

BS EN ISO 16610-71:2014



BSI Standards Publication

Geometrical product specifications (GPS) — Filtration

Part 71: Robust areal filters: Gaussian regression filters

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National foreword

This British Standard is the UK implementation of EN ISO 16610-71:2014.

The UK participation in its preparation was entrusted to Technical Committee TDW/4, Technical Product Realization.

A list of organizations represented on this committee can be obtained on request to its secretary.

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Date	Text affected
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English Version

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Robust areal filters: Gaussian regression filters (ISO 16610-
71:2014)

Spécification géométrique des produits (GPS) - Filtrage -
Partie 71: Filtres surfaciques robustes: Filtres de régression
gaussiens (ISO 16610-71:2014)

Geometrische Produktspezifikation (GPS) - Filterung - Teil
71: Robuste Flächenfilter: Gaußsche Regressionsfilter (ISO
16610-71:2014)

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Foreword

This document (EN ISO 16610-71:2014) has been prepared by Technical Committee ISO/TC 213 "Dimensional and geometrical product specifications and verification" in collaboration with Technical Committee CEN/TC 290 "Dimensional and geometrical product specification and verification" the secretariat of which is held by AFNOR.

This European Standard shall be given the status of a national standard, either by publication of an identical text or by endorsement, at the latest by March 2015, and conflicting national standards shall be withdrawn at the latest by March 2015.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN [and/or CENELEC] shall not be held responsible for identifying any or all such patent rights.

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Endorsement notice

The text of ISO 16610-71:2014 has been approved by CEN as EN ISO 16610-71:2014 without any modification.

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

- *Part 1: Overview and basic concepts [Technical Specification]*
- *Part 20: Linear profile filters: Basic concepts [Technical Specification]*
- *Part 21: Linear profile filters: Gaussian filters*
- *Part 22: Linear profile filters: Spline filters [Technical Specification]*
- *Part 28: Profile filters: End effects [Technical Specification]*
- *Part 29: Linear profile filters: Spline wavelets [Technical Specification]*
- *Part 30: Robust profile filters: Basic concepts [Technical Specification]*
- *Part 31: Robust profile filters: Gaussian regression filters [Technical Specification]*
- *Part 32: Robust profile filters: Spline filters [Technical Specification]*
- *Part 40: Morphological profile filters: Basic concepts [Technical Specification]*
- *Part 41: Morphological profile filters: Disk and horizontal line-segment filters [Technical Specification]*
- *Part 49: Morphological profile filters: Scale space techniques [Technical Specification]*
- *Part 60: Linear areal filters — Basic concepts*
- *Part 61: Linear areal filters — Gaussian filters*
- *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 85: Areal Morphological: Segmentation*

The following parts are planned:

- *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*
- *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*
- *Part 42: Morphological profile filters: Motif filters*
- *Part 62: Linear areal filters: Spline filters*
- *Part 69: Linear areal filters: Spline wavelets*
- *Part 70: Robust areal filters: Basic concepts*
- *Part 72: Robust areal filters: Spline filters*
- *Part 80: Morphological areal filters: Basic concepts*
- *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*
- *Part 82: Morphological areal filters: Motif filters*
- *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO 16610 is a Geometrical Product Specification (GPS) standard and is to be regarded as a Global GPS standard (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards.

The ISO/GPS Masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this standard is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this standard and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this standard, unless otherwise indicated.

For more detailed information of the relation of this document to the GPS matrix model, see [Annex C](#).

This part of ISO 16610 specifies the metrological characteristics of robust areal Gaussian regression filters, for the rotationally symmetric filtration of nominal planar surfaces and the filtration of nominal cylindrical surfaces.

The filter is insensitive against specific phenomena in the input data (e.g. spike discontinuities as well as deep valleys and high peaks, etc.). The boundaries of the measured surface are still usable.

Geometrical product specifications (GPS) — Filtration —

Part 71:

Robust areal filters: Gaussian regression filters

1 Scope

This part of ISO 16610 specifies the characteristics of the robust areal Gaussian regression filter for the evaluation of surfaces that may contain spike discontinuities as well as deep valleys and high peaks. It specifies in particular how to separate large scale lateral components and short scale lateral components of a surface.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-30:—¹, *Geometrical product specifications (GPS) — Filtration — Part 30: Robust profile filters: Basic concepts*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-30, ISO/IEC Guide 99 and ISO/IEC Guide 98-3 and the following apply.

3.1

robust planar filter

non linear areal filter to separate a planar surface with specific phenomena (e.g. spike discontinuities as well as deep valleys and high peaks etc.) into large scale lateral components and short scale lateral components

3.2

robust cylindrical filter

non linear areal filter to separate a cylindrical surface with specific phenomena (e.g. spike discontinuities as well as deep valleys and high peaks etc.) into large scale lateral components and short scale lateral components

1) To be published. (Revision of ISO/TS 16610-30:2009)

3.3 biweight function

asymmetric influence function defined by

$$\psi_B(u, c) = \begin{cases} u \left(1 - \left(\frac{u}{c} \right)^2 \right)^2 & \text{for } |u| \leq c \\ 0 & \text{for } |u| > c \end{cases} \quad (1)$$

where c is the real scale parameter and u is a real number

Note 1 to entry: See ISO 16610-30:—, Figure 4.

3.4 robust areal regression filter

weighted M-estimator based on the areal local complete polynomial modelling of the surface

Note 1 to entry: See ISO 16610-30 for the definition of the weighted M-estimator.

Note 2 to entry: See [Annex A](#) for the mathematical definition of the robust areal regression filter.

