Standard Guide for Statistical Analysis of Accelerated Service Life Data[1]

This standard is issued under the fixed designation G172; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

ε^1 NOTE—Editorially corrected designation and footnote 1 in November 2013

1. Scope

1.1 This guide briefly presents some generally accepted methods of statistical analyses that are useful in the interpretation of accelerated service life data. It is intended to produce a common terminology as well as developing a common methodology and quantitative expressions relating to service life estimation.

1.2 This guide covers the application of the Arrhenius equation to service life data. It serves as a general model for determining rates at usage conditions, such as temperature. It serves as a general guide for determining service life distribution at usage condition. It also covers applications where more than one variable act simultaneously to affect the service life. For the purposes of this guide, the acceleration model used for multiple stress variables is the Eyring Model. This model was derived from the fundamental laws of thermodynamics and has been shown to be useful for modeling some two variable accelerated service life data. It can be extended to more than two variables.

1.3 Only those statistical methods that have found wide acceptance in service life data analyses have been considered in this guide.

1.4 The Weibull life distribution is emphasized in this guide and example calculations of situations commonly encountered in analysis of service life data are covered in detail. It is the intention of this guide that it be used in conjunction with Guide G166.

1.5 The accuracy of the model becomes more critical as the number of variables increases and/or the extent of extrapolation from the accelerated stress levels to the usage level increases. The models and methodology used in this guide are shown for the purpose of data analysis techniques only. The fundamental requirements of proper variable selection and measurement must still be met for a meaningful model to result.

2. Referenced Documents

2.1 *ASTM Standards:*[2]

G166 Guide for Statistical Analysis of Service Life Data

G169 Guide for Application of Basic Statistical Methods to Weathering Tests

3. Terminology

3.1 *Terms Commonly Used in Service Life Estimation:*

3.1.1 *accelerated stress, n*—that experimental variable, such as temperature, which is applied to the test material at levels higher than encountered in normal use.

3.1.2 *beginning of life, n*—this is usually determined to be the time of delivery to the end user or installation into field service. Exceptions may include time of manufacture, time of repair, or other agreed upon time.

3.1.3 *cdf, n*—the cumulative distribution function (cdf), denoted by $F(t)$, represents the probability of failure (or the population fraction failing) by time = (t). See 3.1.7.

3.1.4 *complete data, n*—a complete data set is one where all of the specimens placed on test fail by the end of the allocated test time.

3.1.5 *end of life, n*—occasionally this is simple and obvious, such as the breaking of a chain or burning out of a light bulb filament. In other instances, the end of life may not be so catastrophic or obvious. Examples may include fading, yellowing, cracking, crazing, etc. Such cases need quantitative measurements and agreement between evaluator and user as to the precise definition of failure. For example, when some critical physical parameter (such as yellowing) reaches a pre-defined level. It is also possible to model more than one failure mode for the same specimen (that is, the time to reach a specified level of yellowing may be measured on the same specimen that is also tested for cracking).

3.1.6 *f(t), n*—the probability density function (pdf), equals the probability of failure between any two points of time $t_{(1)}$

[1] This guide is under the jurisdiction of ASTM Committee G03 on Weathering and Durability and is the direct responsibility of Subcommittee G03.08 on Service Life Prediction.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

and $t_{(2)}$; $f(t) = \frac{dF(t)}{dt}$. For the normal distribution, the pdf is the "bell shape" curve.

3.1.7 *F(t), n*—the probability that a random unit drawn from the population will fail by time (t). Also $F(t)$ = the decimal fraction of units in the population that will fail by time (t). The decimal fraction multiplied by 100 is numerically equal to the percent failure by time (t).

3.1.8 *incomplete data, n*—an incomplete data set is one where (*1*) there are some specimens that are still surviving at the expiration of the allowed test time, or (*2*) where one or more specimens is removed from the test prior to expiration of the allocated test time. The shape and scale parameters of the above distributions may be estimated even if some of the test specimens did not fail. There are three distinct cases where this might occur.

3.1.8.1 *multiple censored, n*—specimens that were removed prior to the end of the test without failing are referred to as left censored or type II censored. Examples would include specimens that were lost, dropped, mishandled, damaged or broken due to stresses not part of the test. Adjustments of failure order can be made for those specimens actually failed.

3.1.8.2 *specimen censored, n*—specimens that were still surviving when the test was terminated after a set number of failures are considered to be specimen censored. This is another case of right censored or type I censoring. See 3.1.8.3.

3.1.8.3 *time censored, n*—specimens that were still surviving when the test was terminated after elapse of a set time are considered to be time censored. Examples would include experiments where exposures are conducted for a predetermined length of time. At the end of the predetermined time, all specimens are removed from the test. Those that are still surviving are said to be censored. This is also referred to as right censored or type I censoring. Graphical solutions can still be used for parameter estimation. A minimum of ten observed failures should be used for estimating parameters (that is, slope and intercept, shape and scale, etc.).

3.1.9 *material property, n*—customarily, service life is considered to be the period of time during which a system meets critical specifications. Correct measurements are essential to produce meaningful and accurate service life estimates.

3.1.9.1 *Discussion*—There exists many ASTM recognized and standardized measurement procedures for determining material properties. These practices have been developed within committees having appropriate expertise, therefore, no further elaboration will be provided.

3.1.10 *R(t), n*—the probability that a random unit drawn from the population will survive at least until time (t). Also $R(t)$ = the fraction of units in the population that will survive at least until time (t); $R(t) = 1 - F(t)$.

3.1.11 *usage stress, n*—the level of the experimental variable that is considered to represent the stress occurring in normal use. This value must be determined quantitatively for accurate estimates to be made. In actual practice, usage stress may be highly variable, such as those encountered in outdoor environments.

3.1.12 *Weibull distribution, n*—for the purposes of this guide, the Weibull distribution is represented by the equation:

$$F(t) = 1 - e^{-\left(\frac{t}{c}\right)^{b}} \quad (1)$$

where:

$F(t)$ = probability of failure by time (t) as defined in 3.1.7,
t = units of time used for service life,
c = scale parameter, and
b = shape parameter.

3.1.12.1 *Discussion*—The shape parameter (b), 3.1.12, is so called because this parameter determines the overall shape of the curve. Examples of the effect of this parameter on the distribution curve are shown in Fig. 1.

3.1.12.2 *Discussion*—The scale parameter (c), 3.1.12, is so called because it positions the distribution along the scale of the time axis. It is equal to the time for 63.2 % failure.

NOTE 1—This is arrived at by allowing t to equal c in Eq 1. This then reduces to Failure Probability = $1 - e^{-1}$. which further reduces to equal 1 − 0.368 or 0.632.

4. Significance and Use

4.1 The nature of accelerated service life estimation normally requires that stresses higher than those experienced during service conditions are applied to the material being evaluated. For non-constant use stress, such as experienced by time varying weather outdoors, it may in fact be useful to choose an accelerated stress fixed at a level slightly lower than (say 90 % of) the maximum experienced outdoors. By controlling all variables other than the one used for accelerating degradation, one may model the expected effect of that variable at normal, or usage conditions. If laboratory accelerated test devices are used, it is essential to provide precise control of the variables used in order to obtain useful information for service life prediction. It is assumed that the same failure mechanism operating at the higher stress is also the life determining mechanism at the usage stress. It must be noted that the validity of this assumption is crucial to the validity of the final estimate.

