# BRITISH STANDARD

**BS EN 12603:2002**

# Glass in building — Procedures for goodness of fit and confidence intervals for Weibull distributed glass strength data

The European Standard EN 12603:2002 has the status of a British Standard

ICS 81.040.20





British Standards

# National foreword

This British Standard is the official English language version of EN 12603:2002.

The UK participation in its preparation was entrusted by Technical Committee B/520, Glass and glazing in building, to Subcommittee B/520/4, Properties and glazing methods, which has the responsibility to:

— aid enquirers to understand the text;

— present to the responsible international/European committee any enquiries on the interpretation, or proposals for change, and keep the UK interests informed;

— monitor related international and European developments and promulgate them in the UK.

A list of organizations represented on this subcommittee can be obtained on request to its secretary.

**Cross-references**

The British Standards which implement international or European publications referred to in this document may be found in the *BSI Catalogue* under the section entitled "International Standards Correspondence Index", or by using the "Search" facility of the *BSI Electronic Catalogue* or of British Standards Online.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

**Compliance with a British Standard does not of itself confer immunity from legal obligations.**

This British Standard, having been prepared under the direction of the Building and Civil Engineering Sector Policy and Strategy Committee, was published under the authority of the Standards Policy and Strategy Committee on 9 January 2003

**Summary of pages**

This document comprises a front cover, an inside front cover, the EN title page, pages 2 to 33 and a back cover.

The BSI copyright date displayed in this document indicates when the document was last issued.

**Amendments issued since publication**

| Amd. No. | Date | Comments |
|---|---|---|
| | | |
| | | |
| | | |
| | | |



ISBN 0 580 41104 4

| EUROPEAN STANDARD<br>NORME EUROPÉENNE<br>EUROPÄISCHE NORM | **EN 12603**<br><br>November 2002 |
|---|---|

ICS 81.040.20

English version

# Glass in building - Procedures for goodness of fit and confidence intervals for Weibull distributed glass strength data

Verre dans la construction - Procédures de validité de l'ajustement et intervalles de confiance des données de résistance du verre au moyen de la loi de Weibull

Glas im Bauwesen - Bestimmung der Biegefestigkeit von Glas - Schätzverfahren und Bestimmung der Vertrauensbereiche für Daten mit Weibull-Verteilung

This European Standard was approved by CEN on 7 September 2002.

CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration. Up-to-date lists and bibliographical references concerning such national standards may be obtained on application to the Management Centre or to any CEN member.

This European Standard exists in three official versions (English, French, German). A version in any other language made by translation under the responsibility of a CEN member into its own language and notified to the Management Centre has the same status as the official versions.

CEN members are the national standards bodies of Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and United Kingdom.

EUROPEAN COMMITTEE FOR STANDARDIZATION
COMITÉ EUROPÉEN DE NORMALISATION
EUROPÄISCHES KOMITEE FÜR NORMUNG

**Management Centre: rue de Stassart, 36 B-1050 Brussels**



Ref. No. EN 12603:2002 E

# Contents

# Foreword

This document (EN 12603:2002) has been prepared by Technical Committee CEN/TC 129 "Glass in building", the secretariat of which is held by IBN.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by May 2003, and conflicting national standards shall be withdrawn at the latest by May 2003.

In this standard the annexes A, B and C are informative.

According to the CEN/CENELEC Internal Regulations, the national standards organizations of the following countries are bound to implement this European Standard: Austria, Belgium, Czech Republic, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom.

# Introduction

This European Standard is based on the assumption that the statistical distribution of the attribute taken into consideration can be represented by one single Weibull distribution function, even where in certain cases (e.g. lifetime measurements) mixed distributions have frequently been observed. For this reason, the user of the standard has to check by a goodness of fit test whether the measured data of a sample can be represented by means of one single Weibull function. Only in this case can the hypothesis be accepted and the procedures described in this standard be applied.

The user decides on this question also considering all previous relevant data and the general state of knowledge in the special field. Every extrapolation into ranges of fractiles not confirmed by measured values requires utmost care, the more so the farther the extrapolation exceeds the range of measurements.

NOTE The three-parameter Weibull function is:

$$G(x) = 1 - \exp\left[-\left(\frac{x - x_0}{\theta}\right)^{\beta}\right] \quad (1)$$

If $x_o = 0$ is assumed, the two-parameter Weibull function results:

$$G(x) = 1 - \exp\left[-\left(\frac{x}{\theta}\right)^{\beta}\right] \quad (2)$$

which can be written as:

$$x = \theta\left[\ln\left(\frac{1}{1 - G(x)}\right)\right]^{\frac{1}{\beta}} \quad (3)$$

The calculation can be based either on an uncensored or a censored sample. There are several methods of censoring. In this standard only the following method of censoring is considered:

- given a number $r < n$ of specimens of which attribute values $x_i$ were measured.

# 1 Scope

This European Standard specifies procedures for the evaluation of sample data by means of a two-parameter Weibull distribution function.

# 2 Normative references

This European Standard incorporates by dated or undated reference, provisions from other publications. These normative references are cited at the appropriate places in the text, and the publications are listed hereafter. For dated references, subsequent amendments to or revisions of any of these publications apply to this European Standard only when incorporated in its amendment or revision. For undated reference, the latest edition of the publications referred to applies (including amendments).

ISO 2854:1976, *Statistical interpretation of data - Techniques of estimation and tests relating to means and variances.*

ISO 3534, *Statistics - Vocabulary and symbols*.

# 3 Terms and definitions

For the purposes of this European Standard, the terms and definitions given in ISO 3534 apply.

# 4 Symbols and abbreviated terms

$X$ attribute taken into consideration;

$x, x_i, x_r$ values of $X$;

$G(x)$ distribution function of $X$ = percentage of failure;

$x_o, \beta, \theta$ parameters of the three-parameter Weibull function;

^ identification label for point estimators (e.g. $\hat{\beta}$, $\hat{\theta}$, $\hat{G}$);

$1-\alpha$ confidence level;

$\ell_i$ value used in the goodness of fit test;

$L$ value used in the goodness of fit test;

$n$ sample size;

$r$ number of specimens of which attribute values $x_i$ were measured;

NOTE The sample is ordered, i.e. $x_1 \le x_2 \le x_3 \ldots\ldots \le x_r r \le n$;

$f, f_1, f_2$ degrees of freedom;

$\kappa_n, \kappa_{r;n}$ factors used in estimating $\hat{\beta}$;

$C_{r;n}$ factor used in estimating $\hat{\theta}$ ;

$s$ int(0,84$n$) = largest integer < 0,84$n$ ;

$\eta,\xi$ ordinate and abcissa of the Weibull diagram;

$\chi^2$ chi-square distribution function;

$y,v,\gamma$ auxiliary factors used in estimating the confidence limits of $G(x)$;

$A,B,C$ constants used in evaluating $v$ ;

$H(f_2)$ variable used in evaluating $\gamma$ ;

$T_{n;\alpha/2}, T_{n;1-\alpha/2}$ coefficients used in estimating the confidence limits of $\theta$ ;

Subscripts:

*un* lower confidence limit;

*ob* upper confidence limit;

*z* confidence interval limited on two sides.

## 5 Goodness of fit

Sort the $r$ values of $x$ into rank ascending order.

Compute for each value from $i$ = 1 to $i$ = $r$ - 1:

$$\ell_i = \frac{\ln(x_{i+1}) - \ln(x_i)}{\ln\left[\ln\left(\frac{4(n-i-1)+3}{4n+1}\right) \Big/ \ln\left(\frac{4(n-i)+3}{4n+1}\right)\right]} \tag{4}$$

Compute the quantity:

$$L = \frac{\displaystyle\sum_{i=\lfloor r/2 \rfloor+1}^{r-1} \frac{\ell_i}{\lfloor (r-1)/2 \rfloor}}{\displaystyle\sum_{i=1}^{\lfloor r/2 \rfloor} \frac{\ell_i}{\lfloor r/2 \rfloor}} \tag{5}$$

where the symbol $\lfloor r/2 \rfloor$ is used to denote the largest integer less than or equal to r/2.

Reject the hypothesis that the data is from a Weibull distribution at the $\alpha$ significance level if:

$$L \geq F_{\alpha}\left(2\lfloor (r-1)/2 \rfloor, 2\lfloor r/2 \rfloor\right) \tag{6}$$

The values of the fractiles of the *F* distribution can be found for example in Table IV of ISO 2854:1976.

## 6 Point estimators for the parameters *β* and *θ* of the distribution

### 6.1 Censored sample

$$\hat{\beta} = \frac{n\,\kappa_{r;n}}{r \ln x_r - \sum_{i=1}^{r} \ln x_i} \tag{7}$$

$$\hat{\theta} = \exp\left[\ln x_r - C_{r;n}\frac{1}{\hat{\beta}}\right] \tag{8}$$

The factors $\kappa_{r;n}$ and $C_{r;n}$ are listed in Table 1 and Table 2.