3.5 robust areal Gaussian regression filter

robust areal regression filter based on the areal Gaussian weighting function, the biweight influence function and a local complete polynomial modelling of the surface with the degree $p=2$ as the default case

Note 1 to entry: to entry: See ISO 16610-61 for the definition of the areal Gaussian weighting function.

Note 2 to entry: to entry: In case of $p=2$, the robust areal Gaussian regression filter follows a complete polynomial up to second degree.

4 Robust planar Gaussian regression filter

4.1 General

Robust planar Gaussian regression filters complying to this document shall conform to [sections 4.2 to 4.4](#).

4.2 Weighting function

The weighting function of the robust planar Gaussian regression filter depends on the surface values (height to the reference surface) and the location of the weighting function on the surface.

4.3 Filter equation

The filter equation is given by

$$w_{ij} = (1 \ 0 \ 0 \ 0 \ 0 \ 0) \left(\mathbf{X}_{ij}^T \mathbf{S}_{ij} \mathbf{X}_{ij} \right)^{-1} \mathbf{X}_{ij}^T \mathbf{S}_{ij} \mathbf{z}, \quad i=1,\dots,m \quad j=1,\dots,n \quad (2)$$

with the surface values

$$\mathbf{z} = (z_{11} \ \dots \ z_{m1} \ \dots \ z_{1n} \ \dots \ z_{mn})^T \quad (3)$$

The regression function is spanned by the matrix:

$$\mathbf{X}_{ij} = \begin{pmatrix} 1 & x_{1i} & y_{1j} & x_{1i} y_{1j} & x_{1i}^2 & y_{1j}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{mi} & y_{1j} & x_{mi} y_{1j} & x_{mi}^2 & y_{1j}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1i} & y_{nj} & x_{1i} y_{nj} & x_{1i}^2 & y_{nj}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{mi} & y_{nj} & x_{mi} y_{nj} & x_{mi}^2 & y_{nj}^2 \end{pmatrix} \quad (4)$$

where

$$x_{ki} = (k-i) \Delta x, \quad k=1,\dots,m \quad (5)$$

and

$$y_{lj} = (l-j) \Delta y, \quad l=1,\dots,n \quad (6)$$

The spatial varying weighting function \mathbf{S}_{ij} is given by:

$$\mathbf{S}_{ij} = \begin{pmatrix} s_{11ij} \delta_{11} & 0 & \dots & & & 0 \\ 0 & \ddots & & & & \\ \vdots & & s_{m1ij} \delta_{m1} & & & \\ & & & \ddots & & \\ & & & & s_{1nij} \delta_{1n} & \vdots \\ & & & & & \ddots & 0 \\ 0 & & & \dots & 0 & s_{mnij} \delta_{mn} \end{pmatrix} \quad (7)$$

with the Gaussian weighting function:

$$s_{klj} = \frac{1}{\gamma^2 \lambda_c^2} \exp \left(-\frac{\pi}{\gamma^2} \left(\frac{x_{ki}^2 + y_{lj}^2}{\lambda_c^2} \right) \right) \quad k=1,\dots,m \quad l=1,\dots,n \quad (8)$$

The constant γ is given by:

$$\gamma = \sqrt{\frac{-1 - W_{-1} \left(-\frac{1}{2e} \right)}{\pi}} \approx 0,7309 \quad (9)$$

with the branch $W_{-1}(u) < -1$ of the "Lambert W" function [Z].

The weights δ_{ij} are derived from the biweight function as follows:

$$\delta_{ij} = \frac{\psi_B(z_{ij} - w_{ij}, c)}{z_{ij} - w_{ij}}, \quad i = 1, \dots, m \quad j = 1, \dots, n \quad (10)$$

In the default case, the scale parameter c is given by:

$$c = \frac{3 \Delta_{\text{MAD}}}{\sqrt{2} \operatorname{erf}^{-1}\left(\frac{1}{2}\right)} \quad (11)$$

Δ_{MAD} is the median absolute deviation of the residuals $z_{ij} - w_{ij}$, and erf^{-1} is the inverse error function [6].

- m number of the surface values in x direction
- n number of the surface values in y direction
- i index of the surface values in x direction $i = 1, \dots, m$
- j index of the surface values in y direction $j = 1, \dots, n$
- z_{ij} surface values before filtering
- w_{ij} filtered surface values
- λ_c cut-off wavelength
- Δ_x sampling interval in x direction
- Δ_y sampling interval in y direction

NOTE 1 See ISO 16610-30 for the definition of Δ_{MAD} .

NOTE 2 w_{ij} gives the surface values of the large scale lateral components. The short scale lateral components r_{ij} can be obtained by the difference vector $r_{ij} = z_{ij} - w_{ij}$.

NOTE 3 The definition for the value c is equivalent to 3σ of a surface with a Gaussian amplitude distribution.

NOTE 4 The number of zeros in Formula (2) is equal to $p(p+3)/2$ (see Annex A).

NOTE 5 The values w_{ij} are generally calculated by iteration starting with $\delta_{ij}^0 = 1$ and updating the weights according to Formula (10). For the calculation of the first updated weights δ_{ij}^1 , the default scale parameter c can be increased by a factor of two.

