Standard Test Method for Integration of Digital Spectral Data for Weathering and Durability Applications[1]

This standard is issued under the fixed designation G214; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method specifies a single relatively simple method to implement, common integration technique, the Modified Trapezoid Rule, to integrate digital or tabulated spectral data. The intent is to produce greater consistency and comparability of weathering and durability test results between various exposure regimes, calculation of materials properties, and laboratories with respect to numerical results that depend upon the integration of spectral distribution data.

1.2 Weathering and durability testing often requires the computation of the effects of radiant exposure of materials to various optical radiation sources, including lamps with varying spectral power distributions and outdoor and simulated sunlight. Changes in the spectrally dependent optical properties of materials, in combination with exposure source spectral data, are often used to evaluate the effect of exposure to radiant sources, develop activation spectra (Practice G178), and classify, evaluate, or rate sources with respect to reference or exposure source spectral distributions. Another important application is the integration of the original spectrally dependent optical properties of materials in combination with exposure source spectral data to determine the total energy absorbed by a material from various exposure sources.

1.3 The data applications described in 1.2 often require the use of tabulated reference spectral distributions, digital spectral data produced by modern instrumentation, and the integrated version of that data, or combinations (primarily multiplication) of spectrally dependent data.

1.4 Computation of the material responses to exposure to radiant sources mentioned above require the integration of measured wavelength dependent digital data, sometimes in conjunction with tabulated wavelength dependent reference or comparison data.

1.5 The term "integration" in the previous sections refers to the numerical approximation to the true integral of continuous functions, represented by discrete, digital data. There are numerous mathematical techniques for performing numerical integration. Each method provides different levels of complexity, accuracy, ease of implementation and computational efficiency, and, of course, resultant magnitudes. Hulstrom, Bird and Riordan (**1**)[2] demonstrate the differences between results for rectangular (963.56 W/m^2), trapezoid rule (962.53 W/m^2), and modified trapezoid rule (963.75 W/m^2) integration for a single solar spectrum. Thus the need for a standard integration technique to simplify the comparison of results from different laboratories, measurement instrumentation, or exposure regimes.

1.6 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[3]

E275 Practice for Describing and Measuring Performance of Ultraviolet and Visible Spectrophotometers
E424 Test Methods for Solar Energy Transmittance and Reflectance (Terrestrial) of Sheet Materials
E490 Standard Solar Constant and Zero Air Mass Solar Spectral Irradiance Tables
E772 Terminology of Solar Energy Conversion
E903 Test Method for Solar Absorptance, Reflectance, and Transmittance of Materials Using Integrating Spheres
E927 Specification for Solar Simulation for Photovoltaic Testing
E971 Practice for Calculation of Photometric Transmittance and Reflectance of Materials to Solar Radiation

[1] This test method is under the jurisdiction of ASTM Committee G03 on Weathering and Durability and is the direct responsibility of Subcommittee G03.09 on Radiometry.

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[2] The boldface numbers in parentheses refer to a list of references at the end of this standard.

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

E972 Test Method for Solar Photometric Transmittance of Sheet Materials Using Sunlight
E973 Test Method for Determination of the Spectral Mismatch Parameter Between a Photovoltaic Device and a Photovoltaic Reference Cell
G113 Terminology Relating to Natural and Artificial Weathering Tests of Nonmetallic Materials
G130 Test Method for Calibration of Narrow- and Broad-Band Ultraviolet Radiometers Using a Spectroradiometer
G138 Test Method for Calibration of a Spectroradiometer Using a Standard Source of Irradiance
G151 Practice for Exposing Nonmetallic Materials in Accelerated Test Devices that Use Laboratory Light Sources
G173 Tables for Reference Solar Spectral Irradiances: Direct Normal and Hemispherical on 37° Tilted Surface
G177 Tables for Reference Solar Ultraviolet Spectral Distributions: Hemispherical on 37° Tilted Surface
G178 Practice for Determining the Activation Spectrum of a Material (Wavelength Sensitivity to an Exposure Source) Using the Sharp Cut-On Filter or Spectrographic Technique
G197 Table for Reference Solar Spectral Distributions: Direct and Diffuse on 20° Tilted and Vertical Surfaces
G207 Test Method for Indoor Transfer of Calibration from Reference to Field Pyranometers

3. Terminology

3.1 *Definitions*—The definitions given in Terminologies E772 and G113 are applicable to this test method.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *first difference, n*—the difference, $d1_i$, between adjacent ordinate values, $d1_i = y_i+1 - y_i$. An approximation of the first derivative of the function represented by the tabulated data.