4.2 Accelerated service life test data often show different distribution shapes than many other types of data. This is due to the effects of measurement error (typically normally distributed), combined with those unique effects which skew service life data towards early failure time (infant mortality failures) or late failure times (aging or wear-out failures). Applications of the principles in this guide can be helpful in allowing investigators to interpret such data.

4.3 The choice and use of a particular acceleration model and life distribution model should be based primarily on how well it fits the data and whether it leads to reasonable projections when extrapolating beyond the range of data. Further justification for selecting models should be based on theoretical considerations.

NOTE 2—Accelerated service life or reliability data analysis packages are becoming more readily available in common computer software packages. This makes data reduction and analyses more directly accessible to a growing number of investigators. This is not necessarily a good thing as the ability to perform the mathematical calculation, without the

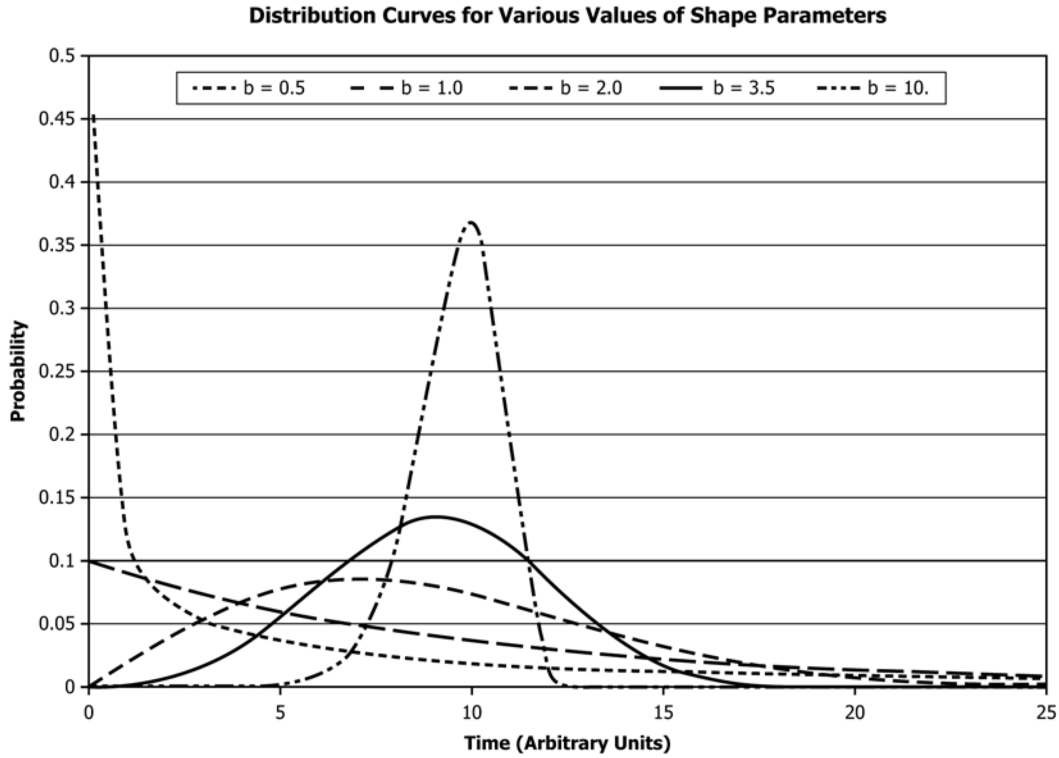

FIG. 1 Effect of the Shape Parameter (b) on the Weibull Probability Density

fundamental understanding of the mechanics may produce some serious errors. See Ref (**1**).[3]

5. Data Analysis

5.1 *Overview*—It is critical to the accuracy of Service Life Prediction estimates based on accelerated tests that the failure mechanism operating at the accelerated stress be the same as that acting at usage stress. Increasing stress(es), such as temperature, to high levels may introduce errors due to several factors. These include, but are not limited to, a change of failure mechanism, changes in physical state, such as change from the solid to glassy state, separation of homogenous materials into two or more components, migration of stabilizers or plasticisers within the material, thermal decomposition of unstable components and formation of new materials which may react differently from the original material.

5.2 A variety of factors act to produce deviations from the expected values. These factors may be of purely a random nature and act to either increase or decrease service life depending on the magnitude and nature of the effect of the factor. The purity of a lubricant is an example of one such factor. An oil clean and free of abrasives and corrosive materials would be expected to prolong the service life of a moving part subject to wear. A contaminated oil might prove to be harmful and thereby shorten service life. Purely random variation in an aging factor that can either help or harm a service life might lead to a normal, or gaussian, distribution. Such distributions are symmetrical about a central tendency, usually the mean.

5.2.1 Some non-random factors act to skew service life distributions. Defects are generally thought of as factors that can only decrease service life (that is, monotonically decreasing performance). Thin spots in protective coatings, nicks in extruded wires, chemical contamination in thin metallic films are examples of such defects that can cause an overall failure even though the bulk of the material is far from failure. These factors skew the service life distribution towards early failure times.

5.2.2 Factors that skew service life towards greater times also exist. Preventive maintenance on a test material, high quality raw materials, reduced impurities, and inhibitors or other additives are such factors. These factors produce lifetime distributions shifted towards increased longevity and are those typically found in products having a relatively long production history.

5.3 *Failure Distribution*—There are two main elements to the data analysis for Accelerated Service Life Predictions. The first element is determining a mathematical description of the life time distribution as a function of time. The Weibull distribution has been found to be the most generally useful. As Weibull parameter estimations are treated in some detail in Guide G166, they will not be covered in depth here. It is the intention of this guide that it be used in conjunction with Guide G166. The methodology presented herein demonstrates how to integrate the information from Guide G166 with accelerated

[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.

test data. This integration permits estimates of service life to be made with greater precision and accuracy as well as in less time than would be required if the effect of stress were not accelerated. Confirmation of the accelerated model should be made from field data or data collected at typical usage conditions.

5.3.1 Establishing, in an accelerated time frame, a description of the distribution of frequency (or probability) of failure versus time in service is the objective of this guide. Determination of the shape of this distribution as well as its position along the time scale axis is the principal criteria for estimating service life.

5.4 *Acceleration Model*—The most common model for single variable accelerations is the Arrhenius model. It was determined empirically from observations made by the Swedish scientist S. A. Arrhenius. As it is one that is often encountered in accelerated testing it will be used as the fundamental model for single variables accelerations in this guide.

5.4.1 Although the Arrhenius model is commonly used, it should not be considered to be a basic scientific law, nor to necessarily apply to all systems. Application of the principles of this guide will increase the confidence of the data analyst regarding the suitability of such a model. There are many instances where its suitability is questionable. Biological systems are not expected to fit this model, nor are systems that undergo a change of phase or a change of mechanism between the usage and some experimental levels.