NOTE 6 For surfaces with big pores or peaks at the surface boundaries the robustness can be increased by setting $p=0$. In this case the nominal form can be eliminated by using the F-operator. The filter equation for $p=0$ results in:

$$w_{ij} = \frac{\sum_{k=1}^m \sum_{l=1}^n s_{kl} \delta_{kl} z_{kl}}{\sum_{k=1}^m \sum_{l=1}^n s_{kl} \delta_{kl}}, \quad \text{with } \gamma = \sqrt{\frac{\ln 2}{\pi}} \quad (12)$$

4.4 Transmission characteristics

The weighting function of the robust planar Gaussian regression filter depends on the surface values and the location on the surface. Therefore no transmission characteristic can be given.

5 Robust cylindrical Gaussian regression filter

5.1 General

Robust cylindrical Gaussian regression filters complying to this document shall conform to [sections 5.2](#) to [5.4](#).

5.2 Weighting function

The weighting function of the robust cylindrical Gaussian regression filter depends on the surface values (height to the reference surface) and the location of the weighting function on the surface.

5.3 Filter equation

The filter equation is given by:

$$\tilde{w}_{ij} = (1 \ 0 \ 0 \ 0 \ 0 \ 0) \left(\tilde{\mathbf{X}}_{ij}^T \tilde{\mathbf{S}}_{ij} \tilde{\mathbf{X}}_{ij} \right)^{-1} \tilde{\mathbf{X}}_{ij}^T \tilde{\mathbf{S}}_{ij} \boldsymbol{\rho} \quad (13)$$

with the radial surface values:

$$\boldsymbol{\rho} = (\rho_{11} \ \dots \ \rho_{m1} \ \dots \ \rho_{1n} \ \dots \ \rho_{mn})^T \quad (14)$$

The regression function is spanned by the matrix:

$$\tilde{\mathbf{X}}_{ij} = \begin{pmatrix} 1 & \varphi_{1i} & z_{1j} & \varphi_{1i} z_{1j} & \varphi_{1i}^2 & z_{1j}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \varphi_{mi} & z_{1j} & \varphi_{mi} z_{1j} & \varphi_{mi}^2 & z_{1j}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \varphi_{1i} & z_{nj} & \varphi_{1i} z_{nj} & \varphi_{1i}^2 & z_{nj}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \varphi_{mi} & z_{nj} & \varphi_{mi} z_{nj} & \varphi_{mi}^2 & z_{nj}^2 \end{pmatrix} \quad (15)$$

where (in the circumferential direction):

$$\varphi_{ki} = \left(\left(k - i + \frac{m}{2} \right) \bmod m - \frac{m}{2} \right) \Delta\varphi, \quad k = 1, \dots, m \quad (16)$$

and (in the axial direction):

$$z_{lj} = (l - j) \Delta z, \quad l = 1, \dots, n. \quad (17)$$

The spatial varying weighting function $\tilde{\mathbf{S}}_{ij}$ is given by:

$$\tilde{\mathbf{S}}_{ij} = \begin{pmatrix} \tilde{s}_{11ij} \tilde{\delta}_{11} & 0 & \dots & & & 0 \\ 0 & \ddots & & & & \\ \vdots & & \tilde{s}_{m1ij} \tilde{\delta}_{m1} & & & \\ & & & \ddots & & \\ & & & & \tilde{s}_{1nij} \tilde{\delta}_{1n} & \vdots \\ & & & & & \ddots & 0 \\ 0 & & & & \dots & 0 & \tilde{s}_{mnij} \tilde{\delta}_{mn} \end{pmatrix} \quad (18)$$

with the Gaussian weighting function:

$$s_{klj} = \frac{n_{upr}}{\gamma^2 2\pi \lambda_{cz}} \exp \left(-\frac{\pi}{\gamma^2} \left(\left(\frac{\varphi_{ki} n_{upr}}{2\pi} \right)^2 + \left(\frac{z_{lj}}{\lambda_{cz}} \right)^2 \right) \right) \quad k = 1, \dots, m \quad l = 1, \dots, n \quad (19)$$

The constant γ is given by:

$$\gamma = \sqrt{\frac{-1 - W_{-1} \left(-\frac{1}{2e} \right)}{\pi}} \approx 0,7309 \quad (20)$$

with the branch $W_{-1}(u) < -1$ of the ‘‘Lambert W’’ function.

The weights $\tilde{\delta}_{ij}$ are derived from the biweight function as follows:

$$\tilde{\delta}_{ij} = \frac{\psi_B(\rho_{ij} - \tilde{w}_{ij}, \tilde{c})}{\rho_{ij} - \tilde{w}_{ij}}, \quad i = 1, \dots, m \quad j = 1, \dots, n \quad (21)$$

In the default case, the scale parameter \tilde{c} is given by:

$$\tilde{c} = \frac{3 \Delta_{\text{MAD}}}{\sqrt{2} \operatorname{erf}^{-1}\left(\frac{1}{2}\right)} \quad (22)$$

Δ_{MAD} is the median absolute deviation of the residuals $\rho_{ij} - \tilde{w}_{ij}$, and erf^{-1} is the inverse error function.

m number of surface values in φ direction (the circumferential direction)

n number of surface values in z direction (the axial direction)

i index of the surface value in φ direction $i = 1, \dots, m$

j index of the surface value in z direction $j = 1, \dots, n$

ρ_{ij} radial surface values before filtering

\tilde{w}_{ij} filtered radial surface values

n_{upr} limiting scale in φ direction in undulations per revolution

λ_{cz} cut-off wavelength in z direction

Δ_{φ} sampling interval in φ direction (the circumferential direction)

Δ_z sampling interval in z direction (the axial direction)

NOTE 1 See ISO 16610-30 for the definition of Δ_{MAD} .