**Table 1 — Coefficient $\kappa_{r;n}$**

| *n* | *r/n* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 |
| 5 | | | | 0,2231 | | 0,4813 | | 0,8018 | |
| 10 | | 0,1054 | 0,2172 | 0,3369 | 0,4667 | 0,6098 | 0,7715 | 0,9616 | 1,202 |
| 20 | 0,0513 | 0,1583 | 0,2721 | 0,3944 | 0,5277 | 0,6756 | 0,8448 | 1,048 | 1,316 |
| 30 | 0,0684 | 0,1759 | 0,2904 | 0,4137 | 0,5482 | 0,6979 | 0,8697 | 1,077 | 1,357 |
| 40 | 0,0770 | 0,1848 | 0,2996 | 0,4233 | 0,5584 | 0,7090 | 0,8822 | 1,092 | 1,378 |
| 50 | 0,0821 | 0,1901 | 0,3051 | 0,4291 | 0,5646 | 0,7158 | 0,8898 | 1,101 | 1,391 |
| 60 | 0,0855 | 0,1936 | 0,3088 | 0,4330 | 0,5687 | 0,7202 | 0,8949 | 1,108 | 1,400 |
| 70 | 0,0879 | 0,1961 | 0,3114 | 0,4357 | 0,5717 | 0,7235 | 0,8985 | 1,112 | 1,406 |
| 80 | 0,0898 | 0,1980 | 0,3134 | 0,4378 | 0,5739 | 0,7259 | 0,9012 | 1,115 | 1,410 |
| 90 | 0,0912 | 0,1995 | 0,3149 | 0,4394 | 0,5756 | 0,7277 | 0,9033 | 1,118 | 1,414 |
| 100 | 0,0924 | 0,2007 | 0,3162 | 0,4407 | 0,5770 | 0,7292 | 0,9050 | 1,120 | 1,417 |
| $\kappa_p$ | 0,10265 | 0,21129 | 0,32723 | 0,45234 | 0,58937 | 0,74274 | 0,92026 | 1,1382 | 1,4436 |
| $d_1$ | -1,0271 | -1,0622 | -1,1060 | -1,1634 | -1,2415 | -1,3540 | -1,5313 | -1,8567 | -2,6929 |
| $d_2$ | 0,000 | 0,030 | 0,054 | 0,089 | 0,145 | 0,242 | 0,433 | 0,906 | 2,796 |
| Asymptotic estimate for large $n$ : $\kappa_{r,n} = \kappa_p + d_1/n + d_2/n^2$ | | | | | | | | | |

**Table 2 — Coefficient $C_{r,n}$**

| $n$ | $r/n$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 |
| 10 | -2,880 | -1,826 | -1,267 | -0,8681 | -0,5436 | -0,2574 | 0,0120 | 0,2837 | 0,5846 |
| 20 | -2,547 | -1,658 | -1,147 | -0,7691 | -0,4548 | -0,1727 | 0,0979 | 0,3776 | 0,7022 |
| 30 | -2,444 | -1,605 | -1,108 | -0,7364 | -0,4253 | -0,1443 | 0,1269 | 0,4098 | 0,7446 |
| 40 | -2,394 | -1,578 | -1,089 | -0,7202 | -0,4106 | -0,1301 | 0,1415 | 0,4262 | 0,7664 |
| 50 | -2,365 | -1,562 | -1,077 | -0,7105 | -0,4018 | -0,1216 | 0,1503 | 0,4360 | 0,7796 |
| 60 | -2,345 | -1,522 | -1,069 | -0,7040 | -0,3959 | -0,1159 | 0,1562 | 0,4426 | 0,7885 |
| 70 | -2,331 | -1,544 | -1,064 | -0,6994 | -0,3917 | -0,1118 | 0,1604 | 0,4473 | 0,7949 |
| 80 | -2,321 | -1,539 | -1,060 | -0,6959 | -0,3886 | -0,1088 | 0,1635 | 0,4509 | 0,7998 |
| 90 | -2,313 | -1,534 | -1,056 | -0,6932 | -0,3861 | -0,1064 | 0,1660 | 0,4537 | 0,8035 |
| 100 | -2,307 | -1,531 | -1,054 | -0,6911 | -0,3841 | -0,1045 | 0,1679 | 0,4559 | 0,8065 |
| $c_p$ | -2,2504 | -1,4999 | -1,0309 | -0,67173 | -0,36651 | -0,08742 | 0,18563 | 0,47589 | 0,83403 |
| $a_1$ | -5,5743 | -3,0740 | -2,2859 | -1,9301 | -1,7619 | -1,7114 | -1,7727 | -2,0110 | -2,7773 |
| $a_2$ | -7,201 | -1,886 | -0, 767 | -0,335 | -0,091 | 0,111 | 0,369 | 0,891 | 2,825 |
| Asymptotic estimate for large $n$ : $C_{r,n} = c_p + a_1/n + a_2/n^2$ | | | | | | | | | |

## 6.2 Uncensored (complete) sample

$$\hat{\beta} = \frac{n\,\kappa_n}{\frac{s}{n-s}\sum_{i=s+1}^{n}\ln x_i - \sum_{i=1}^{s}\ln x_i} \qquad (10)$$

$$\hat{\theta} = \exp\left[\frac{1}{n}\sum_{i=1}^{n}\ln x_i + 0{,}5772\frac{1}{\beta}\right] \qquad (11)$$

The factors $\kappa_n$ are listed in Table 3.

**Table 3 — Coefficient $\kappa_n$**

| *n* | $\kappa_n$ | *n* | $\kappa_n$ |
|---|---|---|---|
| 2 | 0,6931 | 32 | 1,4665 |
| 3 | 0,9808 | 33 | 1,4795 |
| 4 | 1,1507 | 34 | 1,4920 |
| 5 | 1,2674 | 35 | 1,5040 |
| 6 | 1,3545 | 36 | 1,5156 |
| 7 | 1,1828 | 37 | 1,5266 |
| 8 | 1,2547 | 38 | 1,4795 |
| 9 | 1,3141 | 39 | 1,4904 |
| 10 | 1,3644 | 40 | 1,5009 |
| 11 | 1,4079 | 41 | 1,5110 |
| 12 | 1,4461 | 42 | 1,5208 |
| 13 | 1,3332 | 43 | 1,5303 |
| 14 | 1,3686 | 44 | 1,4891 |
| 15 | 1,4004 | 45 | 1,4984 |
| 16 | 1,4293 | 46 | 1,5075 |
| 17 | 1,4556 | 47 | 1,5163 |
| 18 | 1,4799 | 48 | 1,5248 |
| 19 | 1,3960 | 49 | 1,5331 |
| 20 | 1,4192 | 50 | 1,5411 |
| 21 | 1,4408 | 51 | 1,5046 |
| 22 | 1,4609 | 52 | 1,5126 |
| 23 | 1,4797 | 53 | 1,5204 |
| 24 | 1,4975 | 54 | 1,5279 |
| 25 | 1,5142 | 55 | 1,5352 |
| 26 | 1,4479 | 56 | 1,5424 |
| 27 | 1,4642 | 57 | 1,5096 |

**Table 3** *(continued)*

| | | | |
|---|---|---|---|
| 28 | 1,4796 | 58 | 1,5167 |
| 29 | 1,4943 | 59 | 1,5236 |
| 30 | 1,5083 | 60 | 1,5304 |
| 31 | 1,5216 | ∞ | 1,5692 |

# 7 Assessment of data and tests

## 7.1 The Weibull diagram

The probability diagram for the Weibull distribution is drawn up in such a way that the distribution function of a two-parameter Weibull distribution is represented by a straight line.

The ordinate axis is graduated according to the function

$$\eta = \ln\left(\ln\left(\frac{1}{1 - G(x)}\right)\right) \tag{12}$$

and the abscissa axis according to the function

$$\xi = \ln x \text{ or } \xi = \log x \tag{13}$$

NOTE Such forms are available. As a rule, diagrams should be used with a range of $G$-values from $G = 1 \times 10^{-3} = 0{,}1$ % to $G = 0{,}999 = 99{,}9$ %. The necessary range of $x$-values depends on the value $\beta$ of the shape parameter.

## 7.2 Graphical representation of the estimated distribution function

The point estimators of the shape parameter $\beta$ and the scale parameter $\theta$ define a straight line in the Weibull diagram; it is appropriate to define this straight line though the following two points:

$$x = \hat{\theta} \qquad G(x) = 0{,}6321 = 63{,}21\% \tag{14}$$

$$x = \hat{\theta} \times 0{,}01005^{\frac{1}{\beta}} \quad G(x) = 0{,}01 = 1\% \tag{15}$$

This straight line shall be plotted into the diagram.

## 7.3 Plotting of sample data in the Weibull diagram

### 7.3.1 Single values

Measurements of a censored or uncensored sample yield $r$ or $n$ values $x_i$, respectively, of the attribute $X$. These values $x_i$ shall be ordered to make up an Ordered Sample.

Each value $\boldsymbol{x_i}$ of the Ordered Sample shall be co-ordinated to an estimated value:

$$\hat{G}(x_i) = \frac{i - 0{,}3}{n + 0{,}4} \tag{16}$$

This way the points representing the measured values of the sample shall be plotted into the Weibull diagram.

