Designation: E646 – 16

Standard Test Method for Tensile Strain-Hardening Exponents (*n* -Values) of Metallic Sheet Materials[1]

This standard is issued under the fixed designation E646; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

INTRODUCTION

This test method for determining tensile strain-hardening exponents *n* utilizes stress-stain data obtained in a uniaxial tension test. Tensile data are obtained in a continuous and rate-controlled manner via displacement or strain control. The strain-hardening exponents are determined from an empirical representation over the range of interest of the true-stress versus true-strain curve. The mathematical representation used in this method is a power curve (Note 1) of the form (**1**)[2]:

$$\sigma = K\varepsilon^n$$

where:

σ = true stress,

ε = plastic component of true strain, but in special cases may be the total true strain. (See 10.2),

K = is a constant, often called the strength coefficient having the units of stress, and

n = strain-hardening exponent

1. Scope*

1.1 This test method covers the determination of a strain-hardening exponent by tension testing of metallic sheet materials for which plastic-flow behavior obeys the power curve given in the Introduction.

NOTE 1—A single power curve may not be a satisfactory fit to the entire stress-strain curve between yield and necking. If such is the case, more than one value of the strain-hardening exponent may be obtained (**2**) by agreement using this test method.

1.2 This test method is specifically for metallic sheet materials with thicknesses of at least 0.005 in. (0.13 mm) but not greater than 0.25 in. (6.4 mm). The method has successfully been and may be applied to other forms and thicknesses by agreement

1.3 The values stated in inch-pound units are to be regarded as standard. The values given in parentheses are mathematical conversions to SI units that are provided for information only and are not considered standard.

NOTE 2—The value of the strain-hardening exponent, *n*, has no units and is independent of the units used in its determination

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[3]

E4 Practices for Force Verification of Testing Machines

E6 Terminology Relating to Methods of Mechanical Testing

E8/E8M Test Methods for Tension Testing of Metallic Materials

E29 Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications

E83 Practice for Verification and Classification of Extensometer Systems

E111 Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E691 Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method

2.2 *ISO Standard*

ISO 10275:2007 Metallic materials -- Sheet and strip -- Determination of tensile strain hardening exponent

[1] This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.02 on Ductility and Formability.

Current edition approved Feb. 1, 2016. Published March 2016. Originally approved in 1978. Last previous edition approved in 2015 as E646 - 15. DOI: 10.1520/E0646-16

[2] The boldface numbers in parentheses refer to the list of references appended to this method.

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

***A Summary of Changes section appears at the end of this standard**

Copyright © ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959. United States

3. Terminology

3.1 For definitions of other terms used in this test method, refer to E6 (Standard Terminology Relating to Methods of Mechanical Testing).

3.2 *Definitions:*

3.2.1 *elastic true strain, ε_e, n*—elastic component of the true strain.

3.2.2 *engineering strain, e, n*—a dimensionless value that is the change in length (ΔL) per unit length of original linear dimension (L_0) along the loading axis of the specimen; that is, $e = \Delta L/L_0$.

3.2.3 *engineering stress, S [FL^{-2}], n*—the normal stress, expressed in units of applied force, F per unit of original cross-sectional area, A_0; that is, $S = F/A_0$

3.2.4 *necking, n*—the onset of nonuniform or localized plastic deformation, resulting in a localized reduction of cross-sectional area.

3.2.5 *plastic true strain, ε_p, n*—the inelastic component of true strain.

3.2.6 *strain hardening, n*—an increase in hardness and strength caused by plastic deformation.

3.2.7 *true strain, ε, n*—the natural logarithm of the ratio of instantaneous gauge length, L, to the original gauge length, L_0; that is, $\varepsilon = \ln (L/L_0)$ or $\varepsilon = \ln (1+e)$.

3.2.8 *true stress,* σ *[FL^{-2}], n*—the instantaneous normal stress, calculated on the basis of the instantaneous cross-sectional area, A; that is, $\sigma = F/A$; if no necking has occurred, $\sigma = S(1+e)$.

3.3 *Definitions of Terms Specific to This Standard:*

3.3.1 *strain-hardening exponent (n), n*—an experimental constant, computed from the least squares best fit, linear slope of log σ versus log ε data over a specific strain range where ε is the plastic component of true strain, but in special cases may be the total true strain (see 10.2).

3.3.2 *strength coefficient (K)* **[FL^{-2}]**, *n*—an experimental constant, computed from the fit of the data to the assumed power curve, that is numerically equal to the extrapolated value of true stress at a true strain of 1.00.

4. Summary of Test Method

4.1 This test method applies to materials exhibiting a continuous stress-strain curve in the plastic region. The displacement or strain is applied in a continuous and rate-controlled manner while the normal tensile load and strain are monitored. The instantaneous cross-sectional area may be monitored or calculated by assuming constancy of volume in the plastic region. Equations are presented that permit the calculation of the true stress, σ, true strain, ε, strain-hardening exponent, *n*, and strength coefficient, *K*, for that continuous portion of the true-stress versus true-strain curve which follows the empirical relationships described.

