Standard Practice for Establishing Consistent Test Method Tolerances[1]

This standard is issued under the fixed designation D 4356; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice should be used in the development of any test method in which the determination value is calculated from measurement values by means of an equation. The practice is not applicable to such determination values as those calculated from counts of nonconformities, ratios of successes to failures, gradings, or ratings.

1.2 The purpose of this practice is to provide guidance in the specifying of realistic and consistent tolerances for making measurements and for reporting the results of testing.

1.3 This practice can be used as a guide for obtaining the minimum test result tolerance that should be specified with a particular set of specified measurement tolerances, the maximum permissible measurement tolerances which should be specified to achieve a specified test result tolerance, and more consistent specified measurement tolerances.

1.4 These measurement and test result tolerances are not statistically determined tolerances that are obtained by using the test method but are the tolerances specified in the test method.

1.5 In the process of selecting test method tolerances, the task group developing or revising a test method must evaluate not only the consistency of the selected tolerances but also the technical and economical feasibility of the measurement tolerances and the suitability of the test result tolerance for the purposes for which the test method will be used. This practice provides guidance only for establishing the consistency of the test method tolerances.

1.6 This practice is presented in the following sections:

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 123 Terminology Relating to Textiles[2]

D 2905 Practice for Statements on Number of Specimens for Textiles[2]

E 29 Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications[3]

E 456 Terminology Relating to Quality and Statistics[3]

3. Terminology

3.1 *Definitions:*

3.1.1 *determination process*, *n*—the act of carrying out the series of operations specified in the test method whereby a single value is obtained. (Syn. *determination.* See Section 4.)

3.1.1.1 *Discussion*—A determination process may involve several measurements of the same type or different types, as well as an equation by which the determination value is calculated from the measurement values observed.

3.1.2 *determination tolerance*, *n—as specified in a test method*, the exactness with which a determination value is to be calculated and recorded.

3.1.2.1 *Discussion*—In this practice, the determination tolerance also serves as the bridge between the test result tolerance and the measurement tolerances. The value of the determination tolerance calculated from the specified test result tolerance is compared with the value calculated from the specified measurement tolerances.

[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.20on Test Method Evaluation and Quality Control.

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[2] *Annual Book of ASTM Standards*, Vol 07.01.

[3] *Annual Book of ASTM Standards*, Vol 14.02.

3.1.3 *determination value*, *n*—the numerical quantity calculated by means of the test method equation from the measurement values obtained as directed in a test method. (Syn. *determination.* See Section 4.)

3.1.4 *measurement process*, *n*—the act of quantifying a property or dimension. (Syn. *measurement.* See Section 4.)

3.1.4.1 *Discussion*—One test method determination may involve several different kinds of measurement.

3.1.5 *measurement tolerance, n—as specified in a test method*, the exactness with which a measurement is to be made and recorded.

3.1.6 *measurement tolerance propagation equation*, *n*—the mathematical formula, derived from the test method equation, which shows the dependence of the determination tolerance on the measurement tolerances. (Syn. *propagation equation*.)

3.1.6.1 *Discussion*—Propagation equations and the propagation of errors are discussed in Annex A1.

3.1.7 *measurement value*, *n*—the numerical result of quantifying a particular property or dimension. (Syn. *measurement.* See Section 4.)

3.1.7.1 *Discussion*—Measurement values in test methods are of two general types: those whose magnitude is specified in the test method, such as the dimensions of a specimen, and those whose magnitude is found by testing, such as the measured mass of a specimen.

3.1.8 *propagation equation*, *n*—Synonym of *measurement tolerance propagation equation.*

3.1.9 *test method equation*, *n*—the mathematical formula specified in a test method, whereby the determination value is calculated from measurement values.

3.1.10 *test method tolerances*, *n—as specified in a test method*, the measurement tolerances, the determination tolerance, and the test result tolerance.

3.1.11 *test result*, *n*—a value obtained by applying a given test method, expressed either as a single determination or a specified combination of a number of determinations.

3.1.11.1 *Discussion*—In this practice the test result is the average of the number of determination values specified in the test method.

3.1.12 *test result tolerance*, *n—as specified in a test method*, the exactness with which a test result is to be recorded and reported.

3.1.13 *tolerance terms*, *n*—the individual members of a measurement tolerance propagation equation in which each member contains only one test method tolerance.

3.1.14 For the definitions of other terms used in this practice, refer to Terminology D 123 and Terminology E 456.

4. Discussion of Terms

4.1 *Test Results, Determinations, and Measurements*:

4.1.1 A test result is always a value (numerical quantity), but *measurement* and *determination* are often used as referring to general concepts, processes or values—the context indicating which meaning is intended. In this practice it is necessary to make these distinctions explicit by means of the terms given in Section 3.

4.1.2 The necessary distinctions can be illustrated by a test method for obtaining the mass per unit area of a fabric. Two kinds of measurement are required for each test specimen, length and mass. Two different length measurements are made, the length and the width of the specimen. One determination value of the mass per unit area is calculated by dividing the mass measurement value by the product of the length measurement value and the width measurement value from one specimen.

4.1.3 If the test method directs that mass per unit area determinations are to be made on three test specimens, the test result is the average of the three determination values, each obtained as directed in 4.1.2.

4.2 *Test Method Tolerances*:

4.2.1 The specified measurement tolerances tell the operator how closely observations are to be made and recorded. "Weigh the specimen to the nearest 0.01 g" and "Measure the length of the specimen to the nearest 0.02 in." are examples of typical measurement tolerance specifications in a test method.

4.2.2 The specified determination and test result tolerances tell the operator how many significant digits should be recorded in the determination value and in the test result, respectively.

5. Expressing Test Method Tolerances

5.1 Tolerances in test methods are commonly specified in one of four ways which are combinations of two general distinctions. A test method tolerance may be absolute or relative and may be expressed either as a range having an upper and a lower limit or as the result of rounding-off. These distinctions are illustrated by the following equivalent instructions that are possible in weighing a 5.00 g test specimen:

	Absolute	Relative
Upper and Lower Limit	within ±0.005 g	within ±0.1 %
Rounding-off	to the nearest 0.01 g	to the nearest 0.2 %

5.2 Within one method, state all test method tolerances in either the rounding-off mode or the upper and lower limit mode. The rounding-off mode is preferred for all test methods. Use a series of absolute tolerances for successive levels of a measurement or determination in preference to a relative tolerance.

5.3 The numerical value of a tolerance expressed in terms of rounding-off is twice that for the same tolerance expressed as an upper and lower limit. A discussion of rounding-off appears in Section 3 of Practice E 29 and in Chapter 4 of Ref **(1)**[4]. Numbers are usually rounded-off to the nearest 1, 2, or 5 units in the last place.

6. Tolerance Symbols

6.1 An absolute tolerance is symbolized by a capital delta, Δ, followed by a capital letter designating a measurement value, a determination value or a test result. Thus, ΔA.

6.2 A relative tolerance is symbolized by the absolute tolerance, ΔA, divided by the corresponding measurement value, determination value, or test result, A. Thus, $\Delta A/A$.

