



# Standard Practice for Reporting Uniaxial Strength Data and Estimating Weibull Distribution Parameters for Advanced Graphites<sup>1</sup>

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## 1. Scope\*

1.1 This practice covers the reporting of uniaxial strength data for graphite and the estimation of probability distribution parameters for both censored and uncensored data. The failure strength of graphite materials is treated as a continuous random variable. Typically, a number of test specimens are failed in accordance with the following standards: Test Methods C565, C651, C695, C749, Practice C781 or Guide D7775. The load at which each specimen fails is recorded. The resulting failure stresses are used to obtain parameter estimates associated with the underlying population distribution. This practice is limited to failure strengths that can be characterized by the two-parameter Weibull distribution. Furthermore, this practice is restricted to test specimens (primarily tensile and flexural) that are primarily subjected to uniaxial stress states.

1.2 Measurements of the strength at failure are taken for various reasons: a comparison of the relative quality of two materials, the prediction of the probability of failure for a structure of interest, or to establish limit loads in an application. This practice provides a procedure for estimating the distribution parameters that are needed for estimating load limits for a particular level of probability of failure.

## 2. Referenced Documents

### 2.1 ASTM Standards:<sup>2</sup>

- C565 Test Methods for Tension Testing of Carbon and Graphite Mechanical Materials
- C651 Test Method for Flexural Strength of Manufactured Carbon and Graphite Articles Using Four-Point Loading at Room Temperature
- C695 Test Method for Compressive Strength of Carbon and Graphite

- C749 Test Method for Tensile Stress-Strain of Carbon and Graphite
- C781 Practice for Testing Graphite and Boronated Graphite Materials for High-Temperature Gas-Cooled Nuclear Reactor Components
- D4175 Terminology Relating to Petroleum Products, Liquid Fuels, and Lubricants
- D7775 Guide for Measurements on Small Graphite Specimens
- E6 Terminology Relating to Methods of Mechanical Testing
- E178 Practice for Dealing With Outlying Observations
- E456 Terminology Relating to Quality and Statistics

<sup>1</sup> This practice is under the jurisdiction of ASTM Committee D02 on Petroleum Products, Liquid Fuels, and Lubricants and is the direct responsibility of Subcommittee D02.F0 on Manufactured Carbon and Graphite Products.

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<sup>2</sup> For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards volume information, refer to the standard's Document Summary page on the ASTM website.

## 3. Terminology

3.1 Proper use of the following terms and equations will alleviate misunderstanding in the presentation of data and in the calculation of strength distribution parameters.

### 3.2 Definitions:

3.2.1 *estimator, n*—a well-defined function that is dependent on the observations in a sample. The resulting value for a given sample may be an estimate of a distribution parameter (a point estimate) associated with the underlying population. The arithmetic average of a sample is, for example, an estimator of the distribution mean.

3.2.2 *population, n*—the totality of valid observations (performed in a manner that is compliant with the appropriate test standards) about which inferences are made.

3.2.3 *population mean, n*—the average of all potential measurements in a given population weighted by their relative frequencies in the population.

3.2.4 *probability density function, n*—the function  $f(x)$  is a probability density function for the continuous random variable  $X$  if:

$$f(x) \geq 0 \tag{1}$$

and

$$\int_{-\infty}^{\infty} f(x) dx = 1 \tag{2}$$

The probability that the random variable  $X$  assumes a value between  $a$  and  $b$  is given by:

\*A Summary of Changes section appears at the end of this standard

$$\Pr(a < X < b) = \int_a^b f(x) dx \quad (3)$$

3.2.5 *sample, n*—a collection of measurements or observations taken from a specified population.

3.2.6 *skewness, n*—a term relating to the asymmetry of a probability density function. The distribution of failure strength for graphite is not symmetric with respect to the maximum value of the distribution function; one tail is longer than the other.

3.2.7 *statistical bias, n*—inherent to most estimates, this is a type of consistent numerical offset in an estimate relative to the true underlying value. The magnitude of the bias error typically decreases as the sample size increases.

3.2.8 *unbiased estimator, n*—an estimator that has been corrected for statistical bias error.

3.2.9 *Weibull distribution, n*—the continuous random variable  $X$  has a two-parameter Weibull distribution if the probability density function is given by:

$$f(x) = \left(\frac{m}{\beta}\right) \left(\frac{x}{\beta}\right)^{m-1} \exp\left[-\left(\frac{x}{\beta}\right)^m\right] \quad x > 0 \quad (4)$$

$$f(x) = 0 \quad x \leq 0 \quad (5)$$

and the cumulative distribution function is given by:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^m\right] \quad x > 0 \quad (6)$$

or

$$F(x) = 0 \quad x \leq 0 \quad (7)$$

where:

$m$  = Weibull modulus (or the shape parameter) ( $> 0$ ), and  
 $\beta$  = scale parameter ( $> 0$ ).

