



# Standard Test Method for Determining Transmissivity of Nonleaky Confined Aquifers by the Theis Recovery Method<sup>1</sup>

This standard is issued under the fixed designation D5269; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

## 1. Scope

1.1 This test method covers an analytical procedure for determining the transmissivity of a confined aquifer. This test method is used to analyze data from the recovery of water levels following pumping or injection of water to or from a control well at a constant rate.

1.2 The analytical procedure given in this test method, along with several others, is used in conjunction with the field procedure in Test Method [D4050](#). Guide [D4043](#) provides information for determining hydraulic properties.

1.3 *Limitations*—The valid use of the Theis recovery method is limited to determination of transmissivities for aquifers in hydrogeologic settings reasonably corresponding to the assumptions of the Theis theory (see [5.2](#)).

1.4 *Units*—The values stated in either SI Units or inch-pound units are to be regarded separately as standard. The values in each system may not be exact equivalents; therefore each system shall be used independently of the other. Combining values from the two systems may result in nonconformance with the standard. Reporting of test results in units other than SI shall not be regarded as nonconformance with this test method.

1.5 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice [D6026](#). All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice [D6026](#), unless otherwise superseded by this standard.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

<sup>1</sup> This test method is under the jurisdiction of ASTM Committee [D18](#) on Soil and Rock and is the direct responsibility of Subcommittee [D18.21](#) on Groundwater and Vadose Zone Investigations.

Current edition approved April 15, 2015. Published May 2015. Originally approved in 1992. Last previous edition approved in 2008 as D5269 – 96 (2008). DOI: 10.1520/D5269-15.

## 2. Referenced Documents

### 2.1 *ASTM Standards*:<sup>2</sup>

- [D653 Terminology Relating to Soil, Rock, and Contained Fluids](#)
- [D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction](#)
- [D4043 Guide for Selection of Aquifer Test Method in Determining Hydraulic Properties by Well Techniques](#)
- [D4050 Test Method for \(Field Procedure\) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems](#)
- [D4105 Test Method for \(Analytical Procedure\) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Modified Theis Nonequilibrium Method](#)
- [D4106 Test Method for \(Analytical Procedure\) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method](#)
- [D6026 Practice for Using Significant Digits in Geotechnical Data](#)

## 3. Terminology

### 3.1 *Definitions*:

3.1.1 For definitions of common Terminology terms used within this guide refer to Terminology [D653](#).

### 3.2 *Definitions of Terms Specific to This Standard*:

3.2.1 *observation well*—a well open to all or part of an aquifer.

### 3.3 *Symbols and Dimensions*:

3.3.1  $b$  [L]—aquifer thickness.

3.3.2  $K$  [ $LT^{-1}$ ]—hydraulic conductivity.

<sup>2</sup> For referenced ASTM standards, visit the ASTM website, [www.astm.org](http://www.astm.org), or contact ASTM Customer Service at [service@astm.org](mailto:service@astm.org). For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.3.2.1 *Discussion*—The use of the symbol  $K$  for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol  $k$  is commonly used for this term in rock mechanics and soil science.

3.3.3  $K_r$ —hydraulic conductivity in the plane of the aquifer, radially from the control well.

3.3.4  $K_z$ —hydraulic conductivity in the vertical direction.

3.3.5  $\ln$ —natural logarithm.

3.3.6  $\log_{10}$ —logarithm to the base 10.

3.3.7  $Q$  [ $L^3T^{-1}$ ]*—*discharge.

3.3.8  $r$  [L]*—*radial distance from control well.

3.3.9  $r_c$  [L]*—*equivalent inside radius of control well.

3.3.10  $S$  [nd]*—*storage coefficient.

3.3.11  $s$  [L]*—*drawdown.

3.3.12  $s_c$  [L]*—*drawdown corrected for the effects of reduction in saturated thickness.

3.3.13  $S_y$  [nd]*—*specific yield.

3.3.14  $s'$  [L]*—*residual drawdown.

3.3.15  $\Delta s'$  [L]*—*change in residual drawdown over one log cycle of  $t/t'$ .

3.3.16  $T$  [ $L^2T^{-1}$ ]*—*transmissivity.

3.3.17  $t$  [T]*—*time since pumping or injection began.

3.3.18  $t'$  [T]*—*time since pumping or injection stopped.

3.3.19  $u$ *—*dimensionless parameter, equal to  $r^2S/4Tt$ .

3.3.20  $u'$ *—*dimensionless parameter, equal to  $r^2S/4Tt'$ .