5.4.2 The Arrhenius model has, however, been found to be of widespread utility and the accuracy has been verified in some systems. Wherever possible, confirmation of the accuracy of the accelerated model should be verified by actual usage data. The form of the equation most often encountered is:

$$\text{Rate} = Ae^{-\Delta H/kT} \tag{2}$$

where:

A = pre-exponential factor and is characteristic of the product failure mechanism and test conditions,
T = absolute temperature in Kelvin (K),
ΔH = activation energy. For the sake of consistency with many references contained in this guide, the symbol ΔH is used. In other recent texts, it has become a common practice to use E for the activation energy parameter. Either symbol is correct, and
k = Boltzmann's constant. Any of several different equivalent values for this constant can be used depending on the units appropriate for the specific situation. Three commonly used values are: (*1*) 8.617×10^{-5} eV/K, (*2*) 1.380×10^{-18} ergs/K, and (*3*) 0.002 kcal/mole·K.

5.4.3 The rate may be that of any reasonable parameter that one wishes to model at accelerated conditions and relate to usage conditions. It could be the rate in color change units per month, gloss loss units per year, crack growth in mm's per year, degree of chalking per year and so forth. It could also be the amount of corrosion penetration per hour, or byte error growth rate on data storage disks.

5.4.4 Because the purpose of this guide is to model service life, the Eq 2 may be rewritten to express the Arrhenius model in terms of time rather than rate. As time and rate are inversely related, the new expression is formed by changing the sign of the exponent so that the time, *t*, is:

$$\text{Time} = A'e^{\Delta H/kT} \tag{3}$$

5.4.5 The time element used in the Eq 3 is arbitrary. It can be the time for the first 5 % failure, time for average failure, time for 63.2 % failure, time for 95 % failure or any other representation that would suit the particular application.

5.4.6 Because Guide G166 emphasizes the utility of the Weibull distribution model, it will be used for the rest of the discussion in this guide as well. Should a different distribution model fit a particular application, simple adjustments permit their use. Therefore, by setting the value for time in the above expression to be the time for 63.3 % failure, the model will predict the scale parameter for the Weibull distribution at the usage stress.

5.4.7 The Weibull model, as given in Eq 1, is also expressed as a function of time. We can, therefore, relate the Weibull distribution model to the Arrhenius acceleration model by:

$$1 - e^{-\left(\frac{t}{c}\right)^{b}} = Ae^{\Delta H/kT} = F(t) \tag{4}$$

5.4.8 By determining the Weibull shape and scale parameters at temperatures above the expected service temperature, and relating these parameters with the Arrhenius model, one may determine an expression to estimate these parameters at usage condition. This integration of the Weibull parameters and an acceleration model such as Arrhenius forms the fundamental structure of this guide.

6. Accelerated Service Life Model—Single Variable

6.1 For the purposes of this discussion, the accelerating stress variable is assumed to be temperature. This is generally true for most systems and is the stress most frequently used in the Arrhenius model. Other ones, such as voltage, may work as well.

6.2 *Temperature Selection*—One of the critical points used in Accelerated Service Life modeling is the choice of the number and levels of the accelerating stress. Theoretically, it takes only two levels of stress to develop a linear model and extrapolate to usage conditions. This does not provide any insight into the degree of linearity, or goodness of fit, of the model. At least three levels of the accelerating stress are necessary to determine an estimate of linearity. These should be chosen such that one can reasonably expect to obtain good estimates for the shape and scale parameters of the Weibull model at the lowest stress temperature and within the allowable time for the experiment.

6.2.1 If the service life of the material is expected to be on the order of years at 25°C, and the time available to collect supporting data is on the order of months, then the lowest temperature chosen might be 60°C. This would reasonably be expected to produce sufficient failures to model the Weibull distribution within the allotted time frame. This is only used as an example. The temperature is system dependent and will vary for each material evaluated.

6.2.2 The highest temperature chosen is one that should allow one to accurately measure the time to failure of each specimen under test. If the selected upper temperature is too high, then all or nearly all of the test specimens may fail before the first test measurement interval. More importantly, if the highest temperature level produces a change in degradation mechanism, the model is not valid.

6.3 *Specimen Distribution*—Whenever the cost of specimens or the cost of analysis is a significant factor, a non-uniform distribution of specimens is recommended over having the same number of specimens at each temperature. The reasons for this are:

6.3.1 Use of more specimens at lower temperatures, compared to the number used at higher temperatures, increases the chance of obtaining sufficient failures within the allotted time for the experiment and improves the accuracy of extrapolation to the usage condition.

6.3.2 If three evenly spaced temperatures are chosen for the number of stress levels, and there are *x* specimens available for the experiment then place x/7 at the high temperature, 2x/7 at the mid temperature and 4x/7 at the lowest temperature. This is only a first order guide (see Ref (**2**)). If the cost of specimens and analysis are not significant, then a more even distribution among the stress conditions may be appropriate.

7. Service Life Estimation

7.1 The Guide G166 may be consulted for methods which may be employed to estimate the service life of a material.

8. Example Calculations—Single Accelerating Variable of Temperature, Weibull Distribution

8.1 Determine Weibull scale and shape parameters for failure times at each accelerated temperature.

8.1.1 Consider a hypothetical case where 55 adhesive coated strips are placed on test. This particular adhesive is one that exhibits a characteristic of thermal degradation resulting in sudden failure from stress. The specimens are divided into three groups with one group being placed in an oven at 80°C, the second group in an oven at 70°C and the third group into an oven at 60°C. The first group contains 10 specimens, the second group contains 15 specimens and the third group contains 30 specimens. This approximates the 1X, 2X, 4X ratio cited above.

TABLE 1 Failure Times for Experimental Adhesive, h

80°C	70°C	60°C	
1465	2375	2407	3590
1384	2259	2521	3703
1177	2399	2727	3764
1857	2062	2820	3806
1998	1773	2903	4018
1244	2367	2954	4087
1506	2606	3102	4210
1424	2348	3122	4230
1595	1869	3221	4254
947	2194	3237	4407
	2115	3239	4560
	3240	3398	4525
	1411	3440	4650
	1707	3524	4680
	2522	3557	4850

8.1.2 The time to failure for this application is defined as the time at which the adhesive strip will no longer support a 5 lb load. The test apparatus is constructed with one end of each strip adhered to a test panel and the other end suspending a 5 lb weight. Optical proximity sensors are used to detect when the strip releases from the panel. The times to fail for each individual strip are recorded electronically to the nearest hour. Table 1 is a summary of the times to fail for each individual strip, by temperature.

8.1.3 From these three sets of data, three sets of Weibull parameters are calculated, one for each temperature. Refer to Guide G166 for detailed examples for these calculations. The values determined from the above sets of data are shown in Table 2.

