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Standard Practice for Process Capability and Performance Measurement[1]

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1. Scope

1.1 This practice provides guidance for determining process capability and performance under several common scenarios of use including: (*a*) normal distribution based capability and performance indices such as C_p, C_{pk}, P_p, and P_{pk}; (*b*) process capability using attribute data for non-conforming units and non-conformities per unit type variables, and (*c*) additional methods in working with process capability or performance.

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics

E2334 Practice for Setting an Upper Confidence Bound For a Fraction or Number of Non-Conforming items, or a Rate of Occurrence for Non-conformities, Using Attribute Data, When There is a Zero Response in the Sample

2.2 *Other Document:*

MNL 7 Manual on Presentation of Data and Control Chart Analysis[3]

3. Terminology

3.1 *Definitions*—Unless otherwise noted, all statistical terms are defined in Terminology E456.

3.1.1 *long term standard deviation,* σ_{LT}*, n*—sample standard deviation of all individual (observed) values taken over a long period of time.

3.1.1.1 *Discussion*—A long period of time may be defined as shifts, weeks, or months, etc.

3.1.2 *process capability, PC, n*—statistical estimate of the outcome of a characteristic from a process that has been demonstrated to be in a state of statistical control.

3.1.3 *process capability index,* C_p*, n*—an index describing process capability in relation to specified tolerance.

3.1.4 *process performance, PP, n*—statistical measure of the outcome of a characteristic from a process that may not have been demonstrated to be in a state of statistical control.

3.1.5 *process performance index,* P_p*, n*—index describing process performance in relation to specified tolerance.

3.1.6 *short term standard deviation,* σ_{ST}*, n*—the inherent variation present when a process is operating in a state of statistical control, expressed in terms of standard deviation.

3.1.6.1 *Discussion*—This may also be stated as the inherent process variation.

3.1.7 *stable process, n*—process in a state of statistical control; process condition when all special causes of variation have been removed.

3.1.7.1 *Discussion*—Observed variation can then be attributed to random (common) causes. Such a process will generally behave as though the results are simple random samples from the same population.

3.1.7.2 *Discussion*—This state does not imply that the random variation is large or small, within or outside of specification, but rather that the variation is predictable using statistical techniques.

3.1.7.3 *Discussion*—The process capability of a stable process is usually improved by fundamental changes that reduce or remove some of the random causes present or adjusting the mean towards the preferred value, or both.

3.1.7.4 *Discussion*—Continual adjustment of a stable process will increase variation.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *lower process capability index,* C_{pkl}*, n*—index describing process capability in relation to the lower specification limit.

3.2.2 *lower process performance index,* P_{pkl}*, n*—index describing process performance in relation to the lower specification limit.

3.2.3 *minimum process capability index,* C_{pk}*, n*—smaller of the upper process capability index and the lower process capability index.

3.2.4 *minimum process performance index,* P_{pk}*, n*—smaller of the upper process performance index and the lower process performance index.

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[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.30 on Statistical Quality Control.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from ASTM Headquarters, 100 Barr Harbor Drive, W. Conshohocken, PA 19428.

3.2.5 *special cause, n*—variation in a process coming from source(s) outside that which may be expected due to chance causes (or random causes).

3.2.5.1 *Discussion*—Sometimes "special cause" is taken to be synonymous with "assignable cause." However, a distinction should be recognized. A special cause is assignable only when it is specifically identified. Also, a common cause may be assignable.

3.2.5.2 *Discussion*—A special cause arises because of specific circumstances which are not always present. As such, in a process subject to special causes, the magnitude of the variation from time to time is unpredictable.

3.2.6 *upper process capability index, C_{pku}, n*—index describing process capability in relation to the upper specification limit.

3.2.7 *upper process performance index, P_{pku}, n*—index describing process performance in relation to the upper specification limit.

4. Significance and Use

4.1 *Process Capability*—Process capability can be defined as the natural or inherent behavior of a stable process that is in a state of statistical control (**1**).[4] A "state of statistical control" is achieved when the process exhibits no detectable patterns or trends, such that the variation seen in the data is believed to be random and inherent to the process. Process capability is linked to the use of control charts and the state of statistical control. A process must be studied to evaluate its state of control before evaluating process capability.

4.2 *Process Control*—There are many ways to implement control charts, but the most popular choice is to achieve a state of statistical control for the process under study. Special causes are identified by a set of rules based on probability theory. The process is investigated whenever the chart signals the occurrence of special causes. Taking appropriate actions to eliminate identified special causes and preventing their reappearance will ultimately obtain a state of statistical control. In this state, a minimum level of variation may be reached, which is referred to as common cause or inherent variation. For the purpose of this standard, this variation is a measure of the uniformity of process output, typically a product characteristic.

4.3 *Process Capability Indices*—The behavior of a process (as related to inherent variability) in the state of statistical control is used to describe its capability. To compare a process with customer requirements (or specifications), it is common practice to think of capability in terms of the proportion of the process output that is within product specifications or tolerances. The metric of this proportion is the percentage of the process spread used up by the specification. This comparison becomes the essence of all process capability measures. The manner in which these measures are calculated defines the different types of capability indices and their use. Two process capability indices are defined in 5.2 and 5.3. In practice, these indices are used to drive process improvement through continuous improvement efforts. These indices may be used to identify the need for management actions required to reduce common cause variation, compare products from different sources, and to compare processes.

