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Standard Practice for Measuring Internal Rate of Return and Adjusted Internal Rate of Return for Investments in Buildings and Building Systems[1]

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INTRODUCTION

The internal rate-of-return (IRR) and adjusted internal rate-of-return (AIRR) methods are members of a family of economic evaluation methods that provide measures of economic performance of an investment over time. Other methods in this family of evaluation methods are life-cycle cost analysis, net benefits and net savings analysis, benefit-to-cost and savings-to-investment ratio analysis, and payback analysis.

The IRR and AIRR methods are the topic of a single standard practice because they both measure economic performance as a compound yield on investment. The IRR is the compound rate of interest that, when applied as a discount rate to a project's stream of dollar benefits and costs, will equate them. The AIRR is the overall yield taking into account earnings on receipts reinvested to the end of the study period. The IRR or AIRR is compared against the investor's minimum acceptable rate of return (MARR), and the investment is considered economically attractive if the calculated yield exceeds the MARR. If an investment entails an initial outlay and a single receipt at the end of the study period, there is no difference between the IRR and the AIRR. But if cash flows occur over multiple time periods, the two will normally be different. This arises because the AIRR includes in its measure the return on reinvestment of receipts, whereas the IRR does not.

The AIRR is recommended for most applications in which a measure of yield is desired. Caution is recommended in applying either measure, however, because problems arise under certain conditions.

1. Scope

1.1 This practice covers a procedure for calculating and interpreting the internal rate of return (IRR) and adjusted internal rate of return (AIRR) measures in the evaluation of building designs, systems, and equipment.

1.2 The values stated in inch-pound units are to be regarded as standard. The values given in parentheses are mathematical conversions to SI units that are provided for information only and are not considered standard.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E631 Terminology of Building Constructions
E833 Terminology of Building Economics
E917 Practice for Measuring Life-Cycle Costs of Buildings and Building Systems
E964 Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios for Buildings and Building Systems
E1074 Practice for Measuring Net Benefits and Net Savings for Investments in Buildings and Building Systems
E1121 Practice for Measuring Payback for Investments in Buildings and Building Systems

[1] This practice is under the jurisdiction of ASTM Committee E06 on Performance of Buildings and is the direct responsibility of Subcommittee E06.81 on Building Economics.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

E1185 Guide for Selecting Economic Methods for Evaluating Investments in Buildings and Building Systems
E1369 Guide for Selecting Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Buildings and Building Systems
E1765 Practice for Applying Analytical Hierarchy Process (AHP) to Multiattribute Decision Analysis of Investments Related to Buildings and Building Systems
E1946 Practice for Measuring Cost Risk of Buildings and Building Systems and Other Constructed Projects
E2204 Guide for Summarizing the Economic Impacts of Building-Related Projects

2.2 *ASTM Adjuncts:*[3]
Discount Factor Tables, Adjunct to Practices E917, E964, E1057, E1074, and E1121

3. Terminology

3.1 *Definitions*—For definitions of general terms related to building construction used in this practice, refer to Terminology E631; and for general terms related to building economics, refer to Terminology E833.

4. Summary of Practice

4.1 This practice is organized as follows:
4.1.1 *Section 1, Scope*—Identifies coverage.
4.1.2 *Section 2, Applicable Documents*—Lists ASTM standards that are referenced.
4.1.3 *Section 3, Terminology*—Addresses definitions of terms.
4.1.4 *Section 4, Summary of Practice*—Outlines the contents.
4.1.5 *Section 5, Significance and Use*—Explains the relevance of the IRR and AIRR and indicates their appropriate uses.
4.1.6 *Section 6, Procedure*—Summarizes the steps in IRR and AIRR analysis.
4.1.7 *Section 7, Objectives, Constraints, and Alternatives*—Discusses the first step in an analysis, that is, the identification of the objectives of the analysis, any constraints that must be taken into account in finding a solution, and technically feasible project alternatives.
4.1.8 *Section 8, Data and Assumptions*—Discusses the second step in an analysis; that is the data and assumptions that are typically required for calculating the IRR and AIRR, and, in particular, the requirement of the AIRR for specification of a reinvestment rate.
4.1.9 *Section 9, IRR Calculation*—Describes the third step, performing calculations, as it applies to the IRR.
4.1.10 *Section 10, AIRR Calculation*—Describes the third step, performing calculations, as it applies to the AIRR.
4.1.11 *Section 11, Choosing Between the IRR and AIRR*—Discusses how to choose between the IRR and the AIRR.
4.1.12 *Section 12, Limitations*—Discusses limitations and shortcomings of the IRR and AIRR.
4.1.13 *Section 13, Analysis of IRR or AIRR Results and the Decision*—Discusses the decision criterion and the treatment of uncertainty, risk, and unqualified effects.
4.1.14 *Section 14, Applications*—Describes the types of decisions to which the IRR and AIRR are applicable.
4.1.15 *Section 15, Report*—Identifies information that shall be included in a report of an IRR or AIRR application.

5. Significance and Use

5.1 The IRR method has been used traditionally in finance and economics to measure the percentage yield on investment.
5.1.1 The IRR method is appropriate in most cases for evaluating whether a given building or building system will be economically efficient, that is, whether its time-adjusted benefits will exceed its time-adjusted costs over the period of concern to the decision maker. However, it has deficiencies that limit its usefulness in choosing among projects competing for a limited budget.

5.2 The AIRR method is a measure of the overall rate of return that an investor can expect from an investment over a designated study period. It is appropriate both for evaluating whether a given building or building system will be economically efficient and for choosing among alternatives competing for a limited budget.
5.2.1 The AIRR method overcomes some, but not all, of the deficiencies of the IRR. The AIRR is particularly recommended over the IRR for allocating limited funding among competing projects.

6. Procedure

6.1 The recommended steps for applying the IRR or the AIRR method to an investment decision are summarized as follows:
6.1.1 Identify objectives, constraints, and alternatives.
6.1.2 Compile data and establish assumptions.
6.1.3 Compute IRR or AIRR based on a comparison of two alternatives (one of which may be to do nothing).
6.1.4 Compare the computed IRR or AIRR against the MARR to determine the acceptability of the alternative with the higher investment cost.
6.1.5 If a limited budget is to be allocated among competing alternatives, select alternatives in descending order of their IRR or AIRR measures until the budget is exhausted.
6.1.6 Report the results.