NOTE 2 \tilde{w}_{ij} gives the surface values of the large scale lateral components. The short scale lateral components \tilde{r}_{ij} can be obtained by the difference vector $\tilde{r}_{ij} = \rho_{ij} - \tilde{w}_{ij}$.

NOTE 3 The definition for the value \tilde{c} is equivalent to 3σ of a surface with a Gaussian amplitude distribution.

NOTE 4 The number of zeros in Formula (13) is equal to $p(p+3)/2$ (see [Annex A](#)).

NOTE 5 The values \tilde{w}_{ij} are generally calculated by iteration starting with $\tilde{\delta}_{ij}^0 = 1$ and updating the weights according to Formula (21). For the calculation of the first updated weights $\tilde{\delta}_{ij}^1$, the default scale parameter \tilde{c} can be increased by a factor of two.

5.4 Transmission characteristics

The weighting function of the robust cylindrical Gaussian regression filter depends on the surface values and the location on the surface. Therefore no transmission characteristic can be given.

6 Nesting Index for planar and cylinder surfaces

It is recommended to choose the nesting index according to ISO 25178-3:2012, Table 1. In this case, the filter is robust against specific phenomena in the input data with a lateral size equal to the chosen nesting index divided by 3.

Example: the nesting index for the robust planar Gaussian regression filter is set to $\lambda_c = 0,8$ mm. The filter will be insensitive against e.g. peaks or valleys with a lateral size of $0,8 \text{ mm}/3 \cong 0,267$ mm.

7 Filter designation

Robust areal Gaussian regression filters according to this part of ISO 16610 are designated

Robust planar filters FARGRP

Robust cylindrical filters FARGRC

Annex A (informative)

Regression filter

An areal robust regression filter using a weighted M-estimator is given by the generalized nonlinear minimization:

$$\min_{\beta_{ij}} \left(f(\mathbf{w} - \mathbf{X}_{ij} \beta_{ij})^T \mathbf{s}_{ij} \right), \quad i=1, \dots, m \quad j=1, \dots, n \quad (\text{A.1})$$

with the matrix:

$$\mathbf{X}_{ij} = \begin{pmatrix} 1 & \cdots & u_{1i}^k & v_{1j}^l \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cdots & u_{mi}^k & v_{1j}^l \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cdots & u_{1i}^k & v_{nj}^l \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \cdots & u_{mi}^k & v_{nj}^l \end{pmatrix}, \quad k=0, \dots, p \quad l=0, \dots, p \quad k+l \leq p \quad (\text{A.2})$$

the parameter vector:

$$\beta_{ij} = \left(\beta_{1,ij} \quad \cdots \quad \beta_{1+\frac{p(p+3)}{2},ij} \right) \quad (\text{A.3})$$

the data values:

$$\mathbf{w} = (w_{11} \quad \cdots \quad w_{m1} \quad \cdots \quad w_{1n} \quad \cdots \quad w_{mn})^T \quad (\text{A.4})$$

and a symmetrical weighting function:

$$\mathbf{s}_{ij} = (s_{11,ij} \quad \cdots \quad s_{m1,ij} \quad \cdots \quad s_{1n,ij} \quad \cdots \quad s_{mn,ij})^T \quad (\text{A.5})$$

The coordinates are given by:

$$u_{ki} = (k-i) \Delta u, \quad k=1, \dots, m \quad (\text{A.6})$$

and:

$$v_{lj} = (l-j) \Delta v, \quad l=1, \dots, n \quad (\text{A.7})$$

- p degree of the complete polynomial
- m number of surface values in u direction
- n number of surface values in v direction
- i index of the surface values in u direction $i = 1, \dots, m$
- j index of the surface values in v direction $j = 1, \dots, n$
- w_{ij} data values before filtering
- $\beta_{k,ij}$ polynomial coefficients at position (i, j)
- $\beta_{1,ij}$ filtered data value at position (i, j)
- Δ_u sampling interval in u direction
- Δ_v sampling interval in v direction

The function f describes the underlying statistic of the deviations Δ_w .

In general, the solution for Formula (A.1) can be expressed as the weighted least squares problem:

$$\beta_{1,ij} = (1 \ 0 \ \dots \ 0) \left(\mathbf{X}_{ij}^T \mathbf{S}_{ij} \mathbf{X}_{ij} \right)^{-1} \mathbf{X}_{ij}^T \mathbf{S}_{ij} \mathbf{w} \quad (\text{A.8})$$

The number of zeros in Formula (A.9) is equal to $p(p+3)/2$. The zeros suppress the higher order polynomial coefficients of the minimization problem as stated in (A.1).

The spatial varying weighting function \mathbf{S}_{ij} is given by:

$$\mathbf{S}_{ij} = \begin{pmatrix} s_{11ij} \delta_{11} & 0 & \dots & & 0 \\ 0 & \ddots & & & \\ \vdots & & s_{m1ij} \delta_{m1} & & \\ & & & \ddots & \\ & & & s_{1nij} \delta_{1n} & \vdots \\ 0 & & & \dots & \ddots & 0 \\ & & & & 0 & s_{mnij} \delta_{mn} \end{pmatrix} \quad (\text{A.9})$$

with s_{klij} the vector elements of the weighting function \mathbf{s}_{ij} and the additional weights δ_{kl} derived from the influence function $\psi(r)$:

$$\delta_{kl} = \frac{\psi(w_{kl} - \beta_{1,kl})}{w_{kl} - \beta_{1,kl}} \quad \text{with} \quad \psi(r) = \frac{\partial f(r)}{\partial r} \quad (\text{A.10})$$