4.4 *Process Performance Indices*—When a process is not in a state of statistical control, the process is subject to special cause variation, which can manifest itself in various ways on the process variability. Special causes can give rise to changes in the short-term variability of the process or can cause long-term shifts or drifts of the process mean. Special causes can also create transient shifts or spikes in the process mean. Even in such cases, there may be a need to assess the long-term variability of the process against customer specifications using process performance indices, which are defined in 6.2 and 6.3. These indices are similar to those for capability indices and differ only in the estimate of variability used in the calculation. This estimated variability includes additional components of variation due to special causes. Since process performance indices have additional components of variation, process performance usually has a wider spread than the process capability spread. These measures are useful in determining the role of measurement and sampling variability when compared to product uniformity.

4.5 Attribute capability applications occur where attribute data are being used to assess a process and may involve the use of non-conforming units or non-conformities per unit.

4.6 Additional measures and methodology to process assessments include the index C_{pm}, which incorporates a target parameter for variable data, and the calculation of Rolled Throughput Yield (RTY), that measures how good a series of process steps are.

5. Process Capability Analysis

5.1 It is common practice to define process behavior in terms of its variability. Process capability, PC, is calculated as:

$$PC = 6\sigma_{ST} \tag{1}$$

where σ_{ST} is the inherent variability of a controlled process (**2, 3**). Since control charts can be used to achieve and verify control for many different types of processes, the assumption of a normal distribution is not necessary to affect control, but complete control is required to establish the capability of a process (**2**). Thus, what is required is a process in control with respect to its measures of location and spread. Once this is achieved, the inherent variability of the process can be estimated from the control charts. The estimate obtained is an estimate of variability over a short time interval (minutes, hours, or a few batches). From control charts, σ_{ST} may be estimated from the short-term variation within subgroups depending on the type of control chart deployed, for example, average-range ($\bar{X} - R$) or individual-moving range ($\bar{X} - MR$). The estimate is:

$$\hat{\sigma}_{ST} = \frac{\bar{R}}{d_2} \text{ or } \frac{\overline{MR}}{d_2} \tag{2}$$

where, $\bar{R}$ is the average range, $\overline{MR}$ is the average moving range, d_2 is a factor dependent on the subgroup size, n, of the

[4] The boldface numbers in parentheses refer to the list of references at the end of this standard.

control chart (see ASTM MNL 7, Part 3). If an average-standard deviation ($\bar{X} - s$) chart is used, the estimate becomes:

$$\hat{\sigma}_{ST} = \frac{\bar{s}}{c_4} \quad (3)$$

where $\bar{s}$ is the arithmetic average of the sample standard deviations, and c_4 is a factor dependent on the subgroup size, n, of the control chart (see ASTM MNL 7, Part 3).

5.1.1 Therefore, PC is estimated by:

$$6\,\hat{\sigma}_{ST} = \frac{6\bar{R}}{d_2} \text{ or } \frac{6\bar{s}}{c_4} \quad (4)$$

5.2 *Process Capability Index, C_P:*

5.2.1 The process capability index relates the process capability to the customer's specification tolerance. The process capability index, C_p, is:

$$C_p = \frac{\text{Specification Tolerance}}{\text{Process Capability}} = \frac{USL - LSL}{6\sigma_{ST}} \quad (5)$$

where USL = upper specification limit and LSL = lower specification limit. For a process that is centered with an underlying normal distribution, Fig. 1, Fig. 2, and Fig. 3 denotes three cases where PC, the process capability, is wider than (Fig. 1), equal to (Fig. 2), and narrower than (Fig. 3) the specification tolerance.

5.2.2 Since the tail area of the distribution beyond specification limits measures the proportion of product defectives, a larger value of C_p is better. The relationship between C_p and the percent defective product produced by a centered process (with a normal distribution) is:

C_p	Percent Defective	Parts per Million	C_p	Percent Defective	Parts per Million
0.6	7.19	71900	1.1	0.0967	967
0.7	3.57	35700	1.2	0.0320	318
0.8	1.64	16400	1.3	0.0096	96
0.9	0.69	6900	1.33	0.00636	64
1.0	0.27	2700	1.67	0.00006	0.57

5.2.3 From these examples, one can see that any process with a $C_p < 1$ is not as capable of meeting customer requirements (as indicated by % defectives) as a process with values of $C_p \geq 1$. Values of C_p progressively greater than 1 indicate more capable processes. The current focus of modern quality is on process improvement with a goal of increasing product uniformity about a target. The implementation of this focus is to create processes with $C_p > 1$. Some industries consider $C_p = 1.33$ (an $8\sigma_{ST}$ specification tolerance) a minimum with a $C_p = 1.66$ preferred (4). Improvement of C_p should depend on a company's quality focus, marketing plan, and their competitor's achievements, etc.

5.3 *Process Capability Indices Adjusted For Process Shift, C_{pk}:*

5.3.1 The above examples depict process capability for a process centered within its specification tolerance. Process centering is not a requirement since process capability is independent of any specifications that may be applied to it. The amount of shift present in a process depends on how far the process average is from the center of the specification spread. In the last part of the above example ($C_p > 1$), suppose that the process is actually centered above the USL. The C_p has a value >1, but clearly this process is not producing as much conforming product as it would have if it were centered on target.