7. Objectives, Constraints, and Alternatives

7.1 Specify clearly the objective of the economic analysis.
7.1.1 Suppose, for example, an individual or organization has funds on hand to invest in real estate projects. The problem is which projects to choose from potential candidates. The objective of the economic analysis in this case is to identify the project or set of projects within the budget that is expected to maximize profits over the long run.

7.2 Identify any constraints that narrow the field of candidates.
7.2.1 Constraints, for example, might include a budget of $1 000 000; a geographical limitation to buildings located

[3] Available from ASTM International Headquarters. Order Adjunct No. ADJE091703.

within 100 km from downtown; and a strong preference for nonresidential property.

7.3 Identify feasible alternatives.

7.3.1 Feasible alternatives include an office building in the suburbs costing \$1 000 000; convenience shopping strips in nearby towns costing a total of \$900 000; two medical/dental offices costing \$500 000 each; and a \$1 000 000 investment share in a downtown shopping complex.

8. Data and Assumption

8.1 To calculate the IRR or AIRR, data are needed.

8.2 Benefit and cost data that are often relevant when calculating the IRR or AIRR are revenues, resale or salvage value, subsidies (for example, grants), and costs of planning, design, engineering, construction, purchase, installation, operation and maintenance, utilities, and repairs and replacement.

8.3 The time of occurrence of each benefit and cost is also needed.

8.4 Taxes such as tax credits, property taxes, and income taxes are also often relevant because they affect benefits and costs. If benefits and costs are adjusted for taxes, the IRR or AIRR measure gives the *after-tax* rate of return.

8.5 If the terms of financing are unique to each alternative, financing costs (and associated tax effects) should also be taken into account.

8.6 Choose a minimum acceptable rate of return (MARR) for comparison against the calculated IRR or AIRR.

8.6.1 The appropriate MARR indicates the investor's opportunity cost of foregoing the return on the next best investment opportunity in order to invest in the project in question.

8.7 If the AIRR is used, a reinvestment rate is needed.

8.7.1 The reinvestment rate is usually set equal to the MARR; hence, it equals the discount rate. This is because the reinvestment rate is an indicator of future opportunity cost, and that is also the purpose of the discount rate. Setting the reinvestment rate and the discount rate equal makes the reinvestment rate assumption in the AIRR method consistent with the reinvestment rate assumption that is implicit in the net benefits (net savings) method (Practice E1074).

9. IRR Calculation

9.1 The IRR is the compound rate of interest that, when used to discount a project's cash flows, will reduce the present value of net benefits (PVNB) to zero. (See Practice E1074 for a discussion of how to compute the PVNB.) The solution value of i^* in Eq 1 is the IRR. It is computed as a decimal, then expressed as a percent.

9.1.1 Find the value of i^* for which:

$$PVNB = \sum_{t=0}^{N} \left(B_t - \bar{C}_t \right) / (1 + i^*)^t = 0 \quad (1)$$

where:

PVNB = present value of net benefits (or, if applied to a cost-reducing investment, present value of net savings (PVNS)),

N = number of discounting periods in the study period,

B_t = dollar value of benefits in period t for the building or system evaluated less the counterpart benefits in period t for the mutually exclusive alternative against which it is compared,

$\bar{C}_t$ = dollar costs, including investment costs, in period t for the building or system evaluated less the counterpart costs in period t for the mutually exclusive alternative against which it is compared, and

i^* = interest rate for which PVNB = 0, that is, the IRR measure expressed as a decimal.

9.2 An algebraic solution of i^* is not possible with Eq 1 for all values of N. Use a computer program with built-in formulas to calculate IRR and AIRR. Or, use a manual approach to approximate the IRR such as the trial-and-error approach, the graphical approach, and an approach that uses simple payback and uniform present value (UPV) factor tables. (See Practice E1121 for a description of payback and the Adjunct on Discount Factor Tables for UPV factors.)

9.2.1 *Trial and Error Solution:*

9.2.1.1 The trial-and-error approach to calculating the IRR entails choosing a trial rate of interest that is expected approximately to balance benefits and costs over the project study period. Then present value calculations are made for that trial rate. (For an illustration of discounting calculations, see Practice E917.) If the PVNB is zero, then the trial rate is the solution value of the internal rate of return. If the PVNB is negative, the trial rate is too high, and a second, lower trial rate is then used. If the PVNB is positive for the original trial rate, then the IRR is higher than the trial rate, and a second, higher trial rate is used. When two trial rates are found such that one yields a PVNB greater than zero and the other a PVNB less than zero, the IRR lies between those rates and can be approximated by interpolation, provided the investment has a unique IRR. Considerable time is saved in the trial-and-error approach if the first trial rate is close to the true rate. One approach is to start with the MARR as the trial rate. If the PVNB is negative with the MARR, then the project is not economically feasible, and no further calculations are necessary. If the PVNB is positive, then select higher trial rates in an attempt to bound the true rate.

9.2.1.2 The UPV factor tables are useful in finding a trial rate. The first step is to sum the undiscounted cash flows (not including the initial cost) and divide the sum by the number of years in the study period (excluding any planning/design/construction period) to obtain an average annual cash flow. Then divide the initial project cost by the average to obtain a rough estimate of simple payback (SPB). The second step is to search the UPV discount factor tables in the row that corresponds to the study period for the UPV factor that is closest to the estimated SPB. (Again exclude any years in the planning/design/construction period.) The rate that appears at the top of the column in which the UPV factor is found is a promising trial rate. The more uniform the annual cash flow, the more likely that this trial rate will be close to the solution rate.

