



Standard Test Method for Plastic Strain Ratio r for Sheet Metal¹

This standard is issued under the fixed designation E517; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers special tension testing for the measurement of the plastic strain ratio, r , of sheet metal intended for deep-drawing applications. 0.05.

1.2 The values stated in inch-pound units are to be regarded as standard. The values given in parentheses are mathematical conversions to SI units that are provided for information only and are not considered standard.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*²

E6 Terminology Relating to Methods of Mechanical Testing

E8 Test Methods for Tension Testing of Metallic Materials

E83 Practice for Verification and Classification of Extensometer Systems

E92 Test Method for Vickers Hardness of Metallic Materials (Withdrawn 2010)³

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E691 Practice for Conducting an Interlaboratory Study to Determine the Precision of a Test Method

3. Terminology

3.1 *Definitions of Terms Specific to This Standard:*

3.1.1 *delta r* (Δr)—measure of the tendency of sheet to draw in nonuniformly and to form ears in the flange of deep-drawn cylindrical parts in the directions of higher r value (see 10.4).

¹ This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.02 on Ductility and Formability.

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

³ The last approved version of this historical standard is referenced on www.astm.org.

3.1.1.1 *Discussion*—In cold-reduced and annealed low-carbon steel sheet, r_0 and r_{90} are usually greater than r_{45} , while in hot-rolled steels r_{45} may be greater. Other earing tendencies occur; thus, for some materials the earing tendency may be better represented by $r_{\max} - r_{\min}$.

3.1.2 *plastic-strain ratio* r (in sheet metal that has been strained by uniaxial tension sufficiently to induce plastic flow) is the ratio of the true strain that has occurred in a width direction w perpendicular to the direction of applied stress and in the plane of the sheet, to the concomitant true strain in the thickness direction t . Thus, r is numerically equal to

$$r = \epsilon_w / \epsilon_t \quad (1)$$

where:

ϵ_w = width strain, and

ϵ_t = thickness strain.

3.1.2.1 *Discussion*—Due to difficulty in measuring thickness changes with sufficient precision, in practice an equivalent relationship is commonly used, based on length and width strain measurements (see 9.1.2).

3.1.3 r_m —weighted average of r values obtained in three directions: 0° (parallel), 45° (diagonal), and 90° (transverse) to the rolling direction (see 10.3).

3.1.3.1 *Discussion*—Some materials may show significantly different values of r for other test directions, in which case an average value may include these when special note is made and another subscript is used to avoid confusion with r_m as defined in 3.1.3. Symbols which are often used interchangeably with r_m are \bar{r} and $r\text{-Bar}$.

3.1.4 *yield point elongation* (for a material that has a yield point) is the total strain associated with discontinuous yielding.

3.2 The definitions relating to tension testing appearing in Terminology E6 shall apply to this test method.

4. Significance and Use

4.1 The plastic strain ratio r is a parameter that indicates the ability of a sheet metal to resist thinning or thickening when subjected to either tensile or compressive forces in the plane of the sheet. It is a measure of plastic anisotropy and is related to the preferred crystallographic orientations within a polycrystalline metal. This resistance to thinning or thickening contributes to the forming of shapes, such as cylindrical flat-bottom cups, by the deep-drawing process. The r value, therefore, is considered a measure of sheet metal drawability. It

is particularly useful for evaluating materials intended for parts where a substantial portion of the blank must be drawn from beneath the blank holder into the die opening.

4.2 For many materials this ratio remains essentially constant over a range of plastic strains up to maximum applied force in a tension test. For materials that give different r values at various strain levels, a superscript is used to designate the percent strain at which the r value was measured. For example, if a 20 % elongation is used, the report would show r^{20} .

4.3 Materials usually have different r values when tested in different orientations relative to the rolling direction. The angle of sampling of the individual test coupon is noted by a subscript. Thus, for a test specimen whose length is aligned parallel to the rolling direction, the r value would be reported as r_0 . If, in addition, the measurement was made at 20 % elongation and it was deemed necessary to note the percent strain at which the value was measured, the value would be reported as r_0^{20} .