NOTE 3—This test method is recommended for use only in the plastic range for metallic sheet material for which the true-stress true-strain data follow the stated relationship.

5. Significance and Use

5.1 This test method is useful for estimating the strain at the onset of necking in a uniaxial tension test (**1**). Practically, it provides an empirical parameter for appraising the relative stretch formability of similar metallic systems. The strain-hardening exponent is also a measure of the increase in strength of a material due to plastic deformation.

5.2 The strain-hardening exponent may be determined over the entire plastic stress-strain curve or any portion(s) of the stress-strain curve specified in a product specification.

NOTE 4—The engineering strain interval 10–20% is commonly used for determining the strain-hardening exponent, *n*, of formable low-carbon steel products

5.3 This test method is not intended to apply to any portion of the true stress versus true strain curve that exhibits discontinuous behavior; however, the method may be applied by curve-smoothing techniques as agreed upon.

NOTE 5—For example, those portions of the stress-strain curves for mild steel, aluminum, or other alloys that exhibit yield point and Lüders band elongation, twinning, or Portevin–Le Chatelier effect (PLC) may be characterized as behaving discontinuously.

NOTE 6—Caution should be observed in the use of curve-smoothing techniques as they may affect the *n*-value.

5.4 This test method is suitable for determining the tensile stress-strain response of metallic sheet materials in the plastic region prior to the onset of necking.

5.5 The *n*-value may vary with the displacement rate or strain rate used, depending on the metal and test temperature.

6. Apparatus

6.1 *Testing Machines*—Machines used for tension testing shall conform to the requirements of Practices E4. The forces used to determine stress shall be within the force range of the testing machine as defined in Practices E4.

6.2 *Strain-Measurement Equipment*—Equipment for measurement of extension shall conform to the requirements of Class C or better as defined in Practice E83.

7. Sampling

7.1 Samples shall be taken from the material as specified in the applicable product specification.

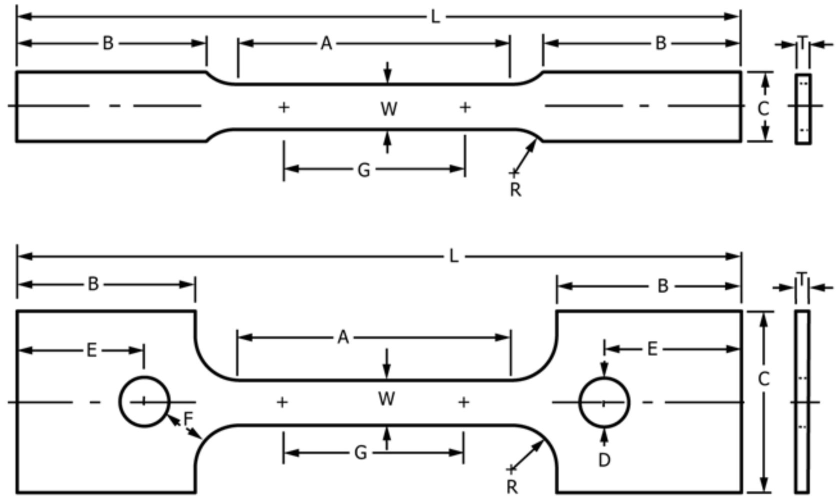
8. Test Specimens

8.1 *Selection and Preparation of Specimens:*

8.1.1 In the selection of specimen blanks, special care shall be taken to assure obtaining representative material that is flat and uniform in thickness.

8.1.2 In the preparation of specimens, special care shall be taken to prevent the introduction of residual stresses.

8.2 *Dimensions*—Recommended metallic sheet specimen configurations are shown in Fig. 1. Specimen configurations shall have sides parallel to 0.001 in. and dimensions shall be reported as stated in 11.1.6.



Dimensions

Required Dimensions for Reduced Section of Specimen			
		Dimensions	
		in.	mm
G	Gage length	2.000 ± 0.005	50.0 ± 0.10
W	Width[A]	0.500 ± 0.010	12.5 ± 0.25
T	Thickness[B]	thickness of material	
R	Radius of fillet, min	½	13
L	Overall length, min	8	200
A	Length of reduced section, min	2¼	60
B	Length of grip section, min	2	50
Suggested Dimensions for Ends of Specimen			
"Plain-End" Specimens			
C	Width of grip section[C and D]	¾	20
"Pin-End" Specimens			
C	Width of grip section, approximate[E]	2	50
D	Diameter of hole for pin[F]	½	13
E	Distance of center of pin from end, approximate	1½	38
F	Distance of edge of hole from fillet, min	½	13

