BRITISH STANDARD

BS EN 1999-1-5:2007

Incorporating corrigendum November 2009

Eurocode 9 — Design of aluminium structures —

Part 1-5: Shell structures

ICS 13.220.50; 91.010.30; 91.080.10

British Standards

National foreword

This British Standard is the UK implementation of EN 1999-1-5:2007, incorporating corrigendum November 2009.

The start and finish of text introduced or altered by corrigendum is indicated in the text by tags. Text altered by CEN corrigendum November 2009 is indicated in the text by AC1 AC1.

The structural Eurocodes are divided into packages by grouping Eurocodes for each of the main materials, concrete, steel, composite concrete and steel, timber, masonry and aluminium, this is to enable a common date of withdrawal (DOW) for all the relevant parts that are needed for a particular design. The conflicting national standards will be withdrawn at the end of the coexistence period, after all the EN Eurocodes of a package are available.

Following publication of the EN, there is a period allowed for national calibration during which the national annex is issued, followed by a further coexistence period of a maximum 3 years. During the coexistence period Member States will be encouraged to adapt their national provisions to withdraw conflicting national rules before the end of the coexistent period in March 2010.

At the end of this coexistence period, the national standard(s) will be withdrawn.

In the UK, the following national standards are superseded by the Eurocode 9 series. These standards will be withdrawn on a date to be announced.

Eurocode	Superseded British Standards
EN 1999-1-1	BS 8118-2:1991 Structural use of aluminium. Specification for materials, workmanship and protection (superseded).
	DD ENV 1999-1-1:2000 Eurocode 9. Design of aluminium structures. General rules. General rules and rules for buildings (superseded).
	BS 8118-1:1991 Structural use of aluminium. Code of practice for design (partially superseded).
EN 1999-1-2	DD ENV 1999-1-2 Design of aluminium structures. General rules. Structural fire design (superseded).
EN 1999-1-3	DD ENV 1999-2:2000 Eurocode 9. Design of aluminium structures. Structures susceptible to fatigue (superseded).
	BS 8118-1:1991 Structural use of aluminium. Code of practice for design (partially superseded).
EN 1999-1-4	BS 8118-1:1991 Structural use of aluminium. Code of practice for design (partially superseded).
EN 1999-1-5	None

This British Standard was published under the authority of the Standards Policy and Strategy Committee on 30 March 2007

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Amendments/corrigenda issued since publication

Date	Comments
31 August 2010	Implementation of CEN corrigendum November 2009

The UK participation in its preparation was entrusted by Technical Committee B/525, Building and civil engineering structures, to Subcommittee B/525/9, Structural use of aluminium.

A list of organizations represented on this subcommittee can be obtained on request to its secretary.

Where a normative part of this EN allows for a choice to be made at the national level, the range and possible choice will be given in the normative text, and a note will qualify it as a Nationally Determined Parameter (NDP). NDPs can be a specific value for a factor, a specific level or class, a particular method or a particular application rule if several are proposed in the EN.

To enable EN 1999 to be used in the UK, the NDPs will be published in a National Annex, which will be made available by BSI in due course, after public consultation has taken place.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Compliance with a British Standard cannot confer immunity from legal obligations.

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EUROPEAN STANDARD

NORME EUROPÉENNE

EUROPÄISCHE NORM

EN 1999-1-5

February 2007

ICS 13.220.50; 91.010.30; 91.080.10

Supersedes ENV 1999-1-1:1998, ENV 1999-1-2:1998, ENV 1999-2:1998

Incorporating corrigendum November 2009

English Version

Eurocode 9 - Design of aluminium structures - Part 1-5: Shell structures

Eurocode 9 - Calcul des structures en aluminium - Partie 1-5 : Coques

Eurocode 9 - Bemessung und Konstruktion von Aluminiumtragwerken - Teil 1-5: Schalen

This European Standard was approved by CEN on 11 October 2006.

CEN members are bound to comply with the CEN/CENELEC Internal Regulations which stipulate the conditions for giving this European Standard the status of a national standard without any alteration. Up-to-date lists and bibliographical references concerning such national standards may be obtained on application to the CEN Management Centre or to any CEN member.

This European Standard exists in three official versions (English, French, German). A version in any other language made by translation under the responsibility of a CEN member into its own language and notified to the CEN Management Centre has the same status as the official versions.

CEN members are the national standards bodies of Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland and United Kingdom.

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COMITÉ EUROPÉEN DE NORMALISATION
EUROPÄISCHES KOMITEE FÜR NORMUNG

Management Centre: rue de Stassart, 36 B-1050 Brussels

Ref. No. EN 1999-1-5:2007: E

Content

Foreword

This European Standard (EN 1999-1-5:2007) has been prepared by Technical Committee CEN/TC250 « Structural Eurocodes », the secretariat of which is held by BSI.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by August 2007, and conflicting national standards shall be withdrawn at the latest by March 2010.

This European Standard supersedes ENV 1999-1-1:1998, ENV 1999-1-2:1998 and ENV 1999-2:1998.

CEN/TC 250 is responsible for all Structural Eurocodes.

According to the CEN/CENELEC Internal Regulations, the national standards organizations of the following countries are bound to implement this European Standard:

Austria, Bulgaria, Belgium, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italia, Latvia, Lithuania, Luxemburg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland and the United Kingdom

Background of the Eurocode programme

In 1975, the Commission of the European Community decided on an action programme in the field of construction, based on article 95 of the Treaty. The objective of the programme was the elimination of technical obstacles to trade and the harmonisation of technical specifications.

Within this action programme, the Commission took the initiative to establish a set of harmonised technical rules for the design of construction works, which, in a first stage, would serve as an alternative to the national rules in force in the Member States and, ultimately, would replace them.

For fifteen years, the Commission, with the help of a Steering Committee with Representatives of Member States, conducted the development of the Eurocodes programme, which led to the first generation of European codes in the 1980s.

In 1989, the Commission and the Member States of the EU and EFTA decided, on the basis of an agreement[1] between the Commission and CEN, to transfer the preparation and the publication of the Eurocodes to the CEN through a series of Mandates, in order to provide them with a future status of European Standard (EN). This links de facto the Eurocodes with the provisions of all the Council's Directives and/or Commission's Decisions dealing with European standards (e.g. the Council Directive 89/106/EEC on construction products - CPD - and Council Directives 93/37/EEC, 92/50/EEC and 89/440/EEC on public works and services and equivalent EFTA Directives initiated in pursuit of setting up the internal market).

The Structural Eurocode programme comprises the following standards generally consisting of a number of Parts:

EN 1990	Eurocode 0:	Basis of Structural Design
EN 1991	Eurocode 1:	Actions on structures
EN 1992	Eurocode 2:	Design of concrete structures
EN 1993	Eurocode 3:	Design of steel structures
EN 1994	Eurocode 4:	Design of composite steel and concrete structures
EN 1995	Eurocode 5:	Design of timber structures
EN 1996	Eurocode 6:	Design of masonry structures
EN 1997	Eurocode 7:	Geotechnical design
EN 1998	Eurocode 8:	Design of structures for earthquake resistance
EN 1999	Eurocode 9:	Design of aluminium structures

[1] Agreement between the Commission of the European Communities and the European Committee for Standardisation (CEN) concerning the work on EUROCODES for the design of building and civil engineering works (BC/CEN/03/89).

Eurocode standards recognise the responsibility of regulatory authorities in each Member State and have safeguarded their right to determine values related to regulatory safety matters at national level where these continue to vary from State to State.

Status and field of application of Eurocodes

The Member States of the EU and EFTA recognise that Eurocodes serve as reference documents for the following purposes:

- as a means to prove compliance of building and civil engineering works with the essential requirements of Council Directive 89/106/EEC, particularly Essential Requirement No.1 – Mechanical resistance and stability, and Essential Requirement No 2 – Safety in case of fire
- as a basis for specifying contracts for the execution of construction works and related engineering services
- as a framework for drawing up harmonised technical specifications for construction products (En's and ETA's)

The Eurocodes, as far as they concern the construction works themselves, have a direct relationship with the Interpretative Documents[2] referred to in Article 12 of the CPD, although they are of a different nature from harmonised product standards[3]. Therefore, technical aspects arising from the Eurocodes work need to be adequately considered by CEN Technical Committees and/or EOTA Working Groups working on product standards with a view to achieving full compatibility of these technical specifications with the Eurocodes.

The Eurocode standards provide common structural design rules for everyday use for the design of whole structures and component products of both a traditional and an innovative nature. Unusual forms of construction or design conditions are not specifically covered and additional expert consideration will be required by the designer in such cases.

National standards implementing Eurocodes

The National Standards implementing Eurocodes will comprise the full text of the Eurocode (including any annexes), as published by CEN, which may be preceded by a National title page and National foreword, and may be followed by a National annex [informative].

The National Annex (informative) may only contain information on those parameters which are left open in the Eurocode for national choice, known as Nationally Determined Parameters, to be used for the design of buildings and civil engineering works to be constructed in the country concerned, i.e. :

– values for partial factors and/or classes where alternatives are given in the Eurocode;
– values to be used where a symbol only is given in the Eurocode;
– geographical and climatic data specific to the Member State, e.g. snow map;
– the procedure to be used where alternative procedures are given in the Eurocode;
– references to non-contradictory complementary information to assist the user to apply the Eurocode.

Links between Eurocodes and harmonised technical specifications (EN's and ETA's) for products

There is a need for consistency between the harmonised technical specifications for construction products and the technical rules for works[4]. Furthermore, all the information accompanying the CE Marking of the construction products, which refer to Eurocodes, shall clearly mention which Nationally Determined Parameters have been taken into account.

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[2] According to Art. 3.3 of the CPD, the essential requirements (ERs) shall be given concrete form in interpretative documents for the creation of the necessary links between the essential requirements and the mandates for harmonised ENs and ETAGs/ETAs.

[3] According to Art. 12 of the CPD the interpretative documents shall :

a) give concrete form to the essential requirements by harmonising the terminology and the technical bases and indicating classes or levels for each requirement where necessary ;
b) indicate methods of correlating these classes or levels of requirement with the technical specifications, e.g. methods of calculation and of proof, technical rules for project design, etc. ;
c) serve as a reference for the establishment of harmonised standards and guidelines for European technical approvals.

The Eurocodes, *de facto*, play a similar role in the field of the ER 1 and a part of ER 2.

[4] see Art.3.3 and Art.12 of the CPD, as well as clauses 4.2, 4.3.1, 4.3.2 and 5.2 of ID 1.

National Annex for EN 1999-1-5

This European Standard gives alternative procedures, values and recommendations for classes with notes indicating where national choices may have to be made. Therefore the National Standard implementing EN 1999-1-5 should have a National Annex containing all Nationally Determined Parameters to be used for the design of aluminium shell structures to be constructed in the relevant country.

National choice is allowed in EN 1999-1-5 through clauses:

- 2.1 (3)
- 2.1 (4)

1 General

1.1 Scope

1.1.1 Scope of EN 1999

(1)P EN 1999 applies to the design of buildings and civil engineering and structural works in aluminium. It complies with the principles and requirements for the safety and serviceability of structures, the basis of their design and verification that are given in EN 1990 – Basis of structural design.

(2)P EN 1999 is only concerned with requirements for resistance, serviceability, durability and fire resistance of aluminium structures. Other requirements, e.g. concerning thermal or sound insulation, are not considered.

(3) EN 1999 is intended to be used in conjunction with:

- EN 1990 Basis of structural design
- EN 1991 Actions on structures
- European Standards for construction products relevant for aluminium structures
- EN 1090-1 Execution of steel structures and aluminium structures – Part 1: Requirements for conformity assessment of structural components[5]
- EN 1090-3 Execution of steel structures and aluminium structures – Part 3: Technical requirements for aluminium structures[5]

(4) EN 1999 is subdivided in five parts:

EN 1999-1-1 Design of Aluminium Structures: General structural rules.

EN 1999-1-2 Design of Aluminium Structures: Structural fire design.

EN 1999-1-3 Design of Aluminium Structures: Structures susceptible to fatigue.

EN 1999-1-4 Design of Aluminium Structures: Cold-formed structural sheeting.

EN 1999-1-5 Design of Aluminium Structures: Shell structures.

1.1.2 Scope of EN 1999-1-5

(1)P EN 1999-1-5 applies to the structural design of aluminium structures, stiffened and unstiffened, that have the form of a shell of revolution or of a round panel in monocoque structures.

(2) The relevant parts of EN 1999 should be followed for specific application rules for structural design.

(3) Supplementary information for certain types of shells are given in EN 1993-1-6 and the relevant application parts which include:

- Part 3-1 for towers and masts;
- Part 3-2 for chimneys;
- Part 4-1 for silos;
- Part 4-2 for tanks;
- Part 4-3 for pipelines.

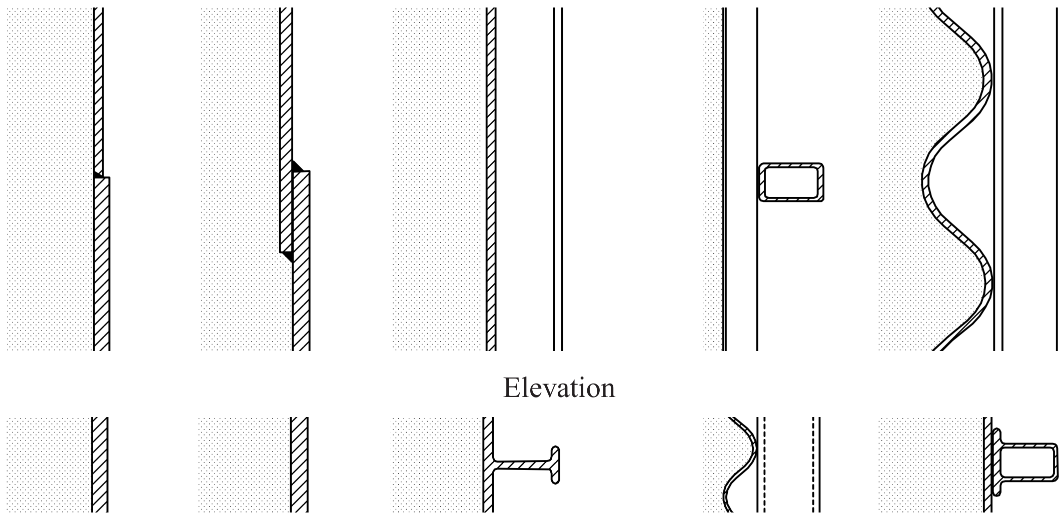
(4) The provisions in EN 1999-1-5 apply to axisymmetric shells (cylinders, cones, spheres) and associated circular or annular plates and beam section rings and stringer stiffeners where they form part of the complete structure.

5 To be published

(5) Single shell panels (cylindrical, conical or spherical) are not explicitly covered by EN 1999-1-5. However, the provisions can be applicable if the appropriate boundary conditions are duly taken into account.

(6) Types of shell walls covered in EN 1999-1-5 can be, see Figure 1.1:

- shell wall constructed from flat rolled sheet, termed ‘isotropic’;
- shell wall with lap joints formed by connecting adjacent plates with overlapping sections, termed ‘lap-jointed;
- shell wall with stiffeners attached to the outside, termed ‘externally stiffened’ irrespective of the spacing of the stiffeners;
- shell wall with the corrugations running up the meridian, termed ‘axially corrugated’;
- shell wall constructed from corrugated sheets with the corrugations running around the shell circumference, termed ‘circumferentially corrugated’.



Isotropic (unstiffened) | lap-jointed | externally stiffened | axially corrugated | circumferentially corrugated

Figure 1.1 - Illustration of cylindrical shell forms

(7) The provisions of EN 1999-1-5 are intended to be applied within the temperature range defined in EN 1999-1-1. The maximum temperature is restricted so that the influence of creep can be neglected. For structures subject to elevated temperatures associated with fire see EN 1999-1-2.

(8) EN 1999-1-5 does not cover the aspects of leakage.

1.2 Normative references

(1) EN 1999-1-5 incorporates by dated or undated reference, provisions from other publications. These normative references are cited at the appropriate places in the text and the publications are listed hereafter. For dated references, subsequent amendments to or revisions of any of these publications apply to this European Standard only if incorporated in it by amendment or revision. For undated references the latest edition of the publication referred to applies (including amendments).

EN 1090-1 Execution of steel structures and aluminium structures – Part 1: Requirements for conformity assessment of structural components[5]

EN 1090-3 Execution of steel structures and aluminium structures – Part 3: Technical requirements for aluminium structures[5]

EN 1990 Basis of structural design

EN 1991 Actions on structures – All parts

EN 1993-1-6 Design of steel structures - Part 1-6: Shell structures

EN 1993-3-2 Design of steel structures - Part 3-2: Chimneys

EN 1993-4-1 Design of steel structures - Part 4-1: Silos

EN 1993-4-2 Design of steel structures - Part 4-2: Tanks

EN 1993-4-3 Design of steel structures - Part 4-3: Pipelines

EN 1999-1-1 Design of aluminium structures - Part 1-1: General rules

EN 1999-1-2 Design of aluminium structures - Part 1-2: Structural fire design

EN 1999-1-3 Design of aluminium structures - Part 1-3: Structures susceptible to fatigue

EN 1999-1-4 Design of aluminium structures - Part 1-4: Cold-formed structural sheeting

1.3 Terms and definitions

(1) Supplementary to EN 1999-1-1, for the purposes of this part, the following definitions apply:

1.3.1 Structural forms and geometry

1.3.1.1
shell
A thin-walled body shaped as a curved surface with the thickness measured normal to the surface being small compared to the dimensions in the other directions. A shell carries its loads mainly by membrane forces. The middle surface may have finite radius of curvature at each point or infinite curvature in one direction, e.g. cylindrical shell.
In EN 1999-1-5, a shell is a structure or a structural component formed from curved sheets or extrusions.

1.3.1.2
shell of revolution
A shell composed of a number of parts, each of which is a complete axisymmetric shell.

1.3.1.3
complete axisymmetric shell
A shell whose form is defined by a meridional generator line rotated around a single axis through 2π radians. The shell can be of any length.

1.3.1.4
shell segment
A part of shell of revolution in the form of a defined shell geometry with a constant wall thickness: a cylinder, conical frustum, spherical frustum, annular plate or other form.

1.3.1.5
shell panel
An incomplete axisymmetric shell: the shell form is defined by a rotation of the generator about the axis through less than 2π radians.

1.3.1.6
middle surface
The surface that lies midway between the inside and outside surfaces of the shell at every point. If the shell is stiffened on only one surface, the reference middle surface is still taken as the middle surface of the curved shell plate. The middle surface is the reference surface for analysis, and can be discontinuous at changes of thickness or shell junctions, leading to eccentricities that are important to the shell response.

1.3.1.7
junction
The point at which two or more shell segments meet: it can include a stiffener or not: the point of attachment of a ring stiffener to the shell may be treated as a junction.

1.3.1.8
stringer stiffener
A local stiffening member that follows the meridian of the shell, representing a generator of the shell of revolution. It is provided to increase the stability, or to assist with the introduction of local loads. It is not intended to provide a primary resistance for bending due to transverse loads.

1.3.1.9
rib
A local member that provides a primary load carrying path for bending down the meridian of the shell, representing a generator of the shell of revolution. It is used to transfer or distribute transverse loads by bending.

1.3.1.10
ring stiffener
A local stiffening member that passes around the circumference of the shell of revolution at a given point on the meridian. It is assumed to have no stiffness in the meridional plane of the shell. It is provided to increase the stability or to introduce axisymmetric local loads acting in the plane of the ring by a state of axisymmetric normal forces. It is not intended to provide primary resistance for bending.

1.3.1.11
base ring.
A structural member that passes around the circumference of the shell of revolution at the base and provides means of attachment of the shell to a foundation or other element. It is needed to ensure that the assumed boundary conditions are achieved in practice.

1.3.2 Special definitions for buckling calculations

1.3.2.1
critical buckling load
The smallest bifurcation or limit load determined assuming the idealised conditions of elastic material behaviour, perfect geometry, perfect load application, perfect support, material isotropy and absence of residual stresses (LBA analysis).

1.3.2.2
critical buckling stress
The nominal membrane stress associated with the elastic critical buckling load.

1.3.2.3
characteristic buckling stress
The nominal membrane stress associated with buckling in the presence of inelastic material behaviour and of geometrical and structural imperfections.

1.3.2.4
design buckling stress
The design value of the buckling stress, obtained by dividing the characteristic buckling stress by the partial factor for resistance.

1.3.2.5
key value of the stress
The value of stress in a non-uniform stress field that is used to characterise the stress magnitude in the buckling limit state assessment.

1.3.2.6
tolerance class
The class of requirements to geometrical tolerances for work execution.

NOTE Geometrical tolerances for work execution are built up from fabrication of components and execution of the components at site.

1.4 Symbols

(1) In addition to the symbols defined in EN 1999-1-1, the following are used.