9.2.1.3 Table 1 illustrates the trial-and-error approach for calculating the IRR for an initial investment that yields an uneven yearly cash flow over four years. Columns 2 and 3 list

TABLE 1 Trial-and-Error Solution for Internal Rate of Return

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year (t)	Benefits (B_t)	Costs ($\bar{C}_t$)	Net Cash Flow ($B_t - \bar{C}_t$) (4) = (2) − (3)	SPV Factor for i = 25 %	PVNB at 25 % (6) = (4) × (5)	SPV Factor for i = 22 %	PVNB at 22 % (8) = (4) × (7)
0	0	$10 000	$–10 000	1.000	$–10 000	1.000	$–10 000
1	$ 4 000	3 000	1 000	0.8000	800	0.8197	820
2	11 500	4 500	7 000	0.6400	4 480	0.6719	4 703
3	10 000	4 000	6 000	0.5126	3 076	0.5507	3 304
4	8 000	5 000	3 000	0.4096	1 229	0.4514	1 354
Total			$7 000		$–415		$181

the dollar values of benefits and costs that accrue in each of the four years, and Column 4 shows the net cash flow for each of those years, including the initial investment.

9.2.1.4 From inspecting Column 4 in Table 1, one might expect a relatively high return over four years. Using the approach described in 9.2.1.2 to select a trial rate, the calculated UPV value for four years corresponds in the Adjunct discount tables most closely to a rate of 25 %. Multiplying yearly net cash flows by single present value (SPV) factors for each year based on a 25 % discount rate (Column 5) converts them to equivalent present values (Column 6). Summing the values in Column 6 produces a PVNB of – $415. Since the PVNB is less than zero, the IRR must be lower than the 25 % trial rate. Therefore another, lower rate is chosen, and the SPV factors for the lower rate are multiplied times yearly net cash flows. Using a second trial rate of 22 % (as illustrated in Columns 7 and 8) yields a positive PVNB of $181. Therefore, the IRR must lie between 22 and 25 %. Linear interpolation yields an IRR of 22.9 % as follows:

$$IRR = 22\,\% + \left(\frac{\$181 - \$0}{\$181 + \$415}\right)(25\,\% - 22\,\%) = 22.9\,\% \quad (2)$$

9.2.2 *Graphical Solution:*

9.2.2.1 Another approach to approximating the IRR is to use a graphical technique. The profile of the PVNB for a given investment is plotted for a range of discount rates. The IRR is approximately that rate where the PVNB curve intersects the discount rate axis, that is, where the PVNB is zero. Fig. 1

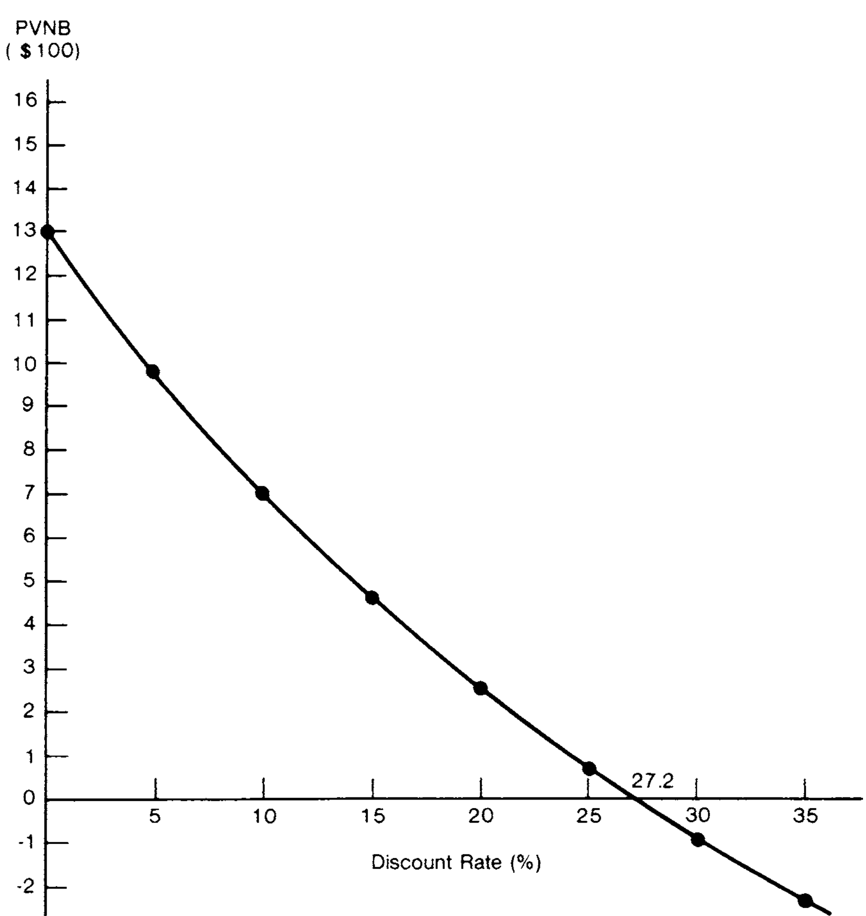

FIG. 1 Graphical Solution of IRR

illustrates a graphical solution of the IRR for the project described in Table 2.

9.2.2.2 Given a discount rate of zero, the PVNB is the arithmetic sum of the net cash flows over time (that is, $1300 for the project described in Table 2). Therefore the function intersects the vertical axis at $1300. Using the discount rates labeled on the horizontal axis of Fig. 1 as trial rates, the PVNB values are calculated as shown in Table 2 and then plotted in Fig. 1.

9.2.2.3 An IRR of 27.2 % is found by visual inspection of the intersection of the profile of PVNB with the horizontal axis. Note that the graphical method usually approximates the solution IRR more closely than does the linear interpolation illustrated earlier with the trial-and-error approach, provided that enough PVNB points above and below the horizontal axis are plotted to form the curve accurately. However, if two trial rates are chosen very close to the true value, the linear interpolation approach can also approximate the true value closely.

9.2.3 *Simple Payback-UPV Factor Table Solution:*

9.2.3.1 A third technique for calculating the IRR works only when the annual net cash flows are constant or change at a constant rate. It works best if there is no planning/design/construction period. The procedure is first to compute simple payback (SPB). The next step is to search through the UPV or UPV* factors in the row corresponding to the study period for the factors that most closely bracket the value of the computed SPB. (UPV* factors, available in the adjunct on Discount Factors, are identified by an asterisk, indicating that annual net cash flows change at a constant rate.) Then see what discount rates those factors represent. Linear interpolation using the bracketing UPV or UPV* factors yields an approximation of the IRR. Note in this approach that the IRR is interpolated directly from the UPV or UPV* factor values; whereas, in the trial-and-error method described in 9.2.1, the IRR is interpolated from PVNB figures. In 9.2.1 the UPV tables are used only to find the first trial rate for calculating the PVNB.