3.2.2 *second difference, n*—the difference $d2_i$, between adjacent first differences (as defined in 3.2.1) in tabulated data; namely $d2_i = d1_i+1 - d1_i$. An approximation of the second derivative of the function represented by the tabulated data.

3.3 For the purposes of this standard, the terms "integral" and "integration" are used in the sense of a computed numerical approximation to a definite integral of continuous functions represented by tabulated or measured numerical (digital) data as functions of wavelength. The approximations are computed as the summation of discrete magnitudes computed according to the method. The data to be integrated may be interpolated to achieve consistent wavelength intervals.

4. Summary of Test Method

4.1 Given a set of *n* digital or numerical (tabulated) data y_i, $1 \leq i \leq n$, as a function of an independent variable, such as wavelength, λ_i, compute the area under each trapezoid, A_i bounded by λ_i and λ_{i+1} with altitudes (heights) y_i and y_{i+1}, for $2 < i < n$-1, respectively.

$$A_i = 0.5 \times (\lambda_{i+1} - \lambda_i) \times (y_{i+1} + y_i) \quad (1)$$

The uniform factor of ½ is needed to compute the area of a general trapezoid.

4.2 Compute the sum, A_0 of the *n*-2 A_i areas over the interval from $i = 2$ to $i = n$-1.

$$A_0 = \Sigma A_i \qquad i = 2....n - 1 \quad (2)$$

4.3 The total area *A*, approximating the integral from λ_1 to λ_n is computed by adding in the start and end values to A_0.

$$\text{Start: } A_i = 0.5 \times 0.5 \times (\lambda_2 - \lambda_1) \times (y_2 + y_1) \quad (3)$$

$$\text{End: } A_n = 0.5 \times 0.5 \times (\lambda_n - \lambda_{n-1}) \times (y_n + y_{n-1}) \quad (4)$$

Eq 1 can be written A_t, of height *h* (in this case each $h = (\lambda_{i+1} - \lambda_i)$) and altitudes $a = y_i$ and $b = y_{i+1}$.

$$A_t = h \times (a + b)/2 \quad (5)$$

Therefore, for uniform step *h*, the total area under curve is expressed as:

$$A = 0.5 \times h \times (y\ 1 + 2 \times \Sigma_2^{n-1} y_i + y_n) \quad (6)$$

NOTE 1—For data with variable *h*, the above calculations must be done independently for each segment of the data with the same *h*.

4.4 To compute the integral of the products of two spectral data sets, such as a reference Spectrum, E(λ), (for example reference spectra such as Standard Tables G173, G177, and G197), or the spectral content of calibration or other sources (as in Test Methods G207, G130, and G138) and measured or tabulated spectral optical property data, R(λ) such as transmittance or reflectance as measured in accordance with Test Method E903 and E424 and Practice E971, or spectral mismatch errors such as in Test Method E973, it is necessary for all data sets to have identical wavelength (λi) and wavelength intervals (λi+1 – λi). Then the appropriate products *E(λi)·R(λi)* are computed and treated using the procedures in 4.1 to 4.3. If the spectral wavelength intervals are different, one data set (usually with the smallest or shortest wavelength interval, should be selected as the data set, M(λ), with which to match all other data sets wavelength intervals. The other data sets should be interpolated, using linear interpolation, to obtain values at wavelength values and intervals identical to the selected M(λ).

4.4.1 When interpolating data sets, it is recommended that the data set with the coarsest or largest wavelength step size or interval be interpolated to the step size of the data set with the smaller step size or interval.

4.5 Compute an estimate for the absolute error in the integration based on the wavelength limits for the integral, the average wavelength interval of the data, and the average of the second differences of the spectral data. Compute the estimated relative (percentage) error in integral approximation based on the total integral and absolute error values (see Section 15 on precision and bias).

5. Significance and Use

5.1 Weathering and durability testing often requires the computation of the effects of radiant exposure of materials to various optical radiation sources, including lamps with varying spectral power distributions and outdoor and simulated sunlight as in Test Methods E972, G130, and G207.

5.2 The purpose of this test method is to foster greater consistency and comparability of weathering and durability test results between various exposure regimes, calculation of materials properties, and laboratories with respect to numerical results that depend upon the integration of spectral distribution data.

5.3 Changes in the optical properties of materials such as spectral reflectance, transmittance, or absorptance are often the measure of material stability or usefulness in various applications. Computation of the material responses to exposure to radiant sources mentioned above requires the integration of measured wavelength-dependent digital data, sometimes in conjunction with tabulated wavelength-dependent reference or comparison data.