[A] The width of the reduced section shall be parallel to within ±0.001 in. (±0.025 mm).
[B] The thickness of the reduced section shall not vary by more than ±0.0005 in. (0.013 mm) or 1 %, whichever is larger, within the gage length, G.
[C] It is desirable, if possible, that the grip sections be long enough to extend into the grips a distance equal to two-thirds or more the length of the grips.
[D] Narrower grip sections may be used. If desired, the width may be 0.500± 0.010 in. (12.5 ± 0.25 mm) throughout the length of the specimen, but the requirement for dimensional tolerance in the central reduced section stated in footnote A shall apply. The ends of the specimen shall be symmetrical with the center line of the reduced section within 0.01 in. (0.25 mm).
[E] The ends of the specimen shall be symmetrical with the center line of the reduced section within 0.01 in. (0.25 mm).
[F] Holes shall be on the centerline of the reduced section, within ±0.002 in. (±0.05 mm).

FIG. 1 Specimen for Determining *n* -Values

NOTE 7—While this standard is specifically for metallic sheet materials, it has been successfully applied to many tensile specimens having a uniform cross-sectional area, for example, round bars and flats where parallel sides have been maintained to within 0.001 in. as required by 8.2. Since other test results may be desired to be obtained, specimens may be intentionally tapered with sides parallel to within the same tolerance of 0.001 in.

9. Procedure

9.1 Measure and record the original thickness T, of the reduced section of the specimen to at least the nearest 0.0005 in. (0.013 mm) and the width, W, of the reduced section to at least the nearest 0.001 in. (0.025 mm).

9.1.1 The method to record observed values, given in 7.2 of Practice E29, shall be used for all measurements.

9.2 Grip the specimen in the testing machine in a manner to ensure axial alignment of the specimen as noted in Test Methods E8/E8M and attach the extensometer.

9.2.1 The order of this step may be reversed if required by the design of the extensometer or the specimen grips, or both.

9.3 *Speed of Testing:*

9.3.1 The speed of testing shall be such that the loads and strains are accurately indicated.

9.3.2 In the absence of any specified limitations on the speed of testing (by, for example, the appropriate product specification), the test speed, defined in terms of rate of separation of heads during tests, free running crosshead speed, or rate of straining shall be between 0.05 in./in. (m/m) and 0.50 in./in. (m/m) of the length of the reduced section per minute (in accord with Test Method E8/E8M, Standard Test Methods for Tension Testing Metallic Materials, 7.6.4 Speed of Testing When Determining Tensile Strength) The speed setting shall not be changed during the strain interval over which the strain hardening exponent, *n*, is to be determined

NOTE 8—The mode of control and the rate used may affect the values obtained.

9.3.3 If the yield point, yield-point elongation, yield strength, or any combination of these is to be determined also, the rate of stress or strain application or crosshead separation during this portion of the test shall be within the range permitted by Test Methods E8/E8M or any other specified value. After exceeding the strain necessary for this information, adjust the crosshead speed to within the range specified by this standard.

9.4 Record the force and corresponding strain for at least five approximately equally spaced levels of strain (Note 9) covering the strain range of interest or required in the product specification. Usually, the greatest of these strains is at or slightly prior to the strain at which the maximum force occurs, and usually the lower bound of these strains is the yield strain (for continuous-yielding material) or the end of yield-point extension (for discontinuous-yielding material). See Fig. 2. The requirement that at least five force-strain data pairs be recorded is met with an autographic recording and the selection of five or more pairs from that curve.

NOTE 9—Since the slope of the curve may vary slightly along its length, there is a statistical basis for choosing points equally spaced out along the strain range.

9.4.1 The test is not valid if fewer than five data pairs are obtained.

9.4.2 If multiple values of the strain-hardening exponent are to be determined (Note 1), use at least five stress and strain values for the calculation of the strain-hardening exponent in each interval of strain.

9.4.3 Other parameters may be recorded in place of forces and strains provided that they can ultimately be transformed into true stress and true strain at least as accurately as those measured using the techniques already described in this test method.

10. Calculations

10.1 The calculations in this section are based on true strain and true stress (**3**, **4**, **5**). The true strain (also called the logarithmic strain) is given by (Terminology E6):

$$\varepsilon = \ln \frac{L}{L_o} = \ln(1 + e)$$

where:

L = is the current length of the gauge length,
L_o = is the initial gauge length, and
e = is the engineering strain.

