Designation: E1369 – 15

Standard Guide for Selecting Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Buildings and Building Systems[1]

1. Scope

1.1 This guide covers techniques for treating uncertainty in input values to an economic analysis of a building investment project. It also recommends techniques for evaluating the risk that a project will have a less favorable economic outcome than what is desired or expected.[2]

1.2 The techniques include breakeven analysis, sensitivity analysis, risk-adjusted discounting, the mean-variance criterion and coefficient of variation, decision analysis, simulation, and stochastic dominance.

1.3 The techniques can be used with economic methods that measure economic performance, such as life-cycle cost analysis, net benefits, the benefit-to-cost ratio, internal rate of return, and payback.

2. Referenced Documents

2.1 *ASTM Standards:*[3]

E631 Terminology of Building Constructions
E833 Terminology of Building Economics
E917 Practice for Measuring Life-Cycle Costs of Buildings and Building Systems
E964 Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios for Buildings and Building Systems
E1057 Practice for Measuring Internal Rate of Return and Adjusted Internal Rate of Return for Investments in Buildings and Building Systems
E1074 Practice for Measuring Net Benefits and Net Savings for Investments in Buildings and Building Systems
E1121 Practice for Measuring Payback for Investments in Buildings and Building Systems
E1185 Guide for Selecting Economic Methods for Evaluating Investments in Buildings and Building Systems
E1946 Practice for Measuring Cost Risk of Buildings and Building Systems and Other Constructed Projects
E2204 Guide for Summarizing the Economic Impacts of Building-Related Projects

2.2 *Adjuncts:*

Discount Factor Tables Adjunct to Practices E917, E964, E1057, E1074, and E1121[4]

3. Terminology

3.1 *Definitions*—For definitions of general terms related to building construction used in this guide, refer to Terminology E631; and for general terms related to building economics, refer to Terminology E833.

4. Summary of Guide

4.1 This guide identifies related ASTM standards and adjuncts. It describes circumstances when measuring uncertainty and risk may be helpful in economic evaluations of building investments. This guide defines uncertainty, risk exposure, and risk attitude. It presents nonprobabilistic and probabilistic techniques for measuring uncertainty and risk exposure. This guide describes briefly each technique, gives the formula for calculating a measure where appropriate, illustrates the techniques with a case example, and summarizes its advantages and disadvantages.

4.2 Since there is no best technique for measuring uncertainty and risk in every economic evaluation, this guide concludes with a discussion of how to select the appropriate technique for a particular problem.

4.3 This guide describes in detail how risk exposure can be measured by probability functions and distribution functions

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[1] This guide is under the jurisdiction of ASTM Committee E06 on Performance of Buildings and is the direct responsibility of Subcommittee E06.81 on Building Economics.

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[2] For an extensive overview of techniques for treating risk and uncertainty, see Marshall, H.E., *Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Building Investments*, National Institute of Standards and Technology, Special Publication 757, 1988.

[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[4] Available from ASTM International Headquarters. Order Adjunct No. ADJE091703. Original adjunct produced in 1984.

(see Annex A1). It also describes how risk attitude can be incorporated using utility theory and other approaches (see Annex A2).

5. Significance and Use

5.1 Investments in long-lived projects such as buildings are characterized by uncertainties regarding project life, operation and maintenance costs, revenues, and other factors that affect project economics. Since future values of these variable factors are generally not known, it is difficult to make reliable economic evaluations.

5.2 The traditional approach to project investment analysis has been to apply economic methods of project evaluation to best-guess estimates of project input variables as if they were certain estimates and then to present results in single-value, deterministic terms. When projects are evaluated without regard to uncertainty of inputs to the analysis, decision makers may have insufficient information to measure and evaluate the risk of investing in a project having a different outcome from what is expected.

5.3 Risk analysis is the body of theory and practice that has evolved to help decision makers assess their risk exposures and risk attitudes so that the investment that is the best bet for them can be selected.

NOTE 1—The decision maker is the individual or group of individuals responsible for the investment decision. For example, the decision maker may be the chief executive officer or the board of directors.

5.4 Uncertainty and risk are defined as follows. Uncertainty (or certainty) refers to a state of knowledge about the variable inputs to an economic analysis. If the decision maker is unsure of input values, there is uncertainty. If the decision maker is sure, there is certainty. Risk refers either to risk exposure or risk attitude.

5.4.1 Risk exposure is the probability of investing in a project that will have a less favorable economic outcome than what is desired (the target) or is expected.

5.4.2 Risk attitude, also called risk preference, is the willingness of a decision maker to take a chance or gamble on an investment of uncertain outcome. The implications of decision makers having different risk attitudes is that a given investment of known risk exposure might be economically acceptable to an investor who is not particularly risk averse, but totally unacceptable to another investor who is very risk averse.

NOTE 2—For completeness, this guide covers both risk averse and risk taking attitudes. Most investors, however, are likely to be risk averse. The principles described herein apply both to the typical case where investors have different degrees of risk aversion and to the atypical case where some investors are risk taking while others are risk averse.

5.5 No single technique can be labeled the best technique in every situation for treating uncertainty, risk, or both. What is best depends on the following: availability of data, availability of resources (time, money, expertise), computational aids (for example, computer services), user understanding, ability to measure risk exposure and risk attitude, risk attitude of decision makers, level of risk exposure of the project, and size of the investment relative to the institution's portfolio.

6. Procedures

6.1 The recommended steps for carrying out an evaluation of uncertainty or risk are as follows:

6.1.1 Determine appropriate economic measure(s) for evaluating the investment (see Guide E1185).

6.1.2 Identify objectives, alternatives, and constraints (see Practices E917, E964, E1057, E1074, and E1121).

6.1.3 Decide whether an uncertainty and risk evaluation is needed, and, if so, choose the appropriate technique (see Sections 5, 7, 8, and 10).

6.1.4 Compile data and establish assumptions for the evaluation.

6.1.5 Determine risk attitude of the decision maker (see Section 7 and Annex A2).

6.1.6 Compute measures of worth[5] and associated risk (see Sections 7 and 8).

6.1.7 Analyze results and make a decision (see Section 9).

6.1.8 Document the evaluation (see Section 11).

7. Techniques: Advantages and Disadvantages

7.1 This guide considers in detail three nonprobabilistic techniques (breakeven analysis, sensitivity analysis, and risk-adjusted discounting) and four probabilistic techniques (mean-variance criterion and coefficient of variation, decision analysis, simulation, and stochastic dominance) for treating uncertainty and risk. This guide also summarizes several additional techniques that are used less frequently.

7.2 *Breakeven Analysis:*

7.2.1 When an uncertain variable is critical to the economic success of a project, decision makers frequently want to know the minimum or maximum value that variable can reach and still have a breakeven project; that is, a project where benefits (savings) equal costs. For example, the breakeven value of an input *cost* variable is the maximum amount one can afford to pay for the input and still break even compared to benefits earned. A breakeven value of an input *benefit* variable is the minimum amount the project can produce in that benefit category and still cover the projected costs of the project.

NOTE 3—Benefits and costs are treated throughout this guide on a discounted cash-flow basis, taking into account taxes where appropriate. (See Practice E917 for an explanation of discounted cash flows considering taxes.)

7.2.2 To perform a breakeven analysis, an equation is constructed wherein the benefits are set equal to the costs for a given investment project, the values of all inputs except the breakeven variable are specified, and the breakeven variable is solved algebraically.

7.2.3 Suppose a decision maker is deciding whether or not to invest in a piece of energy conserving equipment for a government-owned building. The deviation of the formula for computing breakeven investment costs for the equipment is as follows:

[5] The NIST Building Life-Cycle Cost (BLCC) Computer Program helps users calculate measures of worth for buildings and building components that are consistent with ASTM standards. The program is downloadable from http://energy.gov/eere/femp/building-life-cycle-cost-programs.

$$S = C \quad (1)$$

$$C = I + \text{O\&M} + R$$

$$S = I + \text{O\&M} + R$$

$$I = S - \text{O\&M} - R$$

where:

S = savings (benefits) in reduced energy costs from using the equipment,
C = all costs associated with the equipment,
I = initial investment costs of the equipment,
O&M = operation and maintenance costs of the equipment, and
R = replacement costs required to keep the equipment functional over the study period, and where all cost and benefit cash flows are discounted to present values.

7.2.4 By rearranging terms, the breakeven investment unknown is isolated on the left side of the equation. Substitution of known values for the terms on the right side allows the analyst to solve for the breakeven value. For example, if S = $20 000, O&M = $2500, and R = $1000, then

$$I = \$20\,000 - \$2500 - \$1000 \quad (2)$$

or

$$I = \$16\,500 \quad (3)$$

7.2.5 This means that $16 500, the breakeven value, is the maximum amount that can be paid for the energy-conserving equipment and still recover all costs through energy savings.

7.2.6 An advantage of breakeven analysis is that it can be computed quickly and easily with limited information. It also simplifies project evaluation in that it gives just one value to decision makers to use as a benchmark for comparison against the predicted performance of that uncertain variable. Breakeven analysis helps decision makers assess the likelihood of achieving the breakeven value and thereby contributes implicitly to the analysis of project risk.

7.2.7 A disadvantage is that it provides no probabilistic picture of input variable uncertainty or of project risk exposure. Furthermore, it includes no explicit treatment of risk attitude.

7.3 *Sensitivity Analysis:*

7.3.1 Sensitivity analysis measures the impact on project outcomes of changing a key input value about which there is uncertainty. For example, choose a pessimistic, expected, and optimistic value for an uncertain variable. Then do an economic analysis for each of the three values to see how the outcome changes as they change, with other things held the same.

7.3.2 Sensitivity analysis also applies to different combinations of input values. That is, alter several variables at once and then compute a measure of worth. For example, one scenario might include a combination of all pessimistic values, another all expected values, and a third all optimistic values; or a combination might include optimistic values for some variables in conjunction with pessimistic or expected values for others. Examining different combinations is required if the uncertain variables are interrelated.