9.2.3.2 Eq 3 shows how to calculate SPB, which is the first step in the procedure.

$$SPB = \frac{C_o}{\left(\overline{B - C}\right)} \tag{3}$$

where:

SPB = simple payback time,
C_o = the initial project costs as of the base time, and
$\left(\overline{B-C}\right)$ = constant annual net cash flow, or initial annual value of a net cash flow changing at a constant rate.

9.2.3.3 This approach is illustrated with the following problem. Find the IRR for a project with a one-time initial cost of $1000 and benefits that accrue in a uniform stream of $200 per year for 16 years. Dividing the one-time initial cost of $1000 by the constant annual net cash flow of $200 yields an SPB of 5. By searching through the row where $N = 16$ in the UPV columns of the adjunct factor tables, two UPV factors are found that most closely bracket the value 5. One factor is 5.162 for a discount rate of 18 %, and the other is 4.938 for a discount rate of 19 %. Having bracketed the IRR between 18 % and 19 %, a single value of 18.7 % is approximated through linear interpolation as follows:

$$IRR = 18\,\% + \frac{5.162 - 5.000}{5.162 - 4.938}(19\,\% - 18\,\%) = 18.7\,\% \tag{4}$$

10. AIRR Calculation

10.1 The solution value of $\bar{i}$ in Eq 5 is the AIRR.

$$\sum_{t=0}^{N} \frac{\left(B_t - \hat{C}_t\right)(1+r)^{N-t}}{(1+\bar{i})^N} = \sum_{t=0}^{N} I_t/(1+r)^t \tag{5}$$

where:

$\hat{C}_t$ = dollar costs, excluding investment costs, in period t for the building or system evaluated less counterpart costs in period t for the mutually exclusive alternative against which it is compared,
r = prescribed rate of return on reinvestment of cash flows,
I_t = investment costs in period t on which return is to be maximized,
$\bar{i}$ = the interest rate that equates the two sides of the equation; that is, the AIRR measure expressed as a decimal, and
B_t = is as previously defined.

10.2 Net cash flows $(B_t - \hat{C}_t)$ are carried forward (compounded) at the specified reinvestment rate (r) to the end of the study period (end of the Nth period) and summed. Future investment costs are discounted to present value using r as the discount rate. The value of i that discounts the resulting

TABLE 2 Data for Graphic Solution of IRR

Year (t)	Benefits (B_t)	Costs ($\bar{C}_t$)	Net Cash Flow ($B_t - \bar{C}_t$)	Trial Rates													
				5 %		10 %		15 %		20 %		25 %		30 %		35 %	
				SPV	PVNB	SPV	PVNB	SPV	PVNB	SPV	PVNB	SPV	PVNB	SPV[A]	PVNB	SPV	PVNB
0	0	$−2200	$−2200	1.000	$−2200	1.000	$−2200	1.000	$−2200	1.000	$−2200	1.000	$−2200	1.000	$−2200	1.000	$−2200
1	$1000	0	1000	0.9524	952	0.9091	909	0.8696	870	0.8333	833	0.8000	800	0.7692	769	0.7407	741
2	1500	0	1500	0.9070	1361	0.8264	1240	0.7561	1134	0.6944	1042	0.6400	960	0.5917	888	0.5487	823
3	1000	0	1000	0.8638	864	0.7513	751	0.6575	658	0.5787	579	0.5126	513	0.4552	455	0.4064	406
			$1300		$977		$700		$462		$254		$73		$−88		$−230

[A] The SPV factors for 30 % and 35 % were calculated from the equation $SPV = \frac{1}{(1+i)^a}$ because factors for 30 % and 35 % were unavailable in the Adjunct discount factor tables.

terminal value of net cash flows to a present value equal to present value investment costs is the AIRR.

10.3 The AIRR equals the IRR if the reinvestment rate, *r*, equals the IRR, that is, the solution value of i^*. If, however, *r* is less than i^*, the AIRR is less than the IRR; and if *r* is greater than i^*, the AIRR is greater than the IRR.[4]

10.4 The solution value of $\bar{\imath}$ in Eq 5, expressed as a decimal, is the AIRR. It can be solved for directly if the equation is rearranged as shown in Eq 6.

$$AIRR = -1.0 + (TV/PVI)^{1/N} \tag{6}$$

where:

TV = $\sum_{t=0}^{N} \left(B_t - \hat{C}_t\right)(1+r)^{N-t}$, the terminal (future) value at the end of the study period of net cash flows excluding investment costs, and

PVI = $\sum_{t=0}^{N} I_t/(1+r)^t$, the present value of investment costs.

10.5 Once the *TV* and *PVI* are computed, the AIRR can be solved for directly by substituting *TV, PVI*, and *N* into Eq 6.

10.6 To illustrate the calculation of the AIRR, Eq 6 is used. In 9.2.2, an approximate IRR of 27.2 % was calculated for the investment data in Table 2. Now an AIRR is computed for the modified case where the reinvestment rate is lower than 27.2 %.

10.7 Table 3 illustrates the calculation of terminal values from a reinvestment of the first and second years' cash flow at 15 %. Note that net cash flows are identical to those for the investment described in Table 2.

10.8 The future value of $1000 earning 15 % for two years is $1323, the future value of $1500 earning 15 % for one year is $1725, and the future value of $1000 received at the end of the final year is $1000. The terminal value (combined future values of the positive net cash flows) is $4048. Solving the problem in Table 3 using Eq 6 yields the following:

$$AIRR = -1.0 + \left(\frac{\$4{,}048}{\$2{,}200}\right)^{1/3} = 0.225 \text{ or } 22.5\,\% \tag{7}$$

10.9 Eq 8, which is equivalent to Eq 6, can be used when the reinvestment rate is constant from year to year and the *PVNB* and *PVI* have already been computed.

$$AIRR = -1.0 + (1+r)\left(1 + \frac{PVNB}{PVI}\right)^{1/N} \tag{8}$$

10.9.1 Not having to derive the *TV* simplifies the calculation of the AIRR. (Note that *r* is a constant reinvestment rate and that the *PVNB* and *PVI* have been computed with a discount rate equal to *r*.)