6.3 Relative tolerances are expressed as percentages by $100\Delta A/A$. All relative tolerances for a specific test method must be expressed in the same way throughout, either as fractions or as percentages.

[4] The boldface numbers in parentheses refer to the list of references at the end of this standard.

SUMMARY AND USES

7. Summary of Practice

7.1 A specific measurement tolerance propagation equation relating the determination tolerance to the measurement tolerances is derived by applying an adaptation of the law of error propagation to the test method equation. In this measurement tolerance propagation equation, the determination tolerance term should equal the sum of individual measurement tolerance terms.

7.2 Tentative measurement and determination tolerance values are substituted in the propagation equation terms, and the consistency of the selected test method tolerances is judged by the relative magnitudes of the tolerance terms.

7.3 Successive adjustments in the selected test method tolerance values are made until a consistent set of test method tolerances is established.

8. Significance and Use

8.1 In any test method, every direction to measure a property of a material should be accompanied by a measurement tolerance. Likewise, determination and test result tolerances should be specified. This practice provides a method for evaluating the consistency of the test method tolerances specified.

8.2 This practice should be used both in the development of new test methods and in evaluating old test methods which are being revised.

8.3 The test result tolerance obtained using this practice is not a substitute for a precision statement based on interlaboratory testing. However, the measurement tolerances selected by means of this practice will be an important part of the test method conditions affecting the precision of the test method.

MATHEMATICAL RELATIONSHIPS

9. Propagation Equations

9.1 The test method equations by which determination values are calculated from measurement values in textile testing usually involve simple sums or differences, products or ratios, or combinations of these. Measurement tolerance propagation equations for each of these types of relationships are derived in Annex A2 by applying the general measurement tolerance propagation equation, developed in Annex A1, to each of the typical test method equations. Propagation equations for a number of textile test method equations are given in Table A2.1.

9.2 In the following discussion, the determination of mass per unit area is used to illustrate the principles involved in obtaining consistent tolerances.

9.2.1 Eq 1 is a typical mass per unit area equation.

$$W = KM/DE \tag{1}$$

where:

W = the mass per unit area,
K = a constant to change W from one set of units to another,
M = the specimen mass,
D = the specimen width, and
E = the specimen length.

9.2.2 The corresponding propagation equation is Eq 2, derived in A2.4.1.

$$(\Delta W/W)^2/2 = (\Delta M/M)^2 + (\Delta D/D)^2 + (\Delta E/E)^2 \tag{2}$$

where:

$(\Delta W/W)^2/2$ = the mass per unit area determination tolerance term,
$(\Delta M/M)^2$ = the mass measurement tolerance term,
$(\Delta D/D)^2$ = the width measurement tolerance term, and
$(\Delta E/E)^2$ = the length measurement tolerance term.

10. Tolerance Terms

10.1 As shown in Annex A2, every propagation equation can be expressed in the form of $r = a + b + c$ . . ., in which each of the terms of this equation contains only one test method tolerance. The r term contains the determination tolerance, ΔR, and the other terms contain such measurement tolerances as ΔA, ΔB, and ΔC. The terms r, a, b, and c are tolerance terms.

10.1.1 For the mass per unit area example $r = (\Delta W/W)^2/2$, $a = (\Delta M/M)^2$, $b = (\Delta D/D)^2$, and $c = (\Delta E/E)^2$, as can be seen from Eq 2.

NOTE 1—The number of measurement tolerance terms is not restricted to 3, of course, but matches the number of measurements, q, for which tolerances are specified.

10.2 The key to this practice is the recognition that there are two ways of calculating the determination tolerance term:

10.2.1 The determination tolerance term, r, can be calculated from a specified value of ΔR using the expression for r given in the propagation equation. For example, in Eq 2 $r = (\Delta W/W)^2/2$. By substituting a typical value for W and a specified value for ΔW, a value of r is obtained.

10.2.2 The determination tolerance term can also be calculated as the sum of the measurement tolerance terms a, b, c, etc., which have been calculated from specified values of ΔA, ΔB, ΔC, etc. For the mass per unit area example, an estimate of the value of r may be obtained from values of a, b, and c found by substituting values of ΔM, M, ΔD, D, ΔE, and E in the tolerance term expressions $(\Delta M/M)^2$, $(\Delta D/D)^2$ and $(\Delta E/E)^2$.

10.3 These two ways of calculating the determination tolerance term usually produce different results, often radically different. In order to deal with this inconsistency, the second way of calculating the determination tolerance term is labelled u, which equals $a + b + c + \ldots$.

10.3.1 Therefore, in the following sections, r is the determination tolerance term value calculated from the specified determination tolerance by means of the expression for r supplied in the propagation equation, and u is the determination tolerance term value calculated from the specified measurement tolerances by means of the expressions for the measurement tolerance terms, a, b, c, etc., supplied in the propagation equation.

10.3.2 The term, r, is the specified determination tolerance term and u is the effective determination tolerance term.

11. Determination Tolerances

11.1 The propagation equation relates the determination tolerance to the specified measurement tolerances. However, in a test method it is usually the test result tolerance that is specified rather than the determination tolerance. Therefore, a bridge from the test result tolerance to the determination tolerance is necessary. This is supplied by Eq 3.

$$\Delta R = \Delta Q\sqrt{n} \tag{3}$$

where:

ΔR = the determination tolerance, to three significant digits,
ΔQ = the test result tolerance, to one significant digit, and
n = the number of determinations per test result.

See A1.3 for a derivation of Eq 3.

NOTE 2—While only one significant digit should be used for test result and measurement tolerances, three significant digits should be retained in the determination tolerance since it is a mathematical extension of the test result tolerance. In an extended calculation it is good practice to protect significant information being transmitted through intermediate stages of the calculation by retaining one or two extra significant digits on intermediate values used in the calculation.

12. Consistency Criteria

12.1 Two types of inconsistencies have been observed in test method tolerances. The first occurs between the specified determination tolerance and the value of the effective determination tolerance actually obtainable from the specified measurement tolerance, as discussed in 10.3. The second occurs between the specified measurement tolerances. In comparing the measurement tolerance terms of two measurements, it is often found that the one term will be more than 10 times the other so that the larger term dominates (and the smaller term is negligible) in its effect on the effective determination tolerance term, u. Such inconsistencies need to be examined. The means used in this practice is to study the tolerance term ratios u/r, a/r, b/r, c/r, etc.

12.2 Corresponding to these two inconsistencies are two norms which are stated in Eq 4 and Eq 5.

$$0.2 \leq u/r \leq 2.0 \tag{4}$$

$$0.2/q \leq a/r,\ b/r,\ c/r,\ \text{etc.} \leq 2.0/q \tag{5}$$

where:

u = the sum of the measurement tolerance terms,
r = the specified determination tolerance term,
q = the number of measurements, and
a, b, c, etc., = the measurement tolerance terms.

12.3 These ranges of acceptable ratio values should not be used rigidly. Rather, they should be taken as guidelines for constructive evaluation of the test method tolerances specified. For instance, an unusually low measurement tolerance term may be acceptable because there is little or no added cost in achieving the specified measurement tolerance instead of a larger one.