#### 4. Summary of Test Method

4.1 This test method describes an analytical procedure for determining transmissivity using data collected during the recovery phase of a withdrawal or injection well test. The field test (see Test Method D4050) requires pumping or injecting a control well that is open to the entire thickness of a confined aquifer at a constant rate for a specified period. The water-levels in the control well, observation wells, or piezometers are measured after pumping is stopped and used to calculate the transmissivity of the aquifer using the procedures in this test method. Alternatively, this test method can be performed by injecting water into the control well at a constant rate. With some modification, this test method can also be used to analyze the residual drawdown following a step test. This test method is used by plotting residual drawdown against either a function of time or a function of time and discharge and determining the slope of a straight line fitted to the points. The solution calculations are shown in Section 8.

#### 5. Significance and Use

5.1 This test method is useful for analyzing data on the recovery of water levels following pumping or injection of water to or from a control well at a constant rate. The analytical procedure given in this test method along with several others is used in conjunction with the field procedure in Test Method D4050.

##### 5.2 Assumptions:

5.2.1 The well discharges at a constant rate,  $Q$ , or at steps of constant rate  $Q_1, Q_2 \dots Q_n$ .

5.2.2 Well is of infinitesimal diameter and is open through the full thickness of the aquifer.

5.2.3 The nonleaky aquifer is homogeneous, isotropic, and extensive in area.

5.2.4 Discharge from the well is derived exclusively from storage in the aquifer.

5.2.5 The geometry of the assumed aquifer and well are shown in Fig. 1.

##### 5.3 Implications of Assumptions:

5.3.1 Implicit in the assumptions are the conditions of radial flow. Vertical flow components are induced by a control well that partially penetrates the aquifer, that is, not open to the aquifer through the full thickness of the aquifer. If vertical flow components are significant, the nearest partially penetrating observation well should be located at a distance,  $r$ , beyond which vertical flow components are negligible. See 5.3.1 of Test Method D4106 for assistance in determining the minimum distance to partially penetrating observation wells and piezometers.

5.3.2 The Theis method assumes the control well is of infinitesimal diameter. The storage in the control well may adversely affect drawdown measurements obtained in the early part of the test. See 5.3.2 of Test Method D4106 for assistance in determining the duration of the effects of well-bore storage on drawdown.

5.3.3 *Application of Theis Recovery Method for Unconfined Aquifers:*

5.3.3.1 Although the assumptions are applicable to artesian or confined conditions, the Theis solution may be applied to unconfined aquifers if (A) drawdown is small compared with the saturated thickness of the aquifer or if the drawdown is corrected for reduction in thickness of the aquifer and (B) the effects of delayed gravity yield are small. See 5.3.3 of Test

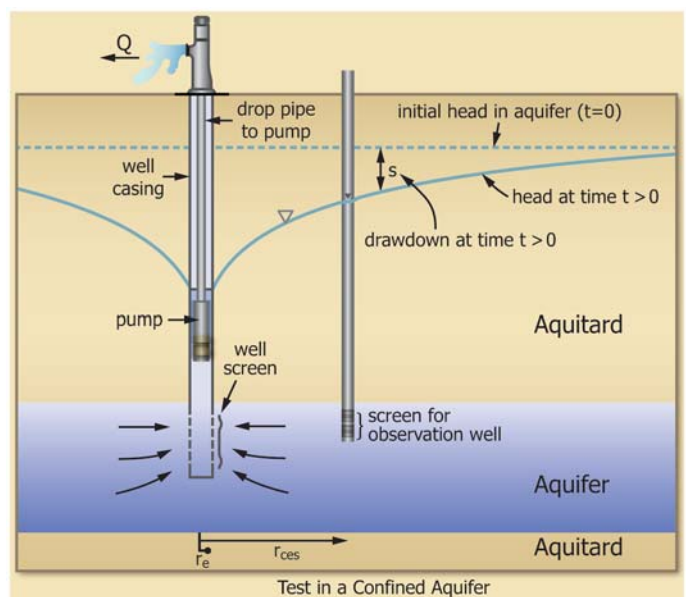


FIG. 1 Cross Section Through a Discharging Well in a Nonleaky Aquifer

Method **D4106** for guidance in treating reduction in saturated thickness and delayed gravity drainage in unconfined aquifers.

**NOTE 1**—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice **D3740** are generally considered capable of competent and objective testing/sampling/inspection. Users of this standard are cautioned that compliance with Practice **D3740** does not in itself assure reliable results. Reliable results depend on many factors; Practice **D3740** provides a means of evaluating some of those factors.