10.10 Eq 9 can be used when the reinvestment rate is constant from year to year and the savings-to-investment ratio (SIR) has already been computed (Practice E964). A comprehensive example linking the SIR method and the AIRR method applied to a building economics problem is provided in Appendix X1.

$$AIRR = -1.0 + (1+r)\,(SIR)^{1/N} \tag{9}$$

10.11 The appropriate value of the reinvestment rate, *r*, is normally best approximated by the MARR, or discount rate, rather than the return on the original investment. Setting *r* higher than the discount rate suggests that the discount rate is too low, that is, it does not adequately reflect the next best investment opportunity. Setting *r* lower than the discount rate suggests that the discount rate is too high to reflect the next best investment opportunity. Thus, the discount rate is normally the appropriate value to substitute for *r* in using Eq 5, Eq 6, Eq 8, or Eq 9 to calculate the AIRR.

11. Choosing Between the IRR and AIRR

11.1 Controversy persists over whether the IRR or the AIRR is a more accurate measure of an investment's return. This controversy centers on how reinvested earnings over the study period are treated by the two methods. The IRR does not explicitly take into account the earnings on an investment's benefit stream or cash payouts during the study period. The AIRR does.

11.2 By incorporating the expected earnings of reinvested net cash flows, the AIRR provides a measure of the rate of return based on the investor's expected position at the end of the study period as compared with the initial position. The AIRR is a more accurate guide than the IRR for selecting that set of projects that will maximize aggregate PVNB over the study period.

11.3 For simple accept/reject decisions, either the IRR or the AIRR will generally suffice, except in cases where there is no unique value of the IRR. For most other decisions, the AIRR is more reliable than IRR for maximizing net benefits. (See Guide E1185 for types of decisions that require economic analysis and recommendations on which economic methods to use.)

12. Limitations

12.1 A limitation of the IRR is that multiple measures can result, such that there is no unique IRR value that serves as a solution value for i^* in Eq 1. This situation can occur when the cash flow shows reversals in sign over the study period. Finding more than one solution value results in confusion when, for example, the MARR falls between two solution values of the IRR. Furthermore, trial-and-error solutions sometimes find only one answer and fail to indicate the existence of

[4] The National Institute of Standards and Technology Building Life-Cycle Cost (BLCC) Computer Program helps users calculate measures of worth for buildings and building components that are consistent with ASTM standards. The program is downloadable from: http://www.eere.energy.gov/femp/information/download_blcc.html.

TABLE 3 Calculation of Terminal Value

Year (t)	Net Cash Flow ($B_t - \hat{C}_t$)	Reinvestment Rate (r)	SCA Factor	Terminal Value
0	$-2200[A]			
1	1000	15 %	1.323	$1323
2	1500	15 %	1.150	1725
3	1000			1000
	$1300			$4048

[A] This is equal to *PVI* in Eq 6.

other mathematically correct solution values. The AIRR, as calculated in Eq 5, Eq 6, Eq 8, and Eq 9, avoids the problem.

12.2 Another limitation of the IRR is that it can result in project selections that do not maximize PVNB of a portfolio for the available budget.

12.3 A limitation that applies to both the IRR and the AIRR is that they tend to be misleading for design and sizing decisions unless they are applied incrementally. For example, the IRR and AIRR on investment in 1 in. thick batt of pipe insulation will usually be higher than on a 2-in. batt, but the 2-in. batt may be more cost effective. This limitation can be overcome by computing the measure on incremental cash flows rather than totals. If, for example, the additional inch of insulation saves more than it costs, its incremental IRR or AIRR will exceed the MARR.

12.4 Unless the reinvestment rate is set equal to the discount rate, the AIRR will not give results consistent with the NB (NS) method, even if the AIRR is applied incrementally. Furthermore, Eq 5 for calculating the AIRR applies only to the case where the reinvestment rate is set equal to the discount rate. If the two rates are different, the equation applies only to the case where there is no more than one sign change in the cash-flow profile.

13. Analysis of IRR or AIRR Results and the Decision

13.1 Take care to interpret correctly the results of the IRR or AIRR computation, as the decision criterion is affected by the objectives of the evaluation. 14.1 treats accept or reject decisions. 14.2 treats sizing and design decisions, where the optimum size or design is based on incremental investment changes. 14.3 treats prioritization decisions, where multiple projects are competing for the available budget.

13.2 In the final investment decision, take into account not only the numerical values of the IRR or AIRR, but also uncertainty of investment alternatives relative to the risk attitudes of the investor, the availability of funding and other cash-flow constraints, any unquantified effects attributable to the alternatives, and the possibility of noneconomic objectives. (These topics are discussed in Section 10 of Practice E917.)

13.2.1 Decision makers typically experience uncertainty about the correct values to use in establishing basic assumptions and in estimating future costs. Guide E1369 recommends techniques for treating uncertainty in parameter values in an economic evaluation. It also recommends techniques for evaluating the risk that a project will have a less favorable economic outcome than what is desired or expected. Practice E1946 establishes a procedure for measuring cost risk for buildings and building systems, using the Monte Carlo simulation technique as described in Guide E1369. Practice E917 provides direction on how to apply Monte Carlo simulation when performing economic evaluations of alternatives designed to mitigate the effects of natural and man-made hazards that occur infrequently but have significant consequences. Practice E917 contains a comprehensive example on the application of Monte Carlo simulation in evaluating the merits of alternative risk mitigation strategies for a prototypical data center.

13.2.2 Describe any significant effects that remain unquantified. Explain how these effects impact the recommended alternative. Refer to Practice E1765 for guidance on how to present unquantified effects along with the computed values of the IRR or AIRR or any other measures of economic performance.

14. Applications[5]

14.1 When the AIRR or the IRR (when it has a unique value) is greater than the MARR, the project is cost effective (that is, the PVNB (PVNS) is greater than zero).