APPLICATION OF PRINCIPLES

13. Procedure

13.1 *Introduction*—The procedure in this practice falls into four steps. In the first step the propagation equation is obtained and all available information on the test method tolerances and measurement values is assembled. This is done only once. The remaining three steps probably will be repeated at least once before an acceptable set of specified test method tolerances and measurement values is obtained.

13.1.1 In Step 2, acceptable measurement tolerance ranges are calculated from the desired determination tolerance, the specified measurement values, and the consistency criterion stated in 12.2.

13.1.2 In Step 3, the selection of new tolerance and measurement values follows after comparing the starting values assembled in the first step with the acceptable ranges calculated in the second step. In making this selection, consideration is given to the feasibility of attaining the selected measurement tolerances with the apparatus and procedure given in the test method.

13.1.3 In Step 4, the selected values from Step 3 are next evaluated for consistency. To do this, these values are put in tolerance term form and the tolerance ratios are compared directly with the consistency criteria.

13.1.4 This consistency evaluation usually will suggest further study of the test method to see what changes can be made to achieve adequate consistency. If changes in any of the test method tolerances or in any of the specified measurement values are made, Steps 2, 3, and 4 must be repeated.

13.2 *Step 1, Preliminaries*:

13.2.1 *Propagation Equation*—Obtain the measurement tolerance propagation equation corresponding to the test method equation. If the equation is not listed in Table A2.1, follow the directions given in Annex A2.

13.2.2 *Tolerance Terms*—Identify the individual tolerance terms in the propagation equation and label them r, a, b, c, etc. as described in Section 10.

13.2.3 *Measurement Values*—When any of the tolerance terms contains measurement value(s) found by testing, select at least two values of R which are representative of the range in which the test method is to be used. Calculate the corresponding measurement values from the selected determination values using the test method equation.

13.2.3.1 As described in 3.1.7, measurement values are of two kinds. One is specified in the test method, for example, the dimensions of a specimen. The other is found by testing and varies with the material being tested, the mass of a specimen, for instance. Changes in the determination value, R, will result from changes in measurement values found by testing.

13.2.3.2 The magnitudes of these measurement values affect the relationship between the measurement tolerances and the determination tolerance. Therefore, specific values of each must be selected.

13.2.3.3 The process of evaluating the tolerances for consistency and feasibility may lead to changes in the specified measurement values in order to achieve one or the other objective.

13.2.4 *Starting Tolerance Values*—List all available test method tolerances, and label them as described in Section 6. Convert relative tolerances to absolute tolerances, using the selected measurement values, so that the effect of the latter may be seen more readily.

13.2.4.1 When the test method specifies that more than one determination is to be made for a test result, calculate the starting determination tolerance value, to three significant digits, from the selected test method tolerance value and the number of determinations specified in the test method, using Eq 3 (11.1). Even if no other test method tolerance is specified, specify a determination tolerance value, ΔR.

NOTE 3—Refer to Practice D 2905 for guidance in using available precision data to obtain starting values of *n* to achieve the selected test method tolerance, ΔQ, from the determination tolerance, ΔR.

13.3 *Step 2, Acceptable Tolerance Ranges*:

13.3.1 Calculate the specified determination tolerance term, *r*, to two significant digits, from ΔR for each of the values of *R* selected.

13.3.2 For each value of *R*, calculate a lower and an upper limit for *a/r, b/r, c/r,* etc. from Eq 5 in 12.2, using a value of *q* equal to the number of measurements in the test method equation. Retain only two significant digits.

13.3.3 Multiply each of the above limit values by the corresponding value of *r* to obtain the lower and upper limit values of *a, b, c,* etc. Retain only two significant digits.

13.3.4 Calculate lower and upper limit values of the individual measurement tolerances from *a, b, c,* etc. using the formulas identified as directed in 13.2. Retain only one significant digit.

13.3.5 Display the values obtained in 13.3 in a table for convenience in evaluating.

13.4 *Step 3, New Values*:

13.4.1 Compare the specified tolerance values with the lower and upper tolerance limits calculated as directed in 13.3.

13.4.2 If any tolerance is less than the lower limit, it is usually wise to delay further consideration until the other tolerances have been dealt with. Then, decide whether there would be any appreciable saving in equipment, material, or labor that would make going to a larger tolerance worthwhile. A small tolerance on one measurement may permit the use of an “oversize” tolerance for another measurement in meeting the consistency criterion for the determination tolerance.

13.4.3 If any measurement tolerance is greater than the upper limit, consider what changes in measurement tolerance are feasible. In general, select the smallest practical tolerance for use in the next step.

13.5 *Step 4, Consistency Evaluation*—After a new set of test method tolerances has been selected and evaluated for feasibility, it is ready for consistency evaluation.

13.5.1 Substitute the selected values of test method tolerances and measurement values, together with the corresponding determination values, in the appropriate tolerance terms and calculate values of *r, a, b, c,* etc. to two significant digits.

13.5.2 Calculate the sum of the measurement tolerance term values, $u = a + b + c + \ldots$.

13.5.3 Calculate the ratios *u/r, a/r, b/r, c/r,* etc. to two significant digits.

13.5.4 Compare *u/r* with 0.2 and 2.0. If *u/r* is greater than 2.0, study the measurement tolerance terms for the cause(s). If *u/r* is less than 0.2, consider reducing the determination and test method tolerances. Also keep in mind that the value of the test result tolerance is determined by the determination tolerance value and the number of determinations specified by the test method according to Eq 3. Thus, *r* can be reduced or increased by changing *n*.

13.5.5 Compare each of *a/r, b/r, c/r,* etc. with 0.2/*q* and 2/*q*.

13.5.5.1 This comparison should disclose no surprises, since a measurement tolerance below the lower limit established in 13.3 will have a ratio lower than 0.2/*q* and vice versa. However, comparing these ratios with the consistency criterion will reveal the extent of the inconsistency that exists.

13.5.5.2 If the ratio is greater than 2.0/*q*, study the tolerance and any measurement value included in the tolerance term. Will a change in measurement value decrease the tolerance term value?

13.6 *Successive Trials*—The remainder of the procedure consists of a succession of trials in which changes are made in test method tolerances and specified measurement values until a set of values is obtained which is both feasible and reasonably consistent. There are three aspects to the feasibility evaluation, which are summed up in the following questions the task group members must answer to their satisfaction.

13.6.1 Is the test result tolerance small enough to meet the needs of the users of the test method? Is it smaller than need be?

13.6.2 Can each measurement tolerance be achieved with the apparatus and procedure given in the test method?

13.6.3 If changes in the test method (new apparatus, larger specimens, more determinations, changes in technique, etc.) are necessary to achieve consistency, will the cost of testing be increased unreasonably?

14. Mass per Unit Area Example

14.1 *Test Method Directions*—The example chosen to illustrate the procedure described in Section 13 starts with the following directions: Cut five specimens 2.5 ± 0.05 in. by 10.0 ± 0.05 in., weigh each specimen to the nearest 0.01 *g*, and calculate the average mass per unit area in ounces per square yard to the nearest 0.1 oz/yd^2. Typical areal densities for this material range from 10 to 60 oz/yd^2.