4.4 A material that has a yield point followed by discontinuous yielding stretches unevenly while this yielding is taking place. In steels, this is associated with the propagation of Lüders' bands on the surface. The accuracy and reproducibility of the determination of r will be reduced unless the test is continued beyond this yield-point elongation. Similarly, the discontinuous yielding associated with large grain size in a material decreases the accuracy and reproducibility of determinations of r made at low strains.

5. Interferences

5.1 Many factors may affect the measurements taken for determining r value. In particular, errors in the measurement of the change in width can cause the reported r value to be invalid. The following phenomena are known to cause severe errors in the measurement of the change in width thus affecting the r value reported.

5.1.1 *Canoeing*—Canoeing is a phenomenon which occurs in some materials when they are stretched. In these materials, the test specimen bows about its longitudinal axis taking on a shape resembling the bottom of a canoe. In this case, unless the measurements of the change in width are compensated for, there will be significant errors in the r value calculated.

5.1.2 *Sharp Knife Edges*—Knife edges, used to measure the change in width automatically, while the specimen is stretched, may cause localized deformation of the specimen under the knife edges. This problem is intensified by the knife edges being sharp and attached to the specimen with high forces. This combination produces a compressive stress 90° to the tensile stress being applied to stretch the specimen, which causes localized deformation. As a result, excessively high r values may be calculated.

6. Apparatus

6.1 Measuring Devices:

6.1.1 Instruments for measuring length and width shall be checked for accuracy and be graduated to permit measurements to be made to ± 0.001 in. (± 0.02 mm) or better.

6.1.2 If the longitudinal strain or the transverse strain, or both, are to be obtained using an extensometer, the extensom-

eter shall conform to Practice E83 as Class C or better. The extensometers shall be verified over a range appropriate for the strains used to determine r value.

6.2 *Testing Machine*—The testing machine used to strain the specimen shall be capable of uniaxially straining the specimen in accordance with the requirements in 9.2.5 or 9.3.4.

7. Test Specimen

7.1 *Size*—The length and width of the specimen are not critical, provided care is used to stretch the gauge section in a uniform manner, avoiding grip effects and anomalous changes along the gauge lengths.

7.1.1 The specimen shall include the full sheet thickness unless otherwise specified.

7.1.2 The thickness of the gauge section of the specimen shall be uniform within 0.0005 in. (0.013 mm) in the gauge section. If the as-received surface is nonuniform, the surface shall be prepared by machining or by grinding to this tolerance.

7.1.3 The distance between a gauge mark and a grip shall be at least twice the width of the reduced section (or gauge width for parallel strips) of the specimen.

7.1.4 Duplicate specimens should be tested and the average r value of these reported for each test direction. If necessary, a third determination may be made, rejecting the extreme.

7.2 *Type*—Any of three types of specimen may be used. Other types including subsize specimens are acceptable provided they give comparable values of equivalent accuracy.

7.2.1 *Specimen A, with reduced section, as shown in Fig. 1*—While this is similar to Fig. 6 of Test Methods E8, the reduced section shall be parallel-sided rather than tapered.

7.2.2 *Specimen B, with a uniform width of 0.75 in. (20 mm), machined edges, and no reduced section, as shown in Fig. 2.*

7.2.3 *Specimen C, precision-sheared a uniform width of 1.125 in. (28.58 mm), or with machined edges and no reduced section, as shown in Fig. 3.*

7.2.3.1 gauge lengths for Specimen C shall be marked on the sheet surface perpendicular to and parallel to the specimen edges. The gauge marks shall be made with Vickers diamond indenters described in Test Method E92, or similar precise marks.

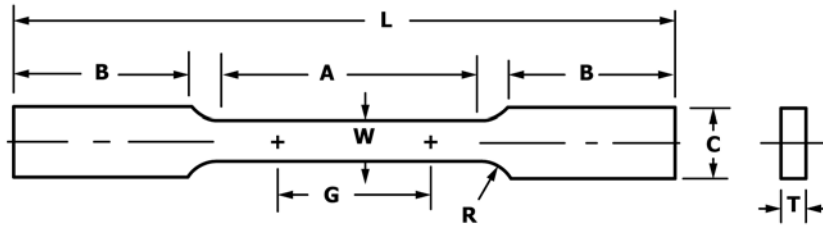
8. Specimen Preparation

8.1 Specimen blanks shall be sheared or sawed individually and with the exception of Specimen C, which may be used as sheared, shall be machined individually or in packs to remove cold-worked edges.