TABLE 2 Summary of Weibull Parameters for the Accelerated Data in Table 1

	80°C	70°C	60°C
Weibull Scale	1580.8	2391.1	3932.9
Weibull Shape	5.39	5.45	6.10

8.2 Plot data on one common Weibull graph.

8.2.1 Graphically display the data before proceeding further with analysis. This simple step allows the analyst to detect abnormal trends, outliers and any other anomalous behavior of the data. The graph in Fig. 2 shows the three sets of accelerated data displayed on one Weibull axis.

8.2.2 From inspection of the graphical display above and the numerical values of the shape parameters in Table 2, it may be seen that the Weibull shapes (slopes of the line) are essentially the same. A significant difference among the shapes may indicate a change in degradation mechanism has occurred. If the shapes are essentially the same, then it is safer to assume that the same mechanism operates at all of the experimental temperatures.

8.2.3 The Weibull scale parameters show a clear trend toward higher values as the temperature decreases. This is what is to be expected if the samples fail sooner at higher temperatures.

8.3 Estimate the Weibull scale parameter at the usage condition.

8.3.1 For the sake of this example, it is assumed that the usage temperature for this tape application is 25°C. We need then to regress the Weibull scale parameters versus temperature to estimate what the scale would be at 25°C. To do this, we use Eq 4, which relates the Arrhenius equation to the Weibull scale parameter. By taking the natural logarithm of both sides of the equation, the following is produced:

$$\ln(A') + \Delta H/k \cdot (1/T) = \ln[F(t)] \quad (5)$$

NOTE 3—It doesn't matter whether natural logarithms (ln) or base 10 logarithms (log) are used, only that one is consistent throughout a calculation. Natural logarithms (ln) are chosen here to be consistent with Guide G166.

8.3.2 In this form, we now have the equation for a straight line ($Y = a_{(1)}X + a_{(2)}$) with ln $[F(t)]$ representing the dependent variable (Y), $\Delta H/k$ is the slope of the line ($a_{(1)}$), $1/T$ is the independent variable (X) and ln (A') is the intercept ($a_{(2)}$). Simple linear regression of ln $[F(t)]$ and $1/T$ will allow us to

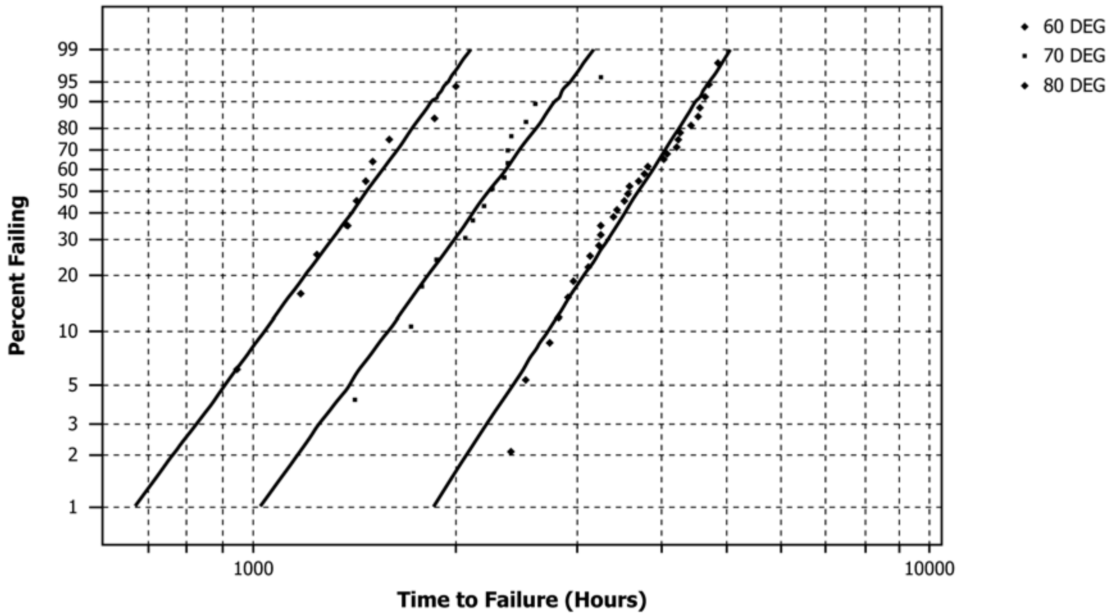

FIG. 2 Weibull Probability Plots for 80°C, 70°C, and 60°C Experimental Adhesive Failure Times, h

solve for the slope and intercept. As we have three equations, and only two unknowns, there is ample information for the solution to be found.

8.3.3 *Convert °C to K*—In order to convert °C to K, the constant 273.1 is added to each centigrade temperature. Thus 80°C become 353.1 K, 70°C becomes 343.1 K and 60°C becomes 333.1 K.

8.3.4 *Regression*—Calculation of the reciprocals of Kelvin temperature and the natural logarithm of the Weibull Scale parameters produces the values shown in Table 3.

TABLE 3 Summary of Estimated Weibull Scale Parameters for Experimental Adhesive

Temp. K	1/K	Weibull Scale, h	ln, (Scale, h)
353.1	0.0028321	1580.8	7.3657
343.1	0.0029146	2391.1	7.7795
333.1	0.0030021	3932.9	8.2771

8.3.4.1 Linear regression of the ln (Scale) versus the reciprocal Kelvin temperature produces the following:

ln Scale = −7.83 + 5363 (1/*T* K)

8.3.4.2 As we wish to calculate the Weibull scale parameter at 298.1 K (25°C) we simply substitute 298.1 for *T* K and solve for ln (Scale). This becomes:

ln (Scale) = −7.83 + 5363 (1/298.1)
ln (Scale) = −7.83 + 17.9906
ln (Scale) = 10.1606
Scale = 25864 h

8.3.4.3 This then translates to the estimate that 63.2 % of the adhesive strips will release by 2.95 years if exposed at 25°C temperature.

8.3.4.4 A graphical display of the ln(Weibull Scale) versus 1/Temperature K is shown in Fig. 3. It may be seen that the three ln(Weibull Scale) values lie along a straight line when plotted against the reciprocal of the temperature in K. It may also be seen that at the value for 1/*T* K for 25°C (0.003354) the ln(Scale) agrees with the calculated value of 10.1606 above.

8.3.5 *Acceleration Factor*—Acceleration factors must be used with extreme caution. They apply only to the system where the specific data sets have been analyzed. They do not extend to other systems. To calculate the acceleration factor for the example data one needs only to ratio the scale factors. The scale factor at 25°C is assigned the acceleration factor value of 1 as it is, by definition for this case, the usage condition. By dividing the scale factor at the usage condition by the scale factor at the accelerated condition, the amount of acceleration provided by the higher temperature may be determined. The result of this operation for the example data is shown in Table 4.

TABLE 4 Estimated Scale Parameters and Acceleration Factor for Experimental Adhesive Data

Temperature	Scale	Acceleration Factor
80	1580.8	16.366
70	2391.1	10.819
60	3932.9	6.578
25	25864.0	1.0

8.3.5.1 As a final check on the entire analysis, the failure times from the accelerated temperatures may be multiplied by their acceleration factor to normalize all of the accelerated date to the usage condition. All of the failure times at 80°C are multiplied by 16.366, all of the failure times at 70°C are multiplied by 10.819 and all of the failure times at 60°C are multiplied by 6.578.