(2) Coordinate system (see Figure 1.2):

r radial coordinate, normal to the axis of revolution;
x meridional coordinate;
z axial coordinate;
θ circumferential coordinate;
ϕ meridional slope: angle between axis of revolution and normal to the meridian of the shell:

(3) Pressures:

p_n normal to the shell;
p_x meridional surface loading parallel to the shell;
p_θ circumferential surface loading parallel to the shell;

(4) Line forces:

P_n load per unit circumference normal to the shell;
P_x load per unit circumference acting in the meridional direction;
P_θ load per unit circumference acting circumferentially on the shell;

(5) Membrane stress resultants (see Figure 1.3a):

n_x meridional membrane stress resultant;
n_θ circumferential membrane stress resultant;
$n_{x\theta}$ membrane shear stress resultant;

(6) Bending stress resultants (see Figure 1.3b):

m_x meridional bending moment per unit width;
m_θ circumferential bending moment per unit width;
$m_{x\theta}$ twisting shear moment per unit width;
q_{xn} transverse shear force associated with meridional bending;
$q_{\theta n}$ transverse shear force associated with circumferential bending;

(7) Stresses:

σ_x meridional stress;
σ_θ circumferential stress;
σ_{eq} von Mises equivalent stress (can be negative in cyclic loading conditions);
τ, $\tau_{x\theta}$ in-plane shear stress;
τ_{xn}, $\tau_{\theta n}$ meridional, circumferential transverse shear stresses associated with bending;

(8) Displacements:

u meridional displacement;
v circumferential displacement;
w displacement normal to the shell surface,
β_ϕ meridional rotation (see 5.3.3);

(9) Shell dimensions:

- d internal diameter of shell;
- L total length of shell;
- l length of shell segment;
- l_g gauge length for measurement of imperfections;
- $l_{g,\theta}$ gauge length for measurement of imperfections in circumferential direction;
- $l_{g,w}$ gauge length for measurement of imperfections across welds;
- l_R limited length of shell for buckling strength assessment;
- r radius of the middle surface, normal to the axis of revolution;
- t thickness of shell wall;
- t_{max} maximum thickness of shell wall at a joint;
- t_{min} minimum thickness of shell wall at a joint;
- t_{ave} average thickness of shell wall at a joint;
- β apex half angle of cone;

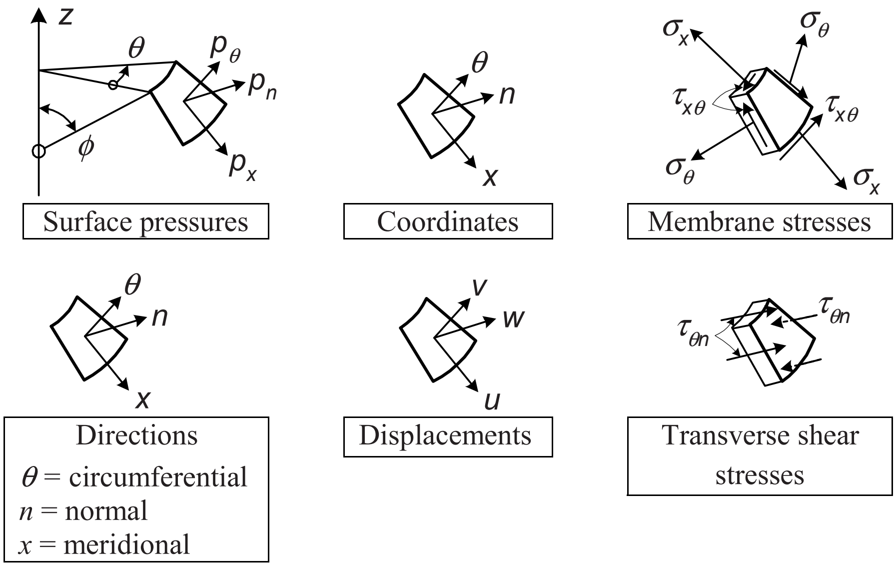
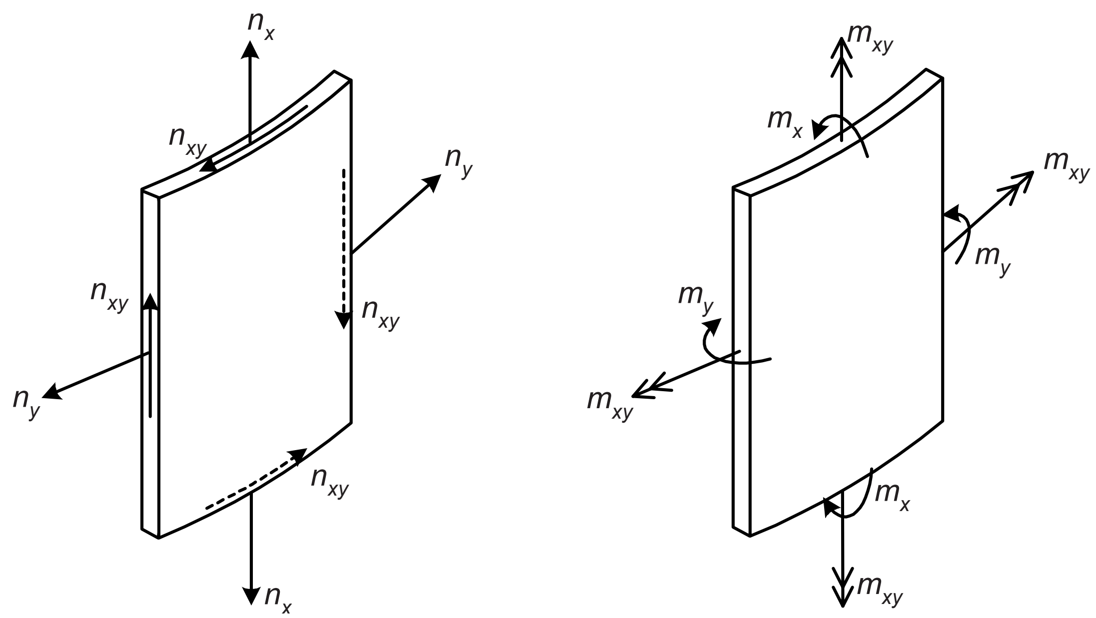


Figure 1.2 - Symbols in shells of revolutions



a) Membrane stress resultants b) Bending stress resultants

Figure 1.3 - Stress resultants in the shell wall (In this figure x is meridional and y is circumferential)

(10) Tolerances (see 6.2.2):

e eccentricity between the middle surfaces of joined plates;

U_e non-intended eccentricity tolerance parameter;

U_r out-of-roundness tolerance parameter;

U_0 initial dent tolerance parameter;

Δw_0 tolerance normal to the shell surface;

(11) Properties of materials:

f_{eq} von Mises equivalent strength;

f_u characteristic value of ultimate tensile strength;

f_o characteristic value of 0,2 % proof strength;

(12) Parameters in strength assessment:

C coefficient in buckling strength assessment;

C_ϕ sheeting stretching stiffness in the axial direction;

C_θ sheeting stretching stiffness in the circumferential direction;

$C_{\phi\theta}$ sheeting stretching stiffness in membrane shear;

D_ϕ sheeting flexural rigidity in the axial direction;

D_θ sheeting flexural rigidity in the circumferential direction;

$D_{\phi\theta}$ sheeting twisting flexural rigidity in twisting;

R calculated resistance (used with subscripts to identify the basis);

R_{pl} plastic reference resistance (defined as a load factor on design loads);

R_{cr} elastic critical buckling load (defined as a load factor on design loads);

k calibration factor for nonlinear analyses;

$k_{(...)}$ power of interaction expressions in buckling strength interaction expressions;

μ alloy hardening parameter in buckling curves for shells;

$a_{(...)}$ imperfection reduction factor in buckling strength assessment;

Δ range of parameter when alternating or cyclic actions are involved;

(13) Design stresses and stress resultants

$\sigma_{x,Ed}$ design values of the buckling-relevant meridional membrane stress (positive when compression);

$\sigma_{\theta,Ed}$ design values of the buckling-relevant circumferential membrane (hoop) stress (positive when compression);

τ_{Ed} design values of the buckling-relevant shear membrane stress;

$n_{x,Ed}$ design values of the buckling-relevant meridional membrane stress resultant (positive when compression);

$n_{\theta,Ed}$ design values of the buckling-relevant circumferential membrane (hoop) stress resultant (positive when compression);

$n_{x\theta,Ed}$ design values of the buckling-relevant shear membrane stress resultant.

(14) Critical buckling stresses and stress resistances:

$\sigma_{x,cr}$ meridional critical buckling stress;

$\sigma_{\theta,cr}$ circumferential critical buckling stress;

τ_{cr} shear critical buckling stress;

$\sigma_{x,Rd}$ meridional design buckling stress resistance;

$\sigma_{\theta,Rd}$ circumferential design buckling stress resistance;

τ_{Rd} shear design buckling stress resistance.

(15) Further symbols are defined where they first occur.

1.5 Sign conventions

(1) In general the sign conventions are the following, except as noted in (2)

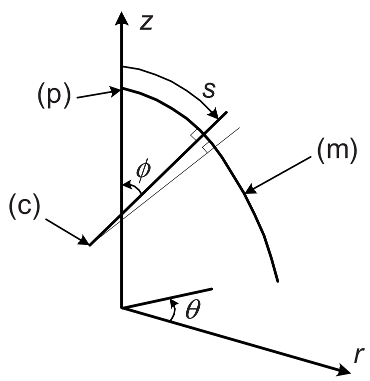
- outward direction positive;
- internal pressure positive;
- outward displacement positive;
- tensile stresses positive;
- shear stresses as shown in Figure 1.2.

(2) For simplicity, for buckling analysis, compressive stresses are treated as positive. For these cases both external pressures and internal pressures are treated as positive.

1.6 Coordinate systems

(1) In general, the convention for the global shell structure axis system is in cylindrical coordinates (see Figure 1.4) as follows:

coordinate along the central axis of a shell of revolution	z
radial coordinate	r
circumferential coordinate	θ



(p) = pole, (m) = shell meridian,
(c) = instantaneous centre of meridional curvature

Figure 1.4 - Coordinate systems for a circular shell

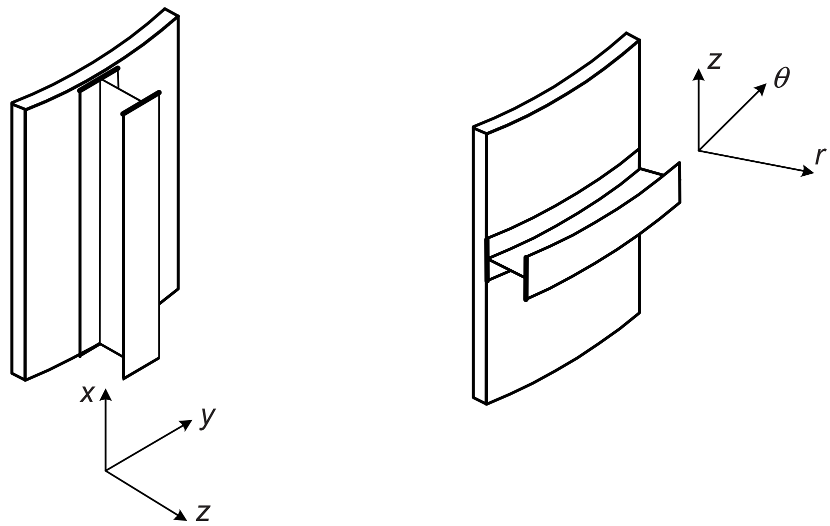
(2) The convention for structural elements attached to the shell wall (see Figure 1.5) is different for meridional and circumferential members.

(3) The convention for meridional straight structural elements (see Figure 1.5(I)) attached to the shell wall is:

meridional coordinate for barrel, hopper and roof attachment	x
strong bending axis (parallel to flanges: axis for meridional bending)	y
weak bending axis (perpendicular to flanges)	z

(4) The convention for circumferential curved structural elements (see Figure 1.5(II)) attached to a shell wall is:

circumferential coordinate axis (curved)	θ
radial axis (axis for bending in the meridional plane)	r
meridional axis (axis for circumferential bending)	z



a) meridional stiffener b) circumferential stiffener

Figure 1.5 - Local coordinate system for meridional and circumferential stiffeners on a shell

2 Basis of design

2.1 General

(1)P The design of shells shall be in accordance with the rules given in EN 1990 and EN 1999-1-1.

(2)P Appropriate partial factors shall be adopted for ultimate limit states and serviceability limit states.

(3)P For verification by calculation at ultimate limit states the partial factor γ_M shall be taken as follows:

- resistance to yielding and instability: γ_{M1}
- resistance of plate in tension to fracture: γ_{M2}
- resistance of joints: see EN 1999-1-1

NOTE Numerical values for γ_{Mi} may be defined in the National Annex. The following numerical values are recommended:

$\gamma_{M1} = 1{,}10$

$\gamma_{M2} = 1{,}25$

(4) For verifications at serviceability limit states the partial factor $\gamma_{M,ser}$ should be used.

NOTE Numerical values for $\gamma_{M,ser}$ may be defined in the National Annex. The following numerical value is recommended:

$\gamma_{M,ser} = 1{,}0.$

2.2 Consequence class and execution class

(1) The choice of Consequence Class 1, 2 or 3, see EN 1999-1-1, should be agreed between the designer and the owner of the construction work in cooperation, taking national provisions into account.

(2) The Execution Class, see EN 1999-1-1, should be defined in the execution specification.

3 Materials and geometry

3.1 Material properties

(1) EN 1999-1-5 applies to wrought materials (alloys and tempers) listed in EN 1999-1-1, Tables 3.2a and b and EN 1999-1-4 Table 2.1 for cold-formed sheeting.

(2) For service temperatures between 80°C and 100°C the material properties should be obtained from EN 1999-1-1.

(3) In a global numerical analysis using material nonlinearity, the appropriate stress-strain curve should be selected from EN 1999-1-1, Annex E.

3.2 Design values of geometrical data

(1) The thickness t of the shell should be taken as defined in 1999-1-1 and 1999-1-4.

(2) The middle surface of the shell should be taken as the reference surface for loads.

(3) The radius r of the shell should be taken as the nominal radius of the middle surface of the shell, measured normal to the axis of revolution.

3.3 Geometrical tolerances and geometrical imperfections

(1) The following geometrical deviations of the shell surface from the nominal shape should be considered:

- out-of-roundness (deviation from circularity);
- eccentricities (deviations from a continuous middle surface in the direction normal to the shell along junctions of plates);
- local dents (local normal deviations from the nominal middle surface).

NOTE EN 1090-3 contains requirements to geometrical tolerances for shell structures.

(2) For geometrical tolerance related to buckling resistance, see 6.2.2.

4 Durability

(1) For basic requirements, see Section 4 of EN 1999-1-1

(2) Special attention should be given to cases in which different materials are intended to act compositely, if these materials are such that electrochemical phenomena might produce conditions leading to corrosion.

NOTE For corrosion resistance of fasteners for the environmental corrosivity categories following EN ISO 12944-2 see EN 1999-1-4.

(3) The environmental conditions prevailing from the time of manufacture, including those during transport and storage on site, should be taken into account.

5 Structural analysis

5.1 Geometry

(1) The shell should be represented by its middle surface.

(2) The radius of curvature should be taken as the nominal radius of curvature.

(3) An assembly of shell segments should not be subdivided into separate segments for analysis unless the boundary conditions for each segment are chosen in such a way as to represent interactions between them in a conservative manner.

(4) A base ring intended to transfer support forces into the shell should be included in the analysis model.

(5) Eccentricities and steps in the shell middle surface should be included in the analysis model if they induce significant bending effects as a result of the membrane stress resultants following an eccentric path.

(6) At junctions between shell segments, any eccentricity between the middle surfaces of the shell segments should be considered in the modelling.

(7) A ring stiffener should be treated as a separate structural component of the shell, except where the spacing of the rings is closer than $1{,}5\sqrt{rt}$.

(8) A shell that has discrete stringer stiffeners attached to it may be treated as an orthotropic uniform shell provided that the stringer stiffeners are no further apart than $5\sqrt{rt}$.

(9) A shell that is corrugated (axially or circumferentially) may be treated as an orthotropic uniform shell provided that the corrugation wavelength is less than $0{,}5\sqrt{rt}$ (see A.5.7).

(10) A hole in the shell may be neglected in the modelling provided its largest dimension is smaller than $0{,}5\sqrt{rt}$.

(11) The overall stability of the complete structure can be verified as detailed in EN 1993 Parts 3-1, 3-2, 4-1, 4-2 or 4-3 as appropriate.

5.2 Boundary conditions

(1) The appropriate boundary conditions should be used in analyses for the assessment of limit states according to the conditions shown in Table 5.1. For the special conditions needed for buckling calculations, reference should be made to 6.2.

(2) Rotational restraints at shell boundaries may be neglected in modelling for plastic limit state. For short shells (see Annex A), the rotational restraint should be included in buckling calculation.

(3) Support boundary conditions should be checked to ensure that they do not cause excessive non-uniformity of transmitted forces or introduced forces that are eccentric to the shell middle surface.

(4) When a global numerical analysis is used, the boundary condition for the normal displacement w should also be used for the circumferential displacement v, except where special circumstances make this inappropriate.

Table 5.1 - Boundary conditions for shells

Boundary condition code	Simple term	Description			Normal displace-ments	Meridional displace-ments	Meridional rotation
		radially	meridionally	rotation			
BC1r	Clamped	restrained	restrained	restrained	$w = 0$	$u = 0$	$\beta_\phi = 0$
BC1f		restrained	restrained	free	$w = 0$	$u = 0$	$\beta_\phi \neq 0$
BC2r		restrained	free	restrained	$w = 0$	$u \neq 0$	$\beta_\phi = 0$
BC2f	Pinned	restrained	free	free	$w = 0$	$u \neq 0$	$\beta_\phi \neq 0$
BC3	Free edge	free	free	free	$w \neq 0$	$u \neq 0$	$\beta_\phi \neq 0$
NOTE The circumferential displacement v is very closely linked to the displacement w normal to the surface so separate boundary conditions are not needed.							

5.3 Actions and environmental influences

(1) Actions should all be assumed to act at the shell middle surface. Eccentricities of load should be represented by static equivalent forces and moments at the shell middle surface.

(2) Local actions and local patches of action should not be represented by equivalent uniform loads unless otherwise stated.

(3) The actions and combinations of actions are given in EN 1991 and EN 1990. In addition, those of the following actions that are relevant for the structure, should be considered in the structural analysis:

- local settlement under shell walls;
- local settlement under discrete supports;
- uniformity of support of structure;
- thermal differentials from one side of the structure to the other;
- thermal differentials from inside to outside the structure;
- wind effects on openings and penetrations;
- interaction of wind effects on groups of structures;
- connections to other structures;

- conditions during erection.

(4) Shells may, due to how the loads are carried by membrane forces, be sensitive to a change in geometry e.g. by dents. In addition to unavoidable deviations in geometry from execution, dents may come from unforeseen actions during service. The sensitivity will be increased where the members consists of relatively thin sections. In case dents are introduced that exceeds those values given in C.4 the consequences for the load bearing capacity should be investigated. A program for periodical check of the geometry is recommended.

(5) When selecting the design concept, means to avoid the risk of unacceptable dents should be considered. Such means may e.g. be using a larger thickness than necessary according to the structural calculations, or to arrange for protective means for areas where the risk is judged to be significant.

5.4 Stress resultants and stresses

(1) Provided that the radius to thickness ratio is greater than $(r/t)_{\text{min}} = 25$, the curvature of the shell may be ignored when calculating the stress resultants from the stresses in the shell wall.

5.5 Types of analysis

(1) The design should be based on one or more of the types of analysis given in Table 5.2 depending on the limit state and other considerations. The types of analysis are further explained in Table 5.3. For more details, reference is made to EN 1993-1-6.

Table 5.2 - Types of shell analysis

Type of analysis		Shell theory	Material law	Shell geometry
Membrane theory analysis	MTA	membrane equilibrium	not applicable	perfect 1)
Linear elastic shell analysis	LA	linear bending and stretching	linear	perfect 1)
Linear elastic bifurcation analysis	LBA	linear bending and stretching	linear	perfect 1)
Geometrically non-linear elastic analysis	GNA	non-linear	linear	perfect 1)
Materially non-linear analysis	MNA	linear	non-linear	perfect 1)
Geometrically and materially non-linear analysis	GMNA	non-linear	non-linear	perfect 1)
Geometrically non-linear elastic analysis with imperfections	GNIA	non-linear	linear	imperfect 2)
Geometrically and materially non-linear analysis with imperfections	GMNIA	non-linear	non-linear	imperfect 2)

1) Perfect geometry means that the nominal geometry is used in the analytical model without taking the geometrical deviations into account.