8.2 The dimensions of each specimen shall be measured for uniformity of thickness and width in the gauge section to meet the requirements of 7.1.2 and 8.3.

8.3 Within the gauge length, parallelism of the edges shall be maintained so that no two width measurements differ by more than 0.1 % of the measured width (Specimens A and B only).

8.4 Reasonable care shall be taken to position the gauge marks symmetrically to the midpoint and centerline of the specimen or reduced section.



Dimensions

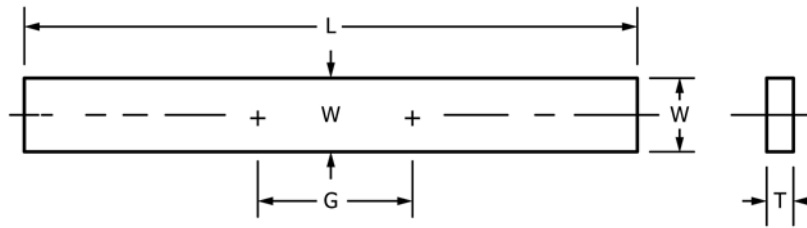
		Specimen A			
		Standard		Alternative	
		in.	mm	in.	mm
G	Gage length	2.00 ± 0.01	50 ± 0.25	1.00 ± 0.005	25 ± 0.13
W	Width (Note 2 and Note 3)	0.500 ± 0.01	12.5 ± 0.25	0.500 ± 0.01	12.5 ± 0.25 [†]
T	Thickness	thickness of material		thickness of material	
R	Radius of fillet, min	1/2	13	1/2	13
L	Overall length, min	8	200	7 1/4	180
A	Length of reduced section, min	3	75	2 1/4	60
B	Length of grip section, min	2	50	2	50
C	Width of grip section, approximate	3/4	20	3/4	20

NOTE 1—The edges of the reduced section shall be machined parallel over the gage length within a tolerance of 0.0005 in. (0.012 mm).

NOTE 2—The ends of the reduced section shall not differ in width by more than 0.005 in. or 0.013 mm. However, the width within the gage length must conform to 8.3.

[†]Editorially corrected in June 2007.

FIG. 1 Rectangular Tension Test Specimens with Reduced Parallel Section, for *r* Determination



Dimensions

		Specimen B			
		Standard		Alternative	
		in.	mm	in.	mm
G	gauge length	2.00 ± 0.01	50 ± 0.25	1.00 ± 0.005	25 ± 0.13
W	gauge width	0.75 ± 0.005	20 ± 0.13	0.75 ± 0.005	20 ± 0.13
T	Thickness	thickness of material		thickness of material	
L	Overall length, min	8	200	7	175
C	Width of specimen (Note)	0.75 ± 0.005	20 ± 0.13	0.75 ± 0.005	20 ± 0.13

NOTE 1—Edges of Specimen B shall be machined parallel over the full length within a tolerance of 0.0008 in. (0.020 mm).

FIG. 2 Machined Rectangular Tension Test Specimens, Parallel Strip, for *r* Determination

8.4.1 Gauge marks shall be lightly scribed or punched in the surface of the specimen or made with a Vickers diamond indenter.

8.4.2 The gauge lengths shall be in compliance with 7.1.3.

8.4.3 For Specimen A, the gauge length shall be centered in the reduced section.

8.4.4 For Specimen C, a double set of gauge marks shall be used in compliance with 7.2.3.1.

9. Procedure

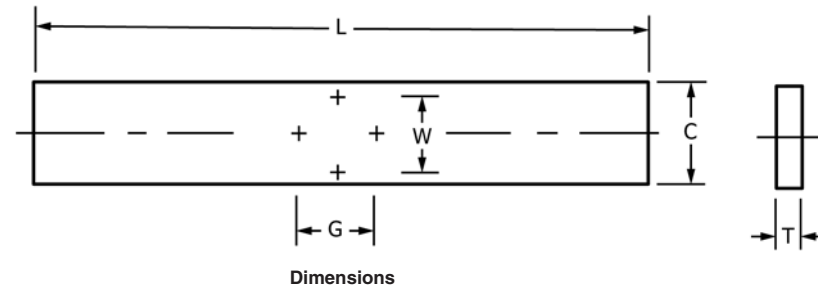
9.1 If the tensile properties of the material are unknown, either make an autographic force/extension record or run a separate tension test to determine the yielding characteristics