14.2 *Step 1, Preliminaries*:

14.2.1 *Propagation Equation*:

14.2.1.1 The test method equation is Eq 6.

$$W = KM/DE \tag{6}$$

where:

W = mass per unit area, oz/yd^2,
K = constant to convert the measurement dimensional units to oz/yd^2 ,
M = specimen mass, *g*,
D = specimen width, in., and
E = specimen length, in.

14.2.1.2 As shown in 9.1.2 and A2.4, the corresponding propagation equation is Eq 7.

$$(\Delta W/W)^2/2 = (\Delta M/M)^2 + (\Delta D/D)^2 + (\Delta E/E)^2 \tag{7}$$

where:

ΔW = specified determination tolerance,
ΔM = specified measurement tolerance for specimen mass,
ΔD = specified measurement tolerance for specimen width, and
ΔE = specified measurement tolerance for specimen length.

14.2.2 *Tolerance Terms*:

14.2.2.1 The tolerance terms in this propagation equation are given by Eq 8, Eq 9, Eq 10, and Eq 11.

$$r = (\Delta W/W)^2/2, \tag{8}$$

$$a = (\Delta M/M)^2, \tag{9}$$

$$b = (\Delta D/D)^2, \quad \text{and} \tag{10}$$

$$c = (\Delta E/E)^2. \tag{11}$$

14.2.2.2 The effective determination tolerance term is given

$$u = a + b + c \tag{12}$$

14.2.2.3 The specified determination tolerance term is *r*, as given by Eq 8.

14.2.3 *Determination and Measurement Values*:

14.2.3.1 The propagation equation contains both absolute tolerances (ΔW, ΔM, ΔD, and ΔE) and determination and measurement values (*W*, *M*, *D*, and *E*) for which values must be selected.

14.2.3.2 The specimen dimensions are specified as $D = 2.5$ in. and $E = 10.0$ in.

14.2.3.3 The specimen mass is calculated from the expected range of test results by means of Eq 1, with $K = 45.72$ to convert from g/in.^{2} to oz/yd^2. For the initial consistency evaluation, choose two values for *W*: 10 oz/yd^2 and 60 oz/yd^2. The corresponding values of *M* are 5.47 g and 32.8 g.

14.2.4 *Starting Tolerance Values*:

14.2.4.1 The rounded-off mode of expressing the absolute tolerances is used.

14.2.4.2 The test method directions in 14.1 give the measurement tolerances as $M = 0.01$ g, $D = 0.1$ in., and $E = 0.1$ in.

14.2.4.3 The specified determination tolerance is calculated from the specified test result tolerance of 0.1 oz/yd^2 and the specified number of determinations, 5, using Eq 8. $\Delta W = 0.1 \sqrt{5} = 0.224$ oz/yd^2.

14.2.5 *Summary of Values*—All of these initial values are given in Table 1.

14.3 *Step 2, Acceptable Tolerance Ranges*:

14.3.1 Calculate the specified determination tolerance term, *r*, from $W = 0.224$ oz/yd^2 for the two values of *W* selected, using Eq 8.

14.3.2 For each value of *W*, calculate a lower and an upper limit value for *a/r*, *b/r*, and *c/r* from Eq 5 in 12.2 using $q = 3$, since there are three measurements.

14.3.3 Multiply each of the values calculated as directed in 14.3.2 by the corresponding value of *r*. These are the lower and upper limits of *a, b,* and *c*.

14.3.4 Obtain lower and upper limit values of the measurement tolerances ΔM, ΔD, and ΔE from the above values of *a*, *b*, and *c*, using Eq 9, Eq 10, and Eq 11. For instance $\Delta M = M \sqrt{a} = 5.47 \text{ x } \sqrt{(17 \times 10^{-6})} = 0.02$ g, for the 10 oz/yd^2 material.

14.3.5 All of the above values in 14.3 are summarized in Table 2.

14.4 *Step 3, New Values*:

14.4.1 The specified mass tolerance of 0.01 g is obviously smaller than need be to meet the consistency criteria. However, with present-day laboratory balances, it is no easier or less expensive to measure to a tolerance of between 0.02 and 0.07 g. Therefore leave ΔM at 0.01 g.

14.4.2 The specified specimen dimension tolerance of 0.1 in. is larger than the upper limit for all but the length measurement at 10 oz/yd^2. A study of the dimension measurement process indicates a tolerance of 0.02 in. should be feasible. This is lower than need be for the length measurement at 10 oz/yd^2 but just below the upper limit for 60 oz/yd^2. For the width measurement, however, 0.02 in. is still much too large at 60 oz/yd^2. An obvious solution to this problem would be to increase the specimen width to 10 in. so that the tolerance ranges for *D* would be the same as for *E*. However, this would quadruple the amount of material required and increase the cost of testing. Alternatively, a 5 by 5 in. specimen would have no longer area than the 2.5 by 10.0 in. specimen, and the tolerance range for the width would be the same as for *E*, better for *D* and worse for *E*.

14.5 *Step 4, Consistency Evaluation*:

14.5.1 Table 3 shows the effect of going to a 5 by 5 in. specimen and measuring the specimen dimensions to the nearest 0.02 in. In the upper half of the table are given the values of the tolerance terms *a, b, c,* and *u*; and in the bottom half, the tolerance ratios *a/r*, *b/r*, *c/r* and *u/r.*

14.5.2 Table 4 shows the effect of going to a 10 by 10 in. specimen and measuring the specimen dimensions to the nearest 0.02 in.

14.5.3 Keeping in mind the ratio criterion range of 0.067 to 0.67, from these two tables it can be seen that while the 5 by 5 in. specimen size is adequate for the material having a mass per unit area of 10 oz/yd^2 (and up to 32 oz/yd^2), it is still not satisfactory for 60 oz/yd^2 material. On the other hand, the 10 by 10 in. specimen size is about right for the 60 oz/yd^2 material, and much larger than need be for 10 oz/yd^2.

14.5.4 Thus, the task group appears to be faced with the choice of using larger specimens for heavier materials or of accepting a greater test result tolerance for heavier materials. A

TABLE 1 Mass Per Unit Area Example—Starting Determination, Measurement, and Tolerance Values

Equation Element	Tolerances	Determination and Measurement Values	
Mass per unit area, *W*	$\Delta W = 0.224$ oz/yd	10 oz/yd^2	60 oz/yd^2
Specimen Mass, *M*	$\Delta M = 0.01$ g	5.47 g	32.8 g
Specimen Width, *D*	$\Delta D = 0.1$ in.	2.5 in.	2.5 in.
Specimen Length, *E*	$\Delta E = 0.1$ in.	10.0 in.	10.0 in.