2) Imperfect geometry means that the geometrical deviations from the nominal geometry (tolerances) are taken into account in the analytical model.

Table 5.3 – Description of types of shell analysis

Membrane theory analysis (MTA)	An analysis of a shell structure under distributed loads assuming a set of membrane forces that satisfy equilibrium with the external loads.
Linear elastic analysis (LA)	An analysis on the basis of the small deflection linear elastic shell bending theory assuming perfect geometry.
Linear elastic bifurcation (eigenvalue) analysis (LBA)	An analysis that calculates the linear elastic bifurcation eigenvalue on the basis of small deflections using the linear elastic shell bending theory, assuming perfect geometry. Note that eigenvalue in this context does not refer to vibration modes.
Geometrically non-linear analysis (GNA)	An analysis on the basis of the shell bending theory assuming perfect geometry, considering non-linear large deflection theory and linear elastic material properties.
Materially non-linear analysis (MNA)	An analysis equal to (LA), however, considering non-linear material properties. For welded structure the material in the heat-affected zone should be modelled.
Geometrically and materially non-linear analysis (GMNA)	An analysis applying the shell bending theory assuming perfect geometry, considering non-linear large deflection theory and non-linear material properties. For welded structure the material in the heat-affected zone should be modelled.
Geometrically non-linear elastic analysis with imperfections included (GNIA) 1)	An analysis equal to (GNA), however, considering an imperfect geometry.
Geometrically and materially non-linear analysis with imperfections included (GMNIA)	An analysis equal to (GMNA), however, considering an imperfect geometry.
1) This type of analyses is not covered in this standard, however, listed here for the purpose of having a complete presentation of types of shell analysis.	

6 Ultimate limit state

6.1 Resistance of cross section

6.1.1 Design values of stresses

(1) At each point in the structure the design value of the stress $\sigma_{\mathrm{eq,Ed}}$ should be taken as the highest primary stress determined in a structural analysis that considers the laws of equilibrium between imposed design load and internal forces and moments.

(2) The primary stress may be taken as the maximum value of the stresses required for equilibrium with the applied loads at a point or along a line in the shell structure.

(3) If a *membrane theory analysis* (MTA) is used, the resulting two dimensional field of stress resultants $n_{x,\mathrm{Ed}}$, $n_{\theta,\mathrm{Ed}}$, $n_{x\theta,\mathrm{Ed}}$ may be represented by the equivalent design stress $\sigma_{\mathrm{eq,Ed}}$ obtained from:

$$\sigma_{\mathrm{eq,Ed}} = \frac{1}{t}\sqrt{n_{x,\mathrm{Ed}}^2 + n_{\theta,\mathrm{Ed}}^2 - n_{x,\mathrm{Ed}} n_{\theta,\mathrm{Ed}} + 3n_{x\theta,\mathrm{Ed}}^2} \tag{6.1}$$

(4) If a *linear elastic analysis* (LA) or a *geometrically non-linear elastic analysis* (GNA) is used, the resulting two-dimensional field of primary stresses may be represented by the von Mises equivalent design stress:

$$\sigma_{\mathrm{eq,Ed}} = \sqrt{\sigma_{x,\mathrm{Ed}}^2 + \sigma_{\theta,\mathrm{Ed}}^2 - \sigma_{x,\mathrm{Ed}}\sigma_{\theta,\mathrm{Ed}} + 3\left(\tau_{x\theta,\mathrm{Ed}}^2 + \tau_{xn,\mathrm{Ed}}^2 + \tau_{\theta n,\mathrm{Ed}}^2\right)} \tag{6.2}$$

in which:

$$\sigma_{x,\mathrm{Ed}} = \frac{1}{\eta}\left(\frac{n_{x,\mathrm{Ed}}}{t} \pm \frac{m_{x,\mathrm{Ed}}}{t^2/4}\right), \quad \sigma_{\theta,\mathrm{Ed}} = \frac{1}{\eta}\left(\frac{n_{\theta,\mathrm{Ed}}}{t} \pm \frac{m_{\theta,\mathrm{Ed}}}{t^2/4}\right), \tag{6.3}$$

$$\tau_{x\theta,\mathrm{Ed}} = \frac{1}{\eta}\left(\frac{n_{x\theta,\mathrm{Ed}}}{t} \pm \frac{m_{x\theta,\mathrm{Ed}}}{t^2/4}\right), \qquad \tau_{xn,\mathrm{Ed}} = \frac{q_{xn,\mathrm{Ed}}}{t}, \quad \tau_{\theta n,\mathrm{Ed}} = \frac{q_{\theta n,\mathrm{Ed}}}{t} \tag{6.4}$$

η being a correction factor due to inelastic behaviour of material and depending on both hardening and ductility features of the alloy.

NOTE 1 The above expressions give a simplified conservative equivalent stress for design purposes.

NOTE 2 Values for η are given in EN 1999-1-1 Annex H as a function of alloy features. Values of η corresponding to a geometrical shape factor $\alpha_0 = 1,5$ should be taken

NOTE 3 The values of $\tau_{xn,\mathrm{Ed}}$ and $\sigma_{xn,\mathrm{Ed}}$ are usually very small and do not affect the resistance, so they may generally be ignored.

6.1.2 Design values of resistance

(1) The von Mises equivalent design strength should be taken from:

$$f_{\mathrm{eq,Rd}} = \frac{f_{\mathrm{o}}}{\gamma_{\mathrm{M1}}} \qquad \text{in section without HAZ} \tag{6.5}$$

$$f_{\mathrm{eq,Rd}} = \min\left(\frac{\rho_{\mathrm{u,haz}} f_{\mathrm{u}}}{\gamma_{\mathrm{M2}}}, \frac{f_{\mathrm{o}}}{\gamma_{\mathrm{M1}}}\right) \qquad \text{in section with HAZ} \tag{6.6}$$

where:

- f_{o} is the characteristic value of the 0,2 % proof strength as given in EN 1999-1-1
- f_{u} is the characteristic value of the ultimate strength as given in EN 1999-1-1
- $\rho_{\mathrm{u,haz}}$ is the ratio between the ultimate strength in the heat affected zone HAZ and in the parent material, as given in EN 1999-1-1
- γ_{M1} is the partial factor for resistance given in 2.1 (3).
- γ_{M2} is the partial factor for resistance given in 2.1 (3).

(2) The effect of fastener holes should be taken into account in accordance with EN 1999-1-1.

6.1.3 Stress limitation

(1) In every verification of this limit state, the design stresses should satisfy the condition:

$$\sigma_{\mathrm{eq,Ed}} \le f_{\mathrm{eq,Rd}} \tag{6.7}$$

6.1.4 Design by numerical analysis

(1) The design plastic limit resistance should be determined as a load ratio R applied to the design values of the combination of actions for the relevant load case.

(2) The design values of the actions F_{Ed} should be determined as detailed in 5.3.

(3) In an *materially non-linear analysis* (MNA) and *geometrically and materially non-linear analysis* (GMNA) based on the design limiting strength $f_{\mathrm{o}}/\gamma_{\mathrm{M}}$, the shell should be subject to the design value of the loads, progressively increased by the load ratio R until the plastic limit condition is reached.

(4) If an *materially non-linear analysis* (MNA) is used, the load ratio R_{MNA} may be taken as the largest value attained in the analysis. The effect of strain hardening may be included provided that a corresponding limit value of allowable material deformation is considered. Guidelines on analytical models for stress-strain relationship to be used in MNA are given in EN 1999-1-1.

(5) If a *geometrically and materially non-linear analysis* (GMNA) is used, if the analysis predicts a maximum load followed by a descending path, the maximum value should be used to determine the load ratio R_{GMNA}. If a GMNA analysis does not predict a maximum load, but produces a progressively rising action-displacement relationship (with or without strain hardening of the material), the load ratio R_{GMNA} should be taken as no larger than the value at which the maximum von Mises equivalent plastic strain in the structure attains the alloy ultimate deformation limit value as given in EN 1999-1-1, Section 3. For design purposes, an ultimate plastic strain value equal to $5(f_o/E)$ or $10(f_o/E)$ can be assumed, depending on the alloy features.

NOTE Values of ultimate plastic strain values ε_u corresponding to $5(f_o/E)$ or $10(f_o/E)$ are given in EN 1999-1-1, Annex H.

(6) The result of the analysis should satisfy the condition:

$$R = \frac{F_{Rd}}{F_{Ed}} \geq 1{,}0 \qquad (6.8)$$

where F_{Ed} is the design value of the action.

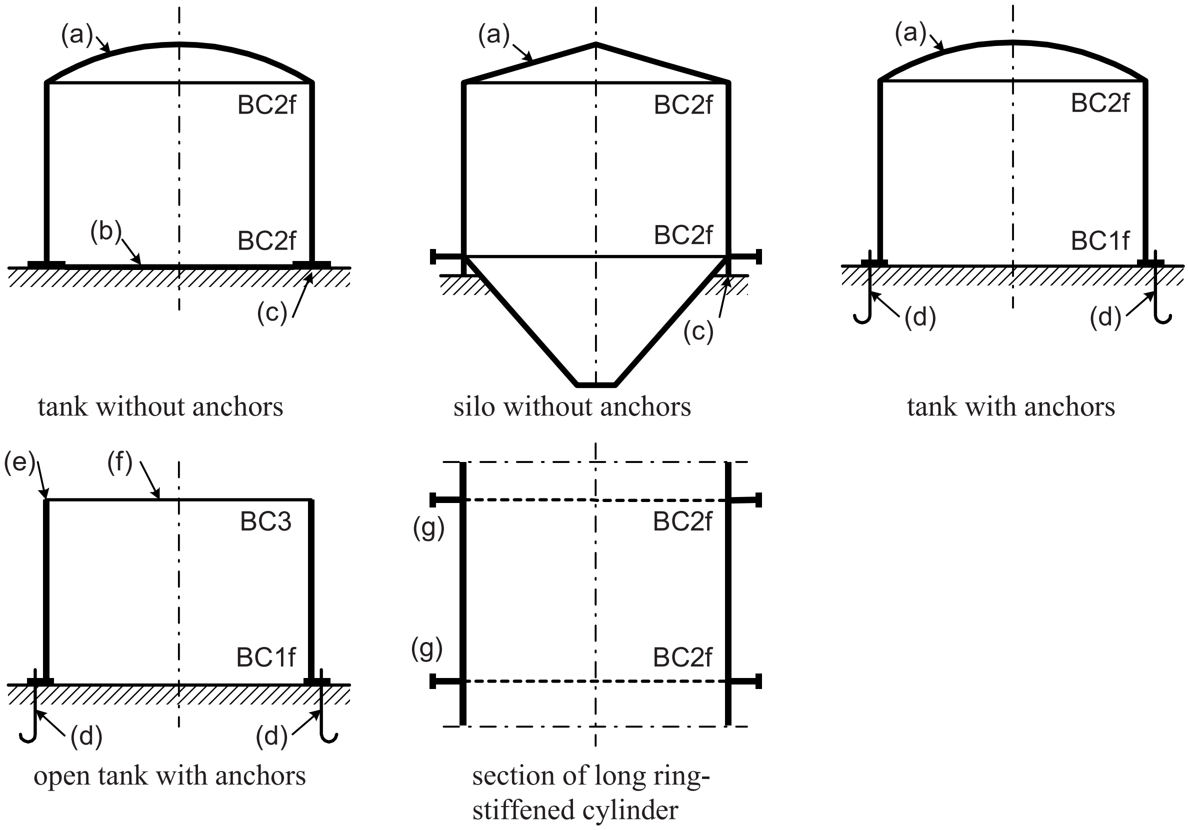
6.2 Buckling resistance

6.2.1 General

(1) All relevant combinations of actions causing compressive membrane stresses or shear membrane stresses in the shell wall should be taken into account.

(2) The sign convention for use in calculation for buckling should be taken as compression positive for meridional and circumferential stresses and stress resultants.

(3) Special attention should be paid to the boundary conditions which are relevant to the incremental displacements due to buckling (as opposed to pre-buckling displacements). Examples of relevant boundary conditions are shown in Figure 6.1.



Keys: (a) roof, (b) bottom plate, (c) no anchoring, (d) closely spaced anchor bolts, (e) no stiffening ring, (f) free edge, (g) ring stiffener.

Figure 6.1 - Schematic examples of boundary conditions for buckling limit state

6.2.2 Buckling-relevant geometrical tolerances

(1) The geometrical tolerance limits given in EN 1090-3 should be met if buckling is one of the ultimate limit states to be considered.

NOTE 1 The design buckling stresses determined hereafter include imperfections that are based on geometric tolerances expected to be met during execution.

NOTE 2 The geometric tolerances given in EN 1090-3 are those that are known to have a large impact on the safety of the structure.

(2) The tolerance class (Class 1, Class 2, Class 3 or Class 4) should be chosen according to both load case and tolerance definitions given in EN 1090-3. The description of each class relates only to the strength evaluation.

(3) Each of the imperfection types should be classified separately; the lowest class should then govern the entire design.

(4) The different tolerance types may each be treated independently, and no interactions need normally be considered.

6.2.3 Shell in compression and shear

6.2.3.1 Design values of stresses

(1) The design values of stresses $\sigma_{x,\mathrm{Ed}}$, $\sigma_{\theta,\mathrm{Ed}}$, and τ_{Ed}, should be taken as the key values of compressive and shear membrane stresses as obtained by *linear shell analysis* (LA). Under purely axisymmetric conditions of loading and support, and in other simple load cases, membrane theory may generally be used.

(2) The key values of membrane stresses should be taken as the maximum value of each stress at that axial coordinate in the structure, unless specific provisions are given in Annex A.

NOTE In some cases (e.g. stepped walls under circumferential compression, see A.2.3), the key values of membrane stresses are fictitious and larger than the real maximum values.

(3) For basic loading cases the membrane stresses may be taken from relevant standard expressions.

6.2.3.2 Buckling strength

(1) The design buckling resistances should be obtained from:

$$\sigma_{x,\mathrm{Rd}} = \alpha_x \rho_{x,\mathrm{w}} \chi_{x,\mathrm{perf}} \frac{f_\mathrm{o}}{\gamma_{\mathrm{M1}}} \tag{6.9}$$

$$\sigma_{\theta,\mathrm{Rd}} = \alpha_\theta \rho_{\theta,\mathrm{w}} \chi_{\theta,\mathrm{perf}} \frac{f_\mathrm{o}}{\gamma_{\mathrm{M1}}} \tag{6.10}$$

$$\tau_{\mathrm{Rd}} = \alpha_\tau \rho_{\tau,\mathrm{w}} \chi_{\tau,\mathrm{perf}} \frac{f_\mathrm{o}}{\sqrt{3}\,\gamma_{\mathrm{M1}}} \quad \text{(also valid for stiffened shells)} \tag{6.11}$$

for unstiffened shells, and

$$n_{x,\mathrm{Rd}} = \alpha_{n,x} \chi_{x,\mathrm{perf}} \frac{n_{\mathrm{x,Rk}}}{\gamma_{\mathrm{M1}}} \tag{6.12}$$

$$p_{n,\mathrm{Rd}} = \alpha_{p,\theta} \chi_{\theta,\mathrm{perf}} \frac{p_{n,\mathrm{Rk}}}{\gamma_{\mathrm{M1}}} \tag{6.13}$$

for stiffened and/or corrugated shells

where:

- $n_{x,\mathrm{Rk}}$ is the axial squash limit of the stiffened shell;
- $p_{n,\mathrm{Rk}}$ is the uniform squash limit pressure of the stiffened shell or the toriconical and torispherical shell;
- α_i is the imperfection reduction factor to be taken from Annex A;
- $\rho_{i,\mathrm{w}}$ is the reduction factor due to heat-affected zones according to 6.2.4.4. For shells without welds $\rho_{i,\mathrm{w}} = 1$;
- $\chi_{i,\mathrm{perf}}$ is the reduction factor due to buckling of a perfect shell given in (2).
- γ_{M1} is the partial factor for resistance given in 2.1 (3).

NOTE 1 Expression (6.13) is also valid for toriconical and torispherical shells, see Annex B

NOTE 2 α_i for toriconical and torispherical shells, see Annex B

(2) The reduction factor due to buckling for a perfect shell is given by:

$$\chi_{i,\text{perf}} = \frac{1}{\phi_i + \sqrt{\phi_i^2 - \bar{\lambda}_i^2}} \quad \text{but} \quad \chi_{i,\text{perf}} \leq 1{,}00 \tag{6.14}$$

with:

$$\phi_i = 0{,}5\left(1 + \mu_i(\bar{\lambda}_i - \bar{\lambda}_{i,0}) + \bar{\lambda}_i^2\right) \tag{6.15}$$

where:

μ_i is a parameter depending on the alloy and loading case, to be taken from Annex A;

$\bar{\lambda}_{i,0}$ is the squash limit relative slenderness , to be taken from Annex A;

i is subscript to be replaced by x, θ or τ depending on loading type.

(3) The shell slenderness parameters for different stress components should be determined from:

$$\bar{\lambda}_x = \sqrt{\frac{f_o}{\sigma_{x,\text{cr}}}} \tag{6.16}$$

$$\bar{\lambda}_\theta = \sqrt{\frac{f_o}{\sigma_{\theta,\text{cr}}}} \tag{6.17}$$

$$\bar{\lambda}_\tau = \sqrt{\frac{f_o}{\sqrt{3}\tau_{\text{cr}}}} \quad \text{(also valid for stiffened shells)} \tag{6.18}$$

for unstiffened shells, and

$$\bar{\lambda}_x = \sqrt{\frac{n_{\text{x,Rk}}}{n_{x,\text{cr}}}} \tag{6.19}$$

$$\bar{\lambda}_\theta = \sqrt{\frac{p_{n,\text{Rk}}}{p_{n,\text{cr}}}} \tag{6.20}$$

for stiffened and/or corrugated shells.

where:

$\sigma_{x,\text{cr}}$, $\sigma_{\theta,\text{cr}}$ and τ_{cr} are the critical buckling stresses as given in Annex A or obtained by *linear elastic bifurcation (eigenvalue) analysis* (LBA);

$n_{x,\text{cr}}$, $p_{n,\text{cr}}$ are the critical buckling stress resultants for stiffened shells or toriconical and torispherical shells as given in Annex A or obtained by *linear elastic bifurcation (eigenvalue) analysis* (LBA).

NOTE 1 Expressions (6.19) and (6.20) are also valid for toriconical and torispherical shells, see Annex B

NOTE 2 $p_{n,\text{cr}}$ for toriconical and torispherical shells, see Annex B

6.2.3.3 Buckling strength verification

(1) Although buckling is not a purely stress-initiated failure phenomenon, the buckling strength verification should be represented by limiting the design values of membrane stresses or stress resultants. The influence of bending stresses on the buckling strength may be neglected provided they arise as a result of boundary compatibility effects. In the case of bending stresses from local loads or from thermal gradients, special consideration should be given.

(2) Depending on the loading and stressing situation, one or more of the following checks for the key values of single membrane stress components should be carried out:

$$\sigma_{x,\mathrm{Ed}} \leq \sigma_{x,\mathrm{Rd}} \tag{6.21}$$

$$\sigma_{\theta,\mathrm{Ed}} \leq \sigma_{\theta,\mathrm{Rd}} \tag{6.22}$$

$$\tau_{\mathrm{Ed}} \leq \tau_{\mathrm{Rd}} \tag{6.23}$$

(3) If more than one of the three buckling-relevant membrane stress components are present under the actions under consideration, the following interaction check for the combined membrane stress state should be carried out:

$$\left(\frac{\sigma_{x,\mathrm{Ed}}}{\sigma_{x,\mathrm{Rd}}}\right)^{k_x} + \left(\frac{\sigma_{\theta,\mathrm{Ed}}}{\sigma_{\theta,\mathrm{Rd}}}\right)^{k_\theta} - k_{\mathrm{i}}\left(\frac{\sigma_{x,\mathrm{Ed}}}{\sigma_{x,\mathrm{Rd}}}\right)\left(\frac{\sigma_{\theta,\mathrm{Ed}}}{\sigma_{\theta,\mathrm{Rd}}}\right) + \left(\frac{\tau_{\mathrm{Ed}}}{\tau_{\mathrm{Rd}}}\right)^{k_\tau} \leq 1{,}00 \tag{6.24}$$

where $\sigma_{x,\mathrm{Ed}}$, $\sigma_{\theta,\mathrm{Ed}}$ and τ_{Ed} are the interaction-relevant groups of the significant values of compressive and shear membrane stresses in the shell and the values of the interaction parameters k_x, k_θ, k_τ and k_{i} are:.

$$\begin{aligned} k_x &= 1 + \chi_x^2 \\ k_\theta &= 1 + \chi_\theta^2 \\ k_\tau &= 1{,}5 + 0{,}5\chi_\tau^2 \\ k_{\mathrm{i}} &= (\chi_x \chi_\theta)^2 \end{aligned} \tag{6.25}$$

NOTE 1 In case of unstiffened cylinder under axial compression and circumferential compression and shear the formulae in A.1.6 for the interaction parameters may be used.