3.2.9.1 *Discussion*—The random variable representing uniaxial tensile strength of graphite will assume only positive values, and the distribution is asymmetrical about the population mean. These characteristics rule out the use of the normal distribution (as well as others) and favor the use of the Weibull and similar skewed distributions. If the random variable representing uniaxial tensile strength of a graphite is characterized by Eq 4, Eq 5, Eq 6, and Eq 7, then the probability that the tested graphite will fail under an applied uniaxial tensile stress,  $\sigma$ , is given by the cumulative distribution function:

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \quad \text{for } \sigma > 0 \quad (8)$$

and

$$P_f = 0 \quad \text{for } \sigma \leq 0 \quad (9)$$

where:

$P_f$  = the probability of failure, and  
 $\sigma_0$  = the Weibull characteristic strength.

3.2.9.2 *Discussion*—The Weibull characteristic strength depends on the uniaxial test specimen (tensile, compression and flexural) and may change with specimen geometry. In addition, the Weibull characteristic strength has units of stress and should be reported using units of MPa or GPa.

3.3 For definitions of other statistical terms, terms related to mechanical testing, and terms related to graphite used in this practice, refer to Terminologies D4175, E6, and E456, or to appropriate textbooks on statistics (1-5).<sup>3</sup>

#### 3.4 Nomenclature:

$F(x)$	= cumulative distribution function
$f(x)$	= probability density function
$\mathcal{L}$	= likelihood function
$m$	= Weibull modulus
$\hat{m}$	= estimate of the Weibull modulus
$\hat{m}_U$	= unbiased estimate of the Weibull modulus
$N$	= number of specimens in a sample
$P_f$	= probability of failure
$t$	= intermediate quantity used in calculation of confidence bounds
$X$	= random variable
$x$	= realization of a random variable $X$
$\beta$	= Weibull scale parameter
$\hat{\mu}$	= estimate of mean strength
$\sigma$	= uniaxial tensile stress
$\sigma_i$	= maximum stress in the $i$ th test specimen at failure
$\sigma_0$	= Weibull characteristic strength (associated with a test specimen)
$\hat{\sigma}_0$	= estimate of the Weibull characteristic strength

## 4. Summary of Practice

4.1 This practice provides a procedure to estimate Weibull distribution parameters from failure data for graphite data tested in accordance with applicable ASTM test standards. The procedure consists of computing estimates of the biased Weibull modulus and Weibull characteristic strength. If necessary, compute an estimate of the mean strength. If the sample of failure strength data is uncensored, compute an unbiased estimate of the Weibull modulus, and compute confidence bounds for both the estimated Weibull modulus and the estimated Weibull characteristic strength. Finally, prepare a graphical representation of the failure data along with a test report.

## 5. Significance and Use

5.1 Two- and three-parameter formulations exist for the Weibull distribution. This practice is restricted to the two-parameter formulation. An objective of this practice is to obtain point estimates of the unknown Weibull distribution parameters by using well-defined functions that incorporate the failure data. These functions are referred to as estimators. It is desirable that an estimator be consistent and efficient. In addition, the estimator should produce unique, unbiased estimates of the distribution parameters (6). Different types of estimators exist, including moment estimators, least-squares estimators, and maximum likelihood estimators. This practice details the use of maximum likelihood estimators.

5.2 Tensile and flexural specimens are the most commonly used test configurations for graphite. The observed strength values depend on specimen size and test geometry. Tensile and

<sup>3</sup> The boldface numbers in parentheses refer to the list of references at the end of this standard.

flexural test specimen failure data for a nearly isotropic graphite (7) is depicted in Fig. 1. Since the failure data for a graphite material can be dependent on the test specimen geometry, Weibull distribution parameter estimates ( $\hat{m}$ ,  $\hat{\sigma}_0$ ) shall be computed for a given specimen geometry.

5.3 Many factors affect the estimates of the distribution parameters. The total number of test specimens plays a significant role. Initially, the uncertainty associated with parameter estimates decreases significantly as the number of test specimens increases. However, a point of diminishing returns is reached where the cost of performing additional strength tests may not be justified. This suggests a limit to the number of test specimens for determining Weibull parameters to obtain a desired level of confidence associated with a parameter estimate. The number of specimens needed depends on the precision required in the resulting parameter estimate or in the resulting confidence bounds. Details relating to the computation of confidence bounds (directly related to the precision of the estimate) are presented in 8.3 and 8.4.

### 6. Outlying Observations

6.1 Before computing the parameter estimates, the data should be screened for outlying observations (outliers). Provided the experimentalist has followed the prescribed experimental procedure, all test results must be included in the computation of the parameter estimates. Given the experimentalist has followed the prescribed experimental procedure, the data may include apparent outliers. However, apparent outliers must be retained and treated as any other observation in the failure sample. In this context, an outlying observation is one that deviates significantly from other observations in the sample and is an extreme manifestation of the variability of the strength due to non-homogeneity of graphite material, or large disparate flaws, given the prescribed experimental procedure

has been followed. Only where the outlying observation is the result of a known gross deviation from the prescribed experimental procedure, or a known error in calculating or recording the numerical value of the data point in question, may the outlying observation be censored. In such a case, the test report should record the justification. If a test specimen is deemed unsuitable either for testing, or fails before the prescribed experimental procedure has commenced, then this should not be regarded as a test result. However, the null test should be fully documented in the test report. The procedures for dealing with outlying observations are detailed in Practice E178.