14.2 The AIRR method, when calculated for incremental investment changes, can be used to evaluate how large an investment to make or what design alternatives are cost effective, subject to the limitation noted in 12.4. The PVNB (PVNS) is maximized at that size or design where the incremental AIRR on the last project increment equals or just exceeds the MARR.

14.3 Use the AIRR method to choose among different purpose projects competing for the same budget, subject to the limitation noted in 12.4. The combination of projects that maximizes aggregate PVNB (PVNS) from a limited budget is generally found by undertaking projects in descending order of their AIRRs until the budget is exhausted. Only those projects with AIRRs greater than the MARR are cost effective. If the AIRR falls below the MARR before the available budget is exhausted, terminate project acceptance with the last project whose AIRR exceeds the MARR. It is economically efficient to hold back part of the budget if the remaining available projects have AIRRs less than the MARR. If projects with high AIRRs cost more than the available budget, substitute lower cost projects with lower AIRRs (but AIRRs greater than the MARR) until the budget is exhausted.

14.4 Use the set of projects selected by the AIRR rankings only as a guideline because the AIRR rankings as described in 14.3 will not indicate the optimal project mix in every case where there is a budget constraint. If resources permit, test all combinations of the candidate projects to determine the optimal portfolio where PVNB (PVNS) is maximized.

15. Report

15.1 Include the following information in a report of an IRR or AIRR analysis:

15.1.1 The objective, constraints, and alternatives considered.

15.1.2 Key data and assumptions including:

15.1.2.1 MARR,

15.1.2.2 Study period,

15.1.2.3 Cost data,

15.1.2.4 Benefits (savings) data,

15.1.2.5 Grants and tax deductions when they apply,

15.1.2.6 Financing terms if they are unique to the alternatives considered,

15.1.2.7 Reinvestment rate if the AIRR is used, and

[5] For a comprehensive description of the appropriate applications of the AIRR and the IRR, see Ruegg, R. T. and Marshall, H. E., *Building Economics: Theory and Practice*, Van Nostrand Reinhold, New York, NY, 1990, pp. 67–91.

15.1.2.8 A caution to the IRR user that the calculated IRR may not be realized over the entire study period when there are multi-year cash flows.

15.2 Guide E2204 presents a generic format for reporting the results of an IRR or AIRR analysis. It provides technical persons, analysts, and researchers a tool for communicating results in a condensed format to management and non-technical persons. The generic format calls for a description of the significance of the project, the analysis strategy, a listing of data and assumptions, and a presentation of the computed values of the IRR or AIRR or any other measures of economic performance.

16. Keywords

16.1 adjusted internal rate of return; building economics; discounting; economic evaluation methods; engineering economics; internal rate of return; net benefits; net savings; overall rate of return; payback; savings-to-investment ratio; terminal value

APPENDIX

(Nonmandatory Information)

X1. USING ADJUSTED INTERNAL RATE OF RETURN (AIRR) TO EVALUATE ENERGY EFFICIENCY IMPROVEMENTS IN A HIGH SCHOOL BUILDING

X1.1 *Background*—A high school constructed in 2009 in the greater St. Louis, MO, metropolitan area is subjected to an economic analysis to determine if energy efficiency improvements would be cost effective. The community where the high school is located does not have an energy code requirement, so the 1999 Edition of the ASHRAE 90.1 Standard (**1**)[6] is used as the basis for all energy-related requirements associated with the base case building design. The alternative against which the base case is analyzed uses the 2007 Edition of the ASHRAE 90.1 Standard (**2**) as the basis for all energy-related requirements associated with its building design. The ASHRAE 90.1 1999 Edition is used as the base case because it is assumed to be "common practice" for building design requirements in states with no state-wide energy code (Kneifel, 2012) (**3**). The ASHRAE 90.1 2007 Edition is used as the alternative because it provided the most comprehensive energy-related design requirements when the school was constructed. In addition, information on a similar school design constructed in Louisville, KY, indicated that the ASHRAE 90.1 2007 Edition design option was cost effective vis-à-vis the ASHRAE 90.1 1999 Edition design option (**3**). Both localities are in the same climate zone and have similar heating degree day and cooling degree day requirements.

X1.2 *Data and Assumptions*—Table X1.1 summarizes key assumptions, data elements and data values for the high school building being analyzed. The two-story building has a floor area of 130 000 ft^2 (12 077 m^2). The length of the study period is 25 years, which is less than the service life of the building but long enough to reflect a typical local government planning horizon. The economic analysis uses a 3 % real discount rate (net of general inflation or deflation) to convert future dollar values to present values. Because a real discount rate is being used, all dollar-denominated annual recurring costs and other future costs are expressed in 2009 constant dollars (dollars of uniform purchasing power exclusive of general inflation or deflation). The appropriate value for the reinvestment rate used in computing the AIRR (see Eq 9 in 10.10) follows the guidance given in 10.11 of setting the reinvestment rate to the minimum acceptable rate of return (MARR), which equals the discount rate. The initial investment cost estimates for the base case, ASHRAE 90.1 1999 Edition, and the alternative, ASHRAE 90.1 2007 Edition, are based on data from RS Means CostWorks (**4**). The timing and values for all maintenance, repair, and replacement costs are based on data from Whitestone Research (**5**).

X1.2.1 *Investment Cost Data*—The investment cost data reported in Table X1.1 cover the initial investment cost, the residual value of the high school building at the end of the study period in year 25, the present value (PV) of the residual value, and the PV of replacement costs for energy-related system upgrades. The initial investment cost is already expressed in PV terms, so no discounting is required. The residual value at the end of the study period is a measure of the economic value of the remaining life of the building. The residual value in year 25 is discounted to a PV through use of a single present value (SPV) factor (ASTM Discount Factor Tables Adjunct). The PV of replacement costs for energy-related system upgrades is calculated by multiplying the appropriate SPV factor based on the timing of each replacement item by the dollar value for each replacement item in that time period and summing over all time periods and all replacement items. All four sets of investment costs are separately tabulated for the base case, ASHRAE 90.1 1999 Edition, and the alternative, ASHRAE 90.1 2007 Edition.

