac{\mathrm{d}^2 \psi}{\mathrm{d}s^2}\right| \cdot v^2 \le \alpha_0 \quad \Rightarrow \quad \left|\frac{\mathrm{d}^2 \psi}{\mathrm{d}s^2}\right| \le \frac{\alpha_0}{v^2} \tag{A.10}$$

Numerically:
$$\left| \frac{\mathrm{d}^2 D}{\mathrm{d}S^2} \right| \le c_{11} \cdot \frac{\alpha_0 \left[\mathrm{rad/s^2} \right]}{V^2}$$
 (A.11)

with constant for track with gauge 1435 mm: $c_{11} = 19440$.

This condition does not permit jumps in the first local derivative of cant, nor, because of the proportionality between horizontal curvature and cant, in the first local derivative of curvature either. As a limit $\alpha_0 \approx 0.1 \text{ rad/s}^2$ can be chosen. Together with limited non-compensated lateral and vertical acceleration this limits acceleration field everywhere within the vehicle.

A.3.3.5 Angular jerk about roll axis

The angular roll jerk equalling the third time derivative of angle of cant can be determined and limited:

$$\left| \frac{\mathrm{d}\alpha}{\mathrm{d}t} \right| = \left| \frac{\mathrm{d}^3 \psi}{\mathrm{d}t^3} \right| = \left| \frac{\mathrm{d}^3 \psi}{\mathrm{d}s^3} \cdot v^3 \right| \le \dot{\alpha}_0 \quad \Rightarrow \quad \left| \frac{\mathrm{d}^3 \psi}{\mathrm{d}s^3} \right| \le \frac{\dot{\alpha}_0}{|v|^3} \tag{A.12}$$

Numerically:
$$\left| \frac{\mathrm{d}^3 D}{\mathrm{d}S^3} \right| \le c_{15} \cdot \frac{\dot{\alpha}_0 \left[\mathrm{rad/s^3} \right]}{\left| V \right|^3}$$
 (A.13)

with constant for track with gauge 1435 mm $c_{15} = 69984$.

This condition does not permit jumps in the second local derivative of cant, nor, because of the proportionality between horizontal curvature and cant, in the second local derivative of curvature either. For a limit for $\dot{\alpha}_0 \approx$ 0,2 rad/s³ can be assumed.

A.3.3.6 Vertical acceleration

The local vertical acceleration normal to the horizontal plane is represented by the ratio of vertical acceleration $\beta_{\rm V}$ and has to be limited:

$$\left|\beta_{V}\right| = \frac{\left|a_{V}\right|}{g} = \frac{\left|\kappa_{V}\right|}{g} \leq \beta_{V0} = \frac{a_{V0}}{g} \implies \left|\kappa_{V}\right| \leq \frac{g}{V^{2}} \frac{\beta_{V0}}{v^{2}} = \frac{a_{V0}}{v^{2}} \tag{A.14}$$

Numerically:
$$|R_V| \ge c_{31} \cdot \frac{V^2}{\beta_{V0}} = c_{v2} \cdot \frac{V^2}{a_{V0} \, [\text{m/s}^2]} \text{ with } c_{31} = \frac{1}{127,0942}$$
 (A.15)

Limits may be derived from 5.2.11 a_{v0} . This is the only kinematic criterion identical to that of the normative part of this European Standard. Due to the small limits of ratio of vertical acceleration β_{v0} up to 6,052 %, vertical alignment design is examined independently from horizontal track alignment geometry.

A.3.3.7 Vertical jerk

For improved comfort, especially if high vertical accelerations are permitted in certain locations, the ratio of vertical jerk normal to the horizontal plane can be determined and limited:

$$\left| \frac{\mathrm{d}\beta_{V}}{\mathrm{d}t} \right| = \frac{1}{g} \cdot \left| \frac{\mathrm{d}a_{V}}{\mathrm{d}t} \right| = \left| \frac{\mathrm{d}\kappa_{V}}{\mathrm{d}s} \cdot \frac{v^{3}}{g} \right| \le \dot{\beta}_{V0} = \frac{\dot{a}_{V0}}{g} \quad \Rightarrow \quad \left| \frac{\mathrm{d}\kappa_{V}}{\mathrm{d}s} \right| \le \frac{g \cdot \beta_{V0}}{|v|^{3}} = \frac{\dot{a}_{V0}}{|v|^{3}} \tag{A.16}$$

Numerically:
$$\left| \frac{dR_{V}}{dS} \right| \le c_{35} \cdot \frac{R_{V}^{2} \cdot \dot{\beta}_{V0} \left[1/S \right]}{\left| V \right|^{3}} = \frac{R_{V}^{2} \cdot \dot{a}_{V0} \left[\text{m/s}^{3} \right]}{c_{v3} \cdot \left| V \right|^{3}} \text{ with } c_{35} = 457,5391$$
 (A.17)

This condition does not permit jumps in vertical curvature. As a limit, $\dot{\beta}_{\rm V0} \approx 0.05~[1/{\rm s}]$ can be assumed. If applied, vertical transition curves are to be supplied. Then jerk field is limited everywhere within the vehicle.

A.3.4 Application

A.3.4.1 General

In comparison to conventional track alignment design rules, the parameter angular velocity about roll axis ω is omitted whilst the parameters angular acceleration α and angular jerk $\frac{\mathrm{d}\alpha}{\mathrm{d}t}$ about roll axis and vertical jerk $\frac{\mathrm{d}a_{\scriptscriptstyle V}}{\mathrm{d}t}$ are included. Non-compensated lateral acceleration and jerk are specified to obtain correct results in also transition curves. In a circular curve nothing changes.

The criteria used in this Annex can be evaluated directly for any possible geometry and gauge by means of the values given in the normative parts of this European Standard and some additional limits. Introducing special track geometry functions for horizontal curvature $\kappa_{_H}(s)$, cant angle $\psi(s)$, and vertical curvature $\kappa_{_V}(s)$, extreme values have to be calculated and compared to their limits. This can be done both for conventional transitions and for any type of progressive track alignment as various types of spirals, track alignment design for centre of gravity as "out-swinging" transition curves, etc.

A.3.4.2 Existing or given geometry

For various conventional transition geometry, the following text gives the evaluations for a transition from a straight line to a circular curve with $|r_{\rm H}|=|r_{\rm C}|\geq r_0$ of subclause 5.2.1 and cant angle $|\psi_{\rm C}|\leq \psi_0=\frac{D_0}{B}$ for track with gauge 1435 mm being lower than the limit of subclause 5.2.2. Also, the angle of non-compensated lateral acceleration $\left|\beta_{\rm QC}\right|\leq\beta_{\rm Q0}=\frac{I_0}{B}$ for track with gauge 1435 mm should here be lower than the limit from

5.2.3. The static criterion for the angle of cant gradient $|\gamma(s)| = \left|\frac{\mathrm{d}\psi}{\mathrm{d}s}\right| = \frac{1}{b} \cdot \left|\frac{\mathrm{d}d}{\mathrm{d}s}\right| \le \gamma_0$, see 5.2.5 for track with gauge 1435 mm, shall be satisfied throughout the transition.

In Table A.3, the maximum values for various items and geometry are given. This makes it possible to check which conditions can be fulfilled and which cannot. The line for angular roll velocity can be used to evaluate formulae in 5.2.6, Rate of change of cant $\frac{dD}{dT}\Big|_0$.

The conventional clothoid with constant cant gradient (Figure A.2) as an idealised transition yields infinite angular roll acceleration and with it, infinite non-compensated lateral acceleration close to straight and curved alignment components and therefore does not meet all the dynamic conditions. With the conventional *Helmert* (*Schramm*) (Figure A.3), *Bloss* (Figure A.4), and Cosine (Figure A.5) transition, all accelerations can be limited, but not the jerks, which are infinite at the beginning and the end, for the two quadratic parabolas also in the middle. Only with the Sine (*Klein*) transition (Figure A.6) can the angular roll jerk and non-compensated lateral jerk be limited, but they are not continuous. In Figure A.7 and Figure A.8 the first and the second derivatives can be compared.

If, to obtain the cant, only the outer rail is elevated above the inner one, Table A.4 give some supplementary maximum expressions containing the gauge as a parameter for various items and geometry. Total line inclination $\theta(s)$ and curvature $\kappa_{\rm V}(s)$ of track centre line, vertical acceleration and vertical jerk are obtained by adding the expressions with the correct signs from the table to those obtained for the track alignment in the diagram of altitudes. It should be checked whether these sums are within the permissible ranges; for vertical acceleration and vertical jerk, this is checked in accordance with the inequalities represented in Equation (A.14) and Equation (A.16).

Table A.3 — Functions of transition curves, positions and extreme expressions for various kinematic parameters and cant gradient

	Clothoid with constant cant gradient	Helmert (Schramm)	Bloss	Cosine	Sine (Klein)
Region: Function $\frac{\kappa_{\rm H}}{\kappa_{\rm C}} = \frac{\psi}{\psi_{\rm C}}$ $\frac{\kappa_{\rm H}}{\kappa_{\rm C}} = \frac{d}{d_{\rm C}}$	Everywhere in the linear part: $\frac{s}{\ell}$	$0 \le \frac{s}{\ell} \le \frac{1}{2}:$ $2 \cdot \left(\frac{s}{\ell}\right)^{2};$ $\frac{1}{2} \le \frac{s}{\ell} \le 1:$ $1 - 2 \cdot \left(1 - \frac{s}{\ell}\right)^{2}$	$0 \le \frac{s}{\ell} \le 1:$ $\left(3 - \frac{s}{\ell}\right) \cdot \left(\frac{s}{\ell}\right)^2$	$0 \le \frac{s}{\ell} \le 1:$ $\frac{1}{2} \cdot \left(1 - \cos \frac{\pi \cdot s}{\ell}\right)$	$0 \le \frac{s}{\ell} \le 1:$ $\frac{s}{\ell} - \frac{1}{2 \cdot \pi} \cdot \sin\left(2 \cdot \pi \cdot \frac{s}{\ell}\right)$
Angle of non compensated lateral acceleration $oldsymbol{eta}_{\mathbb{Q}}$	ll .	$\frac{\frac{1}{2}\Big _{-}}{\frac{\beta_{QC}}{2}} + \frac{4 \cdot h \cdot \psi_{C} \cdot v^{2}}{g \cdot \ell^{2}}$	$0_{+}:$ $\frac{6 \cdot h \cdot \psi_{C} \cdot v^{2}}{g \cdot \ell^{2}}$	$0_{+}:$ $\frac{\pi^{2} \cdot h \cdot \psi_{C} \cdot v^{2}}{2 \cdot g \cdot \ell^{2}}$	$\frac{s_1}{\ell} = \frac{1}{2 \cdot \pi} \cdot \frac{\beta_{QC}}{\beta_{QC}}$ $\cdot \arccos \frac{\beta_{QC}}{\beta_{QC}} - \frac{4 \cdot \pi^2 \cdot h \cdot v^2 \cdot \psi_C}{g \cdot \ell^2}$ $< \frac{1}{2} :$ $\beta_{Q1} = \beta_Q \left(\frac{s_1}{\ell} \right)$

Table A.3 (continued)