#### 7.3.2 Classified values

In the case of a very large sample, the range of measured *x*-values can be subdivided into classes, usually containing the same number of values. The proportion of *x*-values summed up in any class considered shall be plotted at the upper limit of that class.

### 7.4 Assessment of sample data

The straight line plotted according to 7.2 and the points which represent the measured values of the sample, plotted according to 7.3 can be compared visually.

Systematic deviations can examined in detail taking into consideration the general knowledge of the basic technical and scientific facts and the results of previous relevant research. For instance, if the distribution of the attribute values can be approximated by segments of straight lines with different slopes, a mixed Weibull distribution may be assumed. This can be taken as a hint that several basic mechanisms determine the attribute values $\boldsymbol{x_i}$. Such a detailed examination is beyond the scope of this standard.

## 8 Confidence intervals

The equations of the following sub-clauses are valid for the case that the confidence intervals are limited on two sides (subscript *z*). Where the confidence intervals are limited only on one side, $\alpha/2$ shall be replaced by $\alpha$ in the following equations.

The confidence level (1 - $\alpha$) is to be chosen by the user of this standard.

### 8.1 Confidence interval for the shape parameter $\beta$

The upper limit of the confidence interval for the shape parameter $\beta$ at the confidence level (1 - $\alpha$) is:

$$\beta_{ob;z} = \hat{\beta}\frac{\chi^2_{f_1;1-\alpha/2}}{f_1} \tag{17}$$

and the lower limit:

$$\beta_{un;z} = \hat{\beta}\frac{\chi^2_{f_1;\alpha/2}}{f_1} \tag{18}$$

$f_1$ shall be obtained by multiplication of the figures from Table 4 by the sample size *n*.

**Table 4 — Values of the function $f_1/n$**

| $n$ | $r/n$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1,0 |
| 10 | | 0,211 | 0,434 | 0,671 | 0,926 | 1,200 | 1,497 | 1,825 | 2,174 | 2,701 |
| 20 | 0,103 | 0,316 | 0,543 | 0,784 | 1,042 | 1,320 | 1,621 | 1,946 | 2,277 | 2,891 |
| 30 | 0,137 | 0,351 | 0,579 | 0,821 | 1,080 | 1,360 | 1,661 | 1,985 | 2,303 | 2,958 |
| 40 | 0,154 | 0,369 | 0,597 | 0,840 | 1,100 | 1,380 | 1,682 | 2,004 | 2,315 | 2,991 |
| 50 | 0,164 | 0,380 | 0,608 | 0,851 | 1,111 | 1,392 | 1,693 | 2,015 | 2,320 | 3,009 |
| 100 | 0,185 | 0,401 | 0,629 | 0,873 | 1,135 | 1,415 | 1,718 | 2,037 | 2,330 | 3,045 |
| $h_0$ | 0,2052 | 0,4218 | 0,6514 | 0,8959 | 1,1577 | 1,4391 | 1,7416 | 2,0598 | 2,3394 | 3,085 |
| $h_1$ | -2,052 | -2,111 | -2,175 | -2,244 | -2,314 | -2,376 | -2,390 | -2,205 | -0,856 | |
| $h_2$ | 0,000 | 0,008 | 0,002 | -0,016 | -0,064 | -0,188 | -0,526 | -1,682 | -7,928 | |
| Asymptotic estimate for large $n$ : $f_1/n = h_0 + h_1/n + h_2/n^2$<br>For uncensored samples ($r/n$=1) a good approximation is $f_1/n$ = 3,085 - 3,84/$n$ | | | | | | | | | | |

The values $\chi^2_{f_1;1-\alpha/2}$ and $\chi^2_{f_1;\alpha/2}$

are quantiles of the chi-square distribution with $f_1$ degrees of freedom. Values are given in Table 5.

**Table 5 — The 2,5 % and 97,5 % quantiles of the $\chi^2$ distribution**

| Degrees of freedom $f$ | $p$ | |
|---|---|---|
| | 2,5 % | 97,5 % |
| 1 | 0,000982 | 5,02 |
| 2 | 0,0506 | 7,38 |
| 3 | 0,216 | 9,35 |
| 4 | 0,484 | 11,1 |

**Table 5** *(continued)*

| Degrees of freedom<br>*f* | *p* | |
|---|---|---|
| | 2,5 % | 97,5 % |
| 5 | 0,831 | 12,8 |
| 6 | 1,24 | 14,4 |
| 7 | 1,69 | 16,0 |
| 8 | 2,18 | 17,5 |
| 9 | 2,70 | 19,0 |
| 10 | 3,25 | 20,5 |
| 11 | 3,82 | 21,9 |
| 12 | 4,40 | 23,3 |
| 13 | 5,01 | 24,7 |
| 14 | 5,63 | 26,1 |
| 15 | 6,26 | 27,5 |
| 16 | 6,91 | 28,8 |
| 17 | 7,56 | 30,2 |
| 18 | 8,23 | 31,5 |
| 19 | 8,91 | 32,9 |
| 20 | 9,59 | 34,2 |
| 21 | 10,3 | 35,5 |
| 22 | 11,0 | 36,8 |
| 23 | 11,7 | 38,1 |
| 24 | 12,4 | 39,4 |
| 25 | 13,1 | 40,6 |
| 26 | 13,8 | 41,9 |
| 27 | 14,6 | 43,2 |
| 28 | 15,3 | 44,5 |
| 29 | 16,0 | 45,7 |

**Table 5** *(continued)*

| Degrees of freedom $f$ | $p$ | |
|---|---|---|
| | 2,5 % | 97,5 % |
| 30 | 16,8 | 47,0 |
| 40 | 24,4 | 59,3 |
| 50 | 32,4 | 71,4 |
| 60 | 40,5 | 83,3 |
| 70 | 48,8 | 95,0 |
| 80 | 57,2 | 106,6 |
| 90 | 65,6 | 118,1 |
| 100 | 74,2 | 129,6 |
| Approximation for $f > 30$ | $p$ 2,5 % | 97,5 % |
| | $u_p$ -1,9600 | 1,9600 |
| $\chi^2_{f;p} = f\,[1 - 2/9f + u_p\sqrt{(2/9f)}]^3$ | | |

## 8.2 Confidence interval for the value of the distribution function *G(x)* at a given value of *x*, of the attribute *X*

The limits of the two-sided confidence interval for $G$ at the confidence level $(1 - \alpha)$ for a considered value $x$ of the attribute $X$ shall be calculated by means of three auxiliary factors $y$, $v$ and $\gamma$.

Equation for auxiliary factor $y$:

$$y = \hat{\beta} \ln \frac{\hat{\theta}}{x} = -\ln\left[\ln\left(\frac{1}{1-G(x)}\right)\right] \quad (19)$$

Equation for auxiliary factor $v$:

$$v = A + By^2 - 2Cy \quad (20)$$

The constants $A$, $B$, $C$ shall be obtained by dividing the values obtained from Table 6 by the sample size $n$.

**Table 6 — Constants *A.n*, *B.n* and *C.n***

| *n* | *r*/*n* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| *B.n* | | | | | | | | | | |
| 10 | | 9,488 | 4,609 | 2,979 | 2,161 | 1,667 | 1,336 | 1,096 | 0,9197 | 0,7405 |
| 20 | 19,49 | 6,324 | 3,686 | 2,552 | 1,920 | 1,515 | 1,234 | 1,028 | 0,8784 | 0.6919 |
| 30 | 14,62 | 5,691 | 3,455 | 2,436 | 1,851 | 1,471 | 1,204 | 1,008 | 0,8683 | 0,6761 |
| 40 | 13,00 | 5,420 | 3,350 | 2,382 | 1,819 | 1,450 | 1,189 | 0,9981 | 0,8641 | 0,6687 |
| 50 | 12,18 | 5,269 | 3,290 | 2,350 | 1,800 | 1,437 | 1,181 | 0,9925 | 0,8619 | 0,6647 |
| 60 | 11,70 | 5,173 | 3,251 | 2,330 | 1,787 | 1,429 | 1,175 | 0,9888 | 0,8605 | 0,6616 |
| 80 | 11,14 | 5,058 | 3,204 | 2,305 | 1,772 | 1,419 | 1,168 | 0,9840 | 0,8590 | 0,6584 |
| 100 | 10,83 | 4,991 | 3,177 | 2,290 | 1,763 | 1,413 | 1,164 | 0,9816 | 0,8580 | 0,6564 |
| ∞ | 9,746 | 4,742 | 3,070 | 2,232 | 1,728 | 1,390 | 1,148 | 0,9710 | 0,8549 | 0,6482 |
| *C.n* | | | | | | | | | | |
| 10 | | 17,58 | 6,109 | 2,868 | 1,474 | 0,7502 | 0,3344 | 0,0826 | -0,0694 | -0,1981 |
| 20 | 49,91 | 10,75 | 4,505 | 2,254 | 1,184 | 0,5975 | 0,2500 | 0,0373 | -0,0856 | -0,2216 |
| 30 | 35,98 | 9,397 | 4,107 | 2,089 | 1,102 | 0,5533 | 0,2253 | 0,0245 | -0,0883 | -0,2206 |
| 40 | 31,36 | 8,819 | 3,927 | 2,012 | 1,064 | 0,5323 | 0,2136 | 0,0185 | -0,0891 | -0,2262 |
| 50 | 29,06 | 8,499 | 3,825 | 1,967 | 1,041 | 0,5200 | 0,2068 | 0,0150 | -0,0894 | -0,2238 |
| 60 | 27,68 | 8,296 | 3,750 | 1,938 | 1,026 | 0,5120 | 0,2023 | 0,0127 | -0,0895 | -0,2271 |
| 80 | 26,10 | 8,050 | 3,680 | 1,900 | 1,008 | 0,5020 | 0,1970 | 0,0100 | -0,089 | -0.2287 |
| 100 | 25,30 | 7,910 | 3,630 | 1,880 | 0,9980 | 0,4960 | 0,1940 | 0,0080 | -0.089 | -0.2292 |
| ∞ | 22,19 | 7,383 | 3,450 | 1,801 | 0,9562 | 0,4734 | 0,1807 | 0,0019 | -0,0891 | -0,2309 |
| *A.n* | | | | | | | | | | |
| 10 | | 39,04 | 12,052 | 5,609 | 3,233 | 2,172 | 1,650 | 1,384 | 1,255 | 1,170 |
| 20 | 140,7 | 23,96 | 9,136 | 4,666 | 2,850 | 2,000 | 1,570 | 1,350 | 1,248 | 1,159 |
| 30 | 100,4 | 20,96 | 8,416 | 4,410 | 2,743 | 1,949 | 1,546 | 1,339 | 1,248 | 1,165 |
| 40 | 87,06 | 19,68 | 8,088 | 4,292 | 2,692 | 1,925 | 1,534 | 1,335 | 1,249 | 1,161 |