5.4 This test method specifies and describes the Modified Trapezoid Rule as a single reasonably accurate and easily implemented integration technique for computing approximations of spectral source and optical property integrals.

5.5 The method includes a procedure for estimating the approximate absolute and relative (percent) error in the estimated spectral integrals.

5.6 The method includes a procedure to construct data sets that match in spectral wavelength and spectral wavelength interval, which does not have to be uniform over the spectral range of interest. Uniform spectral intervals simplify some of the calculations, but are not required.

6. Interferences

6.1 Closed form expressions such as simple functions, spectral properties, and source functions are rarely available, preventing analytical solution to integration of those functions.

6.2 Digitized or tabulated data are only approximations to the continuous spectral property and source functions found in nature.

6.3 Mismatched spectral abscissae and spectral data intervals (steps) for two or more spectral data sets must be adjusted to match at least one of the spectral data sets. Simple linear interpolation is suggested as a means of putting data sets in a form where they can be multiplied or otherwise combined. The data sets should then all match a selected (usually the highest resolution, or smallest step interval) data set. The wavelength intervals do not need to be uniform, just consistent between the multiple data sets.

6.4 Interpolation to produce matching spectral wavelengths and data intervals can introduce additional uncertainty in integrated data, above and beyond the error due to the integration technique and measurement and instrumentation uncertainty.

7. Apparatus

7.1 A digital computer with computing power, storage capacity, and capable of ingesting the spectral data in question and processing it with applications suitable for analyzing data, such as spreadsheet software or mathematical analysis software.

7.2 For applications requiring measurement of spectral distribution of sources (such as Specification E927, Practice G151, or Test Methods G130 and G207), a spectroradiometer calibrated in accordance with Test Method G138 is required.

7.3 For applications requiring measurement of spectral absorptance, reflectance, and transmittance of materials such as Test Method G138, a spectrophotometer is used.

7.3.1 If the measured data alone is to be integrated, this method applies directly.

7.3.2 If the measured data is to be used in conjunction with other measured or tabulated data, it is recommended that the spectral step interval and data point wavelengths match the data set with the smallest wavelength interval as closely as possible.

7.3.3 If possible, use the smallest wavelength step interval available for the spectroradiometer measurements that is compatible with the smallest interval step size in the other data sets. The other data sets (with larger data intervals) can then be interpolated to the measured data intervals.

7.3.3.1 It is recommended that simple linear interpolation, if needed, be accomplished in accordance with subsection 12.3.1.

8. Hazards

8.1 Hazardous levels of ultraviolet or concentrated solar or artificial optical radiation may be encountered in the process of measuring source spectra.

8.2 Electrical (high voltage, current) and thermal (hot surfaces, intense infrared radiation) hazards may be encountered, especially when using high intensity optical radiation sources.

9. Sampling, Test Specimens, and Test Units

9.1 Care must be taken to ensure that the units of wavelength and amplitude of the data under analysis are consistent. Any scaling or unit conversion applied to the original data shall be documented. Examples are conversion from wavelength units of microns (10^{-6} m) to nanometres (10^{-9} m) for units of wavelength; or microwatts per square metre to watts per square metre for flux density.

9.2 Sampling of data at uniform wavelength intervals or step sizes will simplify the computations described in the Procedure, Section 12.

9.3 As mentioned in subsection 6.3, the wavelength interval between data points is not required to be uniform or constant, just consistent between the multiple data sets. Eq 1-6 applied to each interval will ensure the correct individual areas between data points are accounted for.

9.4 When combinations of several spectral data sets (such as products of spectral source data and optical property data) are desired, the wavelength interval or step size between data points should match. If not, the spectral data should be interpolated to match the data set with the shortest (smallest) step size. Alternatively, all data sets can be interpolated to a single, consistent wavelength step size selected by the user. The technique for matching up the wavelength step size must be reported.

10. Preparation of Apparatus

10.1 If spectral data or optical properties are to be measured, the spectroradiometer(s) used should be properly calibrated and configured for the appropriate measurements.

10.2 If spectral properties of materials are to be measured, the spectrophotometer(s) used should be calibrated as recommended by the manufacturer or in accordance with Practice E275.

10.3 If only tabulated or modeled spectral data are to be analyzed, the data should be incorporated in the appropriate digital form for processing by the chosen analysis software. Tabulated data can be entered by hand or copied and pasted from electronic documents.