7.3.3 The following illustration of sensitivity analysis treats an accept/reject decision. Consider a decision on whether or not to install a programmable time clock to control heating, ventilating, and air conditioning (HVAC) equipment in a building. The time clock reduces electricity consumption by turning off that part of the HVAC equipment that is not needed during hours when the building is unoccupied. Using the benefit-to-cost ratio (BCR) as the economic method, the time clock is acceptable on economic grounds if its BCR is greater than 1.0. The energy reduction benefits from the time clock, however, are uncertain. They are a function of three factors: the initial price of energy, the rate of change in energy prices over the life cycle of the time clock, and the number of kilowatt hours saved. Assume that the initial price of energy and the number of kilowatt-hours saved are relatively certain, and that the sensitivity of the BCR is being tested with respect to the following three values of energy price change: a low rate of energy price escalation (slowly increasing benefits from energy savings); a moderate rate of escalation (moderately increasing benefits); and a high rate of escalation (rapidly increasing benefits). These three assumed values of energy price change might correspond to our projections of pessimistic, expected, and optimistic values. Three BCR estimates result from repeating the BCR computation for each of the three energy price escalation rates. For example, BCRs of 0.8, 2.0, and 4.0 might result. Whereas a deterministic approach might have generated a BCR estimate of 2.0, now it is apparent that the BCR *could be* significantly less than 2.0, and even less than 1.0. Thus accepting the time clock could lead to an inefficient outcome.

7.3.4 There are several advantages of sensitivity analysis. First, it shows how significant a single input variable is in determining project outcomes. Second, it recognizes the uncertainty associated with the input. Third, it gives information about the range of output variability. And fourth, it does all of these when there is little information, resources, or time to use more sophisticated techniques.

7.3.5 Disadvantages of sensitivity analysis in evaluating risk are that it gives no explicit probabilistic measure of risk exposure and it includes no explicit treatment of risk attitude. The findings of sensitivity analysis are ambiguous. How likely is a pessimistic or expected or optimistic value, for example, and how likely is the corresponding outcome value? Sensitivity analysis can in fact be misleading if all pessimistic assumptions or all optimistic assumptions are combined in calculating economic measures. Such combinations of inputs are unlikely in the real world.

7.3.6 Sensitivity results can be presented in text, tables, or graphs. One type of graph that is useful in showing the sensitivity of project worth to a critical variable is illustrated in Fig. 1. Net benefits (NB) for Projects A and B decrease as the discount rate increases. The slopes of the functions show that NB is more sensitive to discount rate changes for Project A than for Project B, assuming other variables remain unchanged. These functions also help in making comparisons as to which project is more cost effective. At a discount rate below 7 %, for

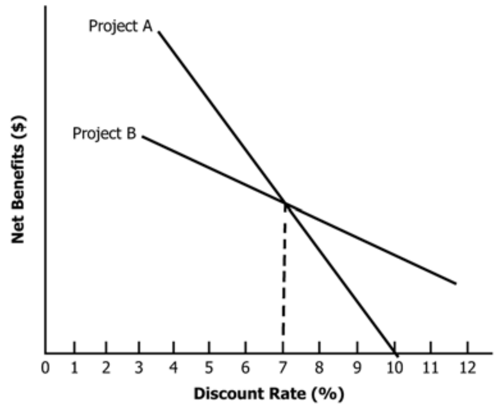
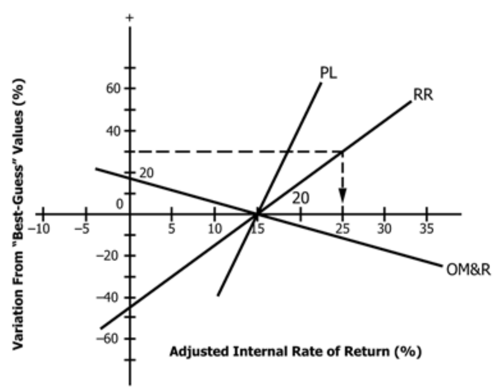


FIG. 1 Sensitivity of Net Benefits of Projects A and B to Discount Rate



NOTE 1—PL = project life,
RR = reinvestment rate, and
OM&R = operation, maintenance, and replacement costs.

FIG. 2 Spider Diagram Showing Sensitivity of the Adjusted Internal Rate of Return to Variations in Uncertain Variables

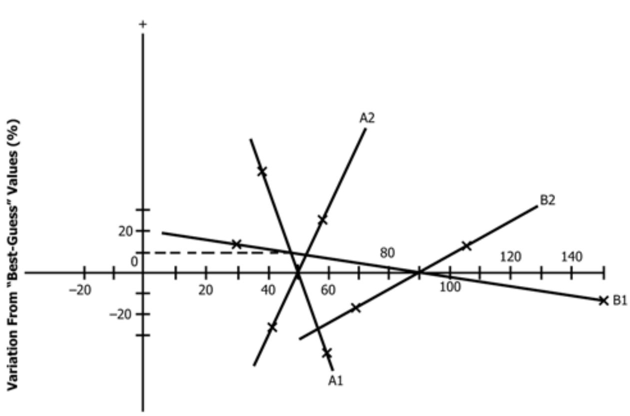


FIG. 3 Spider Diagrams for Competing Projects

example, Project A has the greater NB. At a rate above 7 %, Project B yields the greater NB. And at 7 %, the two projects provide identical NB.

7.3.7 Note that the functions indicate the potential values of NB *if* different values of the discount rate occur. If decision makers have some idea as to the likelihood of specific discount rates, the graph will help them evaluate the NB implications for these two projects. The sensitivity graph in this sense contributes to an implicit description of risk exposure. Yet the graph fails to provide a quantitative measure of the probability of any given NB occurring.

7.3.8 Another special graph for sensitivity analysis that presents a snapshot of potential impacts of uncertain input variables on project outcomes is the spider diagram. The one illustrated in Fig. 2 shows for a prospective commercial building investment the sensitivity of the adjusted internal rate of return (AIRR) to three variables: operation, maintenance, and replacement costs (OM&R); project life (PL); and the reinvestment rate (RR). Each variable is represented by a labeled function that shows what AIRR values would result from different values of the uncertain variable. For example, the downward-sloping OM&R function indicates that the AIRR is inversely proportional to OM&R costs. By design, the OM&R function (as well as the other two functions) passes through the horizontal axis at the best-guess estimate of the AIRR (15 % in this case), based on the best-guess estimates of the three uncertain variables. Since each of the variables is measured by different units (money, years, and percent), the vertical axis is denominated in positive and negative percent changes from the best-guess values fixed at the horizontal axis. The AIRR value corresponding to any given percent variation indicated by a point on the function is found by extending a line perpendicular to the horizontal axis and reading directly the AIRR value. Thus a 30 % increase in the best-guess reinvestment rate would yield a 25 % AIRR, assuming other values remain unchanged.

7.3.9 The contribution of the spider diagram is its picture of the relative importance of the different uncertain variables. It shows immediately that the lesser the slope of a function, the more sensitive is the AIRR to that variable. For example, any given percent change in OM&R will have a greater impact on the AIRR than will an equal percent change in RR or PL.

7.3.10 Spider diagrams can be helpful when comparing competing projects as long as the decision maker keeps in mind that extreme values of the measure of worth reflect variations in one variable only. For example, look at the spider diagram for Projects A and B in Fig. 3. The NB of Project A is a function of variables A1 and A2, and the NB of Project B is a function of variables B1 and B2. The horizontal axis suggests that Project B has a higher present value net benefits ($90 000) than Project A ($50 000). That is, if only best-guess values were used in a single-value, deterministic approach, Project B would be the preferred project. However, if we assign, say a 50 % confidence interval about the uncertain variables A1, A2, B1, and B2, as shown by X's on the functions, there appears the possibility that Project A could yield a higher NB than Project B. That is, within that confidence interval, if the extreme B1 value to the left were to occur, Project B would yield a lesser NB than would Project A for A1 or A2 extreme values to the left. Furthermore, if A1 and B1 were the same input variable, we would know that Project A would be preferred at values of A1 and B1 above 10 % over the best-guess value, and Project B would be preferred at values of A1 and B1 below 10 %.

7.3.11 Once again, however, sensitivity analysis gives no indication of the probability of any given value of NB. Furthermore, because only one variable is allowed to change at a time, and NB is a function of more than one variable, sensitivity analysis gives an incomplete description of the possible outcomes.

7.4 *Risk-Adjusted Discounting:*

7.4.1 One technique used by the business community to account for risk is the risk-adjusted discount rate (RADR). The objective of using the RADR technique is to raise the likelihood that the investor will earn a return over time sufficient to compensate for extra risk associated with specific projects.

7.4.2 Projects with anticipated high variability in distributions of project worth have their net benefits or returns discounted at higher rates than projects with low variability. Thus in computing net benefits or the benefit-to-cost ratio, the discount rate is higher for benefit streams of risky projects than for those with certain outcomes. Or when applying rate-of-return methods, the minimum acceptable rate of return (MARR) is raised above the risk-free rate to compensate for the higher variability of returns in risky projects.

7.4.3 Calculate the RADR as follows:

$$RADR = RF + AR1 + AR2 \tag{4}$$

where:

RF = risk-free rate,
AR1 = adjustment for normal risk encountered in the firm's operations, and
AR2 = adjustment for extra risk above or below normal risk.

All terms are expressed as percents.

7.4.4 The risk-free rate (RF) component accounts for the time value of money. It is what might be earned, for example, on government treasury bills, the closest thing to a riskless investment available to most investors. The adjustment for normal risk (AR1) is the risk premium that a firm might impose to cover the average riskiness of its normal operations. The sum of RF and AR1 should equal the MARR the firm requires on typical investments. The AR2 component adjusts for projects with more or less risk than what is normally associated with the firm. The adjustment can be positive or negative.