and the elongation in accordance with Test Methods E8, using the specimen shown in Fig. 6 of Test Methods E8. This will establish strain limits within which the *r* determination may be made.

9.1.1 The plastic strain ratio *r* may be determined from width and thickness changes resulting from plastic deformation provided these changes can be measured with sufficient accuracy in a tension test.

9.1.2 For most thin sheet metals, however, it is preferable to measure length and width changes and, assuming constant volume, calculate *r* by one of the following procedures:

9.2 Manual Procedure:



		Specimen C	
		in.	mm
G	Gage Length	0.75 ± 0.005	20 ± 0.13
W	Gage Width	0.75 ± 0.005	20 ± 0.13
T	Thickness	thickness of material	
L	Overall Length, min	7	175
C	Width of specimen	1.125 ± 0.125	28.58 ± 3.17

FIG. 3 Sheared Rectangular Tension Test Specimen, Parallel Strip, for r Determination

9.2.1 Determine the original width of the specimen, w_0 , within ± 0.0005 in. (± 0.013 mm). If a gauge length of 0.75 in. (20 mm) is used, as for Specimen C, one width measurement is sufficient. If a gauge length of 1.00 in. (25 mm) or longer is used, make width measurements at a minimum of three evenly spaced places within the gauge length and use the average.

9.2.2 Measure the original gauge length, l_0 , within ± 0.001 in. (± 0.025 mm) in a 1.00-in. (25-mm), or a 0.75-in. (20-mm) gauge section, and within ± 0.002 in. (± 0.05 mm) in a 2.00-in. (50-mm) gauge section.

9.2.3 When gage marks are made with two indenters mounted a known distance apart in a fixture, only final gauge length and width measurements are needed.

9.2.4 Pull the specimen axially until it is stretched beyond any yield-point elongation but not exceeding the strain at maximum applied force. Measurement accuracy is improved as the strain is increased within the above limits, as explained in X3.3.1.

NOTE 1—Strains of 15–20% are commonly utilized for determining the r -value of formable low carbon steel products.

9.2.5 The rate of straining shall be 0.5/min or less, unless otherwise specified.

9.2.6 Measure the final width, w_f , and gauge length, l_f , in the same manner and tolerance as the initial values. Make these measurements with no tensile force applied to the specimen.

9.3 Automatic Procedure:

9.3.1 Attach extensometers conforming to 6.1.2 to the specimen to measure the longitudinal and transverse strain. Transverse extensometers must not cause and must not measure additional transverse deformation due to the knife edges deforming the specimen during the test.

9.3.2 Record the gauge length of the extensometers. In the case of the extensometer measuring transverse strain, the gauge length may be the width of the specimen.

9.3.3 Pull the specimen axially.

9.3.4 The rate of straining shall be 0.5/min or less, unless otherwise specified.

9.3.5 Determine the change in width corresponding to a change in length from the data created by the extensometers, when the specimen is stretched beyond any yield-point elongation, but has not exceeded the strain at maximum applied force. Measurement accuracy is improved as the strain is increased within the above limits, as explained in X1.3.3.1.