### 7. Maximum Likelihood Parameter Estimators

7.1 The likelihood function for the two-parameter Weibull distribution of a censored sample is defined by the expression (8):

$$\mathcal{L} = \left\{ \prod_{i=1}^r \left( \frac{\hat{m}}{\hat{\sigma}_0} \right) \left( \frac{\sigma_i}{\hat{\sigma}_0} \right)^{\hat{m}-1} \exp \left[ - \left( \frac{\sigma_i}{\hat{\sigma}_0} \right)^{\hat{m}} \right] \right\} \prod_{j=r+1}^N \exp \left[ - \left( \frac{\sigma_j}{\hat{\sigma}_0} \right)^{\hat{m}} \right] \quad (10)$$

7.1.1 For graphite material, this expression is applied to a sample where outlying observations are identified under the conditions given in Section 6. When Eq 10 is used to estimate the parameters associated with a strength distribution containing outliers, then  $r$  is the number of data points retained in the sample, that is, data points not considered outliers, and  $i$  is the associated index in the first product. In this practice, the second product is carried out for the outlying observations. Therefore the second product is carried out from ( $j = r + 1$ ) to  $N$  (the total number of specimens) where  $j$  is the index in the second summation. Accordingly,  $\sigma_i$  is the maximum stress in the  $i$ th test specimen at failure. The parameter estimates (the Weibull modulus  $\hat{m}$  and the characteristic strength  $\hat{\sigma}_0$ ) are determined

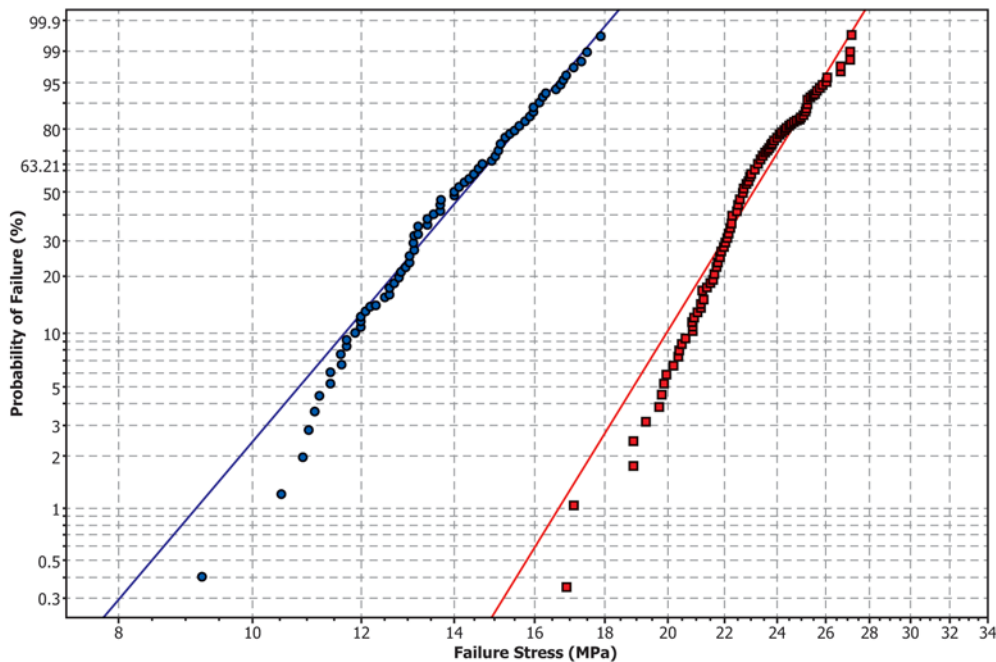


FIG. 1 Failure Strengths for Tensile Test Specimens (left) and Flexural Test Specimens (right) for a Nearly Isotropic Graphite (7)

by taking the partial derivatives of the logarithm of the likelihood function with respect to  $\hat{m}$  and  $\hat{\sigma}_0$  then equating the resulting expressions to zero. Finally, the likelihood function for the two-parameter Weibull distribution for a sample free of outlying observations is defined by the expression:

$$\mathcal{L} = \prod_{i=1}^N \left( \frac{\hat{m}}{\hat{\sigma}_0} \right) \left( \frac{\sigma_i}{\hat{\sigma}_0} \right)^{\hat{m}-1} \exp \left[ - \left( \frac{\sigma_i}{\hat{\sigma}_0} \right)^{\hat{m}} \right] \quad (11)$$

where  $r$  was taken equal to  $N$  in Eq 10.

7.2 The system of equations obtained by differentiating the log likelihood function for a censored sample is given by (9):

and

$$\hat{\sigma}_0 = \left[ \left( \sum_{i=1}^N (\sigma_i)^{\hat{m}} \right) \frac{1}{r} \right]^{1/\hat{m}} \quad (13)$$

where:

$r$  = the total number of observations ( $N$ ) minus the number of outlying observations in a censored sample.

7.3 For a censored sample Eq 12 is solved first for  $\hat{m}$ . Subsequently,  $\hat{\sigma}_0$  is computed from Eq 13. Obtaining a closed form solution of Eq 12 for  $\hat{m}$  is not possible. This expression must be solved numerically.

