



# Standard Practice for Determining Data Criteria and Processing for Liquid Drop Size Analysis<sup>1</sup>

This standard is issued under the fixed designation E799; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

## 1. Scope

1.1 This practice gives procedures for determining appropriate sample size, size class widths, characteristic drop sizes, and dispersion measure of drop size distribution. The accuracy of and correction procedures for measurements of drops using particular equipment are not part of this practice. Attention is drawn to the types of sampling (spatial, flux-sensitive, or neither) with a note on conversion required (methods not specified). The data are assumed to be counts by drop size. The drop size is assumed to be the diameter of a sphere of equivalent volume.

1.2 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.3 The analysis applies to all liquid drop distributions except where specific restrictions are stated.

## 2. Referenced Documents

### 2.1 ASTM Standards:<sup>2</sup>

[E1296 Terminology for Liquid Particle Statistics](#) (Withdrawn 1997)<sup>3</sup>

### 2.2 ISO Standards:<sup>4</sup>

[13320-1 Particle Size Analysis-Laser Diffraction Methods](#)  
[9276-1 Representation of Results of Particle Size Analysis-Graphical Representation](#)  
[9272-2 Calculation of Average Particle Sizes/Diameters and Moments from Particle Size Distribution](#)

## 3. Terminology

### 3.1 Definitions of Terms Specific to This Standard:

<sup>1</sup> This practice is under the jurisdiction of ASTM Committee E29 on Particle and Spray Characterization and is the direct responsibility of Subcommittee E29.02 on Non-Sieving Methods.

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<sup>2</sup> For referenced ASTM standards, visit the ASTM website, [www.astm.org](http://www.astm.org), or contact ASTM Customer Service at [service@astm.org](mailto:service@astm.org). For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

<sup>3</sup> The last approved version of this historical standard is referenced on [www.astm.org](http://www.astm.org).

<sup>4</sup> Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, <http://www.ansi.org>.

3.1.1 *spatial, adj*—describes the observation or measurement of drops contained in a volume of space during such short intervals of time that the contents of the volume observed do not change during any single observation. Examples of spatial sampling are single flash photography or laser holography. Any sum of such photographs would also constitute spatial sampling. A spatial set of data is proportional to concentration: number per unit volume.

3.1.2 *flux-sensitive, adj*—describes the observation of measurement of the traffic of drops through a fixed area during intervals of time. Examples of flux-sensitive sampling are the collection for a period of time on a stationary slide or in a sampling cell, or the measurement of drops passing through a plane (gate) with a shadowing on photodiodes or by using capacitance changes. An example that may be characterized as neither flux-sensitive nor spatial is a collection on a slide moving so that there is measurable settling of drops on the slide in addition to the collection by the motion of the slide through the swept volume. Optical scattering devices sensing continuously may be difficult to identify as flux-sensitive, spatial, or neither due to instantaneous sampling of the sensors and the measurable accumulation and relaxation time of the sensors. For widely spaced particles sampling may resemble temporal and for closely spaced particles it may resemble spatial. A flux-sensitive set of data is proportional to flux density: number per (unit area  $\times$  unit time).

3.1.3 *representative, adj*—indicates that sufficient data have been obtained to make the effect of random fluctuations acceptably small. For temporal observations this requires sufficient time duration or sufficient total of time durations. For spatial observations this requires a sufficient number of observations. A spatial sample of one flash photograph is usually not representative since the drop population distribution fluctuates with time. 1000 such photographs exhibiting no correlation with the fluctuations would most probably be representative. A temporal sample observed over a total of periods of time that is long compared to the time lapse between extreme fluctuations would most probably be representative.

3.1.4 *local, adj*—indicates observations of a very small part (volume or area) of a larger region of concern.

### 3.2 Symbols—Representative Diameters:

3.2.1 ( $\bar{D}_{pq}$ ) is defined to be such that:<sup>5</sup>

$$\bar{D}_{pq}^{(p-q)} = \frac{\sum_i D_i^p}{\sum_i D_i^q} \quad (1)$$

where:

- $\bar{D}$  = the overbar in  $\bar{D}$  designates an averaging process,
- $(p - q) p > q$  = the algebraic power of  $\bar{D}_{pq}$ ,
- $p$  and  $q$  = the integers 1, 2, 3 or 4,
- $D_i$  = the diameter of the  $i$ th drop, and
- $\sum_i$  = the summation of  $D_i^p$  or  $D_i^q$ , representing all drops in the sample.
- $0 = p$  and  $q$  = values 0, 1, 2, 3, or 4.