7.4.5 For discounting *benefit* streams, AR2 is an increasing function of (*1*) the perceived variability in project outcomes (risk exposure) and (*2*) the degree to which the decision maker is risk averse (risk attitude). For *cost* streams, AR2 is a decreasing function of those same risk factors.

7.4.6 For computing the RADR, each benefit and cost stream should be discounted with a unique RADR that includes AR1 and AR2 values that describe that stream's uncertainty. For benefit or savings streams, AR1 and AR2 are adjusted upwards as perceived risk increases; that is, as future benefits become more uncertain, the RADR technique requires raising the discount rate to make the project look less desirable. For cost streams, AR1 and AR2 are adjusted downwards as perceived risk increases; that is, as future costs become more uncertain, the correct application of the RADR technique requires lowering the discount rate to make the project look less cost effective. It follows then that the appropriate adjustment for risk when using life cycle cost (LCC) analysis is a decrease in the discount rate for each cost stream to make project costs appear higher. Otherwise LCC analysis will be biased in favor of projects with a greater risk of higher-than-anticipated costs.

7.4.7 Let us look once again at the BCR of the time clock for an illustration of the RADR when making an accept/reject decision. If no unusual risk is associated with the time clock, the discount rate is equal to the sum of RF and AR1 as shown under Eq 4. Let us suppose that the BCR for the time clock is 1.1 in this case. Thus it appears economically sound.

7.4.8 Now let us assume instead that the economic performance of the time clock is more risky than average. This might arise, for example, from the impact of uncertain kilowatt-hour reductions or uncertain future energy prices. Furthermore, let us assume that the decision maker is risk averse. Using the RADR technique, we raise the discount rate for evaluating energy cost savings by some positive value of AR2. If the resulting BCR falls below 1.0, the project no longer appears economically acceptable.

7.4.9 Advantages of the RADR technique are that it is relatively simple to understand; it is easy to compute; and it accounts to some extent for uncertainty of inputs, risk exposure, and risk attitude.

7.4.10 A major limitation in using the RADR is the lack of any accepted procedure for establishing the RADR value. It is typically estimated based on the decision maker's best judgment. One common approach is to simply lump projects into risk categories, each of which has an assigned RADR. There is little fine tuning. Furthermore, there is no distinction between adjustments for handling risk exposure and risk attitude.

7.4.11 A common mistake in application is to use a constant AR2 over the entire study period. This distorts risk adjustment when there are periods for which no special adjustment is necessary above or below what is considered normal risk. A constant AR2 also distorts risk adjustment because it implies in effect that returns become exponentially more uncertain over time, which is often not the case. Thus a discount rate that includes a constant AR2 severely reduces the weight of net benefits accrued in later years, regardless of the certainty of their occurrence. This biases selection towards projects with early payoffs. To avoid this common mistake in application and its resulting bias, use a variable AR2.

7.5 *Mean-Variance Criterion and Coefficient of Variation:*

7.5.1 Comparing mean values and standard deviations of measures of project worth can help decision makers evaluate returns and risk exposure of one project versus another and determine stochastic dominance. If two projects competing for limiting funds are compared on the basis of BCRs, for example, the mean-variance criterion dictates that the one with the higher mean (that is, expected value) and lower standard deviation be chosen. This presumes that decision makers prefer higher BCRs to lower BCRs and less risk to more risk.

7.5.2 If one project has a higher mean and higher standard deviation of the measure of project worth, then the choice is not clear with the mean-variance criterion. In this case, the coefficient of variation can be computed to determine the relative risk of the two projects. The coefficient of variation is found by dividing the standard deviation by the mean as follows:

$$CV = \sigma/\mu, \quad (5)$$

where:

CV = coefficient of variation,
σ = standard deviation, and
μ = mean or expected value.

7.5.3 The project with the lower coefficient of variation has the lesser risk per unit of return or project worth. It will be preferred by risk-averse decision makers. Risk-taking decision makers, on the other hand, will prefer the project with the higher coefficient.

7.5.4 An advantage of the coefficient of variation is that it provides an explicit measure of relative risk exposure. Another is that risk attitude is considered when the decision maker evaluates the coefficients of variation to choose among alternative projects. The major limitation is in acquiring the σ and μ values for the measure of project worth.

7.6 *Decision Analysis:*

7.6.1 Decision analysis is one of the few techniques for making economic decisions in an uncertain environment that treats formally both risk exposure and risk attitude. It provides a methodology that allows a decision maker to include alternative outcomes, risk attitudes, and subjective impressions about uncertain events in an evaluation of investments.

7.6.2 Decision analysis typically uses decision trees to represent all possible outcomes, costs, and probabilities associated with a given decision problem. A decision tree is a decision-flow diagram that serves as a road map to clarify possible alternatives and outcomes of sequential decisions. A decision tree is used in this section to illustrate how it helps bring order to complex decisions about risky investments.

7.6.3 In general, the decision analysis approach has three steps. The first is to structure the problem. This includes defining variables, describing with models their relationships, assigning values to possible outcomes, and measuring the importance of variables through sensitivity analysis. The second step is to assign subjective probabilities to important variables and possible outcomes, and to find the best bet alternative. This includes describing uncertainty with subjective probability distributions, describing risk attitude with a utility function (see Annex A2), and finding the alternative that is expected to yield the greatest economic return (or utility if the decision maker is not risk neutral). The third step, which is not always taken, is to determine whether obtaining additional information is worth the cost. If it is, then the information is collected, and steps 1 and 2 are repeated.

NOTE 4—Subjective probability distributions are developed by the decision analyst asking the decision maker or an expert(s) designated by the decision maker a series of probing questions designed to reveal the best judgments available on the likelihood of uncertain events.

7.6.4 *Decision Analysis of Energy Conservation Investment:*

7.6.4.1 This illustration examines an energy investment problem facing a state energy office. The office has been directed to make a choice regarding an energy conservation project from among six possibilities for retrofitting two public buildings. The purpose of the conservation project is to demonstrate to private companies that energy conservation is profitable. The objective of the decision analysis exercise is to choose the retrofit package that yields the maximum expected net benefits (NB), that is, shows the greatest profit potential. If none of the packages yields a positive NB, the choice will be not to invest at all.

7.6.4.2 Two types of retrofit costs are considered. The first is a fixed retrofit investment cost that is incurred for energy conservation work in each building regardless of which retrofit package is chosen. The second is the cost of implementing the individual retrofits in each package. The present value fixed investment (F1 and F2) costs and retrofit package (R1 through R6) costs are shown in Table 1. All costs are assumed to be *certain*.

7.6.4.3 The predicted benefit outcomes (dollar energy savings in present value terms) are *uncertain* for the different retrofit packages. Table 2 shows estimates of these possible benefit outcomes with their respective probabilities of occurrence. Since the state is assumed to be risk neutral and act so as to maximize the expected monetary value of its investments, there is no need to consider risk attitude and the corresponding utility measures of outcomes. Furthermore, since the state pays no taxes, they are not included in the analysis.

7.6.4.4 The decision tree in Fig. 4 clarifies the possible alternatives and outcomes listed in Table 1 and Table 2. The following explanation describes the potential paths of the decision tree starting from the left side.

NOTE 5—The procedure for finding the package that yields maximum net benefits requires starting from the right side of the tree, as will be shown later. It is easier to explain the tree structure, however by starting from the left.

7.6.4.5 The basic alternative of not investing is indicated by the top line segment coming out of the box on the left side of Fig. 4. The fixed investment of $500 000 in Building I is shown by the next line, and the investment of $800 000 in Building II is shown by the bottom line. Each box in a decision tree represents a decision juncture or node, and the line segments represent alternative branches on the decision tree. The state energy office will select that branch sequence that they expect will maximize the present value of net benefits from conservation.

7.6.4.6 Associated with each building is another decision node, requiring a decision regarding a specific set of retrofit choices, R1 through R3 or R4 through R6. The known costs of each retrofit package are shown under each alternative branch.

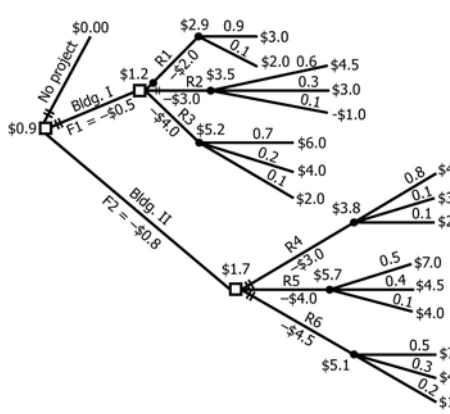
7.6.4.7 The benefit outcomes (dollar energy savings) are uncertain for the different retrofit packages. Thus at the end of each retrofit package branch is a chance node or juncture followed by alternative outcomes. Retrofit package R1, for example, is followed by a chance node, indicated by a circle,

TABLE 1 Fixed Investment and Retrofit Package Cost for Buildings I and II (Cost in Millions of Dollars)

Building I				Building II			
F1	R1	R2	R3	F2	R4	R5	R6
0.5	2.0	3.0	4.0	0.8	3.0	4.0	4.5

TABLE 2 Possible Benefit Outcomes and Their Estimated Probabilities of Occurrence for the Six Retrofit Packages

Retrofit Packages	Possible Benefit Outcomes, millions of dollars	Estimated Probabilities
R1	3.0	0.9
	2.0	0.1
R2	4.5	0.6
	3.0	0.3
	–1.0	0.1
R3	6.0	0.7
	4.0	0.2
	2.0	0.1
R4	4.0	0.8
	3.0	0.1
	2.5	0.1
R5	7.0	0.5
	4.5	0.4
	4.0	0.1
R6	7.0	0.5
	4.5	0.3
	1.0	0.2



NOTE 1—□ = decision node,
O = chance node, and
R = retrofit package.

FIG. 4 Decision Tree for Conservation Investment (Dollar Values are in Millions)

with two potential outcomes. The probability of each alternative outcome is indicated on top of its line and the value of each alternative outcome at the tip of its line.