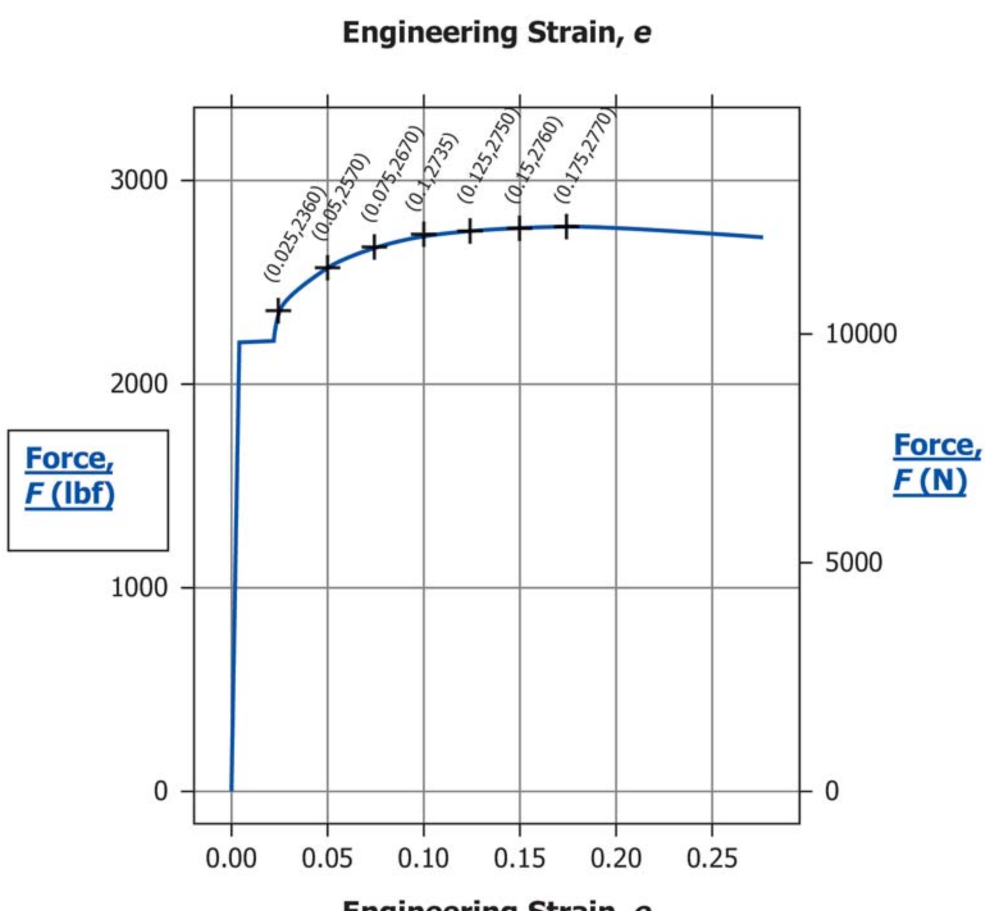


FIG. 2 Example Showing Load-Strain Data Pairs: material with initial discontinuous yielding region. These data pairs are used for a sample calculation in Appendix X1