Element Item	Clothoid with constant cant gradient	Helmert (Schramm)	Bloss	Cosine	Sine (Klein)
Change of angle of non compensated lateral jerk $\frac{\mathrm{d}\boldsymbol{\beta}_{\mathrm{Q}}}{\mathrm{d}t}$	0, 1:	$0, \frac{1}{2}, 1$:	0, 1: ∞	0, 1: ∞	$\begin{array}{c} 0_{+}\text{,}1_{-}\text{:}\\ \frac{4\cdot\pi^{2}\cdot h\cdot\psi_{C}\cdot v^{3}}{g\cdot\ell^{3}}\\ \text{in the middle }\frac{1}{2}\text{:}\\ \frac{2\cdot v}{\ell}\cdot\left(\beta_{\mathrm{QC}}-\frac{2\cdot\pi\cdot h\cdot\psi_{C}\cdot v^{2}}{g\cdot\ell^{2}}\right) \end{array}$
Angular roll velocity $\omega = \frac{\mathrm{d} \psi}{\mathrm{d} t}$	everywhere: $\frac{\psi_{\text{C}} \cdot v}{\ell}$	$\frac{\frac{1}{2}:}{\frac{2 \cdot \psi_{C} \cdot v}{\ell}}$	$\frac{\frac{1}{2}:}{\frac{3 \cdot \psi_C \cdot v}{2 \cdot \ell}}$	$ \frac{\frac{1}{2}:}{\frac{\pi \cdot \psi_C \cdot v}{2 \cdot \ell}} $ $ 0_+, 1: $	$\frac{\frac{1}{2}:}{\frac{2 \cdot \psi_C \cdot v}{\ell}}$
Angular roll acceleration $\alpha = \frac{d\omega}{dt}$	0, 1: ±∞	everywhere: $\pm \frac{4 \cdot \psi_C \cdot v^2}{\ell^2}$	$ \frac{0_{+},1_{-}:}{\pm \frac{6 \cdot \psi_{C} \cdot v^{2}}{\ell^{2}}} $	$\pm \frac{0_+, 1:}{2 \cdot \psi_C \cdot v^2}$	$ \frac{2 \cdot \psi_C \cdot v}{\ell} $ $ \frac{1}{4}, \frac{3}{4} \cdot \dots $ $ \pm \frac{2 \cdot \pi \cdot \psi_C \cdot v^2}{\ell^2} $
Angular roll jerk $\frac{\mathrm{d}\alpha}{\mathrm{d}t}$	0, 1: ∞	$0, \frac{1}{2}, 1:$	0, 1: ∞	0, 1: ∞	$\pm \frac{2 \cdot \pi \cdot \psi_C \cdot v^2}{\ell^2}$ $0_+, \frac{1}{2}, 1:$ $\pm \frac{4 \cdot \pi^2 \cdot \psi_C \cdot v^3}{\ell^3}$
Maximum cant gradient $\gamma = \frac{\mathrm{d} \psi}{\mathrm{d} s}$	everywhere: $\frac{\psi_C}{\ell}$	$\frac{\frac{1}{2}:}{\frac{2 \cdot \psi_C}{\ell}}$	$\frac{\frac{1}{2}}{\frac{3 \cdot \psi_C}{2 \cdot \ell}}$	$rac{1}{2}$: $rac{\pi \cdot \psi_{\scriptscriptstyle C}}{2 \cdot \ell}$	$ \frac{0_{+}, \frac{1}{2}, 1_{-}:}{\pm \frac{4 \cdot \pi^{2} \cdot \psi_{C} \cdot v^{3}}{\ell^{3}}} $ $ \frac{1}{2}: $ $ \frac{2 \cdot \psi_{C}}{\ell} $

Table A.4 — Transition curves: Some absolute extreme expressions of certain static and remaining kinematic parameters; only the outer rail is raised

Element Item	Clothoid with constant cant gradient	Helmert (Schramm)	Bloss	Cosine	Sine (<i>Klein</i>)
Inclination of track centre line $ heta $	everywhere in the linear part: $\frac{b \cdot \psi_C }{2 \cdot \ell}$	$rac{1}{2}$: $b\cdot oldsymbol{\psi}_C $	$\frac{\frac{1}{2}:}{3 \cdot b \cdot \psi_C } \frac{3 \cdot b \cdot \psi_C }{4 \cdot \ell}$	$\frac{\frac{1}{2}:}{\frac{\pi \cdot b \cdot \psi_C }{4 \cdot \ell}}$	$\frac{\frac{1}{2}:}{\frac{b\cdot \psi_{\scriptscriptstyle C} }{\ell}}$
Vertical curvature of track centre line $\left \mathcal{K}_{\mathrm{V}} \right $	0, 1: ∞ between: 0	everywhere: $\frac{2 \cdot b \cdot \psi_C }{\ell^2}$	$\frac{0_+, 1:}{3 \cdot b \cdot \psi_C } \frac{1}{\ell^2}$	$\frac{0_{+}, 1_{-}:}{\pi^{2} \cdot b \cdot \psi_{C} }$ $\frac{\pi^{2} \cdot b \cdot \psi_{C} }{4 \cdot \ell^{2}}$	$\frac{\frac{1}{4}, \frac{3}{4}:}{\frac{\pi \cdot b \cdot \psi_C }{\ell^2}}$
Vertical acceleration ratio $\left oldsymbol{eta}_{ ext{ iny V}} ight $	0, 1: ∞	everywhere: $\frac{2 \cdot b \cdot \psi_C \cdot v^2}{g \cdot \ell^2}$	$\frac{0_{+}, 1_{-}:}{3 \cdot b \cdot \psi_{C} \cdot v^{2}}$ $\frac{g \cdot \ell^{2}}{g \cdot \ell^{2}}$	$\frac{0_{+}, 1_{-}:}{\pi^{2} \cdot b \cdot \psi_{C} \cdot v^{2}}$ $\frac{4 \cdot g \cdot \ell^{2}}{4 \cdot g \cdot \ell^{2}}$	$\frac{\frac{1}{4}, \frac{3}{4}:}{\frac{\pi \cdot b \cdot \psi_C \cdot v^2}{g \cdot \ell^2}}$
Vertical jerk ratio $\frac{\left \frac{d \beta_{V}}{d t} \right }{\left \frac{d \beta_{V}}{d t} \right }$	0, 1: ∞	$0, \frac{1}{2}, 1$:	0, 1: ∞	0, 1: ∞	$\frac{0_{+}, \frac{1}{2}, 1_{-}:}{\frac{2 \cdot \pi^{2} \cdot b \cdot \psi_{C} \cdot v^{3}}{g \cdot \ell^{3}}}$

If the outer rail is raised and the inner rail is lowered, the height of the track centre line is influenced only by the track alignment design in the diagram of altitudes. The expressions in Table A.4 will be zero.

To overcome the difficulties for a cant gradient with constant slope between a straight and a circle or between two circles, the outer rail or both rails are smoothed by a vertical radius $r_{\rm R}=\frac{1}{\kappa_{\rm R}}$ to avoid kinks. In these two rounded areas, curvature and cant are no longer proportional.

If only the outer rail is smoothed, the curvature of the inner rail is zero; if the inner rail is also smoothed, it is in the opposite direction and it has the same amount of curvature but with changed sign.

The following equations combine geometrically the second local derivative of angle of cant with the curvatures of the rails:

Only one rail is smoothed:
$$\left| \frac{\mathrm{d}^2 \psi}{\mathrm{d}s^2} \right| = \frac{\kappa_R}{b}$$
 (A.18)

Both rails are smoothed:
$$\left| \frac{\mathrm{d}^2 \psi}{\mathrm{d}s^2} \right| = 2 \cdot \frac{\kappa_R}{b}$$
 (A.19)

There are jumps in vertical curvature of at least one rail at both sides of every rounded area where the expressions change from zero to those in the above equations. There, all the static and dynamic criteria should be applied to determine the transition.

For the transition from a straight to a circle, the first maximum angle of non-compensated lateral acceleration is reached at the end of the first rounded area, the second at the end of the second rounded area. The latter non-compensated lateral acceleration is always lower then that in the circular curve, only the first one has to be checked as indicated in A.3.3.2:

Only the outer rail is smoothed:
$$\left| \beta_{\underline{Q}} \right| = \frac{\left| a_{\underline{Q}} \right|}{g} = \frac{\left| i \right|}{b} = \left| \frac{b \cdot \beta_{\underline{Q}C} \cdot \psi_C}{2 \cdot \kappa_R \cdot \ell^2} + \frac{h \cdot \kappa_R \cdot v^2}{g \cdot b} \right| \le \beta_{\underline{Q}0} = \frac{a_{\underline{Q}0}}{g} = \frac{i_0}{b}$$
 (A.20)

Both rails are smoothed:
$$\left|\beta_{\mathcal{Q}}\right| = \frac{\left|a_{\mathcal{Q}}\right|}{g} = \frac{\left|i\right|}{b} = \left|\frac{b \cdot \beta_{\mathcal{Q}C} \cdot \psi_{C}}{4 \cdot \kappa_{R} \cdot \ell^{2}} + 2 \cdot \frac{h \cdot \kappa_{R} \cdot v^{2}}{g \cdot b}\right| \leq \beta_{\mathcal{Q}0} = \frac{a_{\mathcal{Q}0}}{g} = \frac{i_{0}}{b}$$
 (A.21)

As non-compensated lateral acceleration at the beginning and at the end of every rounded area jumps to another value, non-compensated lateral jerk $\frac{\mathrm{d}\beta_\mathrm{Q}}{\mathrm{d}t}$ is infinite there.

The kinematic values for roll movement depend only on the vertical curvature of the rail(s) and not directly on cant gradient. In the two areas of transition where the rails are smoothed, the following angular roll acceleration results and has to be limited, as indicated in A.3.3.4:

Only one rail is smoothed:
$$\left|\alpha\right| = \frac{\kappa_{\rm R} \cdot v^2}{b} \le \alpha_0$$
 (A.22)

Both rails are smoothed:
$$|\alpha| = 2 \cdot \frac{\kappa_R \cdot v^2}{b} \le \alpha_0$$
 (A.23)

Due to the jumps of angular roll acceleration at both sides of every rounded area, the angular roll jerk $\frac{\mathrm{d}\alpha}{\mathrm{d}t}$

and the non-compensated lateral jerk $\frac{\mathrm{d}\beta_{\mathrm{Q}}}{\mathrm{d}t}$ are infinite.

If only the outer rail is rounded, in these areas Table A.4 is not be applied and the expressions of the following equations have to be used:

Maximum inclination of track centre line
$$\theta$$
: $|\theta| = \frac{b \cdot |\psi_C|}{2 \cdot \ell}$ (A.24)

Vertical curvature of track centre line
$$\kappa_{\rm V}$$
: $\left|\kappa_{\rm V}\right| = \frac{\kappa_{\rm R}}{2}$ (A.25)

Vertical acceleration ratio
$$\beta_{\rm V}$$
: $\left|\beta_{\rm V}\right| = \frac{\kappa_{\rm R} \cdot {\rm V}^2}{2 \cdot g}$ (A.26)

In addition, for this transition, the total inclination and curvature of the track centre line and vertical acceleration are obtained by adding the expressions from the above equations to those obtained for the track alignment in the diagram of altitudes. It should be checked whether these sums are within the permissible ranges.

Hence, transitions with a linear cant have non-straight areas between the constant slope and the neighbouring alignment elements. The cant gradient is determined by the curvature of the smoothed rail(s), K_R .

NOTE According to bending theory, jumps in curvature cannot occur for an elastically bedded beam. In reality, the segments with different curvatures are separated by long stretches with non-constant curvature. Therefore, all cant gradients with jumps in curvature within the transition or at the boundaries, that are cant gradients with constant slope and smoothed rails, *Helmert* (*Schramm*), *Bloss*, Cosine, represent an idealised geometry and cannot be assumed to have this geometry in reality. For a cant gradient with constant slope in particular, the segment with linear cant is only a part of the whole transition with significant other parts. Therefore, all the cant transitions become somewhat similar and have a high order cant gradient with variable slope.