7.6.4.8 One way to establish the probability and outcome values is for the analyst to discuss with engineers, architects, building managers, equipment manufacturers, and other knowledgeable people the implications of alternative retrofit packages in Buildings I and II. The outcome values at the branch tips will be based on anticipated potential impacts of changes in uncertain input variables, including energy prices, length of system life, performance of the conservation retrofits, and the quantity of energy saved.

7.6.4.9 Let us trace out one set of decisions with its possible outcomes. If the state energy office chooses the R1 package of retrofits in Building I for a total cost of $2 500 000, there is a 90 % probability that the outcome (payoff) will be $3 000 000 and a 10 % chance it will be only $2 000 000.

7.6.4.10 Let us also examine how some of the outcome and probability values might have been derived. The 90 % probability associated with R1 for a $3 000 000 payoff might be due to the R1 conservation package being a well-tested one with predictable results. On the other hand, R2 might contain conservation options that are new and untried, thereby explaining the spread of possible outcomes and the lower probabilities. And since there is no record of performance, and there is some chance of the conservation options not working, a 10 % probability of a loss of $1 000 000 is included in R2, as shown on the bottom outcome branch.

7.6.4.11 The outcome values at the tips of the outcome branches, the probabilities on the outcome branches, and the retrofit and fixed building costs on the alternative branches are estimated (see Table 1 and Table 2). The values shown at each decision node and chance node, on the other hand, must be calculated. The following steps describe the calculation process for the node values and how to determine the retrofit choice that maximizes expected net benefits. Note that the calculation process starts from the right side of the tree and works backwards to the left side.

7.6.4.12 Starting from the right-hand side of the tree, average out for each chance node its expected value; that is, calculate the weighted average for each probability fan by summing the products of the possible outcomes weighted by their respective probabilities. The expected value of the probability fan of R1, for example, is computed as follows:

$$0.9(\$3\,000\,000) + 0.1(\$2\,000\,000) = \$2\,900\,000 \quad (6)$$

Write the expected value atop each chance node, as shown in Fig. 4.

7.6.4.13 Next, fold back to the next preceding stage. That is, at each square decision node, compare the alternative branches with respect to their costs and expected benefits. Choose the one with the highest expected net benefits and write it atop the decision node box. For example, if you fold back the decision node values on the Building II path sequence, expected NB values (before subtracting the $800 000 Building II fixed cost) for retrofit packages R4 through R6 are as follows:

$$R4_{NB} = \$3\,800\,000 - \$3\,000\,000 = \$800\,000 \quad (7)$$

$$R5_{NB} = \$5\,700\,000 - \$4\,000\,000 = \$1\,700\,000 \quad (8)$$

$$R6_{NB} = \$5\,100\,000 - \$4\,500\,000 = \$600\,000 \quad (9)$$

7.6.4.14 The preferred (that is, maximum expected NB) alternative branch is R5. Write its value, $1 700 000, atop the decision node box. Truncate the other two paths by parallel slash marks to indicate that they are less economical choices.

7.6.4.15 The final step is to fold back one more time. Retrofit package R5 in Building II is the most efficient choice because its expected value of net benefits is $900 000 (that is, $1 700 000 – $800 000) compared to $700 000 (that is, $1 200 000 – $500 000) for retrofit package R3 in Building I and zero dollars for having no project. Enter the maximum expected value at the initial decision node box at the far left of the decision tree. Use parallel slash marks to truncate the no project and Building I alternatives. The decision tree, once all values are written in, shows explicitly the economically efficient path sequence (Building II/R5 in this case) and the expected value of net benefits ($900 000) for that path sequence.

7.6.4.16 Note that risk attitude was not addressed explicitly with utility values in this example because the state is assumed to be risk neutral. If the decision maker were risk averse or a risk taker, however, the projected earnings and costs associated with each decision branch could be converted to utility values. A utility function (see Fig. A2.5) is used to find the utility value corresponding to these benefits and costs. The averaging out to find expected values (now expected utility values) and the rolling back process are the same as described earlier. Once the alternative that maximizes utility is identified, the certain equivalent dollar value corresponding to that alternative's utility is found on the utility function. The certain equivalent value shows what the risky investment is worth, taking into consideration the decision maker's risk attitude.

7.7 *Simulation:*

7.7.1 Simulation is a well-documented technique used to determine risk exposure from an investment decision. To perform a simulation, probability functions of significant input variables must be estimated. The simulation process for building a probability density function (pdf) and cumulative distribution function (cdf) of the measure of project worth is as follows: draw a value for each input variable randomly from its probability function, substitute the set of input values for that round of draws into the formula for computing the measure of economic worth, and repeat the process over and over until a pdf and cdf can be constructed for the measure of worth.

7.7.2 For example, in analyzing the time clock, the initial energy price, the rate of energy price escalation, and the kilowatt hour savings are uncertain input variables. If each of these inputs could be described by a probability distribution, a simulation could be used to arrive at a probability distribution of the time clock's BCR (or some other measure of worth). Specifically, a random combination of each of the three variables would be selected and combined with constant inputs to compute a BCR. By repeating this random sampling over and over, typically 500 to 1000 times, and computing the BCR for each combination, a pdf and cdf can be generated for evaluating the cost effectiveness of the time clock.

7.7.3 *Construction Contingency Simulation Example:*

7.7.3.1 Contingency analysis is routinely used by cost engineers in estimating the costs of construction projects. A contingency is a cost element included in project cost estimation to cover costs that have some likelihood of occurrence, but whose amounts cannot be predicated with certainty. By adding the contingency to the line-item estimate of project cost, the cost engineer hopes to project the most likely final cost. Typical uses of contingencies are to cover possible increases in material or labor costs beyond normal escalation, unanticipated developments in applying a first-time technology, changes in project scope due to omission or error, or unforeseen work disruptions from operating in a volatile foreign country.

7.7.3.2 Contingencies are often estimated simply as a percent of the base estimate of project cost. Historical data on the differences between actual and estimated costs for similar projects can be used to determine an average percent of underestimation (or overestimation). The percent can apply to the overall project or to specific elements of the project that are estimated separately. This simple approach is typical in estimating costs of small projects. There is no distinction, however, between accounting for risk exposure and risk attitude in the contingency estimate.

7.7.3.3 For large construction projects with many uncertain variables, a sophisticated risk-analysis technique based on simulations is sometimes employed in estimating contingencies. It provides decision makers with the probabilities of cost overruns (that is, risk exposure) associated with every possible contingency markup in the relevant range. The following example adapted from S. H. Zaheer[6] illustrates how to use simulation to measure risk exposure when making a cost estimate for a specific construction project. Note that the intent here is to show how useful simulation can be in describing risk of an investment and not to describe every step the computer program takes to do the simulation.

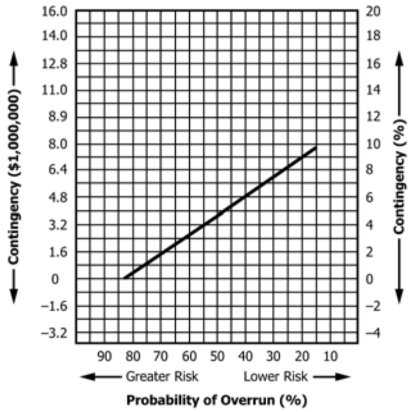
7.7.3.4 Construction cost is being estimated for Project X. It is expected to cost $140 million exclusive of contingencies. Of the $140 million, $60 million are spent dollars or firm commitments. Being relatively certain, they require no consideration for contingency. The other $80 million are uncertain and make up the base on which the contingency is calculated.

7.7.3.5 The process for carrying out a contingency/risk analysis is as follows. Generate subjective probability distributions for every activity that is deemed particularly uncertain. The distributions describe the percent of estimated costs of these activities, where the midpoint is 100 % of the estimated value. Enter these data, along with the estimated dollar costs of both certain and uncertain activities into a computer simulation package. It generates a probability distribution of the contingency percent of total project cost and a graph that plots probability of cost overrun against contingency percent and amount.

7.7.3.6 Fig. 5 shows how the probability of a cost overrun (that is, risk exposure) varies with the contingency adjustment for this construction project. To use the contingency/risk analysis to select a single cost estimate, the decision maker considers risk exposure and risk attitude. Risk exposure, as indicated by the rising probability of cost overrun, increases as the percent contingency markup goes down. Risk exposure decreases as the percent contingency markup increases. Risk attitude enters when the decision maker chooses a contingency amount, thereby establishing a probability of overrun that will be acceptable.

7.7.3.7 The risk neutral decision maker will choose the most likely cost estimate of $144 million, which includes the $140 million without contingency plus a contingency of $4 million (0.05·$80 million of uncertain costs). That is, the vertical axis on the right side of the graph shows the most likely contingency percent (where the probability of overrun is 50 %) to be about 5 %. This assumes an underlying probability distribution that is symmetric.

[6] Zaheer, S.H., "Contingency and Capital Cost Estimates," *Cost Engineers' Notebook*, American Association of Cost Engineers, Morgantown, WV, March 1983, p. 13.



NOTE 1—Adapted from Zaheer, S.H., "Contingency and Capital Cost Estimates," *Cost Engineers' Notebook*, American Association of Cost Engineers, Morgantown, WV, March 1983, p. 13.

FIG. 5 Contingency/Risk Graph for Construction Project X

7.7.3.8 A more risk-averse decision maker might opt for a lower risk of overrun by choosing a larger contingency. For example, if an overrun probability of only 20 % were acceptable, the contingency would be $7.2 million (0.09·$80 million), and the total cost estimate would be $147.2 million ($140 million plus $7.2 million contingency).