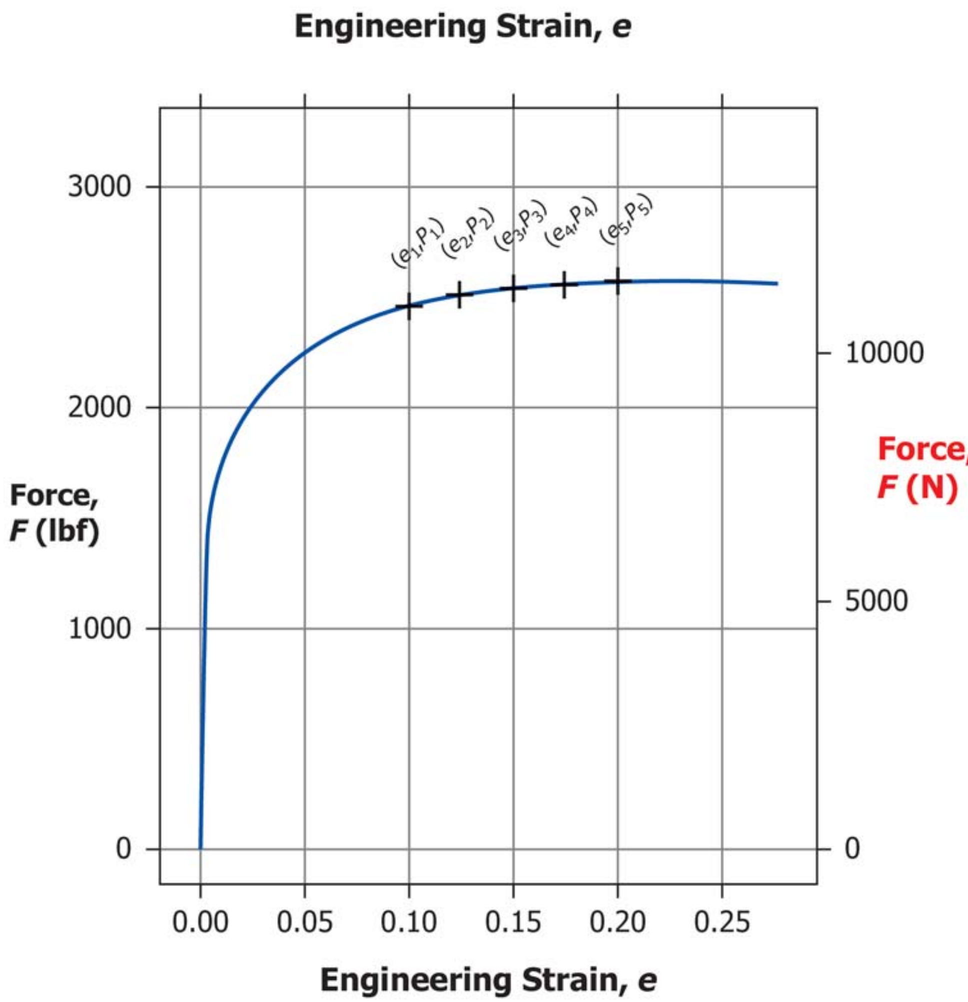


FIG. 3 Example Showing Force-Strain Data Pairs: material with no discontinuous yielding

If L or e is measured under load, then the true strain is the total true strain. If L or e is measured in the unloaded state, or if the elastic component of L or e is subtracted from the loaded L or e, then the true strain given by the equation is the plastic true strain. The true stress is the applied force divided by the instantaneous area (see Terminology E6). If the strain in the gauge length is uniform, the true stress is given by:

$$\sigma = \frac{FL}{A_o L_o} = S(1 + e)$$

where A_o is the initial cross sectional area and S is the engineering stress. This equation assumes constancy of volume and a Poissons ratio of ½ . These assumptions may not always be strictly met, but they introduce a negligible error in the result.

10.2 Determine the strain-hardening exponent from the logarithmic form of the power curve representation of

(1) the true stress versus plastic true strain curve within the plastic range (Method A) or

(2) the true stress versus the total true strain curve within the plastic range (Method B).

Method B is an option only when the elastic strain is less than 10% of the total true strain for all points in the chosen strain range. Method A has no such restriction and may be used everywhere that Method B may be used as well as in strain ranges where the elastic strain is greater than or equal to 10% of the total true strain and Method B would be invalid. The method used shall be reported (see 11.1.5).

NOTE 10—Method A is consistent with the current version of ISO 10275:2007 and with earlier versions of this ASTM test method (**5**).

NOTE 11—Method B is consistent with earlier versions of this ASTM test method and earlier versions of ISO 10275:2007, when the elastic strain was less than 10 % of the total true strain (**5**). Method B provides a means of comparison with data bases generated using these earlier versions. Method B does not involve the subtraction of the elastic strain and is therefore simpler to implement and has a reduced uncertainty.

NOTE 12—The difference between strain-hardening exponents determined by Method A and Method B is usually less than 5% for steel, but may be higher for low modulus material with high *n*-value. For a specific material, values of the strain-hardening exponent determined by Method A are always less than those determined by Method B. Users of this standard should be aware that the value of the strength coefficient, *K*, will be different depending on the method used. Since this test method does not report *K*, nor purport to provide a standard measurement of *K*, this difference is not addressed here.

10.2.1 In Method A, the elastic strain shall be subtracted from the total true strain. The elastic strain shall be determined from the slope of the elastic loading line (see E111) or calculated by dividing the true stress by the nominal value of Young's modulus of elasticity. In any case, the method used to arrive at the elastic strain and the value of Young's modulus or slope of the elastic loading line shall be reported.

10.2.2 All data pairs used to calculate the strain-hardening exponent, *n* shall be treated in the same manner

10.3 Obtain the logarithms of the true stress versus true strain pairs calculated in 10.1 where any data pair having the stress or strain equal to zero must be excluded for mathematical reasons. From these paired sets of (log σ, log ε), calculate, via linear regression analysis of log σ versus log ε, the slope, *n*, and the standard error of the slope (**6**, **7**, **8**, **9**).

10.4 The equation for calculating the strain-hardening exponent, *n*, is as follows:

$$n = \frac{N\sum_{i=1}^{N}\left(\log \varepsilon_i \log \sigma_i\right) - \left(\sum_{i=1}^{N}\log \varepsilon_i\right)\left(\sum_{j=1}^{N}\log \sigma_j\right)}{N\sum_{i=1}^{N}(\log \varepsilon_i)^2 - \left(\sum_{i=1}^{N}\log \varepsilon_i\right)^2}$$

where:

N = the number of (σ_i, ε_i) data pairs and the use of the term "log" does not require the use of base 10 in this and following equations.

