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Standard Practice for
Sampling and Data-Analysis for Structural Wood and Wood-Based Products[1]

INTRODUCTION

Sampling and data analysis should be integrated in the design and evaluation of wood and wood-based structural products. This practice is useful in assessing the appropriateness of the assigned properties and for checking the effectiveness of grading procedures. Statistical methodologies are provided to serve as a basis for the empirical establishment and evaluation of mean and near minimum property estimates. These population estimates are then used by product standards to assign structural design values for use with an established design methodology (that is, allowable stress design, load and resistance factor design, limit states design, etc.). Near-minimum property estimates are typically used by the product standards to define the performance for a variety of structural properties where strength is a primary consideration (that is, extreme fiber stress in bending, axial tension, axial compression, shear, and elasticity for buckling concerns). Population mean estimates are often used to assess serviceability design criteria where strength is not the primary design concern (that is, elasticity estimates used for deformation calculations, permissible compression stress at a deformation, etc.).

For situations where a manufactured product is sampled repeatedly or lot sizes are small, alternative test methods as described in Ref (**1**)[2] may be more applicable.

1. Scope

1.1 This practice covers sampling and analysis procedures for the investigation of specified populations of wood and wood-based structural products referred to in this standard as products. Appropriate product standards should be referenced for presentation requirements for data. Depending on the interest of the user, the population from which samples are taken may range from the products produced at a specific manufacturing site to all the products produced in a particular grade from a particular geographic area, during some specified interval of time. This practice generally assumes that the population is sufficiently large so that, for sampling purposes, it may be considered infinite. Where this assumption is inadequate, that is, the population is assumed finite, many of the provisions of this practice may be employed but the sampling and analysis procedure must be designed to reflect a finite population. The statistical techniques embodied in this practice provide procedures to summarize data so that logical judgments can be made. This practice does not specify the action to be taken after the results have been analyzed. The action to be taken depends on the particular requirements of the user of the product.

1.2 The values stated in inch-pound units are to be regarded as the standard.

1.3 This practice does not purport to address the adjustment factors needed to adjust test data to standardized mechanical and environmental conditions (that is, temperature, moisture, test span, or load duration). Additionally, it provides a basis for statistical estimates that will typically require further adjustment to determine design values for use with an accepted design methodology (that is, allowable stress, limit states, or load and resistance factor design). It shall be the responsibility of the user to seek out the appropriate adjustments in specific product standards.

1.4 *This practice does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

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[1] This practice is under the jurisdiction of ASTM Committee D07 on Wood and is the direct responsibility of Subcommittee D07.02 on Lumber and Engineered Wood Products.

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[2] The boldface numbers in parentheses refer to the list of references at the end of this practice.

2. Referenced Documents

2.1 *ASTM Standards:*[3]

D9 Terminology Relating to Wood and Wood-Based Products

D198 Test Methods of Static Tests of Lumber in Structural Sizes

D245 Practice for Establishing Structural Grades and Related Allowable Properties for Visually Graded Lumber

D1990 Practice for Establishing Allowable Properties for Visually-Graded Dimension Lumber from In-Grade Tests of Full-Size Specimens

D2555 Practice for Establishing Clear Wood Strength Values

D3737 Practice for Establishing Allowable Properties for Structural Glued Laminated Timber (Glulam)

D5055 Specification for Establishing and Monitoring Structural Capacities of Prefabricated Wood I-Joists

D5456 Specification for Evaluation of Structural Composite Lumber Products

D6570 Practice for Assigning Allowable Properties for Mechanically Graded Lumber

E29 Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications

E105 Practice for Probability Sampling of Materials

3. Terminology

3.1 *Definitions*—For definitions of terms related to wood, refer to Terminology D9.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *established design methodology, n*—methodology used to determine if a structure will perform adequately using structural design values.

3.2.1.1 *Discussion*—Established design methods currently used include allowable stress design, load and resistance factor design, limit states design.

3.2.2 *products, n*—wood and wood-based structural products.

3.2.3 *serviceability, n*—condition other than the building strength under which a building is still considered useful.

3.2.3.1 *Discussion*—Serviceability limit state design of structures includes factors such as durability, overall stability, fire resistance, deflection, cracking, and excessive vibration.

3.2.4 *strength, n*—level of stress expressed in terms of force per area being evaluated for design.

3.2.5 *structural design values, n*—unit stresses and stiffness values utilized in design.

3.2.5.1 *Discussion*—Structural design values are test results adjusted for duration of load, factor of safety, and expected service conditions.

3.2.6 *tolerance limit (TL), n*—tolerance limit with 95 % content and 75 % confidence.

4. Statistical Methodology

4.1 Two general analysis procedures are described under this practice: parametric and nonparametric. A nonparametric approach requires fewer assumptions and is generally more conservative than a parametric procedure. The parametric approach assumes a known distribution of the underlying population, an assumption which, if incorrect, may lead to inaccurate results. Some examples of parametric distributions are normal, lognormal and Weibull. Therefore, if a parametric approach is used, appropriate statistical tests shall be employed to substantiate this choice along with measures of test adequacy (**2**). For parametric approaches in this practice, the examples provided are based on assuming normality.

NOTE 1—The assumption of "normality" in the examples is not a given and should be verified before using in real cases. A nonparametric approach requires fewer assumptions and is generally more conservative than a parametric procedure.

4.2 *Population:*

4.2.1 It is imperative that the population to be evaluated be clearly defined, as inferences made pertain only to that population. In order to define the population, it may be necessary to specify (*1*) grade name and description, (*2*) geographical area over which sampling will take place (nation, state, manufacturing site, etc.), (*3*) species or species group, (*4*) time span for sampling (a day's production, a month, a year, etc.), (*5*) material dimensions, and (*6*) moisture content.

4.2.2 The sampling program should consider the population from which the test specimens originated, including types of processing methods or marketing practices with respect to any influence they may have on the representative nature of the sample. Test specimens may be collected from stock at manufacturing sites, centers of distribution, at points of end use or directly from current production. Sampling programs should consider potential effects of the sample source, timing, and location on the variability of specimen properties.

4.3 *Sampling Procedure:*

4.3.1 *Random Sampling*—The sampling unit is commonly the individual test specimen. When this is not the case, see 4.3.3. The sampling shall assure random selection of sampling units from the population described in 4.2 with all members of the population sharing equal probability of selection. The principles of Practice E105 shall be maintained. When sampling current production, refer to Practice E105 for a recommended sampling procedure (see Appendix X3 of this practice for an example of this procedure). If samples are selected from inventory, random number tables may be used to determine which pieces will be taken for the sample.

4.3.2 *Sampling with Unequal Probabilities*—Under some circumstances, it may be advisable to sample with unequal but known probabilities. Where this is done, the general principles of Practice E105 shall be maintained, and the sampling method shall be completely reported.