A.3.4.3 Planned geometry

To calculate necessary length ℓ of a transition, the criteria in A.3.3 are applied to the variables in Table A.3 for the transition curves and cant gradients. Extreme expressions have to be inverted to yield minimum length conditions in Table A.5. Lengths have to be compared to take at least the longest.

Table A.5 — Minimum lengths for various criteria and transition curves dependent on the limits

Element	Minimum length of transition
Clothoid with constant cant gradient and smoothed	$\left \mathcal{K}_{\mathbf{R}} \right \leq \frac{b \cdot \alpha_0}{v^2}$ (one rail smoothed)
rail(s)	$\left \kappa_{\rm R} \right \le \frac{b \cdot \alpha_0}{2 \cdot v^2}$ (both rails smoothed)
	$\ell \ge \sqrt{b \cdot \left \frac{\psi_{\rm C}}{\kappa_{\rm R}} \right }$ (without linear part)
	$2 \cdot \left(g \cdot b \cdot \beta_{\mathrm{Q}0} - h \cdot v^2 \cdot \left \kappa_{\mathrm{R}}\right \right) \cdot \left \kappa_{\mathrm{R}}\right \cdot \ell^2 - g \cdot b^2 \cdot \left \beta_{\mathrm{QC}} \cdot \psi_{\mathrm{C}}\right \ge 0$ (only outer rail smoothed)
	$4 \cdot \left(g \cdot b \cdot \beta_{\text{Q}0} - 2 \cdot h \cdot v^2 \cdot \kappa_{\text{R}} \right) \cdot \kappa_{\text{R}} \cdot \ell^2 - g \cdot b^2 \cdot \beta_{\text{QC}} \cdot \psi_{\text{C}} \ge 0$ (outer and inner rail smoothed)
	$\ell \ge \frac{ \psi_{\rm C} }{\gamma_0}$
Helmert (Schramm)	$\ell \ge 2 \cdot v \cdot \sqrt{ \psi_{C} } \cdot \max \left(\sqrt{\frac{2 \cdot h}{g \cdot \left(2 \cdot \beta_{Q0} - \left \beta_{QC}\right \right)}}, \sqrt{\frac{1}{\alpha_{0}}} \right)$
	$\ell \geq 2 \cdot \frac{ \psi_{\rm C} }{\gamma_0}$

Table A.5 (continued)

Element	Minimum length of transition
Bloss	$\ell \ge v \cdot \sqrt{6 \cdot \psi_{C} } \cdot \max \left(\sqrt{\frac{h}{g \cdot \beta_{Q0}}}, \sqrt{\frac{1}{\alpha_{0}}} \right)$
	$\ell \ge \frac{3 \cdot \left \psi_{\rm C} \right }{2 \cdot \gamma_0}$
Cosine	$\ell \ge \pi \cdot \nu \cdot \sqrt{\frac{ \psi_C }{2}} \cdot \max \left(\sqrt{\frac{h}{g \cdot \beta_{Q0}}} , \sqrt{\frac{1}{\alpha_0}} \right)$
	$\ell \ge \frac{\pi \cdot \psi_{\rm C} }{2 \cdot \gamma_0}$
Sine (<i>Klein</i>)	$\ell \ge v \sqrt{\frac{2 \pi \psi_{C} }{\alpha_{0}}} , \left \beta_{Q1}\right = \left \beta_{Q} \frac{s_{1}}{\ell}\right \le \beta_{Q0}$ $\ell \ge 2 \cdot \frac{\left \psi_{C}\right }{\gamma}$
	$\ell \ge 2 \cdot \frac{ \psi_{\rm C} }{\gamma_0}$
Sine (<i>Klein</i>) (with jerk conditions)	$\ell \geq v \cdot \max \left(\sqrt[3]{\frac{4 \cdot \pi^2 \cdot h \cdot \psi_C }{g \cdot \dot{\beta}_{Q0}}} \right), \frac{2}{\dot{\beta}_{Q0}} \cdot \left(\left \beta_{QC} \right - \frac{2 \cdot \pi^2 \cdot h \cdot \psi_C \cdot v^2}{g \cdot \ell^2} \right), \sqrt[3]{\frac{4 \cdot \pi^2 \cdot \psi_C }{\dot{\alpha}_0}} \right)$

If only one rail is smoothed, also all the limiting expressions of Table A.4 have to be evaluated and checked, and, if a condition is not met, the transition length has to be increased or curvature of the rounded areas decreased.

For a special transition curve with proportionality between cant and horizontal curvature, for given limits and for a neighbouring circular curve within the limits, particularly Equation (A.5), $\left|\beta_{\mathcal{QC}}\right| \leq \beta_{\mathcal{Q}0}$, only a reduced number of equations becomes critical because of the proportionality between some of them. Therefore, by fulfilling this set, the other equations are always satisfied automatically.

If for a Helmert (Schramm), Bloss, Cosine and Sine (Klein) transition

$$\frac{a_{\mathrm{Q}0}}{h} \le \alpha_0 \tag{A.27}$$

is verified, the angular acceleration, Equation (A.10), about the roll axis need not be checked; otherwise it has to be done. For these transitions, non-compensated lateral acceleration, Equation (A.6), is usually within the range.

If for a Sine (Klein) transition

$$\frac{1}{h} \cdot \left(\dot{a}_{Q0} + \frac{2 \cdot a_{QC} \cdot v}{\ell} \right) \le \dot{\alpha}_0 \tag{A.28}$$

is verified, the angular jerk, Equation (A.12), about the roll axis needs not be checked. If

$$\frac{\dot{a}_{\mathrm{Q}0}}{h} \ge \dot{\alpha}_0 \tag{A.29}$$

is verified, the non-compensated lateral jerk, Equation (A.9), needs not be checked. If none is fulfilled, the roll jerk and lateral jerk conditions have to be checked.

If only the outer rail is rounded, similar relations as above are also applicable to the vertical acceleration ratio $\beta_{\scriptscriptstyle V}$ and the vertical jerk ratio $\frac{{
m d}\beta_{\scriptscriptstyle V}}{{
m d}t}$. For conventional track geometry (h=0), there is also this relationship between the angular velocity ω about the roll axis and the non-compensated lateral jerk.

As a numerical example for the reduction of criteria for limits for track with gauge 1435 mm: $I_0 = 110 \sim \beta_{\rm Q0} = 0.07\dot{3} \sim a_{\rm Q0} = 0.719154 \, {\rm m/s^2}$; $a_{\rm V0} = 0.31 \, {\rm m/s^2} \sim \beta_{\rm V0} \approx 0.0316112$; and if only the outer rail is elevated, a *Bloss* transition is evaluated:

$$\frac{\psi_C \cdot v^2}{\ell^2} \le \frac{\alpha_0}{6} = \frac{0,1}{6} = 0,01\dot{6} \le \frac{a_{\varrho 0}}{6 \cdot h} = \frac{0,719154}{6 \cdot 1,8} = 0,066588 \le \frac{a_{v0}}{3 \cdot b} = \frac{0,31}{3 \cdot 1,5} = 0,06\dot{8}$$

Meeting the angular acceleration criterion automatically fulfils the non-compensated lateral and vertical acceleration conditions for these limits. Therefore, application of progressive track alignment design rules does not add any specific restraints compared to application of traditional rules.

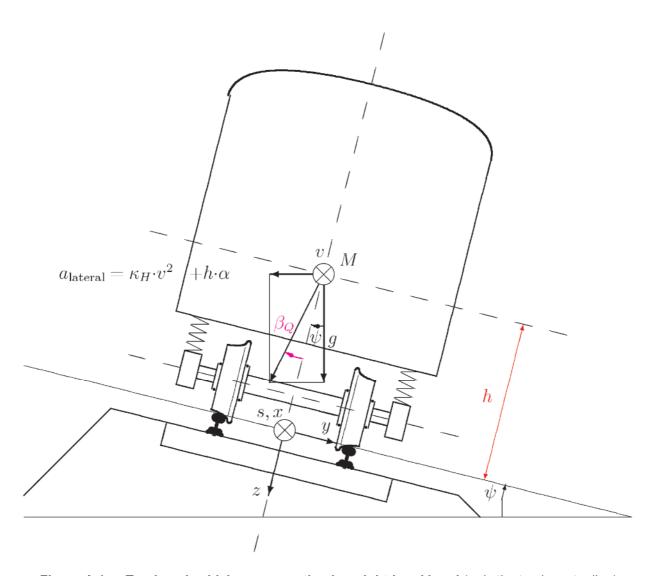
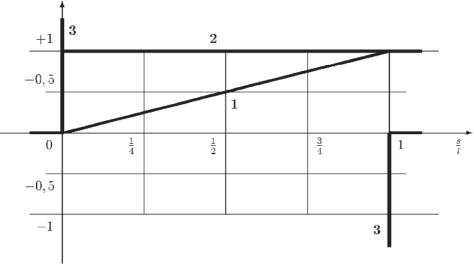


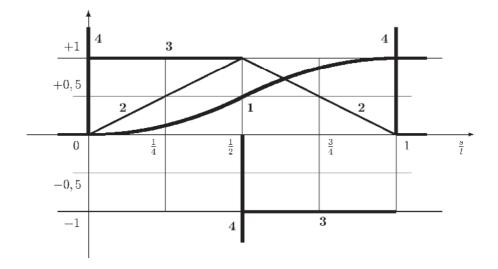
Figure A.1 — Track and vehicle cross section in a right-hand bend (s is the track centre line)



Key

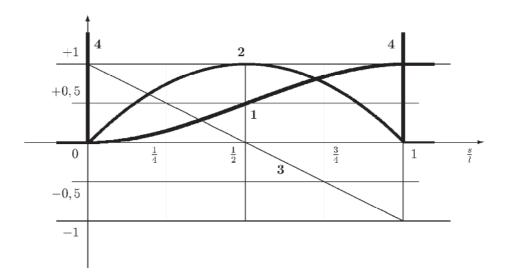
- 1 cant and curvature
- 2 first derivative
- 3 second derivative (infinite)

Figure A.2 — Clothoid with linear cant transition – Normalised cant and curvature in the horizontal plane, normalised derivatives



- 1 cant and curvature
- 2 first derivative
- 3 second derivative
- 4 third derivative (infinite)

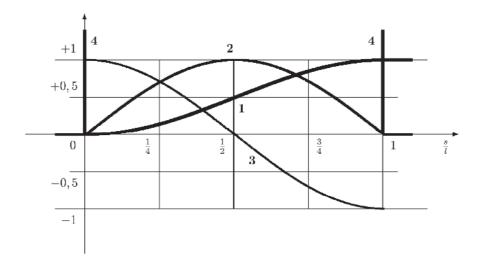
Figure A.3 — Helmert (Schramm) transition – Normalised cant and curvature in the horizontal plane, normalised derivative



Key

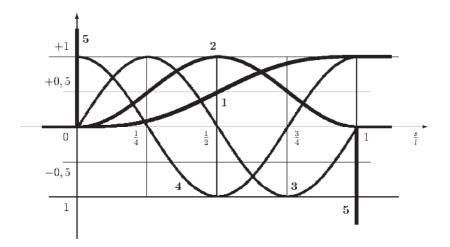
- 1 cant and curvature
- 2 first derivative
- 3 second derivative
- 4 third derivative (infinite)

Figure A.4 — Bloss transition – Normalised cant and curvature in the horizontal plane, normalised derivatives



- 1 cant and curvature
- 2 first derivative
- 3 second derivative
- 4 third derivative (infinite)

