Designation: E561 – 15a

Standard Test Method for
K_R Curve Determination[1]

This standard is issued under the fixed designation E561; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 This test method covers the determination of the resistance to fracture of metallic materials under Mode I loading at static rates using either of the following notched and precracked specimens: the middle-cracked tension M(T) specimen or the compact tension C(T) specimen. A K_R curve is a continuous record of toughness development (resistance to crack extension) in terms of K_R plotted against crack extension in the specimen as a crack is driven under an increasing stress intensity factor, K. (**1**)[2]

1.2 Materials that can be tested for K_R curve development are not limited by strength, thickness, or toughness, so long as specimens are of sufficient size to remain predominantly elastic to the effective crack extension value of interest.

1.3 Specimens of standard proportions are required, but size is variable, to be adjusted for yield strength and toughness of the materials.

1.4 Only two of the many possible specimen types that could be used to develop K_R curves are covered in this method.

1.5 The test is applicable to conditions where a material exhibits slow, stable crack extension under increasing crack driving force, which may exist in relatively tough materials under plane stress crack tip conditions.

1.6 The values stated in SI units are to be regarded as the standard. The values given in parentheses are for information only.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[3]

E4 Practices for Force Verification of Testing Machines

E399 Test Method for Linear-Elastic Plane-Strain Fracture Toughness K_{Ic} of Metallic Materials

E1823 Terminology Relating to Fatigue and Fracture Testing

2.2 *Other Document:*

AISC Steel Construction Manual[4]

3. Terminology

3.1 *Definitions*—Terminology E1823 is applicable to this method.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *apparent plane-stress fracture toughness, K_{app}*—The value of K calculated using the initial crack size and the maximum force achieved during the test. K_{app} is an engineering estimate of toughness that can be used to calculate residual strength. K_{app} depends on the material, specimen size, and specimen thickness and as such is not a material property.

3.2.2 *effective modulus, E_{eff} [FL^{-2}]*—an elastic modulus that allows a theoretical (modulus normalized) compliance to match an experimentally measured compliance for an actual initial crack size a_o.

3.2.3 *plane-stress fracture toughness, K_c*—The value of K_R at instability in a force-controlled test corresponding to the maximum force point in the test. K_c depends on the material, specimen size, and specimen thickness and as such is not a material property.

3.2.3.1 *Discussion*—See the discussion of plane-strain fracture toughness in Terminology E1823.

4. Summary of Test Method

4.1 During slow-stable fracturing, the developing crack extension resistance K_R is equal to the applied stress intensity factor K. The crack is driven forward by continuously or incrementally increasing force or displacement. Measurements

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[1] This test method is under the jurisdiction of ASTM Committee E08 on Fatigue and Fracture and is the direct responsibility of Subcommittee E08.07 on Fracture Mechanics.

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[2] The boldface numbers in parentheses refer to the list of references at the end of this standard.

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[4] Available from American Institute of Steel Construction (AISC), One E. Wacker Dr., Suite 700, Chicago, IL 60601-1802, http://www.aisc.org.

***A Summary of Changes section appears at the end of this standard**

are made periodically for determination of the effective crack size and for calculation of K values, which are individual data points that define the K_R curve for the material under those test conditions.

4.2 The crack starter is a low-stress-level fatigue crack.

4.3 The method covers two techniques for determination of effective crack size: *(1)* direct measurement of the physical crack size which is then adjusted for the developing plastic zone size, and *(2)* compliance measurement techniques that yield the effective crack size directly. Methods of measuring crack extension and of making plastic-zone corrections to the physical crack size are prescribed. Expressions for the calculation of crack-extension force K_R are given. Criteria for determining if the specimen conditions are predominantly elastic are provided.

5. Significance and Use

5.1 The K_R curve characterizes the resistance to fracture of materials during slow, stable crack extension and results from the growth of the plastic zone ahead of the crack as it extends from a fatigue precrack or sharp notch. It provides a record of the toughness development as a crack is driven stably under increasing applied stress intensity factor K. For a given material, K_R curves are dependent upon specimen thickness, temperature, and strain rate. The amount of valid K_R data generated in the test depends on the specimen type, size, method of loading, and, to a lesser extent, testing machine characteristics.

5.2 For an untested geometry, the K_R curve can be matched with the crack driving (applied K) curves to estimate the degree of stable crack extension and the conditions necessary to cause unstable crack propagation **(2)**. In making this estimate, K_R curves are regarded as being independent of initial crack size a_o and the specimen configuration in which they are developed. For a given material, material thickness, and test temperature, K_R curves appear to be a function of only the effective crack extension Δa_e **(3)**.

5.2.1 To predict crack behavior and instability in a component, a family of crack driving curves is generated by calculating K as a function of crack size for the component using a series of force, displacement, or combined loading conditions. The K_R curve may be superimposed on the family of crack driving curves as shown in Fig. 1, with the origin of the K_R curve coinciding with the assumed initial crack size a_o. The intersection of the crack driving curves with the K_R curve shows the expected effective stable crack extension for each loading condition. The crack driving curve that develops tangency with the K_R curve defines the critical loading condition that will cause the onset of unstable fracture under the loading conditions used to develop the crack driving curves.

5.2.2 Conversely, the K_R curve can be shifted left or right in Fig. 1 to bring it into tangency with a crack driving curve to determine the initial crack size that would cause crack instability under that loading condition.

5.3 If the K-gradient (slope of the crack driving curve) of the specimen chosen to develop the K_R curve has negative characteristics (see Note 1), as in a displacement-controlled test condition, it may be possible to drive the crack until a maximum or plateau toughness level is reached **(4, 5, 6)**. When a specimen with positive K-gradient characteristics (see Note 2) is used, the extent of the K_R curve which can be developed is terminated when the crack becomes unstable.

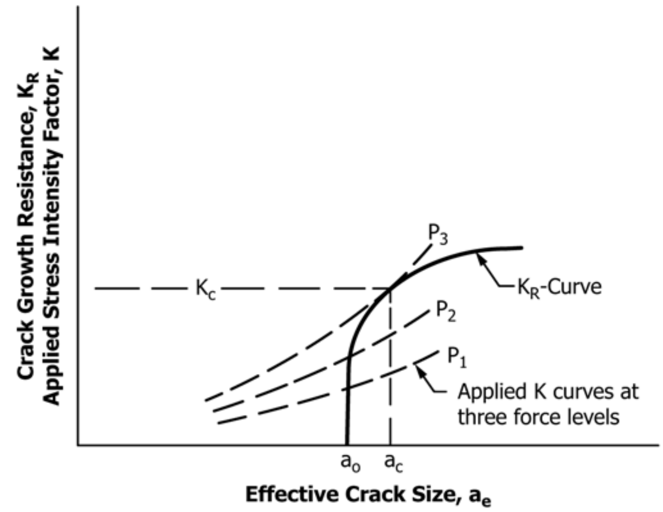


FIG. 1 Schematic Representation of K_R curve and Applied K Curves to Predict Instability; K_c, P_3, a_c, Corresponding to an Initial Crack Size, a_o

NOTE 1—Fixed displacement in crack-line-loaded specimens results in a decrease of K with crack extension.

NOTE 2—With force control, K usually increases with crack extension, and instability will occur at maximum force.

6. Apparatus

6.1 *Testing Machine*—Machines used for K_R curve testing shall conform to the requirements of Practices E4. The forces used in determining K_R values shall be within the verified force application range of the testing machine as defined in Practices E4.

6.2 *Grips and Fixtures for Middle-Cracked Tension (M(T)) Specimens*—In middle-cracked tension specimens, the grip fixtures are designed to develop uniform stress distribution in the central region of the specimen. Single pin grips can be used on specimens less than 305 mm (12 in.) wide if the specimen is long enough to ensure uniform stress distribution in the crack plane (see 8.5.3.) For specimens wider than 305 mm (12 in.), multiple-bolt grips such as those shown in Fig. 2 or wedge grips that apply a uniform displacement along the entire width of the specimen end shall be used if the stress intensity factor and compliance equations in Section 11 are to be used. Other gripping arrangements can be used if the appropriate stress intensity factor and compliance relationships are verified and used. Grips should be carefully aligned to minimize the introduction of bending strain into the specimen. Pin or gimbal connections can be located between the grips and testing machine to aid the symmetry of loading. If extra-heavy-gauge, high-toughness materials are to be tested, the suitability of the grip arrangement may be checked using the *AISC Steel Construction Manual*.