4.3.3 *Sequential Sampling*—When trying to characterize how a certain population may perform in a structure, it may be deemed more appropriate to choose a sampling unit, such as a package, that is more representative of how the product will be selected for use. Such a composite sampling unit might consist

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[3] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

of a sequential series of pieces chosen to permit estimation of the properties of the unit as well as the pieces. Where this is done, the principles in 4.3.1 and 4.3.2 apply to these composite sampling units and the sampling method shall be completely reported.

4.4 *Sample Size:*

4.4.1 Selection of a sample size depends upon the property or properties to be estimated, the actual variation in properties occurring in the population, and the precision with which the property is to be estimated. For any property, strength values, or the modulus of elasticity, various percentiles of the population may be estimated and for all properties, nonparametric or parametric techniques are applicable. Commonly, the mean is estimated for properties which will eventually be used by the product standard to evaluate a serviceability design concern. Near minimum property estimates are typically evaluated for properties where strength is the primary objective.

4.4.2 Determine sample size sufficient for estimating the mean by a two-stage method, with the use of the following equation. This equation assumes the data is normally distributed and the mean is to be estimated to within 5 % with specified confidence:

$$n = \left(\frac{ts}{\alpha \bar{X}} \right)^2 = \left(\frac{t}{\alpha} CV \right)^2 \qquad (1)$$

where:

n = sample size,
s = standard deviation of specimen values,
$\bar{X}$ = specimen mean value,
CV = coefficient of variation, $s/\bar{X}$,
α = estimate of precision, (0.05), and
t = value of the t statistic from Table 1.

Often, the values of s, $\bar{X}$, and t or CV and t are not known before the testing program begins. However, s and $\bar{X}$, or CV, may be approximated by using the results of some other test program, or they may simply be guessed.

NOTE 2—An example of initial sample size calculation is:

Sampling a grade of lumber to determine its mean modulus of elasticity (*E*). Assuming a 95 % confidence level, the *t* statistic can be approximated by 2.

s = 300 000 psi (2067 MPa)
$\bar{X}$ = assigned *E* of the grade = 1 800 000 psi (12 402 MPa)
CV = (300 000/1 800 000) = 0.167
t = 2
n = $\left(\frac{2}{0.05} \times 0.167 \right)^2 = 44.622$ (45 pieces)

Calculate the sample mean and standard deviation and use them to estimate a new sample size from Eq 1, where the value of *t* is taken from Table 1. If the second sample size exceeds the first, the first sample was insufficient; obtain and test the additional specimens.

NOTE 3—More details of this two-stage method are given in Ref (**3**).

4.4.3 Tolerance intervals and their associated tolerance limits can be one-sided or two-sided. In the examples of this standard, it is assumed that the limits are one-sided lower limits. To determine sample size based on a tolerance limit (TL), the desired content (C) and associated confidence level must be selected (Note 4). The choice of a specified content and confidence is dependent upon the end-use of the material, economic considerations, current design practices, code requirements, etc. For example, a content of 95 % and a confidence level of 75 % may be appropriate for a specific property of structural lumber. Different confidence levels may be suitable for different products or specific end uses. Appropriate content and confidence levels shall be selected before the sampling plan is designed.

TABLE 1 Values of the *t* Statistics Used in Calculating Confidence Intervals[A]

df *n* – 1	*CI* = 75 %	*CI* = 95 %	*CI* = 99 %
1	2.414	12.706	63.657
2	1.604	4.303	9.925
3	1.423	3.182	5.841
4	1.344	2.776	4.604
5	1.301	2.571	4.032
6	1.273	2.447	3.707
7	1.254	2.365	3.499
8	1.240	2.306	3.355
9	1.230	2.262	3.250
10	1.221	2.228	3.169
11	1.214	2.201	3.106
12	1.209	2.179	3.055
13	1.204	2.160	3.012
14	1.200	2.145	2.977
15	1.197	2.131	2.947
16	1.194	2.120	2.921
17	1.191	2.110	2.898
18	1.189	2.101	2.878
19	1.187	2.093	2.861
20	1.185	2.086	2.845
21	1.183	2.080	2.831
22	1.182	2.074	2.891
23	1.180	2.069	2.807
24	1.179	2.064	2.797
25	1.178	2.060	2.787
26	1.177	2.056	2.779
27	1.176	2.052	2.771
28	1.175	2.048	2.763
29	1.174	2.045	2.756
30	1.173	2.042	2.750
40	1.167	2.021	2.704
60	1.162	2.000	2.660
120	1.156	1.980	2.617
∞	1.150	1.960	2.576

[A] Adapted from Ref (**3**). For calculating other confidence levels, see Ref (**3**).

NOTE 4—The content is an estimate of the proportion of the population that lies above the tolerance limit. For example, a tolerance limit with a content of 95 % describes a level at which 95 % of the population lies above the tolerance limit. The confidence level is the percentage of time that the desired content is expected to be achieved through sampling.

4.4.3.1 To determine the sample size for near-minimum properties, the nonparametric tolerance limit concept of Ref (**3**) may be used (Table 2). This will provide the sample size suitable for several options in subsequent near-minimum analyses. Although the frequency with which the tolerance limit will fall above (or below) the population value, corresponding to the required content, is controlled by the confidence level selected, the larger the sample size the more likely the tolerance limit will be close to the population value. It is, therefore, desirable to select a sample size as large as possible commensurate with the cost of sampling and testing (see also 5.4).

TABLE 2 Sample Size and Order Statistic for Estimating the 5 % Nonparametric Tolerance Limit, NTL[A]

75 % Confidence[B]		95 % Confidence		99 % Confidence	
Sample Size[C]	Order Statistic[D]	Sample Size	Order Statistic	Sample Size	Order Statistic
28	1	59	1	90	1
53	2	93	2	130	2
78	3	124	3	165	3
102	4	153	4	198	4
125	5	181	5	229	5
148	6	208	6	259	6
170	7	234	7	288	7
193	8	260	8	316	8
215	9	286	9	344	9
237	10	311	10	371	10
259	11	336	11	398	11
281	12	361	12	425	12
303	13	386	13	451	13
325	14	410	14	478	14
347	15	434	15	504	15
455	20	554	20	631	20
562	25	671	25	755	25
668	30	786	30	877	30
879	40	1013	40	1115	40
1089	50	1237	50	1349	50

[A] Adapted from Ref **(8)**. For other tolerance limits or confidence levels, see Ref **(5)**, **(8)**, or **(9)**.

[B] The shaded columns indicate the tolerance levels traditionally used for most wood and wood-based products.

[C] Where the sample size falls between two order statistics (for example, 27 and 28 for the first order statistic at 75 confidence), the larger of the two is shown in the table, and the confidence is greater than the nominal value.

[D] The rank of the ordered observations, beginning with the smallest.