10.4 Output data from spectral models should be generated and formatted for electronic processing. The spectral model inputs and details of the configuration(s) of the model should be documented.

10.5 All data should be double checked for consistent units of wavelength and amplitude.

11. Calibration and Standardization

11.1 A spectroradiometer and a spectrophotometer used to collect spectral source or optical property data must be calibrated according to manufacturer's specifications and traceability to recognized National Measurement Institution reference standards. Examples are reference standard lamps or standards of reflectance. See Test Methods G138 or E903 for details.

11.2 Standardization of the wavelength step size or interval is required, as mentioned in subsections 10.2 and 10.3. Simple linear interpolation of the data to the selected consistent wavelength interval is suggested, as described in Eq 7 in subsection 12.3.1.

11.3 The source of tabulated or digitized data from standards, such as Standard Tables G173, G177, G197, or E490, spectral model computations; or from data tabulated in specifications, digitized from graphs, or selected from hardcopy or electronic publications should be cited. Any mathematical manipulation of such data, such as interpolation, rescaling, unit conversions, etc., shall be documented.

12. Procedure

12.1 Given a set of n digital or numerical (tabulated) data y_i as a function of an independent variable, such as wavelength, λ_i, the area under each trapezoid, A_i bounded by λ_i and λ_{i+1} with altitudes (heights) y_i and y_{i+1}, and $i \geq 2$ and $i \leq n$-1, respectively, is computed as in Section 4, Eq 1-6.

As described in Eq 3 and Eq 4, the beginning and ending trapezoids are added to the result to approximate the error caused by the discrete sampling of the spectral irradiance data.. Appendix X1 and Appendix X2 show examples of computation of spectral power distribution integration and the integration of the product of the spectral transmission data and spectral data with interpolation.

12.2 To compute the integral of the products of two spectral data sets, such as a reference Spectrum, E(λ), (for example Standard Tables G173 and G197) and measured or tabulated spectral optical property data, R(λ), (for example transmittance or reflectance as measured according to Test Method E903), it is necessary for both tabulated data sets to have identical wavelength (λ_i) and wavelength intervals ($\lambda_{i+1} - \lambda_i$) so the appropriate products $E(\lambda_i)\cdot R(\lambda_i)$ can be computed and treated as in Eq 1-6. At least one data set should be interpolated, using linear interpolation, to obtain values at wavelengths identical to the other.

12.3 When interpolating data sets, it is recommended that the data set with the coarsest or largest wavelength step size or interval be interpolated to the step size of the data set with the smaller step size or interval.

12.3.1 Linear interpolation of a value y for an abscissa value λ_i denoted as $y(\lambda)$ between tabulated or digitized data (λ_j,y_j) and (λ_{j+1}, y_{j+1}) is computed using:

$$y(\lambda) = (y_{j+1} - y_j) \cdot (\lambda_j - \lambda_i)/(\lambda_{j+1} - \lambda_j) + y_j \quad (7)$$

where:

$\lambda_j < \lambda_i < \lambda_{j+1}$

12.4 Compute an estimate for the absolute error in the integration based on the wavelength limits for the integral, the

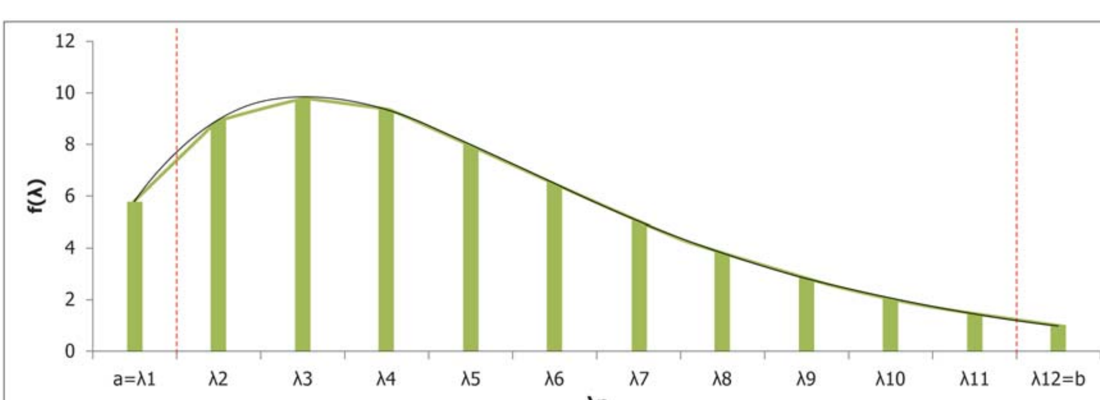

FIG. 1 Shows the Modified Trapezoidal Method for λ_n.