TABLE 2 Mass per Unit Area Example—Acceptable Tolerance Ranges

W ΔW *r*	10 oz/yd^2 0.224 oz/yd^2 250×10^{-6}		60 oz/yd^2 0.224 oz/yd^2 7.0×10^{-6}	
	Lower	Upper	Lower	Upper
a/r, b/r, c/r	0.067	0.67	0.067	0.67
a, b, c	17×10^{-6}	170×10^{-6}	0.47×10^{-6}	4.7×10^{-6}
ΔM	0.02 g	0.07 g	0.02 g	0.07 g
ΔD	0.01 in.	0.03 in.	0.002 in.	0.005 in.
ΔE	0.04 in.	0.13 in.	0.007 in.	0.022 in.

TABLE 3 Mass per Unit Area Example—Consistency Evaluation 5 by 5 in. Specimen

Measurement	Measurement Values	Tolerances	Tolerance Terms	
			10 oz/yd^2	60 oz/yd^2
M	5.47, 32.8 g	0.01 g	a 3.3×10^{-6}	a 0.093×10^{-6}
D	5 in.	0.02 in.	b 16×10^{-6}	b 16×10^{-6}
E	5 in.	0.02 in.	c 16×10^{-6}	c 16×10^{-6}
			u 35×10^{-6}	*u* 32×10^{-6}
	Ratio		Tolerance Ratios	
M	*a/r*		0.013	0.013
D	*b/r*		0.064	2.3
E	*c/r*		0.064	2.3
	u/r		0.14	4.6

TABLE 4 Mass per Unit Area Example—Consistency Evaluation 10 by 10 in. Specimen

Measurement	Measurement Values	Tolerances	Tolerance Terms	
			10 oz/yd^2	60 oz/yd^2
M	5.47, 32.8 g	0.01 g	a 3.3×10^{-6}	a 0.093×10^{-6}
D	10 in.	0.02 in.	b 4.0×10^{-6}	b 4.0×10^{-6}
E	10 in.	0.02 in.	c 4.0×10^{-6}	c 4.0×10^{-6}
			u 11.3×10^{-6}	*u* 8.1×10^{-6}
	Ratio		Tolerance Ratios	
M	*a/r*		0.013	0.013
D	*b/r*		0.016	0.57
E	*c/r*		0.016	0.57
	u/r		0.045	1.16

way of dealing with this latter option is to specify a relative tolerance instead of an absolute tolerance for reporting the test result.

14.6 *Relative Test Result Tolerance*:

14.6.1 The relative test result tolerance could be given in a statement such as: Report the mass per unit area to the nearest 0.5 %.

14.6.2 For evaluating the effect of specifying a relative test method tolerance, express the relative tolerance as a fraction rather than a percent. The determination tolerance corresponding to 0.5 % is $0.005 \sqrt{5} = 0.0112$ (by Eq 3, since the test result value equals the average of the individual determination values) and $r = 63 \times 10^{-6}$ for all levels of mass per unit area. Table 5 shows the effect of the relative test method tolerance on the acceptable tolerance ranges, using a 5 by 5 in. specimen size. Table 6 shows the effect on the toleranceratios.

14.6.3 This approach, using 5 by 5 in. specimens and a test result tolerance of 0.5 %, brings the dimension tolerances and the tolerance ratios both within the acceptable ranges established by the consistency criteria at the expense of accepting larger absolute test result tolerances for heavier materials.

TABLE 5 Mass per Unit Area Example—Acceptable Tolerance Ranges with 5 by 5 in. Specimens and a Relative Test Result Tolerance of 0.5 %

	10 oz/yd^2		60 oz/yd^2	
	Lower	Upper	Lower	Upper
r	63×10^{-6}	63×10^{-6}	63×10^{-6}	63×10^{-6}
a/r, b/r, c/r	0.067	0.67	0.067	0.67
a, b, c	4.2×10^{-6}	42×10^{-6}	4.2×10^{-6}	42×10^{-6}
ΔM	0.01 g	0.04 g	0.07 g	0.2 g
ΔD	0.01 in.	0.03 in.	0.01 in.	0.03 in.
ΔE	0.01 in.	0.03 in.	0.01 in.	0.03 in.

14.7 *Additional Work*:

14.7.1 The task group may, at this point, decide that enough work had been done and choose one of the above options to include in the test method standard.

14.7.2 The next step will be to conduct an interlaboratory study, using the selected measurement tolerances, in order to establish the precision of the test result obtained under these conditions.

TABLE 6 Mass per Unit Area Example—Consistency Evaluation with 5 by 5 in. Specimens and a Relative Test Result Tolerance of 0.5 %

Measurement	Measurement Values	Tolerances	Tolerance Terms	
			10 oz/yd^2	60 oz/yd^2
M	5.47, 32.8 g	0.01 g	a 3.3×10^{-6}	a 0.093×10^{-6}
D	5 in.	0.02 in.	b 16×10^{-6}	b 16×10^{-6}
E	5 in.	0.02 in.	c 16×10^{-6}	c 16×10^{-6}
			u 35×10^{-6}	*u* 32×10^{-6}
	Ratio		Tolerance Ratios	
M	*a/r*		0.05	0.001
D	*b/r*		0.25	0.25
E	*c/r*		0.25	0.25
	u/r		0.56	0.51

ANNEXES

(Mandatory Information)

A1. GENERAL MEASUREMENT TOLERANCE PROPAGATION EQUATION

A1.1 Statement of General Equation

A1.1.1 For any specific test method, the test method equation relating the measurement tolerances to the determination tolerance is obtained by applying Eq A1.1, the general measurement tolerance propagation equation, to the test method equation.

$$\Delta R^2/2 = \sum_{i=1}^{q} (\partial R/\partial X_i)^2 \, \Delta X_i^{\,2} \qquad \text{(A1.1)}$$

where:

X_i = the measurement made on a test specimen,
q = the number of independent measurements,
ΔX_i = the specified measurement tolerances,
ΔX_i^2 = the square of ΔX_i,
R = the determination value, a function of the q measurements, $X_1, X_2, X_3 \ldots X_q$,
ΔR = the determination tolerance,
ΔR^2 = the square of ΔR,
$\partial R/\partial X_i$ = the partial differential of R with respect to X_i, and
$\sum_{i=1}^{q}$ = the operation of summing the q terms of the form $(\partial R/\partial X_i)^2 \Delta X_i^2$.

A1.2 Derivative of General Equation

A1.2.1 This general measurement tolerance propagation equation is derived from the well known law of error propagation **(2)** given in Eq A1.2.

$$\text{Var } R = \sum_{i=1}^{q} (\partial R/\partial X_i)^2 \text{ Var } X_i \qquad \text{(A1.2)}$$

where:

Var R = the variance of R = the square of the standard deviation of R, and
Var X_i = the variance of X_i = the square of the standard deviation of X_i.

A1.2.2 Since the X_i are rounded values, their distributions are rectangular **(1).** The range of each distribution is ΔX_i, and the uniform probability density of the distribution is $1/\Delta X_i$. The variance of this rectangular distribution **(1)** is $\Delta X_i^2/12$.