4.4.3.2 If a parametric approach is used, then a tolerance limit with stated content and confidence can be obtained for any sample size; however, the limitation expressed in 4.4.3.1 applies. That is, although the frequency that the tolerance limit falls above (or below) the population value, corresponding to the required content is controlled, the probability that the tolerance limit will be close to the population value depends on the sample size. For example, if normality is assumed, the parametric tolerance limit (PTL) will be of the form $PTL = \bar{X} - Ks$, (see Ref **(3)**), and the standard error (SE) of this statistic may be approximated by the following equation:

$$SE = s\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}} \tag{2}$$

where:

s = standard deviation of specimen values,
n = sample size, and
K = confidence level factor from Table 3.

The sample size, n, may be chosen to make the standard error sufficiently small for the intended end use of the material.

NOTE 5—An example of sample size calculation where the purpose is to estimate a near minimum property is shown in the following calculation based on the assumption of normality of population.

Estimate the sample size, n, for a compressive strength parallel to grain test in which normality will be assumed. A CV of 22 % and a mean strength of 4600 psi are assumed based on other tests. The target PTL of the grade is 2700 psi. The PTL is to be estimated with a content of 95 % (5 % PTL) and a confidence of 75 %.

CV = 0.22
$\bar{X}$ = 4600 psi (31.7 MPa)
s = (0.22) (4600) = 1012 psi (7.0 MPa)
K = $(\bar{X} - PTL)/s = (4600-2700)/1012 = 1.877$

From Table 3:

K = 1.869 for $n = 30$

Therefore, $n \approx 30$ specimens.

$$SE = 1012\sqrt{\frac{1}{30} + \frac{1.877^2}{2(30-1)}} \tag{3}$$

$$= 310 \text{ psi } (2.14 \text{ MPa})$$

Consequently, although 30 specimens is sufficient to estimate the 5 % PTL with 75 % confidence, the standard error (approximately 12 % of the PTL) illustrates that, with this size sample, the PTL estimated by test may not be as close to the true population fifth percentile as desired. A larger n may be desirable.

4.4.4 Often, the objective of the evaluation program will be to estimate mean and near-minimum properties simultaneously. When this is the case, only one sample size need be used. It should be the greater of the two obtained in accordance with 4.4.2 and 4.4.3.

4.4.5 If a sampling unit other than an individual test specimen is to be used, as provided for in 4.3.3, then the required sample size must be determined by procedures that are statistically appropriate for the sampling method chosen. In the case of multisource data, as in the sampling of some or all manufacturing sites in a defined region, special procedures may be required, for example, those based on the methodology introduced in Ref **(4)**. In all cases, the procedures shall be fully described.

5. Analysis and Presentation of Results

5.1 If the goal is to evaluate the data against published design properties or to develop new design properties, the data shall be adjusted for moisture content, test conditions (environment and mechanical arrangement), and other factors in accordance with the subject product requirements. Depending on the product standard used to establish allowable properties for the particular product, these adjustments to the data may be required before data analysis and presentation. Table 4 shows a listing of currently developed product standards that use adjustment procedures applied to test data developed in accordance with this procedure. An example of an adjustment of various properties are given in -.

5.1.1 Properties shall be adjusted to a single moisture content if appropriate for the objective of the testing program and relevant product standard. Although test results can be adjusted for moisture content, these adjustments decrease in accuracy with increasing change in moisture content. For this reason, it is suggested that the specimens be conditioned as closely as possible to the target moisture content prior to test. Properties also must be adjusted for the duration of load condition anticipated.

5.2 The results of the tests performed in accordance with consensus standard testing procedures shall be analyzed and presented as (*1*) a set of summarizing statistics, and (*2*) an appendix of unadjusted individual test specimen results. If parametric procedures are to be used, a description of the