**Table 6** *(continued)*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 80,39 | 18,97 | 7,901 | 4,223 | 2,662 | 1,911 | 1,528 | 1,332 | 1,249 | 1,165 |
| 60 | 76,40 | 18,52 | 7,781 | 4,179 | 2,643 | 1,902 | 1,524 | 1,331 | 1,249 | 1,162 |
| ∞ | 60,53 | 16,50 | 7,219 | 3,967 | 2,550 | 1,859 | 1,503 | 1,323 | 1,251 | 1,162 |

$v = By^2 - 2Cy + A$

*B*, *C* and *A* are obtained by dividing the values in the table by *n*.

For uncensored samples ($r/n$=1) good approximations are:

$B = 0{,}6482/n + 0{,}805/n^2 + 1{,}13/n^3$;  $C = -0{,}2309/n + 0{,}15/n^2 + 1{,}78/n^3$;  $A = 1{,}162/n$

Equation for auxiliary factor $\gamma$:

$$\gamma = \exp\left(-y + H(f_2)\right) \quad (21)$$

where $f_2$ and $H(f_2)$ are determined from Table 7.

NOTE $\gamma$ and $f_2$ depend on the value of $\hat{G}(x)$, the sample size, *n*, and the ratio $r/n$. $\gamma$ and $f_2$ are independent of $\hat{\beta}$.

**Table 7 — $f_2$ and $H(f_2)$ as a function of $v$**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $v$ | 0,221 | 0,490 | 1,645 | 1,774 | 1,923 | 2,096 | 2,299 | 2,541 | 2,681 |
| $f_2$ | 10,00 | 5,000 | 2,000 | 1,900 | 1,800 | 1,700 | 1,600 | 1,500 | 1,450 |
| $H(f_2)$ | 0,103 | 0,213 | 0,577 | 0,611 | 0,650 | 0,693 | 0,742 | 0,798 | 0,830 |
| $v$ | 2,834 | 3,003 | 3,191 | 3,401 | 3,636 | 3,901 | 4,201 | 4,543 | 4,935 |
| $f_2$ | 1,400 | 1,350 | 1,300 | 1,250 | 1,200 | 1,150 | 1,100 | 1,105 | 1,000 |
| $H(f_2)$ | 0,863 | 0,900 | 0,940 | 0,983 | 1,030 | 1,081 | 1,138 | 1,201 | 1,270 |

Mathematical functions:-

$v \leq 2$ : $f_2 = (8v + 12)/(v^2 + 6v)$

$H(f_2) = (15 f_2^2 + 5 f_2 + 6)/(15 f_2^3 + 6 f_2)$

$2 < v \leq 5$ : $f_2 = 3{,}509 - 1{,}3055v + 0{,}2480v^2 - 0{,}0175v^3$

$H(f_2) = 0{,}08832 + 0{,}3218v - 0{,}0167v^2$

Then the limits of the confidence interval for *G* are:

upper limit:

$$G_{ob;z} = 1 - \exp\left[ -\gamma \frac{\chi_{f_2;1-\alpha/2}}{f_2} \right] \tag{22}$$

lower limit:

$$G_{un;z} = 1 - \exp\left[ -\gamma \frac{\chi_{f_2;\alpha/2}}{f_2} \right] \tag{23}$$

## 8.3 Confidence interval for the scale parameter $\theta$

### 8.3.1 Method for all samples

The limits of the two-sided confidence interval for the scale parameter $\theta$ at the confidence level (1 - $\alpha$) shall be calculated by iteration:

$$\theta_{ob;z;j+1} = \frac{\theta_{ob;z;j}}{\left[ \ln \frac{1}{1 - G_{un;z}(x = \theta_{ob;z;j})} \right]^{\frac{1}{\hat{\beta}}}} \tag{24}$$

$$\theta_{un;z;j+1} = \frac{\theta_{un;z;j}}{\left[ \ln \frac{1}{1 - G_{ob;z}(x = \theta_{un;z;j})} \right]^{\frac{1}{\hat{\beta}}}} \tag{25}$$

The iteration can be started with $\theta_{ob;z;0} = \theta_{un;z;0} = \hat{\theta}$.

After each iteration, new values of $G_{ob,z}(x = \theta_{un;z;j})$ and $G_{un,z}(x = \theta_{ob;z;j})$ shall be calculated by the method in 8.2.

The iteration shall be stopped when two successive values of both $\theta_{ob;z}$ and $\theta_{un;z}$ are equal to within the required accuracy. For instance, for evaluation of strength test results, a difference less than 0,1 % of should give sufficient accuracy.

### 8.3.2 Method for uncensored samples

In the case of uncensored (complete) samples, the following simpler equations can be used:

$$\theta^{*}_{ob;z} = \hat{\theta} \exp\left[ -\frac{T_{n;\alpha/2}}{\hat{\beta}} \right] \tag{26}$$

$$\theta^{*}_{un;z} = \hat{\theta} \exp\left[ -\frac{T_{n;1-\alpha/2}}{\hat{\beta}} \right] \qquad (27)$$

with the coefficients $T_{n;\alpha/2}$ and $T_{n;1-\alpha/2}$ taken from Table 8.

**Table 8 — Confidence factors $T_{n;p}$**

| *n* | $p = 1 - \alpha/2$ | | | | $p = \alpha/2$ | | | |
|---|---|---|---|---|---|---|---|---|
| | 0,975 | 0,95 | 0,9 | 0,75 | 0,25 | 0,1 | 0,05 | 0,025 |
| 5 | 1,4897 | 1,107 | 0,772 | 0,349 | -0,444 | -0,888 | -1,247 | -1,5675 |
| 6 | 1,2233 | 0,939 | 0,666 | 0,302 | -0,385 | -0,740 | -1,007 | -1,3247 |
| 7 | 1,0642 | 0,829 | 0,598 | 0,272 | -0,344 | -0,652 | -0,874 | -1,1437 |
| 8 | 0,9548 | 0,751 | 0,547 | 0,251 | -0,313 | -0,591 | -0,784 | -1,0096 |
| 9 | 0,8738 | 0,691 | 0,507 | 0,235 | -0,289 | -0,544 | -0,717 | -0,9122 |
| 10 | 0,8114 | 0,644 | 0,475 | 0,222 | -0,269 | -0,507 | -0,665 | -0,8387 |
| 11 | 0,7603 | 0,605 | 0,448 | 0,211 | -0,253 | -0,477 | -0,622 | -0,7790 |
| 12 | 0,7176 | 0,572 | 0,425 | 0,202 | -0,239 | -0,451 | -0,587 | -0,7326 |
| 13 | 0,6815 | 0,544 | 0,406 | 0,194 | -0,228 | -0,429 | -0,557 | -0,6894 |
| 14 | 0,6502 | 0,520 | 0,389 | 0,187 | -0,217 | -0,410 | -0,532 | -0,6572 |
| 15 | 0,6235 | 0,499 | 0,374 | 0,180 | -0,208 | -0,393 | -0,509 | -0,6266 |
| 16 | 0,5989 | 0,480 | 0,360 | 0,175 | -0,200 | -0,379 | -0,489 | -0,6016 |
| 17 | 0,5778 | 0,463 | 0,348 | 0,170 | -0,193 | -0,365 | -0,471 | -0,5795 |
| 18 | 0,5577 | 0,447 | 0,338 | 0,165 | -0,187 | -0,353 | -0,455 | -0,5566 |
| 19 | 0,5405 | 0,433 | 0,328 | 0,161 | -0,181 | -0,342 | -0,441 | -0,5356 |
| 20 | 0,5254 | 0,421 | 0,318 | 0,157 | -0,175 | -0,332 | -0,428 | -0,5187 |
| 22 | 0,4958 | 0,398 | 0,302 | 0,150 | -0,166 | -0,314 | -0,404 | -0,4907 |
| 24 | 0,4719 | 0,379 | 0,288 | 0,144 | -0,158 | -0,299 | -0,384 | -0,4669 |
| 26 | 0,4509 | 0,362 | 0,276 | 0,138 | -0,150 | -0,286 | -0,367 | -0,4450 |
| 28 | 0,4326 | 0,347 | 0,265 | 0,134 | -0,144 | -0,274 | -0,352 | -0,4249 |