7.7.3.9 On the other hand, a risk taker might choose a relatively low contingency. For example, if an overrun probability of 70 % were acceptable, the contingency would be $1.6 million (0.02·$80 million), and the total cost estimate would be $141.6 million ($140 million plus $1.6 million contingency). Note that Fig. 5 is really a cdf, although it differs in two respects from what is typically used in risk analysis. First, Fig. 5 has the axes reversed. The item whose distribution is being measured (contingency percent and amount) is on the vertical axis instead of the horizontal axis, and the cumulative distribution measure (probability of overrun) is on the horizontal axis. Second, the cdf in Fig. 5 measures cumulative probabilities of a value being greater than instead of less than. Thus, for example, there would be about a 35 % probability of a cost overrun with a contingency of 7 %. That is, there is a 35 % probability that the necessary contingency to avoid an overrun would be >7 %.

7.7.4 There are several advantages of the simulation technique applied to risk analysis. First, simulation works with any distribution of input variables, so it is not limited to certain classes of well-behaved distributions.

7.7.5 Second, it can handle interdependencies between inputs; that is, where one input is related to another, the two can be tied together.

7.7.6 Third, simulation can be used in generating pdfs and cdfs for any of the economic measures described in 1.3.

7.7.7 Fourth, while it is true that the cdf describes only risk exposure, it also facilitates incorporation of risk attitude by the decision maker in the decision process. That is, when the decision maker selects projects on the basis of cdfs, inherent in that selection is an implicit assessment of risk attitude.

7.7.8 The disadvantage of simulation is that it requires many calculations, and is therefore practical only when used with a computer.

7.8 *Stochastic Dominance:*

7.8.1 Comparing the stochastic dominance of project worth can help decision makers evaluate returns and risk across competing projects, particularly when decision makers are risk averse. Stochastic dominance investment rules are consistent with the maximum expected utility criterion, which is the optimal investment decision rule for investors.[7] A necessary condition for stochastic dominance is that the expected mean return of a dominant investment be no less than the expected mean of a dominated investment; however, no such condition exists regarding the variances of competing investments.[7] Advantages of stochastic dominance over the mean-variation criterion is that the mean-variance criterion has been found to (*1*) accept projects no risk averse decision maker would select, and to (*2*) reject projects some risk averse decision makers would find acceptable.[8] A disadvantage is stochastic dominance requires information about the probability distribution of the returns of each project, and more than just their means and variances.[8]

7.8.2 *First Degree Stochastic Dominance:*

7.8.2.1 An investment dominates another by first degree stochastic dominance (FSD) if for every possible return, the probability of a lower return for the FSD dominant investment is equal to or smaller (with a strict inequality for at least one return) than the FSD dominated investment. The expected return will be the largest for an investment that dominates by FSD. An investor, regardless of risk preference (neutral, taking, averse), will prefer a FSD dominant investment over a dominated investment.

7.8.3 *Second Degree Stochastic Dominance:*

7.8.3.1 An investment dominates another by second degree stochastic dominance (SSD) if for every possible return, the probability of a lower return for the SSD dominant investment is equal to or smaller (with a strict inequality for at least one return) than the SSD dominated investment, on average. A SSD dominant investment will be less risky than a dominated investment. In the absence of a FSD dominant investment, a risk averse investor will prefer a SSD dominant investment over a dominated investment. An investment that is FSD dominant is SSD dominant.

7.8.4 *Third Degree Stochastic Dominance:*

7.8.4.1 An investment dominates another by third degree stochastic dominance (TSD) if for every possible return, the average probability of lower returns (over each lesser return) for the TSD dominant investment is equal to or smaller (with a strict inequality for at least one payoff level) than the TSD dominated investment, on average. A TSD dominant investment will have a lower risk of small returns, on average, over a TSD dominated investment. In the absence of a FSD or SSD dominant investment, a risk averse investor will prefer a TSD dominant investment over a dominated investment. An investment that is SSD dominant is TSD dominant.

[7] Levy, H., *Stochastic Dominance: Investment Decision Making under Uncertainty*, Springer, New York, NY, 2006.

[8] Aharony, J., and Loeb, M., "Mean-Variance vs. Stochastic Dominance," *Journal of Banking and Finance*, Vol 1, No. 1, 1977, pp. 95–102.

8. Other Techniques

8.1 The following additional techniques can be used for measuring uncertainty, risk, or both.

8.2 *Certainty Equivalent Technique:*

8.2.1 With the certainty equivalent technique, the decision maker determines a certainty equivalent factor (CEF) by which estimated project net benefits are adjusted to reflect risk exposure and risk attitude. If the CEF is based on risk exposure, for example, it will be exactly 1.0 if the net benefits outcome is perceived as certain, and less than 1.0 if net benefits are perceived as uncertain. Multiplying estimated net benefits by the CEF adjusts the net benefits outcome according to the decision maker's perceived risk exposure. A CEF adjustment yields a certainty equivalent amount a decision maker finds equally acceptable to making that same investment with its uncertain outcome.

8.2.2 If the CEF is also based on risk attitude, then the CEF becomes smaller as aversion to risk increases, thereby making the certainty equivalent net benefits smaller.

8.2.3 One advantage of the CEF technique is that it accounts for risk by a factor that can include both the decision maker's risk attitude and assessment of risk exposure. Another advantage of the CEF technique is that it separates the discounting procedure that accounts for the time value of money from adjustments for risk. By so doing, it allows for differential risk weighting over time, which might be more appropriate than the increasingly heavy discounting for risk over time implicit in risk-adjusted discounting.

8.2.4 A limitation of the CEF technique is the lack of a rigorous, theoretically defensible, mathematical expression for establishing a CEF that combines risk attitude and risk exposure.

8.3 *Input Estimation Using Expected Values:*

8.3.1 Expected value analysis (also known as probability analysis) can be applied to the estimation of uncertain input data. An expected value of an input is found by summing the products of possible input values and their respective probabilities of occurrence. The values of any number of inputs can be estimated by this technique. The expected input values are ultimately used to compute a single-value measure of project worth.

8.3.2 If expected values were used over repeated applications, the differences between actual values and predicted values would tend to be less than if point estimates were used. This is the primary contribution of using expected value analysis to estimate input values.

8.3.3 A major shortcoming of this technique is that a single-value measure of project worth provides no measure of risk exposure. Also, there is no treatment of risk attitude. Finally, applications are limited to problems where input distributions can be developed.

8.4 *Mathematical/Analytical Technique:*

8.4.1 The mathematical/analytical (M/A) technique allows one to generate probability functions for economic measures of worth without the repeated trials of simulation. The M/A technique allows one to generate pdfs and cdfs of life-cycle costs, net benefits, and unadjusted internal rate-of-return measures. The M/A technique requires that input values be normally distributed.

8.4.2 For example, by using the mean and variance of net benefits earned each period on an investment and the probability tables for the normal distribution, pdfs and cdfs can be derived for net benefits of the investment. Also, the probability of investments yielding net benefits less than, equal to, or greater than zero (or some other specified target value) can be computed directly.

8.4.3 A major advantage of the M/A technique is that means and variances (and therefore pdfs and cdfs) of project worth can be generated directly by hand calculations without repeated simulations on a computer. Thus, risk exposure can be determined without the use of a computer.

8.4.4 A disadvantage of the M/A technique is that it presumes that the underlying probability distributions associated with the inputs are normally distributed.

8.4.5 Another disadvantage of the M/A technique is that the input distributions in each period must be of the measure of economic worth. That is, uncertainty expressed in input distributions pertains exclusively to the final measure of project worth for that period. This means that the analyst has to determine how all of the uncertain factors that affect benefits and costs combine in a given period to make that period's net benefits distribution. Since benefits and costs may be correlated among periods, this means the analyst must also make assumptions about how net benefits in other periods affect the distribution for any given period.

8.5 *Portfolio Analysis:*

8.5.1 Portfolio analysis is used to seek the combination of assets with the maximum return for any given degree of risk (that is, variance of the return), or the minimum risk for any given rate of return. By diversifying its assets so that returns are not perfectly positively correlated, a firm's overall investment risk can be reduced. The risk reduction from this diversification of assets is called the portfolio effect.

8.5.2 The evaluation of building investments seems a logical application of portfolio theory. Yet few applied efforts have been made. One reason is that it is difficult to get data to measure the correlation of building investments.

8.5.3 Another reason portfolio analysis is applied infrequently to building investments is that they involve large chunks of capital, large and lumpy chunks of real estate, cumbersome markets, lengthy transaction times, and large transaction costs.

9. Analyze Results and Make a Decision

9.1 For techniques that generate single-value measures of worth, consult ASTM standard guides and practices on the various economic methods to determine how to analyze results and make a decision.

9.2 When using breakeven or sensitivity analysis, the decision maker implicitly incorporates risk exposure and risk attitude when considering how likely it is that the breakeven or assumed value(s) for a project will be achieved and when deciding whether the likelihood of failing to achieve that value is an acceptable risk.

9.3 When using risk-adjusted discounting and decision analysis, no special attention to risk attitude and risk exposure is necessary, since risk evaluation is incorporated in the calculations of project worth.

9.4 The mean-variance criterion, coefficient of variation, and simulation analysis treat measures of worth as distributions rather than as single values. This provides extra information, but it also requires user skill in interpreting the distributions.

9.5 Using the mean-variance criterion, risk averse decision makers will favor the project with the higher mean and lower variance, other things equal.

9.6 The coefficient of variation is helpful when one project has a higher mean and variance than its alternative. Risk averse decision makers will prefer the project with the lower coefficient of variation.

9.7 Simulation analysis provides a cumulative distribution function that gives the probability of a measure of worth being less than or equal to specific values. The function is in effect a graph of risk exposure. Decision makers implicitly consider their risk attitude when deciding if that risk exposure, and therefore the project, is acceptable. Practice E1946 establishes a procedure for measuring cost risk for buildings and building systems using the simulation technique.

10. Choosing the Appropriate Technique

10.1 If it can be agreed that better decisions come from more complete information, then accounting for uncertainty and risk enhances decision making. Yet there is no best technique for handling uncertainty and risk in evaluating every investment. What is best depends on circumstances of the organization. The analyst and decision maker should consider the following items of information before selecting a technique for a given investment problem.