$\sum_i D_i^0$  is the total number of drops in the sample, and some of the more common representative diameters are:

- $\bar{D}_{10}$  = linear (arithmetic) mean diameter,
- $\bar{D}_{20}$  = surface area mean diameter,
- $\bar{D}_{30}$  = volume mean diameter,
- $\bar{D}_{32}$  = volume/surface mean diameter (Sauter), and
- $\bar{D}_{43}$  = mean diameter over volume (De Broukere or Herdan).

See **Table 1** for numerical examples.

<sup>5</sup> This notation follows: Mugele, R.A., and Evans, H.D., "Droplet Size Distribution in Sprays," *Industrial and Engineering Chemistry*, Vol 43, No. 6, 1951, pp. 1317-1324.

3.2.2  $D_{Nf}$ ,  $D_{Lf}$ ,  $D_{Af}$ , and  $D_{Vf}$  are diameters such that the fraction,  $f$ , of the total number, length of diameters, surface area, and volume of drops, respectively, contain precisely all of the drops of smaller diameter. Some examples are:

- $D_{N0.5}$  = number median diameter,
- $D_{L0.5}$  = length median diameter,
- $D_{A0.5}$  = surface area median diameter,
- $D_{V0.5}$  = volume median diameter, and
- $D_{V0.9}$  = drop diameter such that 90 % of the total liquid volume is in drops of smaller diameter.

See **Table 2** for numerical examples.

3.2.3

$$\log(\bar{D}_{gm}) = \sum_i \log(D_i)/n \quad (2)$$

where:

- $n$  = number of drops,
- $\bar{D}_{gm}$  = the geometric mean diameter

3.2.4

$$D_{RR} = D_{VF} \quad (3)$$

where:

- $f$  =  $1 - 1/e \approx 0.6321$ , and
- $D_{RR}$  = Rosin-Rammler Diameter fitting the Rosin-Rammler distribution factor (see Terminology **E1296**).

**TABLE 1 Sample Data Calculation Table**

Size Class Bounds (Diameter in Micrometres)	Class Width	No. of Drops in Class	Sum of $D_i^k$ in Each Size Class <sup>A</sup>				Vol. % in Class <sup>B</sup>	Cum. % by Vol.
			$D_i$	$D_i^2$	$D_i^3$	$D_i^4$		
240-360	120	65	$19.5 \times 10^3$	$5.9 \times 10^6$	$1.8 \times 10^9$	$1. \times 10^{12}$	0.005	0.005
360-450	90	119	48.2	19.6	8.0	3	0.021	0.026
450-562.5	112.5	232	117.4	59.7	30.5	16	0.081	0.107
562.5-703	140.5	410	259.4	164.8	105.2	67	0.280	0.387
703-878	175	629	497.2	394.7	314.5	252	0.837	1.224
878-1097	219	849	838.4	831.3	827.6	827	2.202	3.426
1097-1371	274	990	1221.7	1513.7	1883.2	2352	5.010	8.436
1371-1713	342	981	1512.7	2342.1	3641.1	5683	9.687	18.123
1713-2141	428	825	1589.8	3076.1	5976.2	11657	15.900	34.023
2141-2676	535	579	1394.5	3372.5	8189.2	19965	21.788	55.811
2676-3345	669	297	894.1	2702.8	8203.5	24999	21.826	77.637
3345-4181	836	111	417.7	1578.2	5987.6	22807	15.930	93.567
4181-5226	1045	21	98.8	466.5	2212.1	10532	5.885	99.453
5226-6532	1306	1	5.9	34.7	348.5	1534	0.547	100.000
Totals of $D_i^k$ in $\sum_k$ entire sample		= 6109 $D_{N0.5} = 1300$	$8915.3 \times 10^3$ $\bar{D}_{10} = 1460$	$16562.6 \times 10^6$ $\bar{D}_{21} = 1860$ $\bar{D}_{20} = 1650$	$37729.0 \times 10^9$ $\bar{D}_{32} = 2280$ $\bar{D}_{31} = 2060$ $\bar{D}_{30} = 1830$ $D_{V0.5} = 2540$	$100695 \times 10^{12}$ $\bar{D}_{43} = 2670$		

Worst case class width

$$\frac{348.5}{37729} = 0.009 \text{ Relative Span} = (D_{V0.9} - D_{V0.5})/D_{V0.5} = (3900 - 14200)/2530 = 0.98$$

$$\frac{669}{2676 + 3345} \times 0.21826 = 0.024$$

Less than 1 %, adequate sample size

Adequate class sizes

<sup>A</sup> The individual entries are the values for each  $k$  as used in **5.2.1 (Eq 1)** for summing by size class.