7.4 When a sample does not require censoring Eq 11 is used for the likelihood function. For uncensored data, the parameter estimates (the Weibull modulus  $\hat{m}$  and the characteristic strength  $\hat{\sigma}_0$ ) are determined by taking the partial derivatives of the logarithm of the likelihood function given by Eq 11 with respect to  $\hat{m}$  and  $\hat{\sigma}_0$  then equating the resulting expressions to zero. The system of equations obtained is given by (9):

and

$$\hat{\sigma}_0 = \left[ \left( \sum_{i=1}^N (\sigma_i)^{\hat{m}} \right) \frac{1}{N} \right]^{1/\hat{m}} \quad (15)$$

For an uncensored sample Eq 14 is solved first for  $\hat{m}$ . Subsequently  $\hat{\sigma}_0$  is computed from Eq 15. Obtaining a closed form solution of Eq 14 for  $\hat{m}$  is not possible. This expression must be solved numerically.

7.5 An objective of this practice is the consistent statistical representation of strength data. To this end, the following procedure is the recommended graphical representation of strength data. Begin by ranking the strength data obtained from laboratory testing in ascending order, and assign to each a ranked probability of failure  $P_f$  according to the estimator:

$$P_f(\sigma_i) = \frac{i - 0.5}{N} \quad (16)$$

where:

$N$  = number of specimens, and

$i$  = the  $i$ th datum.

Compute the natural logarithm of the  $i$ th failure stress, and the natural logarithm of the natural logarithm of  $[1/(1 - P_f)]$  (that is, the double logarithm of the quantity in brackets), where  $P_f$  is associated with the  $i$ th failure stress.

7.6 Create a graph representing the data as shown in Fig. 2. Plot  $\ln\{\ln[1/(1 - P_f)]\}$  as the ordinate, and  $\ln(\sigma)$  as the abscissa. A typical ordinate scale assumes values from +2 to -6. This approximately corresponds to a range in probability of failure from 0.25% to 99.9%. The ordinate axis must be labeled as probability of failure  $P_f$ , as depicted in Fig. 2. Similarly, the abscissa must be labeled as failure stress (flexural, tensile, etc.), preferably using units of MPa.

7.7 Included on the plot should be a line defined by the following mathematical expression:

$$P_f = 1 - \exp \left[ - \left( \frac{\sigma}{\hat{\sigma}_0} \right)^{\hat{m}} \right] \quad (17)$$

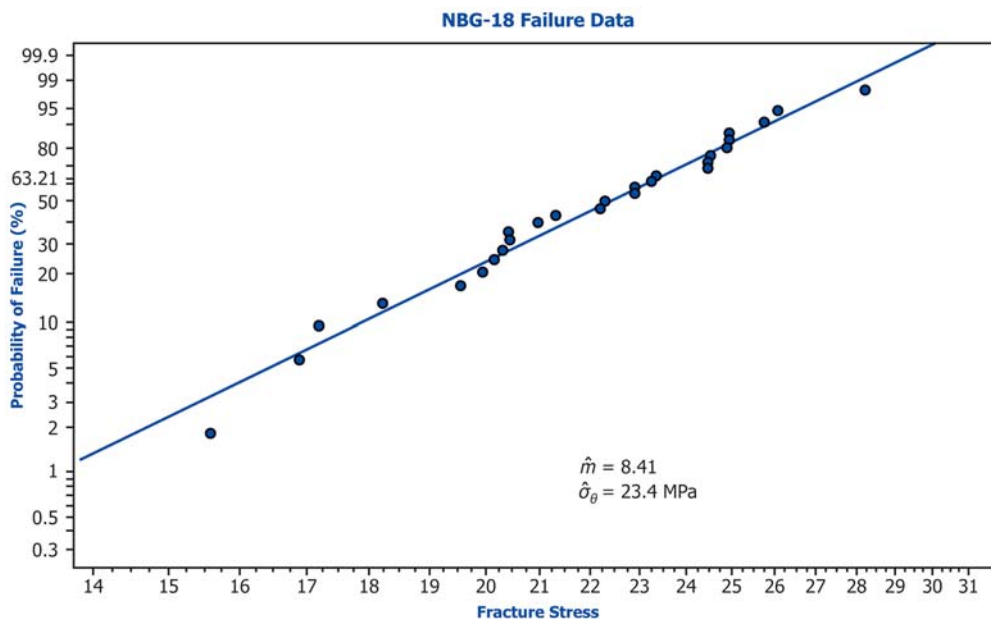


FIG. 2 Failure Strength for the Tensile Test Specimen Geometry Oriented to the Axial Direction of the Billet, End Edge Location (10)



The slope of the line, which is the estimate of the Weibull modulus  $\hat{m}$ , and the characteristic strength  $\hat{\sigma}_0$  should be explicitly identified, as shown in Fig. 2. The estimate of the characteristic strength corresponds to a  $P_f$  of 63.2%, or a value of zero for  $\ln\{\ln[1/(1 - P_f)]\}$ . A test report (that is, a data sheet) that contains information regarding the type of material characterized, the test procedure (preferably designating an appropriate standard), the number of failed specimens with the failure load and method of calculating the failure stress, the maximum likelihood estimates of the Weibull parameters, the unbiasing factor, and the information that allows the construction of 90% confidence bounds should be prepared. This data sheet should accompany the graph to provide a complete representation of the failure data. Insert a column on the graph (in any convenient location), or alternatively provide a separate table that identifies the individual strength values in ascending order. In addition, the experimentalist should include a separate sketch of the specimen geometry that includes all pertinent dimensions. An estimate of mean strength can also be depicted in the graph. The estimate of mean strength is calculated by using the arithmetic mean as the estimator:

$$\hat{\mu} = \left( \sum_{i=1}^N \sigma_i \right) \left( \frac{1}{N} \right) \quad (18)$$