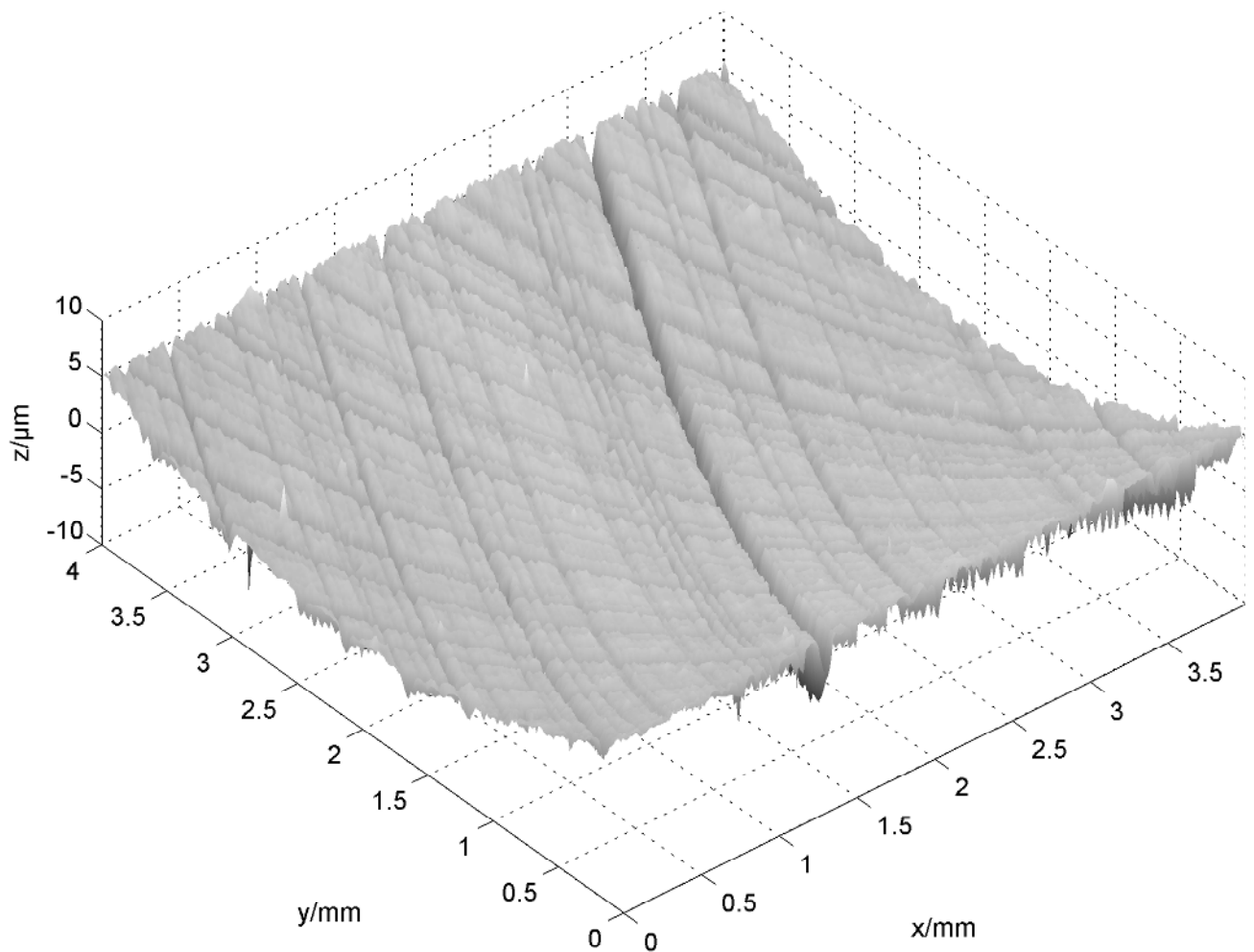
NOTE 1 In case of deviations with a Gaussian amplitude distribution, the best maximum log likelihood estimation is given if $f(r)$ is the quadratic loss function r^2 . In this case Formula (A.1) is equal to a least square sum and the filter operation is linear.

NOTE 2 This general formulation of the filter ensures that it follows complete polynomials of degree p .