8.3.5.2 After performing this operation, all of the normalized failure time data may be combined into one data set and the Weibull shape and scale parameters may be recalculated

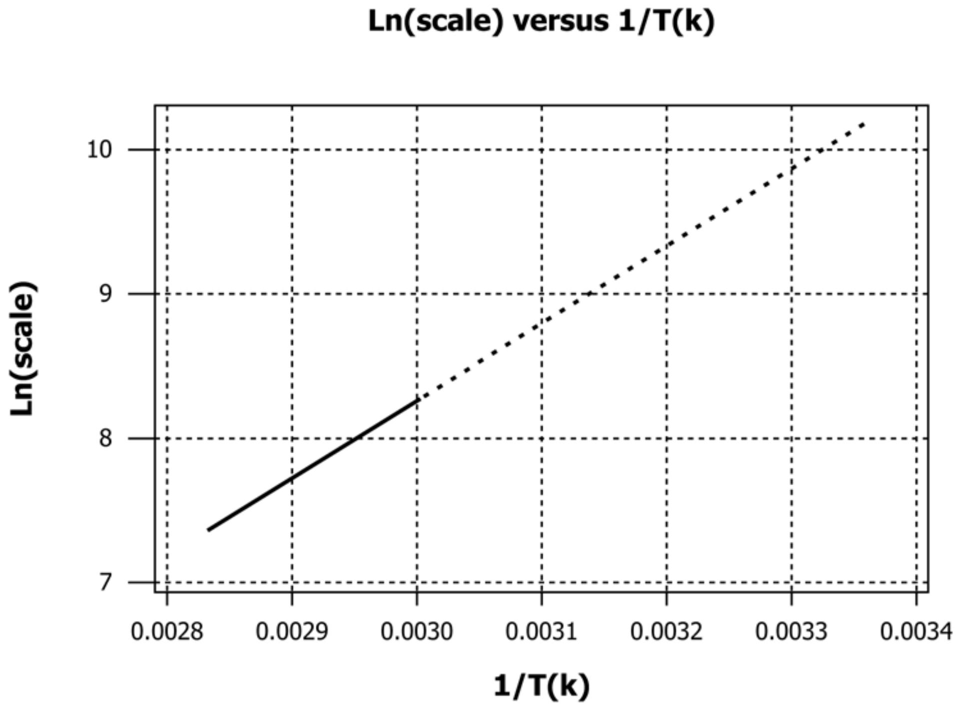

FIG. 3 ln (Weibull Scale) versus 1/T °K for Experimental Adhesive

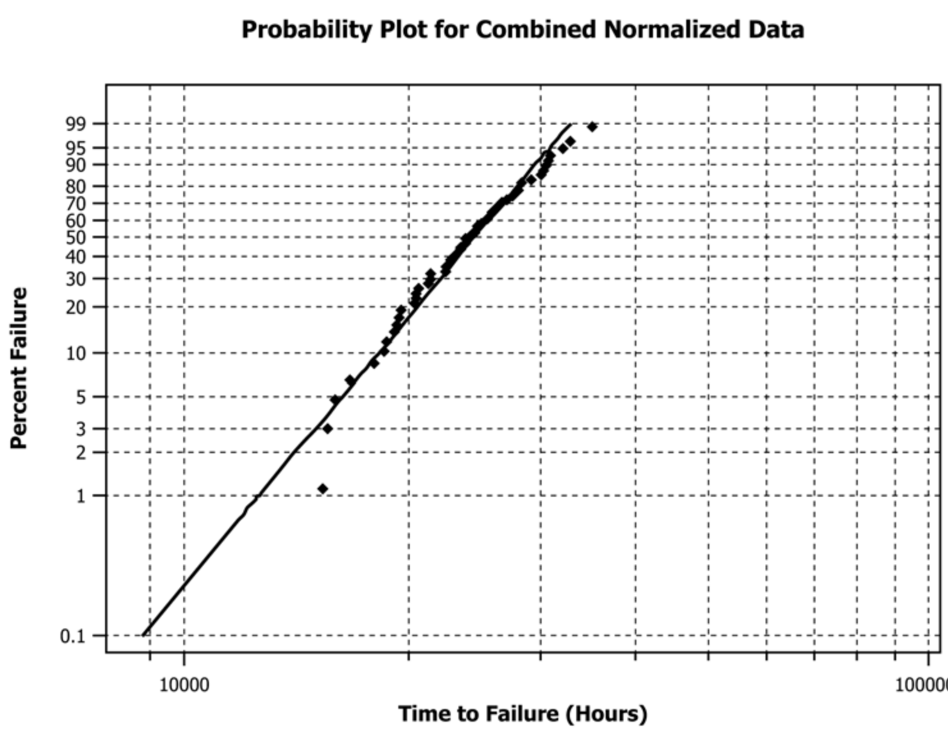

FIG. 4 Combined Normalized Data Plotted on a Single Probability Plot

based on the combined data set. Fig. 4 shows the result of plotting all of the normalized failure times on one axis.

8.3.5.3 It may be seen that there is excellent fit for all of the normalized data to one line, even though it derives from three different temperatures. Also, the Weibull scale for the combined data is 25 870, which is excellent agreement with the estimated value of 25 871 estimated from the Arrhenius equation. The Weibull shape parameter of 5.74 from the normalized data is also in excellent agreement with the individual scale parameters calculated from the three accelerated conditions above.

8.3.5.4 As a final calculation, a survival plot, as described in Guide G166 may be calculated for the above data. This is shown in Fig. 5.