TABLE A1.1 Determination Tolerance Term Divisor As Function of Measurement Tolerance Ratio for Eq A1.5

Ratio *p*	Divisor $(1 + p)^2/(1 + p^2)$
0.0	1.00
0.1	1.20
0.2	1.38
0.3	1.55
0.4	1.69
0.5	1.80
0.6	1.88
0.7	1.94
0.8	1.98
0.9	1.99
1.0	2.00

A1.2.3 The sum, $R(X_1, X_2)$ of two variables, X_1 and X_2, having rectangular distributions of the same range ($\Delta X_1 = \Delta X_2$) has a triangular distribution **(3).** When the ranges are different ($\Delta X_1 \neq \Delta X_2$), the distribution of the sum is an isosceles trapezoid.[5] By setting $\Delta X_2 = p\Delta X_1$ *with* $0 \leq p \leq 1$, *the range of R can be expressed by* $\Delta R = (1 + p)\Delta X_1$ *and the variance of the trapezoidal distribution is given by Eq A1.3.*

$$\text{Var } R = \frac{\Delta R^2}{12} \cdot \frac{(1 + p^2)}{(1 + p)^2} \qquad \text{(A1.3)}$$

Eq A1.3 is derived by applying the integration for the second moment about a zero mean to the probability density equations of an isosceles trapezoidal distribution. The distribution of R is rectangular at $p = 0$, trapezoidal for $0 < p < 1$ and triangular at $p = 1$. The corresponding variances for $p = 0$ and $p = 1$ from Eq A1.3 are $\Delta R^2/12$ and $\Delta R^2/24$, respectively. The trapezoidal variances are intermediate to those for rectangular and triangular distributions as shown in Table A1.1.

A1.2.4 Substituting the measurement variances, $\Delta X_1^2/12$ and $\Delta X_2^2/12$, and the determination variance expressed by Eq A1.3 in Eq A1.2 produces Eq A1.4.

[5] The isosceles trapezoidal probability density curve is determined by convolutions as described in Ref **(4).**

$$\Delta R^2 (1 + p^2)/(1 + p)^2 = (\partial R/\partial X_1)^2 \Delta X_1^2 + (\partial R/\partial X_2)^2 \Delta X_2^2 \quad \text{(A1.4)}$$

Since $\partial R/\partial X_1$ and $\partial R/\partial X_2$ are both 1, Eq A1.4 reduces to Eq A1.5.

$$\Delta R^2 /[(1 + p)^2 /(1 + p^2)] = \Delta X_1^2 + \Delta X_2^2 \quad \text{(A1.5)}$$

From Table A1.1 we see that 2 is a good approximation for $(1 + p)^2 /(1 + p^2)$ for values of p greater than 0.5.

A1.2.5 The distribution of the sum, R, of four or more rectangularly distributed variables of the same range is essentially normal **(5).** The effective range of a normal distribution at a given probability level is $\Delta R = 2z \sqrt{Var\ R}$, where z is the number of standard deviation units associated with the given probability level, and so Var $R = \Delta R^2/4z^2$. Therefore, for a determination having a normal distribution the test method tolerances are related as shown in Eq A1.6.

$$\Delta R^2 = (4z^2/12) \sum_{i=q}^{q} (\partial R/\partial X_i)^2 \Delta X_i^2 \quad \text{(A1.6)}$$

This equation is derived by substituting the above value of Var R in Eq A1.2 as well as Var $X_i = \Delta X_i^2/12$. For $4z^2/12 = 2$, as suggested in A1.2.4, $z = 2.45$. This value of z corresponds to a probability level of 98.6 % that the determination value, R, lies within the range ΔR.

A1.2.6 The above discussions indicate that since the measurement values, X_i, have rectangular distributions and the determination value, R, may have a trapezoidal, triangular, normal or some intermediate distribution, the relationship between ΔR and the ΔX_i has the form shown in Eq A1.7.

$$\Delta R^2 /k = \sum_{i=1}^{q} (\partial R/\partial X_i)^2 \Delta X_i^2 \quad \text{(A1.7)}$$

Furthermore, for accomplishing the purposes of this practice setting, $k = 2$ is considered an adequate allowance for the differences in distribution between R and the X_i. Substituting 2 for k in Eq A1.7 produces the general measurement tolerance propagation equation used in this practice.

A1.3 Test Result and Determination Tolerances

A1.3.1 When a test result is calculated as the average of a number of determination values, Eq A1.1 does not apply because the distributions of the determination values are not rectangular but are approximately normal and the distribution of the test result is also normal.

A1.3.2 The variance of a normally distributed variable is given by Eq A1.8 (see A1.2.5).

$$\text{Var } R = {'}\Delta R^2/4z^2 \quad \text{(A1.8)}$$

where:

R	=	the normally distributed variable,
Var R	=	the variance of R = the square of the standard deviation of R,
ΔR	=	the expected range of R, and
z	=	the number of standard deviation units associated with a given probability level.

A1.3.3 The equation relating the test result to the determination value is Eq A1.9.

$$Q = \sum_{i=1}^{n} R_i/n \quad \text{(A1.9)}$$

where:

Q	=	the test result,
R_i	=	the ith determination value,
n	=	the number of determination values, and
$\sum_{i=1}^{n}$	=	the operation of summing the n determination values.

A1.3.4 Applying Eq A1.2-A1.9 we obtain Eq A1.10.

$$\text{Var } Q = \sum_{i=1}^{n} (\partial Q/R_i)^2 \text{ Var } R_i \quad \text{(A1.10)}$$

Applying Eq A1.8 to Q and the R_i in Eq A1.10, and deriving the $\partial Q/\partial R_i$ from Eq A1.9, we obtain Eq A1.11.

$$\Delta Q^2 = \Delta R^2/n \quad \text{(A1.11)}$$

where:

ΔQ	=	the specified test result tolerance,
ΔR	=	the specified determination tolerance, and
n	=	the number of determinations averaged.

A1.3.5 The specified determination tolerance used in the procedure of this practice is calculated from the specified test result tolerance by means of Eq A1.12 which is merely a rearrangement of Eq A1.11.

$$\Delta R = \Delta Q\sqrt{n} \quad \text{(A1.12)}$$

A2. SPECIFIC MEASUREMENT TOLERANCE PROPAGATION EQUATIONS

A2.1 Introduction

A2.1.1 *Test Method Equations*—The equations by which determination values are calculated from measurement values in textile testing usually involve simple sums or differences, products or ratios, or combinations of these.

A2.1.2 *Propagation Equations*—The following sections present typical examples of such test method equations and the derivation of the corresponding specific measurement tolerance propagation equations by applying the general measurement tolerance propagation equation, Eq A2.1, to the test method equations.

$$\Delta R^2 /2 = \sum_{i=1}^{q} (\partial R/\partial X_i)^2 \Delta X_i^2 \quad \text{(A2.1)}$$

where:

ΔR	=	tolerance expected for the determination value, R,
ΔX_i	=	tolerances specified for the measurement values, X_i,
$\partial R/X_i$	=	partial derivatives of R by X_i, and
$\sum_{i=1}^{q}$	=	operation of summing the q terms of the form $(\partial R/\partial X_i)^2\Delta X_i^2$.