NOTE 2—For complete compatibility with the manual method, only the plastic component of the strain values measured should be used in the determination of the r value by the automatic method, unless it can be shown that the elastic component of the total strain is negligible. (The error in calculated r value decreases with increasing strain, higher modulus, lower strength, and at lower r values.) Plastic strains can be determined by reducing the tensile force on the specimen to zero with the extensometers in place, or they can be calculated by subtracting the elastic strains from the total strains indicated by the extensometers. The longitudinal elastic strain at any point may be calculated by dividing the true stress at that point by the nominal value of the modulus of elasticity. The transverse elastic strain may be calculated by multiplying the longitudinal elastic strain by the nominal value of Poisson's ratio. Examples: If an r value of 2.0 at 15 % strain is determined for steel using longitudinal and transverse strain measurements measured at a true stress of 50 000 psi (345 MPa), subtracting the elastic strain component from these measurements would increase the r value calculated from 2.0 to approximately 2.03. If an r value of 1.0 at 10 % strain is determined for aluminum using longitudinal and transverse strain measurements measured at a true stress of 50 000 psi (345 MPa), subtracting the elastic strain component from these measurements would increase the r value calculated from 1.0 to approximately 1.03.

9.3.6 l_o = the gauge length of the longitudinal extensometer.

9.3.7 w_o = the gauge length of the transverse extensometer.

9.3.8 $l_f = l_o +$ the change in length determined in 9.3.5.

9.3.9 $w_f = w_o +$ the change in width determined in 9.3.5.

NOTE 3—The manual procedure shall be used as a referee.

10. Calculation

10.1 Calculate r as follows:

$$r = \varepsilon_w / \varepsilon_l \quad (2)$$

where:

$$\varepsilon_w = \ln(w_f / w_o) \text{ and}$$

$$\varepsilon_l = \ln(l_f / l_o)$$

Assuming the volume remains constant:

$$\varepsilon_t = \ln(l_0 w_0 / l_f w_f) \quad (3)$$

10.1.1 Invert as follows to eliminate negative values:

$$r = \frac{\ln(w_0/w_f)}{\ln(l_f w_f / l_0 w_0)} \quad (4)$$

10.1.2 Nomographs based on ln-ln relationships of original and final gauge lengths and widths may be constructed and used to read r directly.

10.2 Use the r value subscripts and superscripts designating test conditions when necessary (see Section 3).

10.3 Calculate r_m as follows:

$$r_m = (r_0 + 2r_{45} + r_{90})/4 \quad (5)$$

10.4 Calculate earing tendency as follows:

$$\Delta r = (r_0 + r_{90} - 2r_{45})/2 \quad (6)$$

11. Report

11.1 The report shall include the following, as agreed between purchaser and supplier:

- 11.1.1 Description of material tested,
- 11.1.2 Specimen used and sampling direction,
- 11.1.3 Method of width and length (or thickness) determinations,
- 11.1.4 Length strain used,
- 11.1.5 Other special conditions during testing,
- 11.1.6 Brief description of testing machine,
- 11.1.7 Number of determinations and r values (significant to 0.01),
- 11.1.8 Average value, r_m , and
- 11.1.9 Earing tendency, Δr , or $r_{\max} - r_{\min}$.

12. Precision and Bias

12.1 The plastic strain ratio, as shown in Section 4, is a derived quantity. Its precision and bias depend on the precision and bias of the strain measurements.

12.1.1 The use of this test method is not believed to introduce any systematic error or bias into the results.

12.1.2 The precision of the r value measurement shall be expressed in terms of its coefficient of variation, $v(r)$, that is, the ratio of the measurement standard deviation, $s(r)$, to the mean observed value, $v(r) = s(r)/r$.

12.2 From basic statistical considerations, as shown in the Appendix, $v(r)$ can be written in terms of the coefficients of variation of the width strain, $v(\varepsilon_w)$, and the length strain, $v(\varepsilon_l)$, as:

$$v(r) = (1+r)[(v(\varepsilon_w))^2 + (v(\varepsilon_l))^2]^{1/2} \quad (7)$$

12.2.1 The required precision of the dimensional measurements leads to approximate coefficients of variation of 0.01 and 0.025 for the length and width strains, respectively, assuming $\varepsilon_l = 0.1$ and $r = 1$. Exact values can be calculated from the

replicate measurements of initial and final dimensions following standard statistical procedures.

12.2.2 These estimates of $v(\varepsilon_l)$ and $v(\varepsilon_w)$ lead to a calculated $v(r)$, for the assumed value of $r = 1$, of:

$$v(r) = 2[(0.025)^2 + (0.01)^2]^{1/2} = 0.054 \quad (8)$$

12.2.3 For most “typical” conditions under which r would be measured, that is, for $0.5 < r < 2$ and $0.1 < \varepsilon_l < 0.2$, a $v(r)$ of about 0.03 to 0.08 would be anticipated.