10.1.1 Assess the level of resources and define the set of feasible techniques given those resources. How much time is available to evaluate the project? How much money is available for staff and computer support? Do the technical analysts have the capability to apply all of the techniques? For example, can they develop probability distributions for uncertain variables? And finally, are computer software and hardware available for applying all of the techniques?

10.1.2 Identify the audience that will use the analysis, and consider their reactions to the techniques. Has executive judgment and intuition been successful in the past? Will management understand information generated from applying the techniques? Will they accept the different types of information produced by the techniques?

10.1.3 Consider the approximate size of the investment relative to the institution's budget or portfolio of other investments. For example, if the investment to be analyzed is small relative to the total portfolio, and affects total profitability only slightly if it yields a poor return, then a sophisticated approach to risk analysis is not needed. On the other hand, if the investment is relatively large, and a poor return could bankrupt the institution, then a sophisticated technique might be needed.

10.1.4 Consider whether to use risk attitude in choosing a technique. If the decision maker is risk neutral, for example, decisions can be made on the basis of maximizing net benefits instead of utility, and the procedure to account for risk attitude is unnecessary. If the decision maker is very risk averse or a risk taker, on the other hand, it becomes appropriate to use techniques that can adjust for risk attitude, such as decision analysis, and the risk-adjusted discount rate. This assumes such techniques are acceptable to the decision maker, and, where the decision maker is a group, that a consensus can be reached on their use.

10.1.5 Table 3 is a quick-reference guide to data and

TABLE 3 Characteristics of Techniques for Treating Uncertainty and Risk

	(1)			(2)			(3)		(4)	
Technique	Form of Input Data[A]			Form of Measure of Worth[A]			Consider Risk Exposure[A]		Consider Risk Attitude[A]	
	(S1)	(M1)	(D1)	(S2)	(M2)	D2)	(Ex)	(Im)	(Ex)	(Im)
Breakeven analysis[B]	*			*				*		*
Sensitivity analysis		*			*			*		*
Risk-adjusted discount rate[C]	*		*	*		*	*	*	*	*
Certainty equivalent[C]	*		*	*		*	*	*	*	*
Input estimation using expected values[D]			*	*						
Mean-variance/coefficient of variation			*			*	*			*
Decision analysis			*	*		*	*		*	
Simulation			*			*	*			*
Mathematical/analytical			*			*	*			*

[A] (S1) = single value for each data,
(M1) = multiple values for each data input,
(D1) = distribution of values for each data input,
(Ex) = explicit, numerical measure of risk, and
(Im) = implicit consideration of risk.
(S2) = single-value measure of worth,
(M2) = multiple-value measure of worth,
(D2) = distribution of measure of worth,

[B] Breakeven analysis solves for the value of the uncertain input value that will just make the investment break even (for example, NB = 0, BCR = 1.0, or AIRR = MARR). Thus the economic measure of worth would be a single value.

[C] Risk-adjusted discount rate and certainty equivalent techniques have both single values and distributions of values indicated for the form of data input and for the form of measure of worth. Distributions are cited because both techniques can be used with the mathematical/analytical technique.

[D] Input estimation using expected values does not treat risk exposure or risk attitude. Expected values are computed only for inputs, yielding no information in the single-value measure of worth to help the decision maker consider even implicitly risk attitude.

measure-of-worth characteristics of techniques described in this guide for treating uncertainty and risk.

10.1.5.1 Column 1 indicates the form in which input data are accepted for each technique. The possibilities include single values (S1), multiple values (M1), and probability distributions (D1). For example, the technique breakeven analysis is shown to require single-value data inputs, whereas the simulation technique requires probability distributions. On the other hand, the risk-adjusted discount rate can be used with either single values or probability distribution.

10.1.5.2 Column 2 indicates the form in which the measure of project worth is expressed. It can be expressed as a single value (S2), multiple value (M2), or distribution of values (D2). Input estimation using expected values displays the measure of worth as a single value, for example, whereas the mathematical/analytical approach yields a cumulative distribution of worth values.

10.1.5.3 Column 3 treats risk exposure. Explicit (Ex) consideration means that a numerical adjustment for or graphical measure of risk exposure is provided by the technique. The cumulative distribution of project worth given by the simulation and mathematical/analytical techniques are explicit measures of risk exposure. An implicit (Im) treatment considers risk exposure, but does not treat it quantitatively or graphically. Sensitivity analysis, for example, suggests a possible range of output values, and thereby implies something about risk exposure, but does not quantify that risk exposure.

10.1.5.4 Column 4 treats risk attitude. Explicit consideration occurs when a numerical adjustment is made for risk attitude in project evaluation. Using utility functions with decision analysis is one example. Implicit treatment considers risk attitude, but provides no measure of it in project evaluation. For example, if a cumulative distribution function of project worth is constructed through simulation, implicit in the decision based on that function is the decision maker's attitude towards risk. That is, a risk-averse decision maker might very well make a different decision than a risk taker, given the same profile of risk exposure.

10.1.5.5 Note that the risk-adjusted discount rate and certainty equivalent techniques can be applied to treat risk exposure and attitude either explicitly or implicitly.

10.1.6 Once the question asked earlier about resources, management acceptance, and risk attitude have been answered, Table 3 can help decision makers or analysts select the appropriate technique. Suppose, for example, a decision maker wants to know the range of possible adjusted internal rates of return (AIRRs) an investment might take and at least enough information for the implicit consideration of risk exposure and risk attitude. Furthermore, assume that the decision maker is uncomfortable with the process of generating probability functions of uncertain events and prefers deterministic over nondeterministic answers. Looking at Table 3, the single technique that satisfies these considerations is sensitivity analysis. Multiple (but deterministic) AIRR values are obtained, and information in the form of an array of possible AIRR values is given that helps the decision maker consider risk exposure and attitude implicitly in making a decision. Any other technique that would meet the criteria for this decision maker would violate the constraint that probability functions not be used.

11. Report

11.1 A report of a project economic evaluation should state the objective, the constraints, the alternatives considered, key assumptions and data, and benefits and costs. The Report Section of Practice E917 describes in detail what should be in a report.

11.2 The rationale for adjusting the discount rate for risk in risk-adjusted discounting, constructing distribution functions, determining risk attitude, or for arriving at any other information pertaining to measuring uncertainty or risk should also be documented.

11.3 A generic format for reporting the impacts of uncertainty and risk on measures of a project's economic performance is described in Guide E2204. It provides technical persons, analysts, and researchers a tool for communicating results in a condensed format to management and non-technical persons. The generic format calls for a description of the significance of the project, the analysis strategy, a listing of data and assumptions, and a presentation of measures of economic performance.

12. Keywords

12.1 breakeven analysis; building economics; certainty equivalent; decision analysis; economics; economic evaluation; economic methods; mathematical/analytical technique; probability distribution; risk; risk-adjusted discount rate; risk analysis; risk attitude; risk exposure; sensitivity analysis; simulation; stochastic dominance; uncertainty

ANNEXES

(Mandatory Information)

A1. RISK EXPOSURE MEASURED BY PROBABILITY AND DISTRIBUTION FUNCTIONS

A1.1 A probability distribution quantifies risk exposure by showing probabilities of achieving different economic worth values. Fig. A1.1 is a discrete probability distribution that shows graphically for a building investment a profile of probabilities for the benefit-to-cost ratio (BCR). The discrete probability distribution is also called probability function and probability mass function. Each bar of the histogram shows on the vertical axis the probability of the investment achieving the corresponding BCR on the horizontal axis. The mean (expected value) of the BCR is 2.0. This suggests that the most likely measure of worth will well exceed the 1.0 BCR that is normally regarded as the minimum hurdle necessary for project acceptance.

NOTE A1.1—Measuring the probability of the project's economic worth being less than the target value reveals the risk of accepting an uneconomic project. Another type of risk exposure that some decision makers are concerned about is the probability of passing up a good investment. For example, measuring the probability of the project's economic worth being greater than the target value reveals the risk of rejecting an economic project. This example focuses on the risk of accepting an uneconomic project.

A1.2 Other values for the BCR are possible, however, including a value less than 1.0. Having the standard deviation and mean for the distribution helps the decision maker determine the likelihood that the actual BCR is within acceptable bounds around the mean. The smaller the spread of the distribution, as measured by the standard deviation, the tighter the distribution is around the mean value and the smaller is the risk exposure associated with the project.

A1.3 In a normal distribution the probability is 68.26 %, 95.46 %, and 99.73 %, respectively, that the actual value will be within one, two, and three standard deviations of the mean. Assuming the discrete probability distribution in Fig. A1.1 approximates a normal distribution, one can estimate the probability of the BCR being within any one of the standard deviation ranges. The standard deviation for Fig. A1.1 is found to be 0.72 by the following equation:

$$\sigma = \left(\sum_{s=1}^{N} (\mathrm{BCR}_s - \mu)^2 \cdot P_s \right)^{1/2} \tag{A1.1}$$

where:

σ = standard deviation,
s = possible state,
N = number of possible states,
BCR_s = BCR in sth state,
μ = mean or expected value of the distribution, and
P_s = probability of sth state.

Thus we know, for example, there is a 68.26 % probability that the BCR will lie in the range from 1.28 (2.0 − 0.72) to 2.72 (2.0 + 0.72).

A1.4 Although the probability distribution in Fig. A1.1 does not reveal directly the probability of choosing a project having a BCR less than some target value (that is, less than the expected value of 2.0 in this case), it easily transforms to the cumulative distribution function (cdf) in Fig. A1.2 which does. Any percent on the vertical axis in Fig. A1.2 is read "less than." The function relating BCRs to cumulative probabilities is upward sloping, indicating that the probability of the BCR being less than any given BCR value on the horizontal axis increases as the given BCR value increases.