See Annex A1. for the derivation of Eq A1.1.

A2.1.3 *Tolerance Terms*—As stated in 10.1, every measurement tolerance propagation equation can be expressed in the form of Eq A2.2

$$r = \sum_{i=1}^{q} x_i \tag{A2.2}$$

where:
r = $\Delta R^2/2$, the tolerance term for the determination value, R, and
x_i = $(\partial R/\partial X_i)^2 \Delta X_i^2$, the tolerance term for the measurement value, X_i.

A2.1.4 *Equation Types and Examples*—For each of three types of test method equation, a specific case having only a few measurements is presented. For the first two equations, the general case having an indefinite number of measurements is also given. A list of propagation equation terms for test method equations commonly occurring in textile testing is given in Table A2.1.

SIMPLE SUMS OR DIFFERENCES

A2.2 Specific Case

A2.2.1 *Test Method Equation*—The net mass of a test specimen is obtained by subtracting the tare mass of a watch glass, on which the specimen is placed for weighing, from the gross mass of the specimen together with the watch glass. The net mass of the specimen is calculated using Eq A2.3.

$$N = G - T \tag{A2.3}$$

where:
N = net mass of the test specimen,
G = gross mass of the specimen together with watch glass, and
T = tare mass of the watch glass.

A2.2.2 *Propagation Equation*—Applying Eq A1.1-A2.3 produces Eq A2.4.

$$\Delta N^2/2 = (\partial N/\partial G)^2 \Delta G^2 + (\partial N/\partial T)^2 \Delta T^2 \tag{A2.4}$$

where:
ΔN = tolerance expected for the net mass determination value, N,
ΔG = tolerance specified for the gross mass measurement value, G,
ΔT = tolerance specified for the tare mass measurement value, T,
$\partial N/\partial G$ = partial derivative of N by G, and
$\partial N/\partial T$ = partial derivative of N by T.

The solutions for the two partial derivatives are:

$$\partial N/\partial G = \partial G/\partial G - \partial T/\partial G = 1, \text{ and} \tag{A2.5}$$
$$\partial N/\partial T = \partial G/\partial T - \partial T/\partial T - 1,$$

since G and T are independent measurements and, thus, $\partial T/\partial G = 0$ and $\partial G/\partial T = 0$. Substituting these partial derivative values in Eq A2.4 produces the specific measurement tolerance propagation equation Eq A2.6.

$$\Delta N^2/2 = \Delta G^2 + \Delta T^2 \tag{A2.6}$$

A2.2.3 *Tolerance Terms*—The tolerance term form of Eq A2.6 is Eq A2.7.

$$r = a + b \tag{A2.7}$$

where:
r = $\Delta N^2/2$, the tolerance term for the net mass determination value,
a = ΔG^2, the tolerance term for the gross mass measurement value, and
b = ΔT^2, the tolerance term for the tare mass measurement value.

A2.3 General Case for Sums or Differences

A2.3.1 *Test Method Equation*—For any number of different measurements on the same specimen, the test method equation is Eq A2.8.

$$R = \sum_{i=1}^{q} a_i X_i \tag{A2.8}$$

where:
R = determination value,
X_i = measurement value of the ith property,

TABLE A2.1 Propagation Equation Tolerance Terms for Typical Test Method Equations for Textiles

Test Method Equation	r	= a	+ b	+ c	+ d	ASTM Standard
$R = K(A - B)$	$\Delta R^2/2$	$= K^2 \Delta A^2$	$+ K^2 \Delta B^2$			D 2654
$R = KA/B$	$(\Delta R/R)^2/2$	$= (\Delta A/A)^2$	$+ (\Delta B/B)^2$			D 1775
$R = KA/(A + B)$	$(\Delta R/R)^2/2$	$= [B/(A + B)]^2(\Delta A/A)^2$	$+ [B/(A + B)]^2(\Delta/B)^2$			D 1574
$R = KA/(B - A)$	$(\Delta R/R)^2/2$	$= [B/(B - A)]^2 (\Delta A/A)^2$	$+ [B/(BA)]^2 + (\Delta B/B)^2$			D 885
$R = A/(B + C - D)$	$(\Delta R/R)^2/2$	$= (\Delta A/A)^2$	$+ [\Delta B/(B + C - D)]^2$	$+ [\Delta C/(B + C - D)]^2$	$+ [\Delta D/(B + C - D)]^2$	D 1585
$R = K(A - B)/A$	$(\Delta R/R)^2/2$	$= [B\Delta A/A(A - B)]^2$	$+ [\Delta B/(A - B)]^2$			D 204
$R = K(A - B)/B$	$(\Delta R/R)^2/2$	$= [\Delta A/(A - B)]^2$	$+ [A\Delta B/B(A - B)]^2$			D 461
$R = K(A - B)/C$	$(\Delta R/R)^2/2$	$= [\Delta A/(A - B)]^2$	$+ [\Delta B/(A - B)]^2$	$+ (\Delta C/C)^2$		D 461
$R = K(A - B)/BC$	$(\Delta R/R)^2/2$	$= [\Delta A/(A - B)]^2$	$+ [A\Delta B/B(A - B)]^2$	$+ (\Delta C/C)^2$		D 76
$R = K(A - B)/(C - B)$	$(\Delta R/R)^2/2$	$= [\Delta A/(A - B)]^2$	$+ [(A - C)\Delta B/(A - B)(C - B)]^2 + [\Delta C/(C - B)]^2$			D 2402

NOTE 1—As described in the text, the lowercase letters represent the tolerance terms of a propagation equation. The uppercase letters symbolize the constants and variables in a test method equation. K is a dimensional constant. R is the determination value. A, B, C and D are measurement values.

a_i = a real number that is the coefficient of the ith term in the test method equation (for example, in Eq A2.3 the coefficient of G is + 1 and that for *T* is − 1, and
q = number of properties measured.

A2.3.2 *Propagation Equation*—Applying Eq A1.1-A2.8 produces the specific measurement tolerance propagation equation Eq A2.9.

$$\Delta R^2/2 = \sum_{i=1}^{q} a_i^{\,2}\,\Delta X_i^{\,2} \qquad \text{(A2.9)}$$

where:
ΔR = tolerance expected for the determination value, *R*,
ΔX_i = tolerance specified for the measurement value, X_i, and
a_i = $\partial R/\partial X_i$, the partial derivative of *R* by X_i.

A2.3.3 *Tolerance Terms*—The tolerance term form of Eq A2.9 is Eq A2.10.

$$r = \sum_{i=1}^{q} x_i \qquad \text{(A2.10)}$$

where:
r = $\Delta R^2/2$, the tolerance term for the determination value, *R*, and
x_i = $a_i^{\,2} X_i^{\,2}$, the tolerance term for the measurement value, X_i.

SIMPLE PRODUCTS, RATIOS, OR POWERS

A2.4 Specific Case

A2.4.1 *Test Method and Equation*—The mass per unit area of a rectangular test specimen obtained by weighing and measuring the length and width of a test specimen is calculated using Eq A2.11.

$$W = KM/DE \qquad \text{(A2.11)}$$

where:
W = mass per unit area,
K = constant to change *W* from one set of units to another,
M = mass of the test specimen,
D = width of the test specimen, and
E = length of the test specimen.