NOTE 2 The above rules may sometimes be very conservative, but they have the two limiting cases which are well established as safe for a wide range of cases: a) in very thin shells the interaction between σ_x and σ_θ is linear; and b) in very thick shells the interaction between stresses may be formulated as that of von Mises equivalent stress or that of alternative interaction formulae as given in EN 1999-1-1.

(4) If $\sigma_{x,\mathrm{Ed}}$ or $\sigma_{\theta,\mathrm{Ed}}$ is tensile, its value should be taken as zero in expression (6.24).

NOTE For axially compressed cylinders with internal pressure (leading to circumferential tension) special provisions are made in Annex A. The resulting value of $\sigma_{x,\mathrm{Rd}}$ accounts for both the strengthening effect of internal pressure on the elastic buckling resistance and the weakening effect of the elastic plastic elephant's foot phenomenon (expression (A.22)). If the tensile stress $\sigma_{\theta,\mathrm{Ed}}$ is then taken as zero in expression (6.24), the buckling strength is accurately represented.

(5) The locations and values of each of the buckling-relevant membrane stresses to be used together in combination in expression (6.24) are defined in Annex A.

6.2.4 Effect of welding

6.2.4.1 General

(1) General criteria and rules for welded structures given in EN 1999-1-1 should be followed in the design of aluminium shell structures.

(2) In the design of welded shell structures using strain hardened or artificially aged precipitation hardening alloys the reduction in strength properties that occurs in the vicinity of welds should be allowed for. This area is named heat affected zone (HAZ). Exceptions to this rule are stated in EN 1999-1-1.

(3) For design purposes it is assumed that throughout the heat affected zone the strength properties are reduced on a constant level.

NOTE 1 Even though the reduction mostly affects the 0,2 % proof strength and the ultimate tensile strength of the material, its effects can be significant on the compressed parts of a shells structures susceptible to buckling depending on structural slenderness and alloy properties.

NOTE 2 The effect of softening due to welding is more significant for buckling of shells in the plastic range. Also local welds in areas with risk of buckling may considerably reduce the buckling resistance due to the HAZ. It is therefore recommended to avoid welds in large unstiffened parts subject to compression.

NOTE 3 For design purposes the welding can be assumed as a linear strip across the shell surface whose affected region extends immediately around the weld. Beyond this region the strength properties rapidly recover to their full unwelded values. A premature onset of yielding lines can occur along these lines when shell buckling takes place.

NOTE 4 The effects of HAZ softening can sometimes be mitigated by means of artificial ageing applied after welding, see EN 1999-1-1.

(4) The effect of softening due to welding on the shell buckling resistance should be checked for all welds directly or indirectly subjected to compressive stress according to the rules given in 6.2.4.2.

6.2.4.2 Severity of softening

(1) The severity of softening due to welding is expressed through the reduction factors $\rho_{o,haz}$ and $\rho_{u,haz}$ given by the ratios:

$$\rho_{o,haz} = \frac{f_{o,haz}}{f_o} \text{ and } \rho_{u,haz} = \frac{f_{u,haz}}{f_u} \tag{6.26}$$

between the characteristic value of the 0,2 % proof strength $f_{o,haz}$ (ultimate strength $f_{u,haz}$) in the heat affected zone and the one f_o (f_u) in the parent material.

(2) The characteristic values of strength $f_{o,haz}$ and $f_{u,haz}$ and the values of $\rho_{o,haz}$ and $\rho_{u,haz}$ are listed in Table 3.2a of EN 1999-1-1 for wrought alumimum alloys in the form of sheet, strip and plate and in Table 3.2b for extrusions.

(3) Recovery times after welding should be evaluated according to provisions stated in EN 1999-1-1.

6.2.4.3 Extent of HAZ

(1) General indications on the HAZ extent given in EN 1999-1-1 should be followed.

(2) For the purposes of buckling checks, the HAZ in shell sheeting in areas at risk of buckling is assumed to extend for a distance b_{haz} in any direction from a weld, measured transversely from the centre line of an in-line butt weld or from the point of intersection of the welded surfaces at fillet welds, as shown in Figure 6.2.

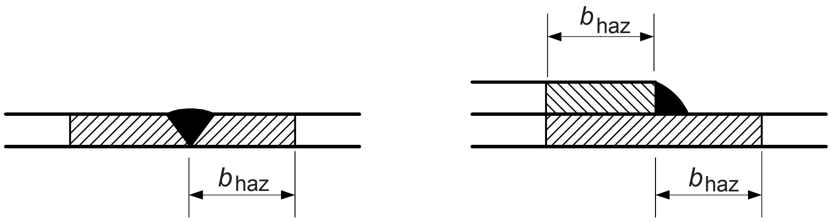


Figure 6.2 - Extend of heat-affected zones (HAZ) in shell sheeting

6.2.4.4 Buckling resistance of unstiffened welded shells

(1) The buckling resistance of unstiffened welded shells should be assessed in any case if compressive stress resultants acting in laterally unrestrained welded panels are present in the shell.

(2) The check of the weld effect on buckling can be avoided if all welds in the shells are parallel to the compressive stress resultants acting in the structure under any load condition, provided that the reduction factor $\rho_{\mathrm{o,haz}}$ due to HAZ is not lower than 0,60.

(3) The effect of welding on the buckling resistance can be evaluated by means of a *geometrically and materially non-linear analysis with imperfections* (GMNIA) analysis and accounting for the actual properties of both parent material and HAZ zones.

(4) If an accurate GMNIA analysis cannot be performed, the shell buckling resistance can be evaluated in a simplified way through the reduction factor given by the ratio $\rho_{i,\mathrm{w}} = \chi_{i,\mathrm{w}} / \chi_i$ between the buckling factor of the welded structure $\chi_{\mathrm{w},i}$ and the one of the unwelded structure χ_i .

NOTE 1 Compressive stress resultants in shells may arise not only due to direct compression, but also to external pressure, shear and localised loads. Whatever the load condition, reduction factors $\chi_{\mathrm{w},i}$ are to be applied if welds which are orthogonal to compressive stress resultants as they can produce a concentrated source of plastic deformation.

NOTE 2 The subscript "i" in clause (4) and (5) should be intended as "x", "θ" or "τ" depending on whether the reduction factors χ and ρ are referred to axial compression, circumferential compression or shear, respectively.

(5) The reduction factor to allow for HAZ softening in shell structures is given by:

$$\rho_{i,\mathrm{w}} = \omega_0 + (1-\omega_0)\frac{\overline{\lambda}_i - \overline{\lambda}_{i,0}}{\overline{\lambda}_{i,\mathrm{w}} - \overline{\lambda}_{i,0}} \quad \text{but} \quad \rho_{i,\mathrm{w}} \le 1 \quad \text{and} \quad \rho_{i,\mathrm{w}} \ge \omega_0 \tag{6.27}$$

where:

$$\omega_0 = \frac{\rho_{\mathrm{u,haz}} f_{\mathrm{u}} / \gamma_{\mathrm{M2}}}{f_{\mathrm{o}} / \gamma_{\mathrm{M1}}} \quad \text{but} \quad \omega_0 \le 1 \tag{6.28}$$

$\rho_{\mathrm{u,haz}}$ and $\rho_{\mathrm{o,haz}}$ are the reduction factors due to HAZ, to be taken from Table 3.2a or Table 3.2b of EN 1999-1-1;

$\overline{\lambda}_{i,0}$ is the relative squash limit slenderness parameter for the load cases under consideration to be taken from Annex A;

$\overline{\lambda}_{i,\mathrm{w}}$ is the limit value of the relative slenderness parameter beyond which the effect of weld on buckling vanishes, given by $\overline{\lambda}_{i,\mathrm{w}} = 1{,}39(1-\rho_{\mathrm{o,haz}})(\overline{\lambda}_{i,\mathrm{w},0} - \overline{\lambda}_{i,0})$, but $\overline{\lambda}_{i,\mathrm{w}} \le \overline{\lambda}_{i,\mathrm{w},0}$, see Figure 6.3;

$\overline{\lambda}_{i,\mathrm{w},0}$ is the absolute slenderness upper limit for the weld effect, depending on load case, structural material and tolerance class of the shell, as given in Table 6.5.

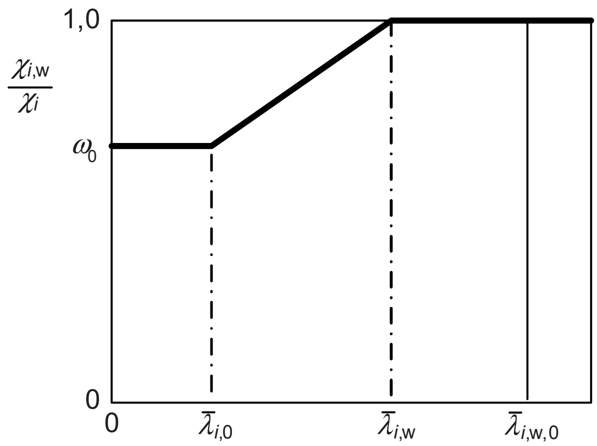


Figure 6.3 - Definition of the reduction factor $\rho_{i,\mathrm{w}}$ due to HAZ

Table 6.5 - Values of $\overline{\lambda}_{i,\mathrm{w},0}$ for relevant load cases allowed for in Annex A

Tolerance class	Axial compression $\overline{\lambda}_{x,\mathrm{w},0}$		Circumferential compression $\overline{\lambda}_{\theta,\mathrm{w},0}$		Torsion and shear $\overline{\lambda}_{\tau,\mathrm{w},0}$	
	Class A material	Class B material	Class A material	Class B material	Class A material	Class B material
Class 1	0,8	0,7	1,2	1,1	1,4	1,3
Class 2	1,0	0,9	1,3	1,2	1,5	1,4
Class 3	1,2	1,1	1,4	1,3	1,6	1,5
Class 4	1,3	1,2	-	-	-	-

6.2.4.5 Buckling resistance of stiffened welded shells

(1) Stiffened welded shells do not need to be checked against the effect of welding if stiffeners have adequate lateral restraint to welded panels. If this is not the case the provisions in 6.2.4.4 apply.

6.2.5 Design by numerical analysis

(1) The procedure given in 5.5 and 6.1.4 for *geometrically and materially non-linear analysis with imperfections* (GMNIA) analysis may be followed. The GMNIA analysis may be performed, as an alternative to the method given in 6.2.3, by assuming as initial geometrical imperfections the maximum values of tolerances given in 6.2.2.

(2) For welded structures the material in the heat-affected zone should be modelled, see 6.2.4.2, 6.2.4.3 and 6.2.4.4.

7 Serviceability limit states

7.1 General

(1) The rules for serviceability limit states given in EN 1999-1-1 should also be applied to shell structures.

7.2 Deflections

(1) The deflections may be calculated assuming elastic behaviour.

(2) With reference to EN 1990 – Annex A1.4 limits for deflections should be specified for each project and agreed with the owner of the project.

Annex A [normative] - Expressions for shell buckling analysis

A.1 Unstiffened cylindrical shells of constant wall thickness

A.1.1 Notations and boundary conditions

(1) General quantities (Figure A.1):

l cylinder length between boundaries;

r radius of cylinder middle surface;

t thickness of shell:

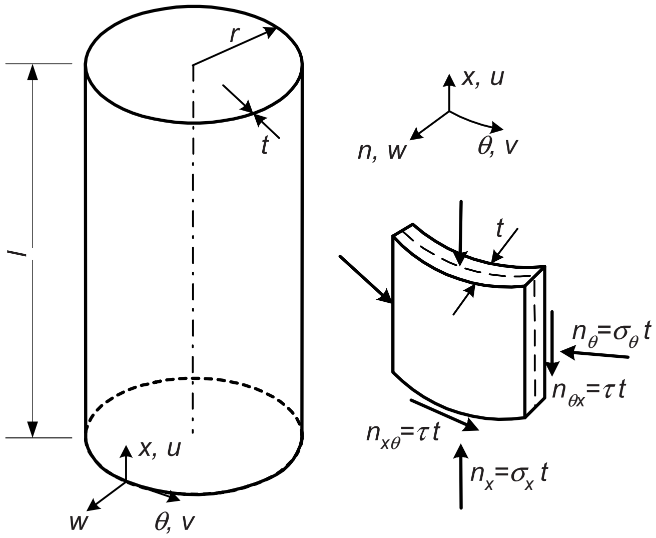


Figure A.1 - Cylinder geometry and membrane stresses and stress resultants

(2) The boundary conditions are set out in 5.2 and 6.2.1.

A.1.2 Meridional (axial) compression

(1) Cylinders need not be checked against meridional shell buckling if they satisfy:

$$\frac{r}{t} \leq 0{,}03 \frac{E}{f_o} \tag{A.1}$$

A.1.2.1 Critical meridional buckling stresses

(1) The following expressions may only be used for shells with boundary conditions BC 1 or BC 2 at both edges.

(2) The length of the shell segment is characterized in terms of the dimensionless parameter ω:

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{r\,t}} \tag{A.2}$$

(3) The critical meridional buckling stress, using values of C_x from Table A.1, should be obtained from:

$$\sigma_{x,\mathrm{cr}} = 0{,}605 E C_x \frac{t}{r} \tag{A.3}$$

Table A.1 - Factor C_x for critical meridional buckling stress

Cylindrical shell	$\omega = \frac{l}{\sqrt{rt}}$	Factor C_x
Short	$\omega \leq 1{,}7$	$C_x = 1{,}36 - \frac{1{,}83}{\omega} + \frac{2{,}07}{\omega^2}$
Medium-length	$1{,}7 < \omega < 0{,}5\frac{r}{t}$	$C_x = 1$
Long	$\omega \geq 0{,}5\frac{r}{t}$	$C_x = 1 - \frac{0{,}2}{C_{xb}}\left(2\omega\frac{t}{r} - 1\right)$ but $C_x \geq 0{,}6$ where C_{xb} is given in Table A.2

Table A.2 - Parameter C_{xb} for the effect of boundary conditions for long cylinder

Case	Cylinder end	Boundary condition	C_{xb}
1	end 1 end 2	BC 1 BC 1	6
2	end 1 end 2	BC 1 BC 2	3
3	end 1 end 2	BC 2 BC 2	1

NOTE BC 1 includes both BC1f and BC1r

(4) For long cylinders as defined in Table A.1 that satisfy the additional conditions:

$$\frac{r}{t} \leq 150 \quad \text{and} \quad \frac{\omega t}{r} \leq 6 \quad \text{and} \quad 500 \leq \frac{E}{f_o} \leq 1000 \tag{A.4}$$

the factor C_{xb} may alternatively be obtained by:

$$C_x = C_{x,N}\frac{\sigma_{x,N,Ed}}{\sigma_{x,Ed}} + \frac{\sigma_{x,M,Ed}}{\sigma_{x,Ed}} \tag{A.5}$$

where:

$C_{x,N}$ is the parameter for long cylinder in axial compression according to Table A.1;

$\sigma_{x,Ed}$ is the design value of the meridional stress ($\sigma_{x,Ed} = \sigma_{x,N,Ed} + \sigma_{x,M,Ed}$);

$\sigma_{x,N,Ed}$ is the stress component from axial compression (circumferentially uniform component);

$\sigma_{x,M,Ed}$ is the stress component from tubular global bending (peak value of the circumferentially varying component).

A.1.2.2 Meridional buckling parameter

(1) The meridional imperfection factor should be obtained from:

$$\alpha_x = \frac{1}{1 + 2{,}60\left(\frac{1}{Q}\sqrt{\frac{0{,}6E}{f_o}}(\bar{\lambda}_x - \bar{\lambda}_{x,0})\right)^{1{,}44}} \quad \text{but} \quad \alpha_x \leq 1{,}00 \tag{A.6}$$

where:

$\bar{\lambda}_{x,0}$ is the meridional squash limit slenderness parameter;

Q is the meridional compression tolerance parameter.

(2) The tolerance parameter Q should be taken from Table A.3 for the specified tolerance class. For tolerance class 4 the tolerance parameter Q depends also on boundary conditions as defined in Table 5.1.

(3) The alloy factor and the meridional squash limit slenderness parameter should be taken from Table A.4 according to the material buckling class as defined in EN 1999-1-1.

Table A.3 - Tolerance parameter *Q*

Tolerance class	Value of Q for boundary conditions	
	BC1r, BC2r	BC1f, BC2f
Class 1	16	
Class 2	25	
Class 3	40	
Class 4	60	50

Table A.4 - Values of $\overline{\lambda}_{x,0}$ and μ_x for meridional compression

Material buckling class	$\overline{\lambda}_{x,0}$	μ_x
A	0,20	0,35
B	0,10	0,20

(4) For long cylinders that satisfy the special conditions of A.1.2.1(4), the meridional squash limit slenderness parameter may be obtained from:

$$\overline{\lambda}_{x,0,1} = \overline{\lambda}_{x,0} + 0{,}10 \frac{\sigma_{x,\mathrm{M,Ed}}}{\sigma_{x,\mathrm{Ed}}} \tag{A.7}$$

where $\overline{\lambda}_{x,0}$ should be taken from Table A.4 and $\sigma_{x,\mathrm{Ed}}$ and $\sigma_{x,\mathrm{M,Ed}}$ are as given in A.1.2.1(4).

A.1.3 Circumferential (hoop) compression

(1) Cylinders need not be checked against circumferential shell buckling if they satisfy:

$$\frac{r}{t} \leq 0{,}21 \sqrt{\frac{E}{f_\mathrm{o}}} \tag{A.8}$$

A.1.3.1 Critical circumferential buckling stresses

(1) The following expressions may be applied to shells with all boundary conditions.

(2) The length of the shell segment is characterized in terms of the dimensionless parameter ω:

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{r\,t}} \tag{A.9}$$

(3) The critical ⟦AC1⟩ circumferential ⟨AC1⟧ buckling stress, using values of C_θ from Table A.5 for medium length cylinders and Table A.6 for short cylinders, should be obtained from:

$$\sigma_{\theta,\mathrm{cr}} = 0{,}92 E \frac{C_\theta}{\omega}\frac{t}{r} \tag{A.10}$$

Table A.5 - External pressure buckling factor C_θ for medium-length cylinders ($20 < \omega / C_\theta < 1{,}63 r / t$)

Case	Cylinder end	Boundary condition	Factor C_θ
1	end 1 end 2	BC 1 BC 1	1,5
2	end 1 end 2	BC 1 BC 2	1,25
3	end 1 end 2	BC 2 BC 2	1,0
4	end 1 end 2	BC 1 BC 3	0,6
5	end 1 end 2	BC 2 BC 3	0
6	end 1 end 2	BC 3 BC 3	0

Table A.6 - External pressure buckling factor C_θ for short cylinders ($\omega / C_\theta \leq 20$)

Case	Cylinder end	Boundary condition	Factor C_θ
1	end 1 end 2	BC 1 BC 1	$C_\theta = 1{,}5 + \dfrac{10}{\omega^2} - \dfrac{5}{\omega^3}$
2	end 1 end 2	BC 1 BC 2	$C_\theta = 1{,}25 + \dfrac{8}{\omega^2} - \dfrac{4}{\omega^3}$
3	end 1 end 2	BC 2 BC 2	$C_\theta = 1{,}0 + \dfrac{3}{\omega^{1{,}35}}$
4	end 1 end 2	BC 1 BC 3	$C_\theta = 0{,}6 + \dfrac{1}{\omega^2} - \dfrac{0{,}3}{\omega^3}$

NOTE In Table A.5 and A.6, BC 1 includes both BC1f and BC1r

(4) For long cylinders ($\omega / C_\theta \geq 1{,}63 r / t$) the circumferential buckling stress should be obtained from:

$$\sigma_{\theta,\mathrm{cr}} = E\left(\frac{t}{r}\right)^2 \left(0{,}275 + 2{,}03\left(\frac{C_\theta r}{\omega t}\right)^4\right) \quad \text{(A.11)}$$

A.1.3.2 Circumferential buckling parameter

(1) The [AC1) circumferential (AC1] imperfection factor should be obtained from:

[AC1) $$\alpha_\theta = \frac{1}{1 + 0{,}2(1 - \alpha_{\theta,\mathrm{ref}})(\bar{\lambda}_\theta - \bar{\lambda}_{\theta,0}) / \alpha_{\theta,\mathrm{ref}}^2} \quad \text{but} \quad \alpha_\theta \leq 1{,}00 \quad \text{(A.12)}$$ (AC1]

(2) The circumferential reference imperfection factor $\alpha_{\theta,\mathrm{ref}}$ should be taken from Table A.7 for the specified tolerance class.