12.3 The definition of r leads to a complex dependence of $v(r)$ on both the applied axial strain, ε_l , and the r value itself. This has been treated in more detail in the Appendix using error propagation theory. This approximate treatment shows the importance of small coefficients of variation of the dimensional measurements (0.1 to 0.2 %) and measurements of r at as high a plastic strain within the uniform strain range as is feasible for the material under investigation.

12.4 *Interlaboratory Test Program*—An interlaboratory study of the r -value of steel was run in 1995. Eight laboratories tested five randomly drawn test specimens from each of two materials. The design of the experiment, similar to that of Practice E691, and a within-between analysis of the data is given in the research report.⁴

12.4.1 *Test Result*—The precision information given below for r -value measured at 15 % elongation is for the comparison of two test results, each of which is the average of five test determinations.

12.4.2 *Precision*—Expressed as a percentage of the average test value:

Manual Procedure	Material A	Material B
Average test value	1.06	1.71
95 % repeatability limit (within laboratory)	5.8 %	7.5 %
95 % reproducibility limit (between laboratories)	8.4 %	10.0 %
Automatic Procedure	Material A	Material B
Average test value	1.14	1.77
95 % repeatability limit (within laboratory)	4.1 %	5.0 %
95 % reproducibility limit (between laboratories)	19.2 %	25.3 %

The terms (repeatability limit and reproducibility limit) are used as specified in Practice E177. The respective standard deviations among test results may be obtained by dividing these limits by 2.8.

12.4.3 *Bias*—No information can be presented on the bias of the procedures in Test Method E517 for measuring r -value because no material having an accepted reference value is available.

13. Keywords

13.1 drawability; earing tendency; plastic strain ratio; r ; r value

⁴ Supporting data have been filed at ASTM International Headquarters and may be obtained by requesting Research Report RR:E28-1016.

APPENDIX
(Nonmandatory Information)
X1. THEORETICAL ESTIMATE OF THE PRECISION OF THE r VALUE
X1.1 Definitions

X1.1.1 From 10.1, an alternative definition of the r value is:

$$r = -\varepsilon_w / (\varepsilon_w + \varepsilon_l) \quad (\text{X1.1})$$

X1.1.2 From the definition of r , it also follows that the plastic width and length strains are related by the expression:

$$-\varepsilon_w = \left(\frac{r}{1+r} \right) \varepsilon_l \quad (\text{X1.2})$$

X1.2 Error Analysis

X1.2.1 The standard deviation of r , $s(r)$, can be approximated from Eq. X1.1, neglecting the covariance of the width and length strains for convenience, as follows:

$$s(r) = \left\{ s(\varepsilon_w)^2 \left[\frac{\partial r}{\partial \varepsilon_w} \right]^2 + s(\varepsilon_l)^2 \left[\frac{\partial r}{\partial \varepsilon_l} \right]^2 \right\}^{1/2} \quad (\text{X1.3})$$

where $s(\varepsilon_w)$ and $s(\varepsilon_l)$ are the standard deviations of the width and length strains. In addition, from the alternative definitions in X1.1.2,

$$\frac{\partial r}{\partial \varepsilon_w} = \frac{r(1+r)}{\varepsilon_w} \quad \text{and} \quad \frac{\partial r}{\partial \varepsilon_l} = \frac{-r(1+r)}{\varepsilon_l} \quad (\text{X1.4})$$

X1.2.2 This leads to the result:

$$\frac{s(r)^2}{r^2} = (1+r)^2 \left[\frac{s(\varepsilon_w)^2}{\varepsilon_w^2} + \frac{s(\varepsilon_l)^2}{\varepsilon_l^2} \right] \quad (\text{X1.5})$$

which can also be restated in terms of the coefficients of variation, v , as:

$$v(r) = (1+r) [v(\varepsilon_l)^2 + v(\varepsilon_w)^2]^{1/2} \quad (\text{X1.6})$$

which is the form used in the precision and bias statement.