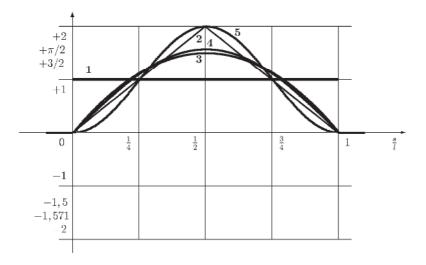
Figure A.5 — Cosine transition – Normalised cant and curvature in the horizontal plane, normalised derivatives



Key

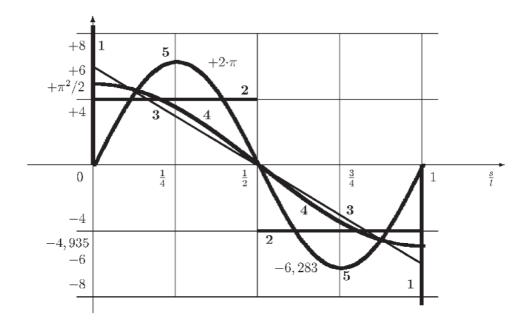
- 1 cant and curvature
- 2 first derivative
- 3 second derivative
- 4 third derivative
- 5 fourth derivative (infinite)

Figure A.6 — Sine (Klein) transition – Normalised cant and curvature in the horizontal plane, normalised derivatives



- 1 clothoid with linear cant transition
- 2 Helmert (Schramm) transition
- 3 Bloss transition
- 4 cosine transition
- 5 sine (Klein) transition

Figure A.7 — Dimensionless first derivatives



- 1 clothoid with linear cant transition (infinite)
- 2 Helmert (Schramm) transition
- 3 Bloss transition
- 4 cosine transition
- 5 sine (Klein) transition

Figure A.8 — Dimensionless second derivatives

Annex B (informative)

Length of alignment elements (circular curves and straights) between two transition curves $L_{\rm i}$

In certain applications, the actual length of any alignment element (other than transition curves) should be set equal to or above a limit given in Table B.1, taking into account the actual alignment design parameters of the neighbouring alignment elements (cant, cant deficiency and their variations); longer elements should be used for higher values of these parameters.

It is desirable where possible to join two reverse circular curves by a continuous transition curve instead of placing a straight line element between the two transitions curves. Hence, in this case, the length of straight line element is zero.

Table B.1 — Minimum length of alignment elements L_i (circular curves and straights)

Range of permissible speed [km/h]	80 ≤ <i>V</i> ≤ 200	200 < V ≤ 300	
Normal limit ^a [m]	$\frac{V_{\mathrm{max}}}{3}$	$\frac{V_{\mathrm{max}}}{1,5}$	
Exceptional limit ^a [m]	$\frac{V_{\mathrm{max}}}{5}$	V _{max} 2,5	
^a Without going under 20 m.			

On high speed lines, a rapid succession of curves and straights may induce a reduction in comfort, particularly when the length of individual alignment elements are such that the passengers are subjected to changing accelerations at a rate which corresponds to the natural frequencies of the vehicles.

There are no special limits for tilting trains.

Annex C (informative)

Rules for converting parameter values for track gauges wider than 1435 mm

C.1 Scope

This annex (informative) describes the rules, which can be applied for converting the values and limits in the standard for gauges wider than 1435 mm.

Annex D (normative) defines the limits of the track alignment parameters, based on the following rules, which shall be applied to tracks with a gauge of 1668 mm and 1524 mm.

C.2 Symbols and abbreviations

Unless otherwise indicated, the symbols and abbreviations of Table C.1 apply to Annex C.

Table C.1 — Symbols and abbreviations

No.	Symbol	Designation	Unit
86	$a_{\mathrm{i}}, a_{\mathrm{i}1}$	quasi-static lateral acceleration, at track level, but parallel to the vehicle floor	m/s ²
87	a_{q}, a_{q1}	non-compensated lateral acceleration in the running plane	m/s ²
88	B, B_1	distance between the spring suspension points on an axle	m
89	C, C_1	factor for calculation of equilibrium cant	mm·m·h²/km²
90	da_i/dt , da_{i1}/dt	rate of change of quasi-static lateral acceleration at track level but parallel to the vehicle floor	m/s ³
91	dD/ds , dD_1/ds	cant gradient	mm/m
92	dD/dt , dD_1/dt	rate of change of cant	mm/s
93	dI/dt , dI_1/dt	rate of change of cant deficiency	mm/s
94	D, D_1	cant	mm
95	e, e_1	distance between the nominal centre points of the two contact patches of a wheelset (e.g. about 1500 mm for track gauge 1435 mm)	mm
96	E, E_1	cant excess	mm
97	$H_{ m s}$	quasi-static lateral force applied by an axle to the track	N
98	I, I_1	cant deficiency	mm
99	K, K_1	suspension stiffness coefficient	N/m
100	ℓ	vehicle wheelbase	m
101	Q_N	Nominal vertical wheel/rail force	N
102	r	ratio e_1/e ($e_1/1500$ mm)	-
103	$\Delta D, \Delta D_1$	overall track cant variation along a cant transition	mm
104	ΔI , ΔI_1	overall cant deficiency (and/or cant excess) variation	mm
105	ΔQ , ΔQ_1	quasi-static vertical wheel/rail force increment	N
106	μ	cross level variation between axles linked by the suspension system	m

In the following, parameters quoted with the index 1 relate to the values converted for track gauges wider than 1435 mm, as opposed to the original 1435 mm gauge values, which are not indexed.

C.3 Basic assumptions and equivalence rules

The conditions are based on the same criteria for the following concepts:

- track forces and safety;
- economic aspects of track maintenance;
- ride comfort and roll flexibility coefficient.

It is assumed that:

the track system and track quality are similar for the wider gauges;

- the composition of the vehicles and their wheel arrangements are similar; the weights, the positions of the centres of gravity, rigidity and dampers are similar; the same safety limits apply with regard to derailment,
 - $\left(\frac{Y}{Q}\right)_{\text{lim}}$; and the same rules apply to the rate of change of the guiding force on an axle on a curve;
- the same levels of safety with regard to track shift limit will be obtained using the same levels for H_S and, in the case of derailment and overturning, by using the same values of reduced load, ΔQ , on the wheels (the guiding wheels in the case of derailment);
- the same degree of track fatigue will be obtained if the H_S , Q values are maintained;
- the ride quality will be similar if the values of a_i and $\frac{da_i}{dt}$ are kept unchanged.

Basic formulae:

$$H_{\rm s} = 2 \cdot Q_{\rm N} \cdot \frac{I}{e}$$

$$\Delta Q = 2 \cdot Q_{\rm N} \cdot I \cdot \frac{h_{\rm g}}{e^2}$$

In the case of cant excess, I is replaced by E (see 5.2.4).

$$\Delta Q = K \cdot \left(\frac{\mu}{4}\right) \cdot \left(\frac{B}{e}\right)^2 \cdot 10^6$$

 $B \text{ [m]: in normal vehicles } B - \frac{e}{1000} = 0.5 \text{ m} \Rightarrow B = 2 \text{ m}$

$$a_{q} = g \cdot \frac{I}{e}$$

$$a_{i} = (1 + s_{r}) \cdot a_{q}$$

Basic data:

Each network system shall take into account the values of e_1 and B_1 . If not available, e_1 can be obtained by adding 65 mm to the gauge and assuming $B_1 - \frac{e_1}{1000} = 0.5$ m.

The value of the parameters for gauges wider than 1435 mm will be designated with the index 1, i.e.:

$$r = \frac{e_1}{e} = \frac{e_1}{1500 \text{mm}}$$

If $e_1 > e$, then $B_1 > B$ and $1 < r < r^2$.

C.4 Detailed conversion rules

The following subclauses define, for each of the track alignment design parameters, the conversion principle to be applied.

C.4.1 Radius of horizontal curve R_1 (5.2.1)

The horizontal radius of curvature derives from the values of cant and cant deficiency D and I in the following relationship:

$$R = \frac{C \cdot V^2}{D + I}$$

When the units used are km/h for the speed V, which corresponds to a reduction factor q_v =3,6 km·s/(h·m) to the unit expressed in m/s (12,96 when squared) and mm for the distance between the nominal centre points of

the two contact patches of a wheelset e, and for cant and cant deficiency D and I, constant $C = \frac{e}{12,96 \cdot g}$

[mm·m·h²/km²], whose value is C = 11.8 mm·m·h²/km², becomes $C_1 = \frac{e_1}{12.96 \cdot g}$ [mm·m·h²/km²], g being the gravity constant (9.81 m/s²).

That is, $C_1 = C \cdot r$; this constant should be used in conjunction with the corresponding values for cant and cant deficiency for the modified gauge value, D_1 and I_1 (see below).

C.4.2 Cant D_1 (5.2.2)

Safety:

- Track shift limit: not relevant.
- Derailment and overturning: in the case of a train which is stopped or travelling at low speed (reduced load on the high rail)

$$\Delta Q = 2 \cdot Q_{\mathsf{N}} \cdot D \cdot \frac{h_{\mathsf{g}}}{e^2}$$

$$\Delta Q_1 = 2 \cdot Q_{\mathsf{N}} \cdot D_1 \cdot \frac{h_{\mathsf{g}}}{e_1^2}$$

If $\Delta Q = \Delta Q_1$, then:

$$D_1 = D \cdot r^2$$

Limit for torsionally-stiff freight wagons on sharp radii curves: $D_{\text{lim}1} = D_{\text{lim}} \cdot r^2 = \frac{R - 50 \text{m}}{1.5 \text{m/mm}} \cdot r^2$.

Track fatigue criteria:

— Additional load on the lower rail (low speed):

$$D_1 = D \cdot r^2$$

— Lateral force (on the track) (low speed):

$$H_s = 2 \cdot Q_N \cdot \frac{D}{e} = 2 \cdot Q_N \cdot \frac{D_1}{e_1}$$

Therefore:

$$D_1 = D \cdot r$$

Ride quality criteria:

In this criterion low speed is fundamental:

$$a_{i} = (1 + s_{r}) \cdot a_{q} = (1 + s_{r1}) \cdot a_{q1} = (1 + s_{r}) \cdot g \cdot \frac{D}{e} = (1 + s_{r1}) \cdot g \cdot \frac{D_{1}}{e_{1}}$$

$$D_{1} = D \cdot \frac{1 + s_{r}}{1 + s_{r1}} \cdot r$$

If $S_r = S_{r1}$, then:

$$D_1 = D \cdot r$$

Rule for converting values:

Limits for cant shall be multiplied by r. However, the limit for torsionally-stiff freight wagons on sharp radii curve will be $D_{\text{lim1}} = \frac{R - 50 \text{m}}{1.5 \text{m/mm}} \cdot r^2$ [mm].

For tracks alongside passenger platforms, the recommended cant restriction will be $110 \cdot \frac{1+s_r}{1+s_{r1}} \cdot r$. If $s_r = s_{r1}$, then this will be: $110 \cdot r$ mm.

In H.3 (Annex H), permissible D_1 and limit D_2 shall be multiplied by $\frac{4 \cdot r^2}{B_1^2}$.