<sup>B</sup> SUM  $D_i^3$  in size class divided by SUM  $D_i^3$  in entire sample.

**TABLE 2 Example of Log Normal Curve with Upper Bound**

Data Collected May 2, 1979		Computer Analysis May 2, 1979	
Upper Bound Diameter (μm)	Normal Curve, %	Adjusted Data, %	Data, %
360.00	0.006	0.005	0.005
450.00	0.027	0.027	0.026
562.50	0.109	0.108	0.107
703.00	0.389	0.387	0.387
878.00	1.227	1.224	1.224
1097.00	3.421	3.426	3.426
1371.00	8.407	8.437	8.436
1713.00	18.109	18.124	18.123
2141.00	34.080	34.024	34.023
2676.00	55.551	55.811	55.811
3345.00	77.828	77.637	77.637
4181.00	93.648	93.568	93.567
5226.00	99.481	99.453	99.453
6532.00	100.000	100.000	100.000

For Computing Curve Averages

Largest drop diameter	=	6532.00 μm
Smallest drop diameter	=	240.00 μm
Fraction of normal curve	=	0.999995

Normal Curve		Simple Calculation	
(Gaussian Limits—4.55457 to 4.53257)			
$D_{10}$	=	1464.91	1459.37 μm (length mean diameter)
$D_{20}$	=	1646.44	1646.57 μm (surface mean diameter)
$D_{30}$	=	1824.85	1832.39 μm (volume mean diameter)
$D_{21}$	=	1850.45	1857.79 μm (surface/length mean diameter)
$D_{31}$	=	2036.73	2053.27 μm (volume/length mean diameter)
$D_{32}$	=	2241.75	2269.32 μm (sauter mean diameter)
$D_{43}$	=	2615.67	2670.75 μm (mean diameter over volume)
$D_{V0.5}$	=	2534.53	2533.31 μm (volume median diameter)
$D_{N0.5}$	=	1303.62	1304.71 μm (number median diameter)

Average of Absolute Relative Deviation from  $D_{V0.5}$  by Volume = 0.311

$$\begin{aligned} \text{Relative Span} &= (D_{V0.900} - D_{V0.100}) / D_{V0.5} \quad (D_{V0.9} - D_{V0.1}) / D_{V0.5} \\ &= (3913.74 - 1437.21) / 2534.53 \\ &= 0.977 \end{aligned}$$

$$\text{Normal curve \% } F(D) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\text{DEL} \ln\left(\frac{AD}{XM^2 D}\right)} e^{-z^2} dz$$

where:

A = 1.8941,

DEL = 1.17206, and

XM = 7335.30.

$F(D)$  = accumulative fraction of liquid volume in drops having diameter less than  $D$ .

3.2.5  $D_{kub}$  = upper-boundary diameter of drops in the  $k$ th size class.

3.2.6  $D_{klb}$  = lower-boundary diameter of drops in the  $k$ th size class.

#### 4. Significance and Use<sup>6</sup>

4.1 These criteria<sup>6</sup> and procedures provide a uniform base for analysis of liquid drop data.

#### 5. Test Data

5.1 Specify the data as temporal or spatial. If the data cannot be so specified, describe the sampling procedure. Also specify whether the data are local (that is, in a very small section of the space of liquid drop dispersion), and whether the data are

representative (that is, a good description of the distribution of concern). Report the fluids, fluid properties, and pertinent operating conditions.

5.1.1 A graph form for reporting data is given in Fig. 1.

5.2 Report the largest and smallest drops of the entire sample, the number of drops in each size class, and the class boundaries. Also report the sampling volume, area, and lapse of time, if available and applicable.

5.3 Estimate the total volume of liquid in the sample that includes measured drops and the liquid in the sample that is not measured. (The volume outside the range of sizes permitted by the measuring technique might be estimated by graphical extrapolation of a histogram or by a curve fitting technique.)

5.4 The ratio of the volume of the largest drop to the total volume of liquid in the sample should be less than the tolerable fractional error in the desired representation. See Table 1. All of the drops in the sample at the large-drop end of the

<sup>6</sup> These criteria ensure that processing probably will not introduce error greater than 5 % in the computation of the various drop sizes used to characterize the spray.