TABLE 3 K Factors for One-Sided Tolerance Limits for Normal Distributions^{A}

	75 % Confidence (g = 0.25)				95 % Confidence (g = 0.05)				99 % Confidence (g = 0.01)			
1-p	0.75	0.90	0.95^{B}	0.99	0.75	0.90	0.95	0.99	0.75	0.90	0.95	0.99
*n*3	1.464	2.501	3.152	4.397	3.805	6.156	7.657	10.555	8.726	13.997	17.374	23.900
4	1.255	2.134	2.681	3.726	2.617	4.162	5.145	7.044	4.714	7.381	9.085	12.389
5	1.151	1.962	2.464	3.422	2.149	3.407	4.203	5.742	3.453	5.362	6.580	8.941
6	1.087	1.859	2.336	3.244	1.895	3.007	3.708	5.063	2.847	4.412	5.407	7.336
7	1.043	1.790	2.251	3.127	1.732	2.756	3.400	4.643	2.490	3.860	4.729	6.413
8	1.010	1.740	2.189	3.042	1.617	2.582	3.188	4.355	2.253	3.498	4.286	5.813
9	0.984	1.702	2.142	2.978	1.532	2.454	3.032	4.144	2.083	3.241	3.973	5.390
10	0.964	1.671	2.104	2.927	1.465	2.355	2.912	3.982	1.954	3.048	3.739	5.075
11	0.946	1.646	2.074	2.886	1.411	2.276	2.816	3.853	1.852	2.898	3.557	4.830
12	0.932	1.625	2.048	2.852	1.366	2.210	2.737	3.748	1.770	2.777	3.411	4.634
13	0.919	1.607	2.026	2.823	1.328	2.156	2.671	3.660	1.702	2.677	3.290	4.473
14	0.908	1.591	2.008	2.797	1.296	2.109	2.615	3.585	1.644	2.593	3.189	4.338
15	0.899	1.577	1.991	2.776	1.267	2.069	2.566	3.521	1.595	2.522	3.103	4.223
16	0.890	1.565	1.977	2.756	1.242	2.033	2.524	3.465	1.552	2.460	3.028	4.124
17	0.883	1.555	1.964	2.739	1.220	2.002	2.487	3.415	1.514	2.405	2.963	4.037
18	0.876	1.545	1.952	2.724	1.200	1.974	2.453	3.371	1.480	2.357	2.906	3.961
19	0.869	1.536	1.942	2.710	1.182	1.949	2.424	3.331	1.450	2.314	2.854	3.893
20	0.864	1.528	1.932	2.697	1.166	1.926	2.396	3.296	1.423	2.276	2.808	3.832
21	0.858	1.521	1.924	2.686	1.151	1.906	2.372	3.263	1.398	2.241	2.767	3.777
22	0.854	1.514	1.916	2.675	1.138	1.887	2.349	3.234	1.376	2.209	2.729	3.727
23	0.849	1.508	1.908	2.666	1.125	1.869	2.329	3.207	1.355	2.180	2.695	3.682
24	0.845	1.502	1.901	2.657	1.113	1.853	2.310	3.182	1.336	2.154	2.663	3.640
25	0.841	1.497	1.895	2.648	1.103	1.838	2.292	3.159	1.319	2.129	2.634	3.602
30	0.825	1.475	1.869	2.614	1.058	1.778	2.220	3.064	1.247	2.030	2.516	3.447
35	0.812	1.458	1.849	2.588	1.025	1.732	2.167	2.995	1.194	1.958	2.430	3.335
40	0.802	1.445	1.834	2.568	0.999	1.697	2.126	2.941	1.154	1.902	2.365	3.249
45	0.794	1.434	1.822	2.552	0.978	1.669	2.093	2.898	1.121	1.857	2.312	3.181
50	0.788	1.426	1.811	2.539	0.960	1.646	2.065	2.863	1.094	1.821	2.269	3.125
60	0.777	1.412	1.795	2.518	0.932	1.609	2.023	2.808	1.051	1.764	2.203	3.039
70	0.769	1.401	1.783	2.502	0.911	1.581	1.990	2.766	1.019	1.722	2.153	2.974
80	0.762	1.393	1.773	2.489	0.894	1.560	1.965	2.733	0.994	1.689	2.114	2.924
90	0.757	1.386	1.765	2.479	0.881	1.542	1.944	2.707	0.974	1.662	2.083	2.884
100	0.753	1.380	1.758	2.470	0.869	1.527	1.927	2.684	0.957	1.639	2.057	2.850
120	0.745	1.371	1.747	2.456	0.851	1.503	1.900	2.650	0.930	1.604	2.016	2.797
140	0.740	1.364	1.739	2.446	0.837	1.485	1.879	2.623	0.909	1.577	1.985	2.758
160	0.736	1.358	1.733	2.438	0.826	1.471	1.862	2.602	0.893	1.556	1.960	2.726
180	0.732	1.353	1.727	2.431	0.817	1.460	1.849	2.585	0.879	1.539	1.940	2.700
200	0.729	1.350	1.723	2.425	0.809	1.450	1.838	2.570	0.868	1.524	1.923	2.679
250	0.723	1.342	1.714	2.414	0.794	1.431	1.816	2.542	0.846	1.496	1.891	2.638
300	0.719	1.337	1.708	2.406	0.783	1.417	1.800	2.522	0.830	1.476	1.868	2.609
350	0.715	1.332	1.703	2.400	0.775	1.407	1.788	2.507	0.818	1.461	1.850	2.586
400	0.712	1.329	1.699	2.395	0.768	1.398	1.778	2.495	0.809	1.449	1.836	2.568
450	0.710	1.326	1.696	2.391	0.763	1.391	1.770	2.484	0.801	1.438	1.824	2.553
500	0.708	1.324	1.693	2.387	0.758	1.385	1.763	2.476	0.794	1.430	1.815	2.541
600	0.705	1.320	1.689	2.382	0.750	1.376	1.753	2.462	0.783	1.416	1.799	2.521
700	0.703	1.317	1.686	2.378	0.745	1.369	1.744	2.452	0.775	1.406	1.787	2.506
800	0.701	1.315	1.683	2.374	0.740	1.363	1.738	2.443	0.768	1.398	1.777	2.493
900	0.699	1.313	1.681	2.371	0.736	1.358	1.732	2.436	0.762	1.391	1.769	2.483
1000	0.698	1.311	1.679	2.369	0.733	1.354	1 728	2.431	0.758	1.385	1.763	2.475
1500	0.694	1.306	1.672	2.361	0.722	1.340	1.712	2.411	0.742	1.365	1.741	2.447
2000	0.691	1.302	1.669	2.356^{C}	0.715	1.332	1.703	2.400^{C}	0.733	1.354	1.727	2.431^{C}
2500	0.689	1.300^{C}	1.666^{C}	2.353^{C}	0.711	1.326	1.697^{C}	2.392^{C}	0.727	1.346	1.719^{C}	2.419^{C}
3000	0.688	1.299^{C}	1.664^{C}	2.351^{C}	0.708	1.323^{C}	1.692^{C}	2.386^{C}	0.722	1.340^{C}	1.712^{C}	2.411^{C}
inf	0.674	1.282	1.645	2.326	0.674	1.282	1.645	2.326	0.674	1.282	1.645	2.326

[A] Obtained from a noncentral *t* inverse approach; see Ref **(10)**.

[B] The shaded column indicates the tolerance level traditionally used for most wood and wood-based products.

[C] Computed using formula X5.2.

selection procedures and a tabulation of distribution parameters shall be provided. Any "best-fit" judgment between competing distributions shall be documented.

NOTE 6—A best-fit procedure should recognize the low power of some published procedures. To check the fit, the series of tests outlined in Ref (**9**) represents several alternatives. Also, tests based on the Anderson-Darling statistic have been shown to be among the more powerful tests (**2**). It should be noted, however, that not all tests are valid for all distributions and that these procedures are effective for checking central tendency. For instance, revised standard tables of the Kolmogorov-Smirnov statistic are presently available for the normal, logistic, and exponential distributions (**2**). Goodness of fit techniques have also been developed for Weibull distributions (**10**).

5.3 Statistics shall be shown with three significant digits. Adequate significant digits shall be maintained in all intermediate calculations in accordance with Practice E29 to avoid rounding errors in the statistics.

5.3.1 The sample mean is calculated as follows:

TABLE 4 Product Standards That Use Adjustment Procedures Applied To Test Data Developed With Practice D2915

Product Standards	Designation
Standard Practice for Establishing structural Grades and Related Allowable Properties for Visually Graded Lumber	D245
Standard Practice for Establishing Allowable Properties for Visually-Graded Dimension Lumber from In-Grade Tests of Full-Size Specimens	D1990
Standard Practice for Establishing Clear Wood Strength Values	D2555
Standard Practice for Establishing Allowable Properties for Structural Glued Laminated Timber (Glulam)	D3737
Standard Specification for Establishing and Monitoring Structural Capacities of Prefabricated Wood I-Joist	D5055
Standard Specification for Evaluation of Structural Composite Lumber Products	D5456
Standard Practice for Assigning Allowable Properties for Mechanically Graded Lumber	D6570

$$\bar{X} = \sum_{i=1}^{n} x_i / n \tag{4}$$

where:

x_i = individual observations, and
n = sample size.

The sample mean is an unbiased estimator of the true population mean.

5.3.2 The sample standard deviation is calculated as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{n} \left(x_i{}^2 - \left[\left(\sum x_i\right)^2/n\right]\right)}{n-1}} \text{ or} \tag{5}$$

$$= \sqrt{\frac{\sum_{i=1}^{n} \left(x_i - \bar{X}\right)^2}{n-1}}$$

5.3.3 The confidence interval (CI) for the mean is calculated as follows:

$$\text{CI} = \bar{X} \pm \left(ts/\sqrt{n}\right) \tag{6}$$

where t depends on the sample size and confidence level, and is given in Table 1. A CI of this type provides that, if the population is normally distributed, a given percent of all intervals found in this manner are expected to contain the true population mean.