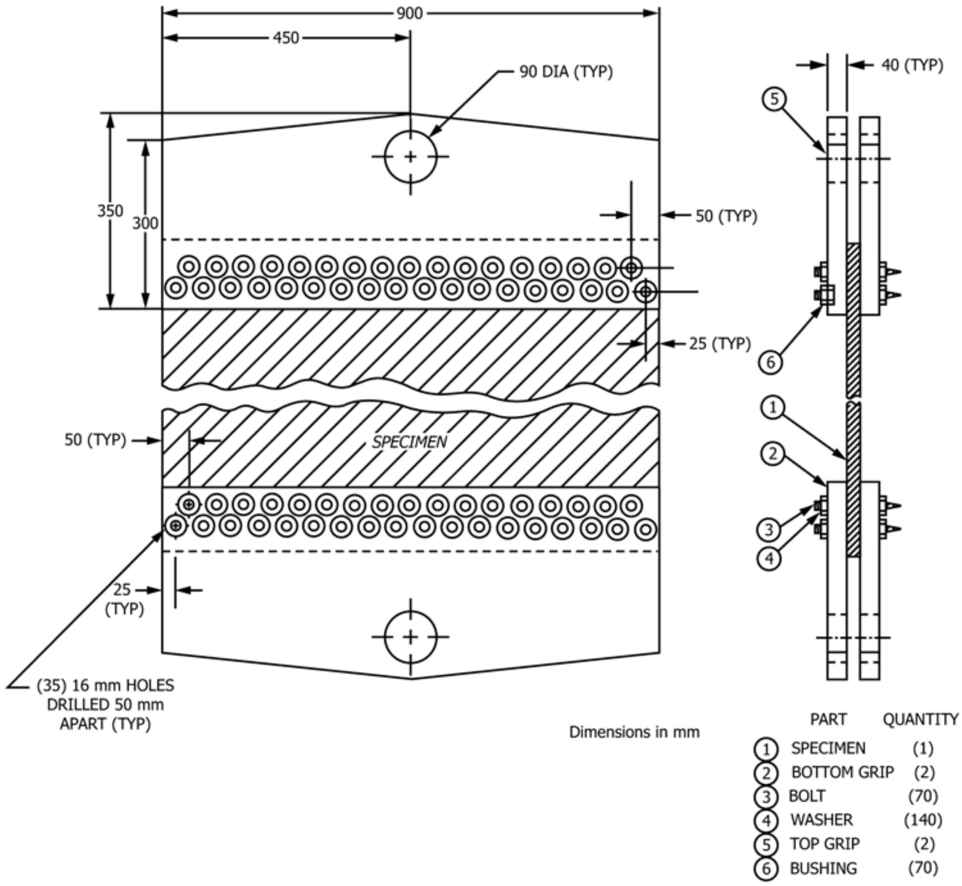


FIG. 2 Middle-Cracked Tension (M(T)) Panel Test Setup

6.3 *Grips and Fixtures for Compact Tension (C(T)) Specimens*—The grips and fixtures described in Test Method E399 are recommended for K_R curve testing where C(T)-type specimens are loaded in tension.

6.4 *Buckling Constraints*—Buckling may develop in unsupported specimens depending upon the specimen thickness, material toughness, crack size, and specimen size (**7**). Buckling seriously affects the validity of a *K* analysis and is particularly troublesome when using compliance techniques to determine crack size (**8**). It is therefore required that buckling constraints be affixed to the M(T) and C(T) specimens in critical regions when conditions for buckling are anticipated. A procedure for the detection of buckling is described in 9.8.3.

6.4.1 For an M(T) specimen in tension, the regions above and below the notch are in transverse compression which can cause the specimen to buckle out of plane. The propensity for buckling increases as *W/B* and 2*a/W* ratios increase and as the force increases. Unless it can be shown by measurement or analysis that buckling will not occur during a test, buckling constraints shall be attached to the central portion of the specimen. The guides shall be so designed to prevent sheet kinking about the crack plane and sheet wrinkling along the specimen width. Buckling constraints should provide a high stiffness constraint against out-of-plane sheet displacements while minimizing friction. Buckling constraints with additional pressure adjustment capability near the center of the specimen are recommended (**7**). Friction between the specimen and the buckling constraints shall not interfere with the in-plane stress distribution in the specimen. Friction can be minimized by using a low-friction coating (such as thin TFE-fluorocarbon sheet) on the contact surfaces of the constraints and by using just enough clamping force to prevent buckling while allowing free movement of the guides along the length of the specimen. A suspension system to prevent the buckling constraint from sliding down the specimen is recommended. Several buckling constraint configurations for M(T) specimens are shown in (**8**) and (**9**).

6.4.2 For C(T) specimens, the portion of the specimen arms and back edge which are in compression may need to be restrained from buckling in thinner specimens of high toughness alloys. It is convenient to use a base plate and cover plate with ports cut at appropriate locations for attaching clip gages and for crack size observations. Friction between buckling restraints and specimen faces is detrimental and should be minimized as much as possible.

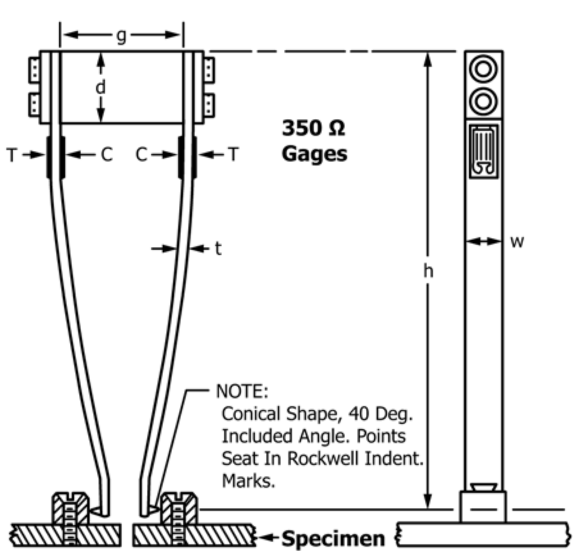
6.4.3 Lubrication shall be provided between the face plates and specimen. Care shall be taken to keep lubricants out of the crack. Sheet TFE-fluorocarbon or heavy oils or both can be used. The initial clamping forces between opposing plates should be high enough to prevent buckling but not high enough to change the stress distribution in the region of the crack tip at any time during the test.

6.5 *Displacement Gages*—Displacement gages are used to accurately measure the crack-mouth opening displacement (CMOD) across the crack at a specified location and span. For small C(T) specimens, the gage recommended in Test Method E399 may have a sufficient linear working range to be used. However, testing specimens with W greater than 127 mm (5 in.) may require gages with a larger working range, such as the gage shown in Fig. 3.

6.5.1 A recommended gage for use in M(T) specimens is shown in Fig. 4 **(10)**. This gage is inserted into a machined hole having a circular knife edge. The diameter d_i, is the gage span 2*Y* used in the calibration. Detail drawings of the gage are given in Fig. 5. Radius of the attachment tip should be less than the radius of the circular knife edge in the specimen.

6.5.2 The gage recommended in 6.5.1 is preferred because of its excellent linearity characteristics and ease of attachment. However, other types of gages used over different span lengths are equally acceptable provided the precision and accuracy requirements are retained. For example, the conventional clip gage of Test Method E399 may be used with screw attached knife edges spanning the crack at a chosen span 2*Y*. In M(T) tests, the proper compliance calibration curve must be used because compliance is a function of *Y/W*. When using the compliance calibration curve given in Eq 5, the proper 2*Y* value to use with screw-on knife edges is the average distance between attachment points across the notch. This is the actual deformation measurement point, not the gage length of the clip gage itself.

6.5.3 The use of point contacts eliminates error in the readings from the hinge-type rotation of C(T) specimens. The precision of all types of gages shall be verified in accordance with the procedure in Test Method E399. In addition, absolute accuracy within 2 % of reading over the working range of the gage is required for use with compliance measurements. Data for compliance measurements must be taken within the verified range of the gage. The gages shall be verified periodically.



Dimensions

g mm (in.)	d mm (in.)	t mm (in.)	h mm (in.)	w mm (in.)
23.3 (0.918)	12.7 (0.500)	1.6 (0.062)	86.4 (3.400)	7.6 (0.300)

FIG. 3 Enlarged Clip Gage for Compliance Measurements on Large Specimens

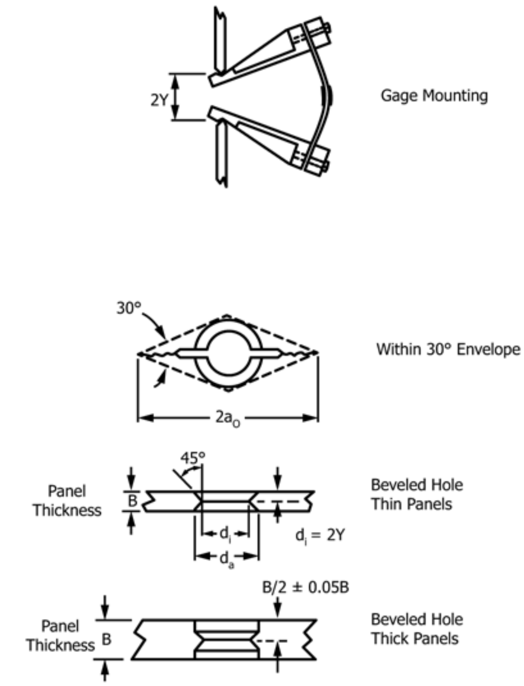


FIG. 4 Recommended Gage for Use in Drilled Hole M(T) Panels

6.6 *Optical Equipment*—If the material being tested is sufficiently thin so that the crack-tip contour does not vary significantly from surface to mid-thickness, crack extension can be followed by surface observations using optical equipment. If force is sustained at given increments so that the crack stabilizes, physical crack size can be determined within 0.2 mm (0.01 in.) using a 30 to 50-power traveling-stage microscope. A digital image correlation system may also be useful for determining in-plane strain distribution and out-of-plane displacements **(11)**.

6.7 *Other Equipment*—Other methods of measuring crack size are available, such as eddy-current probes, which are most useful with nonferrous material, or electrical-resistance measurements, where the extension of the crack is determined from electrical potential differences.

6.8 *Data Recording Equipment*—When running a continuous monotonic test, a system capable of recording force and displacement signals with high fidelity at data rates to capture at least 200 force-CMOD data pairs during the test should be used. Appropriate data filtering can be used provided it does not introduce errors into the data.

7. Specimen Compliance Measurement Requirements

7.1 In the K_R test, the effective crack size is determined either by direct measurement of the physical crack size and adjusting for the crack tip plastic zone, or by specimen compliance techniques which can determine effective crack size directly. This section provides background and requirements for the use of compliance techniques.