Table A.7 - Factor $\alpha_{\theta,\mathrm{ref}}$ based on tolerance class

Tolerance class	Parameter $\alpha_{\theta,\mathrm{ref}}$
Class 1	0,50
Class 2	0,65
Class 3 and 4	0,75

(3) The alloy factor and the AC1 circumferential AC1 squash limit slenderness parameter should be taken from Table A.8 according to the material buckling class as defined in EN 1999-1-1.

Table A.8 - Values of $\overline{\lambda}_{\theta,0}$ and μ_θ for circumferential compression

Material buckling class	$\overline{\lambda}_{\theta,0}$	μ_θ
A	0,30	0,55
B	0,20	0,70

(4) The non-uniform distribution of pressure q_{eq} resulting from external wind loading on cylinders (see Figure A.2) may, for the purpose of shell buckling design, be substituted by an equivalent uniform external pressure:

$$q_{\mathrm{eq}} = k_{\mathrm{w}} q_{\mathrm{w,m}ax} \tag{A.13}$$

where $q_{\mathrm{w,max}}$ is the maximum wind pressure and k_{w} should be found as follows:

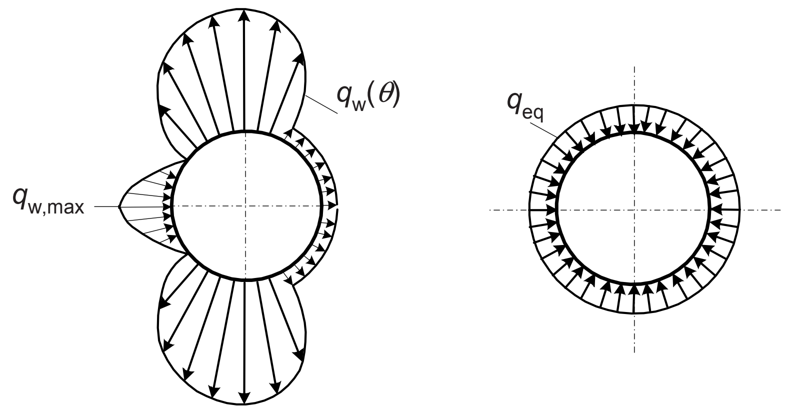
$$k_{\mathrm{w}} = 0{,}46\left(1 + 0{,}1\sqrt{\frac{C_\theta r}{\omega t}}\right) \tag{A.14}$$

with the value of k_{w} not outside the range $0{,}65 \le k_{\mathrm{w}} \le 1{,}0$, and with C_θ taken from Table A.5 according to the boundary conditions.

(5) The circumferential design stress to be introduced into 6.2.3.3 follows from:

$$\sigma_{\theta,\mathrm{Ed}} = (q_{\mathrm{eq}} + q_{\mathrm{s}})\frac{r}{t} \tag{A.15}$$

where q_{s} is the internal suction caused by venting, internal partial vacuum or other phenomena.



a) Wind pressure distribution around shell circumference

b) Equivalent axisymmetric pressure distribution

Figure A.2 - Transformation of typical wind external pressure load distribution

A.1.4 Shear

(1) Cylinders need not be checked against shear buckling if they satisfy:

$$\frac{r}{t} \le 0{,}16\left(\frac{E}{f_o}\right)^{0{,}67} \tag{A.16}$$

A.1.4.1 Critical shear buckling stresses

(1) The following expressions may only be used for shells with boundary conditions BC 1 or BC 2 at both edges.

(2) The length of the shell segment is characterized in terms of the dimensionless parameter ω:

$$\omega = \frac{l}{r}\sqrt{\frac{r}{t}} = \frac{l}{\sqrt{rt}} \tag{A.17}$$

(3) The critical shear buckling stress, using values of C_τ from Table A.9, should be obtained from:

$$\tau_{cr} = 0{,}75 E C_\tau \frac{t}{r} \tag{A.18}$$

Table A.9 - Factor C_τ for critical shear buckling stress

Cylindrical shell	$\omega = \frac{l}{\sqrt{rt}}$	Factor C_τ
Short	$\omega \le 10$	$C_\tau = \sqrt{1 + \frac{42}{\omega^3}}$
Medium-length	$10 < \omega < 8{,}7\frac{r}{t}$	$C_\tau = 1$
Long	$\omega \ge 8{,}7\frac{r}{t}$	$C_\tau = \frac{1}{3}\sqrt{\frac{\omega t}{r}}$

A.1.4.2 Shear buckling parameters

(1) The shear imperfection factor should be obtained from:

$$\alpha_\tau = \frac{1}{1+0{,}2(1-\alpha_{\tau,\mathrm{ref}})(\bar{\lambda}_\tau - \bar{\lambda}_{\tau,0})/a^2_{\tau,\mathrm{ref}}} \quad \text{but} \quad a_\tau \le 1{,}00 \tag{A.19}$$

(2) The shear imperfection factor $\alpha_{\tau,\mathrm{ref}}$ should be taken from Table A.10 for the specified tolerance class.

Table A.10 - Factor $\alpha_{\tau,\mathrm{ref}}$ based on tolerance

Tolerance class	Parameter $\alpha_{\tau,\mathrm{ref}}$
Class 1	0,50
Class 2	0,65
Class 3 and 4	0,75

(3) The alloy factor and the AC1 shear AC1 squash limit slenderness parameter should be taken from Table A.11 according to the material buckling class as defined in EN 1999-1-1.

Table A.11 - Values of $\bar{\lambda}_{\tau,0}$ and μ_τ for shear

Material buckling class	$\bar{\lambda}_{\tau,0}$	μ_τ
A	0,50	0,30
B	0,40	0,40

A.1.5 Meridional (axial) compression with coexistent internal pressure

A.1.5.1 Pressurised critical meridional buckling stress

(1) The critical meridional buckling stress $\sigma_{x,\mathrm{cr}}$ may be assumed to be unaffected by the presence of internal pressure and may be obtained as specified in A.1.2.1.

A.1.5.2 Pressurised meridional buckling parameters

(1) The pressurised meridional buckling strength should be verified analogously to the unpressurised meridional buckling strength as specified in 6.2.3.3 and A.1.2.2. However, the unpressurised imperfection factor α_x may be replaced by the pressurised imperfection factor $\alpha_{x,\mathrm{p}}$.

(2) The pressurised imperfection factor $\alpha_{x,\mathrm{p}}$ should be taken as the smaller of the two following values:

$\alpha_{x,\mathrm{pe}}$ is a factor covering pressure-induced elastic stabilisation;

$\alpha_{x,\mathrm{pp}}$ is a factor covering pressure-induced plastic destabilisation.

(3) The factor $\alpha_{x,\mathrm{pe}}$ should be obtained from:

$$\alpha_{x,\mathrm{pe}} = \alpha_x + (1-\alpha_x)\frac{\bar{p}}{\bar{p} + 0{,}3/\alpha_x^{0{,}5}} \tag{A.20}$$

$$\bar{p} = \frac{pr}{t\sigma_{x,\mathrm{cr}}} \tag{A.21}$$

where:

$\bar{p}$ is the smallest value of internal pressure at the location of the point being assessed, and guaranteed to coexist with the meridional compression;

α_x is the unpressurised meridional imperfection factor according to A.1.2.2;

$\sigma_{x,\text{cr}}$ is the elastic critical meridional buckling stress according to A.1.2.1(3).

(4) The factor $\alpha_{x,\text{pe}}$ should not be applied to cylinders that are long according to A.1.2.1(3), Table A.1. Further, it should not be applied unless:

- the cylinder is medium-length according to A.1.2.1(3), Table A.1;
- the cylinder is short according to A.1.2.1(3), Table A.1 and $C_x = 1$ has been adopted in A.1.2 1(3).

(5) The factor $\alpha_{x,\text{pp}}$ should be obtained from:

$$\alpha_{x,\text{pp}} = \left(1 - \frac{\bar{p}^2}{\bar{\lambda}_x^4}\right)\left(1 - \frac{1}{1{,}12 + s^{1{,}5}}\right)\frac{s^2 + 1{,}21\bar{\lambda}_x^2}{s(s+1)} \tag{A.22}$$

$$\bar{p} = \frac{pr}{t\sigma_{x,\text{cr}}} \tag{A.23}$$

$$s = \frac{r}{400t} \tag{A.24}$$

where:

$\bar{p}$ is the largest value of internal pressure at the location of the point being assessed, and possibly coexistent with the meridional compression;

$\bar{\lambda}_\text{x}$ is the non-dimensional shell slenderness parameter according to 6.2.3.2 (3);

$\sigma_{x,\text{cr}}$ is the elastic critical meridional buckling stress according to A.1.2.1(3).

A.1.6 Combinations of meridional (axial) compression, circumferential (hoop) compression and shear

(1) The buckling interaction parameters to be used in 6.2.3.3(3) may be obtained from:

$$\begin{aligned} k_x &= 1{,}25 + 0{,}75\chi_x \\ k_\theta &= 1{,}25 + 0{,}75\chi_\theta \\ k_\tau &= 1{,}25 + 0{,}75\chi_\tau \\ k_\text{i} &= (\chi_x \chi_\theta)^2 \end{aligned} \tag{A.25}$$

where χ_x, χ_θ and χ_τ are the buckling reduction factors defined in 6.2.3.2, using the buckling parameters given in A.1.2 to A.1.4.

(2) The three membrane stress components should be deemed to interact in combination at any point in the shell, except those adjacent to the boundaries. The buckling interaction check may be omitted for all points that lie within the boundary zone length l_s adjacent to either end of the cylindrical segment. The value of l_s is the smaller of:

$$l_\text{s} = 0{,}1L \quad \text{and} \quad l_\text{s} = 0{,}16r\sqrt{r/t} \tag{A.26}$$

(3) If checks of the buckling interaction at all points are found to be onerous, the following provisions of (4) and (5) permit a simpler conservative assessment. If the maximum value of any of the buckling-relevant membrane stresses in a cylindrical shell occurs in a boundary zone of length l_s adjacent to either end of the cylinder, the interaction check of 6.2.3.3 (3) may be undertaken using the values defined in (4).

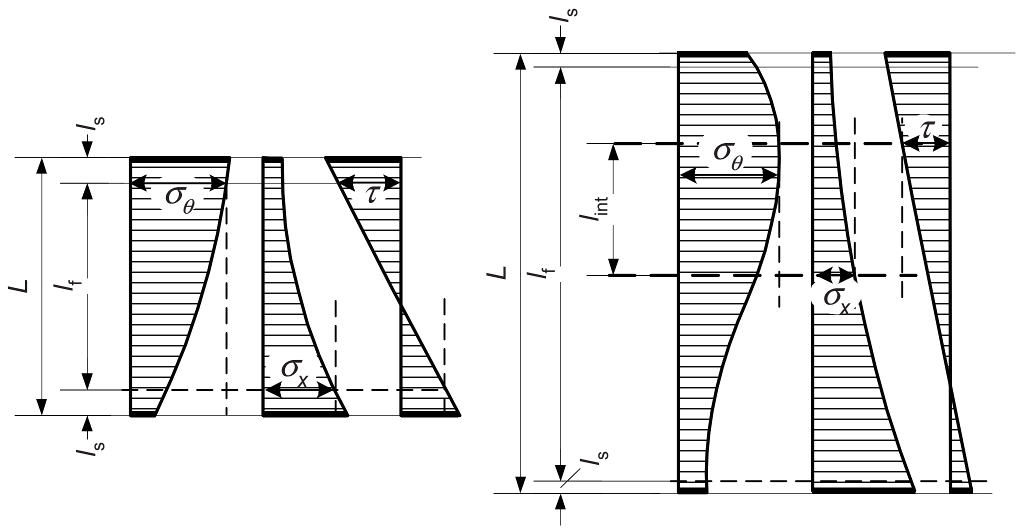
(4) If the conditions of (3) are met, the maximum value of any of the buckling-relevant membrane stresses occurring over the free length l_f that is outside the boundary zones (see Figure A.3a) may be used in the interaction check of 6.2.3.3 (3). where:

$$l_f = L - 2l_s \tag{A.27}$$

(5) For long cylinders as defined A.1.2.1(3), Table A.1, the interaction-relevant groups introduced into the interaction check may be restricted further than the provisions of paragraphs (3) and (4). The stresses deemed to be in interaction-relevant groups may then be restricted to any section of length l_{int} falling within the free remaining length l_f for the interaction check (see Figure A.3b), where:

$$l_{int} = 1{,}3r\sqrt{r/t} \tag{A.28}$$

(6) If (3) to (5) above do not provide specific provisions for defining the relative locations or separations of interaction-relevant groups of membrane stress components, and a simple conservative treatment is still required, the maximum value of each membrane stress, irrespective of location in the shell, may be adopted into expression (6.24).



a) short cylinder b) long cylinder

Figure A.3 - Examples of interaction-relevant groups of membrane stress components

A.2 Unstiffened cylindrical shells of stepwise wall thickness

A.2.1 General

A.2.1.1 Notations and boundary conditions

(1) In this clause the following notations are used:

- L overall cylinder length between boundaries;
- r radius of cylinder middle surface;
- j an integer index denoting the individual cylinder sections with constant wall thickness (from j = 1 to $j = n$);
- t_j the constant wall thickness of section j of the cylinder;
- l_j the length of section j of the cylinder.

(2) The following expressions may only be used for shells with boundary condition BC 1 and BC 2 at both edges (see 5.2), with no distinction made between them.

A.2.1.2 Geometry and joint offsets

(1) Provided that the wall thickness of the cylinder increases progressively stepwise from top to bottom (see Figure A.4a), the procedures given in this clause may be used. Alternatively, *linear elastic bifurcation analysis* LBA may be used to calculate the critical circumferential buckling stress $\sigma_{\theta,\mathrm{cr,eff}}$ in A.2.3.1(7).

(2) Intended offsets e_0 between plates of adjacent sections (see Figure A.4) may be treated as covered by the following expressions provided that the intended value e_0 is less than the permissible value $e_{0,\mathrm{p}}$ which should be taken as the smaller of:

$$e_{0,\mathrm{p}} = 0{,}5(t_{\max} - t_{\min}) \quad \text{and} \quad e_{0,\mathrm{p}} = 0{,}5 t_{\min} \tag{A.29}$$

where:

$t_{\max}$ is the thickness of the thicker plate at the joint;

$t_{\min}$ is the thickness of the thinner plate at the joint.

(3) For cylinders with permissible intended offsets between plates of adjacent sections according to (2), the radius r may be taken as the mean value of all sections.

(4) For cylinders with overlapping joints (lap joints), the provisions for lap-jointed construction given in A.3 should be used.

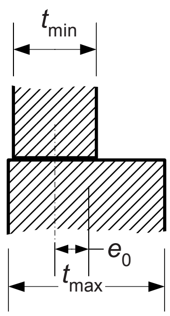


Figure A.4 - Intended offset e_0 in a butt-jointed shell

A.2.2 Meridional (axial) compression

(1) Each cylinder section j of length l_j should be treated as an equivalent cylinder of overall length $l = L$ and of uniform wall thickness $t = t_j$ according to A.1.2.

(2) For long equivalent cylinders, as governed by A.1.2.1(3), Table A.1, the parameter $C_{x\mathrm{b}}$ should be conservatively taken as $C_{x\mathrm{b}} = 1$, unless a better value is justified by more rigorous analysis.

A.2.3 Circumferential (hoop) compression

A.2.3.1 Critical circumferential buckling stresses

(1) If the cylinder consists of three sections with different wall thickness, the procedure according to (4) to (7) should be applied, see Figure A.5(II)

(2) If the cylinder consists of only one section (i.e. constant wall thickness), A.1 should be applied.

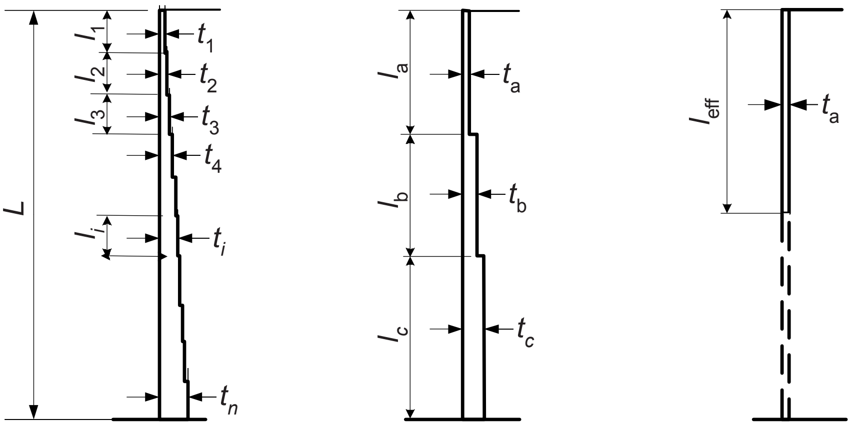
(3) If the cylinder consists of two sections of different wall thickness, the procedure of (4) to (7) should be applied, treating two of the three fictitious sections, a and b, as being of the same thickness.

(4) If the cylinder consists of more than three sections with different wall thicknesses (see Figure A.5(I)), it should first be replaced by an equivalent cylinder comprising three sections a, b and c (see Figure A.5(II)). The length of its upper section, l_a, should extend to the upper edge of the first section that has a wall

thickness greater than 1,5 times the smallest wall thickness t_j, but should not comprise more than half the total length L of the cylinder. The length of the two other sections l_b and l_c should be obtained as follows:

$$l_b = l_a \quad \text{and} \quad l_c = L - 2l_a \qquad \text{if } l_a \le L/3 \tag{A.30}$$

$$l_b = l_c = 0{,}5(L - l_a) \qquad \text{if } L/3 < l_a \le L/2 \tag{A.31}$$



(I) Cylinder of stepwise variable wall thickness

(II) Equivalent cylinder comprising of three sections

(III) Equivalent single cylinder with uniform wall thickness

Figure A.5 - Transformation of stepped cylinder into equivalent cylinder

(5) The fictitious wall thickness t_a, t_b and t_c of the three sections should be determined as the weighted average of wall thickness over each of the three fictitious sections:

$$t_a = \frac{1}{l_a} \sum_a l_j t_j \tag{A.32}$$

$$t_b = \frac{1}{l_b} \sum_b l_j t_j \tag{A.33}$$

$$t_c = \frac{1}{l_c} \sum_c l_j t_j \tag{A.34}$$

(6) The three-section-cylinder (i.e. the equivalent one or real one respectively) should be replaced by an equivalent single cylinder of effective length l_{eff} and of uniform wall thickness $t = t_a$ (see Figure A.5(III)). The effective length should be determined from:

$$l_{eff} = \frac{l_a}{\kappa} \tag{A.35}$$

in which κ is a dimensionless factor obtained from Figure A.6.

(7) For cylinder sections of moderate or short length, the critical circumferential buckling stress of each cylinder section j of original cylinder of stepwise variable wall thickness should be determined from:

$$\sigma_{\theta,cr,j} = \frac{t_a}{t_j} \sigma_{\theta,cr,eff} \tag{A.36}$$

where $\sigma_{\theta,cr,eff}$ is the critical circumferential buckling stress derived from A.1.3.1(3) or A.1.3.1(4) as appropriate, of the equivalent single cylinder of length l_{eff} according to (6). The factor C_θ in these expressions should be given the value $C_\theta = 1{,}0$.

(8) The length of the shell segment is characterised in terms of the dimensionless parameter ω_j:

$$\omega_j = \frac{l_j}{r}\sqrt{\frac{r}{t_j}} = \frac{l_j}{\sqrt{rt_j}} \tag{A.37}$$

(9) If the cylinder section j is long, a second additional assessment of the buckling stress should be made. The smaller of the two values derived from (7) and (10) should be used for the buckling design of the cylinder section j.