### 8. Unbiasing Factors and Confidence Bounds

8.1 Sections 8.2 through 8.4 outline methods to correct for statistical bias errors in the estimated Weibull parameters, and they outline methods to calculate confidence bounds for the distribution parameters. The procedures described herein to correct for statistical bias errors and to compute confidence bounds are appropriate only for data sets where all failures originate from an uncensored sample. The statistical bias associated with the estimate for  $\hat{\sigma}_0$  is minimal, for example, <0.3% for 20 test specimens, as opposed to  $\cong 7\%$  bias for  $\hat{m}$  with the same number of specimens. Therefore, this practice allows the assumption that  $\hat{\sigma}_0$  is an unbiased estimator of the true population parameter. The parameter estimate of the Weibull modulus ( $\hat{m}$ ) generally exhibits statistical bias. The magnitude of statistical bias depends on the number of specimens in the sample. An unbiased estimate of  $m$  shall be obtained by multiplying  $\hat{m}$  by an unbiasing factor (11). This procedure is discussed in the following sections. Statistical bias associated with the maximum likelihood estimators presented in this practice can be reduced by increasing the sample size.

8.2 An unbiased estimator produces nearly zero statistical bias between the value of the true parameter and the point estimate. The amount of deviation can be quantified either as a percent difference or with unbiasing factors. In keeping with the accepted practice in the open literature, this standard quantifies statistical bias through the use of unbiasing factors, denoted here as  $UF$ . Depending on the number of specimens in a given sample, the point estimate of the Weibull modulus  $\hat{m}$  may exhibit significant statistical bias. An unbiased estimate of the Weibull modulus (denoted as  $\hat{m}_U$ ) is obtained by multiplying the biased estimate with an appropriate unbiasing factor. Unbiasing factors for  $\hat{m}$  are listed in Table 1. The example in section 9.2 demonstrates the use of Table 1 in correcting a biased estimate of the Weibull modulus. As a final note, this

TABLE 1 Unbiasing Factors for the Maximum Likelihood Estimate of the Weibull Modulus (11)

Number of Specimens, $N$	Unbiasing Factor, $UF$	Number of Specimens, $N$	Unbiasing Factor, $UF$
5	0.700	42	0.968
6	0.752	44	0.970
7	0.792	46	0.971
8	0.820	48	0.972
9	0.842	50	0.973
10	0.859	52	0.974
11	0.872	54	0.975
12	0.883	56	0.976
13	0.893	58	0.977
14	0.901	60	0.978
15	0.908	62	0.979
16	0.914	64	0.980
18	0.923	66	0.980
20	0.931	68	0.981
22	0.938	70	0.981
24	0.943	72	0.982
26	0.947	74	0.982
28	0.951	76	0.983
30	0.955	78	0.983
32	0.958	80	0.984
34	0.960	85	0.985
36	0.962	90	0.986
38	0.964	100	0.987
40	0.966	120	0.990

procedure is not appropriate for censored samples. The theoretical approach was developed for uncensored samples.

8.3 Confidence bounds quantify the uncertainty associated with a point estimate of a population parameter. The size of the confidence bounds for maximum likelihood estimates of both Weibull parameters will decrease with increasing sample size. The values used to construct confidence bounds are based on percentile distributions obtained by Monte Carlo simulation. For example, the 90% confidence bound on the Weibull modulus is obtained from the 5 and 95 percentile distributions of the ratio of  $\hat{m}$  to the true population value  $m$ . For the point estimate of the Weibull modulus, the normalized values ( $\hat{m}/m$ ) necessary to construct the 90% confidence bounds are listed in Table 2. The example in 9.2 demonstrates the use of Table 2 in constructing the upper and lower bounds on  $\hat{m}$ . The statistical biased estimate of the Weibull modulus must be used here.

8.3.1 Table 2 has values for up to 120 specimens. Beyond 120 specimens equations have been numerically fitted to the data in the table. The equations for the bounds are given by:

$$q_{0.02} = -0.313101 - \frac{0.3453}{N^{0.5}} + N^{0.0410319} \quad (19)$$

$$q_{0.05} = -0.255704 - \frac{0.277943}{N^{0.5}} + N^{0.0348039} \quad (20)$$

$$q_{0.10} = -0.275117 - \frac{0.0523232}{N^{0.5}} + N^{0.0389672} \quad (21)$$

$$q_{0.90} = 0.934956 + \frac{19.343}{N^2} + \frac{1}{N^{0.327436}} - 0.000479117N \quad (22)$$

$$q_{0.95} = 1.0311 + \frac{30.3502}{N^2} + \frac{1}{N^{0.330832}} - 0.00116078N \quad (23)$$

$$q_{0.98} = 1.13215 + \frac{47.5464}{N^2} + \frac{1}{N^{0.332061}} - 0.0018067N \quad (24)$$