NOTE 1—Low spectral resolution provides higher error (λ_1, λ_2, λ_3) in the integrated area calculation.

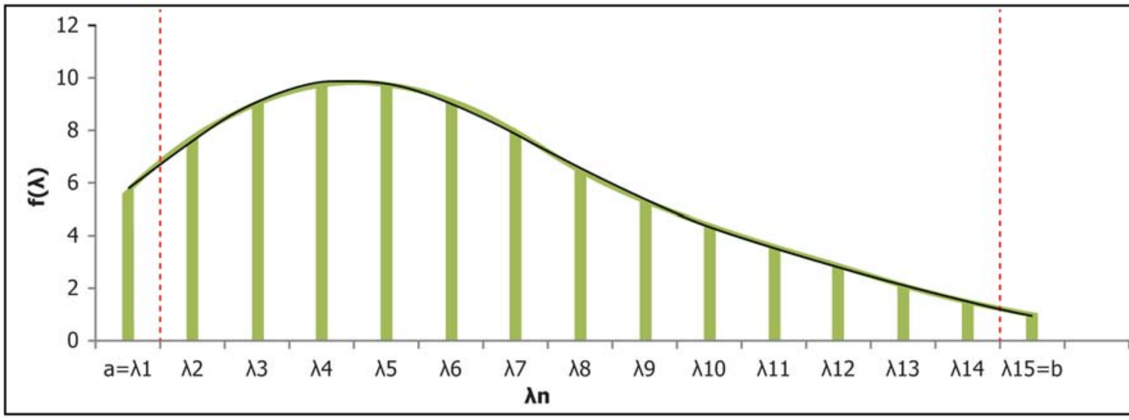

FIG. 2 Higher Resolution Spectral Dataset Provides Less Error When Calculating Area Under Curve. Compare Figure 2 with Figure 1.

average wavelength interval of the data, and the average of the second differences of the spectral data (see Section 15). Appendix X2 contains an example of the integration of the product of a spectral transmittance curve and a reference solar spectral data set.

13. Calculation or Interpretation of Results

13.1 The calculation of spectral integrals, including spectral integrals of the product of optical property data and spectral data and the interpolation of data to a common wavelength interval is described in Section 12.

13.2 The calculation of the estimated error in the integrations is described in Section 15. That section discusses only the estimated error in the integrations, and not the uncertainty in the associated measurement instrumentation or data.

13.3 The results of the calculations, along with any modifications or adjustments to procedures described here are documented in the report, as described in Section 14.

14. Report

14.1 When reporting results and analysis of spectral data integration the following minimum information shall be provided.

14.2 Date, location, contact information for analyst,

14.3 Purpose/application of analysis or result,

14.4 Spectral power distribution source (illuminate), if used;

14.4.1 Lamp type (Xenon, Carbon Arc, Fluorescent, etc.) manufacturer, make and model, if used, and

14.4.2 Natural sunlight (time, date, location, component (direct, diffuse, hemispherical), if applicable;

14.4.2.1 Geometry (tilted, horizontal, vertical, direct beam);

14.5 Measurement instrumentation, if used;

14.5.1 Manufacturer, make, model spectroradiometer and spectrophotometer, if used;

14.5.2 Date and source of calibration with estimated measurement uncertainty, if used;

14.5.3 Spectral wavelength range, nominal bandpass, step size (measurement interval);

14.5.4 Measurement geometry or configuration description, or both;

14.5.5 Ancillary or test article instrumentation, if applicable;

14.5.5.1 Data collection system associated with test units, if used,

14.5.5.2 Date of calibration and accuracy/uncertainty with data collection system, if applicable, and

14.5.5.3 Units or samples under test (make, model, serial number, sample label, etc.), if applicable;

14.6 Tabulated or modeled spectral data source;

14.6.1 Citation or reference,

14.6.2 Spectral model name and reference, if spectral model used,

14.6.3 User input parameters provided to model, if spectral model used,

14.6.4 Original wavelength step interval of tabulated data or spectral model output, and

14.6.5 Modified wavelength step interval used if interpolation needed to match other spectral data.

15. Precision and Bias

15.1 For this method, an approximation of the error in the computed sums with respect to the actual integral of a continuous function *f* over the interval from *a* to *b* is a function of the second derivative of the function *f*, *f''*, within each step interval (at some point, ε_i between (λ_i and λ_{i+1})), the interval (*b*-*a*), and the step size $h = \lambda_{i+1} - \lambda_i$ (**2**,**3**); namely

$$E = [(\lambda_n - \lambda_1)\cdot(h)^3]\cdot f(\varepsilon)/24 \quad (8)$$

where:

$a \leq \varepsilon \leq b$

NOTE 2—The average second difference (f') is used to approximate f'(ε) for $a \leq \varepsilon \leq b$.