(10) The cylinder section j should be treated as long if:

$$\omega_j \geq 1{,}63\frac{r}{t_j} \tag{A.38}$$

in which case the critical circumferential buckling stress should be obtained from:

$$\sigma_{\theta,\mathrm{cr},j} = E\left(\frac{t_j}{r}\right)^2\left(0{,}275 + 2{,}03\left(\frac{C_\theta r}{\omega_j t_j}\right)^4\right) \tag{A.39}$$

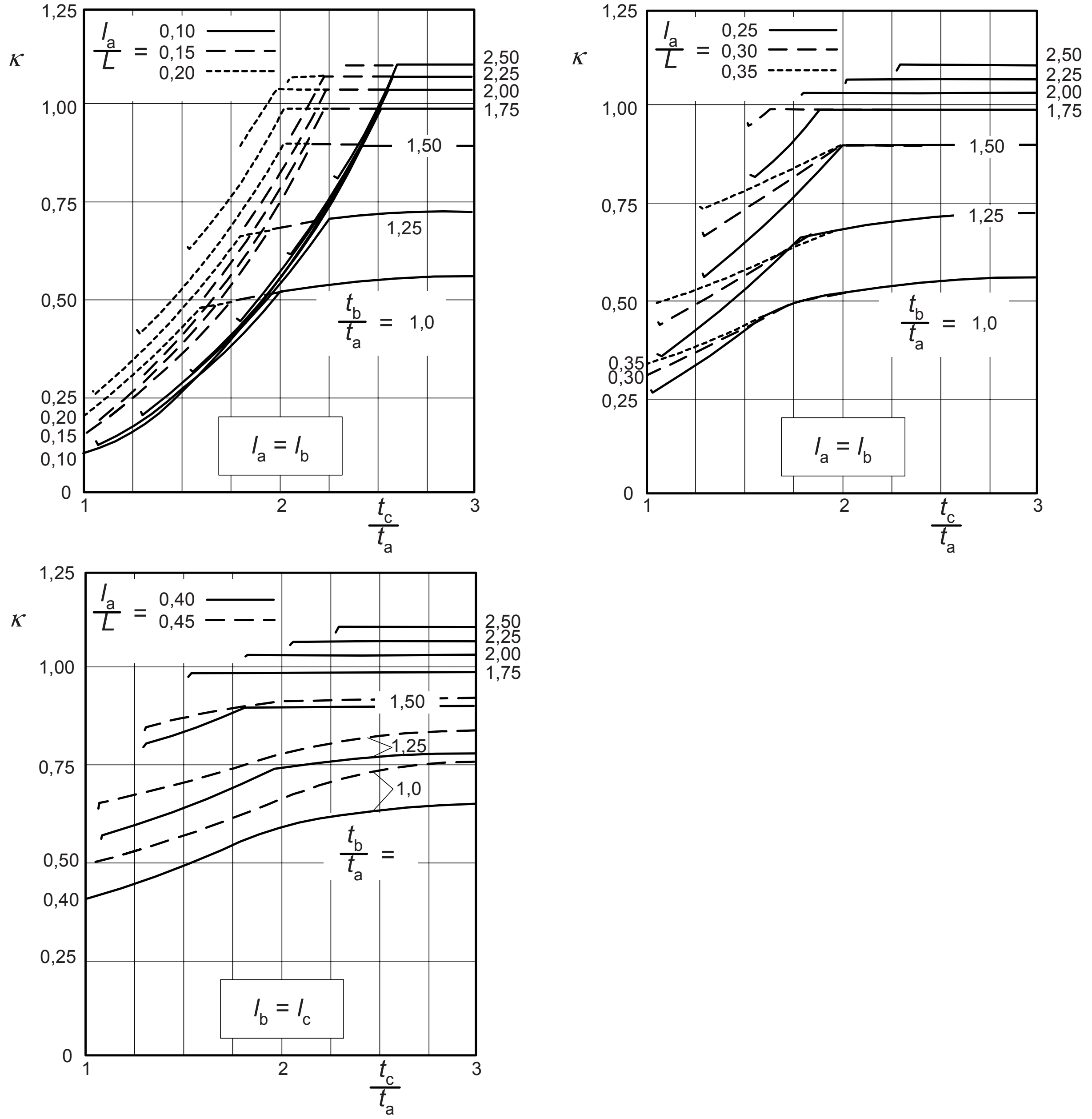


Figure A.6 - Factor κ for determining of the effective length l_{eff}

A.2.3.2 Buckling strength verification for circumferential compression

(1) For each cylinder section j, the conditions of 6.2.3 should be met, and the following check should be carried out:

$$\sigma_{\theta,\mathrm{Ed},j} \leq \sigma_{\theta,\mathrm{Rd},j} \tag{A.40}$$

where:

$\sigma_{\theta,\mathrm{Ed},j}$ is the key value of the circumferential compressive membrane stress, as detailed in the following clauses;

$\sigma_{\theta,\mathrm{Rd},j}$ is the design circumferential buckling stress, as derived from the critical circumferential buckling stress according to A.1.3.2.

(2) Provided that the design value of the circumferential stress resultant $n_{\theta,\mathrm{Ed}}$ is constant throughout the length L the key value of the circumferential compressive membrane stress in the section j, should be taken as the value:

$$\sigma_{\theta,\mathrm{Ed},j} \leq \frac{n_{\theta,\mathrm{Ed}}}{t_j} \tag{A.41}$$

(3) If the design value of the circumferential stress resultant $n_{\theta,\mathrm{Ed}}$ varies within the length L, the key value of the circumferential compressive membrane stress should be taken as a fictitious value $\sigma_{\theta,\mathrm{Ed},j,\mathrm{mod}}$ determined from the maximum value of the circumferential stress resultant $n_{\theta,\mathrm{Ed}}$ anywhere within the length L divided by the local thickness t_j (see Figure A.7), determined as:

$$\sigma_{\theta,\mathrm{Ed},j,\mathrm{mod}} = \frac{\max(n_{\theta,\mathrm{Ed}})}{t_j} \tag{A.42}$$

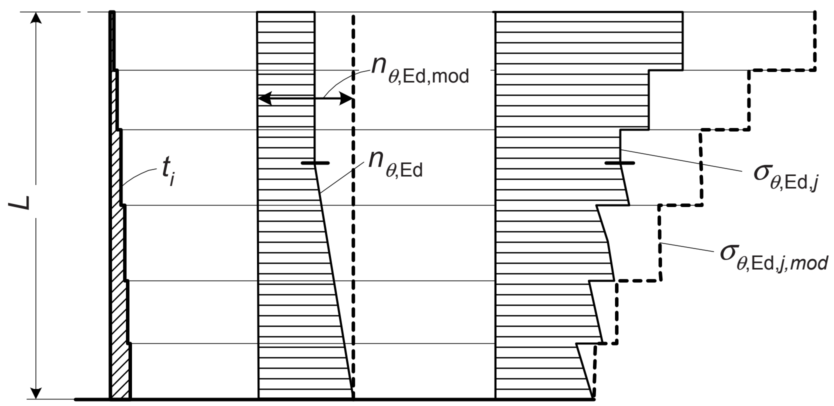


Figure A.7 - Key values of the circumferential compressive membrane stress in cases where $n_{\theta,\mathrm{Ed}}$ varies within the length L

A.2.4 Shear

A.2.4.1 Critical shear buckling stress

(1) If no specific rule for evaluating an equivalent single cylinder of uniform wall thickness is available, the expressions of A.2.3.1(1) to (6) may be applied.

(2) The further determination of the critical shear buckling stresses may on principle be performed as in A.2.3.1(7) to (10), but replacing the circumferential compression expressions from A.1.3.1 by the relevant shear expressions from A.1.4.1.

A.2.4.2 Buckling strength verification for shear

(1) The rules of A.2.3.2 may be applied, but replacing the circumferential compression expressions by the relevant shear expressions.

A.3 Unstiffened lap jointed cylindrical shells

A.3.1 General

A.3.1.1 Definitions

1. circumferential lap joint

a lap joint that runs in the circumferential direction around the shell axis.

2. meridional lap joint

a lap joint that runs parallel to the shell axis (meridional direction).

A.3.1.2 Geometry and stress resultants

(1) If a cylindrical shell is constructed using lap joints (see Figure A.8), the following provisions may be used in place of those set out in A.2.

(2) The following provisions apply both to lap joints that increase, and to lap joints that decrease the radius of the middle surface of the shell. If the lap joint runs in a circumferential direction around the shell axis (circumferential lap joint), the provisions of A.3.2 should be used for meridional compression. If many lap joints run in a circumferential direction around the shell axis (circumferential lap joints) with changes of plate thickness down the shell, the provisions of A.3.3 should be used for circumferential compression. If a single lap joint runs parallel to the shell axis (meridional lap joint), the provisions of A.3.3 should be used for circumferential compression. In other cases, no special consideration need be given for the influence of lap joints on the buckling resistance.

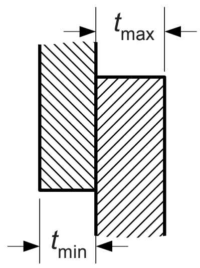


Figure A.8 - Lap jointed shell

A.3.2 Meridional (axial) compression

(1) If a lap jointed cylinder is subject to meridional compression, with meridional lap joints, the buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, but with the design resistance reduced by the factor 0,70.

(2) If a change of plate thickness occurs at the lap joint, the design buckling resistance may be taken as the same value as for that of the thinner plate as determined in (1).

A.3.3 Circumferential (hoop) compression

(1) If a lap jointed cylinder is subject to circumferential compression across meridional lap joints, the design buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate, but with a reduction factor of 0,90.

(2) If a lap jointed cylinder is subject to circumferential compression, with many circumferential lap joints and a changing plate thickness down the shell, the procedure of A.2 should be used without the geometric restrictions on joint eccentricity, and with the design buckling resistance reduced by the factor 0,90.

(3) If the lap joints are used in both directions, with staggered placement of thc meridional lap joints in alternate strakes or courses, the design buckling resistance should be evaluated as the lower of those found in (1) or (2). No further resistance reduction is needed.

A.3.4 Shear

(1) If a lap jointed cylinder is subject to membrane shear, the buckling resistance may be evaluated as for a uniform or stepped-wall cylinder, as appropriate.

A.4 Unstiffened conical shells

A.4.1 General

A.4.1.1 Notation

(1) In this clause the following notations are used:

- h is the axial 1ength (height) of the truncated cone;
- L is the meridional length of the truncated cone;
- r is the radius of the cone middle surface, perpendicular to axis of rotation, that vanes linearly down the length;
- r_1 is the radius at the small end of the cone;
- r_2 is the radius at the large end of the cone;
- β is the apex half angle of cone.

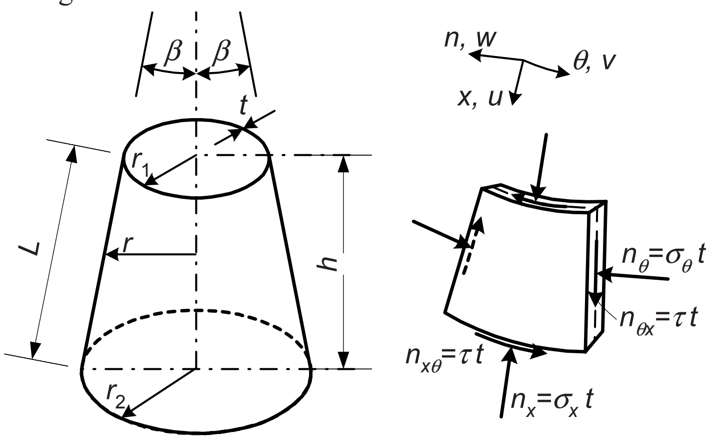


Figure A.9 - Cone geometry, membrane stresses and stress resultants

A.4.1.2 Boundary conditions

(1) The following expressions should be used only for shells with boundary conditions BC 1 or BC 2 at both edges (see 5.2 and 6.2), with no distinction made between them. They should not be used for a shell in which any boundary condition is BC 3.

(2) The rules in this clause A.4.1 should be used only for the following two radial displacement restraint boundary conditions, at either end of the cone:

"cylinder condition" $w = 0$;

"ring condition" $u \sin\beta + w \cos\beta = 0$

A.4.1.3 Geometry

(1) Only truncated cones of uniform wall thickness and with apex half angle $\beta \leq 65°$ (see Figure A.9) are covered by the following rules.

A.4.2 Design buckling stresses

A.4.2.1 Equivalent cylinder

(1) The design buckling stresses that are needed for the buckling strength verification according to 6.2.3 may be derived from an equivalent cylinder of length l_e and of radius r_e in which l_e and r_e depend on the type of stress according to Table A.12.

Table A.12 - Equivalent cylinder length and radius

Loading	Equivalent length	Equivalent cylinder radius
Meridional compression	$l_e = L$	$r_e = \dfrac{r}{\cos\beta}$
Circumferential (hoop) compression	$l_e = L$	$r_e = \dfrac{r_1 + r_2}{2\cos\beta}$
Uniform external pressure q Boundary conditions: Either BC 1 at both ends or BC 2 at both ends	l_e is the lesser of $l_{e,1} = L$ and $l_{e,2} = \dfrac{r_2(0{,}53 + 0{,}125\beta)}{\sin\beta}$ (β in radians, see Figure A.9)	$r_e = \dfrac{0{,}55r_1 + 0{,}45r_2}{\cos\beta}$ if $l_e = l_{e,1}$ (shorter cones) $r_e = 0{,}71r_2 \dfrac{1 - 0{,}1\beta}{\cos\beta}$ if $l_e = l_{e,2}$ (longer cones)
Shear	$l_e = h$	$r_e = \left(1 + \rho - \dfrac{1}{\rho}\right) r_1 \cos\beta$ in which $\rho = \sqrt{\dfrac{r_1 + r_2}{2r_1}}$
Uniform torsion	$l_e = L$	$r_e = r_1 \cos\beta (1 - \rho^{2,5})^{0,4}$ in which $\rho = \dfrac{L\sin\beta}{r_2}$

(2) For cones under uniform external pressure q, the buckling strength verification should be based on the membrane stress:

$$\sigma_{\theta,\mathrm{Ed}} = q r_e / t \tag{A.43}$$

A.4.3 Buckling strength verification

A.4.3.1 Meridional compression

(1) The buckling design check should be carried out at that point of the cone if the combination of acting design meridional stress and design buckling stress according to A.3.2.2 is most critical.

(2) In the case of meridional compression caused by a constant axial force on a truncated cone, both the small radius r_1 and the large radius r_2 should be considered as possibly the location of the most critical position.

(3) In the case of meridional compression caused by a constant global bending moment on the cone, the small radius r_1 should be taken as the most critical.

(4) The design buckling stress should be determined for the equivalent cylinder according to A.1.2.

A.4.3.2 Circumferential (hoop) compression

(1) If the circumferential compression is caused by uniform external pressure, the buckling design check should be carried out using the acting design circumferential stress $\sigma_{\theta,\mathrm{Ed,env}}$ determined using expression (A.43) and the design buckling stress according to A.3.2.1 and A.3.2.3.

(2) If the circumferential compression is caused by actions other than uniform external pressure, the calculated stress distribution $\sigma_{\theta,\mathrm{Ed}}(x)$ should be replaced by a stress distribution $\sigma_{\theta,\mathrm{Ed,env}}(x)$ that everywhere exceeds the calculated value, but which would arise from a fictitious uniform external pressure. The buckling design check should then be carried out as in (1), but using $\sigma_{\theta,\mathrm{Ed,env}}$ instead of $\sigma_{\theta,\mathrm{Ed}}$.

(3) The design buckling stress should be determined for the equivalent cylinder according to A.1.3.

A.4.3.3 Shear and uniform torsion

(1) In the case of shear caused by a constant global torque on the cone, the buckling design check should be carried out using the acting design shear stress τ_{Ed} at the point with $r = r_{\mathrm{e}} \cos\beta$ and the design buckling stress τ_{Rd} according to A.3.2.1 and A.3.2.4.

(2) If the shear is caused by actions other than a constant global torque (such as a global shear force on the cone), the calculated stress distribution $\tau_{\mathrm{Ed}}(x)$ should be replaced by a fictitious stress distribution $\tau_{\mathrm{Ed,env}}(x)$ that everywhere exceeds the calculated value, but which would arise from a fictitious global torque. The buckling design check should then be carried out as in (1), but using $\tau_{\mathrm{Ed,env}}$ instead of τ_{Ed}.

(3) The design buckling stress τ_{Rd} should be determined for the equivalent cylinder according to A.1.4.

A.5 Stiffened cylindrical shells of constant wall thickness

A.5.1 General

(1) Stiffened cylindrical shells can be made of either:

- isotropic walls stiffened with meridional and circumferential stiffeners;

- corrugated walls stiffened with meridional and circumferential stiffeners.

(2) In both cases, buckling checks can be made by assuming the stiffened wall to behave as an equivalent orthotropic shell according to the rules given in A.5.6, provided that conditions issued in A.5.6 are met.

(3) In case of circumferentially corrugated sheeting without meridional stiffeners the plastic buckling resistance can be calculated according to rules given in A.5.4.2(3), (4) and (5).

(4) If the circumferentially corrugated sheeting is assumed to carry no axial load, the buckling resistance of an individual stiffener can be evaluated according to A.5.4.3.

A.5.2 Isotropic walls with meridional stiffeners

A.5.2.1 General

(1) If an isotropic wall is stiffened by meridional (stringer) stiffeners, the effect of compatibility of the shortening of the wall due to internal pressure should be taken into account in assessing the meridional compressive stress in both the wall and the stiffeners.

(2) The resistance against rupture on a meridional seam should be determined as for an isotropic shell.

(3) If a structural connection detail includes the stiffener as part of the means of transmitting circumferential tensions, the effect of this tension on the stiffener should be taken into account in evaluating the force in the stiffener and its susceptibility to rupture under circumferential tension.

A.5.2.2 Meridional (axial) compression

(1) The wall should be designed for the same axial compression buckling criteria as the unstiffened wall unless the maximum meridional distance between stiffeners $d_{s,max}$ (Figure A.10) is lower than $2\sqrt{rt}$, where t is the local thickness of the wall.

(2) Where meridional stiffeners are placed at closer spacing than $2\sqrt{rt}$, the buckling resistance of the complete wall should be assessed by using the procedure given in A.5.6.

(3) The axial compression buckling strength of the stiffeners themselves should be evaluated using the provisions of EN 1999-1-1.

(4) The eccentricity of the stiffener to the shell wall should be taken into account, where appropriate.

A.5.2.3 Circumferential (hoop) compression

(1) The wall should be checked for the same external pressure buckling criteria as the unstiffened wall unless a more rigorous calculation is carried out.

(2) In a more rigorous calculation the meridional stiffeners may be smeared to give an orthotropic wall, and the buckling stress assessment carried out using the provisions of A.5.6, assuming a stretching stiffness $C_\phi = C_\theta = Et$ and a shear membrane stiffness $C_{\phi\theta} = 0{,}38Et$.

A.5.2.4 Shear

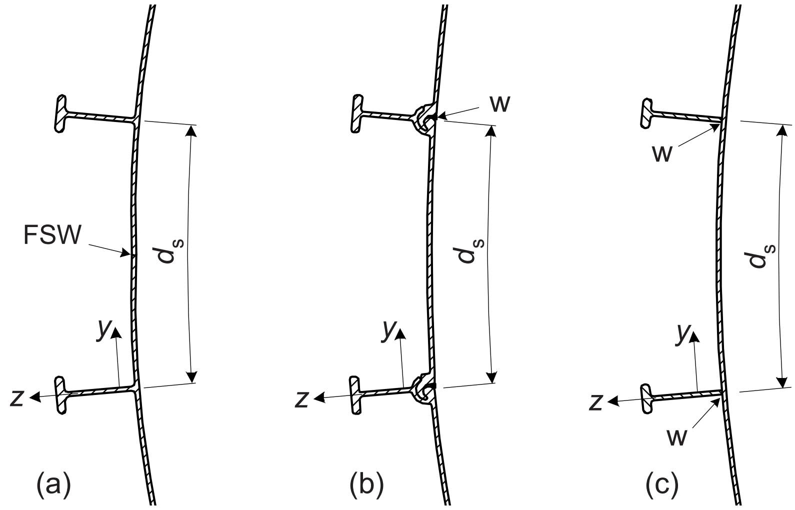
(1) If a major part of the shell wall is subjected to shear loading (as with eccentric filling, earthquake loading etc.), the membrane shear buckling resistance should be found as for an isotropic unstiffened wall (see A.l. 4), but the resistance may be increased by taking account of the stiffeners. The equivalent length l of shell in shear may be taken as the lesser of the height between stiffening rings or boundaries and twice the meridional separation of the meridional stiffeners, provided that each stiffener has a flexural rigidity EI_y for bending in the meridional direction (about a circumferential axis) greater than:

$$EI_{y,\min} = 0{,}1Et^3\sqrt{rl} \tag{A.44}$$

where the values of l and t are taken as the same as those used in the most critical buckling mode.

(2) If a discrete stiffener is abruptly terminated part way up the shell, the force in the stiffener should be taken to be uniformly redistributed into the shell over a length not exceeding $4\sqrt{rt}$.

(3) If the stiffeners are terminated as above, or used to introduce local forces into the shell, the assessed resistance for shear transmission between the stiffener and the shell should not exceed the value given in A.1.4.



Key: w = weld, FSW = friction stir welding

Figure A.10 – Typical axially stiffened shells made of (a) and (b) extrusions and (c) plates and extrusions

A.5.3 Isotropic walls with circumferential stiffeners

(1) For the purpose of buckling checks, rules given in A.5.6 apply assuming the stiffened wall to behave as an orthotropic shell.

A.5.4 Circumferentially corrugated walls with meridional stiffeners

A.5.4.1 General

(1) All calculations should be carried out with thickness exclusive of coatings and geometric tolerances.

(2) The minimum core thickness for the corrugated sheeting of the wall should be 0,68 mm.