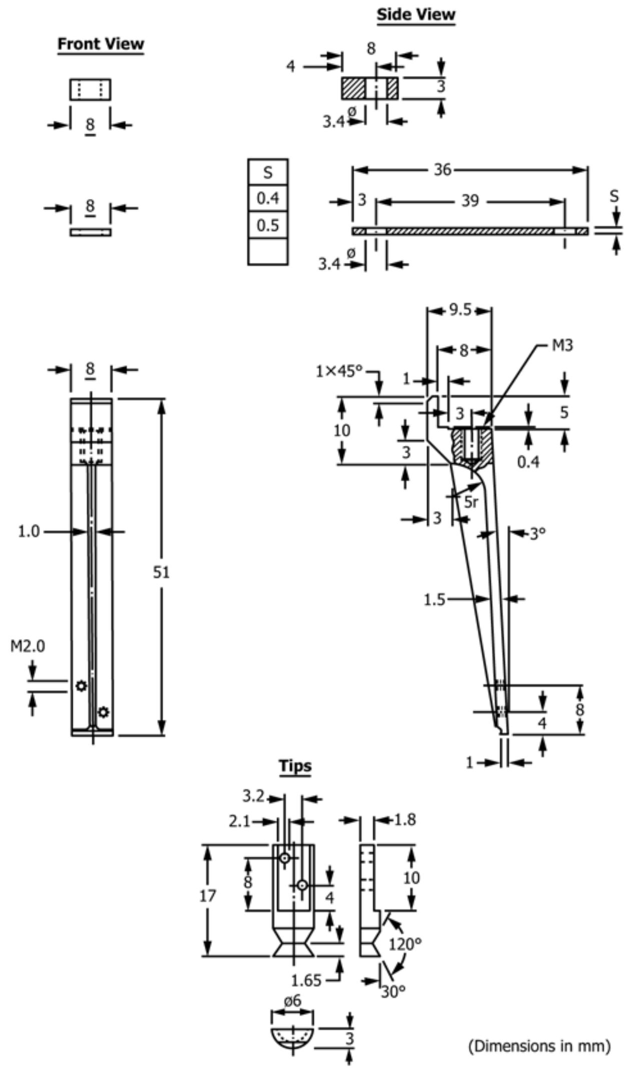


FIG. 5 Detail Drawings of Clip Gage for Use with the M(T) Specimen

7.2 Specimen compliance is the ratio of the change in specimen displacement to the change in force carried by the specimen ($\Delta v/\Delta P$) during the test. The loading (secant) compliance technique and the calibration information are used to determine effective crack size a_e directly (see Fig. 6). The crack size is automatically corrected for the plastic-zone and these values of a_e can be used directly in the appropriate stress intensity factor solutions to determine K_R. Unloading compliance can also be used to determine physical crack size a_p. In this technique, the specimen compliance is measured during periodic load reversals during the test. Specimen unloading compliance values are substituted into the appropriate calibration curve or compliance expression to determine physical crack size a_p. In this case, effective crack size can be computed by adding the plastic zone size at each measurement point.

7.3 The compliance technique uses specimen displacement measured at a single location, for example the front face mouth opening for C(T) specimens or spanning the notch at the specimen midplane for M(T) specimens.

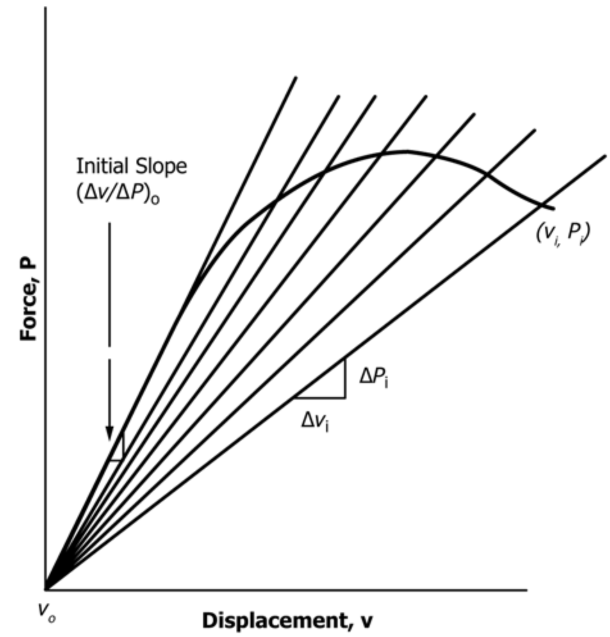


FIG. 6 Schematic Test Record and Secant Compliance Constructions for M(T) or C(T) Specimens

7.4 Specimen compliance is measured by simultaneously recording the force and CMOD during the test. The effective crack size can be determined directly by calculating $\Delta v/\Delta P$ in the single compliance method. Crack size is determined from compliance measurements using the compliance equations or tables for the specimen tested as described in Section 11.

7.5 The compliance technique uses elastic characteristics of the specimen calibrated over a variety of crack sizes (**12**). Compliance calibration curves have been developed for various specimen geometries analytically using finite element methods or experimentally using specimens containing various crack sizes. The change in CMOD (Δv) of specific measurement points on the specimen is determined as a function of the change in force (ΔP). The slopes are normalized for material thickness and elastic modulus and plotted against the ratio of crack size to specimen width, providing a calibration curve of $EB(\Delta v/\Delta P)$ as a function of a/W for the C(T) specimens or $2a/W$ for the M(T) specimen. Analytical expressions for the normalized compliance of the two specimen types covered in this method are given in Section 11 for specified displacement measurement points.

8. Specimen Configuration, Dimensions, and Preparation

8.1 *Specimen Type*—This method covers two specimen types: M(T) and C(T). The choice of specimen type depends on the amount of material available, the type of test to be run, and the type of equipment available. Ideally, the K_R curve should not depend on the specimen type, although the amount of valid K_R curve generated will depend on the specimen type and size. If the material is highly anisotropic, it may be preferable to use the M(T) specimen because the high stress gradient of the C(T) specimen may be more prone to exhibit crack deviation. The following sections provide information about each specimen type.

NOTE 3—Difficulties in the interpretation of test records will be encountered if the specimens are not flat prior to testing or if the specimen contains substantial residual stress.

8.2 *Number of Tests*—Replicate K_R curves can be expected to vary as with other mechanical properties. Test-to-test variability in K_R curves also depends on the material being tested. It is recommended that at least duplicate tests on multiple lots of material be performed when developing design data. For quality assurance testing, a single test can be performed.

8.3 *Specimen Size*—In order for a given calculated K_R value to be valid, the remaining uncracked ligament in the plane of the crack must be predominantly elastic at the value of applied force and physical crack size corresponding to that value of K_R. Methods for estimating specimen size to ensure predominantly elastic conditions over a wide range of Δa_e values are provided for each specimen type below. Methods for determining invalid data points are provided in subsequent sections of the method.

8.4 *Starting Notch and Precrack*—The machined starter notch for either of the recommended specimens may be made by electrical-discharge machining, end milling, or saw cutting. It is advisable to have a root radius at the ends of the notch of 0.08 mm (0.003 in.) or less to facilitate fatigue precracking. Fatigue precracking is highly recommended and may be omitted only if it has been demonstrated for the material and thickness of interest that the machined notch root radius effectively simulates the sharpness of a fatigue precrack. The starter notch should be extended by fatigue precrack not less than 1.3 mm (0.05 in.) in length. The procedure for precracking is given in Testing Procedures, Section 9.

8.5 *Middle-Cracked Tension (M(T)) Specimen:*

8.5.1 The middle-cracked tension (M(T)) specimen is a rectangular specimen containing a centrally-located starter notch that is pulled in tension in the length direction of the specimen.

8.5.2 The ends of the specimen may contain a single pin-loading hole or may be configured for gripping with multiple-bolt grips or wedge grips along the two ends of the specimen as shown in Fig. 2.

8.5.3 To ensure uniform stress entering the crack plane when single-pin grips are used, the distance between the loading pins shall be at least three specimen widths, $3W$. For specimens wider than 305 mm (12 in.), multiple-bolt grips such as those shown in Fig. 2, or wedge grips that apply a uniform displacement along the entire width of the specimen end, shall be used. In this case, the minimum required distance between the innermost gripping points is relaxed to $1.5W$.

8.5.4 A starter notch is machined perpendicular to the tension direction, centered at mid-width and located midway along the length of the specimen. The machined notch shall be centered with respect to the specimen width within $0.002W$ and its length shall be such that after precracking the required minimum amount, the initial crack size, $2a_o$ (machined notch plus fatigue precrack) shall be within the range of 0.25 to $0.40W$. The machined notch must lie within the envelope shown in Fig. 7. A fatigue precrack shall be initiated from each end of the starter notch using the procedure in 9.2. The fatigue precrack shall extend from the starter notch by at least 1.3 mm (0.05 in.) and must extend beyond the envelope shown in Fig. 7.

8.5.5 In the M(T) specimen, crack size a in the equations of Section 11 is the dimension from the specimen centerline to the crack tip. This assumes that the crack is perfectly symmetrical with respect to the specimen centerline. In practice, this is one-half of the average tip-to-tip crack length measurement.

8.5.6 For specimen compliance determination, CMOD measurements are made between points spanning the machined notch at the mid-width of the specimen. This can be done by attaching knife edges to the specimen with screws or cement to accept a commercial clip gage or the one shown in Fig. 3. The specimen can also be machined with integral knife edges using beveled holes as shown in Fig. 4. The CMOD gage shown in Fig. 5 fits into these knife edges.

8.5.7 To ensure predominantly elastic conditions in the M(T) specimen, the net section stress based on the physical crack size must be less than the yield strength of the material at the test temperature. The M(T) specimen width W and initial crack size a_o should be selected to provide valid K_R data up to effective crack extension values of interest. In general, a wider specimen will provide valid data up to a larger value of effective crack extension than a narrow specimen.

8.5.8 The required width to maintain predominantly elastic conditions for a given value of K_R may be estimated from the maximum expected plastic-zone size, r_Y (see Section 10), which is directly proportional to the square of the material toughness-to-yield strength ratio. As a guide, a specimen $27r_Y$ wide and with an initial crack size $2a_o$ of $0.33W$ is expected to fail at a net section stress equal to the yield strength **(13)**. It therefore is desirable to have an estimate of the maximum value of K_R expected in the test before designing the specimen. As an aid, the following table lists minimum recommended M(T) sizes for assumed ratios of K_{Rmax} to yield strength.