10.5 The equation for calculating the logarithm of the strength coefficient, *K*, is as follows:

$$\log K = \frac{\sum_{i=1}^{N}\left(\log \sigma_i\right) - n\sum_{i=1}^{N}\log \varepsilon_i}{N} \quad (1)$$

10.6 The calculation of the standard deviation of the strain-hardening exponent, SD_n, is based upon the variance of the slope of the regression line. This measure of variability contains the computed value of n (10.3) and the computed logarithm of the strength coefficient (10.4) as follows:

$$SD_n = \left[\frac{N\sum_{i=1}^{N}(\log \sigma_i - \log K - n \log \varepsilon_i)^2}{(N-2)\left(N\sum_{i=1}^{N}(\log \varepsilon_i)^2 - \left(\sum_{i=1}^{N}\log \varepsilon_i\right)^2\right)}\right]^{1/2}$$

10.7 An example of a worksheet for manually calculating these values is found in Appendix X1.

11. Report

11.1 The report shall include the following:

11.1.1 The material represented by commercial standard nomenclature. Materials that have no commercial standard shall be so indicated.

11.1.2 The strain interval(s) over which the strain-hardening exponent(s) *n*(s) were determined.

11.1.2.1 The strain-hardening exponent *n*-value and its associated standard deviation SD_n. These values shall be considered applicable only over the strain interval for which it was determined.

11.1.2.2 The number of data pairs selected for computing the strain hardening exponent *n*.

11.1.3 The direction of testing relative to the principal rolling direction.

11.1.4 Any special conditions that are believed to have affected the test result, such as, the strain rate or temperature.

11.1.5 The method used (A or B). These indicate whether the plastic component of true strain (Method A) or the total true strain (Method B) was used.

11.1.5.1 If Method A was used, also report the method used to arrive at the elastic strain and the value of Young's modulus or slope of the elastic loading line.

11.1.6 Test specimen type and relevant dimensions.

12. Precision and Bias[4]

12.1 The precision of this test method is based on an interlaboratory study of E646, Standard Test Method for Tensile Strain-hardening Exponents (*n*-Values) of Metallic Sheet Materials. The study data was a subset of results reported during the May 2006 ASTM Committee E28 Proficiency Testing Program for Mechanical Properties of Steel. Each of eight laboratories participating in the program tested the two different stabilized ultra-low carbon steel sheet materials provided.

12.2 The eight laboratories conducted triplicate tests on each of the materials. *n*-Value was determined over the 5-15% strain range for all tests using Method B. Every test result represented an individual determination. Details on the composition of the materials used are listed in Table 1. Details on the mechanical properties of the material used are listed in Table 2. Table 3 lists a statistical summary of strain hardening exponent, *n* test results.

12.3 The terms s_r, s_R, *r*, and *R* are calculated as described in Section 15 of Practice E691. The terms repeatability limit, *r*, and Reproducibility limit, *R*, are used as specified in Practice E177. The respective standard deviations, s_r and s_R are obtained by dividing the limit values by 2.8

12.4 *Repeatability "r"*—(repeatability limit) is the interval representing the critical difference between two test results for the same material, obtained by the same operator using the same equipment on the same day in the same laboratory. When comparing two test results made under these conditions, a measurement difference less than the value for "*r*" is an indication that the results may be equivalent.

12.5 *Reproducibility "R"*—(Reproducibility limit) is the interval representing the difference between two test results for the same material, obtained by different operators using different equipment in different laboratories. When comparing two test results made under these conditions, a measurement

[4] Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:E28-1027.

TABLE 1 Chemical Analysis (mass%)

Sample	C	Mn	P	S	Si	Cu	Ni	Cr	Al	Cb	Ti	V	N	Sn	Mo
Material B	0.003	0.61	0.042	0.011	0.01	0.01	0.01	0.02	0.040	0.013	0.001	0.001	0.0037	0.001	0.003
Material C	0.004	0.60	0.042	0.011	0.01	0.01	0.01	0.02	0.039	0.013	0.001	0.001	0.0041	0.001	0.003

TABLE 2 Mechanical Properties

Sample	0.2% offset Yield ksi (MPa)	Tensile Strength ksi (MPa)	Total Elongation in 2 in. (in 50 mm)(%)	*n*-value (5–15% strain)	Rockwell Hardness (HRB)
Material B	38.6 (266)	51.9 (358)	37	0.1822	61
Material C	35.7 (246)	50.6 (349)	36	0.1860	56

TABLE 3 Statistics of *n*-values at 5–15% Strain

Material	Average *n*-value by Method B	Repeatability Standard Deviation	Reproducibility Standard Deviation	Repeatability Limit	Reproducibility Limit
	$\bar{n}$	S_r	S_R	r	R
B	0.1822	0.0025	0.0032	0.0070	0.0091
C	0.1860	0.0068	0.0083	0.0190	0.0234

difference less than the value for "*R*" is an indication that the results may be equivalent.

12.6 Any judgment in accordance with these two statements would have an approximate 95% probability of being correct.

12.7 *Bias*—At the time of the study, there was no accepted reference material suitable for determining the bias for this test method, therefore no statement on bias is being made.

12.8 The precision statement was determined through statistical examination of 48 results, from eight laboratories, on two steel sheet materials.