**TABLE 2 Normalized Upper and Lower Bounds on the Maximum Likelihood Estimate of the Weibull Modulus (11)**

Number of Specimens <i>N</i>	<i>q</i> <sub>0.02</sub>	<i>q</i> <sub>0.05</sub>	<i>q</i> <sub>0.10</sub>	<i>q</i> <sub>0.90</sub>	<i>q</i> <sub>0.95</sub>	<i>q</i> <sub>0.98</sub>
5	0.604	0.683	0.766	2.277	2.779	3.518
6	0.623	0.697	0.778	2.03	2.436	3.067
7	0.639	0.709	0.785	1.861	2.183	2.64
8	0.653	0.72	0.792	1.747	2.015	2.377
9	0.665	0.729	0.797	1.665	1.896	2.199
10	0.676	0.738	0.802	1.602	1.807	2.07
11	0.686	0.745	0.807	1.553	1.738	1.972
12	0.695	0.752	0.811	1.513	1.682	1.894
13	0.703	0.759	0.815	1.48	1.636	1.83
14	0.71	0.764	0.819	1.452	1.597	1.777
15	0.716	0.77	0.823	1.427	1.564	1.732
16	0.723	0.775	0.826	1.406	1.535	1.693
17	0.728	0.779	0.829	1.388	1.51	1.66
18	0.734	0.784	0.832	1.371	1.487	1.63
19	0.739	0.788	0.835	1.356	1.467	1.603
20	0.743	0.791	0.838	1.343	1.449	1.579
22	0.752	0.798	0.843	1.32	1.418	1.538
24	0.759	0.805	0.848	1.301	1.392	1.504
26	0.766	0.81	0.852	1.284	1.37	1.475
28	0.772	0.815	0.856	1.269	1.351	1.45
30	0.778	0.82	0.86	1.257	1.334	1.429
32	0.783	0.824	0.863	1.246	1.319	1.409
34	0.788	0.828	0.866	1.236	1.306	1.392
36	0.793	0.832	0.869	1.227	1.294	1.377
38	0.797	0.835	0.872	1.219	1.283	1.363
40	0.801	0.839	0.875	1.211	1.273	1.351
42	0.804	0.842	0.877	1.204	1.265	1.339
44	0.808	0.845	0.88	1.198	1.256	1.329
46	0.811	0.847	0.882	1.192	1.249	1.319
48	0.814	0.85	0.884	1.187	1.242	1.31
50	0.817	0.852	0.886	1.182	1.235	1.301
52	0.82	0.854	0.888	1.177	1.229	1.294
54	0.822	0.857	0.89	1.173	1.224	1.286
56	0.825	0.859	0.891	1.169	1.218	1.28
58	0.827	0.861	0.893	1.165	1.213	1.273
60	0.83	0.863	0.894	1.162	1.208	1.267
62	0.832	0.864	0.896	1.158	1.204	1.262
64	0.834	0.866	0.897	1.155	1.2	1.256
66	0.836	0.868	0.899	1.152	1.196	1.251
68	0.838	0.869	0.9	1.149	1.192	1.246
70	0.84	0.871	0.901	1.146	1.188	1.242
72	0.841	0.872	0.903	1.144	1.185	1.237
74	0.843	0.874	0.904	1.141	1.182	1.233
76	0.845	0.875	0.905	1.139	1.179	1.229
78	0.846	0.876	0.906	1.136	1.176	1.225
80	0.848	0.878	0.907	1.134	1.173	1.222
85	0.852	0.881	0.91	1.129	1.166	1.213
90	0.855	0.883	0.912	1.124	1.16	1.206
95	0.858	0.886	0.914	1.12	1.155	1.199
100	0.861	0.888	0.916	1.116	1.15	1.192
110	0.866	0.893	0.92	1.11	1.141	1.181
120	0.871	0.897	0.923	1.104	1.133	1.171

percentile distributions are listed in Table 3. The example in 9.2 demonstrates the use of Table 3 in constructing upper and lower bounds on  $\hat{\sigma}_0$ . The biased estimate of the Weibull modulus must be used here.

8.4.2 Table 3 has values for up to 120 specimens. Beyond 120 specimens equations have been numerically generated using the data in the table. The equations for the bounds on the function *t* are given by:

$$t_{0.02} = -0.134788 - \frac{7.34535}{N} - 1.00427N + N^{1.0009} \quad (26)$$

$$t_{0.05} = -0.148236 - \frac{5.20828}{N} - 1.00203N + N^{1.0005} \quad (27)$$

**TABLE 3 Normalized Upper and Lower Bounds on the Function *t* — 90% Confidence Interval**

Number of Specimens <i>N</i>	<i>t</i> <sub>0.02</sub>	<i>t</i> <sub>0.05</sub>	<i>t</i> <sub>0.10</sub>	<i>t</i> <sub>0.90</sub>	<i>t</i> <sub>0.95</sub>	<i>t</i> <sub>0.98</sub>
5	-1.631	-1.247	-0.888	0.772	1.107	1.582
6	-1.396	-1.007	-0.74	0.666	0.939	1.291
7	-1.196	-0.874	-0.652	0.598	0.829	1.12
8	-1.056	-0.784	-0.591	0.547	0.751	1.003
9	-0.954	-0.717	-0.544	0.507	0.691	0.917
10	-0.876	-0.665	-0.507	0.475	0.644	0.851
11	-0.813	-0.622	-0.477	0.448	0.605	0.797
12	-0.762	-0.587	-0.451	0.425	0.572	0.752
13	-0.713	-0.557	-0.429	0.406	0.544	0.714
14	-0.683	-0.532	-0.41	0.389	0.52	0.681
15	-0.651	-0.509	-0.393	0.374	0.499	0.653
16	-0.624	-0.489	-0.379	0.36	0.48	0.627
17	-0.599	-0.471	-0.365	0.348	0.463	0.605
18	-0.578	-0.455	-0.353	0.338	0.447	0.584
19	-0.558	-0.441	-0.342	0.328	0.433	0.566
20	-0.54	-0.428	-0.332	0.318	0.421	0.549
22	-0.509	-0.404	-0.314	0.302	0.398	0.519
24	-0.483	-0.384	-0.299	0.288	0.379	0.494
26	-0.46	-0.367	-0.286	0.276	0.362	0.472
28	-0.441	-0.352	-0.274	0.265	0.347	0.453
30	-0.423	-0.338	-0.264	0.256	0.334	0.435
32	-0.408	-0.326	-0.254	0.247	0.323	0.42
34	-0.394	-0.315	-0.246	0.239	0.312	0.406
36	-0.382	-0.305	-0.238	0.232	0.302	0.393
38	-0.37	-0.296	-0.231	0.226	0.293	0.382
40	-0.36	-0.288	-0.224	0.22	0.285	0.371
42	-0.35	-0.28	-0.218	0.214	0.278	0.361
44	-0.341	-0.273	-0.213	0.209	0.271	0.352
46	-0.333	-0.266	-0.208	0.204	0.264	0.344
48	-0.325	-0.26	-0.203	0.199	0.258	0.336
50	-0.318	-0.254	-0.198	0.195	0.253	0.328
52	-0.312	-0.249	-0.194	0.191	0.247	0.321
54	-0.305	-0.244	-0.19	0.187	0.243	0.315
56	-0.299	-0.239	-0.186	0.184	0.238	0.309
58	-0.294	-0.234	-0.183	0.181	0.233	0.303
60	-0.289	-0.234	-0.179	0.177	0.229	0.297
62	-0.284	-0.226	-0.176	0.174	0.225	0.292
64	-0.279	-0.222	-0.173	0.171	0.221	0.287
66	-0.274	-0.218	-0.17	0.169	0.218	0.282
68	-0.27	-0.215	-0.167	0.166	0.214	0.278
70	-0.266	-0.211	-0.165	0.164	0.211	0.274
72	-0.262	-0.208	-0.162	0.161	0.208	0.269
74	-0.259	-0.205	-0.16	0.159	0.205	0.266
76	-0.255	-0.202	-0.158	0.157	0.202	0.262
78	-0.252	-0.199	-0.155	0.155	0.199	0.258
80	-0.248	-0.197	-0.153	0.153	0.197	0.255
85	-0.241	-0.19	-0.148	0.148	0.19	0.246
90	-0.234	-0.184	-0.144	0.143	0.185	0.239
95	-0.227	-0.179	-0.139	0.139	0.179	0.232
100	-0.221	-0.174	-0.136	0.136	0.175	0.226
110	-0.211	-0.165	-0.129	0.129	0.166	0.215
120	-0.202	-0.158	-0.123	0.123	0.159	0.205

8.3.2 The *R*<sup>2</sup> value for the fit of these equations with the table values is 0.999. These equations are not appropriate for use with censored samples.

8.4 Confidence bounds can be constructed for the estimated Weibull characteristic strength. However, the percentile distributions needed to construct the bounds do not involve the same normalized ratios or the same tables as those used for the Weibull modulus. Define the function:

$$t = \hat{m} \ln(\hat{\sigma}_0 / \sigma_0) \quad (25)$$

8.4.1 The 90% confidence bound on the characteristic strength is obtained from the 5 and 95 percentile distributions of *t*. For the point estimate of the characteristic strength, these

$$t_{0.10} = -0.182162 - \frac{3.45837}{N} - 0.997697N + N^{0.999678} \quad (28)$$

$$t_{0.90} = 0.111372 + \frac{1.56654}{N} + \frac{1}{N^{0.671389}} - 0.000378813N \quad (29)$$

$$t_{0.95} = 0.18327 + \frac{3.226}{N} + \frac{1}{N^{0.825572}} - 0.000654297N \quad (30)$$

$$t_{0.98} = 0.247705 + \frac{6.06723}{N} + \frac{1}{N^{1.67665}} - 0.000834172N \quad (31)$$

8.4.3 The  $R^2$  value associated with the fit of the equations above to the table values is 0.999. Again, this procedure is not appropriate for censored statistics.