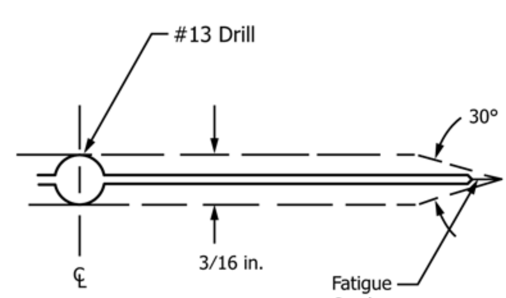


FIG. 7 Enlarged View of the Right Half of the Permitted Notch Envelope in M(T) Panels

Table of Minimum M(T) Specimen Geometry for Given Conditions

K_{Rmax}/σ_{YS}		Width		$2a_o$		Length[A]	
$\sqrt{m}$	$\sqrt{in.}$	m	in.	m	in.	m	in.
0.08	0.5	0.076	3.0	0.025	1.0	0.229	9
0.16	1.0	0.152	6.0	0.051	2.0	0.457	18
0.24	1.5	0.305	12.0	0.102	4.0	0.914	36
0.32	2.0	0.508	20.0	0.170	6.7	0.762	30
0.48	3.0	1.219	48.0	0.406	16.0	1.829	72

[A] Distance between pin centers of single pin loaded M(T) specimens is nominally 3*W*. Specimens wider than 305 mm (12 in.) will require multiple pin grips or full-width gripping and the length requirement for the distance between nearest gripping points is relaxed to 1.5*W*.

8.6 *Compact Tension (C(T)) Specimen:*

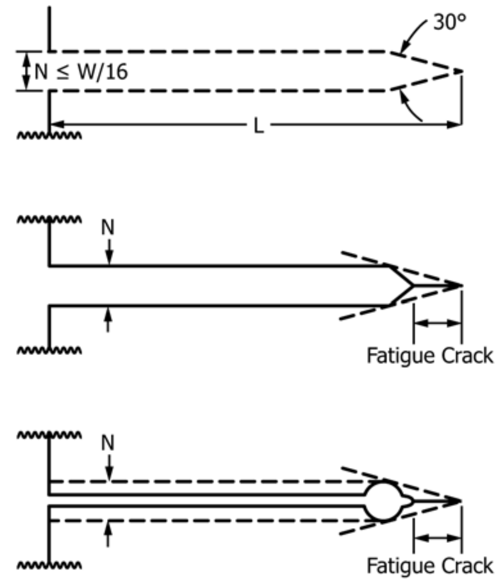
8.6.1 The recommended C(T) specimen is shown in Fig. 8. The specimen is loaded in tension with clevis grips using pins inserted through the loading holes. The loading hole size is proportional to the specimen width.

8.6.2 Fig. 9 shows the allowable notch types and envelope sizes for this specimen. The notch is machined perpendicular to the loading axis and is centered with respect to the top and bottom edges of the specimen. A fatigue precrack shall be initiated from the notch tip using the procedure in 9.2. The fatigue precrack shall extend from the starter notch by at least 1.3 mm (0.05 in.) and must extend beyond the envelope shown in Fig. 9.

8.6.3 The initial crack size a_o (that is, machined notch plus fatigue precrack) in the C(T) specimens shall be between 0.35 and 0.55*W*.

8.6.4 For specimen compliance determination, CMOD measurements are made across the notch at either location V_0 or V_1 in Fig. 8 (0.25*W* ± 0.0006*W* or 0.1576*W* ± 0.0006*W* in advance of the loading hole centerline). Span of the gage is not critical so long as it is less than *W*/4. Alternative location of the gage is permitted but displacement values must be linearly extrapolated to 0.1576*W* in order to use the expressions given in Section 11 for compliance measurement.

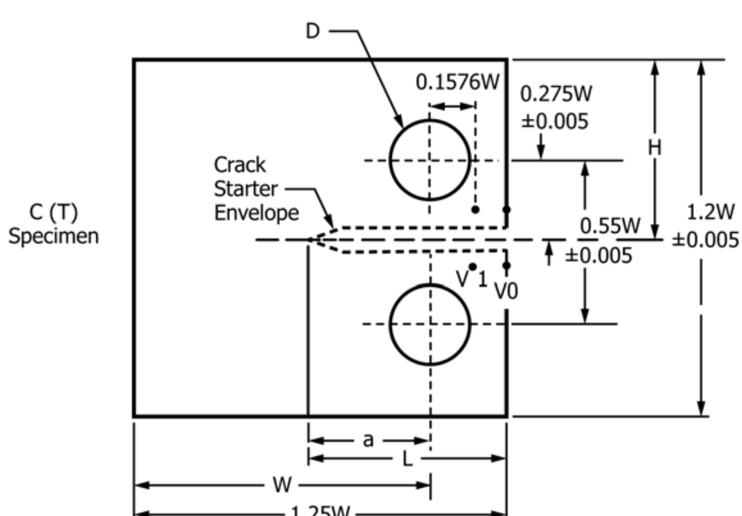
8.6.5 To ensure that a given calculated value of K_R is considered valid for the C(T) specimen, the remaining uncracked ligament must remain predominantly elastic. This



NOTE 1—*N* need not be less than 1.6 mm (1⁄16 in.) but must not exceed *W*/16.

NOTE 2—The intersection of the crack-starter tips with the two specimen faces shall be equidistant from the top and bottom edges of the specimen within 0.005*W*.

FIG. 9 Envelope for Crack-Starter Notches and Examples of Notches Extended with Fatigue Cracks



Specimen Width W (mm)	D (mm)	d (mm)	Specimen Width W (in.)	D (in.)
75 < W ≤ 125	25	10	3 < W ≤ 5	1.0
125 < W ≤ 250	40	20	5 < W ≤ 10	1.5
250 < W	65	20	10 < W	2.5

NOTE 1—Specimen thickness B shall not vary by more than 0.127 mm (0.005 in.) or 0.01*W*, whichever is greater.

FIG. 8 Compact Tension (C(T)) Specimen

condition is considered to be met in this method as long as the length of the remaining uncracked ligament, $W\text{-}a_p$, at that point in the test is greater than or equal to eight plastic zone sizes. This is met with the condition given in Eq 1.

$$(W - a_p) \geq \frac{4}{\pi}\left(\frac{K_R}{\sigma_{YS}}\right)^2 \tag{1}$$

8.6.5.1 In this expression, W is the specimen width as shown in Fig. 8, a_p is the physical crack size corresponding to the K_R point being considered, and σ_{YS} is the 0.2 % offset yield strength of the material. By substituting the maximum expected or desired K_R for a test, an estimate of the required specimen size can be made. As an aid, the following table shows maximum final crack size to width ratios for several normalized K_{Rmax} values:

Table of Minimum C(T) Specimen Width W for Given Conditions, m (in.)

K_{Rmax}/σ_{YS}		Maximum a_p/W				
√m	√in.	0.4	0.5	0.6	0.7	0.8
0.10	0.6	0.02	0.03	0.03	0.04	0.06
		(0.8)	(1.0)	(1.3)	(1.7)	(2.5)
0.20	1.3	0.08	0.10	0.13	0.17	0.25
		(3.3)	(4.0)	(5.0)	(6.7)	(10.0)
0.30	1.9	0.19	0.23	0.29	0.38	0.57
		(7.5)	(9.0)	(11.3)	(15.0)	(22.6)
0.40	2.5	0.34	0.40	0.51	0.67	1.01
		(13.3)	(15.9)	(19.9)	(26.5)	(39.8)
0.50	3.1	0.53	0.64	0.80	1.06	1.59
		(20.9)	(25.1)	(31.3)	(41.8)	(62.7)

9. Testing Procedures

9.1 *Specimen Measurements*—Measure specimen thickness B to ±0.5 % of B at two locations in the plane of the notch between the notch tip and the specimen edge. Measure specimen width, W, to ±0.5 % of W.

9.2 *Specimen Precracking*—All specimens shall be precracked in the final heat-treated condition. The length of the fatigue crack extension shall not be less than 1.3 mm (0.05 in.). The precrack must also extend beyond the applicable envelope boundary shown in Fig. 7 or Fig. 9 depending on the specimen being tested.

9.2.1 Precracking may include two or more stages: crack initiation, intermediate propagation, and finishing. To avoid temporary growth retardation from a single step of load shedding, one or more intermediate levels may be added. The reduction in maximum force from the final intermediate stage to the finishing stage shall not be more than 30 %.

9.2.2 As a guide, crack initiation can be started in most commercial materials at K_{max}/E = 0.00013 $m^{1/2}$ (0.00083 $in.^{1/2}$). Many commercial materials can be finished at K_{max}/E = 0.0001 $m^{1/2}$ (0.0006 $in.^{1/2}$). Most aluminum alloys can be precracked at ΔK = 10 to 12 MPa·√m (9 to 11 ksi·√in.). Stress ratio selection is optional, but R = 0.1 is recommended.

NOTE 4—Elastic (Young's) modulus, E, in units of MPa will yield K_{max} in units of MPa·√m. Elastic (Young's) modulus, E, in units of ksi will yield K_{max} in units of ksi·√in.

9.2.3 The finishing stage shall extend the precrack by at least 0.65 mm (0.025 in.), and shall be performed at fixed cyclic load. The finishing stage should be completed in no less than 5×10^3 cycles.

NOTE 5—It may be advantageous, and is allowed in this method, to precrack the specimen in a different machine than that used to run the K_R test. Because the maximum force required for precracking is substantially less than that required for the K_R test, a smaller test machine capable of higher precracking frequency can be used.