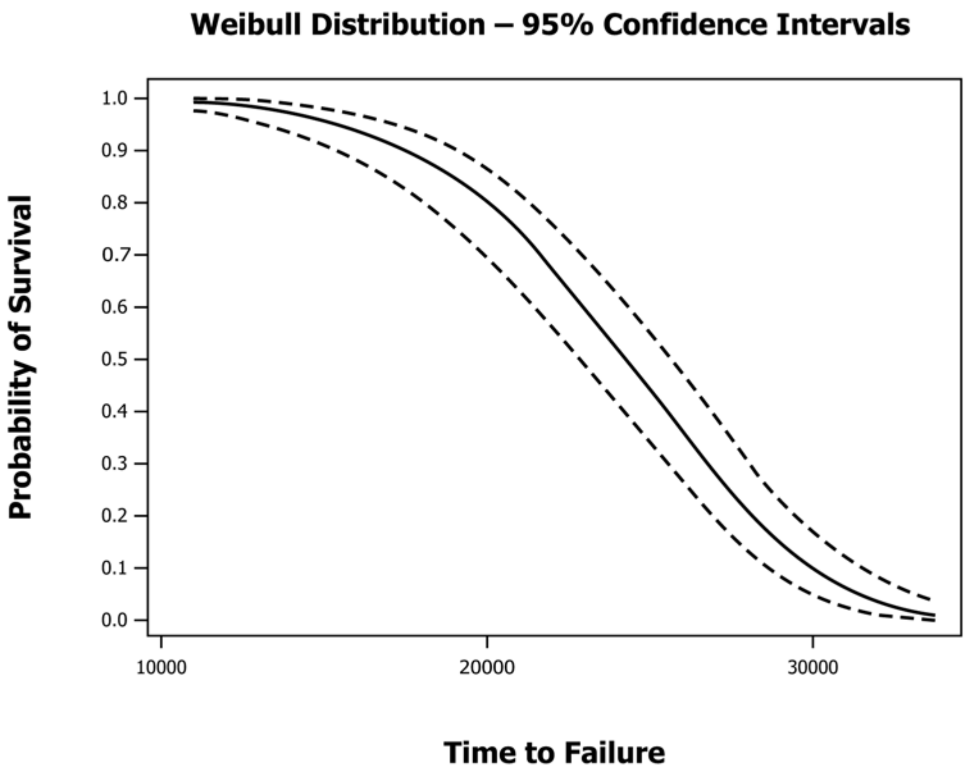

FIG. 5 Survival Plot of Combined Normalized Data

9. Accelerated Service Life Estimation With More Than One Variable

9.1 Often there is a need to accelerate the effects of more than one variable. A common set of variables used in a two variable model is temperature and relative humidity (rh). An example where this applies is the oxidation of metals. The rate is not only dependent on the temperature but on the availability of water in the form of humidity. Here again it is imperative that one be alert to make sure that the accelerated conditions do not alter the mechanism of the reaction. If the temperature of the specimen were to exceed a critical value of around 100°C, the ability of the atmospheric moisture to interact with the metal surface as a liquid interface would be prevented.

9.2 *Acceleration Model (More Than One Variable)*—There have been many specialized models that have been found useful for specific systems. A generalized model was derived from the laws of thermodynamics and, as such, should be a useful starting place for an experimenter. The model is known as the Eyring model after the physical chemist Henry Eyring. In its complete form the expression is:

$$tc = A''T^{\alpha}e^{\Delta H/kT_e(B+C/T)rh} \quad (6)$$

This may be simplified, however, to the expression:

$$tc = A''e^{\Delta H/kT_e(B)rh} \quad (7)$$

where:

A'' = pre-exponential factor and is characteristic of the product failure mechanism and test conditions,

ΔH = activation energy. For the sake of consistency with references contained in this guide, the symbol ΔH is used throughout this guide. In other recent texts, it has become a common practice to use E for the activation energy parameter. Either symbol is correct,

k = Boltzmann's constant. Any of several different equivalent values for this constant can be used depending on the units appropriate for the specific situation. Three commonly used values are: (*1*) 8.617×10^{-5} eV/ K, (*2*) 1.380×10^{-18} ergs / K, and (*3*) 0.002 kcal/mole-K,

T = absolute temperature in Kelvin,

B = pre-exponential factor relating Relative Humidity to failure time, and

rh = relative humidity expressed as a decimal, (20 % = 0.20).

9.2.1 The simplification, as described in Tobias and Trindade's "Applied Reliability" is applicable for models covering only a small range of temperatures and relative humidities as would be used in service life studies.

9.2.2 It may be noted that the reduced expression is very similar to the Arrhenius model but with the one extra exponential term $e^{(B)rh}$. The *rh* parameter used in this expression may be replaced with other stress parameters that better suit different systems. For example, temperature and applied voltage may be the appropriate choices for accelerated aging of capacitors, transistors, resistors and other electronic applications. Temperature and inflation pressure may be the choices for tire wear; engine rpm and lubricant viscosity may be the choices for engine life.

9.2.3 The Eyring model may be extended to include third, fourth and even further stress parameters, according to the original derivation. The example used in 9.2.4 for two stresses may be easily extended as well. There are, however, no known examples where more than two stresses have been used in practice.

9.2.4 As in Section 5, the time, t_c, from the Eyring Model and $F(t)$ from the Weibull distribution expression are equated to produce the expression:

$$1 - e^{-\left(\frac{t}{c}\right)^b} = A''e[\Delta H/kT]e[B \cdot rh] = F(t) \tag{8}$$

9.2.5 It can be seen that there are now two variables, T for temperature and rh for relative humidity and that there are three constants for which to solve, A'', ΔH and B. The solution of this equation may be found by experimentally determining the Weibull distribution at a minimum of three different conditions. A minimum of five different combinations of conditions is required to determine the linearity of the mathematical model. This is accomplished by using three different temperatures and three different relative humidities. This permits examination for linearity along both the temperature and the relative humidity variables.

10. Example Calculations—Multiple Accelerating Variables (*T* and *rh*)

10.1 The same consideration for selection of the acceleration stress levels should be used as described in Section 6. In addition, the distribution of specimens should be skewed towards the usage condition. In the example calculation shown below, there are 10 specimens per stress at the higher temperature/relative humidity condition, 15 specimens per stress for the mid level stress and 30 specimens for the lowest temperature.

10.1.1 For this example, consider another hypothetical case where 80 test specimens of electric motors are exposed to three levels of temperature, 80°C, 70°C and 60°C and three levels of relative humidity, 85 %, 70 %, and 55 %. The motors are divided into five groups of 10, 10, 15, 15 and 30 and placed into environmental chambers operated continuously at the specified conditions. The current required for each motor is monitored separately. The time to failure for this application is the time at which the current required to turn the motors at 3600 rpm exceeds 0.2 amps. This was an arbitrary current draw agreed upon by the buyer and seller of these motors.

10.1.2 The time to fail (hours) for each motor is recorded, along with the temperature and relative humidity to which it was exposed during the test. Table 5 shows the time to fail, in operating hours, for each motor as a function of stress conditions.

10.1.3 As in 8.1, the Weibull parameters of shape and scale (in hours) were determined for each of the five data sets in Table 5. These values are tabulated in Table 6.

10.2 Again in a manner similar to that used in 8.2, and as per Guide G166, the Weibull data, by stress condition, is plotted on the same axis. This is done to look for abnormal trends, outliers and any anomalous behavior of the data. Fig. 6 shows the five sets of accelerated data displayed on one Weibull axis.

TABLE 5 Summary of Life Test Data for Electric Motors

80°C/ 85 % rh	80°C/ 70 % rh	80°C/ 55 % rh	70°C/ 85 % rh	60°C/ 85 % rh	
1465	2296	2307	2872	2181	3590
1384	1855	2682	2732	2242	3703
1177	1480	2590	2901	2545	3764
1957	1542	2324	2493	2624	3920
1998	1740	1812	2144	2706	4136
1244	2402	1938	2862	2954	4220
1506	2070	1702	3151	3102	4301
1424	893	2450	2839	3122	4343
1522	2097	1351	2260	3221	4354
844	1366	2421	2653	3237	4507
		1793	2558	3239	4560
		3144	3918	3398	4525
		2146	1706	3440	4650
		2199	2064	3524	4680
		2414	3050	3557	4747

TABLE 6 Weibull Shape and Scale Parameters for Experimental Electric Motor Data

Stress Conditions	80°C/ 85 % rh	80°C/ 70 % rh	80°C/ 55 % rh	70°C/ 85 % rh	60°C/ 85 % rh
Weibull Shape	4.91	4.79	5.62	5.45	5.75
Weibull Scale	1582	1942	2395	2891	3938

10.2.1 It may be seen that the slopes of the five lines are essentially parallel, indicating a common mechanism that is a requirement for accelerated service life prediction. The Weibull scale parameters indicate a clear trend towards higher numbers (longer life) as the accelerated stress conditions become closer to the usage condition.