5.3.4 The sample nonparametric percent point estimate (NPE) may be interpolated from the sample. To perform the interpolation, arrange the test values in ascending order. Symbolically, call them x_1, x_2, x_3, ... x_n. Beginning with the lowest value (that is, first order statistic, see Note 7), calculate $i/(n + 1)$, where i is the order of the value, for each successively higher value until $i/(n + 1) \geq k/100$, call it the j^{th} value, equals or exceeds the sample k percentile point estimate. Interpolate the nonparametric k percentage point estimate by:

$$\text{NPE} = \left[\frac{k}{100}(n+1) - (j-1)\right]\left[x_j - x_{(j-1)}\right] + x_{(j-1)} \tag{7}$$

where k is the desired percentile point estimate.

NOTE 7—Order statistics are ranked test values from the lowest to the highest. For example, the first order statistic is the lowest test value or the weakest piece in the sample, the second order statistic is the second weakest piece, etc.

5.3.5 The nonparametric lower tolerance limit (NTL) of a specified content is the m^{th} order statistic, where m depends upon the sample size and confidence level. Table 2 depicts the order statistic required to determine the lower-5 % NTL at a given sample size and three confidence levels. For example, if the sample size was 93 and the confidence level was chosen to be 95 %, $m = 2$. That is, the lower-5 % NTL with at least 95 % confidence would be the second order statistic. If other lower percentiles are estimated, the corresponding NTLs can be determined (**3, 11**).

5.3.6 If parametric methods are used, the parametric point estimate (PPE) and lower parametric tolerance limit (PTL) shall be estimated by procedures documented as adequate for the method adopted (**1, 3, 11**).

NOTE 8—Two examples of typical test data and a summary of the results that meet the requirements of 5.1 – 5.3 are given in Appendix X1 and Appendix X2.

5.3.7 If a sampling unit other than an individual piece is used, then the calculation of sample means, standard deviations, confidence intervals, tolerance limits, and exclusion limits must be made in a manner statistically consistent with the sampling procedure chosen.

5.4 If the purpose of the testing program is to establish properties for the population that are further adjusted to design values by the appropriate product standard, this is done using the results of 5.3.1, 5.3.4, 5.3.5, 5.3.6, or 5.3.7. The value of modulus of elasticity used for serviceability calculations shall be the sample mean of 5.3.1, if the width of the confidence interval is a sufficiently small fraction of the mean (for example, if $ts/\left(\bar{X}\sqrt{n}\right) \leq \lambda$, where λ, predetermined by the user will normally be in the range from 0.01 to 0.10). If this condition is not satisfied, additional samples must be taken as described in 4.3 until the condition holds. Generally, the value of any near-minimum strength property shall be the sample 5 % NPE of Section 5.3.4, if the relative difference between the NPE and the NTL is sufficiently small (that is, if (NPE − NTL-)/NPE < δ, where δ will normally be in the range from 0.01 to 0.10). This condition is essentially that of having sufficiently narrow confidence interval for the NPE. If this condition is not satisfied, additional samples may be taken until the condition holds, or the NTL may be used for the property value. If the latter course is chosen, one should be cognizant of the imprecision in the NTL consequent on the sample size (see 4.4.3.1). Alternatively, the PPE and PTL of the parametric procedures provided for in 5.3.6 may be employed in a parallel manner.

6. Reporting of Data

6.1 Enough detail shall be provided to repeat the analysis conducted. The actual data shall be reported. The sampling technique used to be in compliance with Section 3 shall be reported. Information shall be provided to justify the choice of a parametric analysis technique in compliance with Section 4.

7. Applications

7.1 The results may be used in combination with the product standards to evaluate the accuracy of existing design properties or to establish new design properties.

7.2 Where properties have been previously assigned to a product population, one purpose of this practice is to provide a format for evaluation of this assignment through full-size product tests. Provisions are made for estimating both the mean and near-minimum property values.

7.3 Results obtained following the procedures and analyses of this practice may also be used to characterize the population sampled for establishing design values. The specific characterization with respect to the population, such as the mean or a near-minimum property, depends on the objective, the content, and confidence associated with the test sample. The representativeness and size of the sample influence how the characterization can be made. Contemporary practice is reflected in 5.4, however, other interpretations may be appropriate.

7.4 The end use of a specific product will dictate the specification requirement. Indeed, this practice addresses itself to the procedures for sampling specified populations and procedures for analyzing the results. It cannot be implemented without the selection of values for the confidence levels and degree of precision needed at various stages of the procedures. These values should be given careful consideration so that they are compatible with the anticipated end use, the risks that surround imprecise estimates, or incorrect decisions, and the costs of sampling, testing, and analysis.

8. Keywords

8.1 confidence level; mean and median properties; parametric and nonparametric properties; sample size; standard error

APPENDIXES

(Nonmandatory Information)

X1. TYPICAL EXAMPLE—COMMODITY LUMBER

X1.1 *Population Description*—Selected at random, from one mill, were 80 No. 2 grade Hem-Fir two-by-fours (current lumber agency grade rules). The 80 test specimens were equilibrated to an average of 15 % moisture content (see Note 5 and Practice D245). For this particular example, an 80-piece sample was considered appropriate.

X1.2 The purpose of the test was to evaluate the bending modulus of elasticity for member deflection calculations, E, and tensile strength, F_t, of a one-mill sample relative to present design values. Consequently, the fifth percentile estimate will be considered for strength and the mean value for E (see 5.4).

X1.3 The design value for the grade and species sampled is given in Table X1.1. A table of test statistics is given in Table X1.2.

X1.4 Histograms and fitted normal, lognormal, and Weibull distributions of edgewise bending E and tensile strength are shown in Fig. X1.1 and Fig. X1.2.

X1.5 Several of the individual test results are shown, only as an example of data that is typically recorded (Table X1.3). It may be desirable to tabulate additional information, such as specific gravity, knot location, etc., depending on purpose. Note that tensile strength data is ordered in ascending order.

X1.6 If appropriate best fit tests have been carried out and documented, only the best fit distribution need be illustrated; however, illustration of other options is instructive (see Table X1.4). Note that the nonparametric estimates in Table X1.4 for tensile strength can be estimated directly from Table X1.3, but the estimates for modulus of elasticity are based on data, most of which, is not shown in Table X1.3.

X1.7 Using Appendix X4, 5.3.1, 5.3.2, 5.3.3, and 5.4, the confidence interval for the mean E value (Table X1.2) did not contain the value as printed in Table X1.1. Consequently, it was decided this sample E did not verify the design E. Analysis of the tension strength values was conducted in accordance with 5.3.4 and 5.3.5. After adjusting the nonparametric lower -5 % tolerance limit to an allowable design value according to Table 8 in practice D245 (that is, 1152/2.1 = 548.6 psi (3.8 MPa)), it can be seen that this value is below the value shown in Table X1.1 ($F_t = 675$ psi (4.6 MPa)); therefore, the sample tension values do not verify the design tension value.

X1.8 Similar analyses could be performed using parametric procedures and employing the values shown in Table X1.4.