APPENDIX

(Nonmandatory Information)

X1. BASIC WORKSHEETS FOR CALCULATING THE STRAIN-HARDENING EXPONENT

X1.1 Table X1.1 is based on the data in Fig. 2and Fig. 3:

X denotes log ε
Y denotes log σ

The sequence of steps in Table X1.2 uses the logic of hand computations. Likewise, the same or similar steps exist in most computer programs.

X1.2 Common mini-computers programs exist to provide logarithmic transformations. Statistical models may readily provide slope and intercept values, but may not immediately provide intercept and slope variances. Spreadsheets may have the same problems too. If such is the case, the equations in Table X1.2 may be used to calculate the desired variances.

X1.3 If data are manually recorded, sufficient decimal places must be carried to avoid losing significant figures in the subtraction of Steps (1) through (9) in Table X1.2. Retention of significant figures follows the method given in Section 7 of Practice E29.

TABLE X1.1 An Example of Calculating the Strain-Hardening Exponent and Standard Deviation by Method B

						Tabulation of Data from Test and Preliminary Evaluation					
Data Pair	Load, F, kips[A]	Engineering Stress, ksi[B]	True Stress, σ, ksi[C]	$Y = \log_{10}\sigma$	Y^2	Extension, in[A]	Engineering Strain, ε[D]	True Strain, ε[E]	$X = \log_{10}\varepsilon$	X^2	XY
1	2.360	44.81	45.93	1.66209	2.76254	0.050	0.025	0.02469	–1.60743	2.58384	–2.67170
2	2.570	48.80	51.24	1.70958	2.92265	0.100	0.050	0.04879	–1.31167	1.72047	–2.24240
3	2.670	50.69	54.50	1.73637	3.01499	0.150	0.075	0.07232	–1.14074	1.30128	–1.98075
4	2.735	51.93	57.12	1.75680	3.08636	0.200	0.100	0.09531	–1.02086	1.04216	–1.79345
5	2.750	52.21	58.74	1.76894	3.12914	0.250	0.125	0.11778	–0.92892	0.86289	–1.64320
6	2.760	52.40	60.26	1.78006	3.16861	0.300	0.150	0.13976	–0.85461	0.73036	–1.52126
7	2.770	52.59	61.80	1.79097	3.20758	0.350	0.175	0.16127	–0.79245	0.62798	–1.41926
$N = 7$				$\sum Y = 12.20481$; $\sum Y^2 = 21.29187$					$\sum X = -7.65668$; $\sum X^2 = 8.86898$; $\sum XY = -13.27200$		

[A]—Values are obtained from Fig. 2.
[B]—Area = 0.504 × 0.1045 = 0.052668 in.^{2}
[C]—True stress = (engineering stress) × (1 + engineering strain).
[D]—Engineering strain = (extension)÷ gage length; gage length = 2.00 in.
[E]—True strain = *ln* (1 + engineering strain). Total strain has been used. Elastic strain was not subtracted in this example.
All calculations shown were performed on a calculator that uses 10 significant figures although only five places to the right of the decimal were displayed.

TABLE X1.2 An Example of a Worksheet for Calculating the Strain Hardening Exponent, Strength Coefficient and Standard Deviation by Method B

Where:
X denotes ε,
Y denotes σ,
n denotes strain hardening exponent, and
b denotes log K
Data operated upon this example are taken from Fig. 2 and evaluated in Table X1.1.
The number of data-pairs N, is 7.
All logarithms used in the example are base 10.
From Table X1.1:

$$\Sigma X = \Sigma(\log \ \varepsilon) = 7.65668; \bar{X} = \frac{\Sigma X}{N} = \frac{-7.65668}{7} = -1.09381$$

$$\Sigma X^2 = 8.86898$$

$$\Sigma Y = \Sigma(\log \ \sigma) = 12.0481; \bar{Y} = \frac{\Sigma Y}{N} = \frac{12.20481}{7} = 1.74354$$

$$\Sigma Y^2 = 21.29187 \text{ and}$$

$$\Sigma X \times Y = -13.27200$$

The calculations for n and b:

$$\frac{\Sigma X \times \Sigma Y}{N} = \frac{(-\ 7.65668)(12.20481)}{7} = -13.34976 \quad \text{Step 1}$$

$$Sxy = \Sigma XY - \text{Step1} = -13.26999 - (-\ 13.349767) = 0.07776 \quad \text{Step 2}$$

$$\frac{(\Sigma\ X)^2}{N} = \frac{(-\ 7.65668)^2}{7} = 8.37496 \quad \text{Step 3}$$

$$Sxx = \Sigma X^2 - \text{Step3} = 8.86898 - 8.3796 = 0.49402 \quad \text{Step 4}$$

$$n = \frac{Sxy}{Sxx} = \frac{\text{Step2}}{\text{Step4}} = \frac{0.07776}{0.49402} = 0.15739 = 0.157 \quad \text{Step 5}$$

$$n\bar{X} = (0.15739)(-\ 1.09381) = -0.17216 \quad \text{Step 6}$$

$$b = \bar{Y} - n\bar{X} = 1.74354 - \text{Step 6} = 1.74354 - (0.17216) = 1.91570 \quad \text{Step 7}$$

$$K = 10^b = 10^{1.91570} = 82.35690 = 82.36; \text{ see Note 1.} \quad \text{Step 7a}$$