(3) If the cylindrical wall is fabricated from corrugated sheeting with the corrugations running circumferentially and meridional stiffeners are attached to the wall, the corrugated wall should be assumed to carry no meridional forces unless the wall is treated as an orthotropic shell, see A.5.6.

(4) Particular attention should be paid to ensure that the stiffeners are flexurally continuous with respect to bending in the meridional plane normal to the wall, because the flexural continuity of the stiffener is essential in developing resistance to buckling.

(5) If the wall is stiffened with meridional stiffeners, the fasteners between the sheeting and stiffeners should be proportioned to ensure that the distributed shear loading on each part of the wall sheeting is transferred into the stiffeners. The sheeting thickness should be chosen to ensure that local rupture at these fasteners is prevented, taking proper account of the reduced bearing strength of fasteners in corrugated sheeting.

(6) The design stress resultants, resistances and checks should be carried out as in 5, 6.1 and A.1, but including the additional provisions set out in (1) to (5) above.

NOTE Example of arrangement for stiffening the wall are shown in Figure A.11.

(7) Bolts for fastenings between panels should satisfy the requirements of EN 1999-1-1. The bolt size should not be less than M8.

(8) The joint detail between panels should comply with the provisions of EN 1999-1-4 for bolts loaded in shear.

(9) The spacing between fasteners around the circumference should not exceed 3° of the circumference.

(10) If penetrations are made in the wall for hatches, doors, augers or other items, a thicker corrugated sheet should be used locally to ensure that the local stress raisers associated with mismatches of stiffness do not lead to local rupture.

NOTE A typical bolt arrangement detail for a panel is shown in Figure A.12.

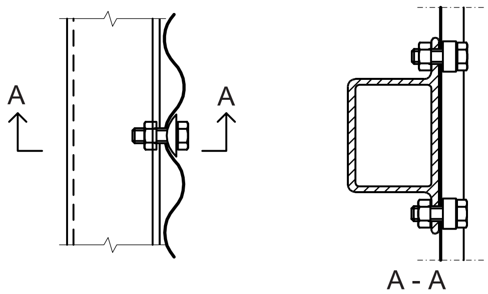


Figure A.11 – Example of arrangement for meridional stiffeners on circumferentially corrugated shells

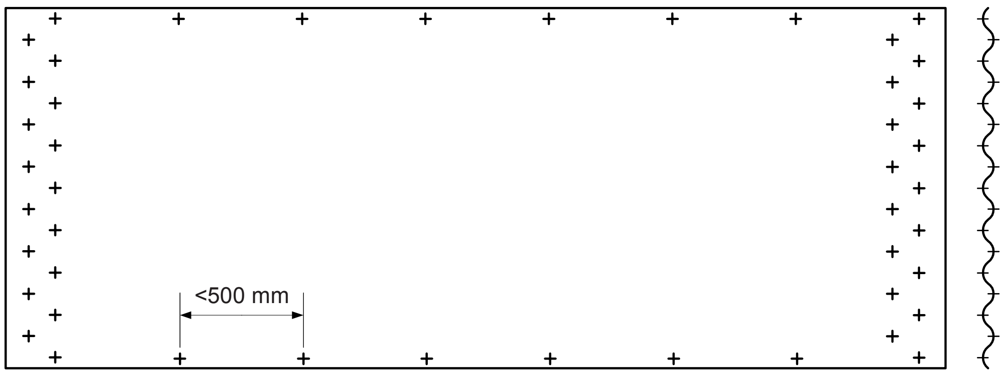


Figure A.12 - Typical bolt arrangement for panel of a corrugated shell

A.5.4.2 Axial compression

(1) Under axial compression, the design resistance should be determined at every point in the shell using the specified tolerance class for execution, the intensity of the guaranteed co-existent internal pressure p and the circumferential uniformity of the compressive stress. The design should consider every point on the shell wall, ignoring the meridional variation of the axial compression, except where the provisions of this Part allow for this.

(2) If the wall is stiffened with meridional stiffeners, the buckling design of the wall should be carried out using one of two alternative methods:

a) buckling of the equivalent orthotropic shell (following A.5.6) if the meridional distance between stiffeners satisfies A.5.6.1(3);

b) buckling of the individual stiffeners (corrugated wall assumed to carry no axial force, but providing restraint to the stiffeners) and following A.5.4.3 if the meridional distance between stiffeners does not satisfy A.5.6.1(3).

(3) If the corrugated shell has no meridional stiffeners, the characteristic value of local plastic buckling resistance should be determined as the greater of:

$$n_{x,\mathrm{Rk}} = \frac{t^2 f_\mathrm{o}}{2d} \tag{A.45}$$

and

$$n_{x,\mathrm{Rk}} = \frac{r_\phi t f_\mathrm{o}}{r} \tag{A.46}$$

where:

- t is the sheet thickness;
- d is the crest to trough amplitude;
- r_ϕ is the local curvature of the corrugation (see Figure A.14);
- r is the cylinder radius.

The local plastic buckling resistance $n_{x,\mathrm{Rk}}$ should be taken as independent of the value of internal pressure p_n.

NOTE The local plastic buckling resistance is the resistance to corrugation collapse or "roll-down".

(4) The design value of the local plastic buckling resistance should be determined as:

$$n_{x,\mathrm{Rd}} = \frac{\alpha_x n_{x,\mathrm{Rk}}}{\gamma_{\mathrm{M1}}} \tag{A.47}$$

in which $\alpha_x = 0{,}80$ and γ_{M1} as given in 2.7.2.

(5) At every point in the structure the design stresses should satisfy the condition:

$$n_{x,\mathrm{Ed}} \leq n_{x,\mathrm{Rd}} \tag{A.48}$$

A.5.4.3 Stiffened wall treated as carrying axial compression only in the stiffeners

(1) If the corrugated sheeting is assumed to carry no axial force (method (b) in A.5.4.3), the sheeting may be assumed to restrain all buckling displacements of the stiffener in the plane of the wall, and the resistance to buckling should be calculated using one of the two following two alternative methods:

(a) ignoring the supporting action of the sheeting in resisting buckling displacements normal to the wall;

(b) allowing for the stiffness of the sheeting in resisting buckling displacements normal to the wall.

(2) Using method (a) in (1), the resistance of an individual stiffener may be taken as the resistance to concentric compression on the stiffener. The design buckling resistance $N_{\mathrm{s,Rd}}$ should be obtained from:

$$N_{\mathrm{s,Rd}} = \frac{\chi A_{\mathrm{eff}} f_\mathrm{o}}{\gamma_{\mathrm{M1}}} \tag{A.49}$$

where A_{eff} is the effective cross-sectional area of the stiffener.

(3) The reduction factor χ should be obtained from EN 1999-1-1 for flexural bucking normal to the wall (about the circumferential axis) according to the type of alloy and using buckling curve 2 irrespective of the alloy adopted (α= 0,32 and $\bar{\lambda}_0 = 0$). The effective length of column used in determining the reduction factor χ should be taken as the distance between adjacent ring stiffeners.

(4) If the elastic restraint provided by the wall against buckling of the stiffener is taken into account, both of the following conditions should be met:

a) The section of wall deemed to provide restraint should be the length of wall as far as the adjacent stiffeners (see Figure A.13), with simply supported conditions at the two ends.

b) No account should be taken of the possible stiffness of stored bulk solid.

(5) Unless more precise calculations are made, the elastic critical buckling load $N_{s,cr}$ should be calculated assuming uniform compression on the cross-section at any level, using:

$$N_{s,cr} = 2\sqrt{EI_s k} \tag{A.50}$$

where:

EI_s is the flexural rigidity of the stiffener for bending out of the plane of the wall (Nmm^2);

k the flexural stiffness of the sheeting (N/mm per mm of wall height) spanning between meridional stiffeners, as indicated in Figure A.13;

(6) The flexural stiffness of the wall plate k should be determined assuming that the sheeting spans between adjacent meridional stiffeners on either side with simply supported boundary conditions, see Figure A.13. The value of k may be found using:

$$k = \frac{6D_\theta}{d_s^3} \tag{A.51}$$

where:

D_θ is the flexural rigidity of the sheeting for circumferential bending;

d_s is the separation of the meridional stiffeners.

(7) If the corrugation is an arc-and-tangent or sinusoidal profile, the value of D_θ may be taken from A.5.7(6). If other corrugation sections are adopted, the flexural rigidity for circumferential bending should be determined for the actual cross section.

(8) At every point in the stiffener, the design stresses should satisfy the condition:

$$N_{s,Ed} \leq N_{s,Rd} \tag{A.52}$$

(9) The resistance of the stiffeners to local and flexural torsional buckling should be determined using EN 1999-1-1.

A.5.4.4 Circumferential (hoop) compression

(1) For the purpose of buckling checks, rules given in A.5.6.3 apply assuming the stiffened wall to behave as an orthotropic shell.

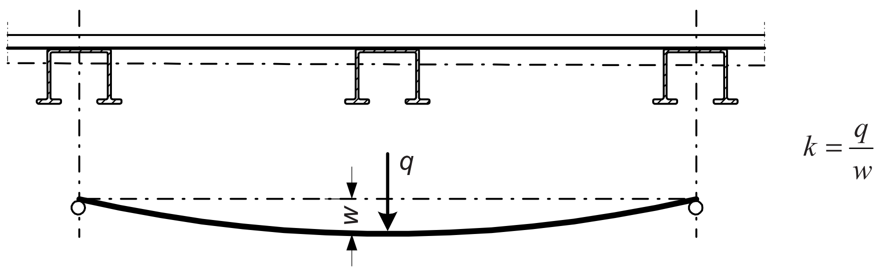


Figure A.13 – Plate restraint stiffness for evaluation of column buckling

A.5.5 Axially corrugated walls with ring stiffeners

A.5.5.1 General

(1) If the cylindrical wall is fabricated using corrugated sheeting with the corrugations running axially, both of the following conditions should be met:

a) the corrugated wall should be assumed to carry no meridional forces;

b) the corrugated sheeting should be assumed to span between attached rings, using the centre to centre separation between rings, and adopting the assumption of sheeting continuity.

(2) The joints between sheeting sections should be designed to ensure that assumed flexural continuity is achieved.

(3) The evaluation of the axial compression force in the wall arising from wall frictional tractions from the bulk solid should take account of the full circumference of the shell, allowing for the profile shape of the corrugation.

(4) If the corrugated sheeting extends to a base boundary condition, the local flexure of the sheeting near the boundary should be considered, assuming a radially restrained boundary.

(5) The corrugated wall should be assumed to carry no circumferential forces.

(6) The spacing of ring stiffeners should be determined using a beam bending analysis of the corrugated profile, assuming that the wall is continuous over the rings and including the consequences of different radial displacements of ring stiffeners that have different sizes. The stresses arising from this bending should be added to those arising from axial compression when checking the buckling resistance under axial compression.

NOTE The meridional bending of the sheeting can be analysed by treating it as a continuous beam passing over flexible supports at the ring locations. The stiffness of each support is then determined from the ring stiffness to radial loading.

(7) The ring stiffeners designed to carry the meridional load should be proportioned in accordance with EN 1999-1-1.

A.5.5.2 Axial compression

(1) For the purpose of buckling checks, rules given in A.5.6.2 apply assuming the stiffened wall to behave as an orthotropic shell.

A.5.5.3 Circumferential (hoop) compression

(1) For the purpose of buckling checks, rules given in A.5.6.3 apply assuming the stiffened wall to behave as an orthotropic shell.

A.5.6 Stiffened wall treated as an orthotropic shell

A.5.6.1 General

(1) If the stiffened wall, either isotropic or corrugated, is treated as an orthotropic shell, the resulting smeared stiffnesses should be taken to be uniformly distributed. In case of corrugated walls, the stiffnesses of the sheeting in different directions should be taken from A.5.7.

(2) The bending and stretching properties of the ring and stringer stiffeners, and the outward eccentricity of the centroid of each from the middle surface of the shell wall should be determined, together with the separation between the stiffeners d_s.

(3) The meridional distance between stiffeners d_s (Figure A.10) should not be more than $d_{s,max}$ given by:

$$d_{s,max} = 7{,}4 \left(\frac{r^2 D_y}{C_y} \right)^{0,25} \tag{A.53}$$

where:

D_y is the flexural rigidity per unit width in the circumferential direction (parallel to the corrugations if circumferentially corrugated sheeting);

C_y is the stretching stiffness per unit width in the circumferential direction (parallel to the corrugations if circumferentially corrugated sheeting).

A.5.6.2 Axial compression

(1) The critical buckling stress resultant $n_{x,\mathrm{cr}}$ per unit circumference of the orthotropic shell should be evaluated at each appropriate level in the shell by minimising the following expression with respect to the critical circumferential wave number j and the buckling height l_i :

$$n_{x,\mathrm{cr}} = \frac{1{,}2}{j^2\omega^2}\left(A_1 + \frac{A_2}{A_3}\right) \tag{A.54}$$

with:

$$A_1 = j^4\left[\omega^4 C_{44} + 2\omega^2(C_{45}+C_{66}) + C_{55}\right] + C_{22} + 2j^2C_{25} \tag{A.55}$$

$$A_2 = 2\omega^2(C_{12}+C_{33})(C_{22}+j^2C_{25})(C_{12}+j^2\omega^2C_{14}) - (\omega^2C_{11}+C_{33})(C_{22}+j^2C_{25})^2 - \\ -\omega^2(C_{22}+\omega^2C_{33})(C_{12}+j^2\omega^2C_{14})^2 \tag{A.56}$$

$$A_3 = (\omega^2C_{11}+C_{33})(C_{22}+C_{25}+\omega^2C_{33}) - \omega^2(C_{12}+C_{33})^2 \tag{A.57}$$

with:

$$C_{11} = C_\phi + EA_\mathrm{s}/d_\mathrm{s} \qquad C_{22} = C_\theta + EA_\mathrm{r}/d_\mathrm{r}$$

$$C_{12} = \nu\sqrt{C_\phi C_\theta} \qquad C_{33} = C_{\phi\theta}$$

$$C_{14} = e_\mathrm{s}EA_\mathrm{s}/(rd_\mathrm{s}) \qquad C_{25} = e_r EA_\mathrm{r}/(rd_\mathrm{r})$$

$$C_{44} = \frac{1}{r^2}(D_\phi + EI_\mathrm{s}/d_\mathrm{s}) \qquad C_{55} = \frac{1}{r^2}(D_\theta + EI_\mathrm{r}/d_\mathrm{r})$$

$$C_{45} = \frac{\nu}{r^2}\sqrt{D_\phi D_\theta} \qquad C_{66} = \frac{1}{r^2}\left[D_{\phi\theta} + 0{,}5(GI_\mathrm{ts}/d_\mathrm{s} + GI_\mathrm{tr}/d_\mathrm{r})\right]$$

$$\omega = \frac{\pi r}{jl_i}$$

where:

l_i is the half wavelength of the potential buckle in the meridional direction;

j number of buckling waves in the circumferential direction

A_s the cross-sectional area of a stringer stiffener;

I_s is the second moment of area of a stringer stiffener about the circumferential axis in the shell middle surface (meridional bending);

d_s is the separation between stringer stiffeners;

I_ts is the uniform torsion constant of a stringer stiffener;

e_s is the outward eccentricity from the shell middle surface of a stringer stiffener;

A_r the cross-sectional area of a ring stiffener;

I_r is the second moment of area of a ring stiffener about the meridional axis axis in the shell middle surface (circumferential bending);

d_r is the separation between ring stiffeners;

I_{tr} is the uniform torsion constant of a ring stiffener;

e_r is the outward eccentricity from the shell middle surface of a ring stiffener;

C_ϕ is the stretching stiffness in the axial direction;

C_θ is the stretching stiffness in the circumferential direction;

$C_{\phi\theta}$ is the stretching stiffness in membrane shear;

D_ϕ is the flexural rigidity in the axial direction;

D_θ is the flexural rigidity in the circumferential direction;

$D_{\phi\theta}$ is the twisting flexural rigidity in twisting;

r the radius of the shell.

NOTE 1 In case of corrugated sheeting, the above properties for the stiffeners (A_s, I_s , I_{ts} etc.) relate to the stiffener section alone: no allowance can be made for an "effective" section including parts of the shell wall.

NOTE 2 For both stretching and bending stiffness of corrugated sheeting, see A.5.7(5) and (6)

NOTE 3 The lower boundary of the buckle can be taken at the point at which either the sheeting thickness changes or the stiffener cross-section changes: the buckling resistance at each such change needs to be checked independently.

(2) The design buckling resistance $n_{x,Rd}$ for the orthotropic shell should be determined as stated in A.1.2 and 6.2.3.2, according to the shell quality class. The critical buckling resistance $n_{x,cr}$ should be obtained from (1) above. An increased quality factor $Q_{stiff} = 1{,}3Q$ may be assumed for stiffened shells made of isotropic walls.

A.5.6.3 Circumferential (hoop) compression

(1) The critical buckling stress for uniform external pressure $p_{n,cr}$ should be evaluated by minimising the following expression with respect to the critical circumferential wave number, j:

$$p_{n,cr} = \frac{1}{rj^2}\left(A_1 + \frac{A_2}{A_3}\right) \tag{A.58}$$

with A_1, A_2 and A_3 as given in A.5.1.2 (3).

(2) If the stiffeners or sheeting change with height up the wall, several potential buckling lengths l_i should be examined to determine which the most critical is, assuming always that the upper end of a buckle is at the top of the zone of thinnest sheeting.

NOTE If a zone of thicker sheeting is used above the zone that includes the thinnest sheeting, the upper end of the potential buckle could occur either at the top of the thinnest zone, or at the top of the wall.

(3) Unless more precise calculations are made, the thickness assumed in the above calculation should be taken as the thickness of the thinnest sheeting throughout.

(4) If the shell has no roof and is potentially subject to wind buckling, the above calculated pressure should be reduced by a factor 0,6.

(5) The design buckling stress for the wall should be determined as stated in 6.2.3.2 and A.1.3 according to the shell quality class. The critical buckling pressure $p_{n,cr}$ should be obtained from (1) above. Coefficient C_θ given in A.1.3.1 should be taken as $C_\theta = 1{,}0$.

A.5.6.4 Shear

(1) Rules given in A.5.2.4 for isotropic walls with meridional stiffeners apply.

A.5.7 Equivalent orthotropic properties of corrugated sheeting

(1) If corrugated sheeting is used as part of the shell structure, the analysis may be carried out treating the sheeting as an equivalent uniform orthotropic wall.

(2) The following properties may be used in a stress analysis and in a buckling analysis of the structure, provided that the corrugation profile has either an arc-and-tangent or a sinusoidal shape. If other corrugation profiles are used, the corresponding properties should be calculated for actual cross section, see EN 1999-1-4.

(3) The properties of the corrugated sheeting should be defined in terms of an *x-y* coordinate system in which the *y* axis runs parallel to the corrugations (straight lines on the surface) whilst *x* runs normal to the corrugations (troughs and peaks). The corrugation should be defined in terms of the following parameters, irrespective of the actual corrugation profile, see Figure A.14, where:

- d is the crest to crest dimension;
- l is the wavelength of the corrugation;
- r_ϕ is the local radius at the crest or trough.

(4) All properties may be treated as one-dimensional, giving no Poisson effects between different directions.

(5) The equivalent membrane properties (stretching stiffnesses) may be taken as:

$$C_x = Et_x = E\frac{2t^3}{3d^2} \tag{A.59}$$

$$C_y = Et_y = Et\left(1+\frac{\pi^2 d^2}{4l^2}\right) \tag{A.60}$$

$$C_{xy} = Et_{xy} = \frac{G2t}{1+\dfrac{\pi^2 d^2}{4l^2}} \tag{A.61}$$

where:

- t_x is the equivalent thickness for smeared membrane forces normal to the corrugations;
- t_y is the equivalent thickness for smeared membrane forces parallel to the corrugations;
- t_{xy} is the equivalent thickness for smeared membrane shear forces.

(6) The equivalent bending properties (flexural stiffnesses) are defined in terms of the flexural rigidity for moments causing bending in that direction (not about an axis), and may be taken as:

$$D_x = EI_x = \frac{Et^3}{12(1-\nu^2)}\frac{1}{1+\dfrac{\pi^2 d^2}{4l^2}} \tag{A.62}$$

$$D_y = EI_y = 0{,}13Etd^2 \tag{A.63}$$

$$D_{xy} = GI_{xy} = \frac{Gt^3}{12}\left(1+\frac{\pi^2 d^2}{4l^2}\right) \tag{A.64}$$

where:

- I_x is the equivalent second moment of area for smeared bending normal to the corrugations;

I_y is the equivalent second moment of area for smeared bending parallel to the corrugations;

I_{xy} is the equivalent second moment of area for twisting.

NOTE 1 Bending parallel to the corrugation engages the bending stiffness of the corrugated profile and is the chief reason for using corrugated construction.