5.3.2 For those cases where the process is not centered, deliberately run off-center for economic reasons, or only a single specification limit is involved, C_p is not the appropriate process capability index. For these situations, the C_{pk} index is used. C_{pk} is a process capability index that considers the process average against a single or double-sided specification limit. It measures whether the process is capable of meeting the customer's requirements by considering:

5.3.2.1 The specification limit(s),

5.3.2.2 The current process average, and

5.3.2.3 The current $\hat{\sigma}_{ST}$.

5.3.3 Under the assumption of normality,[5] C_{pk} is calculated as:

$$C_{pk} = \min[C_{pku}, C_{pkl}] \quad (6)$$

and is estimated by:

$$\hat{C}_{pk} = \min[\hat{C}_{pku}, \hat{C}_{pkl}] \quad (7)$$

where the estimated upper process capability index is defined as:

$$\hat{C}_{pku} = \frac{USL - \bar{X}}{3\,\hat{\sigma}_{ST}} \quad (8)$$

and the estimated lower process capability index is defined as:

[5] Testing for the normality of a set of data may range from simply plotting the data on a normal probability plot (2) to more formal tests, for example, Anderson-Darling test (which can be found in many statistical software programs, for example, Minitab).

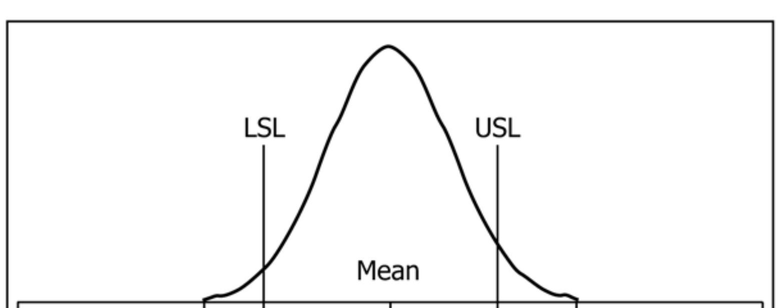

FIG. 1 Process Capability Wider Than Specifications, $C_p < 1$

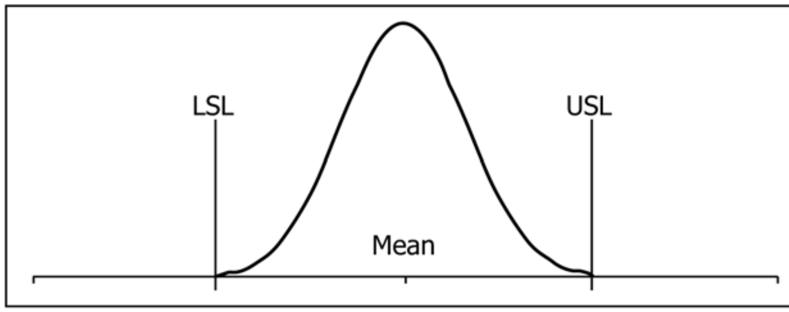

FIG. 2 Process Capability Equal to Specification Tolerance, $C_p = 1$

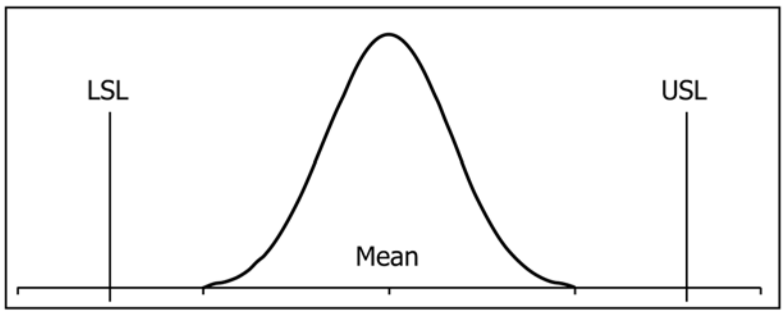

FIG. 3 Process Capability Narrower Than Specifications, $C_p > 1$

$$\hat{C}_{pkl} = \frac{\bar{X} - LSL}{3\,\hat{\sigma}_{ST}} \tag{9}$$

5.3.4 These one-sided process capability indices (C_{pku} and C_{pkl}) are useful in their own right with regard to single-sided specification limits. Examples of this type of use would apply to impurities, by-products, bursting strength of bottles, etc. Once again, the meaning of C_{pk} is best viewed pictorially in Fig. 4.

5.3.5 The relationship between C_p and C_{pk} can be summarized (**2**) as:

5.3.5.1 C_{pk} can be equal to but never larger than C_p,

5.3.5.2 C_p and C_{pk} are equal only when the process is centered on target,

5.3.5.3 If C_p is larger than C_{pk}, then the process is not centered on target,

5.3.5.4 If both C_p and C_{pk} are >1, the process is capable and performing within the specifications,

5.3.5.5 If both C_p and C_{pk} are <1, the process is not capable and not performing within the specifications, and

5.3.5.6 If C_p is >1 and C_{pk} is <1, the process is capable, but not centered and not performing within the specifications.

5.4 *Caveats on the Practical Use of Process Capability Indices:*

5.4.1 One must keep the theoretical aspects and assumptions underlying the use of process capability indices in mind when calculating and interpreting the corresponding values of these indices. To review:

5.4.1.1 For interpretability, C_{pk} requires a Gaussian (normal or bell-shaped) distribution or one that can be transformed to a normal. Definition of C_{pk} requires a normal distribution with a spread of three standard deviations on either side of the mean (**2**, **5**).

5.4.1.2 The process must be in a state of statistical control (stable over time with constant short-term variability).

5.4.1.3 Large sample sizes (preferably >200 or a minimum of 100) are required to estimate C_{pk} with a high level of confidence (at least 95 %).