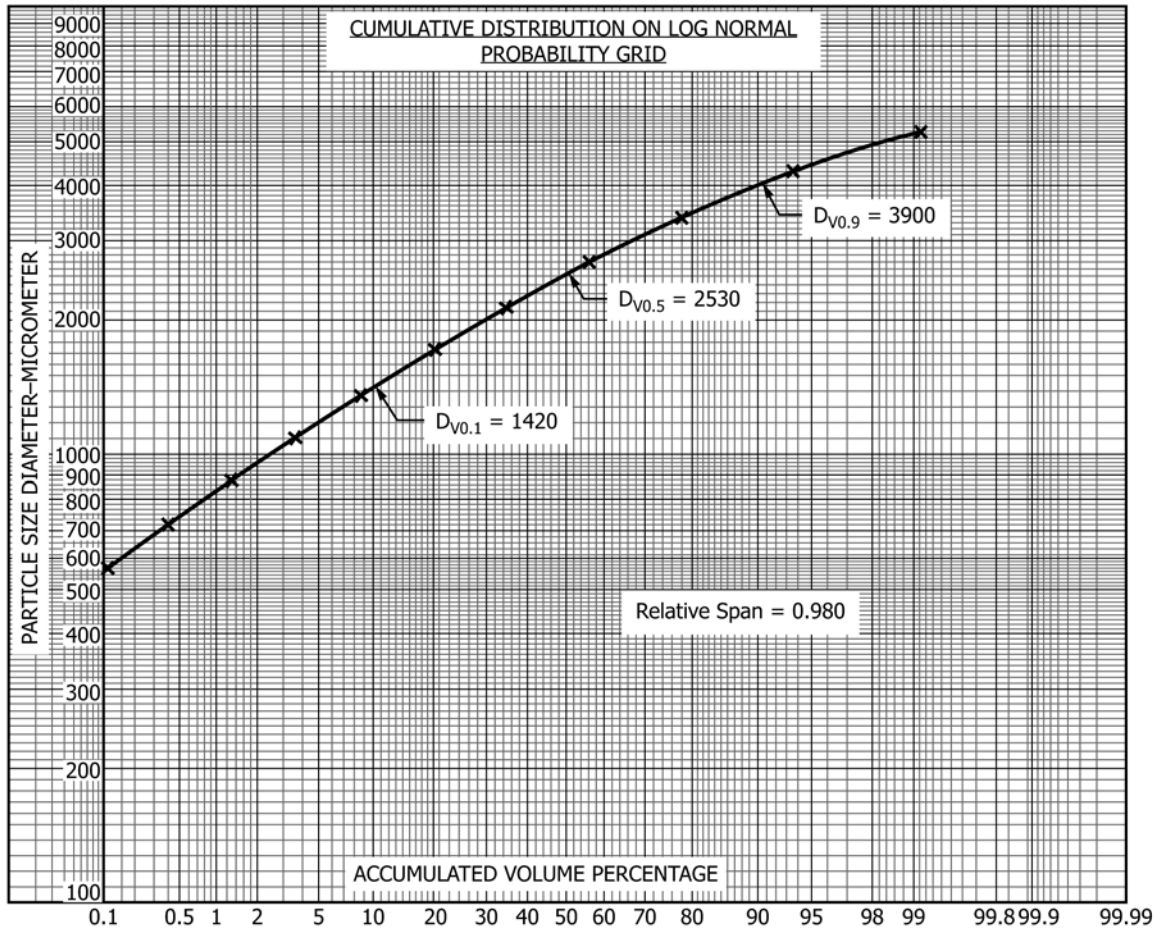


FIG. 1 Sample Data Graph

distribution should be measured. This criterion is a good “rule of thumb” to determine a minimum sample size. The value of  $D_{10}$  is greatly affected by the smallest drops measured.

5.5 Ninety-nine percent of the volume of liquid represented by the data should be in size classes such that no size class has boundaries with a ratio greater than 3:2. For the majority of size classes, this ratio should not exceed 5:4. The 99 % condition exempts size classes having diameters smaller than  $D_{V0.01}$ . These criteria assure that processing probably will not introduce errors greater than 5 % in the computation of the various drop diameters cited in this practice. The criteria may be relaxed for measurements where this degree of accuracy is unattainable.

5.6  $(D_{kub} - D_{klb}) / (D_{kub} + D_{klb})$  multiplied by the liquid volume in the  $k$ th class and divided by the total volume of liquid in the sample shall be less than 0.05 for every class. See Table 1. Use of the same criterion for a size class created by lumping the estimated volume below the boundary of measurement provides a test for determining the need for some type of curve fitting. It may be necessary to relax this requirement for cases where this degree of accuracy is unattainable.

## 6. Data Processing

### 6.1 Transformations of Data:

6.1.1 If drop motions are essentially free from recirculation through the region of observation, spatial data can be transformed to flux-sensitive data by multiplying the number of drops in each size class by the average velocity of drops for that size class at the sample location. If this transformation is performed, the exact procedure shall be noted.