## 9. Examples

9.1 For the first example, consider the failure data in [Table 4](#). The data is uncensored and represents a tensile test specimen geometry oriented to the axial direction of the billet, at an edge location ([12](#)). The solution of [Eq 14](#) requires an iterative

**TABLE 4 Uncensored Failure Strengths—Example 1**

Specimen Number	Strength $\sigma_i$ , MPa	Specimen Number	Strength $\sigma_i$ , MPa
1	11.1	25	16.6
2	12.4	26	16.9
3	12.6	27	17.0
4	12.9	28	17.1
5	13.7	29	17.1
6	13.8	30	17.1
7	14.2	31	17.3
8	14.5	32	17.4
9	14.6	33	17.6
10	14.7	34	17.6
11	14.7	35	17.7
12	15.0	36	17.8
13	15.6	37	17.8
14	15.7	38	17.9
15	15.9	39	17.9
16	15.9	40	18.1
17	16.0	41	18.2
18	16.0	42	18.4
19	16.0	43	18.5
20	16.4	44	18.6
21	16.4	45	19.2
22	16.4	46	19.6
23	16.6	47	19.8
24	16.6	48	20.5

numerical scheme. Using a numerical algorithm a parameter estimate of  $\hat{m} = 9.77$  was obtained. Subsequent solution of [Eq 15](#) yields a value of  $\hat{\sigma}_0 = 17.3$  MPa. [Fig. 3](#) depicts the individual failure data and a curve based on the estimated values of the parameters.

9.2 As an example of computing unbiased estimates of the Weibull modulus, and bounds on both the Weibull modulus and the Weibull characteristic strength, consider the uncensored data presented in [9.1](#). The sample contained 48 specimens and the biased estimate of the Weibull modulus was determined to be  $\hat{m} = 9.77$ . The unbiasing factor corresponding to this sample size is  $UF = 0.972$ , which is obtained from [Table 1](#). Thus, the unbiased estimate of the Weibull modulus is given as:

$$\begin{aligned} \hat{m}_U &= \hat{m} \times UF \\ &= (9.77)(0.972) \\ &= 9.50 \end{aligned} \quad (32)$$

9.2.1 The upper bound on  $\hat{m}$  for this example is:

$$\begin{aligned} \hat{m}_{upper} &= \hat{m} \times q_{0.05} \\ &= 9.77/0.850 \\ &= 11.49 \end{aligned} \quad (33)$$

where  $q_{0.05}$  is obtained from [Table 2](#) for a sample size of 48 failed specimens. The lower bound is:

$$\begin{aligned} \hat{m}_{lower} &= \hat{m} \times q_{0.95} \\ &= 9.77/1.242 \\ &= 7.87 \end{aligned} \quad (34)$$

where  $q_{0.95}$  is also obtained from [Table 2](#). Similarly, the upper bound on  $\hat{\sigma}_0$  is:

where  $t_{0.05}$  is obtained from [Table 3](#) for a sample size of 48 failed specimens. The lower bound on  $\hat{\sigma}_0$  is:

where  $t_{0.95}$  is also obtained from [Table 3](#).

## 10. Keywords

10.1 confidence bounds; graphite; maximum likelihood; strength; unbiasing factors; Weibull characteristic strength; Weibull modulus; Weibull scale; Weibull statistics

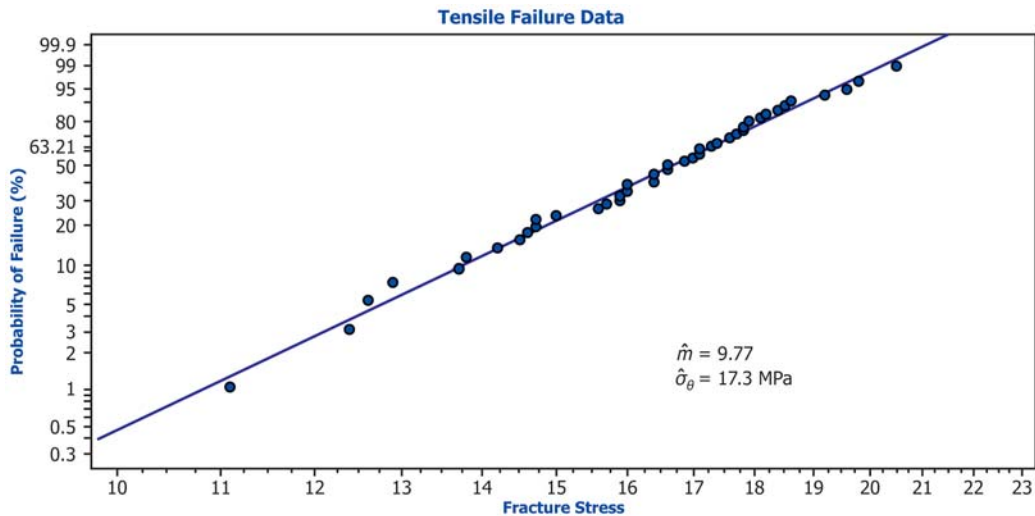


FIG. 3 Example of Failure Stress for Small Tensile Test Specimen Geometry Oriented to the Axial Direction of the Billet, End Edge Location (12)

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## SUMMARY OF CHANGES

Subcommittee D02.F0 has identified the location of selected changes to this standard since the last issue (D7846 – 12) that may impact the use of this standard. (Approved Jan. 1, 2016.)

- (1) Revised Section 3, Terminology.



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