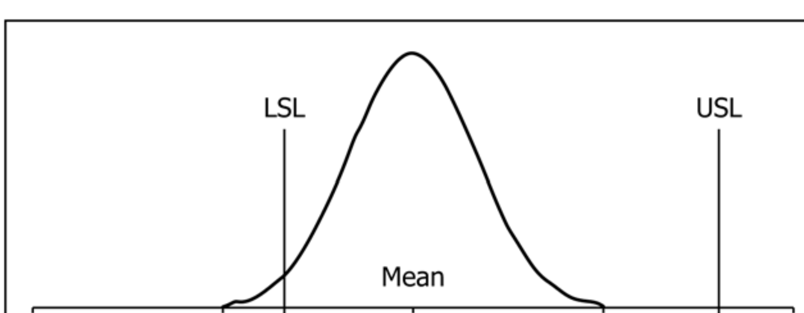

FIG. 4 Noncentered Process, $C_p > 1$ and $C_{pk} < 1$

5.4.1.4 C_p and C_{pk} are affected by sampling procedures, sampling error, and measurement variability. These effects have a direct bearing on the magnitude of the estimate for inherent process variability, the main component in estimating these indices.

5.4.1.5 C_p and C_{pk} are statistics and as such are subject to uncertainty (variability) as found in any statistic.

5.4.2 For additional information about process capability and process capability indices, see Refs (**2**, **5**, **6**).

6. Process Performance Analysis

6.1 *Process Performance:*

6.1.1 Process performance represents the actual distribution of product and measurement variability over a long period of time, such as weeks or months. In process performance, the actual performance level of the process is estimated rather than its capability when it is in control.

6.1.2 As in the case of process capability, it is important to estimate correctly the process variability. For process performance, the long-term variation, σ_{LT}, (**2**, **3**) is estimated. Thus, the accumulated individual production measurements from a process over a long time period, X_1, X_2, …, X_n, has an overall sample standard deviation estimated as:

$$\hat{\sigma}_{LT} = \sqrt{\frac{\Sigma\left(X_i - \bar{X}\right)^2}{n-1}} \tag{10}$$

6.1.3 This standard deviation contains the following "components" of variability: (**6**)

6.1.3.1 Lot-to-lot variability over the long term,

6.1.3.2 Within-lot variability over the short term,

6.1.3.3 Measurement system variability over the long term, and

6.1.3.4 Measurement system variability over the short term.

6.1.4 If the process were in the state of statistical control, one would expect the estimate of σ_{LT}, $\hat{\sigma}_{LT}$, to be very close to the estimate of σ_{ST}, $\hat{\sigma}_{ST}$. One would expect that the two estimates would be almost identical if a perfect state of control were achieved. According to Ott, Schilling, and Neubauer (**2**) and Gunter (**5**), this perfect state of control is unrealistic since control charts may not detect small changes in a process. Such changes give rise to values of $\hat{\sigma}_{LT}$ that are nearly equal but slightly larger than $\hat{\sigma}_{ST}$.

6.1.5 Process performance or process spread is:

$$PP = 6\sigma_{LT} \tag{11}$$

6.2 *Process Performance Index:*

6.2.1 Comparisons of process performance to specification spread result in performance indices that are analogous to process capability indices. The simplest process performance index is P_p, where:

$$P_p = \frac{\text{Specification Tolerance}}{\text{Process Performance}} \tag{12}$$

and is estimated by:

$$\frac{USL - LSL}{6\,\hat{\sigma}_{LT}}$$

6.2.2 The interpretation of P_p is similar to that of C_p. That is, a $P_p \geq 1$ represents a process that has no trouble meeting customer requirements in the long term. A process with $P_p < 1$ cannot meet specifications all the time. In either case, there is no assumption that the process is in the state of statistical control or centered.

6.3 *Process Performance Indices Adjusted For Process Shift:*

6.3.1 For those cases where the process is not centered, deliberately run off-center for economic reasons, or only a single specification limit is involved, P_{pk} is the appropriate process performance index. P_{pk} is a process performance index adjusted for location (process average). It measures whether the process is actually meeting the customer's requirements by considering:

6.3.1.1 The specification limit(s),

6.3.1.2 The current process average, and

6.3.1.3 The current value of $\hat{\sigma}_{LT}$.

6.3.2 Under the assumption of normality, P_{pk} is calculated as:

$$P_{pk} = \min\left[P_{pku}, P_{pkl}\right] \tag{13}$$

and is estimated by:

$$\hat{P}_{pk} = \min\left[\hat{P}_{pku}, \hat{P}_{pkl}\right] \tag{14}$$

where:

$$\hat{P}_{pku} = \frac{USL - \bar{X}}{3\,\hat{\sigma}_{LT}} \tag{15}$$

and

$$\hat{P}_{pkl} = \frac{\bar{X} - LSL}{3\,\hat{\sigma}_{LT}} \tag{16}$$

which are the estimates of the one-sided process performance indices.

6.3.3 Values of P_{pk} have an interpretation similar to those for C_{pk}. The difference is that P_{pk} represents how the process is running with respect to customer requirements over a specified long time period. One interpretation is that P_{pk} represents what the producer *makes* and C_{pk} represents what the producer *could make* if its process were in a state of statistical control. The relationship between P_p and P_{pk} are also similar to that of C_p and C_{pk}.