## 6. Apparatus

6.1 Analysis of data by this test method from the field procedure given in Test Method **D4050** requires that the control well and observation wells meet the requirements specified in the following subsections.

6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate, or several steps at constant rate, for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precautions in the placement or design of observation wells (see **5.3.1**).

6.3 *Construction of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers at a distance from the control well. Observation wells may be open through all or part of the thickness of the aquifer.

6.4 *Location of Observation Wells and Piezometers*—Wells may be located at any distance from the control well within the area of influence of pumping. However, if vertical flow components are significant and if piezometers or partially penetrating observation wells are used, locate them at a distance beyond the effect of vertical flow components. If the aquifer is unconfined, constraints are imposed on the distance to partially penetrating observation wells and the validity of early time measurements (see **5.3.1**).

## 7. Procedure

7.1 The overall procedure process consists of conducting the field procedure for withdrawal or injection well tests (described in Test Method **D4050**) and then the analysis of the field data, which is addressed by this test method in Section 8.

## 8. Calculation and Interpretation of Results

8.1 *Solution*—The solution given by Theis (**1**)<sup>3</sup> can be expressed as follows:

$$s = \frac{Q}{4\pi T} \int_u^{\infty} \frac{e^{-y}}{y} dy \quad (1)$$

and:

$$u = \frac{r^2 S}{4Tt} \quad (2)$$

<sup>3</sup> The boldface numbers in parentheses refer to a list of references at the end of this standard.

8.2 At a control well, observation well, or piezometer, for large values of time,  $t$ , and small values of radius,  $r$ , the Theis equation reduces, as shown by Cooper and Jacob (**2**) and Jacob (**3**) to the following:

$$s' = \frac{Q}{4\pi T} \ln(t/t') \quad (3)$$

where:

$t$  = the time after pumping began and

$t'$  = the time after pumping ceases. From which it can be shown that:

$$T = \frac{2.3Q}{4\pi \Delta s'} \quad (4)$$

where:

$\Delta s'$  = the measured or projected residual drawdown over one  $\log_{10}$  cycle of  $t/t'$ .

8.3 A similar analysis (see **8.2**) may also be used for a step-drawdown test in which a well is pumped at a constant rate for an initial period, and then the pumping rate is increased through several new constant rates in a series of steps. Harrill

(**4**) shows that:  $s' = \frac{2.3\Delta Q_1}{4\pi T} \left( \log_{10} \frac{t_1}{t'} \right) + \frac{2.3\Delta Q_2}{4\pi T} \left( \log_{10} \frac{t_2}{t'} \right) + \dots + \frac{2.3\Delta Q_n}{4\pi T} \left( \log_{10} \frac{t_n}{t'} \right)$

where:

$t_1, t_2, t_n$  = the elapsed times since either pumping was begun or the discharge rate was increased,

$Q_1, Q_2, Q_n$  = the well discharge rates, and

$\Delta Q_1, \Delta Q_2, \Delta Q_n$  = the incremental increases in discharge.

Eq 5 can be rewritten as follows:

$$T = \frac{2.3Q_n}{4\pi s'} \log_{10} f(t, Q) \quad (5)$$

where:

$$f(t, Q) = \frac{t_1^{\Delta Q_1/Q_n} t_2^{\Delta Q_2/Q_n} t_3^{\Delta Q_3/Q_n} \dots t_n^{\Delta Q_n/Q_n}}{t'} \quad (6)$$

and:

$$T = \frac{2.3Q_n}{4\pi \Delta s'_h} \quad (7)$$

where:

$\Delta s'_h$  = the residual drawdown over one log cycle of the expression  $f(t, Q)$  in Eq 6.

Eq 8 can also be used to analyze the residual drawdown following a test in which discharge varies significantly, so long as the discharge can be generalized as a series of constant-discharge steps.

8.4 The Theis recovery method gives satisfactory results when properly used. However, the method is valid only for small values of  $u$ , that is:

for confined aquifers:

$$u' = \frac{r^2 S}{4Tt} \quad (8)$$

or for unconfined aquifers:

$$u' = \frac{r^2 S_y}{4Tr'} \quad (9)$$

NOTE 2—The limiting value for  $u$  of less than 0.01 may be excessively restrictive in some applications. The errors for small values of  $u$ , from Kruseman and De Ridder (5) are:

Error less than, %	1	2	5	10
For $u$ smaller than	0.03	0.05	0.1	0.15

8.4.1 This test method allows only the calculation of transmissivity,  $T$ , not storage coefficient,  $S$ , or specific yield,  $S_y$ . Therefore, to determine whether the assumption in Eq 9 or Eq 10 has been violated it is necessary to estimate a value for storage coefficient for confined aquifers or specific yield for unconfined aquifers. If data are available during the pumping period, the storage may be computed using the procedures in Test Method D4105. Storage coefficients can be estimated as about  $3 \times 10^{-5}b$ , where  $b$  is aquifer thickness in meters. Whereas the specific yield of unconfined aquifers averages about 0.2 according to Lohman (6). After calculating  $T$ , substitute the appropriate values into Eq 9 or Eq 10 and solve for  $u'$ . It is not adequate to simply note that the data described a straight line on semi-log graph paper.