**Table 8** *(continued)*

| *n* | *p* = 1 - α/2 | | | | *p* = α/2 | | | |
|---|---|---|---|---|---|---|---|---|
| 30 | 0,4156 | 0,334 | 0,256 | 0,129 | -0,139 | -0,264 | -0,338 | -0,4098 |
| 32 | 0,4014 | 0,323 | 0,247 | 0,125 | -0,134 | -0,254 | -0,326 | -0,3951 |
| 34 | 0,3879 | 0,312 | 0,239 | 0,122 | -0,129 | -0,246 | -0,315 | -0,3801 |
| | 0,975 | 0,95 | 0,9 | 0,75 | 0,25 | 0,1 | 0,05 | 0,025 |
| 36 | 0,3755 | 0,302 | 0,232 | 0,118 | -0,125 | -0,238 | -0,305 | -0,3687 |
| 38 | 0,3648 | 0,293 | 0,226 | 0,115 | -0,121 | -0,231 | -0,296 | -0,3578 |
| 40 | 0,3544 | 0,285 | 0,220 | 0,113 | -0,118 | -0,224 | -0,288 | -0,3479 |
| 42 | 0,3450 | 0,278 | 0,214 | 0,110 | -0,115 | -0,218 | -0,280 | -0,3394 |
| 44 | 0,3346 | 0,271 | 0,209 | 0,108 | -0,112 | -0,213 | -0,273 | -0,3289 |
| 46 | 0,3286 | 0,264 | 0,204 | 0,105 | -0,109 | -0,208 | -0,266 | -0,3219 |
| 48 | 0,3210 | 0,258 | 0,199 | 0,103 | -0,106 | -0,203 | -0,260 | -0,3136 |
| 50 | 0,3136 | 0,253 | 0,195 | 0,101 | -0,104 | -0,198 | -0,254 | -0,3073 |
| 52 | 0,3067 | 0,247 | 0,191 | 0,099 | -0,102 | -0,194 | -0,249 | -0,3019 |
| 54 | 0,3012 | 0,243 | 0,187 | 0,097 | -0,100 | -0,190 | -0,244 | -0,3939 |
| 56 | 0,2953 | 0,238 | 0,184 | 0,096 | -0,098 | -0,186 | -0,239 | -0,2887 |
| 58 | 0,2895 | 0,233 | 0,181 | 0,094 | -0,096 | -0,183 | -0,234 | -0,2840 |
| 60 | 0,2839 | 0,229 | 0,177 | 0,092 | -0,094 | -0,179 | -0,230 | -0,2788 |
| 62 | 0,2791 | 0,225 | 0,174 | 0,091 | -0,092 | -0,176 | -0,226 | -0,2735 |
| 64 | 0,2743 | 0,221 | 0,171 | 0,089 | -0,091 | -0,173 | -0,222 | -0,2687 |
| 66 | 0,2697 | 0,218 | 0,169 | 0,088 | -0,089 | -0,170 | -0,218 | -0,2647 |
| 68 | 0,2656 | 0,214 | 0,166 | 0,087 | -0,088 | -0,167 | -0,215 | -0,2612 |
| 70 | 0,2618 | 0,211 | 0,164 | 0,085 | -0,086 | -0,165 | -0,211 | -0,2573 |
| 72 | 0,2573 | 0,208 | 0,161 | 0,084 | -0,085 | -0,162 | -0,208 | -0,2530 |
| 74 | 0,2542 | 0,205 | 0,159 | 0,083 | -0,084 | -0,160 | -0,205 | -0,2495 |
| 76 | 0,2504 | 0,202 | 0,157 | 0,082 | -0,083 | -0,158 | -0,202 | -0,2456 |
| 78 | 0,2466 | 0,199 | 0,155 | 0,081 | -0,081 | -0,155 | -0,199 | -0,2427 |

**Table 8** *(continued)*

| *n* | *p* = 1 - α/2 | | | | *p* = α/2 | | | |
|---|---|---|---|---|---|---|---|---|
| 80 | 0,2438 | 0,197 | 0,153 | 0,080 | -0,080 | -0,153 | -0,197 | -0,2391 |
| 85 | 0,2352 | 0,190 | 0,148 | 0,077 | -0,078 | -0,148 | -0,190 | -0,2326 |
| 90 | 0,2286 | 0,185 | 0,143 | 0,075 | -0,075 | -0,144 | -0,184 | -0,2260 |
| 95 | 0,2218 | 0,179 | 0,139 | 0,073 | -0,073 | -0,139 | -0,179 | -0,2197 |
| 100 | 0,2162 | 0,175 | 0,136 | 0,071 | -0,071 | -0,136 | -0,174 | -0,2132 |
| | 0,975 | 0,95 | 0,9 | 0,75 | 0,25 | 0,1 | 0,05 | 0,025 |
| 110 | 0,2056 | 0,166 | 0,129 | 0,067 | -0,067 | -0,129 | -0,165 | -0,2027 |
| 120 | 0,1962 | 0,159 | 0,123 | 0,064 | -0,064 | -0,123 | -0,158 | -0,1946 |

## 8.4 Confidence interval for the value *x* of the attribute *X* at a given value *G*(*x*) of the distribution function

### 8.4.1 Method for all samples

The confidence interval for *x* at a given *G*(*x*) can be calculated by solving the transcendent equations:

$$G_{un;z}\left(x = x_{ob;z}\right) = G_{ob;z}\left(x = x_{un;z}\right) = G \tag{28}$$

These equations can be solved by varying *x* in the procedure described in 8.3.1, as a method of successive approximation.

However, in most cases it is less time consuming to read the confidence intervals for *x* at a given value of *G*(*x*) from the Weibull diagram. For this purpose, the limits of the confidence intervals determined according to 8.2, i.e. $G_{ob;z}(x)$ and $G_{un;z}(x)$, shall be calculated for a limited number of *x*-values and plotted into the Weibull diagram. Within the range of the plot in the Weibull diagram, the confidence limits for *x* at a given *G*(*x*) can be read directly.

This procedure becomes very inaccurate for small values *G*(*x*), as in these cases the degree of freedom $f_2$ of the chi-square distribution assumes values smaller than 1. The limiting curves of the confidence interval of the distribution function *G*(*x*) shall then be linearly extrapolated, either graphically or numerically.

Graphical extrapolation allows for the immediate determination of the limits of the confidence interval of *x* from the Weibull diagram.

To determine the confidence interval of a given value $\hat{x}_2$ of the point estimation numerically, a value $\hat{x}_1 > \hat{x}_2$ shall be chosen, and for this value $\hat{x}_1$ the confidence interval of the distribution function $G(x_1)$ shall be calculated according to 8.2, to obtain the confidence limits $G_{ob;z}\left(\hat{x}_1\right)$ and $G_{un;z}\left(\hat{x}_1\right)$.

The chosen value $\hat{x}_1$ shall correspond to approximately the lower limit of the range of measured values of *x*.

Then the limits of the confidence interval of the value $\hat{x}_2$ shall be calculated, using the following equations:

$$x_{2;ob;z} = \hat{x}_1 \left[ \frac{\ln\left(1 - \hat{G}(\hat{x}_2)\right)}{\ln\left(1 - G_{un;z}(\hat{x}_1)\right)} \right]^{\frac{1}{\beta_{ob;z}}} \tag{29}$$

$$x_{2;un;z} = \hat{x}_1 \left[ \frac{\ln\left(1 - \hat{G}(\hat{x}_2)\right)}{\ln\left(1 - G_{ob;z}(\hat{x}_1)\right)} \right]^{\frac{1}{\beta_{un;z}}} \tag{30}$$

### 8.4.2 Method for uncensored samples

For $G \leq 0{,}632$, the following simpler equations can be used.

$$x_{ob;z} = \theta_{ob;z} \left[ \ln \frac{1}{1 - \hat{G}(\hat{x})} \right]^{\frac{1}{\beta_{ob;z}}} \tag{31}$$

$$x_{un;z} = \theta_{un;z} \left[ \ln \frac{1}{1 - \hat{G}(\hat{x})} \right]^{\frac{1}{\beta_{un;z}}} \tag{32}$$

The values of $\theta_{ob;z}$ and $\theta_{un;z}$ shall be calculated according to 8.3.1 or 8.3.2.

This simplified method of calculation results in a more conservative estimate for the confidence limits of $x$, compared with the more exact method of extrapolation as described in 8.4.1.