6.4 *Interpretation of Process Performance Indices:*

6.4.1 The caveats around process performance indices are similar to those for capability indices. Of course, two obvious differences pertain to the lack of statistical control and the use of long-term variability estimates.

7. Confidence Bounds for Process Capability Indices

7.1 Capability indices are based on sample statistics and should not be considered as absolute measures of process capability or performance. All of the indices discussed in this standard are based on sample estimates, and are therefore subject to sampling error. The sampling error will be a function of the sample size, n. Generally, the larger the sample size, the more accurate will be the sample estimates $\hat{C}_p$, $\hat{C}_{pk}$, $\hat{P}_p$, or $\hat{P}_{pk}$. It is recommended that some measure of the sampling error be

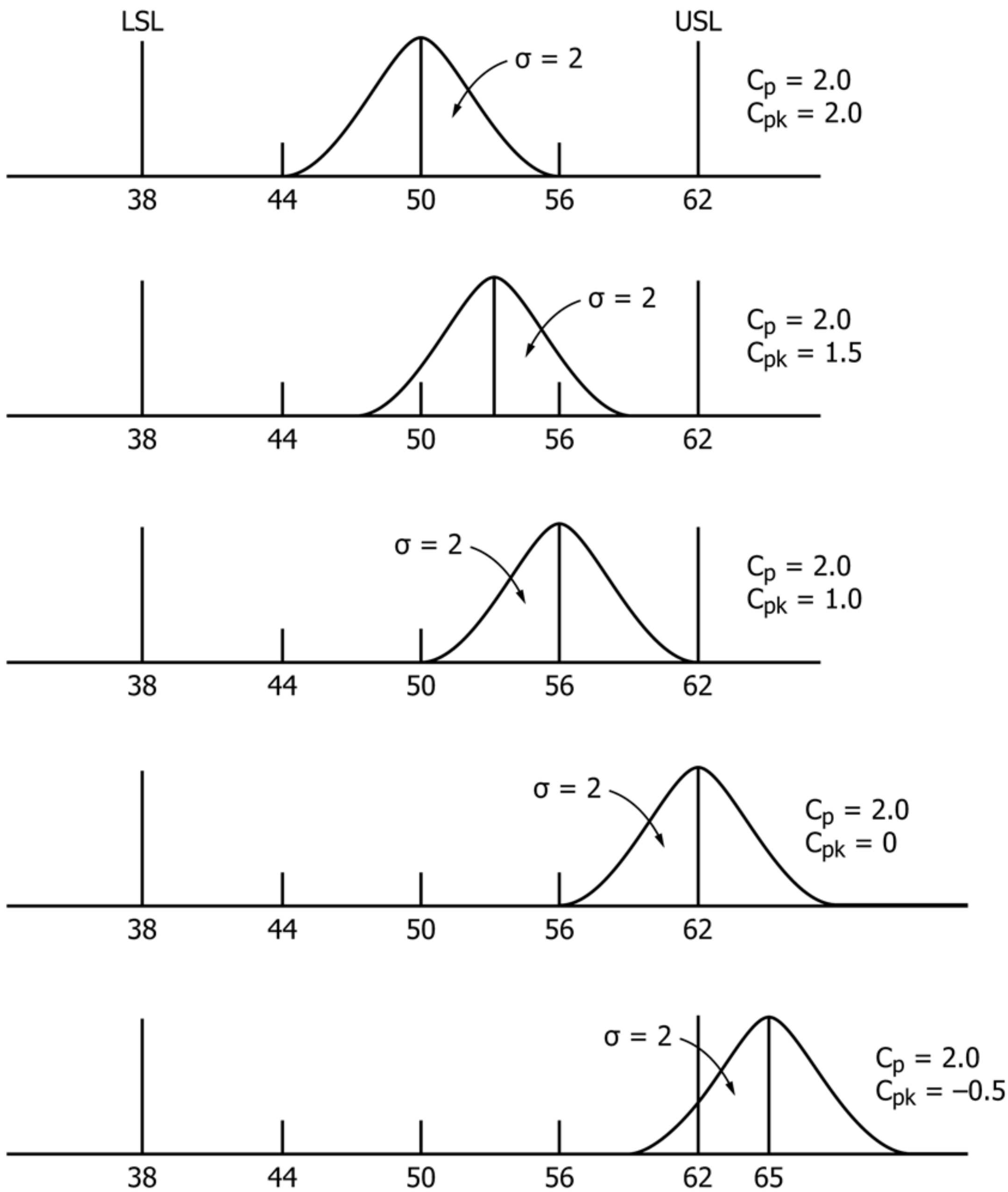

FIG. 5 Theoretical Process Capability Scenarios

calculated whenever these indices are used. Either a standard error of the estimate or a lower confidence bound is the preferred method. These statistics give the user of a capability index some idea of the resulting uncertainty for a given sample size. A lower confidence bound for a process capability index is a statistic that one can claim as the smallest value for the process index, with some stated confidence, say 95 %. It is the lower bound that is of primary interest since it favors the consumer. A consumer is usually interested in the question, "How small might the true process index be?" For example, suppose a consumer requires a P_{pk} of at least 1.33 for a large batch of product. Based on a sample, the supplier shows that the lower 95 % confidence bound for P_{pk} is 1.38. The consumer then has 95 % assurance that the accepted product meets the process index requirement of 1.33. In accepting the product, the consumer is willing to take a 5 % risk that the true P_{pk} is really less than 1.38; however, this risk is minimal and manageable. To claim that the process index is at least some derived quantity with a high degree of confidence is the assurance that the process is not worse that being claimed as the lower bound.