15.2 Compute the estimated error in the trapezoid rule approximation of the integral.

15.2.1 Compute the average spectral wavelength interval:

$$h = (\lambda_n - \lambda_1)/n \quad (9)$$

where λ is the wavelength in appropriate units.

15.2.2 Compute the average second difference, *f'*, in y_i from first differences $k_i = (y_{i+1} - y_i)$ as:

$$F = (1/(n - 2)) \cdot \Sigma - k_i$$

$$k_i = 1 \text{ to } n - 2 \tag{10}$$

15.2.3 Compute the estimated absolute error, *E*, in the integral approximation:

$$E = f' \cdot h^3 \cdot (\lambda_n - \lambda_1)/24 \tag{11}$$

15.2.4 Compute the relative or percentage error, *P(%)*, in the approximation to the integral as:

$$P(\%) = 100(E/A) \tag{12}$$

where *A* is the value of the estimated integral approximation from Eq 1-6 in Section 4. Appendix X1 and Appendix X2 show computational examples.

16. Keywords

16.1 absorptance; integration; optical properties; reflectance; solar; spectral data; spectrum; transmittance

APPENDIXES

(Nonmandatory Information)

X1. EXAMPLE INTEGRATION OF SPECTRAL IRRADIANCE FILE TO CALIBRATE ULTRAVIOLET RADIOMETER IN ACCORDANCE WITH TEST METHOD G138

X1.1 Example

X1.1.1 The nominal passband of a UV-B ultraviolet radiometer specified by the manufacturer is 280 nm to 320 nm.

X1.1.2 A spectroradiometer, calibrated in accordance with Test Method G130, and traceable to the National Institute of Standards and Technology (NIST) Scale of Spectral Irradiance, is used to measure solar spectra from 280 nm to 320 nm at 2 nm intervals.

X1.1.3 Table X1.1 is an example of a measured spectrum produced by the spectroradiometer. Column 1 is the wavelength, λi. Column 2 is the spectral irradiance $E(\lambda_i)$ at wavelength λ_i, in watts per square metre per nm. Column 3 is the area between wavelength λ_i and λ_{i+1}. Column 4 is the first differences for column 2. Column 5 is the second differences for column 2 (first differences for column 4). Note that the first and last irradiance values are entered using Eq 3 and Eq 4 of Section 4 to compute the total integral *A*.

X1.1.4 At the bottom of the table is shown the result of the integration calculations according to Eq 1 as described in Section 4.

X1.1.5 The estimated absolute and percentage error in the integral is computed according to Eq 5 and Eq 6 in subsection 4.3.

X1.1.6 To compute the responsivity of the UV-B radiometer, the total integral of the measured spectrum (1.55 W/m^2) is divided by the recorded signal of the UV-B radiometer at the time of the spectral scan.

X1.1.7 The estimated uncertainty in the resulting responsivity of the UV-B radiometer is a combination of the estimated