9.3 *Specimen Installation*—Prior to gripping the specimen for running the K_R test, zero the load cell. Carefully align the precracked specimen in the testing machine to eliminate eccentricity of loading. Misalignment can result in uncontrolled or spurious stress distribution in the specimen which could be troublesome, particularly if compliance measurements are used to determine crack extension. Fixtures for measuring crack extension may be affixed to the specimen after applying a small preload. Buckling constraints shall also be installed if necessary.

9.4 *Testing Machine Setup*—The testing machine should be operated in displacement control to generate K_R curve data points beyond maximum force. If using a servo-controlled machine in force control, specimen fracture will occur at maximum force and the machine will not be in control after that point.

9.4.1 If used, attach displacement transducers, apply excitation, and warm up instrumentation. Initialize and zero instrumentation and start any data acquisition systems prior to starting the test.

9.5 *Testing Speed*—To maintain a static deformation rate, the testing machine should be set up to apply a displacement rate during the initial linear portion of the force-CMOD curve that will result in a rate of change of K between 0.55 and 2.75 MPa·√m/s (0.50 to 2.5 ksi·√in./s), and this deformation rate should be used throughout the test.

NOTE 6—For an M(T) specimen with W = 400 mm (15.75 in.), $2a_o/W$ from 0.25 to 0.33, and a length between grips of 815 mm (32 in.), a deformation rate of between 0.025 and 0.050 mm/s (0.001 and 0.002 in./s) has been used to achieve the desired static deformation rates.

9.6 *Crack Size Measurements*—Depending on the crack measurement technique chosen, perform the steps in either 9.7 or 9.8. Complete the test procedure by performing the procedure in 9.9 and subsequent sections.

9.7 *Procedure for Tests Using Direct Measurement of Physical Crack Size:*

9.7.1 Apply an increment of displacement to the specimen at a rate that meets the requirements of 9.5, allowing time for the crack to stabilize. Cracks stabilize in most materials within a short time of stopping the deformation. However, when stopping near an instability condition, the crack may take several minutes to stabilize, depending upon the stiffness of the loading frame and other factors.

NOTE 7—Static K_R cannot be determined when the crack is steadily creeping or accelerating at or near instability.

9.7.2 After the crack stabilizes, measure and record the physical crack size. For the M(T) or C(T) specimen, record the force.

9.7.2.1 Measure the physical crack size accurately to 0.2 mm (0.01 in.) at each step using suitable measuring devices described in Section 6.

9.7.2.2 Physical crack size can also be measured with compliance techniques by partial unloading of the specimen after each increment, a technique described in the Section 10.

9.7.3 Continue to apply increments of displacement, allowing the crack to stabilize, and record physical crack size and force or displacement, or both, until the specimen fractures or until no useful data can be collected.

9.7.3.1 *Number of Data Points*—While K_R curves can be developed with as few as four or five data points, ten to fifteen give improved confidence, and tougher materials usually require more data points.

9.7.3.2 If it is desired to check for specimen buckling or friction when using compliance techniques, slowly reduce the specimen deformation to unload the specimen while recording force and displacement. See discussion in 9.8.3.

9.7.4 At the conclusion of the test, carefully unload the specimen and remove buckling constraints and measuring instruments.

9.8 *Procedure for Tests Using Compliance Measurement of Effective Crack Size:*

9.8.1 The test can be run by incremental deformation, but it is permitted to apply a continuous monotonic deformation if the force and displacement measurements can be recorded accurately and simultaneously.

9.8.2 Begin recording data, if necessary, and apply deformation to the specimen at a constant rate that meets the requirements of 9.5. If incremental loading is used, periodically hold the deformation and record the force and displacement values after the crack has stabilized as described in 9.7.1. Otherwise, monitor and record the force versus CMOD while continuously applying deformation.

9.8.3 It may be possible to detect whether buckling or friction are affecting the test by performing a periodic partial unload of the specimen by reversing the deformation direction as shown schematically in Fig. 10, unloading to about 80 % of the test force at the time of the unload. The initial part of the force-CMOD record should have a linear portion which can be substantially retraced upon partial unloading. Should buckling or friction problems develop during the test, the unloading and reloading slopes will tend to diverge. If the slopes differ by more than 2 %, or if one or both have no linear range, or if the unload-reload trace forms a loop, then buckling or friction may be affecting the test results sufficiently to cause significant error in compliance-measured crack sizes and calculated K value. Added confidence can be obtained by comparing the crack sizes predicted from unloading slopes to physical crack size measured with other more direct methods.

NOTE 8—Buckling can also be detected in an M(T) specimen by watching for a difference in the CMOD measured on both faces of the specimen (indicating symmetric buckling) and by watching for clip gage rotation (indicating anti-symmetric buckling).

9.8.4 If desired, physical crack size can be determined by partial unloading of the specimen at selected times during the test. The unloading slope in the force-CMOD trace at any given point represents the unloading compliance of the specimen corresponding to the physical crack size. If the unloading compliance is determined, the force reversal shall be only enough to establish the return slope accurately. Unloading to about 80 % of the test force at the time of the unload has been used successfully. Should the test record not return linearly immediately upon unloading, factors such as buckling or friction are influencing the test record and results should be considered suspect.

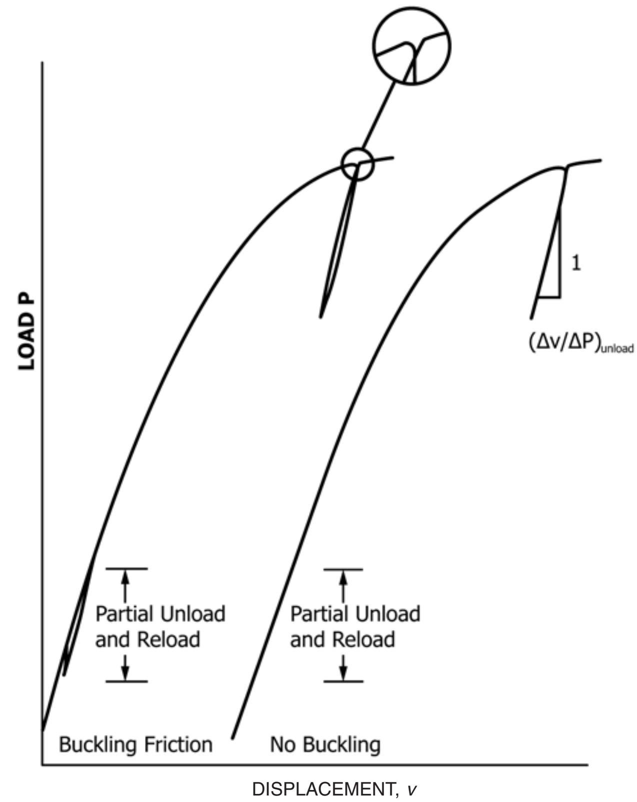


FIG. 10 Detection of Buckling from Compliance Test Records of M(T) and C(T) Specimens

9.8.5 At the conclusion of the test, carefully unload the specimen and remove buckling constraints and measuring instruments.

9.9 *Initial Crack Size Measurement, a_o*—After specimen fracture, inspect the precrack area of the fracture surfaces and determine if excessive crack tunneling occurred. Determine the initial crack size a_o at the precrack mark as the average of three interior crack size measurements taken at the specimen midplane and two quarter planes. Alternatively, the initial crack size a_o at the precrack mark can be taken as the average surface crack size measurements if that value results in no more than a 1 % error in any of the final results. Make crack size measurements to the nearest 0.2 mm (0.01 in.). Refer to the appropriate specimen drawing to determine the reference plane from which the crack size is determined. If excessive tunneling occurred, correct any surface crack measurements made during the test by that amount, so that the observations represent the average of the interior crack sizes.

9.10 *Crack Deviation Measurements*—When testing materials with strong toughness anisotropy, the stable crack extension may deviate from the intended crack direction (**14**). This usually occurs when the test is run in the higher-toughness orientation. Accuracy of the specimen K solution and the elastic compliance relationships decrease with the amount of crack deviation from the intended crack direction. Therefore, note any data points where the physical crack tip at the specimen midplane extends outside a ± 10° deviation envelope originating at machined notch tip.

10. Calculation and Interpretation

10.1 *Construction of the K_R curve*—The K_R curve determined in accordance with this method is a plot of crack extension resistance K_R as a function of effective crack extension Δa_e. Because the crack extension can be measured in several ways, the following sections describe several procedures for determining data pairs of K_R and Δa_e from the test record depending on the type of test run. The physical crack size and plastic zone size also need to be determined for the net section stress validity criteria. A sample tabulation of analysis data is shown in Table 1.

10.1.1 There are three methods for determination of effective crack size, each requiring a slightly different calculation approach: *(1)* Measurement of physical crack size by direct observation and then calculating the effective crack size a_e by adding the plastic zone size, *(2)* Measurement of physical crack size by unloading compliance and calculating a_e by adding the plastic zone size, and *(3)* Measurement of the effective crack size directly by secant compliance, then calculating the physical crack size needed for determining validity.

10.1.2 Depending on the measurement technique chosen, perform the steps in either 10.2 for tests using direct measurement of physical crack size or 10.3 for tests using compliance methods. Use the appropriate sections of 10.3 for the particular compliance method used. Complete the test analysis by using the procedures in 10.4 and subsequent sections. Equations and tables for calculating the stress intensity factor, compliance, force limits, and validity criteria for the three specimen types are described in Section 11.

10.2 *Data Reduction Procedures for Tests Using Direct Visual Measurement of Physical Crack Size:*

10.2.1 For tests where the physical crack size a_p is measured visually, the effective crack size a_e is determined by adding the plastic zone size r_y to the physical crack size.