X1.1.3 The calculations for the standard deviation:

$$\frac{(S\ x\ y)^2}{Sxx} = \text{Step5} \times \text{Step2} = (0.15739) \times (0.07776) = 0.0122386 \text{ see X1.3.} \quad \text{Step 8}$$

$$Syy = \Sigma Y^2 - \frac{\Sigma Y^2}{N} = 21.29187 - \frac{(12.20481)^2}{7} = 0.0122433 \quad \text{Step 9}$$

$$S^2y = \frac{\text{Step9} - \ \text{Step8}}{N-2} = \frac{0.01224 - 0.01224}{7-2} = \frac{4.7 \times 10^{-6}}{5} = 9.4 \times 10^{-7} \quad \text{Step 10}$$

$$S_n^2 = \frac{\text{Step10}}{\text{Step4}} = \frac{9.4 \times 10^{-7}}{4.9402 \times 10^{-1}} = 1.9 \times 10^{-6} \quad \text{Step 11}$$

$$\text{Standard Deviation} = S_n = \sqrt{\text{Step11}} = 1.3 \times 10^{-3}$$

$$S_b^2 = S^2y\left\{\frac{1}{N} + \frac{\bar{X}^2}{Sxx}\right\} = 9.4 \times 10^{-7}\left\{\frac{1}{7} + \frac{(-\ 1.09381)^2}{0.49402}\right\} = 2.41078 \times 10^{-6} \quad \text{Step 12}$$

$$S_b = 1.55 \times 10^{-3}$$

REFERENCES

(1) Kleemola, H. J., and Nieminen, M. A., "On the Strain-Hardening Parameters of Metals," *Metallurgical Transactions,* Vol 5, August 1974, pp. 1863–1866. DOI: 10.1007/BF02644152 http://dx.doi.org/10.1007/BF02644152

(2) Morrison, W. B., "Effect of Grain Size on the Stress-Strain Relationship of Low Carbon Steel," *Transactions of the American Society for Metals,* TASEA, Vol 59, 1966, pp. 824–846.

(3) Dowling, Norman E., *Mechanical Behavior of Materials*, 2nd Ed., Prentice Hall, 1999, pp. 125-128.

(4) Dieter, G., *Mechanical Metallurgy*, McGraw-Hill, 1961, Chapter 9, The Tension Test, pp. 237-272.

(5) Matusevich, A.E., Massa, J.C. and Mancini, R.A., "Computation of Tensile Strain-Hardening Exponents through the Power-Law Relationship," *Journal of Testing and Evaluation*, Vol. 40, No. 4 (2012) Paper ID JTE104226 www.astm.org.

(6) Crow, E. L., Davis, F. A., and Maxfield, M. W.," Statistics Manual," U.S. Naval Ordinance Test Station, *NAVORD Report 3369,* NOTS 948, 1955, p. 165. http://handle.dtic.mil/100.2/AD0149334

(7) Bowker and Lieberman, *Engineering Statistics,* Prentice-Hall, Inc., Englewood Cliffs, NJ, 1959, Chapter 9, Fitting Straight Lines.

(8) Volk, William, *Applied Statistics for Engineers,* McGraw-Hill Book Company, Inc., New York, NY, 1958.

(9) NIST/SEMATECH e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/pmd/section4/pmd431.htm 2012.

SUMMARY OF CHANGES

Committee E28 has identified the location of selected changes to this standard since the last issue (E646–15) that may impact the use of this standard.

(1) 8.2 and Note 7 were revised.

ASTM International takes no position respecting the validity of any patent rights asserted in connection with any item mentioned in this standard. Users of this standard are expressly advised that determination of the validity of any such patent rights, and the risk of infringement of such rights, are entirely their own responsibility.

This standard is subject to revision at any time by the responsible technical committee and must be reviewed every five years and if not revised, either reapproved or withdrawn. Your comments are invited either for revision of this standard or for additional standards and should be addressed to ASTM International Headquarters. Your comments will receive careful consideration at a meeting of the responsible technical committee, which you may attend. If you feel that your comments have not received a fair hearing you should make your views known to the ASTM Committee on Standards, at the address shown below.

This standard is copyrighted by ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, United States. Individual reprints (single or multiple copies) of this standard may be obtained by contacting ASTM at the above address or at 610-832-9585 (phone), 610-832-9555 (fax), or service@astm.org (e-mail); or through the ASTM website (www.astm.org). Permission rights to photocopy the standard may also be secured from the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, Tel: (978) 646-2600; http://www.copyright.com/