C.4.3 Cant deficiency I_1 (5.2.3)

The formulae in the standard will become:

$$I_1 = \frac{C_1 \cdot V_{\text{max}}^2}{\mathsf{R}} - D_1 = I \cdot r \le (I_1)_{\text{lim}}$$

Where $C_1 = C \cdot r$

$$a_{q} = \frac{V_{\text{max}}^{2}}{12,96 \cdot \mathsf{R}} - g \cdot \frac{D_{1}}{e_{1}} = g \cdot \frac{I_{1}}{e_{1}} = \frac{I_{1}}{\left(\frac{e_{1}}{g}\right)} \le \left(a_{\mathsf{q}}\right)_{\mathsf{lim}} = \frac{\left(I_{1}\right)_{\mathsf{lim}}}{\left(\frac{e_{1}}{g}\right)}$$

$$a_{i1} = (1 + s_r) \cdot (1 - s_t) \cdot g \cdot \frac{I_1}{e_1}$$

Safety:

— Track shift limit:

The adopted values for cant deficiency are not critical. If the values of $H_{\rm s}$ are maintained, then:

$$I_1 = I \cdot r$$

Derailment and overturning:

This value is limited by the above track shift limit criteria.

Track fatigue criteria:

The same degree of fatigue applies to the same values of H_s , therefore, the most restrictive values correspond to H_s and the result is:

$$I_1 = I \cdot r$$

Ride quality criteria:

The ride quality (for a geometrically-similar quality track) will be the same for the same values of a_i .

This becomes:

$$I_1 = I \cdot \frac{1 + s_r}{1 + s_{r1}} \cdot r$$

If $s_{r1} = s_r$ then:

$$I_1 = I \cdot r$$

Rule for converting values:

The limits of Table 1, including the values governed by the footnotes, should be multiplied by r.

C.4.4 Cant excess E_1 (5.2.4)

The formula
$$E = D - 11.8 \cdot \frac{V^2}{R}$$
 [mm], becomes $E_1 = D_1 - 11.8 \cdot r \cdot \frac{V^2}{R}$ [mm].

This parameter mainly affects track fatigue and specifically the increase in load on the lower rail. Therefore the normal limit shown as 110 mm would become $110 \cdot r^2$ mm. However, the value for cant excess should not be greater than that for cant deficiency and the corresponding limit would be $110 \cdot r$ mm (see C.4.3).

C.4.5 Cant gradient dD_1/ds (5.2.5)

The limits for this parameter are associated with safety from the point of view of derailment of slow trains as a result of flange climbing. The same degree of safety with regard to derailment will be obtained with the same reduction in load on the guiding wheel. $\frac{\Delta D}{\Delta s}$ mm/m represents the average cant variation corresponding to the

length of the vehicle. The reduction in vertical wheel/rail force corresponding to $\mu = \ell \cdot \frac{\Delta D}{\Delta L} \cdot 10^{-3}$ mm, where ℓ is the vehicle wheel base, is $\Delta Q = K \left(\frac{\ell}{4}\right) \left(\frac{\Delta D}{\Delta L}\right) \left(\frac{B}{e}\right)^2 \cdot 10^{-3}$ N

The wheel-base ℓ shall be taken as the distance between axles for two-axle wagons, and separately as the bogie axle distance and the distance between bogie pivots for bogie vehicles.

Similarly,
$$\Delta Q_1 = K \left(\frac{\ell}{4}\right) \left(\frac{\Delta D_1}{\Delta L}\right) \left(\frac{B_1}{e_1}\right)^2 \cdot 10^3$$
 N; with $\Delta Q = \Delta Q_1$

Therefore:

$$\left(\frac{\Delta D_1}{\Delta L}\right) = \frac{\left(\frac{\Delta D}{\Delta L}\right) \left(\frac{B}{e}\right)^2}{\left(\frac{B_1}{e_1}\right)^2} = \left(\frac{\Delta D}{\Delta L}\right) \cdot \frac{4r^2}{B_1^2}$$

Rule for converting limits:

Normal limit: $2,25 \cdot \frac{4 \cdot r^2}{B_1^2}$ mm/m

Exceptional limit: $2,50 \cdot \frac{4 \cdot r^2}{B_1^2}$

NOTE Being Δl the longitudinal measurement base and ΔD the track twist, in Annex H limiting g_1 and limiting g_2 (H.3), g^+ and g^* (H.4) should be multiplied by $\frac{4 \cdot r^2}{B_s^2}$.

C.4.6 Rate of change of cant dD_1/dt (5.2.6 of the main body of the standard)

The reason for limiting this parameter is due exclusively to the ride comfort based on the suspension (delay or lag in the vehicle body tilt and excitation of roll oscillations). The inclination speed due to cant is restricted. The amount of inclination due to cant is:

$$\frac{D}{e}$$
 or $\frac{D}{e}$ and that perceived inside the vehicle is $(1+s_r)\frac{D}{e}$ or $(1+s_{r1})\frac{D}{e}$ therefore:

$$(1+s_r)\cdot\frac{\left(\frac{dD}{dt}\right)}{e} = (1+s_{r1})\cdot\frac{\left(\frac{dD_1}{dt}\right)}{e_1}$$

i.e.:

$$\left(\frac{dD_1}{dt}\right) = \left(\frac{dD}{dt}\right) \cdot \left(\frac{1+s_r}{1+s_{r1}}\right) \cdot r$$

If
$$S_{r1} = S_r$$

$$\left(\frac{dD_1}{dt}\right) = \left(\frac{dD}{dt}\right) \cdot r$$

Rule for converting values:

The limits in 5.2.6 should be multiplied by r.

C.4.7 Rate of change of cant deficiency dI_1/dt (5.2.7)

The limits of this parameter are associated to the ride comfort.

lf:

$$\frac{da_{i1}}{dt} = \frac{(1+s_{r1})da_{q1}}{dt} = \frac{(1+s_{r1})\left(\frac{dI_1}{dt}\right)}{e_1} \cdot g$$

$$\frac{da_i}{dt} = \frac{\left(1 + s_r\right)\left(\frac{dI}{dt}\right)}{e} \cdot g$$

The equation $\frac{da_{i1}}{dt} = \frac{da_i}{dt}$, implies:

$$\frac{dI_1}{dt} = r \cdot \left[\frac{(1+s_r)}{1+s_{r1}} \right] \cdot \frac{dI}{dt}$$

If $s_{r1} = s_r$, we have:

$$\frac{dI_1}{dt} = r \cdot \frac{\mathsf{d}I}{\mathsf{d}t}$$

Rule for converting values:

The limits in 5.2.7 shall be multiplied by r.

C.4.8 Length of transition curves in the horizontal plane $L_{\rm K}$ (5.2.8)

(valid only for the linear transition curves, and with transition curves coinciding with cant transitions, $L_K = L_D$,)

The length of transition curves in the horizontal plane depends on the cant gradient and the rates of change of cant and cant deficiency, which correspond to the following formulae:

$$L \ge \Delta D_1 \cdot \left(\frac{dD_1}{ds}\right)_{\text{lim}}^{-1} \text{ m}$$

$$L \ge \left(\frac{\mathsf{V}_{\mathsf{max}}}{q_{v}}\right) \cdot \Delta D_{1} \cdot \left(\frac{dD_{1}}{dt}\right)_{\mathsf{lim}}^{-1} \mathsf{m}$$

$$L \ge \left(\frac{\mathsf{V}_{\mathsf{max}}}{q_{v}}\right) \cdot \Delta I_{1} \cdot \left(\frac{dI_{1}}{dt}\right)_{\mathsf{lim}}^{-1} \mathsf{m}$$

The length of transition curve shall comply with all the above criteria.

Where there is no transition curve or it is of insufficient length with respect to the dI/dt criterion, the limits of the abrupt change of cant deficiency, defined in EN 13803-2, shall be complied with.

C.4.9 Other parameters (5.2.9, 5.2.10 and 5.2.11)

These parameters are not gauge-dependent. The limits given in the main part of this standard (see 5.2.9, 5.2.10 and 5.2.11) are applicable to a track gauge wider than 1435 mm.

Annex D

(normative)

Track alignment design parameter limits for track gauges wider than 1435 mm

D.1 Scope

This annex defines the relevant limits for the track alignment design parameters to be applied for tracks laid with gauges wider than 1435 mm.

The following limits have been derived, for the parameters covered in the main body of the standard, by application of the transposition rules described in Annex C.

D.2 Requirements for a gauge of 1668 mm

D.2.1 General

The limits defined in D.2.2 to D.2.10 apply for the tracks laid in Portugal and Spain with a track gauge of 1668 mm.

Base values are: e_1 = 1733 mm; B_1 = 2,233 m.

D.2.2 Cant D_1

Normal limit for cant is 185 mm.

NOTE It is recommended that cant should be restricted to 125 mm for tracks adjacent to passenger platforms. Some other track features, such as level crossings, bridges and tunnels may also, in certain local circumstances, impose cant restrictions

Exceptional limit for cant is 205 mm.

To avoid the risk of derailment of torsionally-stiff freight wagons on small radius curves (R < 320 m), cant should be restricted to the following limit.

$$D_{\text{lim}} = (R - 50\text{m}) \cdot 0.9\text{mm/m}$$
 [mm]

For further information, see informative Annex H.

Some other track features, such as level crossings, bridges and tunnels may also impose cant restrictions in specific local circumstances.

D.2.3 Cant deficiency I_1

Normal and exceptional limits for cant deficiency are given in Table D.1. These limits apply to all trains operating on a line. It is assumed that every vehicle has been tested and approved according to the procedures in EN 14363, EN 15686 and/or EN 15687 in conditions covering its own range of operating cant deficiency.

Table	D.1 —	- Cant	deficiency	$I_{ m lim}$
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	Normal limits	Exceptional limits	
Non-tilting	trains		
80 km/h < V ≤ 200 km/h	150 mm	211 mm	
200 km/h < V ≤ 230 km/h	150 mm	194 mm ^{ab}	
230 km/h < V ≤ 250 km/h	150 mm	177 mm ^{ab}	
250 km/h < V ≤ 300 km/h	115 mm	150 mm ^{abc}	
Tilting trains			
80 km/h ≤ V ≤ 260 km/h ^d	318 mm ^b	354 mm ^b	
1			

^a Trains complying with EN 14363, equipped with a cant deficiency compensation system other than tilt, may be permitted by the Infrastructure Manager to run with higher cant deficiency values.

- NOTE 1 The European signalling system ERTMS includes vehicle limits of non-compensated lateral acceleration in the running plane of 0,60 m/s², 0,65 m/s², 0,75 m/s², 0,80 m/s², 0,85 m/s², 1,00 m/s², 1,10 m/s², 1,20 m/s², 1,60 m/s², 1,80 m/s² and 2,00 m/s². These values reflect the current practice of operating different train categories in Europe.
- NOTE 2 Freight vehicles are normally approved for a cant deficiency in the range 106 mm to 150 mm.
- NOTE 3 Non-tilting passenger vehicles are normally approved for a cant deficiency of 150 mm to 194 mm.
- NOTE 4 Depending on the characteristics of certain special features in track, such as certain switches and crossings in curves, bridges carrying direct-laid ballastless track, tracks with jointed rails, certain sections of line exposed to very strong cross winds, etc., it may be necessary to restrict the permissible cant deficiency. Rules in respect of these restrictions cannot be formulated beforehand since they will be dictated by the design of the special features; definition of such a frame of reference can only be left to the initiative of the infrastructure manager.

D.2.4 Cant excess E_1

Normal limit for cant excess $E_{1 \text{lim}}$ is 125 mm.

D.2.5 Cant gradient dD_1/ds

Normal limit is 2,4 mm/m.

Exceptional limit is 2,7 mm/m.

NOTE For permissible speed lower than 80 km/h, a higher cant gradient may be used after a safety-case analysis, see Annex H.

D.2.6 Rate of change of cant dD_1/dt

For non-tilting trains, normal and exceptional limits for rate of change of cant are given in Table D.2.

^b The Infrastructure Manager may require qualification of a part of a line for the introduction of trains running at these or higher cant deficiencies, taking into account the required track quality and other conditions.

^c The limit may be raised to 177 mm for non ballasted track.