10.2.2 For each observation point where physical crack size a_p and force were recorded, determine the plastic zone size by calculating $K(a_p)$, the stress intensity factor using the physical crack size a_p in Eq 4 for the M(T) specimen or Eq 10 for the C(T) specimen. Substitute $K(a_p)$ for K in Eq 2 along with the yield strength σ_{YS} to determine the plastic zone size r_y.

$$r_Y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{YS}}\right)^2 \quad (2)$$

NOTE 9—The expression for r_y is most accurate for high-strength materials of yield strength-to-density ratios above 174 kPa/(kg·m^{-3}) (700 000 psi/(lbm·in.$^{-3}$)). Lower-strength, high-toughness materials require increasing reliance on unloading compliance methods to correct for plastic-zone effects. Compliance methods are discussed in 10.3.

10.2.3 Add the value of r_y calculated at each observation point to the physical crack size a_p to determine the effective crack size a_e.

10.2.4 Calculate K_R, the stress intensity factor based on the effective crack size, using the appropriate equation for the

TABLE 1 Sample Data Analysis Set

Material and Specimen Information		
Specmen ID		999-888-L-T-1
Test date		2004-08-04
Alloy		XXXX
Temper		YYYY
Data points		1162
σ_{YS}	(MPa)	325
E	(MPa)	71018.5
W	(mm)	761.5
B	(mm)	6.72
a_o	(mm)	125.8
y_o	(mm)	14.1

Linear Slope Analysis		
r_{y1}	(mm)	0.01
r_{y2}	(mm)	1.25
P_{LIM1}	(kN)	19.5213
P_{LIM2}	(kN)	218.255
Init. Slope	(kN/mm)	612.092
Y-int	(kN)	5.88368
X-int	(mm)	–0.0096
r^2		0.99996
# pts in fit		261
E_{eff}	(MPa)	65557.7
E/E_{eff}		1.08

Obs	Secant Slope (kN/mm)	Force (kN)	CMOD (mm)	Δa_{eff} (mm)	K_R (MPa·√m)	K_{rate} (MPa·√m/s)	K_{app} (MPa·√m)	r_Y (mm)	σ_{net} (MPa)	$R_v = \sigma_{net}/\sigma_{YS}$	$R_v \leq 1$?
296	609.2	218.7	0.359	0.00	28.8	0.4	28.8	1.24	63.55	0.20	Y
335	606.4	254.0	0.419	1.03	33.7	0.4	33.5	1.68	73.95	0.23	Y
407	599.4	324.0	0.541	2.35	43.2	0.4	42.7	2.74	94.44	0.29	Y
471	590.2	392.5	0.665	4.10	52.9	0.4	51.8	4.05	114.62	0.35	Y
530	581.2	459.3	0.790	5.87	62.4	0.4	60.6	5.56	134.26	0.41	Y
585	571.4	524.7	0.918	7.84	72.0	0.4	69.2	7.27	153.52	0.47	Y
636	560.6	588.0	1.049	10.06	81.6	0.4	77.5	9.16	172.26	0.53	Y
686	547.5	650.0	1.187	12.85	91.5	0.4	85.7	11.27	190.95	0.59	Y
900	532.6	708.2	1.330	16.17	101.3	0.4	93.4	13.52	208.91	0.64	Y
935	513.3	759.1	1.479	20.67	110.9	0.5	100.1	15.88	225.83	0.70	Y
967	495.0	807.1	1.631	25.20	120.6	0.5	106.4	18.33	242.15	0.75	Y
997	474.5	851.6	1.795	30.56	130.5	0.5	112.3	20.97	258.32	0.80	Y
1024	453.3	890.1	1.964	36.49	140.3	0.6	117.4	23.66	273.62	0.84	Y
1049	431.3	923.8	2.142	43.08	150.2	0.6	121.8	26.45	288.49	0.89	Y
1072	409.5	952.6	2.326	50.06	160.0	0.6	125.6	29.29	302.76	0.93	Y
1093	384.1	974.2	2.536	58.86	170.5	0.8	128.5	32.37	317.39	0.98	Y
1111	358.1	987.5	2.757	68.67	181.0	0.9	130.2	35.45	331.49	1.02	N
1125	335.2	993.5	2.964	78.11	190.4	1.0	131.0	38.21	343.84	1.06	N
1137	307.5	986.7	3.209	90.52	200.7	1.2	130.1	41.18	357.18	1.10	N
1147	278.1	966.2	3.474	105.11	211.3	1.8	127.4	44.05	370.88	1.14	N
1152	259.0	945.2	3.650	115.44	218.0	1.6	124.6	45.73	379.79	1.17	N

specimen being tested (Eq 4 or Eq 10). Use values of effective crack size a_e and the force applied to the specimen at that observation point to calculate K_R. Complete the analysis by following the steps starting at 10.4.

10.3 *Data Reduction Procedures for Tests Using Compliance Methods:*

10.3.1 Compliance methods use values of Δv/ΔP to determine crack size using the appropriate compliance expression. The effective modulus E_{eff} is first determined from the initial linear slope of the force-CMOD curve to initialize the calibration curve or compliance expression and to check the experimental setup.

10.3.2 Check for data integrity by inspecting the force-CMOD curve and, if desired, by plotting force and CMOD as functions of time. A sudden drop in force accompanied by a drop in CMOD usually indicates grip slippage. A small amount of slippage will not be detrimental to the test, but large drops in force, especially near maximum force, would put the test results in doubt. A drop in force accompanied by an increase in CMOD indicates pop-in crack extension, or short bursts of unstable crack extension. Large amounts of pop-in crack extension may contribute to variability in K_R curve results or invalidate the interpretation of data.

10.3.3 The test record of force versus CMOD for the compliance method will have an initial linear region that corresponds to the specimen compliance associated with the initial crack size a_o. Fig. 10 shows a schematic diagram of the test record. Compliance construction lines for determining $\Delta v/\Delta P$ at several points on the force versus CMOD curve are also shown.

10.3.4 *Compliance Initialization*—For tests using the compliance method, determine the effective modulus E_{eff} using the following steps.

10.3.4.1 Determine lower and upper force limits to select the initial linear slope of the force-CMOD curve. This initial linear slope can be determined from digital data by first establishing lower and upper limits of force for the linear regression. These limits can be based on visual estimates from an *X-Y* chart, on statistical determination of the "best" linear region, or on theoretical plastic zone sizes (see Notes 10 and 11). With digital data, a linear regression of at least 20 data pairs between those limits is recommended.

NOTE 10—For relatively high-toughness specimens, the shape of the initial portion of the K_R curve is sensitive to the portion of the force-CMOD curve selected as the initial linear region. This is because there is slight curvature at the beginning of the force-CMOD curve due to the growth of the plastic zone as K increases. The K_c value can also be affected by the region selected. To establish a consistent basis that is applicable to a variety of specimens and specimen sizes, the use of lower and upper plastic zone size limits to determine the lower and upper limits of the initial region of the force-CMOD curve has been found to avoid the problems with other methods for determining the initial linear region. The lower and upper plastic zone sizes can be used to determine the force limits between which the linear region is determined. The force limits can be determined by substituting in the lower and upper plastic zone size limits for r_Y in Eq 9 for the M(T) specimen or Eq 16 for the C(T) specimen.

NOTE 11—Lower and upper plastic zone size limits of 0.050 mm (0.002 in.) and 1.25 mm (0.05 in.) have been found to work well with K_R testing of aluminum alloys.

10.3.4.2 Determine the initial elastic slope $(\Delta v/\Delta P)_o$ of the force-CMOD curve by fitting a line to the force-CMOD data between the lower and upper force limits. Determine the CMOD origin v_o, which is the intersection of the initial elastic slope and the CMOD axis. This can be done using linear regression of the digital force-CMOD data or manually from an *X-Y* chart of force-CMOD.

10.3.4.3 Determine the effective modulus E_{eff} from the initial crack size a_o, the initial elastic slope $(\Delta v/\Delta P)_o$, and the appropriate compliance calibration curve or equation. For the M(T) specimen, E_{eff} can be calculated from Eq 5. For the C(T) specimen, E_{eff} can be calculated using the compliance expressions given in Section 11. The effective modulus is the value of E_{eff} that brings the calibration curve into agreement with the initial crack size a_o to within 0.001W.

10.3.4.4 Check that E_{eff} is within 10 % of the material modulus. This provides a check of the experimental setup and initializes the compliance calibration curve. If E_{eff} is not within 10 % of the material modulus, check the specimen dimensions and conversion factors for force and CMOD. Also, if an algorithm is used to search for the best linear region, make sure that the region selected is reasonably low on the force-CMOD curve. If sufficient digital data is collected during the test, overlapping subsets of the force-CMOD curve can be fit by linear regression and plotted as a function of force or CMOD to see if the region selected is appropriate.

10.3.5 *Effective Crack Size Determination from Secant Compliance (see Fig. 6)*—Use the steps in this section if the effective crack size is to be determined from secant compliance data.

10.3.5.1 *Secant Compliance Curve Analysis*—For the secant compliance method, select a series of at least 20 analysis points along the force-CMOD curve beyond the initial linear region. For each analysis point (v_i, P_i), calculate the secant slope from the CMOD origin v_o to each selected point using Eq 3.

$$\left(\frac{\Delta v}{\Delta P}\right)_i = \frac{(v_i - v_o)}{P_i} \tag{3}$$

Use the secant slope, specimen geometry, and effective modulus E_{eff} to calculate an effective crack size a_e at each selected analysis point using the compliance expressions for the M(T) or C(T) specimen (see Note 12) in Section 11.

NOTE 12—Eq 5 is the preferred equation but must be solved for crack size by iteration. Eq 6 and 7 can be used to estimate the normalized crack size to begin the iteration.

10.3.5.2 Calculate K_R, the stress intensity factor based on the effective crack size using the appropriate equation for the specimen being tested (Eq 4 or Eq 10). Use values of effective crack size a_e and the force applied to the specimen at that selected analysis point to calculate K_R.