In samples where $n \geq 20$ and $\beta \geq 5$, and for values of $G < 0{,}1$ the following equations shall be used. These give a better approximation to the exact method described in 8.4.1.

$$x_{ob;z} = \hat{\theta} \left[ \ln \frac{1}{1 - \hat{G}(\hat{x})} \right]^{\frac{1}{\beta_{ob;z}}} \tag{33}$$

$$x_{un;z} = \hat{\theta} \left[ \ln \frac{1}{1 - \hat{G}(\hat{x})} \right]^{\frac{1}{\beta_{un;z}}} \tag{34}$$

# Annex A
(informative)

# Examples

## A.1 Uncensored sample

### A.1.1 Data

**Table A.1 — Results of an experiment to determine breakage stress**

| Specimen number | Breakage stress, N/mm$^2$ |
|---|---|
| 1 | 41,26 |
| 2 | 42,54 |
| 3 | 44,31 |
| 4 | 44,43 |
| 5 | 44,67 |
| 6 | 45,02 |
| 7 | 45,37 |
| 8 | 46,08 |
| 9 | 46,08 |
| 10 | 46,55 |
| 11 | 47,86 |
| 12 | 48,21 |
| 13 | 48,21 |
| 14 | 48,31 |
| 15 | 49,63 |
| 16 | 50,34 |
| 17 | 50,43 |
| 18 | 50,69 |
| 19 | 50,78 |
| 20 | 51,05 |

**Table A.1** *(continued)*

| Specimen number | Breakage stress, N/mm$^2$ |
|---|---|
| 21 | 51,05 |
| 22 | 51,05 |
| 23 | 51,76 |
| 24 | 53,17 |

## A.1.2 Statistical evaluation

### A.1.2.1 Point estimations

The method is described in 6.2

From Table 3, for $n$ = 24, $\kappa_n$ = 1,4975

$s$ = int(0,84 x 24) = 20

Hence $\hat{\beta}$ = 18,67 and $\hat{\theta}$ = 49,26 N/mm$^2$.

### A.1.2.2 Estimation of confidence intervals

For the 95 % confidence intervals, 1 - $\alpha/2$ = 0,975 and $\alpha/2$ = 0,025

a) The method for determining the confidence interval for the shape parameter is given in 8.1

From Table 4, by linear interpolation, $f_1/n$ = 2.918, so $f_1$ = 70,03.

From Table 5, $\chi^2_{70,03;0,975}$ = 95,05 and $\chi^2_{70,03;0,025}$ = 48,78.

Hence, $\beta_{ob;z}$ = 25,34 and $\beta_{un;z}$ = 13,01

b) The method for determining the confidence intervals for ***G*(*x*)** is given in 8.2 and the results of the computations are given in Table A.2.

**Table A.2 — Results of the computations according to 8.2**

| $G(x)$ % | $\hat{x}$ N/mm² | $y$ | $\nu$ | $f_2$ | $H(f_2)$ | $\gamma$ | $\chi^2_{f_2;0,975}$ | $\chi^2_{f_2;0,025}$ | $G_{ob;z}$ % | $G_{un;z}$ % |
|---|---|---|---|---|---|---|---|---|---|---|
| 99 | 53,46 | -1,5276 | 0,08670 | 24,054 | 0,04215 | 4,8054 | 39,433 | 12,440 | 99,96 | 91,67 |
| 95 | 52,24 | -1,0966 | 0,06243 | 33,026 | 0,03058 | 3,0869 | 50,757 | 19,067 | 99,13 | 83,17 |
| 80 | 50,53 | -0,4752 | 0,04608 | 44,395 | 0,02269 | 1,6452 | 64,679 | 27,887 | 90,90 | 64,42 |
| 63,21 | 49,26 | 0 | 0,04838 | 42,331 | 0,02381 | 1,0241 | 62,179 | 26,259 | 77,78 | 47,02 |
| 10 | 43,67 | 2,2488 | 0,2343 | 9,4985 | 0,1090 | 0,1177 | 19,751 | 2,973 | 21,71 | 3,62 |
| 1 | 38,5 | 4,6013 | 0,7380 | 3,6005 | 0,3027 | 0,01359 | 10,426 | 0,377 | 3,86 | 0,14 |

Figure 1 shows the values of $G(x)$, $G_{ob;z}$ and $G_{un;z}$ from Table 10 plotted onto Weibull paper against $\hat{x}$ from Table A.2, together with strength results data from Table A.1.

c) The method for determining the confidence intervals of the scale parameter is given in 8.3.

Using the method of 8.3.1, Table A.3 shows the results of successive iterations to determine $\theta_{ob;z}$ and $\theta_{un;z}$.

**Table A.3 — Results of successive iterations according to 8.3.1**

| Iteration number | $\theta_{ob;z}$ | $\theta_{un;z}$ |
|---|---|---|
| 0 | 49,26 | 49,26 |
| 1 | 50,47 | 48,19 |
| 2 | 50,44 | 48,08 |
| 3 | 50,44 | 48,06 |

After 3 iterations, the difference is sufficiently small to stop the iteration.

Hence $\theta_{ob;z}$ = 50,44 N/mm² and $\theta_{un;z}$ = 48,06 N/mm².

However, since it is an uncensored sample, the simplified procedure in 8.3.2 can also be used.

From Table 8, $T_{24;0,025}$ = -0,4669 and $T_{24;0,025}$ = 0,4719.

Hence $\theta^*_{ob;z}$ = 50,51 N/mm² and $\theta^*_{un;z}$ = 48,03 N/mm².

d) The 95 % confidence intervals for $x$ when $G$ = 0,1 % can be determined either graphically from Figure A.1, or numerically by the method in 8.4.1.

The graphical extrapolation in Figure A.1 gives

$x_{ob;z}$ = 38,0 N/mm$^2$ $\hat{x}$ = 34,0 N/mm$^2$ $x_{un;z}$ = 30,1 N/mm$^2$

For the numerical method, it is convenient to assume $\hat{x}_1$ = 38,50 N/mm$^2$, corresponding to $\hat{G}$ = 1 %. From Table A.2, the calculations have already been performed to obtain:

$$G_{ob;z}(\hat{x}_1) = 3{,}86 \text{ \% and } G_{un;z}(\hat{x}_1) = 0{,}14 \text{ \%.}$$

Hence $x_{2;ob;z}$ = 38,00 N/mm$^2$ and $x_{2;un;z}$ = 29,03 N/mm$^2$.

There is good correspondence between the graphical method and the numerical method.

For an uncensored sample, the simplified method in 8.4.2 can be used.

Since $n > 20$, $G < 0{,}1$ and from B.1.2.1, $\hat{\beta} > 5$, the value of $\hat{\theta}$ can be used in the equations in 8.4.2.

Hence $x_{ob;z}$ = 37,51 N/mm$^2$ and $x_{un;z}$ = 28,97 N/mm$^2$.

This also gives good correspondence with the graphical method and the full numerical method.