TABLE X1.1 Integrating Spectral Irradiance for UV-B Calibration

Wavelength λ_i, nm	Irradiance $E(\lambda_i)$ $W/m^2/nm$	Area A_i λ_i to λ_{i+1}	1st Difference Irradiance	2nd Difference Irradiance
280.0	160E-23	2.85E-19	...	...
282.0	5.69E-19	5.69E-19	5.69E-19	...
284.0	4.66E-16	4.67E-16	4.66E-16	4.65E-16
286.0	1.09E-13	1.09E-13	1.08E-13	1.08E-13
288.0	6.06E-11	6.07E-11	6.05E-11	6.03E-11
290.0	6.18E-09	6.24E-09	6.12E-09	6.06E-09
292.0	2.84E-07	2.90E-07	2.78E-07	2.72E-07
294.0	4.30E-06	4.59E-06	4.02E-06	3.74E-06
296.0	4.81E-05	5.24E-05	4.38E-05	3.97E-05
298.0	2.83E-04	3.31E-04	2.35E-04	1.91E04
300.0	9.47E-04	1.23E-03	6.64E-04	4.29E–04
302.0	2.90E-03	3.85E-03	1.95E-03	1.29E-03
304.0	1.02E-02	1.31E-02	7.26E-03	5.31E-03
306.0	2.07E-02	3.09E-02	1.06E-02	3.32E-03
308.0	4.35E-02	6.42E-02	2.27E-02	1.22E-02
310.0	5.62E-02	9.97E-02	1.27E-02	-9.99E-03
312.0	1.02E-01	1.58E-01	4.56E-02	3.29E-02
314.0	1.29E-01	2.31E-01	2.73E-02	-1.83E-02
316.0	1.29E-01	2.58E-01	-2.70E-04	-2.76E-02
318.0	1.76E-01	3.05E-01	4.76E-02	4.79E-02
320.0	2.07E-01	3.84E-01	3.07E-02	-1.69E-02
320.0		1.92E-01		
Total Area A_t		1.74E+00 W/m^2	Avg 2nd Difference f'	1.61E-03
Total Interval b-a		4.00E+01 nm	Absolute Error	(Estimated)
Average Interval		h = 2 nm	f'●(h^3/24)●(b-a)	2.15E-02
			% error	1.25 % = 100●(0.0215/1.74)

uncertainty in the integration (about 1.4 %, considered as two standard deviation standard uncertainty) and the standard uncertainty in the measured spectra as determined from an uncertainty analysis for the measurement equipment.

X2. EXAMPLE INTEGRATION OF PRODUCT OF SPECTRAL TRANSMITTANCE AND SPECTRAL IRRADIANCE DATA WITH INTERPOLATION TO COMMON WAVELENGTH INTERVALS

X2.1 Example

X2.1.1 The nominal transmittance passband of an UV ultraviolet filter is provided as tabular data by a manufacturer from 300 nm to 320 nm in 2 nm steps.

X2.1.2 The analyst desires to calculate the total integrated UV-B irradiance transmitted by the filter with respect to the Standard Tables G177 Reference UV spectral distribution.

X2.1.3 Since the Standard Tables G177 reference spectrum has step sizes of 0.5 nm from 280 nm to 400 nm, the transmittance data is interpolated to 0.5 nm steps using linear interpolation in accordance with subection 12.3.1.

X2.1.4 The product of the reference spectrum data and the interpolated transmittance data are computed.

X2.1.5 The integral of the product of the filter transmittance and the reference spectrum is computed using Eq 1 of Section 4.

X2.1.6 Table X2.1 shows the raw 2 nm UV filter transmittance data (column 2), the interpolation wavelengths (column 3) and the resulting interpolated data (column 4), the Standard Tables G177 Reference UV spectral distribution data (column 5) and the product of the interpolated and spectral distribution data (column 6) and the area (integral) calculations (column 7).

X2.1.7 The result for computing the total irradiance transmitted by the filter according to Eq 1 of Section 4 is 0.9021 W/m^2. The results of integrating just the spectral

TABLE X2.1 Example of Integration of Product of Transmittance and Spectral Irradiance

Wavelength, nm	UV Transmittance	Wavelength, nm	Interpolated *T*	G177 Irradiance, *E*	Product $T \times E$	Area A_i Product
300	0.003	300.0	0.003	0.0116	0.000	0.0000
		300.5	0.007	0.0130	0.000	0.0000
		301.0	0.011	0.0177	0.000	0.0001
		301.5	0.015	0.0222	0.000	0.0001
302	0.019	302.0	0.019	0.0229	0.000	0.0002
		302.5	0.038	0.0307	0.001	0.0004
		303.0	0.056	0.0459	0.003	0.0009
		303.5	0.074	0.0546	0.004	0.0016
304	0.092	304.0	0.092	0.0556	0.005	0.0023
		304.5	0.139	0.0646	0.009	0.0035
		305.0	0.186	0.0798	0.015	0.0059
		305.5	0.233	0.0848	0.020	0.0086
306	0.280	306.0	0.280	0.0819	0.023	0.0107
		306.5	0.346	0.0892	0.031	0.0134
		307.0	0.412	0.1080	0.044	0.0188
		307.5	0.478	0.1277	0.061	0.0264
308	0.545	308.0	0.545	0.1334	0.073	0.0334
		308.5	0.578	0.1406	0.081	0.0385
		309.0	0.612	0.1334	0.082	0.0407
		309.5	0.646	0.1310	0.085	0.0416
310	0.680	310.0	0.680	0.1482	0.101	0.0464
		310.5	0.646	0.1867	0.121	0.0554
		311.0	0.612	0.2288	0.140	0.0652
		311.5	0.578	0.2283	0.132	0.0680
312	0.545	312.0	0.545	0.2380	0.130	0.0654
		312.5	0.478	0.2420	0.116	0.0613
		313.0	0.412	0.2564	0.106	0.0553
		313.5	0.346	0.2608	0.090	0.0490
314	0.280	314.0	0.280	0.2768	0.077	0.0419
		314.5	0.233	0.2842	0.066	0.0359
		315.0	0.186	0.2926	0.054	0.0301
		315.5	0.139	0.2604	0.036	0.0226
316	0.092	316.0	0.092	0.2589	0.024	0.0150
		316.5	0.074	0.3026	0.022	0.0115
		317.0	0.056	0.3446	0.019	0.0104
		317.5	0.038	0.3693	0.014	0.0083
318	0.019	318.0	0.019	0.3463	0.007	0.0052
		318.5	0.015	0.3480	0.005	0.0030
		319.0	0.011	0.3733	0.004	0.0024
		319.5	0.007	0.3699	0.003	0.0017
320	0.003	320.0	0.003	0.3889	0.001	0.0009
320						0.0004