X1.3 Application of the Error Analysis

X1.3.1 The coefficients of variation of the strain measurements can then be expressed in terms of the coefficients of variation of the specimen dimensions. This format shows the importance of precise dimensional measurements.

X1.3.1.1 The standard deviation of the width strain is:

$$s(\varepsilon_w) = v(W_o) [1 + \exp(-2\varepsilon_w)]^{1/2} \quad (\text{X1.7})$$

where $v(W_o)$ is the coefficient of variation of the original width measurement. This follows from the same basic error propagation analysis used above. Thus,

$$\varepsilon_w = \ln(W_i/W_o) = \ln W_i - \ln W_o \quad (\text{X1.8})$$

$$s(\varepsilon_w)^2 = s(W)^2 \left(\frac{\partial \varepsilon_w}{\partial W} \right)^2 + s(W_o)^2 \left(\frac{\partial \varepsilon_w}{\partial W_o} \right)^2$$

$$\text{but} \quad \frac{\partial \varepsilon_w}{\partial W} = \frac{1}{W} \quad \text{and} \quad \frac{\partial \varepsilon_w}{\partial W_o} = \frac{-1}{W_o}$$

$$\text{so} \quad s(\varepsilon_w)^2 = s(W)^2 \left[\frac{1}{W_i^2} + \frac{1}{W_o^2} \right] \quad \text{if} \quad s(W) = s(W_o)$$

$$\text{and then} \quad s(\varepsilon_w)^2 = \frac{s(W)^2}{W_o^2} \left[\left(\frac{W_o}{W} \right)^2 + 1 \right] = \frac{s(W)^2}{W_o^2} [\exp(-2\varepsilon_w) + 1]$$

X1.3.1.2 Using the same approach, the standard deviation of the length strain in terms of the true axial strain level, ε_l , and the coefficient of variation of the original length measurement, $v(l_o)$, is:

$$s(\varepsilon_l) = v(l_o) [1 + \exp(-2\varepsilon_l)]^{1/2} \quad (\text{X1.9})$$

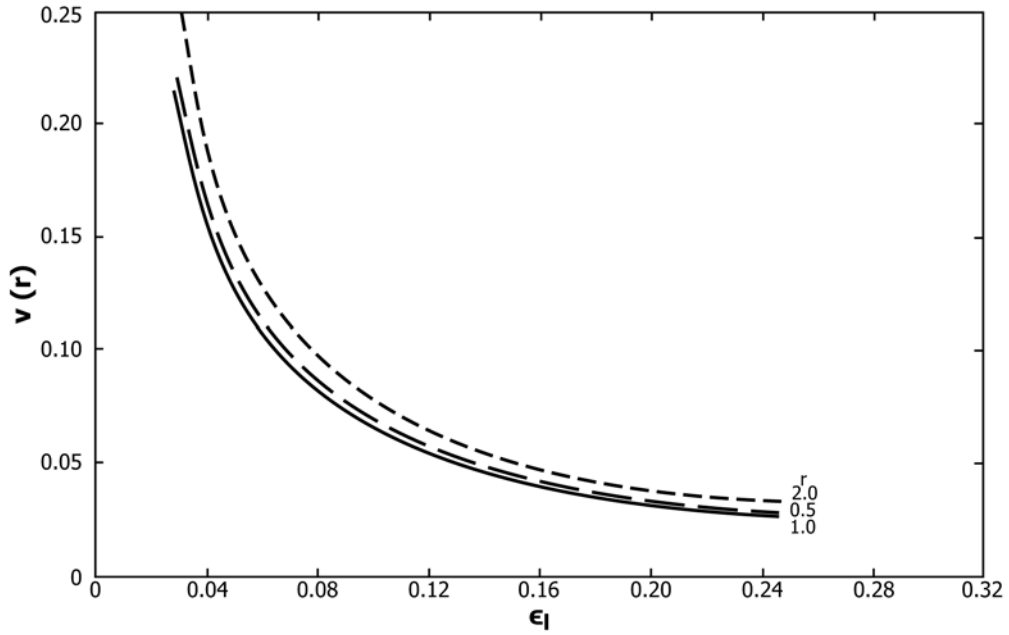
X1.3.2 The coefficient of variation of r , $v(r)$, is then expressed in terms of the relative dimensional measurement uncertainties from Eqs X5 –X7:

$$v(r) = \frac{1+r}{\varepsilon_l} \left\{ v(W_o)^2 \left(\frac{1+r}{r} \right)^2 \cdot [1 + \exp(-2\varepsilon_w)] + v(l_o)^2 [1 + \exp(-2\varepsilon_l)] \right\}^{1/2} \quad (\text{X1.10})$$