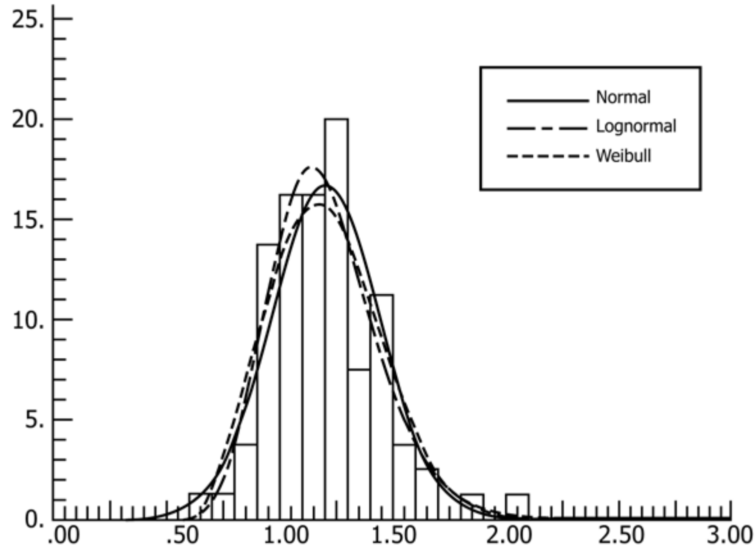

FIG. X1.1 Static Edgewise Modulus of Elasticity (10^6 psi)

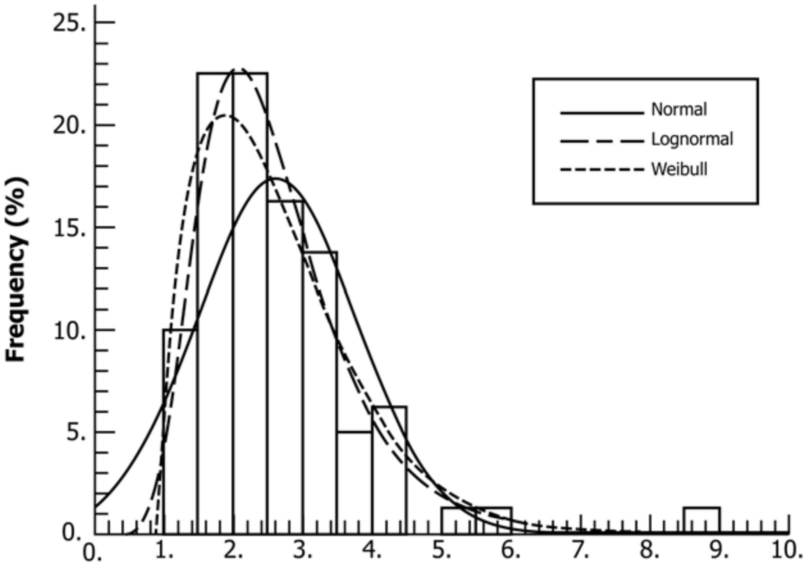

FIG. X1.2 Tensile Strength (1000 psi)

TABLE X1.1 Design Values for No. 2 Grade Hem-Fir Two-by-Fours[A]

Species/Grade	Design Values	
	F_t, psi (MPa)	E, psi (MPa)
Hem-Fir No. 2	675 (4.6)	1 400 000 (9646)

[A] National Design Specification for Wood Construction.

TABLE X1.2 Example Test Results for No. 2 Grade Hem-Fir Two-by-Fours[A]

Property	Mean, psi (MPa)	Confidence Interval for Mean, psi (MPa)[B]	Standard Deviation, psi (MPa)	Sample Size
Static edgewise modulus of elasticity[C]	1 201 600 (8279)	1 148 500 (7113) –1 254 700 (8645)	238 500 (1643)	80
Tensile strength	1250 (8.6)	1100 (7.6) – 1350 (9.3)	547 (3.8)	80

[A] All statistics in psi; all adjusted to 15 % moisture content in accordance with D245 or D1990 (not rounded).

[B] 95 % confidence.

[C] Adjusted to ℓ/d of 21 and uniform load.

TABLE X1.3 Example of Test Results Ordered by Tensile Strength—Two-by-Four Sample

Specimen Number	Moisture Content at Test, %	Tensile Strength, psi (MPa)	Edgewise Modulus of Elasticity 10^2, psi (MPa)[A]	Width, in. (mm)[B]	Thickness, in. (mm)[B]	Bending Strength Ratio, %[C]
1 P 43	15.0	1004 (6.9)	994 (6849)	3.47 (88)	1.47 (37)	13
1 P 1	15.0	1092 (7.5)	959 (6607)	3.47 (88)	1.51 (38)	13
1 P 15	13.0	1152 (7.9)	1061 (7310)	3.42 (87)	1.50 (38)	52
1 P 28	15.0	1169 (8.0)	667 (4596)	3.45 (88)	1.46 (37)	47
1 P 22	16.0	1257 (8.7)	950 (6545)	3.46 (88)	1.49 (38)	52

[A] Test ℓ/d of 44, quarter-point load, corrected to ℓ/d of 21 and a uniform load.

[B] At test moisture content.

[C] Obtained by 5.3.4.1 of Practice D245.

TABLE X1.4 Estimates of Population Parameters for Two-by-Four Sample

Parameter	Static Edgewise Modulus Elasticity 10^6, psi (MPa)	Tensile Strength[A], psi (MPa)
Weibull:		
5 % point estimate	0.8255 (5688)	1.230 (8.5)
Lognormal:		
5 % point estimate	0.8549 (5890)	1.270 (8.7)
5 % TL (75 %)	0.8340 (5746)	1.208 (8.3)
Normal:		
Mean	1.2016 (8279)	2.616 (18.0)
Standard deviation	0.2385 (1643)	1.149 (7.9)
5 % point estimate	0.8091 (5575)	0.726 (5.0)
5 % TL (75 %)	0.7790 (5367)	0.580 (4.0)
Nonparametric:		
5 % point estimate	0.8745 (6025)	1.169 (8.0)
5 % TL (75 %)	0.8490 (5850)	1.152 (7.9)

[A] Not reduced to allowable property.

X2. TYPICAL EXAMPLE—LADDER RAIL STOCK

X2.1 *Population Description*—(Species) ladder rail stock graded in accordance with the (Grading Rules) as "V.G. Ladder Rails." Two hundred pieces of 1⅜ by 2¾-in. by 8 ft were selected randomly from stock at a ladder manufacturer in (location). Specimens were equilibrated in a conditioning room. Actual average moisture content of specimens equaled 11.2 %. The standard deviation was 1.4 %. The purpose of the sampling, testing, and analysis was to obtain the bending modulus of rupture (MOR) and modulus of elasticity (*E*) of typical ladder rails for use in a research study on ladder rail properties. Only mean and lower tail properties estimated by nonparametric procedures were of interest. The 95 % confidence level was deemed appropriate for both *E* and MOR in this study.