8.5 Plot either residual drawdown,  $s'$ , or water level, on the arithmetic axis of semilogarithmic graph paper versus either  $t/t'$  (for recovery from a constant-discharge test) (see Fig. 2) or  $f(t, Q)$  (for recovery from a step-drawdown test) (see Fig. 3) on the logarithmic axis. Fit a straight line to the linear part of the data plot, usually at smaller values of  $t/t'$ . Extend the straight line to intercept the  $t/t' = 1$  axis. At  $t/t' = 1$ , residual drawdown should be approximately equal to zero, or if water levels were plotted, the intercept should be equal to the prepumping water levels corrected for prepumping water-level trends. Substitute the values for  $\Delta s'$  or  $\Delta s'_h$  in Eq 3 or Eq 8 and solve for transmissivity. Check that values of  $t'$  for the points used in defining the straight line meet the criterion that  $u' < 0.01$  (Eq 9 and Eq 10), as described in 8.4.

### 9. Report: Test Data Sheet(s)/Forms

9.1 Prepare a report including as a minimum the information described below. The report of the analysis will include information from the field testing procedure.

9.1.1 *Introduction*—The introductory section is intended to present the scope and purpose of the Theis recovery method for

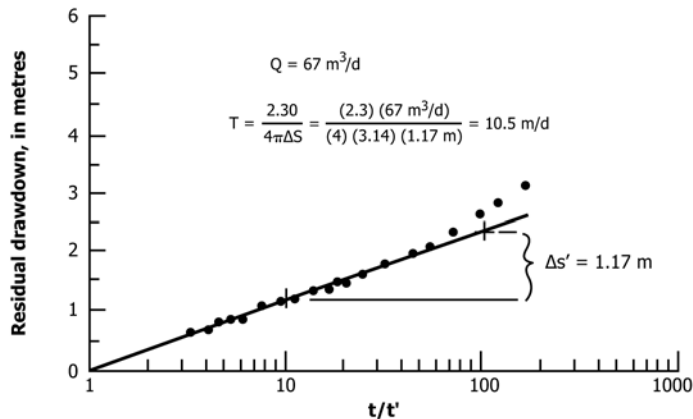


FIG. 2 Example Analysis Using the Theis Recovery Method

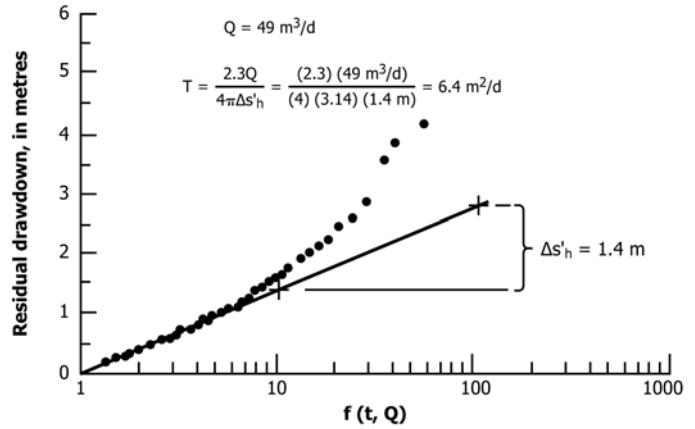


FIG. 3 Example Analysis Using Harrill's Method

determining transmissivity in a confined nonleaky aquifer. Summarize the field hydrogeologic conditions and the field equipment and instrumentation including the construction of the control well and observation wells and piezometers, the method of measurement of discharge and water levels, and the duration of the test and pumping rates. Discuss rationale for selecting the Theis recovery method.

9.1.2 *Hydrogeologic Setting*—Review the information available on the hydrogeology of the site. Include driller's logs and geologist's description of drill cuttings. Interpret and describe the hydrogeology of the site as it pertains to the selection of this method for conducting and analyzing an aquifer test. Compare the hydrogeologic characteristics of the site as it conforms and differs from the assumptions in the solution to the aquifer test method.