^d Currently, there are no lines used or planned where maximum speed for tilting trains exceeds 260 km/h.

Table D.2 — Rate of change of cant $\mathrm{d}\textit{D}/\mathit{dt}_{\mathrm{lim}}$ for constant cant gradients

	Normal limits	Exceptional limits	
Non-tilting trains <i>V</i> ≤200 km/h			
<i>I</i> ≤194 mm	55 mm/s	80 mm/s ^a	
194 mm <u -211 mm	55 mm/s	55 mm/s	
Non-tilting trains 200 km/h< <i>V</i> <300 km/h			
	55 mm/s	70 mm/s	
^a Where I <177 mm and dI/dt <80 mm/s, the exceptional limit for dD/dt may be raised to 95 mm/s.			

For tilting trains, the normal limit is $\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}} = 85\,$ mm/s , and the exceptional limit is $\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}} = 110\,$ mm/s .

D.2.7 Rate of change of cant deficiency dI_1/dt

The limits in Table D.3 apply to all types of transition curves.

Table D.3 — Rate of change of cant deficiency dI/dt_{lim}

	Normal limits	Exceptional limits	
Non-tilting trains <i>V</i> ≤200 km/h			
<i>I</i> ≤194 mm	60 mm/s	115 mm/s	
194 mm <u <211 mm	60 mm/s	105 mm/s	
Non-tilting trains 200 km/h< <i>V</i> ≤300 km/h			
	60 mm/s	85 mm/s	

In case of using tilting trains on given alignment the values of dI/dt are higher. The tilt control system creates transient states at the entry to curves, which may give rise to even more pronounced jerks. Both active and the passive tilt systems need certain time to adapt the angle of tilt to the curve radius and it is for this reason that curves shall include transition sections of sufficient length.

Normal limit:
$$\left(\frac{dI}{dt}\right)_{lim} = 115 \text{ mm/s}$$

D.2.8 Length of circular curves and straights between two transition curves L_{ii}

The limits for the length alignment elements L_{i1} are not gauge-dependent.

The normal limit for the length of a straight or a circular curve placed between two transition curves is 20 m, $L_{i1} \ge 20$ m.

NOTE As an alternative to a short length of a straight or a circular curve, this alignment element may be omitted and the two transition curves connected directly to each other.

For an alternative method to define the minimum lengths, see informative Annex B.

D.2.9 Vertical curves

The limits for vertical curves are not gauge-dependent (see 5.2.10 and 5.2.11).

Vertical curves should be at least 20 m long and may be designed without vertical transition curves.

NOTE Vertical curves are normally designed as parabolas (2nd degree polynomials) or as circular curves.

A vertical curve shall be provided where the difference in slope between adjacent gradients is more than:

- 2 mm/m for permissible speeds up to 230 km/h;
- 1 mm/m for permissible speeds over 230 km/h.

The limits for the vertical radii are defined in D.2.10.

D.2.10 Radius of vertical curve

The normal limit for radius of vertical curve is $R_{\nu, \rm lim} = q_{R, \rm lim} \cdot V^2$ [m], where $q_{\rm R, lim} = 0.35 \, \rm m \cdot h^2/km^2$, without going under 2000 m vertical radius.

NOTE 1 On lines where most of the passengers may be standing, it is recommended that q_R should be greater than 0.77 m·h²/km².

The exceptional limit for radius of vertical curve is $R_{\nu, \rm lim} = q_{R, \rm lim} \cdot V^2$ [m], where $q_{\rm R, lim} = 0.13 \, \rm m \cdot h^2/km^2$ for hollow, and $q_{\rm R} = 0.16 \, \rm m \cdot h^2/km^2$ for crest.

NOTE 2 For sections with switches and crossings laid in vertical curves, the limits defined in EN 13803-2 should be complied with.

D.3 Requirements for a gauge of 1524 mm

D.3.1 General

The limits defined in D.3.2 to D.3.10 apply for the tracks laid in Finland with a track gauge of 1524 mm.

Base values are: e_1 = 1585 mm; B_1 = 2,085 m.

D.3.2 Cant D_1

Normal limit for cant is 170 mm.

NOTE It is recommended that cant should be restricted to 115 mm for tracks adjacent to passenger platforms. Some other track features, such as level crossings, bridges and tunnels may also, in certain local circumstances, impose cant restrictions.

Exceptional limit for cant is 190 mm.

To avoid the risk of derailment of torsionally-stiff freight wagons on small radius curves (R < 320 m), cant should be restricted to the following limit.

$$D_{1 \text{lim}} = (R - 50 \text{m}) \cdot 0.7 \text{mm/m}$$
 [mm]

For further information, see informative Annex H.

Some other track features, such as level crossings, bridges and tunnels may also impose cant restrictions in specific local circumstances.

D.3.3 Cant deficiency I_1

Normal and exceptional limits for cant deficiency are given in Table D.4. These limits apply to all trains operating on a line. It is assumed that every vehicle has been tested and approved according to the procedures in EN 14363, EN 15686 and/or EN 15687 in conditions covering its own range of operating cant deficiency.

Table D.4 — Cant deficiency $I_{\rm lim}$

	Normal limits	Exceptional limits	
Non-tilting	Non-tilting trains		
80 km/h < V ≤ 200 km/h	140 mm	193 mm	
200 km/h < V ≤ 230 km/h	140 mm	178 mm ^{ab}	
230 km/h < V ≤ 250 km/h	140 mm	162 mm ^{ab}	
250 km/h < V ≤ 300 km/h	105 mm	140 mm ^{abc}	
Tilting trains			
80 km/h ≤ V ≤ 260 km/h ^d	290 mm ^b	323 mm ^b	

^a Trains complying with EN 14363, equipped with a cant deficiency compensation system other than tilt, may be permitted by the Infrastructure Manager to run with higher cant deficiency values.

- NOTE 1 The European signalling system ERTMS includes vehicle limits of non-compensated lateral acceleration in the running plane of 0,60 m/s 2 , 0,65 m/s 2 , 0,75 m/s 2 , 0,80 m/s 2 , 0,85 m/s 2 , 1,00 m/s 2 , 1,10 m/s 2 , 1,20 m/s 2 , 1,60 m/s 2 , 1,80 m/s 2 and 2,00 m/s 2 . These values reflect the current practice of operating different train categories in Europe.
- NOTE 2 Freight vehicles are normally approved for a cant deficiency in the range 97 mm to 140 mm.
- NOTE 3 Non-tilting passenger vehicles are normally approved for a cant deficiency of 140 mm to 178 mm.
- NOTE 4 Depending on the characteristics of certain special features in track, such as certain switches and crossings in curves, bridges carrying direct-laid ballastless track, tracks with jointed rails, certain sections of line exposed to very strong cross winds, etc., it may be necessary to restrict the permissible cant deficiency. Rules in respect of these restrictions cannot be formulated beforehand since they will be dictated by the design of the special features; definition of such a frame of reference can only be left to the initiative of the infrastructure manager.

D.3.4 Cant excess E_1

Normal limit for cant excess $E_{1 \text{lim}}$ is 115 mm.

^b The Infrastructure Manager may require qualification of a part of a line for the introduction of trains running at these or higher cant deficiencies, taking into account the required track quality and other conditions.

^c The limit may be raised to 162 mm for non ballasted track.

^d Currently, there are no lines used or planned where maximum speed for tilting trains exceeds 260 km/h.

D.3.5 Cant gradient dD_1/ds

Normal limit is 2,3 mm/m.

Exceptional limit is 2,6 mm/m.

NOTE For permissible speed lower than 80 km/h, a higher cant gradient may be used after a safety-case analysis, see Annex H.

D.3.6 Rate of change of cant dD_1/dt

For non-tilting trains, normal and exceptional limits for rate of change of cant are given in Table D.5.

Table D.5 — Rate of change of cant dD/dt_{lim} for constant cant gradients

	Normal limits	Exceptional limits	
Non-tilting trains <i>V</i> ≤200 km/h			
<i>I</i> ≤178 mm	50 mm/s	75 mm/s ^a	
178< <i>I</i> <u><</u> 193 mm	50 mm/s	50 mm/s	
Non-tilting trains 200 km/h <v<300 h<="" km="" td=""></v<300>			
	50 mm/s	65 mm/s	
$^{\rm a}$ Where $\it I$ <162 mm and $\rm d\it I/d\it t$ <75 mm/s, the exceptional limit for $\rm d\it D/d\it t$ may be raised to 90 mm/s.			

For tilting trains, the normal limit is $\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}} = 80 \, \mathrm{mm/s}$, and the exceptional limit is $\left(\frac{\mathrm{d}D}{\mathrm{d}t}\right)_{\mathrm{lim}} = 100 \, \mathrm{mm/s}$.

D.3.7 Rate of change of cant deficiency dI_1/dt

The limits in Table D.6 apply to all types of transition curves.

Table D.6 — Rate of change of cant deficiency dI/dtlim

	Normal limits	Exceptional limits	
Non-tilting trains	Non-tilting trains V<200 km/h		
<i>I</i> ≤178 mm	60 mm/s	105 mm/s	
178 mm <u <193 mm	60 mm/s	95 mm/s	
Non-tilting trains 200 km/h< <i>V</i> ≤300 km/h			
	60 mm/s	80 mm/s	

In case of using tilting trains on given alignment the values of dI/dt are higher. The tilt control system creates transient states at the entry to curves, which may give rise to even more pronounced jerks. Both active and the passive tilt systems need certain time to adapt the angle of tilt to the curve radius and it is for this reason that curves shall include transition sections of sufficient length.

Normal limit:
$$\left(\frac{dI}{dt}\right)_{lim} = 105 \text{ mm/s}$$

D.3.8 Length of circular curves and straights between two transition curves $L_{\rm il}$

The limits for the length alignment elements L_{i1} are not gauge-dependent.

The normal limit for the length of a straight or a circular curve placed between two transition curves is 20 m, $L_{i1} \ge 20$ m.

NOTE As an alternative to a short length of a straight or a circular curve, this alignment element may be omitted and the two transition curves connected directly to each other.

For an alternative method to define the minimum lengths, see informative Annex B.

D.3.9 Vertical curves

The limits for vertical curves are not gauge-dependent (see 5.2.10 and 5.2.11).

Vertical curves should be at least 20 m long and may be designed without vertical transition curves.

NOTE Vertical curves are normally designed as parabolas (2nd degree polynomials) or as circular curves.

A vertical curve shall be provided where the difference in slope between adjacent gradients is more than:

- 2 mm/m for permissible speeds up to 230 km/h;
- 1 mm/m for permissible speeds over 230 km/h.

The limits for the vertical radii are defined in D.3.10.

D.3.10 Radius of vertical curve

The normal limit for radius of vertical curve is $R_{\nu, \rm lim} = q_{R, \rm lim} \cdot V^2$ [m], where $q_{\rm R, lim} = 0.35 \, \rm m \cdot h^2/km^2$, without going under 2000 m vertical radius.

NOTE 1 On lines where most of the passengers may be standing, it is recommended that q_R should be greater than 0,77 m·h²/km².

The exceptional limit for radius of vertical curve is $R_{\nu, \rm lim} = q_{R, \rm lim} \cdot V^2$ [m], where $q_{\rm R, lim} = 0.13 \, \rm m \cdot h^2/km^2$ for hollow, and $q_{\rm R} = 0.16 \, \rm m \cdot h^2/km^2$ for crest.

NOTE 2 For sections with switches and crossings laid in vertical curves, the limits defined in EN 13803-2 should be complied with.