X1.2.2 *Energy Cost Data*—The energy fuel types used in the building are natural gas for heating and electricity for cooling and lighting. Unit cost data for electricity and natural gas are based on values reported in (**3**). The product of the annual energy requirement for each fuel type and the unit cost for the fuel type equals the annual fuel cost in the first year. Although both electricity and natural gas are treated as annual expenditures, the rate at which their prices change fluctuates over time. These fluctuations are referred to as escalation rates. The escalation rates used in this analysis and the associated

[6] The boldface numbers in parentheses refer to a list of references at the end of this standard.

TABLE X1.1 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Data and Assumptions

Data Element	Value
Floor Area	130 000 ft^2 (12 077 m^2)
Study Period	25 Years
Discount Rate	3 % (real)
Reinvestment Rate (MARR)	3 % (real)
Investment Cost Data	
Initial Investment Cost	
ASHRAE 90.1 1999 Edition	$15 922 252
ASHRAE 90.1 2007 Edition	$15 967 212
Residual Value (Year 25)	
ASHRAE 90.1 1999 Edition	$5 412 217
ASHRAE 90.1 2007 Edition	$5 422 416
PV Residual Value	
ASHRAE 90.1 1999 Edition	$2 584 905
ASHRAE 90.1 2007 Edition	$2 589 776
PV Replacement Costs for Energy-Related System Upgrades	
ASHRAE 90.1 1999 Edition	$366 257
ASHRAE 90.1 2007 Edition	$388 167
Energy Cost Data	
Electricity	
Electricity Unit Cost	6.96¢/kWh
Annual Electricity Cost	
ASHRAE 90.1 1999 Edition	$98 358
ASHRAE 90.1 2007 Edition	$84 515
Electricity UPV*	17.60
PV Electricity Cost	
ASHRAE 90.1 1999 Edition	$1 731 096
ASHRAE 90.1 2007 Edition	$1 487 459
Natural Gas	
Natural Gas Unit Cost	$10.80/$kft^3$ ($305.82/$m^3$)
Annual Natural Gas Cost	
ASHRAE 90.1 1999 Edition	$53 351
ASHRAE 90.1 2007 Edition	$53 144
Natural Gas UPV*	19.92
PV Natural Gas Cost	
ASHRAE 90.1 1999 Edition	$1 062 757
ASHRAE 90.1 2007 Edition	$1 058 629
PV Energy Cost	
ASHRAE 90.1 1999 Edition	$2 793 853
ASHRAE 90.1 2007 Edition	$2 546 088
Future Maintenance and Repair Cost Data	
PV Baseline Maintenance and Repair Costs	
ASHRAE 90.1 1999 Edition	$4 311 735
ASHRAE 90.1 2007 Edition	$4 311 735
PV Maintenance and Repair Costs for Energy-Related System Upgrades	
ASHRAE 90.1 1999 Edition	$1 152 319
ASHRAE 90.1 2007 Edition	$1 099 783

discount factors used to convert an annual stream of fuel costs to a PV are based on future fuel prices projected by the Energy Information Administration of the U.S. Department of Energy as reported in (**6**). The Modified Uniform Present Value (UPV*) factor for each fuel type is based on a 25-year study period; it is reported in Table X1.1 as 17.60 for electricity and 19.92 for natural gas. The UPV* factor is applied to the corresponding annual fuel cost to convert the annual fuel cost in the first year to a PV over the 25-year study period. The annual energy requirements for electricity and natural gas are based on simulations from the EnergyPlus software program (**7**) as reported in Kneifel (2011) (**8**) and Lippiatt et al. (2013) (**9**). The EnergyPlus software program takes into account the integrated design nature of a building's systems. Specifically, as the thermal integrity of the building envelope is improved, the load on the HVAC system is reduced. Thus, the capacity requirements for the HVAC system may be reduced. Consequently, some of the increased investment cost for improving the thermal integrity of the building envelope may be partially offset by reductions in HVAC system cost. All energy-related costs are separately tabulated for the base case, ASHRAE 90.1 1999 Edition, and the alternative, ASHRAE 90.1 2007 Edition.

X1.2.3 *Maintenance and Repair Cost Data*—The PV of maintenance and repair costs is broken into two categories. The first category, referred to as Baseline Maintenance and Repair Costs, corresponds to the basic building; these costs exclude all energy-related system upgrades and are independent of any energy-related system upgrades. The second category covers all Energy-Related System Upgrades maintenance and repair costs. The timing and values for each category of maintenance and repair costs, baseline and energy-related system upgrades, are based on data from Whitestone Research (5). All maintenance and repair costs are separately tabulated for the base case, ASHRAE 90.1 1999 Edition, and the alternative, ASHRAE 90.1 2007 Edition.