7.2 When a process is in good statistical control, the short term capability estimates $\hat{C}_p$ and $\hat{C}_{pk}$ and their long term performance equivalent estimates, and will give similar results for any fixed sample size. The results stated below are cast in terms of the long term measures $\hat{P}_p$ and $\hat{P}_{pk}$, but they could just as well be applied to the short term measures when the process is in statistical control and normally distributed. It is assumed that the variable being measured is normally distributed, and that the process was in a state of statistical control when the sample was taken and that the sample reasonably represents the population or process. Further, the estimate of the standard deviation is the ordinary estimate, s, as specified in Eq 10. For normally distributed variables, the distribution theory for these statistics has been worked out. See Refs (**7, 8, 9**) for details. Under these conditions approximate standard errors and lower 100(1-α) % confidence bounds may be stated as a function of sample size and the point estimate of the process index.

7.3 Let n be the sample size and let the tolerance be T = USL – LSL, for upper and lower specification limits USL and LSL. Let u be a point on a chi-square distribution with n – 1 degrees of freedom such that $P(\chi^2 > u) = 100(1 - \alpha)$ %. Let $z_{1\text{-}\alpha}$ be a point on a standard normal distribution such $P(Z > z_{1\text{-}\alpha})$ = 100α %. For the statistic, $\hat{P}_p$, an exact result for the lower confidence bound may be given (Ref (**8**)). The lower 100(1-α) % confidence bound for process capability index P_p is:

$$P_p \geq \hat{P}_p \sqrt{\frac{u}{n-1}} \tag{17}$$

The approximate standard error for the statistic $\hat{P}_p$ is:

$$se\left(\hat{P}_p\right) = \frac{\hat{P}_p}{\sqrt{2(n-1)}} \tag{18}$$

For the process capability index P_{pk}, the approximate 100(1-α) % lower confidence bound is:

$$P_{pk} \geq \hat{P}_{pk} - z_{1-\alpha}\sqrt{\frac{1}{9\,n} + \frac{\hat{P}_{pk}^2}{2n-2}} \tag{19}$$

The approximate standard error for the statistic $\hat{P}_{pk}$ is:

$$se\left(\hat{P}_{pk}\right) = \sqrt{\frac{1}{9n} + \frac{\hat{P}_{pk}^2}{2n-2}} \tag{20}$$

Results (Eq 19) and (Eq 20) are approximate and useful for practical purposes.

7.4 It is sometimes desirable to ask for a combination of sample size and minimum sample process capability index that is required to state that the true process capability is at least some specified value at some specified confidence. For selected sample sizes, these questions and others of a similar nature are answered in Ref (**8**). Suppose we are using a sample of n = 40 and want to state that the true process capability is at least 1.2, ($P_{pk} \geq 1.2$), with 95 % confidence. What is the minimum sample P_{pk} one would need to achieve this? Table 6 in Ref (**8**) shows this value to be 1.54. Therefore the sample value needs to be at least 1.54. For arbitrary sample size an approximate formula may be developed by inverting Eq 19. Using this method, it is possible to derive a general expression for the minimum sample P_{pk} that one would need, for specified sample size, confidence level, and value, k for which we want to claim that $P_{pk} \geq k$. Let n be the sample size, k the desired P_{pk} we want to state and C the confidence coefficient. Let Z be the associated 100C % quantile on a standard normal distribution. For example, when CC = 0.90, Z = 1.282; when C = 0.95, then Z = 1.645; when C = 0.99, then Z = 2.326. It may be shown that the sample value must be at least as large as h as specified below as a function of n, Z, and k.

$$h = \frac{k + \sqrt{k^2 - \left(1 - \frac{Z^2}{2(n-1)}\right)\left(k^2 - \frac{Z^2}{9n}\right)}}{\left(1 - \frac{Z^2}{2(n-1)}\right)} \tag{21}$$

Using the previous inputs (n = 40, C = 95 %, and k = 1.2) we find that the sample P_{pk} needs to be at least approximately 1.5. This value is reasonably close to the exact value obtained from Ref (**8**).

7.5 *Examples:*

7.5.1 Suppose we want to state that the process capability for a certain process is at least 1.33 with confidence 95 %. A sample size of n = 40 units will always be used. How large must the sample process capability be (h) in order to make this claim? Here C = 0.95, making Z = 1.645. Substituting in the appropriate numbers in equation (Eq 21) (Z = 1.645, n = 40, and k = 1.33) we find that h =1.65; therefore the sample P_{pk} needs to be at least 1.65.

7.5.2 A consumer requires that the supplier always state the standard error of the estimate when reporting a P_{pk} value. What is the standard error if a sample of size n = 50 is used and the sample $\hat{P}_{pk}$ is 1.49. Use Eq 20 with =1.49 and n = 50. The standard error is: 0.158.

7.5.3 What is the lower confidence bound for P_p with 90 % confidence, where a sample of n = 30 is used and the sample value is $\hat{P}_p$ = 1.8. Use Eq 17. The value of u is the lower 10 % point on a chi-square distribution with 29 degree of freedom – this value is 19.7677. The lower bound is: 1.49.