A1.5 The probability (or risk of exposure) of the BCR being less than 1.0 is 5 % in Fig. A1.2. Or, stated another way, the probability of the project earning positive net benefits or at least breaking even is 95 %. The probability that the BCR is less than the target value (expected value) of 2.0 is 35 %.

A1.6 Probability and distribution functions provide considerably more information about risk exposure than deterministic approaches that assume certainty and provide single-value measures of project worth. But the functions in themselves do not treat risk attitude, that is, they show only risk exposure. Different decision makers, individuals or institutions, may

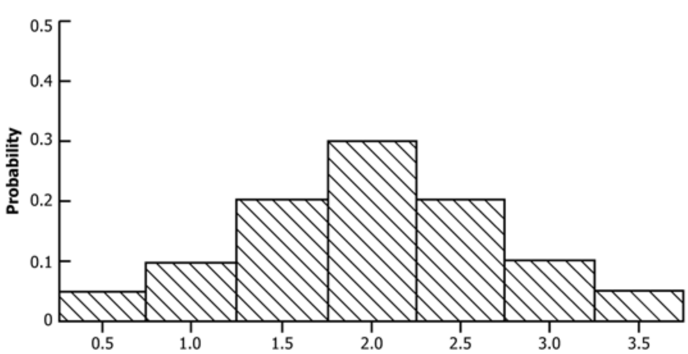


FIG. A1.1 Probability Distribution of Benefit-to-Cost Ratio

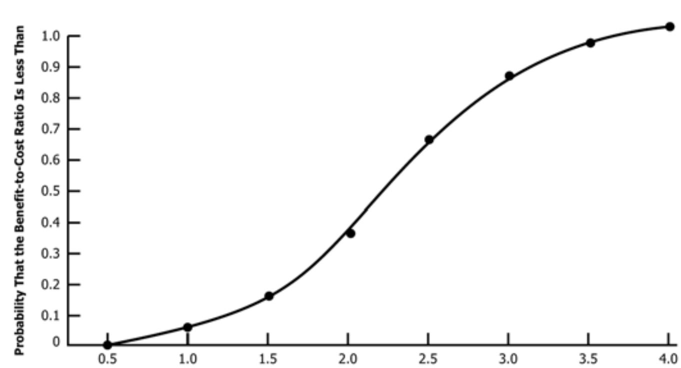


FIG. A1.2 Cumulative Distribution Function of Benefit-to-Cost Ratio respond differently to any given profile of risk exposure. Thus, to make efficient choices when investment outcomes are uncertain, decision makers also need to consider their unique risk attitudes.

NOTE A1.2—An entity such as a corporation has a risk attitude just like an individual. To determine it, however, may be difficult. One approach is to use the risk attitude of the chief executive officer as a proxy for the corporation. Another is to develop a corporate risk policy by getting agreement among the board of directors as might be done for any consensus decision.

A2. RISK ATTITUDE AND UTILITY THEORY

A2.1 There are two general approaches decision makers might follow to incorporate risk attitudes in their project evaluations. First, they can examine the distribution profile, mean, and standard deviation of the measure of project worth to assess their risk exposure, and then make a decision on the basis of their subjective or intuitive perception of whether they are prepared to accept the degree of risk exposure indicated. This informal approach allows for the consideration of risk attitude, but lacks any standard procedure for measuring personal or institutional risk attitude when making a choice. For example, if the investment decision is to accept or reject a single project, this approach is often adequate. Thus the project described by Fig. A1.2 is likely to be deemed cost effective by all but the most risk averse decision makers since there is little probability of a BCR less than 1.0.

A2.2 Even where several projects are being compared, the informal approach for considering risk attitude may be adequate. Although the preferred choice may not be obvious from an examination of probability density functions (pdfs) for individual projects, it may become obvious when functions for alternative projects are superimposed as shown in Fig. A2.1 and Fig. A2.2. Here the probability profiles are a good index to project choice because Project A clearly has stochastic dominance over Project B. That is, for every BCR value in Fig. A2.1 and Fig. A2.2, there is as high or higher probability that Project A will exceed that BCR than will Project B. In other words, for every BCR, there is as high or higher probability that Project B will provide a lower or equal BCR than Project A. Thus the project alternative whose function is farthest to the right is the preferred alternative.

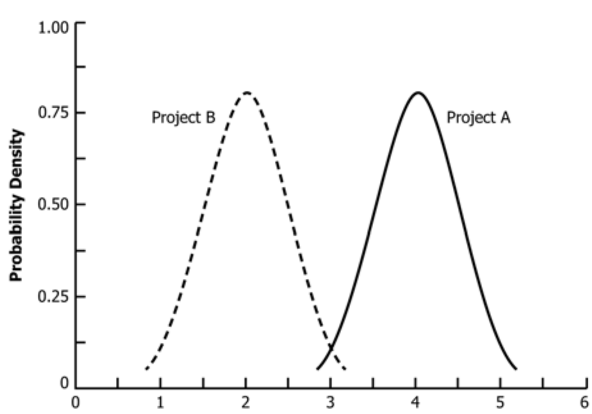


FIG. A2.1 Probability Density Functions of Benefit-to-Cost Ratio for Projects A and B

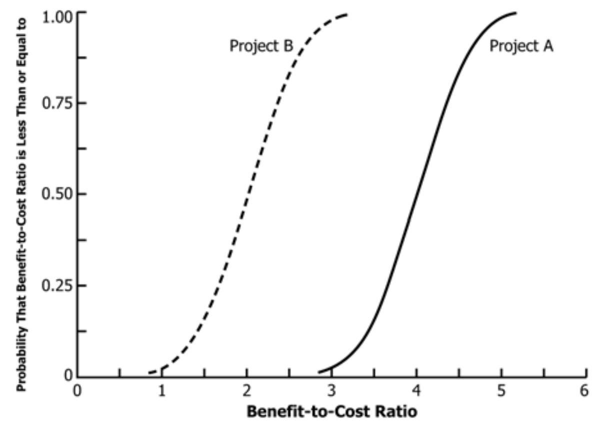


FIG. A2.2 Cumulative Distribution Functions of Benefit-to-Cost Ratio for Projects A and B

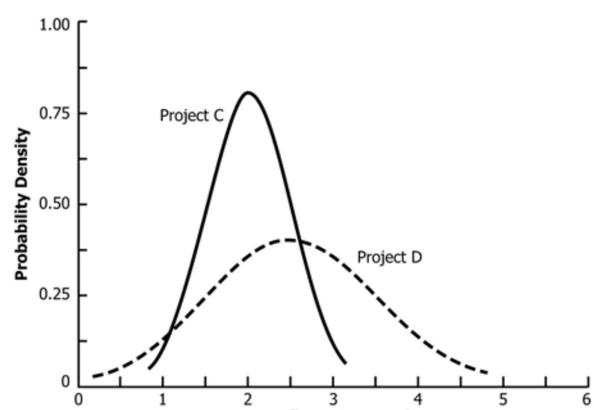


FIG. A2.3 Intermingled Probability Density Functions of Benefit-to-Cost Ratio for Projects C and D

NOTE A2.1—If life-cycle costs of alternatives were measured on the horizontal axis instead of BCRs, the alternative farthest to the left would be preferred because the objective function would be to minimize life-cycle costs rather than to maximize the BCR.

A2.3 The second approach for considering risk attitude is considered formal because it employs a standard procedure for measuring the decision maker's attitude towards risk and then uses that measure in evaluating the economic worth of a risky project. The need for a formal technique is illustrated by the intermingled distributions shown in Fig. A2.3 and Fig. A2.4. Although Project D has the larger mean, it also has the larger variance. That is, the project with the greater expected return also has the greater variance or risk of exposure. There is no

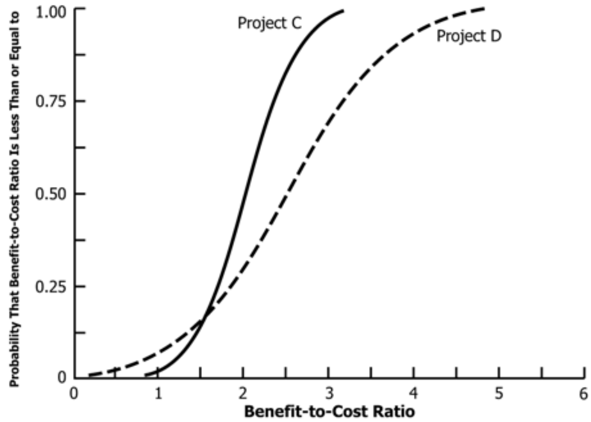
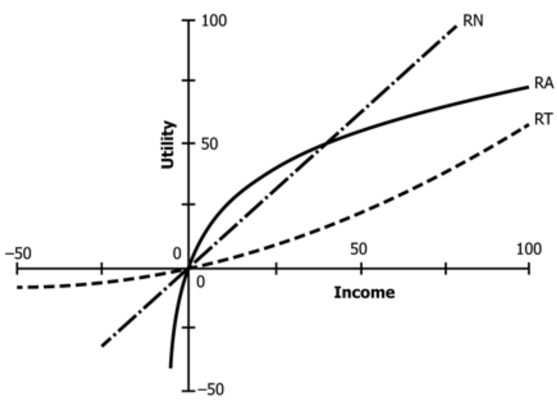


FIG. A2.4 Cumulative Distribution Functions of Benefit-to-Cost Ratio for Projects C and D



NOTE 1—RN = risk neutral,
RA = risk averse, and
RT = risk taking.

FIG. A2.5 Three Types of Risk Attitude

clear indication of stochastic dominance or project preference. Some procedure for including risk attitude in project evaluation is required to establish project preference in this case.