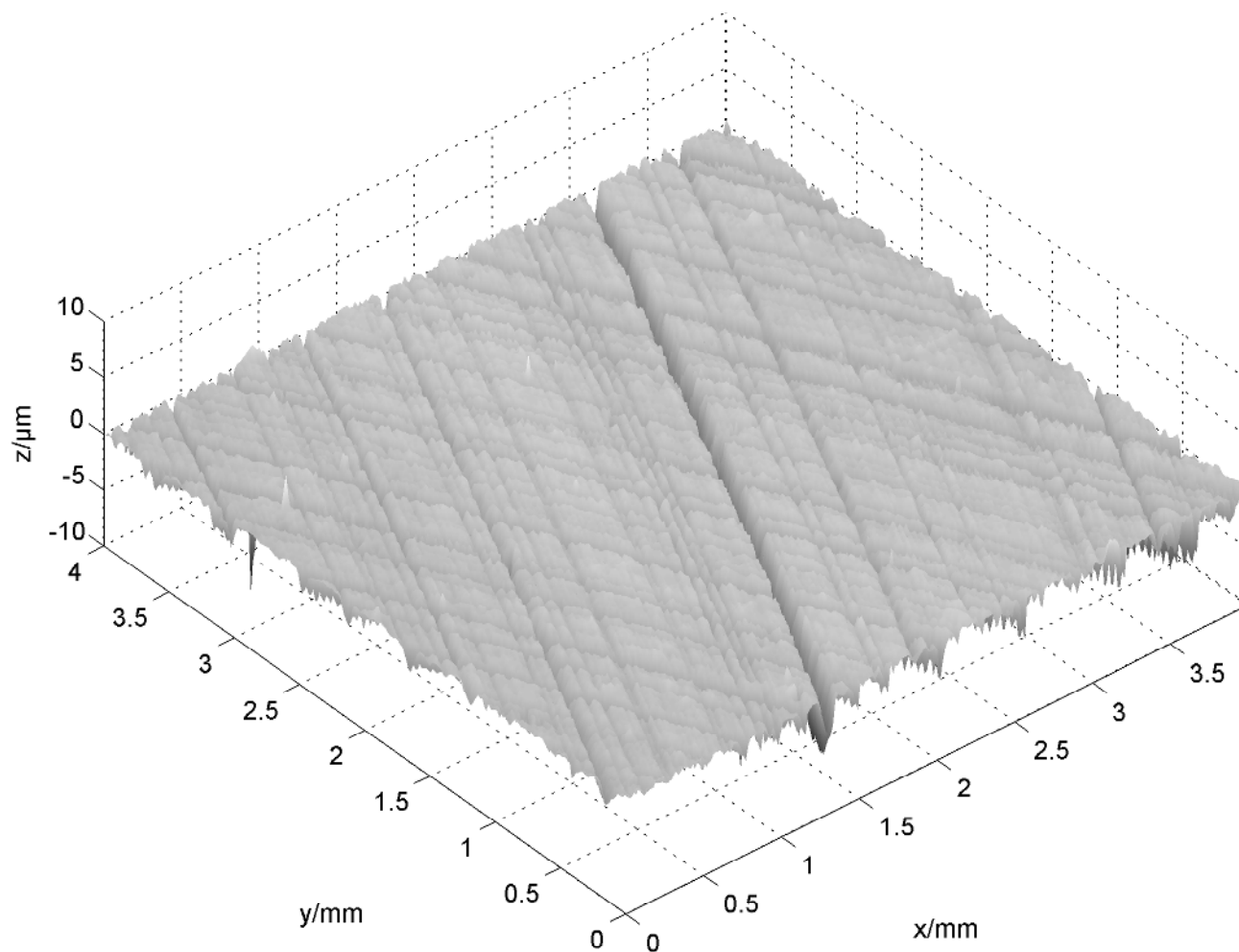
Annex B (informative)

Examples

Examples for the application of the robust planar and robust cylindrical Gaussian regression filter ($p=2$) are for information only. See [Figures B.1, B.2](#) and [B.3](#).