8. The C_{pm} Index

8.1 When there is an emphasis for running to a target, not necessarily on center, the C_{pm} index may be used. C_{pm} is a measure of process capability, similar to the standard C_p calculation, except that the standard deviation is calculated relative to deviations from a target rather than the sample mean. The C_{pm} index was originally defined by Chan et al. (**10**). It is noted that the index continues to apply to a process, normally distributed, and in a state of statistical control.

8.2 Let USL and LSL be the upper and lower specification limits, let T be the target, and let μ and σ be the process mean and standard deviation, respectively. The formula for the C_{pm} index and its relation to the ordinary C_p is shown below in Eq 22-24.

$$C_{pm} = \frac{USL - LSL}{6\sigma'} \tag{22}$$

where:

$$\sigma' = \sqrt{\sigma^2 + (\mu - T)^2} \tag{23}$$

Substituting Eq 23 into Eq 22 and simplifying gives:

$$C_{pm} = C_p\left(\frac{\sigma}{\sigma'}\right) \tag{24}$$

It is clear from Eq 23 that σ ≤ σ', and hence that $C_{pm} \leq C_p$. When the process mean is equal to the target we get $C_{pm} = C_p$.

The quantity $(\mu - T)^2$ is measuring the degree of departure or variation of the mean from the target, T, while σ^2 is measuring variation in the process, believed to be all natural since the process is assumed to be in statistical control. When we add these two quantities, we are calculating the total variation when a target is used. C_{pm} is interpreted in the same way as C_p. It is a measure of process potential that is a maximum when $\mu = T$, that is, when you are on target.

8.3 C_{pm} is estimated using a sample of size n independent measurements from the process and is calculated as follows:

$$\hat{C}_{pm} = \frac{USL - LSL}{6\hat{\sigma}'} = \hat{C}_p \frac{\hat{\sigma}_{ST}}{\hat{\sigma}'} \tag{25}$$

where:

$$\hat{\sigma}' = \sqrt{\frac{\sum_{i=l}^{n}(x_i - T)^2}{n-1}} = \sqrt{s^2 + \frac{n(\bar{x} - T)^2}{n-1}} \tag{26}$$

When a control chart based on measurements of X is being used, a convenient estimator of σ can be used where we replace the sample standard deviation, s, with the estimate based on the average range or average standard deviation of the several subgroups. Such an estimator represents short-term variation in the process which is deemed the same as s for an in-control process. The formula using the subgroup range is given as:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \tag{27}$$

It is noted that the value of T can be anywhere within the specification limits but that, generally, a process targeted at the center of a specification may be optimal in many cases. The difference between using C_{pm} or C_{pk} is in the importance of being on target versus being between the specification limits. C_{pk} is a metric for determining how well the process is within the specification limits based on its variability and its mean relative to the limits. Such a metric penalizes the user for being too close to a specification limit. C_{pm} is a metric that puts its focus on how well the process conforms to the target value, so being off target produces a penalty.

A modified version of the C_{pm} index that is calculated similarly to the C_{pk} index is available and given by:

$$\hat{C}^*_{pm} = \frac{\min\{USL - T, T - LSL\}}{3\hat{\sigma}'} \tag{28}$$

Note that when the target is the midpoint of the specification interval, then $C^*_{pm} = C_{pm}$. As is the case for C_p and C_{pk}, neither C_{pm} nor C^*_{pm} should be considered as absolute metrics of process capability. Both of these metrics are also based on sample estimates, so they are subject to sampling error as well. For details on this topic see Ref **(11)**.

9. Attribute Capability

9.1 For simple binomial counts in a sample of size n, if r defective units are found use the estimate r/n as the point estimate of process capability. Note that this is estimating the true proportion non-conforming, p, being generated by the process:

$$\hat{p} = \frac{r}{n} \tag{29}$$

The statistic Eq 29 is an unbiased estimator of p. The estimated standard error of Eq 29 is:

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n-1}} \tag{30}$$

Confidence interval estimates for the parameter p may also be calculated from standard formulas under certain assumptions and conditions. For details, see Ref **(12)**. When $r = 0$ is observed, it may be misleading to state the process capability as 0, using Eq 29. In $r = 0$ cases we can calculate an upper confidence bound for the unknown proportion p. The standard formula (Practice E2334) for this, using a sample size n and confidence coefficient C, is:

$$p \le \hat{p} = 1 - \sqrt[n]{1 - C} \tag{31}$$

In some industrial practice, $r = 1$ is conservatively assumed when $r = 0$ is observed. In that case, $1/n$ is the estimate of p. When this is done, the capability estimate $1/n$ has an attached confidence coefficient that is not less than 63.2 % as n increases. With larger sample sizes and $r = 0$, we sometimes see $r = 3$ assumed. This would give no less than 95 % confidence with increasing n. In that case, the capability estimate is $3/n$, known as the "Rule of 3" **(13)**.