NOTE 2 Alternative expressions for the equivalent orthotropic properties of corrugated sheeting are available in the references given in EN 1993-4-1.

(7) In circular shells, where the corrugations run circumferentially, the directions x and y in the above expressions should be taken as the axial ϕ and circumferential θ directions respectively. When the corrugations run meridionally, the directions x and y in the above expressions should be taken as the circumferential θ and axial ϕ directions respectively, see Figure A.14.

(8) The shearing properties should be taken as independent of the corrugation orientation. The value of G may be taken as E/2,6.

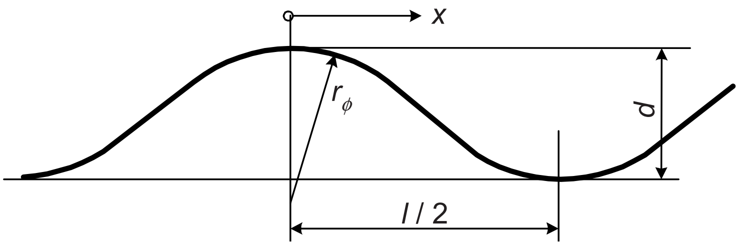


Figure A.14 - Corrugation profile and geometric parameters

A.6 Unstiffened spherical shells under uniform circumferential compression

A.6.1 Notations and boundary conditions

(1) General quantities (Figure A.15):

r radius of sphere middle surface;

t thickness of shell:

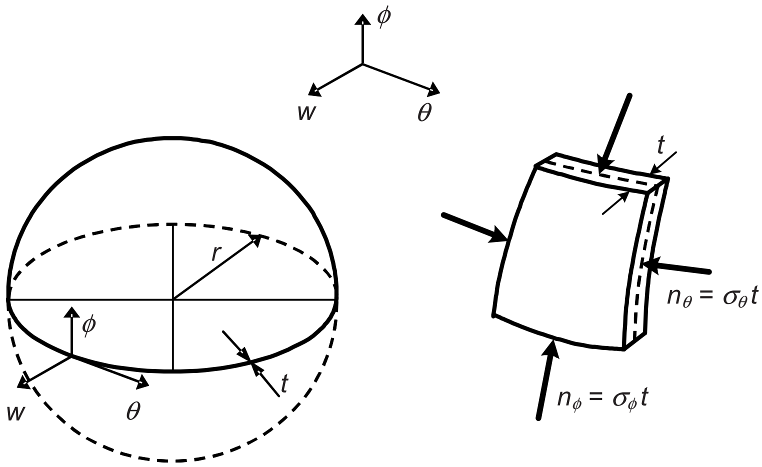


Figure A.15 - Sphere geometry and membrane stresses and stress resultants

(2) The boundary conditions are set out in 5.2 and 6.2.2.

A.6.2 Critical buckling stresses

(1) The following expressions may only be used for complete spheres or spherical caps with boundary conditions BC1r or BC1f at the base edge.

(2) Uniform circumferential compression in spheres or spherical caps is induced by uniform external pressure or may result from blowing action on circular silos or tank roof occurring during download.

(3) In case of circumferential compression due to uniform external pressure p the corresponding stress can be evaluated from:

$$\sigma_\theta = \sigma_\phi = \frac{pr}{2t} \tag{A.65}$$

(4) The critical buckling stress under uniform circumferential compression should be obtained from:

$$\sigma_{\theta,\mathrm{cr}} = \sigma_{\phi,\mathrm{cr}} = 0{,}605 E \frac{t}{r} \tag{A.66}$$

A.6.3 Circumferential buckling parameter

(1) The imperfection factor should be obtained from:

$$\alpha_\theta = \frac{1}{1 + 2{,}60\left(\frac{1}{Q}\sqrt{\frac{0{,}6E}{f_\mathrm{o}}}(\bar{\lambda}_\theta - \bar{\lambda}_{\theta,0})\right)^{1,44}} \quad \text{but} \quad \alpha_\theta \le 1{,}00 \tag{A.67}$$

where:

$\bar{\lambda}_{\theta,0}$ is the squash limit slenderness parameter;

Q is the tolerance parameter.

(2) The tolerance parameter Q should be taken from Table A.13 for the specified tolerance class.

(3) The alloy factor and the squash limit slenderness parameter should be taken from Table A.14 according to the material buckling class as defined in EN 1999-1-1.

Table A.13 - Tolerance parameter *Q*

Tolerance class	Q
Class 1	16
Class 2	25
Class 3 and 4	40

Table A.14 - Values of $\bar{\lambda}_{\theta,0}$ and μ_θ for uniform circumferential compression

Material buckling class	$\bar{\lambda}_{\theta,0}$	μ_θ
A	0,20	0,35
B	0,10	0,20

Annex B [informative] - Expressions for buckling analysis of toriconical and torispherical shells

B.1 General

(1) The rules in this section are valid for conical and spherical ends of cylindrical shells or equivalent structures connected by means of a torus or directly to the cylinder ($r_T = 0$).

B.2 Notations and boundary conditions

(1) In this clause the following notations are used, see Figure B.1:

r radius of middle surface of the cylindrical shell

r_S radius of spherical shell

α angle of the torus shell or half apex angle of the conical shell

r_T radius of torus

t_T thickness of torus, cone or spherical shell

l length of connecting cylinder

t_C wall thickness of connecting cylinder

(2) The rules are valid for constant external pressure acting orthogonal on the shell surface.

(3) The range of applicability is the following:

$$t_T \leq t_C \tag{B.1}$$

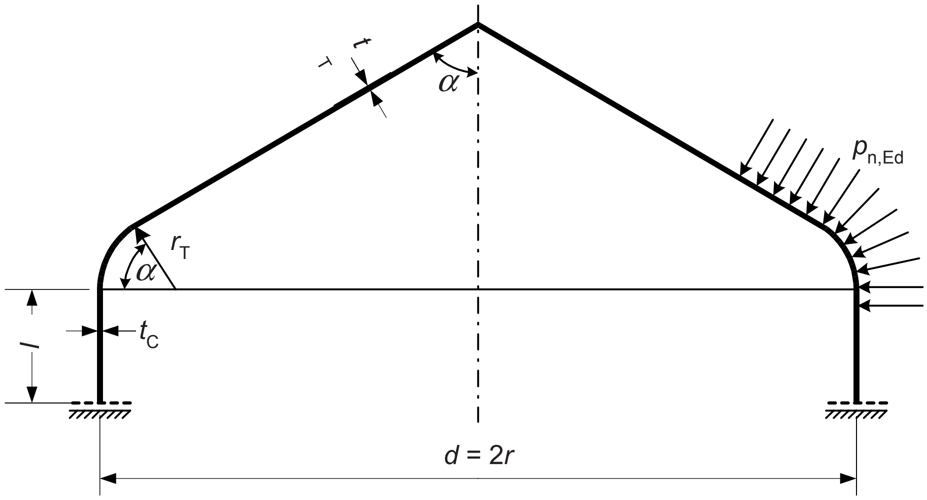
$$35 \leq r / t_C \leq 1250 \tag{B.2}$$

$$45° \leq \alpha \leq 75° \tag{B.3}$$

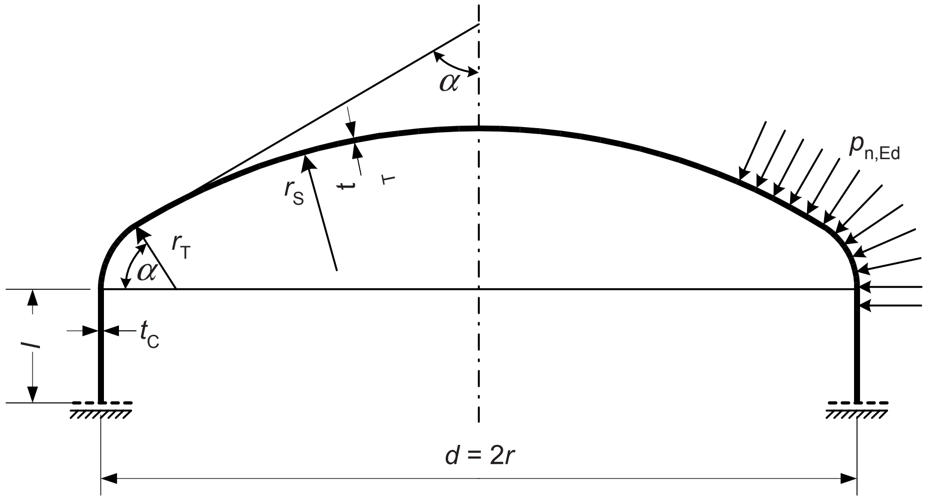
$$0 \leq r_T / r \leq 0{,}4 \tag{B.4}$$

$$1{,}2 \leq r_S / r \leq 3 \tag{B.5}$$

$$1 \leq 1000 f_o / E \leq 4 \tag{B.6}$$



a) toriconical shape



b) torispherical shape

Figure B.1 - Geometry and loads on vessel ends

B.3 External pressure

B.3.1 Critical external pressure

(1) The critical (buckling) external pressure for a toriconical shell is

$$p_{\mathrm{n,cr}} = \frac{2{,}42}{(1-\nu^2)^{0{,}75}} E \sin\alpha (\cos\alpha)^{1{,}5} \left(\frac{t_{\mathrm{T}}}{\bar{r}}\right)^{2{,}5} \quad \text{or} \tag{B.7}$$

$$p_{\mathrm{n,cr}} = 2{,}60 E \sin\alpha (\cos\alpha)^{1{,}5} (t_{\mathrm{T}} / \bar{r})^{2{,}5} \quad \text{for } \nu = 0{,}3$$

where

$$\vec{r} = r - r_{\mathrm{T}}(1-\cos\alpha) + \sqrt{r_{\mathrm{T}} t_{\mathrm{T}}}\,\sin\alpha \quad \text{but} \quad \vec{r} \le r$$

(2) The critical buckling external pressure for a torispherical shell is

$$p_{\mathrm{n,cr}} = 1{,}21 C_{\mathrm{k}} E \left(\frac{t_{\mathrm{T}}}{r_{\mathrm{S}}}\right)^2 \tag{B.8}$$

with $C_k = (r_S / r)^2 \beta^{0,7\sqrt{r_S / r - 1}}$

where β is the larger of

$$\beta = 0{,}105\left(\frac{t_C}{r}\right)^{0,19} \quad \text{and} \quad \beta = 0{,}088\left(\frac{r_T}{r}\right)^{0,23}$$

B.3.2 Uniform squash limit external pressure

(1) The uniform squash limit external pressure for toriconical and torispherical shells is given by expression (B.9) or may be found in the graph in Figure B.2 or may, for $r_T = 0$, be approximated by formula (B.10) or (B.11)

$$p_{n,Rk} = f_o\left(14{,}5 - 450\frac{f_o}{E}\right)\left(1 + 2\frac{r_T}{r} + 7{,}13\left(\frac{r_T}{r}\right)^2\right)\frac{\cos\alpha}{\left(\frac{2r}{t}\right)^{1,5}} \tag{B.9}$$

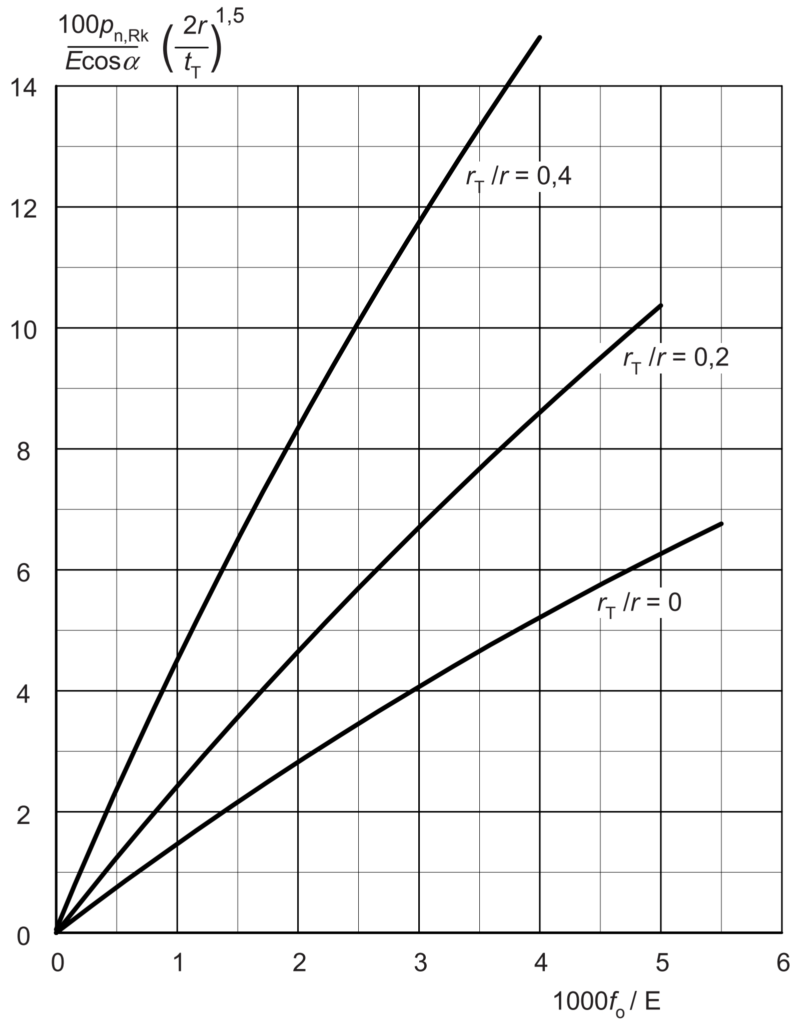


Figure B.2 - Plastic external pressure for toriconical and torispherical shells

- for a toriconical shell

$$p_{n,Rk} = 4{,}4\sqrt{\frac{t_T}{r}}f_o\frac{t_T}{r/\cos\alpha} \tag{B.10}$$

- for a torispherical shell

$$p_{\mathrm{n,Rk}} = 4{,}4\sqrt{\frac{t_{\mathrm{T}}}{r}} f_{\mathrm{o}} \frac{t_{\mathrm{T}}}{r_{\mathrm{S}}} \tag{B.11}$$

B.3.3 External pressure buckling parameter

(1) The imperfection factor should be obtained from:

$$\alpha_\theta = \frac{1}{1 + 2{,}60\left(\frac{1}{Q}\sqrt{\frac{0{,}6E}{f_{\mathrm{o}}}}(\bar{\lambda}_\theta - \bar{\lambda}_{\theta,0})\right)^{1{,}44}} \quad \text{but} \quad \alpha_\theta \le 1{,}00 \tag{B.12}$$

where:

$\bar{\lambda}_{\theta,0}$ is the squash limit slenderness parameter;

Q is the tolerance parameter.

(2) The tolerance parameter Q should be taken from Table B.1 for the specified tolerance class.

(3) The alloy factor and the squash limit slenderness parameter should be taken from Table B.2 according to the material buckling class as defined in EN 1999-1-1.

Table B.1 - Tolerance parameter *Q*

Tolerance class	Q
Class 1	16
Class 2	25
Class 3 and 4	40

Table B.2 - Values of $\bar{\lambda}_{\theta,0}$ and μ_θ for external pressure

Material buckling class	$\bar{\lambda}_{\theta,0}$	μ_θ
A	0,20	0,35
B	0,10	0,20

B.4 Internal pressure

B.4.1 Critical internal pressure

(1) The critical (buckling) internal pressure for a toriconical shell is

$$p_{\mathrm{n,cr}} = 1000\,E\left(\frac{56300}{\alpha^{2,5}} - 0{,}71\right)\left(\frac{t}{2r}\right)^3 \quad \text{if} \quad \frac{r_T}{2r} = 0 \tag{B.13}$$

$$p_{\mathrm{n,cr}} = 1000\,\eta\,E\frac{r_{\mathrm{T}}}{2\mathrm{r}}\left(\frac{t}{2r}\right)^3 \quad \text{if} \quad \frac{r_T}{2r} \ne 0 \tag{B.14}$$

where the parameter η should be taken from Figure B.3.

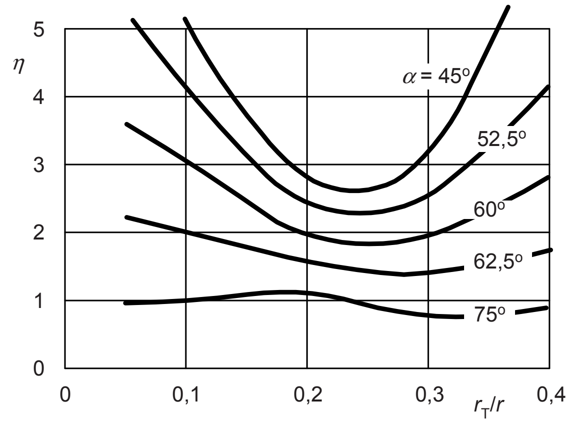


Figure B.3 – Parameter η for expression (B.14)

(2) The critical buckling internal pressure for a torispherical shell is

$$p_{\mathrm{n,cr}} = 100E\left(1{,}85\frac{r_{\mathrm{T}}}{r} + 0{,}68\right)\left(\frac{t}{r_{\mathrm{S}}}\right)^{2{,}45} \tag{B.15}$$

B.4.2 Uniform squash limit internal pressure

(1) The uniform squash limit internal pressure for toriconical and torispherical shells is given by expression (B.16) or may be found in the graph in Figure B.4.

$$p_{\mathrm{n,Rk}} = f_{\mathrm{o}}\left(1{,}2 - 120\frac{f_{\mathrm{o}}}{E}\right)\left(1 + 3{,}9\frac{r_{\mathrm{T}}}{r} + 67\left(\frac{r_{\mathrm{T}}}{r}\right)^2\right)\frac{\cos\alpha}{\left(\frac{2r}{t}\right)^{1{,}25}} \tag{B.16}$$

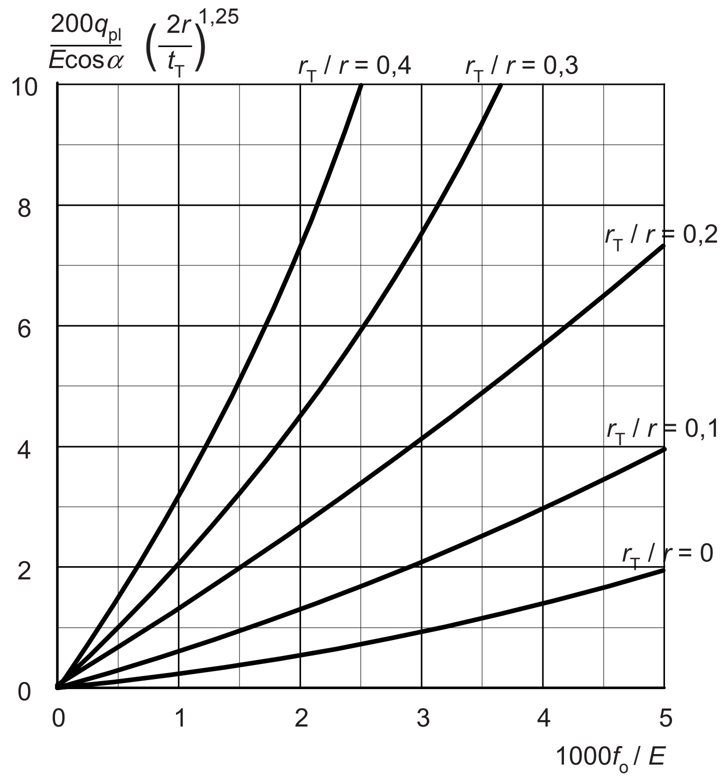


Figure B.4 - Plastic internal pressure for toriconical and torispherical shells

B.4.3 Internal pressure buckling parameter

(1) The imperfection factor should be obtained from:

$$\alpha_\theta = \frac{1}{1 + 2{,}60\left(\frac{1}{Q}\sqrt{\frac{0{,}6E}{f_o}}(\overline{\lambda}_\theta - \overline{\lambda}_{\theta,0})\right)^{1{,}44}} \quad \text{but} \quad \alpha_\theta \le 1{,}00 \tag{B.17}$$

where:

$\overline{\lambda}_{\theta,0}$ is the squash limit slenderness parameter;

Q is the tolerance parameter.

(2) The tolerance parameter Q should be taken from Table B.3 for the specified tolerance class.

(3) The alloy factor and the squash limit slenderness parameter should be taken from Table B.4 according to the material buckling class as defined in EN 1999-1-1.

Table B.3 - Tolerance parameter *Q* for internal pressure

Tolerance class	Q
Class 1	16
Class 2	25
Class 3 and 4	40

Table B.4 - Values of $\overline{\lambda}_{\theta,0}$ and μ_θ for internal pressure

Material buckling class	$\overline{\lambda}_{\theta,0}$	μ_θ
A	0,20	0,35
B	0,10	0,20

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