Annex E (informative)

Track resistance to lateral forces generated by the rolling stock

E.1 Introduction

The operating safety of any rail vehicle is ensured as long the coupling between the wheels and the track is maintained (see EN 14363, EN 15686, EN 15687, UIC 518, UIC 518-1 and UIC 518-2).

Three main causes of failure are:

- mon-compliance with the derailment criterion due to wheel-climb: derailment caused by the fact that a wheel, usually the wheel situated on the outer rail of a curve, leaves the rail head and rises above it. Since the wheel/rail interface is governed by friction forces, this derailment mode is characterised in practice by the $\frac{Y}{Q}$ ratio, where Y is the lateral wheel/rail force and Q the vertical wheel/rail force on the guiding wheel.
- exceeding the lateral strength limit of a track under loading (*Prud'homme* limit), which is the limit of the forces generated by the axle onto the track beyond which a gradual and irreversible deterioration process of the track in the ballast is initiated.
- the vehicle overturning: if the track lateral strength is sufficient, the lateral acceleration of a vehicle body may, at a very high speed, be high enough to cause a vehicle to overturn in a curve, while the outer wheel is not actually derailed. The determining criterion for this risk is then the permanent wheel load existing on the low rail.

E.2 The effect of alignment design parameters on lateral forces generated by the rolling stock

E.2.1 Cant deficiency

The lateral force generated by vehicles onto the track is dependent on cant deficiency, which may be broken down into two components:

- the quasi-static force in a circular curve, the level of which is directly proportional to the cant deficiency and the axle load;
- the dynamic force resulting from the vehicle response to the inputs from track alignment design and defects, is basically dependent upon:
 - the vehicle performance characteristics (specific stability of the vehicle);
 - the quality of track alignment and cross-level;
 - speed.

In addition, the increase in cant deficiency increases the coupling between the vehicle and the track geometrical defects (i.e. the wheel follows the alignment defect).

It therefore seems logical that vehicles possessing special mechanical characteristics (low axle load, reduced unsprung masses, low roll coefficient), under an equivalent loading level, may be allowed to operate over higher cant deficiencies than the conventional rolling stock.

E.2.2 Cant excess

In order to minimise the cant deficiency for fast passenger trains, the track alignment designer may, to a certain extent, increase the level of track cant.

However, the increase in cant may result in higher cant excess for slow trains and thus a higher quasi-static vertical force on the low rail. As the lateral force on the low rail of the curve is proportional to the vertical load, the curving forces of the vehicle axles may be higher. Consequently, rail wear and tear may increase as well the loadings on the track fastening systems.

For cant and cant deficiency, the designer should select the best compromise to suit the types and tonnage of traffic using a route. The designer should aim at balancing the loadings between the two rails.

E.2.3 The lateral strength limit of a track under loading (*Prud'homme* limit)

Beyond the comfort limit, there is another limit: as curving speed increases, the coupling forces exerted in the track plane also increase until they cause plastic strains in the track panel and/or a track lateral displacement. This displacement will cause a track geometrical defect which will gradually expand with the succession of axle loadings. This process may lead to the derailment of a vehicle.

This limit of the lateral track strength is expressed in the form of the *Prud'homme* limit which corresponds to the lower limit for the various types of track tested.

Since the *Prud'homme* limit applies to freshly-tamped track, temporary speed restrictions can be raised during the track consolidation period.

E.2.4 Factors influencing the resistance to track lateral displacement

Track components (rail profile, type of sleeper, type of fastening, ballast characteristics) and other factors as track consolidation after tamping, thermal loads in rails, proximity of two axles, dynamic vertical wheel/rail forces, influence the resistance to track lateral displacement.

These aspects are considered in DT 66, DT 150, RP4, RP5 and RP7 of the ORE C 138 Committee.

Annex F (informative)

Consequences on track resistance, stress and fatigue resulting from tilting body train systems

F.1 General

The tilting body is a vehicle design technique which allows the vehicle body of a railway passenger coach to rotate around a longitudinal axis and thus limit the lateral vehicle body acceleration, as perceived by the passenger. Tilting body techniques can therefore allow this type of vehicle to negotiate curves at higher speeds than a conventional non-tilting vehicle whilst still remaining within safety limits and without compromising passenger comfort.

It is necessary to underline that the tilting behaviour in curves is conditioned by the existence of the sufficient length of the transition curves. By short transition curves or by curves with an abrupt change of the curvature (see EN 13803-2), the tilting body systems cannot be used with the whole efficiency as in the plain line curves with complete transition curves or are even not active.

It is important to verify that the vehicle gauge can be guaranteed by infrastructure even for the false position of the tilting body due to potential failure of the steering system.

F.2 Basic principles applying to tilting body techniques

In the same way as the conventional railway system (non-tilting vehicles), the implementation of tilting body vehicles should be based on dedicated specifications covering:

- safety requirements;
- comfort requirements; running behaviour;
- economic assessment of the system (including both vehicle and infrastructure requirements).

The acceptance criteria are described in EN 14363, EN 15686, UIC 518 and UIC 518-1. It is necessary to point out that in those documents the range of lower radii 150 m $\leq R_{min} < 250$ m is not defined for tests. Each country is therefore responsible in function of the characteristic parameters of its own network to decide whether supplementary tests are necessary to prove the safety requirements in the mentioned range of the lower radii.

The currently used requirements on infrastructure applying the tilting train are described in detail in the UIC 705. The limits for the track alignment design parameters are presented in the main body of this standard.

F.3 Safety requirements

F.3.1 Lateral axle force and track lateral resistance (track lateral shift)

Tilting vehicles should comply with the same safety limit for applied lateral axle load on the track as for-tilting vehicles. This limit is known as the *Prud'homme* limit and expresses the lateral resistance of the loaded track to the lateral forces, which depends on the vertical axle load of the vehicle and on the type of track system (see Annex E). This safety limit is defined for conventional non-tilting vehicles in the EN 14363 and currently in

UIC 518 and UIC 518-1. It is important to keep in mind that the *Prud'homme* limit, as defined in the above documents, corresponds to a reference ballasted track system with 46 kg/m rails laid on wooden sleepers and that higher strength track system types may provide higher resistance to the loaded track (see Annex E).

In practice, lateral forces exerted on the track by a vehicle depend, mainly, on the following factors:

- design of the vehicle, including its suspended (and to some extent, non-suspended) masses (axle loads), suspension and damping characteristics;
- speed;
- track features, including switches and crossings;
- wheel/rail contact profiles;
- track alignment;
- track irregularities, especially in terms of alignment, twist and cant defects.

The introduction of tilting stock on a given line means an increase in the maximum train speed and therefore, in certain cases, a change in the requirements regarding track irregularities. The test procedure should include a set of parameters and test values, such as cant deficiency, speed and track geometrical quality. This procedure should take into consideration the maximum cant deficiency that could be encountered during service. Other parameters, such as the minimum curve radius, length and different types of transition curves, are equally important and should be included in the test procedure, especially when combined with high cant deficiency. This combination of parameters, in conjunction with the design of the suspension and tilting systems, has a major influence on vehicle response and acceptance. The acceptance requirements for tilting vehicles are defined in EN 15686 and UIC 518-1.

F.3.2 Vehicle overturning

Overturning conditions result when a vehicle on a curve, under high lateral acceleration in a curve (usually resulting from high cant deficiency being experienced in the running plane) initiates a rotation around the high rail. This situation increases the risk of the vehicle moving outwards and overturning. The likelihood of this type of event occurring with non-tilting conventional vehicles is small, due to the fact that the limit of the track lateral shift resistance is usually reached at a lower cant deficiency values than for overturning.

However, for tilting train operation, the combination of lateral inertia forces and the loading arising from a cross wind acting at the centre of pressure of the vehicle body may result in zero vertical force of the inner wheels. This can occur for cant deficiency values which are relatively nearer to the allowable service values for tilting vehicles than for non-tilting ones. Supposing that cant deficiencies that can induce overturning lie in the range between 450 mm and 500 mm depending upon the influence of wind loading and are similar for conventional non-tilting and tilting vehicles. These values represent 3 times the maximum cant deficiency limit for non-tilting vehicles, but may be as low as 1,5 times the limit for tilting vehicles (depending on their specific dynamic behaviour; some vehicles may have higher overturning limits). In a typical 400 m radius curve with 160 mm applied cant, a 300 mm cant deficiency is reached at 125 km/h, and a 450 mm cant deficiency at 144 km/h, which may require a special speed control system.

If it can be demonstrated that the mean bogie wheel load of a vehicle on the lower rail always keeps above a minimum at the maximum cant deficiency allowed, then the risk of overturning is minimal.

EN 15686 and UIC 518-1 describe therefore the necessary procedure and the limits for the defined so-called "Overturning criterion". This criterion requires that the tests shall prove that the critical, unstable state with no quasi-static forces on the curve inner wheel cannot be reached by cant deficiency values lower than $l \le 1.5 \cdot l_{\text{adm}}$.

Specifications for the influence of cross wind are in EN 14067-6 and in the TSI HST Infrastructure; and characteristic wind curves are defined in the High-Speed Rolling Stock TSI.

F.3.3 Comfort requirements

Several standards have been used for comfort evaluation:

- ISO 2631, which is based on an evaluation of weighted accelerations; this method can be separately
 applied to each vibration direction, vertical, lateral and longitudinal, and the rms value of accelerations
 can be converted to an index expressing a time duration of acceptable vibration;
- the CEN methods for comfort evaluation (EN 12299) are also based upon the evaluation of weighted acceleration levels (although it uses different weighting functions than the older ISO method which mainly concentrates on the low frequency range of vibration). The CEN methods evaluate the mean comfort level through a ride index incorporating all three vibration directions (i.e. lateral, vertical and longitudinal), comfort on curve transitions (discomfort due to high lateral acceleration, rate of change of lateral acceleration and roll velocity on transition curves) and comfort on discrete events (discomfort due to the lateral dynamic behaviour of the vehicle on local track irregularities).

An important aspect concerning the perceived feeling of comfort and/or the level of wheel/rail forces is the reaction time of the tilting system on changes in the track alignment elements. First of all, the entry and the exit of transition curves are very sensitive, because the reaction of the tilting mechanism on such changes can be retarded especially for the first vehicle. Depending on the control-command system, the consequences of such delayed reactions are discontinuities in the perceived comfort.

A sufficient length of the transition curve is, a further very important factor.

In normal cases, the transition curves with linear gradient of cant and curvature should be used. Within the transition curves in form of clothoide, the roll velocity of the vehicle case can be kept constant.

If transition curves with gradients with variable slope are in use or unavoidable, it is necessary to reach a vehicle body rotation velocity which corresponds to the variable slope of the mentioned gradients. In such cases, the transfer of the alignment indications to the vehicle is mostly based on localisers on the ground and not on on-board sensors fixed in the vehicles.

Under certain circumstances, it is possible for a tilting train to be tilting one way when the curve requires it to tilt the other way (or not tilt). This adverse effect is amplified by some alignment designs, especially in a series of reverse curves and instantaneous changes of curve radius without transition curve. In such configurations, especially within the diverging tracks of the switches and crossings with abrupt changes of curvature (see EN 13803-2), the tilting movement of the vehicle floor should be eliminated.

Consequently, the maximum speed of the tilting trains in curves without or with too short transition curves cannot be higher than that of the conventional non-tilting vehicles.

Annex G (informative)

Constraints and risks associated with the use of exceptional limits

The use of exceptional limits results in a reduced level of comfort for the passengers and may lead to higher track maintenance costs, particularly if associated with undesirable track geometry and equipment quality. Therefore, the designer should avoid unnecessary use of the exceptional limits for the permissible speed, either by complying with the normal limits specified in this standard or by using a margin with respect to design speed.