X2.2 Data reduced to summary statistics are shown in Table X2.1. Examples of individual specimen data are shown in Table X2.2; Table X2.3 contains estimates of near-minimum values. Histograms of test results are shown in Fig. X2.1 and Fig. X2.2. Empirical cumulative distribution functions are shown in Fig. X2.3 and Fig. X2.4.

X2.3 Following the procedures of 4.4 it was determined that the dispersion of *E* (static edgewise) measurements met the 5 % requirement (that is, $ts/\left(\bar{X}\sqrt{n}\right) = 1.96 \times 301\,500/(1\,755\,300 \times 200^{0.5}) = 0.024 \leq 0.05$) with 95 % confidence. Consequently, the research suggested an edgewise E of 1.7×10^6 psi could be used as a design value (Practice D245 rounding rule would round the test value to 1.8 but this would be out of the confidence interval for the mean, thus 1.7 was chosen).

X2.4 Continuing the procedures of 5.4 for the MOR, comparisons between the NPE and several NTL's can be made (Table X2.3). Maintaining the 10 % relative difference criterion (NPE-NTL/NPE < 0.10) the relative difference for the NTL at a 95 % confidence level does not meet the criterion (6518-5364/6518 = 0.17 > 0.10). Therefore, the 95 % confidence level goal of X2.1 for MOR is not met. Either more sampling (see 5.4) is required or the NTL (5364 psi (37 MPa)) may be used as the best estimate of the population MOR.

TABLE X2.1 Ladder Rail Test Statistics[A]

Property	Mean, psi (MPa)	Confidence Interval for Mean, psi (MPa)[B]	Standard Deviation, psi (MPa)	Sample Size
Static edgewise modulus of elasticity[C]	1 755 300 (12 094)	1 713 200 (11 804)–1 797 400 (12 384)	301 500 (2077)	200
Modulus of rupture[D]	9758 (67)	9520 (66)–10 014 (69)	1836 (12.6)	200

[A] All statistics in psi.

[B] 95 % confidence.

[C] Adjusted to ℓ/d of 21, uniform load, and 12 % moisture content.

[D] Adjusted to 12 % moisture content.

TABLE X2.2 Sample Test Results—Ladder Rail

Specimen	Moisture Content at Test, %	Modulus of Rupture, psi (MPa)[A]	Edgewise Static E, 10^6psi (MPa) [A,B]	Width at Test, in. (mm)	Thickness at Test, in. (mm)
103	12.8	14 343 (99)	2.51 (17 294)	2.753 (70)	1.366 (35)
111	11.0	11 423 (79)	1.80 (12 402)	2.760 (70)	1.381 (35)
114	8.6	6505 (45)	1.37 (9439)	2.784 (71)	1.406 (36)
121	11.6	9708 (69)	2.17 (14 951)	2.762 (70)	1.386 (35)

[A] Statistics adjusted to 12 % moisture content in accordance with D1990; not adjusted to allowable properties.

[B] Adjusted to ℓ/d of 21 and uniform load; actual conditions were ℓ/d of 33 and center point load.

TABLE X2.3 Estimates of Near-Minimum Population Parameters of Ladder Rail

Property	5 % Point Estimates	5 % Tolerance Limits 75 % Confidence	5 % Tolerance Limits 95 % Confidence	5 % Tolerance Limits 99 % Confidence
Edgewise modulus of elasticity[A]	1.30 (8957)	1.29 (8888)	1.23 (8475)	1.16 (7992)
Modulus of rupture[B]	6518 (45)	6072 (42)	5364 (37)	5353 (37)

[A] 10^6 psi (MPa); adjusted to ℓ/d of 21 and uniform load in accordance with 5.3.1; adjusted to 12 % moisture content in accordance with D1990; not reduced to allowable property.

[B] psi (MPa); adjusted to 12 % in accordance with D1990; not reduced to allowable property.

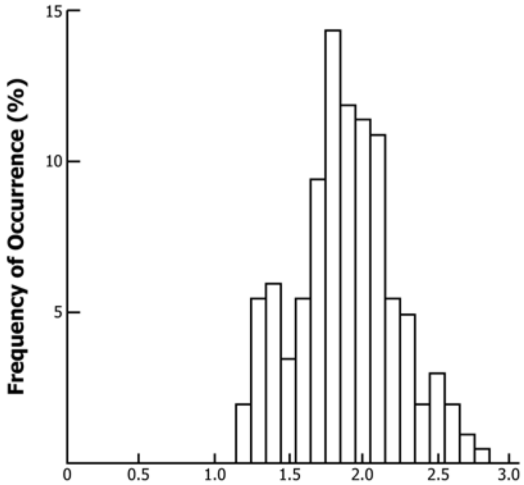

FIG. X2.1 Edgewise *E* (10^6 psi)

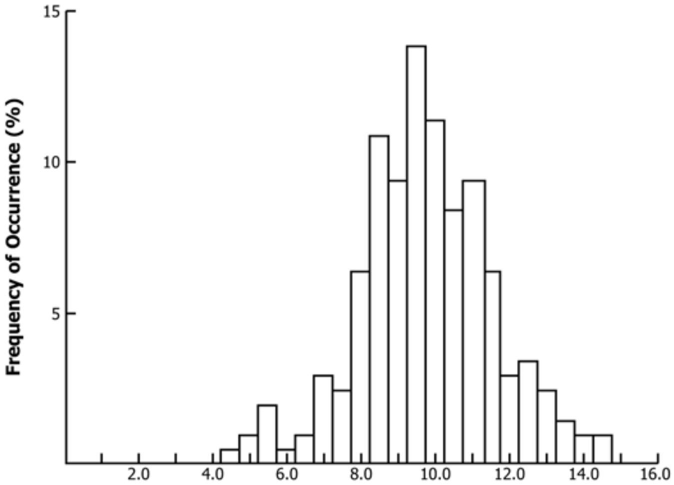

FIG. X2.2 Bending Strength MOR (1000 psi)