10.3 Estimate the Weibull scale parameter at the usage condition.

10.3.1 For the sake of this example, the buyer and seller have agreed that the motors will operate at 25°C and 50 % relative humidity. We need to regress the Weibull scale parameters for the motor examples versus the accelerated temperature and relative humidities at which the data was collected. To do this we refer back to the Eyring model given in Eq 8. By setting the failure percentage at 62.3, $F(t)$ is equal to the Weibull scale parameter. By taking the natural logarithm of both sides of the equal sign we have a linear equation of:

$$\ln(F(t)) = \ln(A'') + \Delta H/k \times 1/T + (B \times rh) = \ln(c) \tag{9}$$

10.3.2 To solve the expression, the temperature in °C must be converted to Kelvin by adding 273.1 to the °C temperatures. Relative humidity is converted to a decimal value from the percent value by dividing by 100. We now have all of the information to solve the regression equation. This is tabulated in Table 7.

TABLE 7 Summary of Weibull Scale Parameter by 1/T Kelvin

T K	1/T Kelvin	rh	Weibull Scale c	ln (Weibull scale c)
353.1	0.0028321	0.85	1582	7.3664
353.1	0.0028321	0.70	1942	7.57147
353.1	0.0028321	0.55	2395	7.78114
343.1	0.0029146	0.85	2891	7.9694
333.1	0.0030021	0.85	3938	8.2784

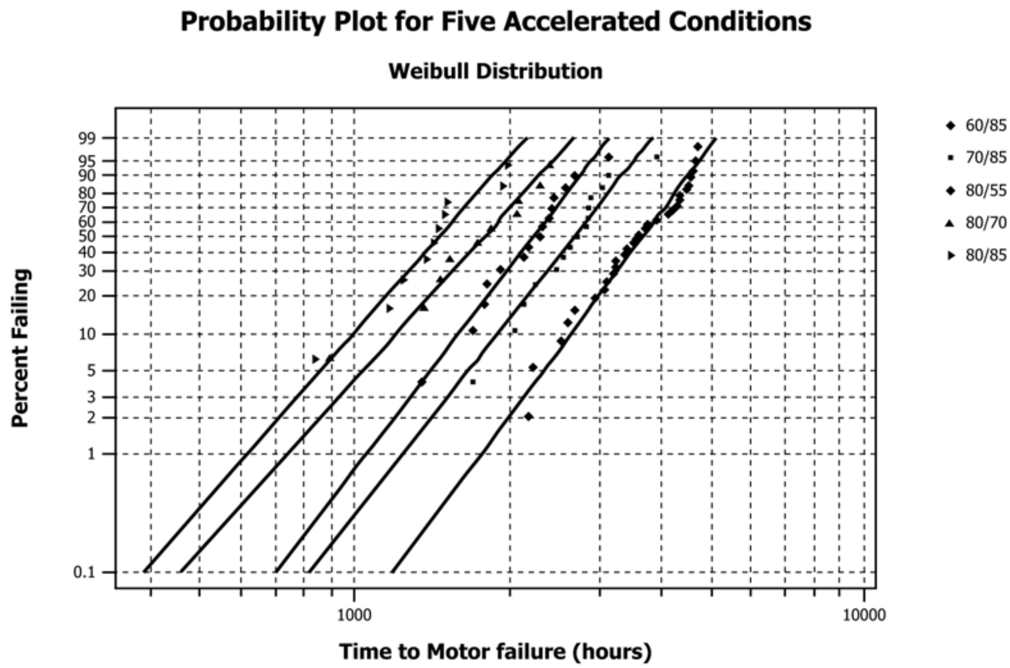

FIG. 6 Weibull Probability Plots for Each Stress Condition Times for Experimental Motor Data

10.3.3 Solution of the regression equation, using Minitab or similar applications, produces the following equation:

$$\ln(\text{Weibull scale}) = -6.879 + (5405.5 \times 1/T\ \text{K}) - (1.1931 \times rh) \quad (10)$$

10.3.3.1 Substituting the usage conditions of 25°C and converting to 298.1 K for temperature and 0.50 for RH we now have:

ln (Weibull scale) = –6.879 + (5405.5 × 0.003354) - (1.1931 × 0.50)
ln (Weibull scale) = –6.879 + 17.5333 – 0.59655
ln (Weibull scale) = 10.6543

10.3.3.2 Therefore, the Weibull scale parameter for motor failure lifetimes at 25°C and 50 % rh = $e^{10.6543}$ or 42 373 h. This means that the estimated time for 63.2 % of the motors to fail by the agreed upon definition, at 25°C and 50 % RH is 42 373 h or 4.84 years.

10.3.4 *Acceleration Factor*—As described in 8.3.3, the acceleration factor for the five accelerated conditions may be calculated by assigning an acceleration factor of 1 to the 25°C, 50 % rh condition. Then by dividing the scale time of 42 373 h by the scale time of the accelerated condition, the amount of acceleration provided by each higher temperature/rh conditions may be calculated. The results of such calculations for the five accelerated conditions of this example are shown in Table 8.

TABLE 8 Estimated Weibull Scale and Acceleration Factors for Experimental Electric Motor Data

Temperature/RH	Scale	Acceleration
80 / 85	1582	26.78
80 / 70	1942	21.82
80 / 55	2395	17.69
70 / 85	2891	14.66
60 / 85	3938	10.76
25 / 50	42373	1.00

10.3.4.1 It is now a simple matter to normalize all of the accelerated data to the usage condition. By multiplying the service life of each individual motor by the acceleration factor associated with the stress at which the motor was tested, the result of all data will be normalized to the usage condition of 25°C and 50 % rh. After such operation has been done, the normalized data may be combined into one dataset and plotted as one curve. The results from the example data have been normalized, combined and plotted as one Weibull plot shown in Fig. 7.

NOTE 4—The application of acceleration factors is system dependent. Factors that are established in one system should not be applied to another system without experimental verification.

10.3.4.2 It may again be seen that there is an excellent fit for all of the normalized data to one common line, even though it derives from five different combinations of temperature and relative humidities. Also, the Weibull scale for the combined data is 42 274 h, which is in excellent agreement with 42 266 h estimated from the Eyring acceleration model. The Weibull shape parameter of 5.402 from the normalized data is also in excellent agreement of the individual shape parameters calculated from the five accelerated stress conditions.