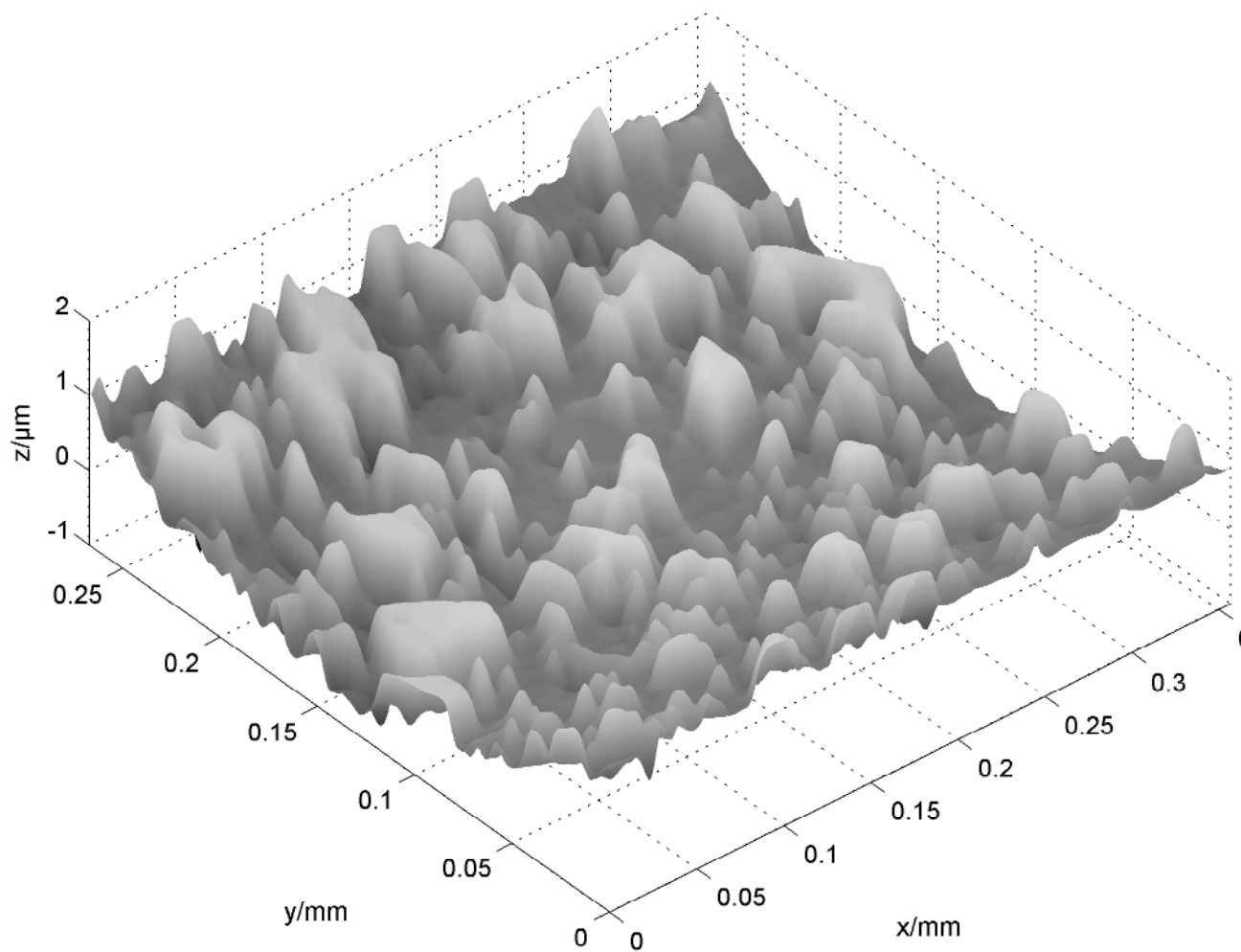


a) Original surface with form

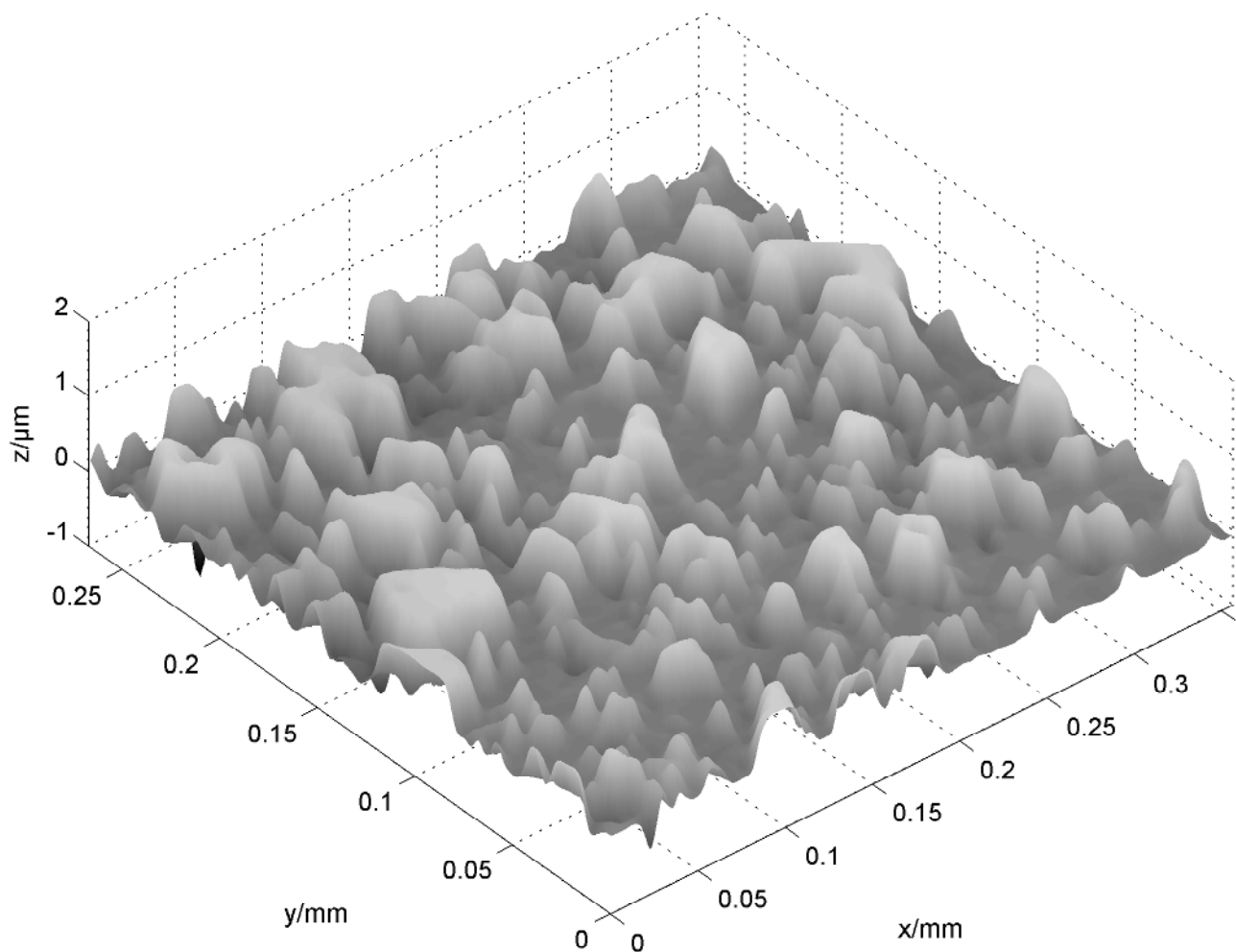


b) Filtered surface. The form is eliminated by the filtering process (regression order $P = 2$)

Figure B.1 — Robust areal Gaussian regression filter with a nesting index of $\lambda_c = 0,8$ mm applied on a honed surface of a cylinder liner

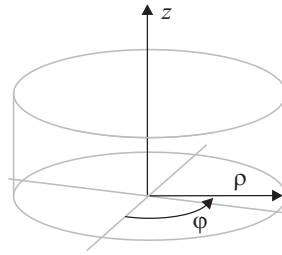


a) Original surface with form



b) Filtered surface. The filter approximates the form (regression order $P = 2$) and is insensitive against the surface particle structures

Figure B.2 — Robust areal Gaussian regression filter with a nesting index of $\lambda_c = 0,8$ mm applied on a planar surface with particle structures



a) Original cylindrical surface with form-deviation and the groove

b) Filtered surface. The filter is insensitive against the groove and approximates the form-deviation only

Figure B.3 — Robust cylindrical Gaussian regression filter with a nesting index of $\lambda_{cz} = 0,8$ mm and $n_{upr} = 15$ applied on a cylindrical surface with a groove

Annex C (informative)

Relationship to the filtration matrix model

C.1 Position in the filtration matrix model

This part of ISO 16610 is a particular filter document. See [Table C.1](#).