X1.3 *Adjusted Internal Rate of Return (AIRR) Calculation*—Tables X1.2-X1.6 provide the information needed to calculate the AIRR for the ASHRAE 90.1 2007 design option. Table X1.7 provides values for the data elements in Eq 9 required for the AIRR calculation and the resultant value of the AIRR. All dollar values reported in Tables X1.2-X1.6 are expressed in PV. A straightforward method for calculating the AIRR requires a value for the SIR. That method, expressed mathematically by Eq 9, is used in this appendix. Thus, the first step in calculating the AIRR for the ASHRAE 90.1 2007 design option is to calculate the SIR for the ASHRAE 90.1 2007 design option. Tables X1.2 and X1.3 provide the basis for calculating the values that go into the numerator (savings) and denominator (investment) of the SIR. The columns in Tables X1.2 and X1.3 are numbered to better illustrate how the resultant values are calculated. Table X1.2 reports the values used to calculate PV Investment Cost for the base case and the alternative. Column 2 contains the initial investment cost, Column 3 contains the PV of all energy-related replacement costs, and Column 4 contains the PV of the residual value. Following the procedure laid out in the life-cycle cost standard (Practice E917), PV Investment Cost equals initial investment cost (Column 2) plus PV replacement costs (Column 3) minus PV residual value (Column 4). The resultant PV Investment Cost is $13 703 604 for the base case and $13 765 603 for the alternative. Note that PV investment cost for the alternative is greater than PV investment cost for the base case. This difference in investment costs between the alternative and the base case equals the PV Incremental Investment Cost associated with the alternative's energy efficiency improvements; it becomes the denominator of the SIR. Table X1.3 reports the values used to calculate PV Non-Investment Cost for the base case and the alternative. Column 2 contains PV energy cost, Column 3 contains the PV of the baseline maintenance and repair costs, and Column 4 contains the PV of maintenance and repair costs for energy-related system upgrades. Following the procedure laid out in the life-cycle cost standard, PV Non-Investment Cost equals PV energy cost (Column 2) plus PV of the baseline maintenance and repair costs (Column 3) plus PV of maintenance and repair costs for energy-related system upgrades (Column 4). The resultant PV Non-Investment Cost is $8 257 907 for the base case and $7 957 606 for the alternative. Note that PV non-investment cost for the alternative is less than PV non-investment cost for the base case. This difference in non-investment costs between the base case and the alternative equals the PV Cost Savings associated with the alternative's energy efficiency improvements; it becomes the numerator of the SIR. Table X1.4 provides the data needed to calculate PV Incremental Investment Cost, the denominator of the SIR. Column 1 of Table X1.4 contains the PV investment cost for the alternative; it is transferred from the appropriate row in Column 5 of Table X1.2. Column 2 of Table X1.4 contains the PV investment cost for the base case; it is transferred from the appropriate row in Column 5 of Table X1.2. PV Incremental Investment Cost recorded in Column 3 of Table X1.4 equals Column 1 minus Column 2. The resultant value is $61 999. Table X1.5 provides the data needed to calculate PV Cost Savings, the numerator of the SIR. Column 1 of Table X1.5 contains the PV non-investment cost for the base case; it is transferred from the appropriate row in Column 5 of Table X1.3. Column 2 of Table X1.5 contains the PV non-investment cost for the alternative; it is transferred from the appropriate row in Column 5 of Table X1.3. PV Cost Savings recorded in Column 3 of Table X1.5 equals Column 1 minus Column 2. The resultant value is $300 301. The numerator of the SIR, PV Cost Savings, is entered in Column 1 of Table X1.6; the denominator of the SIR, PV Incremental Investment Cost, is entered in Column 2 of Table X1.6. The resultant value of 4.84 for the SIR, recorded in Column 3 of Table X1.6, equals Column 1 divided by Column 2. Table X1.7 provides the values for the three data elements (the reinvestment rate expressed as a decimal (r), the number of years in the study period (N) and the savings-to-investment ratio (SIR)) in Eq 9 required to calculate the value of the AIRR expressed as decimal. The values of r, N, and SIR are recorded in Columns 1, 2, and 3 of Table X1.7. Inserting these values into Eq 9 produces an AIRR Value of 0.0971; it is recorded in Column 4 of Table X1.7. Converting the decimal value of the AIRR to a percentage yields a value of 9.71 %; it is recorded in Column 5 of Table X1.7.

TABLE X1.2 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Calculation of Investment Costs

Energy-Related Design Option	Initial Investment Cost	Present Value Replacement Costs for Energy-Related System Upgrades	Present Value Residual Value	Present Value Investment Costs
(1)	(2)	(3)	(4)	(5) = (2) + (3) - (4)
ASHRAE 90.1 1999 Edition	$15 922 252	$366 257	$2 584 905	$13 703 604
ASHRAE 90.1 2007 Edition	$15 967 212	$388 167	$2 589 776	$13 765 603

TABLE X1.3 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Calculation of Non-Investment Costs

Energy-Related Design Option	Present Value Energy Cost	Present Value Baseline Maintenance and Repair Costs	Present Value Maintenance and Repair Costs for Energy-Related System Upgrades	Present Value Non-Investment Costs
(1)	(2)	(3)	(4)	(5) = (2) + (3) + (4)
ASHRAE 90.1 1999 Edition	$2 793 853	$4 311 735	$1 152 319	$8 257 907
ASHRAE 90.1 2007 Edition	$2 546 088	$4 311 735	$1 099 783	$7 957 606

TABLE X1.4 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Calculation of Incremental Investment Cost

Present Value Investment Cost Alternative	Present Value Investment Cost Base Case	Present Value Incremental Investment Cost
(1)	(2)	(3) = (1) - (2)
$13 765 603	$13 703 604	$61 999

TABLE X1.5 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Calculation of Cost Savings

Present Value Non-Investment Cost Base Case	Present Value Non-Investment Cost Alternative	Present Value Cost Savings
(1)	(2)	(3) = (1) - (2)
$8 257 907	$7 957 606	$300 301

TABLE X1.6 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Calculation of Savings-to-Investment Ratio (SIR)

Present Value Cost Savings	Present Value Incremental Investment Cost	Savings-to-Investment Ratio (SIR)
(1)	(2)	(3) = (1)/(2)
$300 301	$61 999	4.84

TABLE X1.7 Economic Evaluation of Energy Efficiency Improvements in a High School Building: Calculation of Adjusted Internal Rate of Return (AIRR)

Reinvestment Rate (r)	Length of Study Period in Years (N)	Savings-to-Investment Ratio (SIR)	Adjusted Internal Rate of Return (Expressed as a Decimal)	Adjusted Internal Rate of Return (Expressed as a Percent)
(1)	(2)	(3)	(4)	(5)
0.03	25	4.84	0.0971	9.71 %

X1.4 *Decision*—An AIRR of 9.71 % demonstrates that the additional investment in energy efficiency associated with the ASHRAE 90.1 2007 design option is cost effective. Recall that cost effectiveness only requires the AIRR to be greater than the minimum acceptable rate of return (see 14.1). Given that the energy-related system upgrades associated with the ASHRAE 90.1 2007 design option are focused on improving energy efficiency, it is instructive to also examine the PV of energy savings associated with the ASHRAE 90.1 2007 design option. Reference to Column 2 of Table X1.3 shows that the PV of energy costs for the base case is $2 793 853 whereas the PV of energy costs for the alternative is $2 546 088. Thus, the PV of energy savings associated with the alternative is $247 765, which translates into an 8.87 % energy cost savings. The magnitude of the PV of energy savings and the percent reduction in the PV of energy costs, in conjunction with the 9.71 % value for the AIRR, underscore the superior performance of the ASHRAE 90.1 2007 design option.

REFERENCES

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