10.3.5.3 *Plastic Zone Size (r_y) Determination*—To be consistent with the technique of direct crack size measurement, the plastic zone size calculation should be based on the physical crack size for validity determination. However, for the secant compliance method, the physical crack size has to be determined from r_y so iteration is required. An overestimate of r_y can be made by substituting the value of K_R from the previous step for K in Eq 2. Estimate the physical crack size $a_p = a_e$ −

r_y and calculate $K(a_p)$, which is the stress intensity factor based on the physical crack size and using the force for this analysis point. Next, determine an underestimate of r_y by substituting $K(a_p)$ for K in Eq 2. Adjust r_y between these limits until $K(a_p)$ results in the same r_y when substituted in Eq 2.

10.3.5.4 Calculate the physical crack size $a_p = a_e - r_y$. This will be used in the net section stress validity calculation. Complete the analysis by going to 10.4.

10.3.6 *Effective Crack Size Determination from Unloading Compliance*—Use the steps in this section if the physical crack size is to be determined directly from unloading compliance data. Effective crack size is computed by adding the plastic zone size to the physical crack size.

NOTE 13—Determination of compliance by digital data collection and analysis is recommended because of the better accuracy compared to manual methods.

10.3.6.1 *Unloading Compliance*—For the unloading compliance method, select unloading data subsets of the force-CMOD curve at each unload point. For each data subset, calculate the unloading slope of the force-CMOD data by manual methods or by linear regression. The slope represents the unloading compliance $(\Delta v/\Delta P)_{unload}$ (see Fig. 10). Use the unloading compliance, specimen geometry, and effective modulus E_{eff} to calculate a physical crack size a_p at each selected unloading point using the compliance expressions for the M(T) or C(T) specimen (see Note 12) in Section 11.

10.3.6.2 For each point where physical crack size a_p was determined, compute the plastic zone size by calculating $K(a_p)$, the stress intensity factor using the physical crack size a_p and the force just prior to the unload point. Use the expressions for K in Eq 4 for the M(T) specimen or Eq 10 for the C(T) specimen. Substitute $K(a_p)$ for K in Eq 2 along with the yield strength σ_{YS} to determine the plastic zone size r_y.

10.3.6.3 For each unloading compliance point, add the value of r_y to the physical crack size a_p to determine the effective crack size a_e.

10.3.6.4 Calculate K_R at each selected unload point using the appropriate equation for the specimen being tested (Eq 4 or Eq 10) and using values of a_e determined in the previous step and the force applied to the specimen just prior to the unload point.

10.4 Calculate the change in effective crack size Δa_e by subtracting the initial crack size a_o from each a_e value calculated.

10.5 Calculate the net section stress validity criteria R_v for each observation point. For the M(T) specimen, this is the ratio of the net stress (using the physical crack size) to the material yield strength. For the C(T) specimens, this is the ratio of eight times the plastic zone size (based on physical crack size a_p) to remaining ligament length. Use Eq 8 for the M(T) specimen or Eq 15 for the C(T) specimen to calculate R_v. Mark as invalid any data points where $R_v > 1.0$ (see sample data in Table 1.)

10.6 *Plotting the K_R curve*—Plot K_R as a function of Δa_e for the data points meeting the net section validity requirements of the specimen tested. This is the valid portion of the K_R curve in accordance with this method provided the other requirements of this method are met.

NOTE 14—Optionally, values of K_R and Δa_e that are invalid according to the net section stress validity can also be plotted but must be clearly marked as such.

10.7 *Lot Release Testing*—For lot release testing where K_R values need to be determined at specified values of effective crack extension, linear interpolation between adjacent points is acceptable as long as there is at least one (K_R-Δa_e) data pair between each specified crack extension point. For this reason it is recommended that at least 50 points be used to accurately define the K_R curve for a lot release test.

11. Specimen-Specific Equations

11.1 For each specimen geometry covered in this method, the equations and calibration tables for calculating K_R and for determining crack size from compliance measurements are tabulated in this section.

11.2 *Middle-Cracked Tension (M(T)) Specimen:*

11.2.1 The general equation for calculating the stress intensity factor K as a function of the crack size for a given specimen geometry is given by:

$$K = \frac{P}{WB} \cdot \sqrt{\pi a \cdot \sec\left(\frac{\pi a}{W}\right)} = \frac{P}{WB} \cdot \sqrt{\frac{\pi a}{\cos\left(\frac{\pi a}{W}\right)}} \quad (4)$$

where:

P = applied force,
B = specimen thickness,
W = total specimen width, and
a = the crack size; depending on the calculation, this could be the effective crack size a_e or the physical crack size a_p.

11.2.2 The preferred analytical equation for calculating normalized compliance $EB(\Delta v/\Delta P)$ as a function of the M(T) specimen geometry and effective crack size **(15)** is given by:

$$EB\left(\frac{\Delta v}{\Delta P}\right) = \frac{2Y}{W} \cdot \sqrt{\frac{\pi a/W}{\sin(\pi a/W)}} \cdot \left\{ \frac{2W}{\pi Y} \cosh^{-1}\left(\frac{\cosh(\pi Y/W)}{\cos(\pi a/W)}\right) - \frac{1+\nu}{\sqrt{1+\left(\frac{\sin(\pi a/W)}{\sinh(\pi Y/W)}\right)^2}} + \nu \right\} \quad (5)$$

which is valid for $0.2 < 2a/W < 0.8$ and $Y/W \leq 0.5$ and where:

E = the specimen material Young's modulus or the effective modulus E_{eff},
$\Delta v/\Delta P$ = specimen compliance (the ratio of the change in CMOD to the change in force),
B = specimen thickness,
W = total specimen width,
Y = half span of the displacement measurement points,
a = effective crack size a_e for increasing load or physical crack size a_p for unloading, and
ν = the material Poisson's ratio.

11.2.3 The compliance calibration curve given in Eq 5 for a M(T) specimen using near-zero gage span is presented in Fig. 11. Note that the analytical curve shown is for a specific gage Y/W ratio.

11.2.4 An analytical inverse function for estimating the normalized crack size from specimen compliance is given in

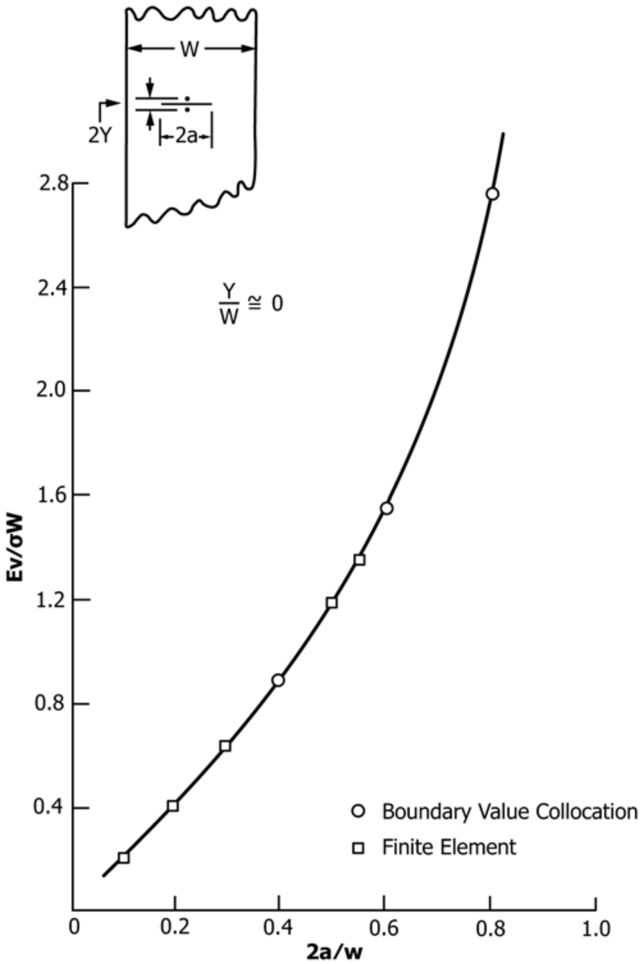


FIG. 11 Compliance Calibration Curve from Eq 5 for a M(T) Specimen with Near Zero Gage Span

Eq 6 and 7. This can be used to estimate an initial guess for iteration of Eq 5 using E_{eff} and the measured specimen compliance. This is a polynomial fit to an inversion of Eq 5.

$$X = 1 - \exp\left[\frac{-\sqrt{[E_{eff}B(\Delta v/\Delta P)]^2 - (2Y/W)^2}}{2.141}\right] \quad (6)$$

$$\frac{2a}{W} = 1.2235X - 0.699032X^2 + 3.25584X^3 - 6.65042X^4 + 5.54X^5 - 1.66989X^6 \quad (7)$$

11.2.5 The following equation is used to calculate the validity ratio for the M(T) specimen at each selected point in the test:

$$R_v = \frac{\sigma_{net}}{\sigma_{YS}} = \frac{P}{\sigma_{YS}\cdot B(W - 2a_p)} \quad (8)$$

where a_p is the physical crack size determined at that point.

11.2.6 The lower and upper force limits for selecting the initial linear region of the force-CMOD curve in an M(T) specimen can be determined by substituting lower and upper plastic zone size limits for r_Y in the following expression:

$$P_{\lim} = \sigma_{YS}\cdot BW\cdot\sqrt{\frac{2}{a_o}\cos\left(\frac{\pi\cdot a_o}{W}\right)}\cdot\sqrt{r_Y} \quad (9)$$

11.3 *Compact Tension (C(T)) Specimen:*

11.3.1 The general equation for calculating the stress intensity factor K as a function of the crack size a for the C(T) specimen geometry (**16**) is given by:

$$K = \frac{P}{B\sqrt{W}}\cdot\frac{\left(2+\frac{a}{W}\right)}{\left(1-\frac{a}{W}\right)^{3/2}}\cdot f\left(\frac{a}{W}\right) \quad (10)$$

where:

$$f\left(\frac{a}{W}\right) = \left[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4\right] \quad (11)$$

which is valid for any $a/W \geq 0.35$ and where:

P = applied force,
B = specimen thickness,
a = crack size; depending on the calculation, this could be the effective crack size a_e or the physical crack size a_p, and
W = specimen width measured from the load line.