Table C.1 — Structure of the ISO 16610 series

	FILTERS : ISO 16610 series					
General	Part 1					
	Profile Filters			Areal Filters		
Fundamental	Part 11			Part 12		
	Linear	Robust	Morphological	Linear	Robust	Morphological
Basic Concepts	Part 20	Part 30	Part 40	Part 60	Part 70	Part 80
Particular Filters	Parts 21-25	Parts 31-35	Parts 41-45	Parts 61-65	Parts 71-75	Parts 81-85
How to Filter	Parts 26-28	Parts 36-38	Parts 46-48	Parts 66-68	Parts 76-78	Parts 86-88
Multiresolution	Part 29	Part 39	Part 49	Part 69	Part 79	Part 89

C.2 Titles of the individual parts in the ISO 16610 series

The individual part titles are:

- Part 1: *Overview and basic concepts*
- Part 20: *Linear profile filters; Basic concepts*
- Part 21: *Linear profile filters; Gaussian filters*
- Part 22: *Linear profile filters; Spline filters*
- Part 26: *Linear profile filters; Filtration on nominally orthogonal grid planar data sets*
- Part 27: *Linear profile filters; Filtration on nominally orthogonal grid cylindrical data sets*
- Part 28: *Linear profile filters; End effects*
- Part 29: *Linear profile filters; Spline wavelets*

- Part 30: *Robust profile filters; Basic concepts*
- Part 31: *Robust profile filters; Gaussian regression filters*
- Part 32: *Robust profile filters; Spline filters*
- Part 40: *Morphological profile filters; Basic concepts*
- Part 41: *Morphological profile filters; Disk and horizontal line segment filters*
- Part 42: *Morphological profile filters; Motif filters*
- Part 45: *Morphological profile filters; Segmentation*
- Part 49: *Morphological profile filters; Scale space techniques*
- Part 60: *Linear areal filters; Basic concepts*
- Part 61: *Linear areal filters; Gaussian filters*
- Part 69: *Linear areal filters; Spline wavelets*
- Part 71: *Robust areal filters; Gaussian regression filters*

Annex D (informative)

Relation to the GPS matrix model

D.1 General

For full details about GPS matrix model, see ISO/TR 14638. The ISO/GPS Masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

D.2 Information on this part of ISO 16610 and its application

This part of ISO 16610 specifies the metrological characteristics of robust areal Gaussian regression filters, for the rotationally symmetric filtration of nominal planar surfaces and the filtration of nominal cylindrical surfaces.

The filter is insensitive against specific phenomena in the input data (e.g. spike discontinuities as well as deep valleys and high peaks etc.). The boundaries of the measured surface are still usable.

D.3 Position in the GPS matrix model

This part of ISO 16610 is a global GPS Technical Specification, that influences the chain links 3 and 5 of all chains of standards, as graphically illustrated on [Table D.1](#).

D.4 Related International Standards

The related International Standards are those of the chain of standards indicated in [Table D.1](#).

Table D.1 — Fundamental and general ISO GPS standards matrix

	Global GPS Standards					
	General GPS Standards					
Chain link number	1	2	3	4	5	6
Size			X		X	
Distance			X		X	
Radius			X		X	
Angle			X		X	
Form of line independent of datum			X		X	
Form of line dependent of datum			X		X	
Form of surface independent of datum			X		X	
Form of surface dependent of datum			X		X	
Orientation			X		X	
Location			X		X	
Circular run-out			X		X	
Total run-out			X		X	
Datums			X		X	
Roughness profile			X		X	
Waviness profile			X		X	
Primary profile			X		X	
Surface imperfections			X		X	
Edges			X		X	

**Fundamental
 GPS
 Standards**

Bibliography

- [1] ISO 8015:2011, *Geometrical product specifications (GPS) — Fundamentals — Concepts, principles and rules*
- [2] ISO 14253-1:1998, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 1: Decision rules for proving conformance or non-conformance with specifications*
- [3] ISO/TR 14638:1995, *Geometrical product specification (GPS) — Masterplan*
- [4] ISO 25178-3:2012, *Geometrical product specifications (GPS) — Surface texture: Areal — Part 3: Specification operators*
- [5] ISO/TS 16610-31:2010, *Geometrical product specifications (GPS) — Filtration — Part 31: Robust profile filters: Gaussian regression filters*
- [6] ISO 80000-2:2009, *Quantities and units — Part 2: Mathematical signs and symbols to be used in the natural sciences and technology*
- [7] CORLESS R.M., GONNET G.H., HARE D.E.G., JEFFREY D.J., KNUTH D.E. On the Lambert W Function. *Adv. Comput. Math.* 1996, **5** pp. 329–359
- [8] HUBER P.J. *Robust Statistics*. John Wiley & Sons, New York, 2004
- [9] SEEWIG J. Linear and robust Gaussian regression filters. *J. Phys. Conf. Ser.* 2005, **13** (1) pp. 254–257

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