Here the X_i of Eq A1.1 are $X_1 = M$, $X_2 = D$ and $X_3 = E$. The determination value $R = W$.

A2.4.2 *Propagation Equation*—Applying Eq A1.1-A2.11 produces Eq A2.12

$$\Delta W^2/2 = (\partial W/\partial M)^2\,\Delta M^2 + (\partial W/\partial D)^2\,\Delta D^2 + (\partial W/\partial E)^2\,\Delta E^2 \qquad \text{(A2.12)}$$

where:
ΔW = tolerance expected for the mass per unit area, *W*,
ΔM = tolerance specified for the test specimen mass, *M*,
ΔD = tolerance specified for the test specimen width, *D*,
ΔE = tolerance specified for the test specimen length, *E*,
$\partial W/\partial M$ = partial derivative of *W* by *M*,
$\partial W/\partial D$ = partial derivative of *W* by *D*, and
$\partial W/\partial E$ = partial derivative of *W* by *E*.

The solutions of the three partial derivatives are: $\partial W/\partial M = K/DE$, $\partial W/\partial D = -KM/D^2\ E$ and $\partial W/\partial E = -KM/DE^2$. Substituting these values in Eq A2.12 produces Eq A2.13.

$$\Delta W^2/2 = (K/DE)^2\,\Delta M^2 + (KM/D^2 E)^2\,\Delta D^2 + (KM/DE)^2\,\Delta E^2 \qquad \text{(A2.13)}$$

Dividing both sides of Eq A2.13 by the square of Eq A2.11, $W^2 = (KM/DE)^2$, simplifies it to Eq A2.14, the specific measurement tolerance propagation equation.

$$(\Delta W/W)^2/2 = (\Delta M/M)^2 + (\Delta D/D)^2 + (\Delta E/E)^2 \qquad \text{(A2.14)}$$

A2.4.3 *Tolerance Terms*—The tolerance term form of Eq A2.14 is Eq A2.15.

$$r = a + b + c \qquad \text{(A2.15)}$$

where:
r = $(\Delta W/W)^2/2$, the tolerance term for the mass per unit area, *W*,
a = $(\Delta M/M)^2$, the tolerance term for the test specimen mass, *M*,
b = $(\Delta D/D)^2$, the tolerance term for the test specimen width, *D*, and
c = $(\Delta E/E)^2$, the tolerance term for the test specimen length, *E*.

A2.5 General Case

A2.5.1 *Test Method Equation*—For any number of different measurements on the same test specimen, the test method equation is Eq A2.16.

$$R = K \prod_{i=1}^{q} X_i^{\,m_i} \qquad \text{(A2.16)}$$

where:
R = determination value,
K = constant to change R from one set of units to another,
X_i = measurement value of the ith property,
m_i = exponent of X_i (Note A2.1),
q = number of properties measured, and
$\prod_{i=1}^{q}$ = operation of multiplying the *q* terms of the form $X_i^{m_i}$.

NOTE A2.1—The value of m_i is +1 for measurement values in the numerator, −1 for those in the denominator; +*k* for positive powers and −*k* for negative powers, where *k* may be any positive integer or real number.

A2.5.2 *Propagation Equation*—Applying Eq A1.1 to Eq A2.16 produces Eq A2.17.

$$\Delta R^2/2 = K^2 \sum_{i=1}^{q} (\partial X_i^{\,m_i}/\partial X_i)^2 \left(\prod_{j=1}^{q} X_j^{\,m_j}\right) \Delta X_i^{\,2}/(X_i^{\,m_i})^2 \qquad \text{(A2.17)}$$

where:

X_i or X_j = one particular measurement.

Since the $\partial X_i^{m_i}/\partial X_i$ all equal $m_i X_i^{m_i-1}$, Eq A2.17 becomes Eq A2.18.

$$\Delta R^2/2 = K^2 \sum_{i=1}^{q} \left(\prod_{j=1}^{q} X_j^{m_j} \right) (m_i X_i^{-1})^2 \Delta X_i^2 \quad \text{(A2.18)}$$

and dividing both sides of this equation by the square of Eq A2.16, $R^2 = K^2 \left(\prod_{i=1}^{q} X_j^{m_j} \right)^2$, simplifies Eq A2.18 to Eq A2.19, the specific measurement tolerance propagation equation.

$$(\Delta R/R)^2/2 = \sum_{i=1}^{q} (m_i \Delta X_i / X_i)^2 \quad \text{(A2.19)}$$

because the K^2 and the $\left(\prod_{j=1}^{q} X_j^{m_j} \right)^2$ factors all cancel out, leaving only the $(m_i X_i^{-1})^2$ factors.

A2.5.3 *Tolerance Terms*—The tolerance term form of Eq A2.19 is Eq A2.20.

$$r = \sum_{i=1}^{q} x_i \quad \text{(A2.20)}$$

where:

r = $(\Delta R/R)^2/2$, the tolerance term for the determination value, R, and

x_i = $(m_i \Delta X_i/X_i)^2$, the tolerance term for the measurement value, X_i.

COMBINATIONS OF SUMS AND PRODUCTS

A2.6 Specific Case

A2.6.1 *Test Method Equation*—The percent change in the length of a yarn skein resulting from immersion in boiling water is calculated using Eq A2.21.

$$P = 100(A - B)/B \quad \text{(A2.21)}$$

where:

P = percent change in skein length,

A = skein length after treatment, and

B = skein length before treatment.

A2.6.2 *Propagation Equation*—Before applying Eq A1.1-A2.21, expand this equation to Eq A2.22 for ease in differentiating.

$$P = 100A/B - 100 \quad \text{(A2.22)}$$

Since $\partial P/\partial A = 100/B$ and $\partial P/\partial B = -100A/B^2$, applying Eq A1.1-A2.22 produces Eq A2.23, the specific measurement tolerance propagation equation.

$$\Delta P^2/2 = (100/B)^2 \Delta A^2 + (-100A/B^2)^2 \Delta B^2 \quad \text{(A2.23)}$$

A2.6.3 *Tolerance Terms*—The tolerance term form of Eq A2.23 is Eq A2.24.

$$r = a + b \quad \text{(A2.24)}$$

where:

r = $\Delta P^2/2$, the tolerance term for the percent change, P,

a = $(100/B)^2 \Delta A^2$, the tolerance term for the after treatment length A, and

b = $(100A/B^2)^2 \Delta B^2$, the tolerance term for the before treatment length, B.

A2.7 *General Case*—The possible combinations of sums and products are too diverse to be represented by one general equation and thus there is no general measurement tolerance propagation equation for such combinations.

OTHER TEST METHOD RELATIONSHIPS

A2.8 *Propagation Equations*—The measurement tolerance propagation equations for other relationships between determination values and measurement values can be derived by anyone having a knowledge of algebra and differential calculus plus a table of derivatives **(6).**

REFERENCES

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