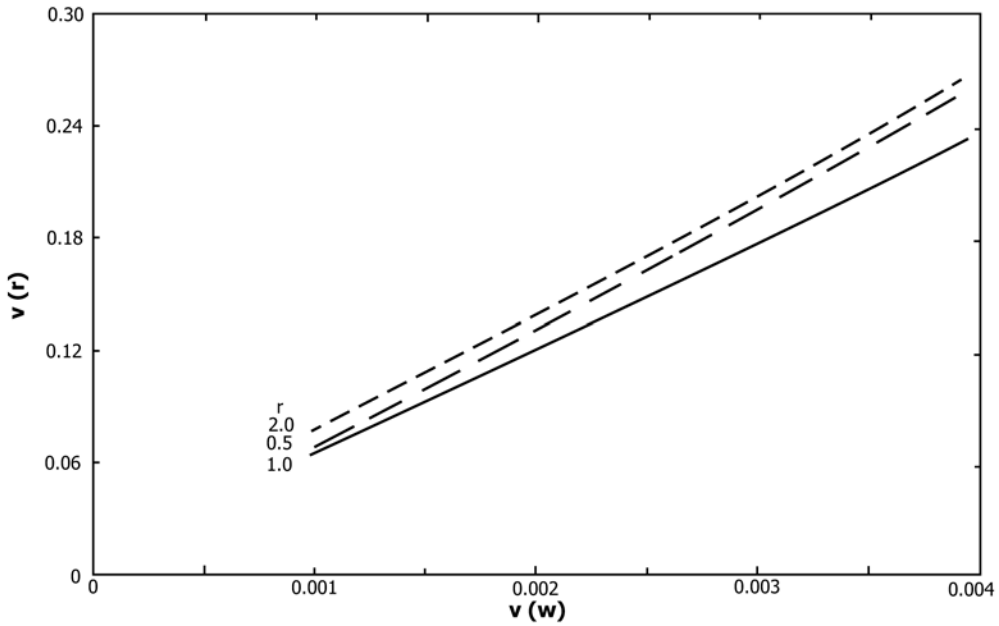
X1.3.3 (Equation X8) can be plotted as shown in Fig. X1.1 and Fig. X1.2 to illustrate the effects of the precision of the axial strain level and the width strain on the r value.

X1.3.3.1 The beneficial effect of increased strain in reducing the coefficient of variation of r is shown in Fig. X1.1. The analysis suggests that measurements at engineering strain levels above 15 % are to be preferred for the highest precision in the determination of r . Understandably, many materials cannot be tested at this strain level; r values can, of course, be measured in materials with low tensile ductility, but such values could be much less precise than r values for more ductile materials.

X1.3.3.2 Increasing the coefficient of variation of the width leads to an increased coefficient of variation of r , as seen in Fig. X1.2. The recommended width measurement tolerances lead to a coefficient of variation of width of about 0.001 (0.1 %) which leads to an estimated coefficient of variation of r of around 4 to 7 %. This further underlines the need for precise dimensional measurements stated in Section 9.



NOTE 1—Calculated for relative length and width measurement errors of 0.001.
FIG. X1.1 Effect of the Axial Strain Level on the Precision of the r Value



NOTE 1—Calculated for an axial strain of 0.10 and a relative length measurement error of 0.001.

NOTE 2—The value of $v(w)$ using the recommended width measurement tolerances in this method is approximately 0.001; at axial strain levels above 0.15 (see Fig. X1.1), the $v(r)$ would be approximately

FIG. X1.2 Effect of Width Measurement Error on the Precision of the r Value

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