A2.4 *Risk Attitude Measurement and Interpretation*—Risk attitude can be measured and interpreted through the application of utility theory, as follows:

A2.4.1 Risk attitude is measured by the tradeoffs decision makers will make between uncertain money payoffs of known probability and sure money payoffs. These tradeoffs are determined by asking decision makers to specify how much sure money (the certainty equivalent) must be received to make them indifferent between the certainty equivalent and the expected value of a given amount that is not certain. For example, if a person were given a choice between a lottery, say with a 50 % probability of winning $1000 and a 50 % probability of winning nothing, and a sure or certain amount of money, there would be some amount of that certain payment at which the decision maker would be indifferent between the lottery and the sure payment. The revealed tradeoffs between the expected value of the lottery and the sure payment determine whether a decision maker is risk neutral, risk averse, or a risk taker. Once several such tradeoffs have been specified, a relationship between money and utility can be determined.

A2.4.2 A graph of this money-utility relationship, called a utility function, can be used to show the decision maker's risk attitude. Formal techniques exist for using utility functions in conjunction with economic methods of project evaluation. These techniques help decision makers choose among competing projects that do not exhibit stochastic dominance like C and D in Fig. A2.3 and Fig. A2.4.

A2.4.3 Fig. A2.5 shows three shapes of utility functions. Each shape represents one of three different risk attitudes—risk neutral, risk averse, and risk taking. Utility values, displayed on the vertical axis, are arbitrary units used to measure the degree of utility or satisfaction associated with a given amount of money shown on the horizontal axis. The utility function reflects a particular relationship between satisfaction, a subjective value, and monetary amounts. Thus the utility function is unique to one individual, firm, or institution. Each decision maker is likely to have a different utility function.

A2.4.4 In using the utility function in an economic analysis, it is assumed that a decision maker will be indifferent among investments with the same expected utility, and prefer Investment A to Investment B if the expected utility is greater for A than B. The procedure for using the utility function to choose among alternative investments is as follows: (*1*) find from the function, for each investment alternative, utility values that correspond to each dollar-valued outcome in the probability distribution of potential outcomes; (*2*) find the expected utility value (the sum of outcome utilities weighed by outcome probabilities) for each investment; and (*3*) select the investment with the maximum expected utility.

A2.4.5 Given this general background on the construction and use of utility functions, the three utility functions in Fig. A2.5 can now be interpreted in some detail. For the straight-line utility function (RN), each additional, fixed increment of income yields a constant increase in utility; that is, the marginal utility of income is constant. The decision maker is considered risk neutral because the gain or loss of a large amount of money would yield the same increase or decrease respectively in utility per dollar as would the gain or loss of a small amount of money.

A2.4.6 Since the shape of the utility function is dependent on tradeoffs between uncertain money payoffs of known probability and sure money payoffs, it is also helpful to consider risk attitude directly in terms of how a decision maker reacts to such gambles. Decision makers who are risk neutral are called EMV'ers because they act on the basis of expected monetary value (EMV). For example, the worth or EMV of the lottery described earlier is calculated as follows:

$$EMV = 0.5(\$1000) + 0.5(\$0.00) = \$500 \tag{A2.1}$$

A2.4.7 An EMV'er would be indifferent to the lottery or a sure cash payment of $500. Hence, the decision maker is risk neutral in the sense of being willing to accept a fair gamble. The utility function for a risk-neutral decision maker is a straight line, because there is a constant tradeoff between satisfaction in utility and dollar amounts. An implicit assumption in many economic analyses is that decision makers are EMV'ers. Thus there is no explicit consideration of risk attitude because maximizing the expected value of net benefits is assumed to be equivalent to maximizing expected utility.

A2.4.8 If the utility function bends over to the right (RA in Fig. A2.5), the decision maker is risk-adverse. Interestingly large dollar amounts are required to achieve constant increments of utility; that is, the marginal utility of income is diminishing. This means that a decision maker would prefer a sure payment that is *less* than the expected value of the lottery to the chance of participating in the lottery. In other words, the amount the decision maker is willing to pay for the lottery ticket will be less than its expected value because of aversion to the risk of the lottery's outcome. This implies that decision makers regard marginal payoffs to be worth less (that is, to be of less utility), per dollar, as total payoffs increase. Thus, to determine the true value of investments for risk-averse decision makers, economic analyses must account for decreasing satisfaction of higher payoffs with corresponding decreases in marginal utility.

A2.4.9 If the utility function bends upward to the left (RT in Fig. A2.5), the decision maker is a risk taker. Successively smaller dollar amounts are required to achieve constant increments in utility; that is, the marginal utility of income is increasing. This implies that the decision maker would actually pay a premium for a lottery ticket, that is, a value greater than the expected value of the lottery. The reason is that the decision maker regards project payoffs to be worth more (that is, to have more utility), per dollar, as the total payoffs increase. Thus, to determine the true value of investments for risk takers, economic analyses must account for increasing marginal satisfaction of higher payoffs with corresponding increases in marginal utility.

A2.4.10 Expected utility analysis based on subjective utility functions will not always predict the way decision makers will actually choose among alternative investments. Furthermore, individual decision makers are not expected to act rationally and consistently in every investment situation with respect to their revealed utility-money functions. And it is even more unlikely that a group of executives representing a firm will always agree upon and act consistently according to a corporate utility function. These are some of the reasons why utility analysis of investment decisions is not widely practiced.

A2.4.11 Utility analysis may still be useful, however, as long as decision makers generally act as if they had compared expected utilities and as if they considered the odds for the economic choices being evaluated. Furthermore, decision makers must be willing to go through the process to establish a utility function and then commit to using it. Under these conditions, a firm or institution can use utility theory in a normative or prescriptive role to establish risk policy that will consistently direct management to investments that support the firm's or institution's risk attitude.

A2.4.12 Several factors limit application of risk attitude adjustments in practice. First, decision makers may consider the technique impractical for their institution. This may be due in part to a lack of understanding and to an unwillingness to give up some opportunities for personal judgment in project evaluation. Second, there is often considerable difficulty in determining an organization's risk attitude. This arises for a couple of reasons. Because individual decision makers may not want to be bound by an organization's risk policy, they may be unwilling to cooperate in defining that policy. Also, because individuals are often risk averters in their personal frame of reference, they may have difficulty in identifying an institutional risk attitude where a more risk-taking attitude might be appropriate.

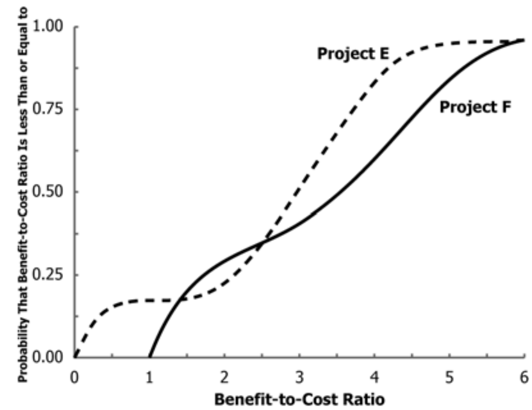


FIG. A2.6 Cumulative Distribution Functions of Benefit-to-Cost Ratio for Projects E and F

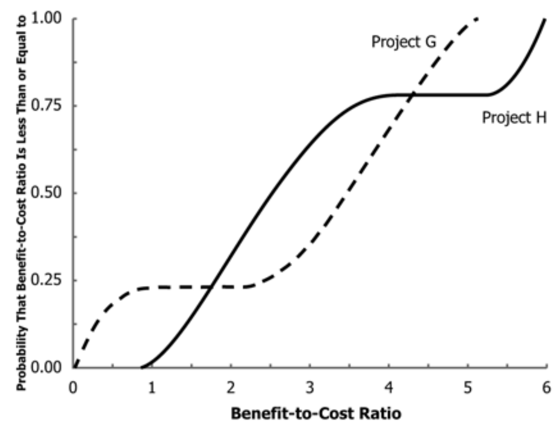


FIG. A2.7 Cumulative Distribution Functions of Benefit-to-Cost Ratio for Projects G and H

A2.4.13 Risk attitude adjustments in project evaluation, however, do have merit. Competent professional assistance in helping decision makers develop and implement a risk policy would overcome many of the limiting factors described earlier. There is a sound theoretical basis for including risk attitude adjustments. And a firm or institution that can establish a risk policy that consistently directs management to investments that support the firm's or institution's risk attitude has an opportunity to select better projects over the long run.

A2.5 Stochastic Dominance Investment Rules for the Risk Averse Decision Maker

A2.5.1 In cases when no investment choice is first degree stochastic dominant (FSD), second (SSD) and third degree stochastic (TSD) dominance investment rules can be used to identify projects that will maximize expected utility of the risk averse decision maker. SSD and TSD can be determined from a comparison of each project's cumulative distribution function (cdf). The following examples use the benefit-to-cost ratio (BCR) to illustrate the degree of stochastic dominance.

A2.5.2 *First Degree Stochastic Dominance:*

A2.5.2.1 The cdf of a FSD dominant investment will lie to the right of the cdf of a FSD dominated investment, and the cdfs will not cross. In Fig. A2.1, Project A dominates Project B by FSD, and by extension, SSD and TSD.

A2.5.3 *Second Degree Stochastic Dominance:*

A2.5.3.1 The cdf of a SSD dominant investment will have a left tail that resides to the right of a SSD dominated investment. In addition, while the cdfs will cross, meaning neither investments exhibit FSD over the other, there will be less total area between the cdfs where a SSD dominant investment lies to the left (or top) of a dominated investment than total area between the cdfs where a SSD dominated investment lies to the left (or top) of a SSD dominant investment—for every payoff level. In Fig. A2.6, Project F dominates Project E by SSD, and by extension TSD, but not by FSD.

A2.5.4 *Third Degree Stochastic Dominance:*

A2.5.4.1 As in the case of SSD, the cdf of a TSD dominant investment will have a left tail that resides to the right of the TSD dominated investment. Also, while the cdfs will cross, there will be less total area between the cdfs where a TSD dominant investment lies to the left of a dominated investment than total area between the cdfs where a TSD dominated investment lies to the left of a TSD dominant investment—on average for every payoff level. In Fig. A2.7, Project H dominates Project G by TSD, but not by FSD or SSD.