6.1.2 If evaporation corrections are applied, the procedure shall be described and the magnitude of the corrections shall be recorded.

6.1.3 If corrections are applied to account for drops outside the boundaries represented by the data, the procedure shall be described. Likewise, if the overall distribution is established by blending several distributions, the procedure shall be described.

6.1.4 If curve fitting (for example, to the upper-limit log normal, Rosin-Rammler or Nukiyama-Tanasawa equation) is used in the data processing, the mathematical function<sup>7</sup> and minimization criteria, including any weighting factors applied

<sup>7</sup> Examples are found in Mugele and Evans, loc. cit.; in Tishkoff, J. M., and Law, C. K., “Applications of a Class of Distribution Functions to Drop Size Data by Logarithmic Least Squares Technique,” *Transactions of ASME*, Vol 99, Ser. A, No. 4, October 1977; and in Goering, C. E., and Smith, D. B., “Equations for Droplet Size Distributions in Sprays,” *Transactions of ASAE*, Vol 21, No. 2, 1978, pp. 209–216.

to the data, shall be given. The quality of fit shall be shown graphically or by tabular comparison with the data. When there are corrections or transformations, the comparison shall be made with the adjusted data.

### 6.2 Calculations Involving Size Classes:

6.2.1 When data are reported by size classes rather than as individual drop diameters, the representative diameters,  $\bar{D}_{pq}$ , may be calculated from summations defined as follows:

$$\sum_i D_i^r = \sum_k \frac{(D_{kub}^{r+1} - D_{kib}^{r+1}) N_k}{(D_{kub} - D_{kib})(r+1)} \quad (4)$$

where:

$r$  = corresponds to the selected value of  $p$  or  $q$  in the expression for  $\bar{D}_{pq}$  as stated in **4.2.1**, and

$N_k$  = the number of drops in the  $k$ th size class.

This calculation is based on the assumption of a linear increase in the accumulation of counts as a function of diameter within each size class. If the data satisfy the criteria in **5.5** and **5.6**, the results based on either of the following two formulas will differ by less than 8 % from that based on the above (preferred) **Eq 1**.

$$\sum_i D_i^r = \sum_k \frac{D_{kub}^r + D_{kib}^r}{2} \times N_k \quad (5)$$

$$\sum_i D_i^r = \sum_k \left( \frac{D_{kub} + D_{kib}}{2} \right)^r \times N_k \quad (6)$$

6.2.2 To obtain the values described in **4.2.2**, the fractional values (number, length, area or volume) accumulated between the minimum drop size in the sample and the upper bounds of the respective size classes shall be plotted against the corresponding upper bound diameters, see **Fig. 1**. The desired values can then be read from the graph. The calculations shall be made for the fractional accumulations based on the procedures from **6.2.1**.

6.2.3 In plotting histograms of the data, the ordinate for each size class shall be the incremental fractional values (number, length, area, or volume) per unit length increase in diameter according to **5.2.1**; that is,

$$k\text{th size class ordinate} = \frac{(D_{kub}^{r+1} - D_{kib}^{r+1}) N_k}{(D_{kub} - D_{kib})^2 (r+1)} / \sum_i D_i^r \quad (7)$$

The bounding abscissae for each vertical bar shall be the diameters corresponding to the lower and upper boundaries of the size class.

### 6.3 Curve Fitting:

6.3.1 If an equation or curve is fitted to the data, the calculations of **3.2.1** and **3.2.2** shall be done with the corresponding quadrature representations for the curve.

6.4 *Measures of Dispersion of Drop Sizes*—(the graph referenced in **6.2.2** is a complete description but the following two measures are easily obtained):

6.4.1 Relative span =  $(D_{V0.9} - D_{V0.1}) / D_{V0.5}$ . (Give values for each of the three diameters used in the calculation.)

6.4.2 *Deviation*—Average relative deviation (from  $D_{V0.5}$ )

$$= \frac{\sum_k |D_{V0.5} - (D_{kub} + D_{kib}) / 2| N_k}{\sum_k N_k D_{V0.5}} \quad (8)$$

6.5 *Modal Values* (diameter of drops for peak frequency of occurrence)—Generally, modal values shall be obtained by drawing smooth curves through the appropriate histograms. If a curve fit is obtained using a mathematical representation, and if it is a good fit, the mode or modes may be computed from the mathematical function.

6.6 *Drop Concentration and Flux Density* shall be computed and reported when possible.

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