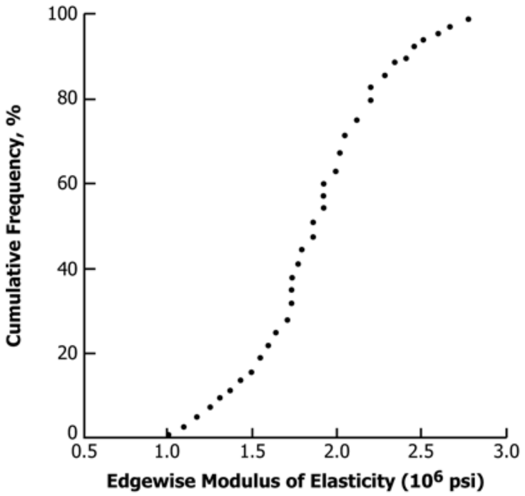

FIG. X2.3 Empirical Cumulative Distribution Function for *E*

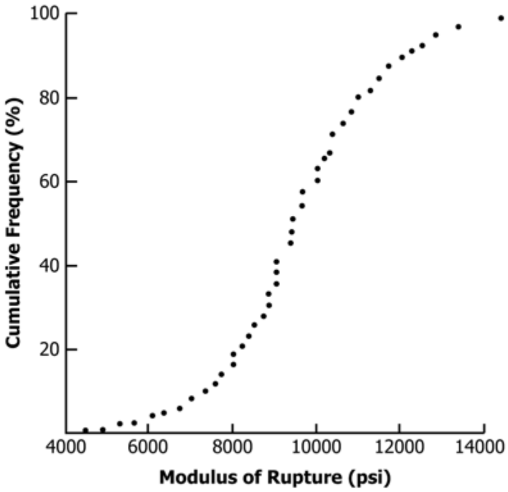

FIG. X2.4 Empirical Cumulative Distribution Function for *R*

X3. EXAMPLE—SAMPLING PROCEDURE

X3.1 When sampling from current production (that is, from the green chain) at a manufacturing facility, the following procedure allows the estimation of a standard error (SE) of the estimate as well as some information about the within-and-between sample variance.

X3.2 Following the procedure outlined in Practice E105 (A1.6) k is generally chosen to be five or greater. Let $k = 5$, therefore, $10k = 50$. Select ten random numbers between 1 and 50. These are the ten random start points; 3, 9, 14, 29, 31, 36, 40, 42, 47, and 50 (Table X3.1). Systemically select test specimens using an interval length of $10k$ beginning at each of the random start points (that is, random start $x + 10k$).

TABLE X3.1 Test Specimens to Be Selected[A]

3	9	14	29	31	36	40	42	47	50
53	59	64	79	81	86	90	92	97	100
103	109	114	129	131	136	140	142	147	150
153	159	164	179	181	186	190	192	197	200
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...

[A] This process is continued until the desired sample size is obtained.

X4. EXAMPLE OF AN ADJUSTMENT TO MODULUS OF ELASTICITY (MOE) FOR DIFFERENT SPAN TO DEPTH AND LOADING CONFIGURATION

X4.1 Modulus of elasticity values of primary concern are apparent values, E_{ai}, used in deflection equations that attribute all deflection to moment. These apparent moduli may be standardized for a specific span-depth ratio and load configuration. Standardization should reflect, as far as possible, conditions of anticipated end use.[4,5] When tests at standardized conditions of load and span are not possible, to adjust E_{ai} to standardized conditions, it is necessary to account for the effect of shear deflection on beam deflection. Factors to adjust E_{ai} for span-depth ratio and load configuration may be derived from Eq X4.1, (Ref (**12**)). To determine the apparent modulus of elasticity, E_{ai2}, based on any set of conditions of span-depth ratio and load configuration, when the modulus, E_{ai}, based on some other set of conditions is known, solve the equation:

TABLE X4.1 K Factors for Adjusting Apparent Modulus of Elasticity of Simply Supported Beams

Loading	Deflection Measured At	K_i
Concentrated at midspan	midspan	1.200
Concentrated at third points	midspan	0.939
Concentrated at third points	load points	1.080
Concentrated at outer quarter-points	midspan	0.873
Concentrated at outer quarter-points	load points	1.20
Uniformly distributed	midspan	0.960

$$E_{ai2} = \frac{1 + K_1 \left(\frac{h_1}{L_1}\right)^2 \left(\frac{E}{G}\right)}{1 + K_2 \left(\frac{h_2}{L_2}\right)^2 \left(\frac{E}{G}\right)} E_{ai} \qquad (X4.1)$$

where:

h = depth of the beam,
L = total beam span between supports,
E = shear free modulus of elasticity,
G = modulus of rigidity, and
K_i = values are given in Table X4.1.

X4.2 The equations were derived using simple beam theory for a simply supported beam composed of isotropic, homogeneous material. Experimental evidence suggests that these equations produce reasonable results with solid wood when converting between load conditions at a fixed span-depth ratio. Care must be exercised when converting between different span-depth ratios to assure that the adjustments are appropriate for the end use.

X4.3 An average apparent modulus of elasticity was obtained by testing simply supported beams loaded at the center and having a span-depth (L/h) ration of 14:1. The MOE value obtained was 1.60 10^6 psi. Assuming an E/G ratio of 16:1, what would be the apparent MOE for loads applied at the one-third points of the span with a span-depth ratio of 21:1? Deflections were measured at the center of the span.

From Table X4.1:

$$h_1/L_1 = 1{:}14 \qquad (X4.2)$$
$$h_2/L_2 = 1{:}21$$
$$E_{a1} = 1.60\ 10^6\ psi$$
$$E/G = 16$$
$$K_1 = 1.20$$
$$K_2 = 0.939$$

Therefore,

$$E_{a2} = (1.09796/1.034070) \times 1.60\ 10^6\ psi \qquad (X4.3)$$
$$E_{a2} = 1.70\ 10^6\ psi$$

---

[4] Spans, which customarily serve as a basis for design range, go from 17 to 21 times the depth of the specimen.

[5] A uniform load distribution is commonly encountered in use. This load configuration is difficult to apply in testing, but may be closely approximated by applying the load at the one-third points of the span, if the span-to-depth ratio is the same.

X5. ONE-SIDED TOLERANCE LIMITS FOR A NORMAL DISTRIBUTION

X5.1 A one-sided tolerance limit, *PTL*, is a value about which it may be said with confidence 1-γ, that at least a proportion, 1-p, of the population is greater than *PTL*. The formula is as follows:

$$PTL = \bar{X} - Ks \qquad \text{(X5.1)}$$

where $\bar{X}$ and s are the mean and the standard deviation, respectively, calculated from the sample data. *K* depends upon sample size *n*, as well as percentile 100-p and confidence 1-γ. *K* values are given in Table 3 or they may be calculated from the following formula:

$$K = \frac{Z_p g + \sqrt{Z_p{}^2 g^2 - \left[g^2 - Z_\gamma{}^2/(2(n-1))\right](Z_p{}^2 - Z_\gamma{}^2/n)}}{g^2 - Z_\gamma{}^2/(2(n-1))} \qquad \text{(X5.2)}$$

where:

g = $(4n - 5)/(4n - 4)$, and

Z_p and Z_γ are calculated with the following formula:

$$Z = T - (b_0 + b_1 T + b_2 T^2)/(1 + b_3 T + b_4 T^2 + b_5 T^3) \qquad \text{(X5.3)}$$

where:

T = $\sqrt{Ln(1/Q^2)}$ ($Q = p$ for Z_p and $Q = \gamma$ for Z_γ)

b_0 = 2.515517

b_1 = 0.802853

b_2 = 0.010328

b_3 = 1.432788

b_4 = 0.189269

b_5 = 0.001308

NOTE X5.1—*K* values computed using Eq X5.2 are approximations (see Ref (**13**)). For small values, the formula can seriously overestimate the *K* factors.

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(5)

(6)

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(8)

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