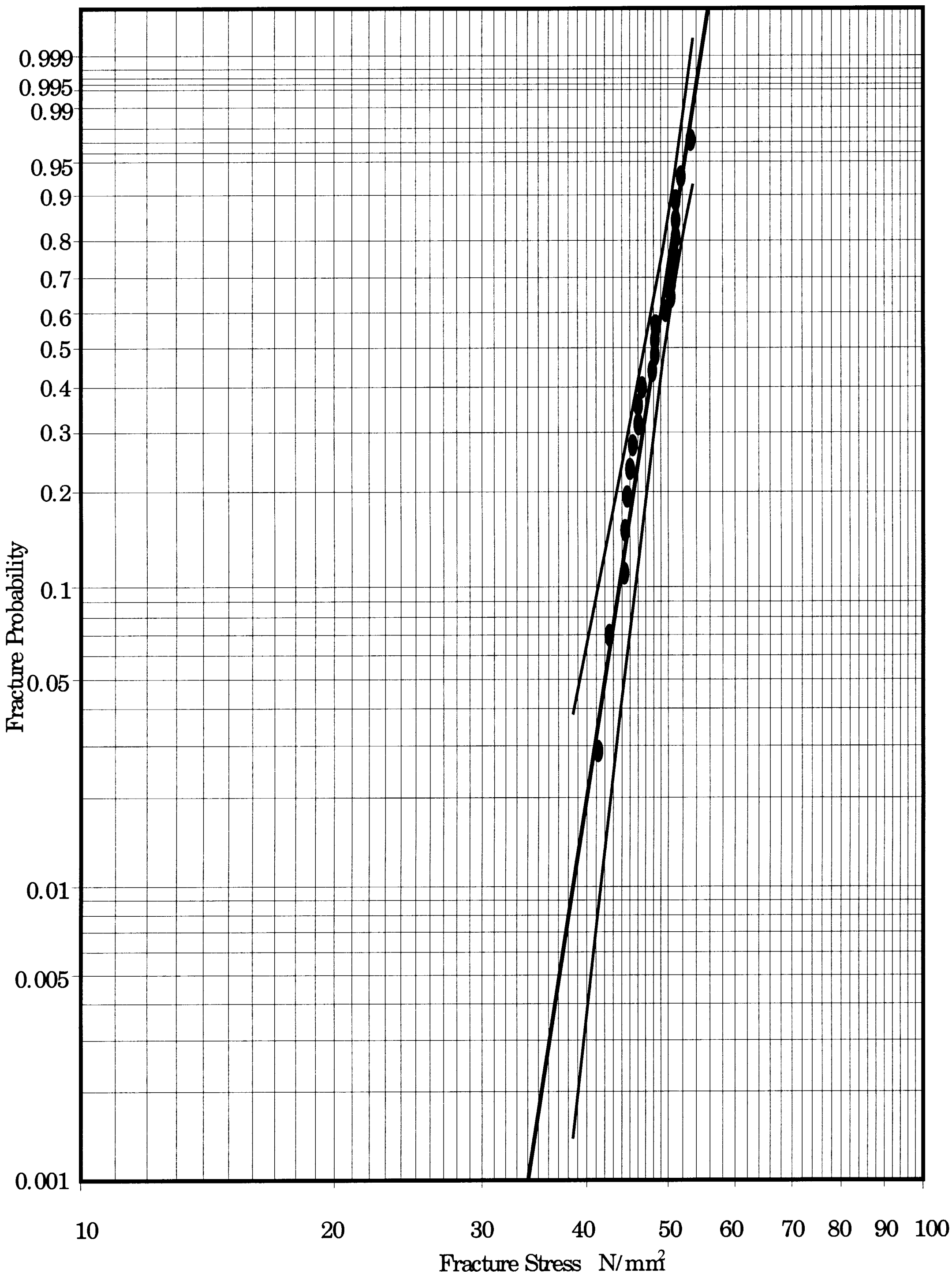



**Figure A.1 — Evaluation of the sample in Table A.1**

## A.2 Censored sample

### A.2.1 Data

The same sample as given in Table A.1 is used for this example, but it is assumed that the specimens could not be stressed to more than 50 $N/mm^2$. The data thus take the form shown in Table A.4.

**Table A.4 — Results of an experiment to determine breakage stress**

| Specimen number | Breakage stress, N/mm$^2$ |
|---|---|
| 1 | 41,26 |
| 2 | 42,54 |
| 3 | 44,31 |
| 4 | 44,43 |
| 5 | 44,67 |
| 6 | 45,02 |
| 7 | 45,37 |
| 8 | 46,08 |
| 9 | 46,08 |
| 10 | 46,55 |
| 11 | 47,86 |
| 12 | 48,21 |
| 13 | 48,21 |
| 14 | 48,31 |
| 15 | 49,63 |
| 16 | > 50 |
| 17 | > 50 |
| 18 | > 50 |
| 19 | > 50 |
| 20 | > 50 |
| 21 | > 50 |
| 22 | > 50 |
| 23 | > 50 |
| 24 | > 50 |

The degree of censoring is given by the values $n = 24$, $r = 15$ and $r/n = 0,625$.

## A.2.2 Statistical evaluation

### A.2.2.1 Point estimations

The method is described in 6.1

For $n = 24$ and $r/n = 0,625$, from Table 1, $\kappa_{r;n} = 0,7271$ and from Table 2, $C_{r;n} = -0,0937$

Hence $\hat{\beta} = 14,67$ and $\hat{\theta} = 49,95$ N/mm$^2$.

### A.2.2.2 Estimation of confidence intervals

For the 95 % confidence intervals, $1 - \alpha/2 = 0,975$ and $\alpha/2 = 0,025$

a) The method for determining the confidence interval for the shape parameter is given in 8.1.

From Table 4, by linear interpolation, $f_1/n = 1,411$, so $f_1 = 33,86$.

From Table 5, $\chi^2_{33,86;0,975} = 51,80$ and $\chi^2_{33,86;0,025} = 19,69$.

Hence, $\beta_{ob;z} = 22,44$ and $\beta_{un;z} = 8,53$

b) The method for determining the confidence intervals for $G(x)$ is given in 8.2.

From Table 6, $B = 0,05951$, $C = 0,02062$ and $A = 0,0781$

The results of the computations are given in Table A.5.

**Table A.5 — Results of the computations according to 8.2**

| $G(x)$ % | $\hat{x}$ N/mm$^2$ | $y$ | $v$ | $f_2$ | $H(f_2)$ | $\gamma$ | $\chi^2_{f_2;0,975}$ | $\chi^2_{f_2;0,025}$ | $G_{ob;z}$ % | $G_{un;z}$ % |
|---|---|---|---|---|---|---|---|---|---|---|
| 99 | 55,43 | -1,5271 | 0,2799 | 8,1008 | 0,1285 | 5,2362 | 13,215 | 2,232 | 99,98 | 76,37 |
| 95 | 53,83 | -1,0966 | 0,1950 | 11,225 | 0,0917 | 3,2841 | 22,239 | 3,948 | 99,85 | 68,50 |
| 80 | 51,60 | -0,4768 | 0,1113 | 18,855 | 0,0540 | 1,7002 | 32,659 | 8,809 | 94,74 | 54,81 |
| 63.21 | 49,95 | 0 | 0,0781 | 26,595 | 0,0381 | 1,0388 | 42,680 | 14,278 | 81,12 | 42,75 |
| 10 | 42,85 | 2,2492 | 0,2864 | 7,9377 | 0,1312 | 0,1203 | 17,440 | 2,149 | 23,23 | 3,20 |
| 3 | 39,37 | 3,4917 | 0,6596 | 3,9331 | 0,2753 | 0,0401 | 11,023 | 0,4661 | 10,63 | 0,474 |
| 2 | 38,28 | 3,9036 | 0,8239 | 3.3067 | 0,3318 | 0,0281 | 9,899 | 0,2982 | 8,07 | 0,253 |
| 1 | 36,50 | 4,6021 | 1,1487 | 2,5804 | 0,4348 | 0,0155 | 8,521 | 0,147 | 4,99 | 0,088 |

Figure 2 shows the values of $G(x)$, $G_{ob;z}$ and $G_{un;z}$ from Table A.5 plotted onto Weibull paper against $\hat{x}$ from Table A.5, together with strength results data from Table A.4.

c) The method for determining the confidence intervals of the scale parameter is given in 8.3.1

Table A.6 shows the results of successive iterations to determine $\theta_{ob;z}$ and $\theta_{un;z}$.

**Table A.6 — Results of successive iterations according to 8.3.1**

| Alteration number | $\theta_{ob;z}$ | $\theta_{un;z}$ |
|---|---|---|
| 0 | 49,95 | 49,95 |
| 1 | 51,98 | 48,24 |
| 2 | 52,54 | 48,30 |
| 3 | 52,75 | 48,30 |
| 4 | 52,84 | 48,30 |
| 5 | 52,88 | 48,30 |

After 5 iterations, the difference is sufficiently small to stop the iteration.

Hence $\theta_{ob;z}$ = 52,88 N/mm$^2$ and $\theta_{un;z}$ = 48,30 N/mm$^2$.

d) The 95 % confidence intervals for $x$ when $G$ = 0,1 % can be determined either graphically from Figure A.1, or numerically by the method in 8.4.1.

The graphical extrapolation in Figure A.2 gives:

$x_{ob;z}$ = 35,8 N/mm$^2$ $\hat{x}$ = 30,2 N/mm$^2$ $x_{un;z}$ = 25,3 N/mm$^2$

For the numerical method, it is convenient to assume $\hat{x}_1$ = 39,37 N/mm$^2$, corresponding to $\hat{G}$ = 3 %. From Table A.5, the calculations have already been performed to obtain:

$$G_{ob;z}(\hat{x}_1) = 10{,}63 \text{ \% and } G_{un;z}(\hat{x}_1) = 0{,}474 \text{ \%.}$$

Hence $x_{2;ob;z}$ = 36,73 N/mm$^2$ and $x_{2;un;z}$ = 22,63 N/mm$^2$.

There is reasonable correspondence between the graphical method and the numerical method.