9.2 For Poisson "event" counts or counting defects over an observation region, the process capability is typically rendered as a rate of event occurrence. We can assume that there is a constant but unknown rate λ operating on the process. When r "events" have been observed in a region of size S, the rate estimate is:

$$\hat{\lambda} = \frac{r}{S} \tag{32}$$

Note that the region S may be an ordinary sample size of discrete objects or a continuum or bulk sample of some type and size. Time can also be used as the observation region in some applications. The statistic Eq 32 is an unbiased estimate of the parameter λ. The estimated standard error is:

$$SE(\hat{\lambda}) = \sqrt{\frac{\hat{\lambda}}{S}} \tag{33}$$

Confidence interval estimates for the parameter λ may also be calculated from standard formulas under certain assumptions and conditions. For details, see Ref **(12)**. When $r = 0$ is observed, it may again be misleading to state the process capability (rate) as 0, using Eq 32. In $r = 0$ cases we can calculate an upper confidence bound for the unknown rate λ. The standard formula (Practice E2334) for this, using a region of size S and confidence coefficient C, is:

$$\lambda \le \hat{\lambda} = \frac{-\ln(1 - C)}{S} \tag{34}$$

In Eq 34, when $C = 0.632$, we find the upper bound is $\lambda \le 1/S$. When $C = 0.95$ we find the upper bound is $\lambda \le 3/S$ (rule of three, again applies here). In some quarters, practitioners assume 1 event and claim a confidence of 63.2 % or assume 3 events and claim a confidence of 95 %. These conservative results harmonize with the discussion of the binomial model above.

9.3 *Rolled Throughput Yield (RTY)*—In many industries where attribute inspection is being used and there are several steps of interest in a process, the RTY metric may be used as a measure of overall throughput yield. RTY is a common practice in 6-sigma applications but is more generally a useful metric for throughput process capability.

9.3.1 Any object inspected is said to possess one or more characteristics that may or may not exhibit a defect. Every defined characteristic presents an opportunity for a defect to occur. It is also possible that multiple defects could occur on the same characteristic within the same object inspected.

9.3.2 *Nomenclature*—The following terms and symbols are used in creating the RTY metric.

9.3.2.1 *unit*—the object inspected.

9.3.2.2 *n*—a sample size equal to the number of product units inspected.

9.3.2.3 *c*—a random variable equal to the number of defects observed in the sample of *n* units.

9.3.2.4 *r*—the number of opportunities or defined characteristics for a defect to occur within the unit. Note that this could possibly include redundancy of characteristics within a unit.

9.3.2.5 *T*—equal to *nr*, the total number of opportunities for a defect in the sample.

9.3.2.6 *DPU*—defects per Unit calculated as *c*/*n*, the total defects observed over the total units inspected.

9.3.2.7 *DPO*—defects per opportunity, calculated as DPU/*r* = *c*/(*nr*) = *c*/*T*.

9.3.2.8 *DPMO*—defects per million opportunities, calculated as DPO*10^6.

9.3.2.9 *y*—first pass yield or the fraction of defect free units produced in a single operation, calculated using the Poisson distribution with approximated rate, λ equal to DPU or *c*/*n*, and using $x = 0$ in the Poisson formula. See Eq 35:

$$y = e^{-DPU} \tag{35}$$

Note that $0 < y < 1$. Alternatively, one can calculate an estimated DPU rate given the yield, *y*. This is Eq 36:

$$-\ln(y) = DPU \tag{36}$$

9.3.3 RTY is the resulting yield from passing product through several (*k*) operations. Each operation has its own capability yield, y_i. When using the Poisson distribution, rates from independent observations are additive, accordingly the calculation for *k* independent operations are:

$$RTY = \prod_{i=1}^{k} y_i = e^{(-DPU_1 + DPU_2 + \ldots)} \tag{37}$$

RTY is interpreted as the probability that a single product unit can withstand *k* operations in series without a defect.

9.3.4 Normalized or average yield, Y_{norm}, is the individual step yield which, if applied to the *k* steps equally, would give the rolled throughput yield. This is equivalent to the geometric mean of the several individual yields (the y_i) and is also equivalent to an overall rate of return on an investment where the annual rates vary from year to year over *k* years. The calculation is:

$$Y_{norm} = \sqrt[k]{RTY} = \sqrt[k]{\prod_{i=1}^{k} y_i} \tag{38}$$

9.3.5 The "total defects per unit" or TDPU that would result from a series of *k* independent steps is calculated as:

$$TDPU = \ln(RTY) \tag{39}$$

TDPU is interpreted as the estimated expected total number of defects per unit considering the entire process of *k* steps.

9.3.6 *RTY Example*—Suppose the following:

n = 10 000 vehicles produced in an operation.

r = 350 characteristics are defined for each vehicle at final inspection in the operation.

k = 12 operations before the units are shipped.

T = *nr* = 10 000(350) = 3.5E6.

Data may be organized in a table as illustrated in Table 1 where the column labeled "*c*" are values of the actual findings for each of the 12 operations. The DPU, DPO, DPMO, and Yield columns are calculated using the formulas previously outlined.

The RTY metric is seen to be 0.778 and the normalized, per operation, throughput is 0.9793. The RTY is equivalent to an overall process one where each of 12 processes involved have a yield of 97.93 %.

10. Keywords

10.1 long-term variability; process capability; process capability indices; process performance; process performance indices; short-term variability

TABLE 1 Example of Rolled Throughput Yield and Associated Calculations

Operation	c	DPU	DPMO	Yield (step)
1	150	0.0150	42.857	0.9851
2	235	0.0235	67.143	0.9768
3	189	0.0189	54.000	0.9813
4	312	0.0312	89.143	0.9693
5	434	0.0434	124.000	0.9575
6	167	0.0167	47.714	0.9834
7	97	0.0097	27.714	0.9903
8	84	0.0084	24.000	0.9916
9	167	0.0167	47.714	0.9834
10	322	0.0322	92.000	0.9683
11	110	0.0110	31.429	0.9891
12	245	0.0245	70.000	0.9758
Product of step yields (RTY)				0.7779
Normalized, per operation (Y_{norm})				0.9793

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