#### 9.1.3 Scope of Aquifer Test:

9.1.3.1 *Equipment*—Report the field installation and equipment for the aquifer test, including the construction, diameter, depth of screened interval, and location of control well and pumping equipment, and the construction, diameter, depth, and screened interval of observation wells or piezometers.

9.1.3.2 *Instrumentation*—Report the field instrumentation for observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to the test. Include a list of measuring devices used during the test, the manufacturer's name, model number, and basic specifications for each major item, and the name and date of the last calibration, if applicable.

9.1.3.3 *Testing Procedures*—State the steps taken in conducting pretest, drawdown, and recovery phases of the test. Include the frequency of measurements of discharge rate, water level in observation wells, and other environmental data recorded during the testing procedure.

#### 9.1.4 Interpretation of Test Results:

9.1.4.1 *Data*—Present tables of data collected during the test. Show methods of adjusting water levels for barometric changes and calculation of drawdown and residual drawdown.

9.1.4.2 *Data Plots*—Present data plots used in analysis of the data. Show data plots with straight line segments and intercepts of the  $t/t' = 1$  axis.

9.1.4.3 Evaluate qualitatively the overall accuracy of the test on the basis of the adequacy of instrumentation and

observations of stress end response, and the conformance of the hydrogeologic conditions and the performance of the test to the assumptions of the test method (see 5.2) and the implications of the assumptions (see 5.3).

## 10. Precision and Bias

10.1 *Precision*—It is not practicable to specify the precision of this test method because the response of aquifer systems during aquifer tests is dependent upon ambient system stresses. Test data on precision is not presented due to the nature of the soil or rock materials tested by this test method. It is either not feasible or too costly at this time to have ten or more

laboratories participate in a round robin testing program. Any variation observed in the data is just as likely to be due to specimen variation as to operator or laboratory testing variation. Subcommittee D18.08 is seeking pertinent data from users of this test method on precision and bias comparisons.

10.2 *Bias*—There is no accepted reference value for this test method, therefore, bias cannot be determined.

## 11. Keywords

11.1 aquifers; aquifer tests; confined aquifers; control wells; groundwater; hydraulic properties; observation wells; step tests; transmissivity; unconfined aquifers

## REFERENCES

- (1) Theis, C. V., “The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage,” *American Geophysical Union Transactions*, Vol 16, Part 2, 1935, pp. 519–524.
- (2) Cooper, H. H., and Jacob, C. E., “A Generalized Graphical Method for Evaluating Formation Constants and Summarizing Well-Field History,” *Trans. Amer. Geophys. Union*, Vol 27, 1946, pp. 526–534.
- (3) Jacob, C. E., “The Recovery Method for Determining the Coefficient of Transmissibility,” in Bentall, Ray, Compiler, “Methods of Determining Permeability, Transmissibility, and Drawdown,” *U.S. Geological Survey Water-Supply Paper 1536-I*, 1963, pp. 288–292.
- (4) Harrill, J. R., “Determining Transmissivity from Water-Level Recovery of a Step-Drawdown Test,” *U.S. Geological Survey Professional Paper 700-C*, 1970.
- (5) Kruseman, G. P., and DeRidder, N. S., “Analysis and Evaluation of Pumping Test Data,” *Intern. Inst. for Land and Reclamation and Improvement, Bull. 47* Wageningen, The Netherlands, 1990.
- (6) Lohman, S. W., “Ground-Water Hydraulics,” *U.S. Geological Survey Professional Paper 708*, 1970.

*ASTM International takes no position respecting the validity of any patent rights asserted in connection with any item mentioned in this standard. Users of this standard are expressly advised that determination of the validity of any such patent rights, and the risk of infringement of such rights, are entirely their own responsibility.*

*This standard is subject to revision at any time by the responsible technical committee and must be reviewed every five years and if not revised, either reapproved or withdrawn. Your comments are invited either for revision of this standard or for additional standards and should be addressed to ASTM International Headquarters. Your comments will receive careful consideration at a meeting of the responsible technical committee, which you may attend. If you feel that your comments have not received a fair hearing you should make your views known to the ASTM Committee on Standards, at the address shown below.*

*This standard is copyrighted by ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959, United States. Individual reprints (single or multiple copies) of this standard may be obtained by contacting ASTM at the above address or at 610-832-9585 (phone), 610-832-9555 (fax), or service@astm.org (e-mail); or through the ASTM website (www.astm.org). Permission rights to photocopy the standard may also be secured from the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, Tel: (978) 646-2600; http://www.copyright.com/*