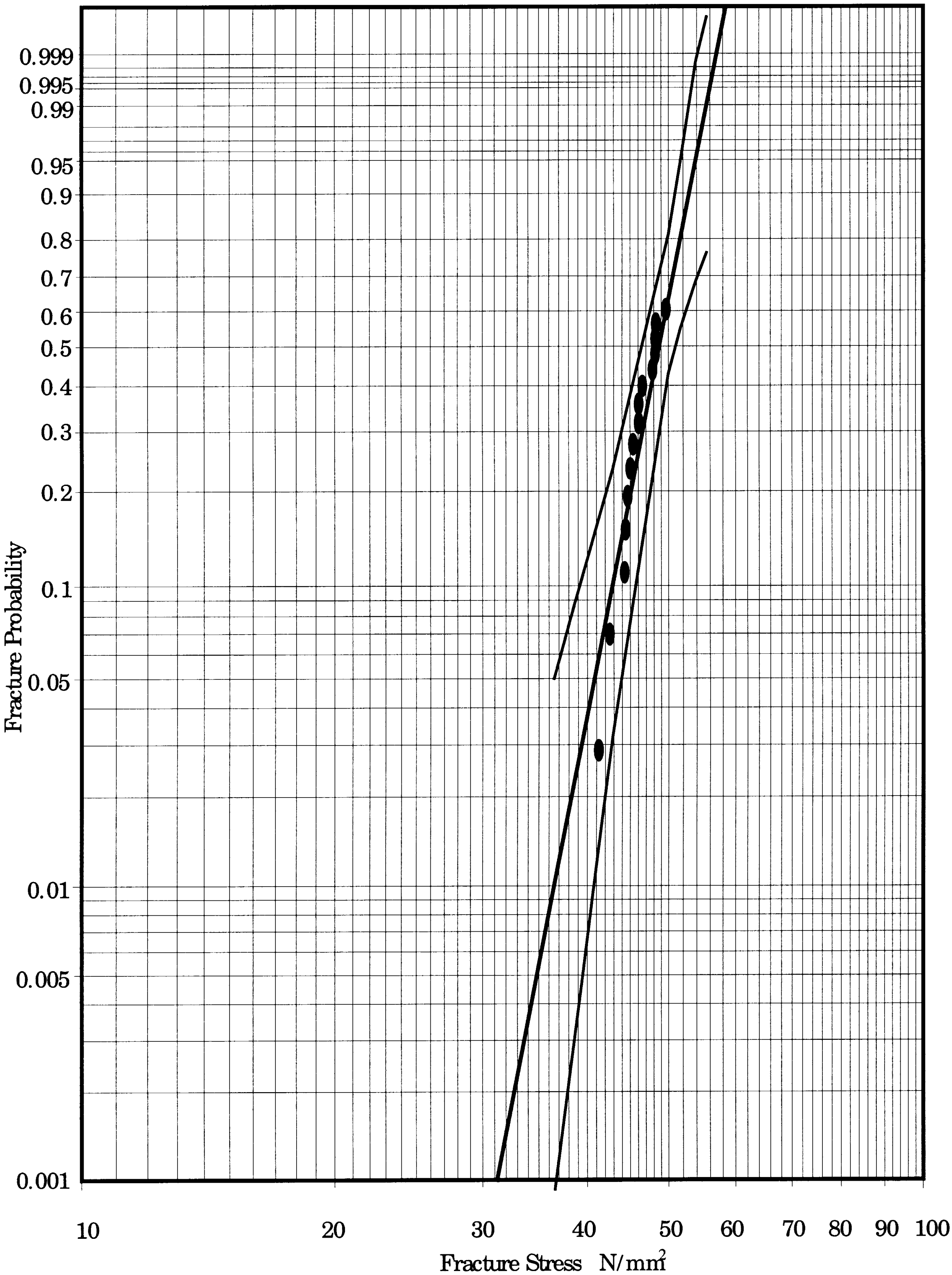
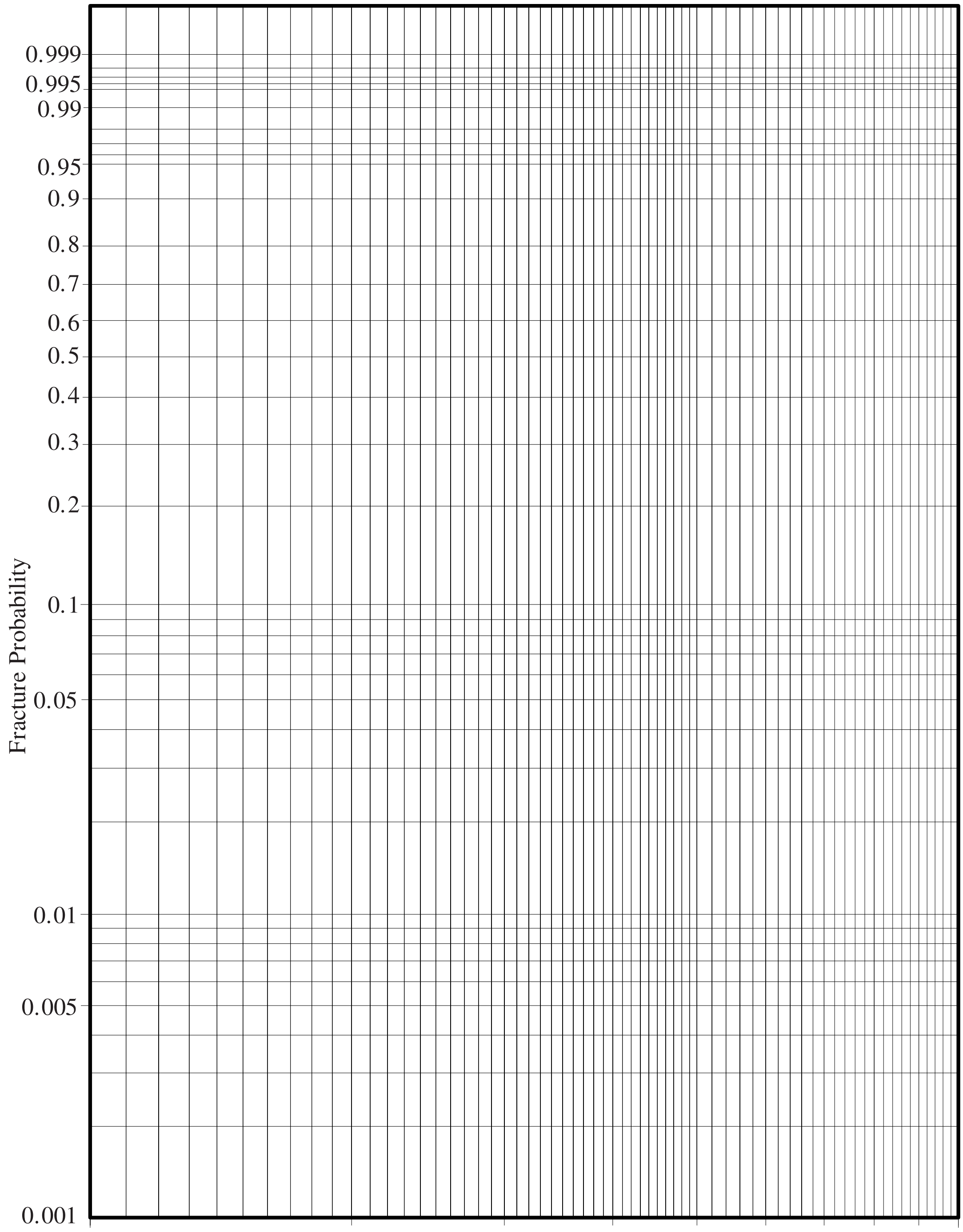



**Figure A.1 — Evaluation of the sample from Table A.4**

# Annex B
(informative)

## Weibull graph




**B.1 – Weibull graph**

# Bibliography

[1] Bain L. J.: Statistical Analysis of Reliability and Life Testing Models; Theory and Methods: Marcel Dekker Publishing Comp.: 1978.

[2] IEC 56 (CO) 162, *Procedures for goodness-of-fit tests, confidence intervals and lower confidence limits for Weibull distributed data.*

[3] DIN 55303-7, *Statistische Auswertung von Daten - Teil 7: Schätz- und Testverfahren bei zweiparametriger Weibull-Verteilung.*

# BSI — British Standards Institution

BSI is the independent national body responsible for preparing British Standards. It presents the UK view on standards in Europe and at the international level. It is incorporated by Royal Charter.

## Revisions

British Standards are updated by amendment or revision. Users of British Standards should make sure that they possess the latest amendments or editions.

It is the constant aim of BSI to improve the quality of our products and services. We would be grateful if anyone finding an inaccuracy or ambiguity while using this British Standard would inform the Secretary of the technical committee responsible, the identity of which can be found on the inside front cover. Tel: +44 (0)20 8996 9000. Fax: +44 (0)20 8996 7400.

BSI offers members an individual updating service called PLUS which ensures that subscribers automatically receive the latest editions of standards.

## Buying standards

Orders for all BSI, international and foreign standards publications should be addressed to Customer Services. Tel: +44 (0)20 8996 9001. Fax: +44 (0)20 8996 7001. Email: orders@bsi-global.com. Standards are also available from the BSI website at http://www.bsi-global.com.

In response to orders for international standards, it is BSI policy to supply the BSI implementation of those that have been published as British Standards, unless otherwise requested.

## Information on standards

BSI provides a wide range of information on national, European and international standards through its Library and its Technical Help to Exporters Service. Various BSI electronic information services are also available which give details on all its products and services. Contact the Information Centre. Tel: +44 (0)20 8996 7111. Fax: +44 (0)20 8996 7048. Email: info@bsi-global.com.

Subscribing members of BSI are kept up to date with standards developments and receive substantial discounts on the purchase price of standards. For details of these and other benefits contact Membership Administration. Tel: +44 (0)20 8996 7002. Fax: +44 (0)20 8996 7001. Email: membership@bsi-global.com.

Information regarding online access to British Standards via British Standards Online can be found at http://www.bsi-global.com/bsonline.

Further information about BSI is available on the BSI website at http://www.bsi-global.com.

## Copyright



Details and advice can be obtained from the Copyright & Licensing Manager. Tel: +44 (0)20 8996 7070. Fax: +44 (0)20 8996 7553. Email: copyright@bsi-global.com.

BSI
389 Chiswick High Road
London
W4 4AL