11.3.2 The expression for calculating normalized compliance $EB(\Delta v/\Delta P)$ as a function of the C(T) specimen geometry and effective crack size (**17**) is given by:

$$EB\frac{\Delta v}{\Delta P} = A_0 + A_1\left(\frac{a}{W}\right) + A_2\left(\frac{a}{W}\right)^2 + A_3\left(\frac{a}{W}\right)^3 + A_4\left(\frac{a}{W}\right)^4 \quad (12)$$

11.3.2.1 The table below shows the coefficients A to be used in Eq 12 for two displacement measurement locations on the C(T) specimen.

Inverse compliance coefficients for the compact tension specimen for two displacement measurement locations V_0 and V_1 shown in Fig. 8

v measurement location	A_0	A_1	A_2	A_3	A_4
V_0	120.7	-1065.3	4098.0	-6688.0	4450.5
V_1	103.8	-930.4	3610.0	-5930.5	3979.0

Accuracy for EBv/P is ±0.04% over the range of $0.35 \leq a/W \leq 0.60$

11.3.3 The expression for calculating the normalized crack size from the normalized compliance in the C(T) specimen (**18**) is given in Eq 13 and 14.

$$\frac{a}{W} = C_0 + C_1U + C_2U^2 + C_3U^3 + C_4U^4 + C_5U^5 \quad (13)$$

where:

TABLE 2 Variability in K_R at Four Selected Levels of Effective Crack Extension, Δa_e Seven Labs—Triplicate Tests

NOTE 1—The standard deviation has been pooled for all laboratories testing a given alloy. Data on the round robin results are on file at ASTM Headquarters, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, USA 19428-2959. Request RR: E-24-1011.

	K_R values for 2024-T351: σ_{YS} = 330 MPa (48 ksi) in MPa·√m (ksi·√in.)			
Effective Crack Extension, Δa_e	2.5 mm (0.1 in.)	5.1 mm (0.2 in.)	7.6 mm (0.3 in.)	10.2 mm (0.4 in.)
Grand Mean of 21 specimens	47.8 (43.5)	61.9 (56.3)	73.4 (66.8)	81.3 (74.0)
Standard Deviation	2.0 (1.8)	2.0 (1.8)	1.5 (1.4)	1.6 (1.5)
	K_R values for 7475-T7351: σ_{YS} = 405 MPa (59 ksi) in MPa·√m (ksi·√in.)			
Effective Crack Extension, Δa_e	2.5 mm (0.1 in.)	5.1 mm (0.2 in.)	7.6 mm (0.3 in.)	10.2 mm (0.4 in.)
Grand Mean of 20 specimens	52.9 (48.1)	65.9 (60.0)	78.2 (71.2)	85.2 (77.5)
Standard Deviation	3.4 (3.1)	4.2 (3.8)	4.1 (3.7)	4.5 (4.1)

$$U = \frac{1}{1+\sqrt{EB\frac{\Delta v}{\Delta P}}} \quad (14)$$

11.3.3.1 The table below contains the coefficients C to be used in Eq 13 for two displacement measurement locations on the C(T) specimen.

Compliance coefficients for the compact tension specimen for two displacement measurement locations V_0 and V_1 shown in Fig. 8

v measurement location	C_0	C_1	C_2	C_3	C_4	C_5
V_0	1.0010	-4.6695	18.460	-236.82	1214.90	-2143.6
V_1	1.0008	-4.4473	15.400	-180.55	870.92	-1411.3

Accuracy for a/W is ±0.0005% over the range of $0.35 \leq a/W \leq 0.60$

11.3.3.2 Fig. 12 shows a plot of the compliance calibration curve for the C(T) specimen for the two displacement measurement locations.

11.3.4 The following equation is used to calculate the validity ratio for the C(T) specimen at each selected point in the test:

$$R_v = \frac{8 \cdot r_Y}{W - a_p} \quad (15)$$

where a_p is the physical crack size determined at that point.

11.3.5 The lower and upper force limits for selecting the initial linear region of the force-CMOD curve in the C(T) specimen can be determined by substituting lower and upper plastic zone size limits for r_Y in the following expression (see Notes 10 and 11):

$$P_{\lim} = \frac{\sigma_{YS} \cdot B \cdot \sqrt{2\pi \cdot W} \cdot \left(1 - \frac{a_0}{W}\right)^{\frac{3}{2}}}{\left(2 + \frac{a_0}{W}\right) \cdot f\left(\frac{a_0}{W}\right)} \cdot \sqrt{r_Y} \quad (16)$$

where $f(a_o/W)$ is given in Eq 11, and where a_o is the initial crack size and σ_{YS} is the yield strength of the material in the orientation corresponding to the force-application direction of the specimen.

12. Report

12.1 Report the following information:

12.1.1 A plot showing the K_R curve, plotted in terms of effective crack extension Δa_e. Clearly indicate any data that are invalid by the net section stress or the crack deviation requirements,

12.1.2 Type and size of specimen used,

12.1.3 Measured specimen dimensions,

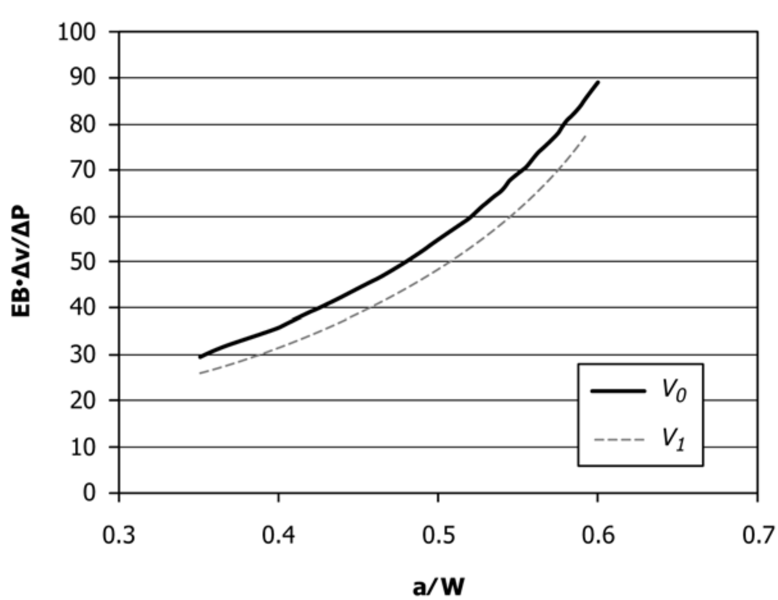


FIG. 12 Compliance Curves for the C(T) Specimen for Two Displacement Measurement Locations V_0 and V_1 shown in Fig. 8

12.1.4 Initial physical crack size a_o,

12.1.5 Crack orientation (see Annex A2 in Terminology E1823 for coding system),

12.1.6 Product form and thickness,

12.1.7 Yield strength,

12.1.8 Material modulus,

12.1.9 Precracking conditions,

12.1.10 Crack measurement technique (direct measurement or single compliance, and whether unloading compliance measurements were used),

12.1.11 Effective modulus, if obtained,

12.1.12 Initial CMOD gage span, if used,

12.1.13 Average K-rate during the initial portion of the test and whether this value meets the requirements of 9.5,

12.1.14 A tabular listing of the K_R and $\Delta\ a_e$ values defining the K_R curve along with the values of r_y and R_v at each point (see sample tabulation of analysis data in Table 1). Note any data points where the physical crack tip is outside the 10° envelope as described in 9.10, and

12.1.15 Test environmental conditions (temperature and humidity).

12.2 The following information can be reported, but is not required:

12.2.1 The CMOD origin v_o,

12.2.2 Force and CMOD data at each selected analysis point,

12.2.3 The rate of change in K_R with respect to time between selected analysis points,

12.2.4 The elapsed time from the start of the test,

12.2.5 The range of data used for the initial linear slope,

12.2.6 The theoretical plastic zone size at the lower and upper ends of the initial linear slope,

12.2.7 Statistical results of the initial linear slope regression,

12.2.8 K_c, which is the K_R value at maximum applied force, and

12.2.9 K_{app}, which is the value of K calculated at maximum applied force, but using the initial crack size a_o instead of the effective crack size a_e.

13. Precision and Bias

13.1 The precision of K_R curve data is a complex synergistic function of the precision and accuracy of the instrumentation used, setup of the test fixtures, and the performance of the test. The latter is a matter of care and skill which cannot be prescribed in a standard method. An example of measurement precision that resulted from interlaboratory testing involving seven laboratories, each testing two materials, is given in Table 2. The two materials represent two levels of uniformity of behavior during stable crack extension; one presenting a slight tendency for crack pop-in. All laboratories participated with the compact, C(T), specimen, but plan-view size and initial crack size were varied as allowed within the scope of this standard.

13.2 A K_R curve is not a single valued quantity, but a series of quantities dependent on crack extension. Hence, K_R curves are not easily analyzed using statistical methods. Bias cannot be evaluated because there exists no reference value by which it is possible to identify a value of K_R at all of the possible levels of the effective crack extension, Δa_e.

14. Keywords

14.1 effective crack extension; fracture mechanics; fracture resistance; fracture toughness; K_R ; K_R curve; linear elastic; plane stress; plastic zone; standard test method; stress intensity factor

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SUMMARY OF CHANGES

Committee E08 has identified the location of selected changes to this standard since the last issue (E561 – 15) that may impact the use of this standard. (Approved December 1, 2015)

(1) Revisions made throughout.

Committee E08 has identified the location of selected changes to this standard since the last issue (E561 – $10^{\varepsilon 2}$) that may impact the use of this standard. (Approved October 15, 2015)

(1) Changed all occurrences of K-R to K_R in the body of the standard.

(2) Revised 8.5.4.