irradiance for the Standard Tables G177 reference spectrum in the interval 300 nm to 320 nm is 3.63 W/m^2.

X2.1.8 Computing the estimated absolute error in the product integral in accordance with Section 15, h = 0.5 nm, (b-a) = 20 nm, the average of the 2nd differences of the product is 0.00004, thus the estimated absolute error = 0.00004 (20) (0.53)/24 = 0.000004 W/m^2 and the relative error in the integral is
100 (0.000004)/0.9021 = 0.0005 %.

X2.1.9 Note that this analysis does not account for the error contribution from the difference between the actual transmittance curve and the interpolated transmittance data. For instance, if the transmittance curve were actually a nearly Gaussian profile, symmetrical about a center wavelength of 310 nm and 6 nm half power bandwidth, the total (integrated) error due to the difference between the Gaussian curve and the interpolated curve can be computed to be 0.03 percent transmittance. One can then estimate that for the computed total transmittance of (0.9021)/(3.63) = 0.248 transmittance, the interpolation error is about 0.003/0.248 or an additional 1.2 % above the 1.2 % estimated error in the integrated product. Fig. X2.1 shows the Standard Tables G177 spectral irradiance, raw and interpolated transmittance data, and Gaussian profile approximation to the transmittance data. Raw transmittance data are at 2 nm intervals.

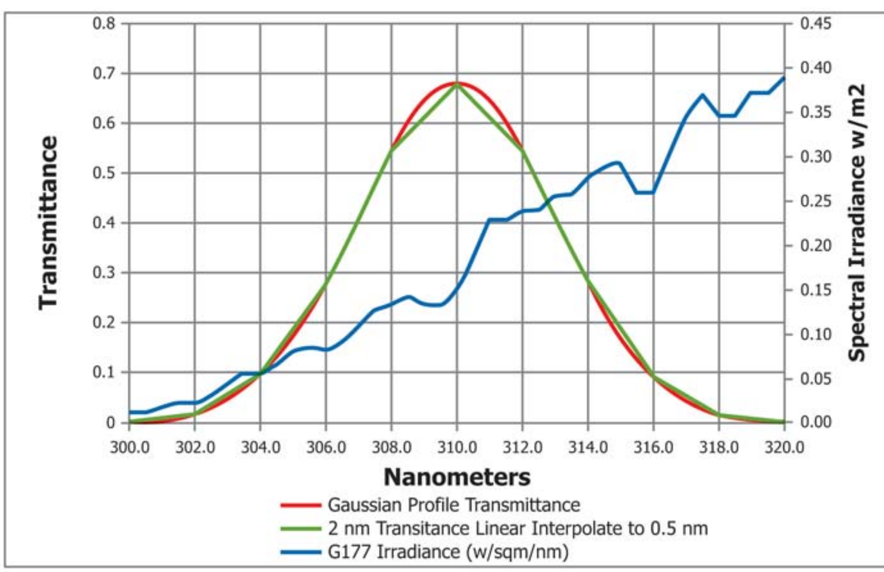

FIG. X2.1 Plots of Standard Tables G177 Spectral Irradiance, Raw, Interpolated, and Gaussian Approximation to Transmittance Data for the Data in Table X2.1

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