10.3.5 As a final calculation, a survival plot, as described in Guide G166 may be calculated for the above data. This is shown in Fig. 8. The upper and lower 95 % confidence limits are displayed as dashed lines on the graph.

10.3.6 It must be emphasized that extrapolation of an accelerated model to a usage condition must be done with care. The model must make technical sense and be scientifically sound. Caution must be used to avoid logic faults where there may be discontinuities brought about by change of phase of a material (gas to liquid to solid) or a change in mechanism of

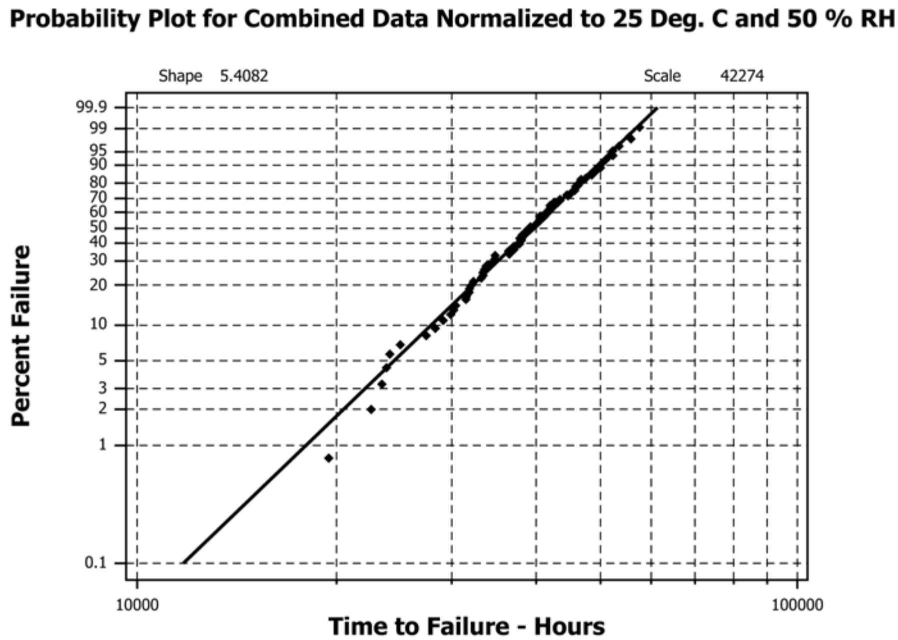

FIG. 7 Combined Normalized Electric Motor Data Plotted on a Single Probability Plot

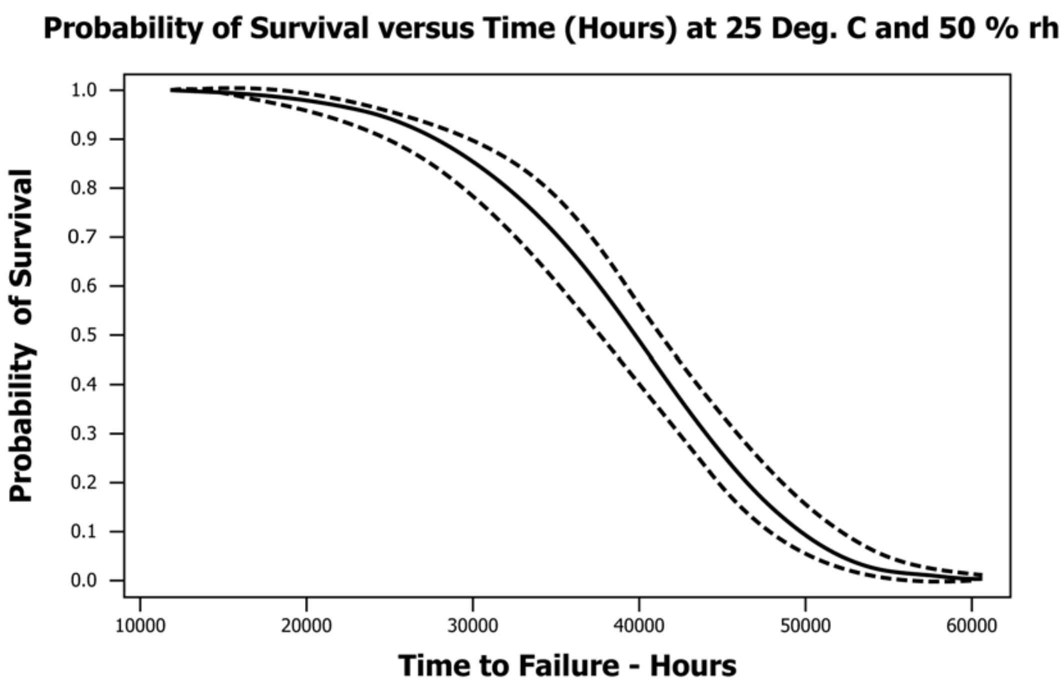

FIG. 8 Survival Plot for Combined Normalized Electric Motor Data

degradation. The material itself must not undergo thermodynamic changes such as crystal structure, solid to glass transitions or other such physical state changes.

11. Modeling the Effects of Weather

11.1 As stated in 9.2, the Eyring model can be used with stresses different than temperature and humidity and may be extended to accommodate more than two variables. This then should make it a comprehensive model allowing its use to include the effects of exposure to the stresses of the weather. Extreme caution must be used in conducting such modeling.

11.2 Degradation of materials caused by weather may be due to a combination of the three major environmental stresses: solar radiation, heat, and moisture. All three of these may be measured in either a real of artificial environment. Acceleration of degradation from these stresses may be obtained by use of devices such as black box or insulated backing, periodic water sprays, angle and direction of exposure to the sun, solar tracking, and multiple reflectors. Other examples of obtaining different levels of weather include use of multiple geographical locations and time of year that exposure occurs.

11.3 Another common means of accelerating these stresses is the use of laboratory weathering devices. Those devices employing artificial light sources must simulate the spectral power distribution of sunlight in the wavelength regions known to that produce degradation of the material being tested. The rate of degradation of each component may depend on several different wavelengths. Altering the relative intensities may significantly accelerate one degradation mechanism and leave one or more of the others unaffected. Exposure to wavelengths, especially short UV, that are not present in solar radiation may initiate degradation mechanisms that would not occur under exposure to the environment.

11.4 Great care must be taken to simulate the effects of the environment. Experimenters are encouraged to use relevant ASTM Standards developed to operate and control accelerated weathering devices. Acceleration factors determined for one material or accelerated device may not be accurate for other materials or accelerated devices.

11.5 Work with photochemical degradation is considered to be a complex system by itself. A technically correct model must consider wavelength, bandwidth and intensity of the incident radiation. If the intent is to model the effects of solar radiation, one must be aware of variations of the solar spectral energy output as a function of many variables including time of year, geographical location and altitude. Artificial light that may accurately reproduce solar effects for one sample system may be totally inaccurate for another sample system. Different dyes, pigments, organic polymers and protective coatings may all have considerably different degradation rates and mechanisms.

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