It is permissible to use the exceptional limits specified in this standard if use of the normal limits incurs unacceptable costs in achieving the maximum desired speed. However, every effort shall be made to design the alignment with substantial margin to the limits.

The above policy is equally valid for the upgrading of existing lines for higher speeds, when observance of normal limits would lead to unacceptable costs being incurred.

The exceptional limits are only acceptable for certain designs of passenger vehicles and even then, it will lead to lower comfort levels for the passengers and almost certainly higher maintenance costs.

The use of exceptional limits has to be agreed by the appropriate body that shall ensure that the vehicle stability and lateral track force criteria are met.

Maintenance should be kept within the limits laid down in the contract and additional inspection of the track may be required.

In comparison with the normal limits, the values used by the designer within a specific project should in normal cases be as low (high) as reasonably possible.

Annex H

(informative)

Recapitulation of the work carried out by the ORE B 55 Committee – Maximum permissible cant

H.1 Introduction

The work of the ORE B 55 Committee addressed the three following topics:

- the definition of the criteria for safety against derailment at low speed;
- the determination of twist values which are permissible in the track and of the resulting rules for cant limits;
- the definition of rules to be observed in the design of, and checks performed on new vehicles with regard to their capability of coping with different track twist values.

H.2 Criteria for safety against derailment at low speed through wheel-climbing

High-speed operation depends mainly on dynamic aspects and comfort. Conversely, at low speed, guiding conditions play a more vital role.

At low speed, curves are effectively negotiated with cant excess and the leading wheel (outer wheel of the first wheelset) exerts a Y force towards the larger radius in the curve while being unloaded because of the cant excess and twist due to the transition when reaching the end of the curve.

The rail guidance principle is a complex subject but, as a criterion for safety against derailment at low speed, the $\frac{Y}{O}$ ratio for the leading wheel has to be less than 1,2 (quasi-static condition).

The Y force applied to the leading wheel is the algebraic sum of four partial forces:

- the Y_1 curving force which depends on the curve radius, the vehicle characteristics and the load of the opposite wheel;
- the Y_2 force due to the non-compensated centrifugal force element in the track plane;
- the Y_3 force which results from the rotational torque between bogie and body for bogie-wagons only;
- the Y_4 force which results from the dynamic interplay between vehicle and track.

The *Q* load is the algebraic sum of five partial forces:

- the nominal constant load Q_N for a given vehicle;
- the initial difference ΔQ_0 mainly due to the hysteresis of leaf-springs and dry-friction dampers;
- the ΔQ_1 difference due to cant deficiency or cant excess;

- the ΔQ_2 variation resulting from track twist, which is related to the vehicle wheel-base but also depends on the vehicle torsional stiffness;
- the ΔQ_3 variation resulting from the dynamic interplay between vehicle and track.

When considering low speeds, the dynamic forces (Y_4 and ΔQ_3) are not significant. The running conditions when the leading wheel is unloaded correspond to:

$$\frac{Y_1 + Y_2 + Y_3}{Q_N - \Delta Q_0 - \Delta Q_1 - \Delta Q_2} \le 1,2$$

This relationship was drawn up for the running conditions, which prevail at low speed, with a 150 m radius curve laid without cant.

H.3 Limits for track twist

Track twist is defined as the difference in cant between two sections of track spaced a distance " $2 \cdot a$ " apart, termed the longitudinal measurement base. Twist is generally expressed in terms of a 3 m base.

The ORE B 55 Committee has used the outcome of its work based on data collected by many railways and based on statistical studies, in order to make the following recommendations that are to be applied to track design and subsequent track maintenance. They recommended:

— using the limit for track twist (g_1) given below, for normal maintenance conditions, based on an actual length of 2 m:

$$\lim g_1 = \frac{20}{2 \cdot a} + 3.0 \le 7 \%_{00}$$

— applying, within the same context, the following rule for the calculation of the cant limit (D_1) to be permitted in the track:

permissible
$$D_1 \le \frac{R-100}{2}$$
 [mm]

— using the reduced track twist limit (g_2) defined by the following relationship when dealing with large cant values and small curve radii:

$$3 \%_{00} \le \lim g_2 = \frac{20}{2 \cdot a} + 1.5 \le 6 \%_{00}$$

— the cant limit (D_2) to be permitted in the track can be inferred from the following relationship:

$$\lim D_2 = \frac{R - 50}{1,5}$$
 [mm]

It should be noted that the twist limit given above is a combination of the designed cant gradient in cant transitions and the errors in cross level.

Reduced speeds on curves of sharp radii can be avoided if exceptional values, exceeding the above, can be accepted with additional safety measures such as fitting rail lubricators on the outside rail of the curves.

H.4 Rules applicable to the design of and checks performed on new vehicles with regard to their capability of coping with track twist values

New vehicles may have to comply with the following twist-absorption conditions (g^+ and g^*) for checking purposes:

$$g^+ = 7 - \frac{5}{2 \cdot a^+} [^{\circ}/_{\circ \circ}]$$

with $2 \cdot a^+ < 4.5$ m, $2 \cdot a^+$ being the bogie wheel-base;

$$g^* = \frac{15}{2 \cdot a^*} + 2.0 \, [^{\circ}/_{\circ \circ}]$$

with 4,5 m \leq 2· a^* \leq 20 m, 2· a^* being the wheel-base for two-axle wagons or the distance between pivots of bogie-wagons.

H.5 List of reports published by the ORE B 55 Committee

- RP 1 (October 1964) Wheel load measurements as a means for testing two-axle goods wagons
- RP 2 (June 1965) Statistical enquiry relating to the permissible track twist
- RP 3 (October 1966) Permissible wheel-load variations on two-axle goods wagons
- RP 4 (October 1970) Two-axle wagons subjected to simultaneous stresses due to track twist and to lateral components of the forces of the automatic coupler; dynamic effects of track twists
- RP 5 (October 1973) Enquiry on the distribution of track twist for base lengths of 1,80 m to 19,80 m
- RP 6 (April 1975) Conditions for negotiating track twist: calculation and measurement of important vehicle parameters
- RP 7 (April 1978) Derailment on curves with high cant and small radius
- RP 8 (Final report April 1983) Conditions for negotiating track twists:
 - recommended values for the track twist and cant;
 - calculation and measurement of the relevant vehicle parameters;
 - vehicle testing.

Annex I (informative)

A-deviation Switzerland

Deviant from the provisions of subclause 5.2 "Normal limits and exceptional limits for track alignment design parameters" of prEN 13803-1, in Switzerland the regulatory statutes of The Swiss railway regulations (SR 742.141.11 / http://www.admin.ch/ch/d/sr/c742_141_11.html) have to be observed in addition.

These regulations specify that the limits defined in article 16N and 17N have to be respected for all track alignment.

Annex ZA (informative)

Relationship between this European Standard and the Essential Requirements of EU Directive 2008/57/EC of the European Parliament and of the council of 17 June 2008 on the interoperability of the rail system within the Community

This European Standard has been prepared under a mandate given to CEN/CENELEC/ETSI by the European Commission and the European Free Trade Association to provide a means of conforming to Essential Requirements of the Directive 2008/57/EC⁴.

Once this standard is cited in the Official Journal of the European Union under that Directive and has been implemented as a national standard in at least one Member State, compliance with the clauses of this standard given in Table ZA 1 for HS Infrastructure and in Table ZA.2 for CR Infrastructure confers, within the limits of the scope of this standard, a presumption of conformity with the corresponding Essential Requirements of that Directive and associated EFTA regulations.

⁴ This Directive 2008/57/EC adopted on 17th June 2008 is a recast of the previous Directives 96/48/EC 'Interoperability of the trans-European high-speed rail system' and 2001/16/EC 'Interoperability of the trans-European conventional rail system' and revisions thereof by 2004/50/EC 'Corrigendum to Directive 2004/50/EC of the European Parliament and of the Council of 29 April 2004 amending Council Directive 96/48/EC on the interoperability of the trans-European high-speed rail system and Directive 2001/16/EC of the European Parliament and of the Council on the interoperability of the trans-European conventional rail system'.

Table ZA.1 — Correspondence between this European standard, the HS TSI INF, published in OJEU dated 19 March 2008, and Directive 2008/57/EC

Clause(s)/ sub- clause(s) of this European Standard	Chapter/ § of the TSI	Essential Requirements of Directive	Comments
	3.4 Essential requirements – Elements of the infrastructure domain corresponding to the essential requirements 4. 2. Description of the infrastructure domain – Functional and technical specifications of the domain 4.2.6. Minimum radius of curvature 4.2.7. Track cant 4.2.8.1 Cant deficiency - Cant deficiency on plain track and on the trough route of switches and crossings 4.8 Register of Infrastructure Annex B 1 – Assessment of the infrastructure subsystem – Table B 1	-	The following track alignment design parameters -Radius of horizontal curve -Cant -Cant deficiency are considered as relevant to satisfy the essential requirements of Directive 2008/57/EC Other track alignment design parameters treated by EN 13803-1: 2010 and not by the HS TSI INF -Cant excess -Rate of change of cant -Cant gradient -Rate of change of cant deficiency -Length of cant transitions -Length of transition curves in the horizontal plane -Length of alignment elements between two transition curves -Radius of vertical curve § 4.2.5 of the HS TSI INF is fixing maximum rising and falling gradients for different line categories

WARNING — Other requirements and other EU Directives may be applicable to the product(s) falling within the scope of this standard.

Table ZA.2 — Correspondence between this European standard, the CR TSI INF (Final draft 3.0 dated 12 December 2008) and Directive 2008/57/EC

Clause(s)/ sub- clause(s) of this European Standard	Chapter/ § of the TSI	Essential Requirements of Directive 2008/57/EC	Comments
Sub clause 5.2.1 Radius of horizontal curve Sub clause 5.2.2 Cant Sub clause 5.2.3 Cant deficiency Sub clause 5.2.11 Radius of vertical curve	3. Essential requirements – Table 1. 4. 2.4. Description of the infrastructure subsystem– Functional and technical specifications of the subsystem - Line layout 4.2.4.4. — Minimum radius of horizontal curve 4.2.4.5. Minimum radius of vertical curve 4.2.5. Track parameters 4.2.5.2. Cant 4.2.5.3. Rate of change of cant (as a function of time) 4.2.5.4.1 Cant deficiency - Cant deficiency on plain track and on the trough route of switches and crossings 4.8 Register of Infrastructure Annex B – Assessment of the infrastructure subsystem – Table 21	Annex III Essential requirements. 1. General requirements 1.1. Safety Clause 1.1.1 1.5. Technical compatibility	The following track alignment design parameters: -Radius of horizontal curve -Cant -Cant deficiency -Radius of vertical curve are considered as relevant to satisfy the essential requirements of Directive 2008/57/EC Other track alignment design parameters treated by EN 13803-1:2010 and not by the CR TSI INF -Cant excess -Rate of change of cant -Cant gradient -Rate of change of cant deficiency -Length of cant transitions -Length of transition curves in the horizontal plane -Length of alignment elements between two transition curves § 4.2.4.3. of the CR TSI INF is fixing maximum gradients for different line categories.

Warning — Other requirements and other Directives may be applicable to the product(s) falling within the scope of this standard.

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- [6] UIC 518-2:2004, Supplement to UIC leaflet 518: Application to wagons with axleloads more than 22,4 t and up to 25 t.
- [7] EN 12299, Railway applications Ride comfort for passengers Measurement and evaluation
- [8] EN 13848-1, Railway applications Track Track geometry quality Part 1: Characterisation of track geometry
- [9] EN 15273-1, Railway applications Gauges Part 1: General Common rules for infrastructure and rolling stock

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